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Proton–proton fusion in effective field theory to fifth order

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Abstract

The proton–proton fusion process $pp \rightarrow de^+\nu_e$ is calculated at threshold to fifth order in pionless effective field theory. There are two unknown two-body currents contributing at the second and fourth orders. Combined with the previous results for $\nu_e d$ and $\bar{\nu}_e d$ scattering, computed to third order in the same approach, we conclude that a $\sim 10\%$ measurement of reactor $\bar{\nu}_e d$ scattering measurement could constrain the $pp \rightarrow de^+\nu_e$ rate to $\sim 7\%$ while a $\sim 3\%$ measurement of $\nu_e d \rightarrow e^- pp$ could constrain the pp rate to $\sim 2\%$. © 2001 Published by Elsevier Science B.V.

The reaction $pp \rightarrow de^+\nu_e$ is of central importance to stellar physics and neutrino astrophysics. Bethe and Critchfield proposed it to be the first reaction to ignite the pp chain nuclear reactions that provided the principal energy and neutrinos in the Sun [1]. Major efforts have been made to provide precise theoretical predictions for this reaction at zero energy [2–13]. No direct experimental constraint is available for this process, and the accuracy of the theoretical predictions must always be weighed against the availability of empirical constraints. However, Schiavilla et al. recently calibrated the matrix element with complementary calculations of tritium beta decay to obtain an estimated uncertainty of less than 1% in the potential model.

Alternatively, effective field theory (EFT) can provide a connection between pp fusion and other processes that might be accessible experimentally. Specifically, there is a direct connection between the reactions $pp \rightarrow de^+\nu_e$ and $\nu_e d \rightarrow e^- pp$ in that they both involve the same matrix elements. Further, in EFT, they can both be shown to depend on a single unknown counterterm (or two-body current) at third order. Otherwise, the nuclear physics input is identical. Further, all four $\nu(\bar{\nu})$–d breakup channels depend on this same counterterm, meaning that a measurement in any one channel implies a measurement in all channels—including pp fusion. To date, measurements are available for $\bar{\nu}_e d$ breakup using reactor antineutrinos with ~ 10–20% uncertainty [14–18] and possibly $\sim 5\%$ in the future [18]. Also, the ORLaND proposal could provide a measurement of $\nu_e d \rightarrow e^- pp$ to a few percent level [19]. It is important to understand the constraints that these measurements could on $pp \rightarrow de^+\nu_e$.

At the present time, $\nu_e d$ and $\bar{\nu}_e d$ breakup processes have been studied to the third order in pionless EFT depending on the unknown two-body counterterm $L_{1,A}$. Through varying $L_{1,A}$, the four channels of potential model results of Refs. [20,21] are reproduced to high accuracy. This confirms that the $\sim 5\%$ difference between Refs. [20,21] is largely due to different assumptions made to short distance physics. We will discuss this difference later in this Letter.

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For pp fusion, an pionless EFT calculation has been performed to second order by Kong and Ravndall [22], with a resulting dependence on $L_{1,A}$. In this Letter, we use the same approach to push the calculation to fifth order, which introduced another unknown two-body counterterm ($K_{1,A}$) that first enters at fourth order. Constraining $K_{1,A}$ using dimensional analysis, we conclude that a measurement of reactor $\bar{\nu}_e d$ with 10% uncertainty can constrain pp fusion to 7% while a measurement of $\nu_e d \to e^- pp$ to 3% would constrain pp fusion to 2%.

The method we use is pionless nuclear effective field theory, EFT($\not{\pi}$) [23], treating the electromagnetic interaction between protons in the manner developed in [22,24]. The dynamical degrees of freedom are nucleons and non-hadronic external currents. Massive hadronic excitations such as pions and the delta are integrated out, resulting in contact interactions between nucleons. The nucleons are non-relativistic but with relativistic corrections built in systematically. Nucleon–nucleon interactions are calculated perturbatively with the small expansion parameter

$$Q \equiv \frac{(1/a_{pp}, \gamma, p, \alpha M_N)}{\widetilde{\Lambda}}, \tag{1}$$

which is the ratio of the light to heavy scales. The light scales include the inverse S-wave nucleon–nucleon scattering length $1/a_{pp}$ ($a_{pp} = -7.82$ fm) in the $^1S_0$, pp channel, the deuteron binding momentum γ ($= 45.69$ MeV) in the $^3S_1$ channel, the proton momentum in the center-of-mass frame p, and the fine structure constant α ($= 1/137$) times the nucleon mass M_N. The heavy scale $\widetilde{\Lambda}$, which dictates the scales of effective range and shape parameters is set by the pion mass m_π. The Kaplan–Savage–Wise renormalization scheme [25] is used to make the power counting [25,26] in Q transparent. This formalism has been successfully applied to many processes involving the deuteron [23,27] including Compton scattering [28,29], $np \to d\gamma$ for big-bang nucleosynthesis [30,31], νd scattering [32] for physics at the Sudbury Neutrino Observatory [33], and parity violation observables [34].

There are other power counting schemes which yield different orderings in the perturbative series and each has certain advantages. For example, the z-parametrization [35] recovers the exact deuteron wave function renormalization at second order; the dibaryon pionless EFT [28] resums the effective range parameter contributions at first order and simplifies the calculation tremendously by cleverly employing the equations of motion to remove redundancies in the theory. While one of these power countings could lead to more rapid convergence in any given calculation, for a high-order calculation as we present here the distinctions between different expansions are negligible.

Ignoring for the moment the weak interaction component, the relevant Lagrangian in EFT($\not{\pi}$) can be written as a derivative expansion

$$
\begin{aligned}
\mathcal{L} = {} & N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M_N} \right) N \\
& - C_0^{(^3S_1)} (N^T P_i N)^\dagger (N^T P_i N) \\
& + \frac{C_2^{(^3S_1)}}{8} \left[(N^T P_i N)^\dagger (N^T \overleftrightarrow{\nabla}^2 P_i N) + \text{h.c.} \right] \\
& - \frac{C_4^{(^3S_1)}}{16} (N^T \overleftrightarrow{\nabla}^2 P_i N)^\dagger (N^T \overleftrightarrow{\nabla}^2 P_i N) \\
& - \frac{\widetilde{C}_4^{(^3S_1)}}{32} \left[(N^T \overleftrightarrow{\nabla}^4 P_i N)^\dagger (N^T P_i N) + \text{h.c.} \right] \\
& + (^3S_1 \to {}^1S_0, \ P_i \to \overline{P}_i) + \cdots,
\end{aligned}
\tag{2}
$$

where $\overleftrightarrow{\nabla} \equiv \overrightarrow{\nabla} - \overleftarrow{\nabla}$ and where $P_i = \sigma_2 \sigma_i \tau_2/\sqrt{8}$ and $\overline{P}_i = \sigma_2 \tau_2 \tau_i/\sqrt{8}$ project out $^3S_1$ and $^1S_0$ channels, respectively, with $\sigma(\tau)$ acting on spin (isospin) indices. The coupling constants have been fit to data in [23,32]. We only perform the calculation at threshold and thus the relativistic corrections and momentum transfer ($\mathbf{q}$) effects are suppressed by factors of γ^2/M_N^2 and $\mathbf{q}^2/\gamma^2$, both of which are $\ll 1\%$. Thus, with the goal of presenting a calculation with a precision of less than 1%, the non-relativistic and zero recoil limits are suitable approximations for us to make.

The weak interaction terms in the Lagrangian can be written as

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} l_+^\mu J_\mu^- + \text{h.c.} + \cdots, \tag{3}$$

where G_F is the Fermi decay constant and $l_+^\mu = \bar{\nu}\gamma^\mu(1 - \gamma_5)e$ is the leptonic current. The hadronic current has vector and axial vector parts,

$$J_\mu^- = V_\mu^- - A_\mu^-.$$

The vector current matrix element vanishes in the zero recoil limit since it yields terms proportional to the

wave function overlapping between two orthogonal states. The time component of the axial current gives center of mass motion corrections which vanish in our approximation. The spatial component of the axial vector is the dominant contribution, and can be written in terms of one-body and two-body currents

$$
\begin{aligned}
A_k^- =\ & \frac{g_A}{2} N^\dagger \tau^- \sigma_k N \\
& + L_{1,A} \big[\big(N^T P_k N \big)^\dagger \big(N^T \overline{P}^- N \big) + \text{h.c.} \big] \\
& + \frac{K_{1,A}}{8} \big[\big(N^T \overset{\leftrightarrow}{\nabla}{}^2 P_k N \big)^\dagger \big(N^T \overline{P}^- N \big) \\
& \qquad + \big(N^T P_k N \big)^\dagger \big(N^T \overset{\leftrightarrow}{\nabla}{}^2 \overline{P}^- N \big) + \text{h.c.} \big] \\
& + \cdots,
\end{aligned}
\tag{4}
$$

where $g_A = 1.26$, $\tau^- = (\tau_1 - i\tau_2)$, and the values of $L_{1,A}$ and $K_{1,A}$ are yet to be determined by data.

The hadronic matrix element is usually parametrized in the form

$$
\big| \langle d; j | A_k^- | pp \rangle \big| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\, \Lambda(p) \delta_k^j,
\tag{5}
$$

where j is the deuteron polarization state, and

$$
C_\eta = \sqrt{\frac{2\pi\eta}{e^{2\pi\eta} - 1}}, \quad \eta = \frac{\alpha M_N}{2p},
\tag{6}
$$

is the well-known Sommerfeld factor. In the center of the Sun, the scale of p is ~ 1 MeV. Thus the Sommerfeld factor C_η^2 changes rapidly with respect to p while $\Lambda(p)$ does not. It is sufficient to keep the p^2 correction for $\Lambda(p)$, since the higher order correction is $\mathcal{O}((p/\alpha M_N)^4)$. Using ϵ to keep track of the Q expansion we find that, to fifth order in the $Q(\epsilon)$ expansion, $\Lambda(0)$ can be written in the compact form

$$
\begin{aligned}
\Lambda(0) =\ & \frac{1}{\sqrt{1 - \epsilon\gamma\rho_d}} \\
& \times \bigg\{ e^\chi - \gamma a_{pp} \big[1 - \chi e^\chi E_1(\chi) \big] \\
& \qquad - \epsilon \gamma^2 a_{pp} \bigg[\overline{L}_{1,A} - \frac{\epsilon^2 \gamma^2}{2} \overline{K}_{1,A} \bigg] \bigg\} \\
& + \mathcal{O}(\epsilon^5).
\end{aligned}
\tag{7}
$$

This expression can be expanded to ϵ^4, and then ϵ should be set to 1. $\rho_d = 1.764$ fm is the effective range

parameter in the $^3S_1$ channel, $\chi = \alpha M_N / \gamma$ and

$$
E_1(\chi) = \int\limits_\chi^\infty dt\, \frac{e^{-t}}{t}.
\tag{8}
$$

$\overline{L}_{1,A}$ and $\overline{K}_{1,A}$ are the renormalization scale μ-independent combinations of the μ-dependent parameters $L_{1,A}$, $K_{1,A}$ and the nucleon–nucleon contact terms C_2 and C_4

$$
\begin{aligned}
\overline{L}_{1,A} &= \frac{-(\mu - \gamma)}{M_N C_{0,-1}^{(pp)}} \bigg[\frac{L_{1,A}}{g_A} - \frac{M_N}{2} \big(C_{2,-2}^{(pp)} + C_{2,-2}^{(d)} \big) \bigg], \\
\overline{K}_{1,A} &= \frac{-(\mu - \gamma)}{M_N C_{0,-1}^{(pp)}} \bigg[\frac{K_{1,A}}{g_A} - M_N \big(\widetilde{C}_{4,-2}^{(pp)} + 2 C_{4,-3}^{(d)} \big) \bigg].
\end{aligned}
\tag{9}
$$

From Ref. [32], we have

$$
\begin{aligned}
C_{0,-1}^{(pp)} &= \frac{4\pi}{M_N} \bigg(\frac{1}{a_{pp}} - \mu \\
& \qquad + \alpha M_N \Big(\ln \frac{\mu\sqrt{\pi}}{\alpha M_N} + 1 - \frac{3}{2}\gamma_E \Big) \bigg)^{-1}, \\
C_{2,-2}^{(pp)} &= \frac{M_N}{8\pi} r_0^{(pp)} C_{0,-1}^{(pp)2}, \\
\widetilde{C}_{4,-2}^{(pp)} &= -\frac{M_N}{4\pi} r_1^{(pp)} C_{0,-1}^{(pp)2}, \\
C_{2,-2}^{(d)} &= \frac{2\pi}{M_N} \frac{\rho_d}{(\mu - \gamma)^2}, \\
C_{4,-3}^{(d)} &= -\frac{\pi}{M_N} \frac{\rho_d^2}{(\mu - \gamma)^3},
\end{aligned}
\tag{10}
$$

where the second subscripts of the C's denote the scaling in powers of Q, $\gamma_E = 0.577$ is Euler's constant, and $r_0^{(pp)} = 2.79$ fm is the pp channel effective range. We take the pp channel shape parameter to be the same as that in the np channel $r_1^{(pp)} = -0.48$ fm^3. Any errors introduced by this are small numerically.

After expanding to ϵ^4 and setting $\mu = m_\pi$, we obtain

$$
\Lambda(0) = 2.58 + 0.011 \left(\frac{L_{1,A}}{1\,\text{fm}^3} \right) - 0.0003 \left(\frac{K_{1,A}}{1\,\text{fm}^5} \right).
\tag{11}
$$

At $\mu = m_\pi$, dimensional analysis as developed in Refs. [23,25] would favour

$$
|L_{1,A}| \approx \frac{1}{m_\pi (m_\pi - \gamma)^2} \approx 6\,\text{fm}^3,
$$

$$|K_{1,A}| \approx \frac{1}{m_\pi^2 (m_\pi - \gamma)^3} \approx 20 \, \text{fm}^5. \tag{12}$$

Two observations follow. First, if we take these naively estimated values (with positive signs, for example), then the expansion of $\Lambda(0)$ converges rapidly

$$\Lambda(0) = 2.51(1 + 0.039 + 0.029 - 0.010 - 0.0001). \tag{13}$$

For other sign combinations, the series also converges rapidly and higher order effects are $\ll 1\%$. Second, Eqs. (11) and (12) show that $K_{1,A}$ is likely to contribute to $\Lambda(0)$ at a level less than 1%. Thus Eq. (11) is precise to 1% even with $K_{1,A}$ set to zero, meaning that we can write

$$\Lambda(0) = 2.58 + 0.011 \left(\frac{L_{1,A}}{1 \, \text{fm}^3} \right) + \mathcal{O}(1\%). \tag{14}$$

This is the central result of this Letter.

Ultimately the value of $L_{1,A}$ must be extracted from experimental data. Before we address this issue, let us look at what values of $L_{1,A}$ are found in fits to potential model calculations. Using the results of Ref. [32], the recent potential model results for $\nu(\bar{\nu})$–d breakup of Nakamura, Sato, Gudkov, and Kubodera (NSGK) [21] are equivalent to

$$L_{1,A}^{\text{NSGK}} = 5.6 \pm 2 \, \text{fm}^3, \tag{15}$$

while the results of Ying, Haxton and Henley (YHH) [20] are equivalent to

$$L_{1,A}^{\text{YHH}} = 0.94 \pm 2 \, \text{fm}^3. \tag{16}$$

The uncertainties represent a conservative estimate from EFT of 3% from Ref. [32] (even though the NSGK results can be reproduced within 1% using the central value of $L_{1,A}$). Based on the size of the third-order contribution, the actual uncertainty may be as small as 1%, but further analysis is required to ascertain this. The (undefinable) errors from the potential models themselves are not included. As mentioned before, the differences between these two calculations are in their treatment of two-body physics. NSGK uses a model to include axial two-body meson exchange currents, while YHH includes vector two-body currents but not the more-important axial two-body currents. Given that $L_{1,A}$ is representative of the dominant axial effects in EFT, it is not surprising to see substantial differences in the value inferred by each calculation.

Similarly, Eq. (14) translates the pp fusion rate ($\Lambda^2(0) = 7.05$–7.06) calculated by Schiavilla et al. into

$$L_{1,A}^{\text{Schiavilla et al.}} = 6.5 \pm 2.4 \, \text{fm}^3, \tag{17}$$

which is consistent with NSGK.

Experimentally, it is easy to relate the EFT $\nu(\bar{\nu})$–d scattering results and pp fusion rate through the third-order results for $\nu(\bar{\nu})$–d provided in [32]. The results for each channel are parameterized in the form

$$\sigma(E_\nu) = a(E_\nu) + b(E_\nu)L_{1,A} + \mathcal{O}(< 3\%), \tag{18}$$

where a and b are functions of neutrino energy. These results, tabulated in Ref. [32], can be easily related to the pp fusion rate through Eq. (14).

For reactor $\bar{\nu}$–d scattering, one expects the rate to be peaked around 8 MeV, which can be interpreted to mean a measurement precise to 10% can determine the pp fusion rate ($\propto \Lambda^2$) to 7%. With the experimental precision further improved to 5%, the pp fusion rate can be determined to 4%. Alternatively, the proposed ORLaND [19] detector might measure $\nu_e d \to e^- pp$ to 3%, in turn constraining the pp fusion rate to 2%.

In strong interaction processes involving external currents, delicate relations between operators are required to guarantee that there are no off-shell ambiguities in the final results. For example, in two-body systems two-body currents serve to absorb the off-shell effects from the two-body strong interactions. This means that different models that reproduce the same on-shell nucleon–nucleon scattering data might need quite distinct two-body currents to deal with off-shell effects. In the case of pp fusion or ν–d scattering, electromagnetic matrix elements cannot be used to constrain weak matrix elements. Their operator structures are quite different at the quark level even they might appear the same in hadronic level. Using tritium β-decay to constrain the two-body current [12] is an excellent idea, in principle. However, contributions from three-body currents that are required to absorbed three-body off-shell effects are not yet constrained. In light of these facts, the need for a precise experimental measurement of $\nu_e(\bar{\nu}_e)$–d breakup cannot be overstated.

Acknowledgements

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