$E0$ emission in $\alpha + {}^{12}$C fusion at astrophysical energies

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We show that $E0$ emission in $\alpha + {}^{12}$C fusion at astrophysically interesting energies is negligible compared to $E1$ and $E2$ emission.

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The ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}$ capture reaction, sometimes called the "Holy Grail" of nuclear astrophysics, determines the ratio of ^{16}O to ^{12}C at the end of helium burning in stars, which is very important for stellar evolution and nucleosynthesis [1]. Nucleosynthesis requires [2] a total S-factor for this reaction of about 170 keV b at a center-of-mass energy $E_{\mathrm{c.m.}} = 0.3$ MeV, the center of the Gamow window. The results of many experiments over more than 3 decades, extrapolated to the Gamow window, show that single-photon emission is dominated by $E1$ and $E2$ decay to the ^{16}O ground state, with approximately equal intensity and a combined S-factor S(0.3) approaching the value quoted above [3]. The corresponding cross sections are $\sigma_{E1}(0.3) \approx \sigma_{E2}(0.3) \approx 1.4 \times 10^{-17}$ b.

In this Brief Report we examine the possible role of $E0$ emission, which has not, to our knowledge, been addressed previously. We note that if $E0$ emission were important, it would have escaped observation in ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}$ capture measurements since they were made by detecting the emitted γ-rays, and the e^+e^- pairs produced by $E0$ emission would not result in a sharp gamma line near the transition energy.

First, we estimate the ratio of direct $E0$ and direct $E2$ emission, following Snover and Hurd [4]. There, a general relation for direct $E0$ emission was derived, and for ${}^{3}\mathrm{He} + {}^{4}\mathrm{He}$ fusion at low energies a simple relation was obtained for the direct cross section ratio σ_{E0}/σ_{E2}, which was shown to be negligibly small. This occurs primarily because $E0$ emission is suppressed by an additional power of α, the fine structure constant, relative to $E2$ emission.

However, in ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}_{\mathrm{g.s.}}$ there are several factors that enhance the relative importance of $E0$ emission: (1) $E0$ emission occurs by s-wave capture, whereas $E1$ and $E2$ emission arise from p-wave and d-wave capture, respectively; (2) $E1$ emission is isospin-inhibited; and (3) the higher transition energy results in larger $E0/E1$ and $E0/E2$ phase-space factor ratios.

In low-energy ${}^{3}\mathrm{He} + {}^{4}\mathrm{He}$ fusion, $E0$ and $E2$ direct capture occur between the same initial and final states (p-waves), and as a result the direct capture radial matrix elements cancel in the cross section ratio. In ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}_{\mathrm{g.s.}}$, however, the radial matrix elements are different since the initial states are different. In analogy with Eq. (11) of [4] we obtain

$$\frac{\sigma_{E0}}{\sigma_{E2}} = \frac{4\pi}{5} \frac{f_{E0}}{f_{E2}} \frac{|R_{00}|^2}{|R_{02}|^2}, \tag{1}$$

where $R_{l_f l_i}$ is the radial integral of r^2 between the initial continuum state with orbital angular momentum l_i and the final bound state with $l_f = 0$.

The quantities f_{EL} are given by [4]

$$f_{E0}(E) = \frac{e^4}{27(\hbar c)^6} b(S)(E - 2mc^2)^3(E + 2mc^2)^2, \tag{2}$$

and

$$f_{E2}(E) = \frac{4\pi e^2}{75(\hbar c)^5} E^5, \tag{3}$$

where $E = E_{\mathrm{c.m.}} + Q$ is the transition energy, $Q = 7.16$ MeV,

$$b(S) = \frac{3\pi}{8}\left(1 - \frac{S}{4} - \frac{S^2}{8} + \frac{S^3}{16} - \frac{S^4}{64} + \frac{5S^5}{512}\right) \tag{4}$$

and $S = (E - 2mc^2)/(E + 2mc^2)$. We estimate $|R_{00}|^2/|R_{02}|^2 = P_0/P_2 = 18$ at $E_{\mathrm{c.m.}} = 0.3$ MeV, where P_{l_i} is the penetrability due to the Coulomb and angular momentum barriers evaluated at the radius $R = 1.3(A_1^{1/3} + A_2^{1/3})$ fm $=$ 5 fm. This yields 4.3×10^{-3} for the direct (i.e., nonresonant) $E0/E2$ cross section ratio at 0.3 MeV.

This estimate for $|R_{00}|^2/|R_{02}|^2$ assumes the capture takes place at the nuclear radius and is not affected by the nuclear interaction between ^{12}C and the α particle in the continuum. However, at low collision energies the effective radius may be larger, due to the importance of extranuclear capture, which would reduce $|R_{00}|^2/|R_{02}|^2$. In addition, the total $E2$ capture cross section in the Gamow window is dominated by the tail of the subthreshold 6.92 MeV 2^+ state, and this effect is also not included above.

We have improved on the above estimate by carrying out potential model calculations of $E0$ and $E2$ emission in ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}_{\mathrm{g.s.}}$. Using a real Woods-Saxon potential with radius parameter $r_0 = 1.25$ fm and diffuseness $a = 0.65$ fm, we find

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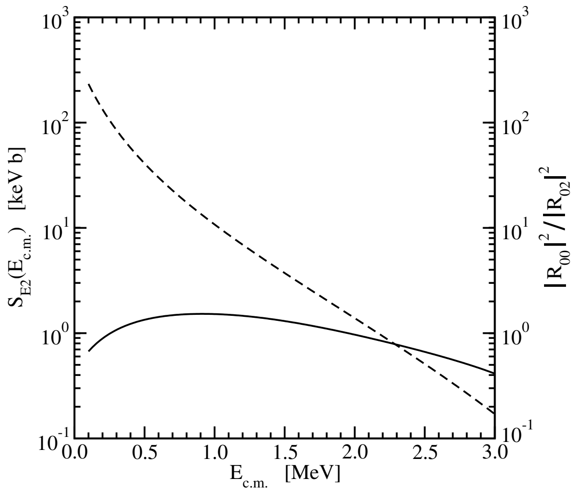

FIG. 1. Dashed curve and left scale: $E2$ S-factor; solid curve and right scale: $E0/E2$ radial matrix element ratio; vs. $E_{\text{c.m.}}$.

$V = 63.87$ MeV to bind the $N = 2$, $L = 0$, 0_1^+ ground state at the measured energy. Here N and L are determined from the relation $2N + L = \Sigma(2n_j + l_j)$ where n_j and l_j are the shell model quantum numbers of the four nucleons (0p or 1s0d) that make up the alpha particle state with quantum numbers N, L in the α-nucleus potential. Since the 6.05 MeV 0_2^+ state and the 6.92 MeV 2_1^+ state are members of the same 4p-4h rotational band, with the particles in the 1s0d shell, they should both have $2N + L = 8$ and hence $N = 4$ for the 0_2^+ state and 3 for the 2_1^+ state. We find $V = 122.74$ MeV (122.03 MeV) to bind the 0_2^+ (2_1^+) states with these node numbers at the correct energy, and thus we use $V(l_i = 0) = 122.74$ MeV and $V(l_i = 2) = 122.03$ MeV for the $l_i = 0$ and 2 scattering states, respectively, and $V(l_f = 0) = 63.87$ MeV for the final state. We note that these scattering potentials are similar to the real Woods-Saxon potential that fits the rainbow scattering region in intermediate energy α-^{12}C elastic scattering [6].

With these potentials, we obtain the $E2$ S-factor shown in Fig. 1. This curve is within a factor of 2 of the measured $E2$ S-factors below $E_{\text{c.m.}} = 2$ MeV, and has $S_{E2}(0.3) = 85$ keV b, in agreement with the value 81 ± 22 keV b obtained by Hammer *et al.* [3] from an extrapolated R-matrix fit to $E2$ data (other modern $E2$ fits that we are aware of yield $S_{E2}(0.3)$ values within a factor of 2 of these values).

Our potential model results for $|R_{00}|^2/|R_{02}|^2$ are also shown in Fig. 1. We obtain a value of 1.1 for the ratio at 0.3 MeV. This may be compared to the value 3.2 calculated with a pure $l_i = 0$ Coulomb scattering wave, indicating that the interior and exterior contributions to the $E0$ matrix element interfere destructively. A calculation with $V(l_i = 0) = 122.03$ MeV, which artificially enhances the contribution of the subthreshold 0_2^+ state by moving it 0.2 MeV closer to threshold, yields a ratio of 2.0 at 0.3 MeV. With $|R_{00}|^2/|R_{02}|^2 = 1.1$,

TABLE I. 0^+ resonance tail and potential model contributions to $E0$ emission at 0.3 MeV.

E_x(MeV)	$\theta^2_{\alpha_0}$	M(fm^2)[a]	$\sigma_{E0}(0.3)$(b)	Ratio[b]
6.05	$\leqslant 0.7$[c]	3.55	$\leqslant 1.6 \times 10^{-21}$	$\leqslant 1.2 \times 10^{-4}$
12.05	0.0036[a,d]	4.03	1.0×10^{-25}	7.8×10^{-9}
14.03	0.031[a,d]	3.3	2.9×10^{-24}	2.2×10^{-7}
25	$\leqslant 1.0$	9.0[e]	$\leqslant 1.0 \times 10^{-22}$	$\leqslant 7.3 \times 10^{-6}$
potential model				2.6×10^{-4}

[a]Monopole decay matrix element [7].
[b]σ_{E0}/σ_{E2}(total) at 0.3 MeV, where $\sigma_{E2}(\text{total}) = 1.4 \times 10^{-17}$ b.
[c]See, e.g., Table IV of [5].
[d]$\Gamma_{\alpha_0}/(2P_0\gamma^2_{W.L.})$ where $\gamma^2_{W.L.} = 3\hbar^2/(2\mu a^2) = 0.82$ MeV.
[e]$M^2 = (0.83)8\hbar^2\langle r^2\rangle_{\text{prot}}/(E_x M_n)$ where $\langle r^2\rangle_{\text{prot}} = 7.34$ fm^2 [7] and M_n = nucleon mass.

our calculated $E0/E2$ cross section ratio is 2.6×10^{-4}. Taking $S_{E2}(0.3) = 80$ keV b, this corresponds to

$$S_{E0}(0.3) = 0.02 \text{ keV b}. \tag{5}$$

Tails of higher lying 0^+ resonances may also contribute to the $E0$ cross section. In Table I we show the 0^+ excited states of ^{16}O with known ground-state monopole decay strengths [7]. Also shown for each state is the reduced α_0 width in units of the Wigner limit, the monopole decay matrix element, the $E0$ cross section at 0.3 MeV based on a Breit-Wigner extrapolation using the s-wave penetrability, and the ratio of the $E0$ cross section to the total $E2$ cross section at 0.3 MeV. We show an estimate for the 6.05 MeV 0_2^+ state for completeness, even though its effect on the cross section is included in the potential model calculations. We also show an upper limit for the contribution of the tail of an isoscalar giant monopole resonance located at $E_x = 25$ MeV with 83% of the isoscalar energy weighted sum rule [8] (the remaining 17% resides in the other 0^+ states shown in Table I). None of the resonance tail contributions from states above 6.05 MeV are significant compared to the $E0$ cross section calculated in the potential model.

$E0$ emission to excited final states in ^{16}O is negligible due to the small phase space factor. Hence our best estimate for the $E0$ contribution to the astrophysical S-factor for ^{12}C $+\alpha$ capture is given by Eq. (5) above.

Two-photon emission is also negligible, based on the measured branching ratio for this process in the decay of the 6.05 MeV 0^+ state [9]. We conclude that electromagnetic processes other than single-photon emission do not contribute significantly to the astrophysical rate for ^{12}C $+ \alpha$ fusion.

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