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Trojan Horse as an indirect technique in nuclear astrophysics

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Abstract

The Trojan Horse method is a powerful indirect technique that provides information to determine astrophysical factors for binary rearrangement processes $x + A \rightarrow b + B$ at astrophysically relevant energies by measuring the cross section for the Trojan Horse reaction $a + A \rightarrow y + b + B$ in quasi-free kinematics. We present the theory of the Trojan Horse method for resonant binary subreactions based on the half-off-energy-shell R matrix approach which takes into account the off-energy-shell effects and initial and final state interactions.

1. Introduction

The presence of the Coulomb barrier for colliding charged nuclei makes nuclear reaction cross sections at astrophysical energies so small that their direct measurement in the laboratory is very difficult, or even impossible. Consequently, indirect techniques often are used to determine these cross sections. The Trojan Horse (TH) method is a powerful indirect technique which allows one to determine the astrophysical factor for rearrangement reactions. The TH method, first suggested by Baur [1], involves obtaining the cross section of the binary $x + A \rightarrow b + B$ process at astrophysical energies by measuring the two-body to three-body ($2 \rightarrow 3$) TH process, $a + A \rightarrow y + b + B$, in the quasi-free (QF) kinematics regime, where the 'Trojan Horse' particle, $a = (xy)$, is accelerated at energies above the Coulomb barrier. After penetrating through the Coulomb barrier, nucleus a undergoes breakup leaving particle x to

interact with target A while projectile y flies away. From the measured $a + A \to y + b + B$ cross section, the energy dependence of the binary subprocess, $x + A \to b + B$, is determined.

The main advantage of the TH method is that the extracted cross section of the binary subprocess does not contain the Coulomb barrier factor. Consequently, the TH cross section can be used to determine the energy dependence of the astrophysical factor, $S(E)$, of the binary process, $x + A \to b + B$, down to zero relative kinetic energy of the particles x and A without distortion due to electron screening [2, 3]. The absolute value of $S(E)$ must be found by normalization to direct measurements at higher energies. At low energies where electron screening becomes important, comparison of the astrophysical factor determined from the TH method to the direct result provides a determination of the screening potential.

Even though the TH method has been applied successfully to many direct and resonant processes (see [4] and references therein), there are still reservations about the reliability of the method due to two potential modifications of the yield from off-shell effects and initial and final state interactions in the TH $2 \to 3$ reaction. Here we will address the theory of the TH method for resonant binary reactions $x + A \to b + B$.

2. Trojan Horse

The TH reaction is a many-body process (at least four-body) and its strict analysis requires many-body techniques. However some important features of the TH method can be addressed in a simple model. Let us consider the TH process assuming that nuclei y, x and B are constituent particles, i.e. we neglect their internal degrees of freedom. For simplicity, we disregard the spins of the particles. The TH reaction amplitude is given in the post form by

$$\tilde{M}(P, \mathbf{k}_{aA}) = \left\langle \chi^{(-)}_{\mathbf{k}_{yF}} \Phi^{(-)}_F \middle| \Delta V_{yF} \middle| \Psi^{(+)}_i \right\rangle. \tag{1}$$

Here, $\Psi^{(+)}_i$ is the exact $a + A$ scattering wavefunction, $\Phi^{(-)}_F$ is the wavefunction of the system $F = b + B = x + A$, $\chi^{(-)}_{\mathbf{k}_{yF}}(\mathbf{r}_{yF})$ is the distorted wave of the system $y + F$, φ_i is the bound state wavefunction of nucleus i, $\mathbf{r}_{ij}$ and $\mathbf{k}_{ij}$ are the relative coordinate and relative momentum of nuclei i and j, $P = \{\mathbf{k}_{yF}, \mathbf{k}_{bB}\}$ is the six-dimensional momentum describing the three-body system y, b and B in the final system, $\Delta V_{yF} = V_{yF} - U_{yF}$, $V_{yF} = V_{yb} + V_{yB} = V_{yx} + V_{yA}$ is the interaction potential of y and the system F and U_{yF} is their optical potential. The surface approximation suggested in [5] was the first serious attempt to address the theory of the TH method. The surface approximation assumes that the TH reaction amplitude has contributions from the external region where the interaction between the fragments b and B (x and A) can be neglected and the wavefunction $\Phi^{(-)}_F$ can be replaced by its leading asymptotic form

$$\Phi^{(+)}_F \approx \varphi_b \left[e^{i\mathbf{k}_{bB}\cdot\mathbf{r}_{bB}} + F_{bB} u^{(+)}_{k_{bB}}(r_{bB}) \right] + \sqrt{\frac{v_{bB}}{v_{xA}}} M_{bB\to xA} \frac{1}{2ik_{bB}} u^{(+)}_{k_{xA}}(r_{xA}), \tag{2}$$

where $\Phi^{(+)}_F \equiv \Phi^{(+)}_{\mathbf{k}_{bB}(F)}$ and $\Phi^{(-)}_{\mathbf{k}_{bB}(F)} = \Phi^{(+)*}_{-\mathbf{k}_{bB}(F)}$, $u^{(+)}_{k_{ij}}(r_{ij})$ is the outgoing spherical wave, F_{bB} is the $b + B$ elastic scattering amplitude, $M_{bB\to xA}$ is the $b + B \to x + A$ reaction amplitude inverse to the binary reaction $x + A \to b + B$ and v_{ij} is the relative velocity of nuclei i and j. The expression for the TH reaction amplitude in the surface approximation is given by

$$\tilde{M}(P, \mathbf{k}_{aA}) \sim M_{bB\to xA} \left\langle \chi^{(-)}_{\mathbf{k}_{yF}} \varphi_A u^{(-)}_{k_{xA}}(r_{xA}) \middle| \Delta V_{yF} \middle| \varphi_a \varphi_A \chi^{(+)}_{\mathbf{k}_{aA}}(\mathbf{r}_{aA}) \right\rangle, \tag{3}$$

where the exact initial scattering wavefunction $\Psi^{(+)}_i$ is replaced by $\varphi_a \varphi_A \chi^{(+)}_{\mathbf{k}_{aA}}(\mathbf{r}_{aA})$ and $\chi^{(+)}_{\mathbf{k}_{aA}} \equiv \chi^{(+)}_{aA}$ is the distorted wave describing the scattering of the nuclei a and A in the initial state of the TH reaction. For simplicity we do not take into account here the Coulomb interactions. However, in the case of the resonant binary reaction $x + A \to b + B$ the dominant

contribution comes from the nuclear interior where both channels $x + A$ and $b + B$ are coupled and where the asymptotic approximation for $\Phi_F^{(+)}$ cannot be applied[7].

In this work we will address the theory of the TH method for the resonant binary subprocesses $x + A \to b + B$ which explicitly takes into account the off-shell character of x. Equation (1) can be used as a starting point to derive the expression for the TH reaction amplitude. We assume that the resonant reaction $x + A \to b + B$ proceeds through the formation of the intermediate compound state Φ_i, i.e. we neglect the direct coupling between the initial $x + A$ and final $b + B$ channels, which contributes dominantly to direct reactions but gives negligible contribution to resonant ones. An important step in deriving the resonant contribution to the TH reaction matrix element is the spectral decomposition for the wavefunction $\Phi_F^{(-)}$ given by equation (3.8.1) [7]. It leads to the shell-model-based resonant R matrix representation for $\Phi_F^{(-)}$ which is similar to the level decomposition for the wavefunction in the internal region in the R matrix approach:

$$\Phi_F^{(-)} \approx \sum_{\nu,\tau=1}^{N} \tilde{V}_\nu^{bB}(E_{bB})[\mathbf{D}^{-1}]_{\nu\tau}\Phi_\tau. \tag{4}$$

Here N is the number of the levels included, E_{bB} is the relative kinetic energy of nuclei b and B, Φ_τ is the bound state wavefunction describing the compound system F excited to the level τ. $\mathbf{D}$ is similar to the level matrix in the R matrix theory and is given by equation (4.2.20b) [7]. Finally,

$$\tilde{V}_\nu^{bB}(E_{bB}) = \left\langle \chi_{bB}^{(-)}\varphi_b \middle| \Delta V_{bB} | \Phi_\nu \right\rangle \tag{5}$$

is the resonant form factor for the decay of the resonance F_ν described by the compound state Φ_ν into the channel $b + B$. The partial resonance width is given by

$$\tilde{\Gamma}_\nu(E_{bB}) = 2\pi \left| \tilde{V}_\nu^{bB}(E_{bB}) \right|^2. \tag{6}$$

Then the TH reaction amplitude is

$$\tilde{M}^{(R)}(P, \mathbf{k}_{aA}) \approx \sum_{\nu,\tau=1}^{N} \tilde{V}_\nu^{bB}(E_{bB})[\mathbf{D}^{-1}]_{\nu\tau}\tilde{M}_\tau(\mathbf{k}_{yF}, \mathbf{k}_{aA}), \tag{7}$$

where $\tilde{M}_\tau(\mathbf{k}_{yF}, \mathbf{k}_{aA})$ is the exact amplitude for the direct transfer reaction $a + A \to y + F_\tau$ populating the compound state F_τ of the system $F = x + A = b + B$:

$$\tilde{M}_\tau(\mathbf{k}_{yF}, \mathbf{k}_{aA}) = \left\langle \chi_{yF}^{(-)}\Phi_\tau \middle| \Delta V_{yF} \middle| \Psi_i^{(+)} \right\rangle. \tag{8}$$

The direct transfer reaction is very well described by the DWBA amplitude, i.e. for the practical analysis we can approximate $\Psi_i^{(+)} \approx \varphi_a\varphi_A\chi_{aA}^{(+)}$. Correspondingly, $\tilde{M}_\tau(\mathbf{k}_{yF}, \mathbf{k}_{aA})$ can be replaced by

$$\tilde{M}_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA}) = \left\langle \chi_{yF}^{(-)}\Phi_\tau \middle| \Delta V_{yF} \middle| \varphi_a\varphi_A\chi_{aA}^{(+)} \right\rangle. \tag{9}$$

Correspondingly for the TH reaction amplitude we get from equation (7)

$$\tilde{M}^{(R)}(P, \mathbf{k}_{aA}) \approx \sum_{\nu,\tau=1}^{N} \tilde{V}_\nu^{bB}(E_{bB})[\mathbf{D}^{-1}]_{\nu\tau}\tilde{M}_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA}). \tag{10}$$

The DWBA amplitude takes into account the rescattering of nuclei a and A in the initial state of the TH reaction and enters as a form factor into the TH resonant reaction amplitude

[7] Generally speaking, one must be very careful in using the asymptotic approximation for the scattering wavefunction $\Phi_F^{(-)}$ because the matrix element with the exact wavefunction in the initial state and ingoing spherical wave $u_{k_{xA}}^{(-)}(r_{xA})$ in the final state vanishes after transformation of the volume integral into a surface integral [6].

reflecting the off-energy shell character of the transferred particle x. Since in the TH method the astrophysical factor determined from the TH method is normalized to the on-energy-shell (OES) S factor, the replacement of the exact transfer amplitude by the DWBA one, as we will see, practically does not affect the final result. The triple differential cross section for the TH process $a + A \to y + b + B$ proceeding through an isolated resonance F_τ is given by

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}E_{bB}\,\mathrm{d}\Omega_{\mathbf{k}_{bB}}\,\mathrm{d}\Omega_{\mathbf{k}_{yF}}} = \lambda_3 \frac{\Gamma_{bB(\tau)}(E_{bB})\left|M_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})\right|^2}{\left(E_{xA} - E_{R_\tau}\right)^2 + \frac{\Gamma_\tau^2(E_{xA})}{4}}. \tag{11}$$

Here, λ_3 is the kinematical factor, $\Gamma_{bB(\tau)}(E_{bB})$ is the observable resonance partial width in the channel $b + B$, $\Gamma_\tau(E_{xA})$ is the total observable width of the resonance F_τ. Note that all functions $T(E)$ are related to $\tilde{T}(E)$ as $T(E) = \tilde{T}(E)\big/\left(1 - \left(\frac{\mathrm{d}\Delta_{\tau\tau}}{\mathrm{d}E}\right)_{E=E_{R_\tau}}\right)$, where $\Delta_{\tau\tau}$ is the τ level shift. Also E_{R_τ} is the resonance energy of the resonance F_τ in the channel $x + A$. Thus the TH triple differential cross section, in contrast to the OES single-level resonance cross section, contains the generalized form factor $\left|M_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})\right|^2$ rather than the entry channel partial resonance width $\Gamma_{xA(\tau)}(E_{xA})$ of the binary process $x + A \to b + B$. A simple renormalization of the TH triple differential cross section allows us to single out the OES astrophysical factor for the resonant binary subprocess $x + A \to b + B$:

$$\begin{aligned} S(E_{xA}) &= \frac{\pi}{k_{xA}^2}\frac{1}{\lambda_3} E_{xA}\, \mathrm{e}^{2\pi\eta_{xA}} \frac{\Gamma_{xA(\tau)}(E_{xA})}{\left|M_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})\right|^2} \frac{\mathrm{d}^3\sigma}{\mathrm{d}E_{bB}\,\mathrm{d}\Omega_{\mathbf{k}_{bB}}\,\mathrm{d}\Omega_{\mathbf{k}_{yF}}} \\ &= \frac{\pi}{2\mu_{xA}}\, \mathrm{e}^{2\pi\eta_{xA}} \frac{\Gamma_{bB(\tau)}(E_{bB})\Gamma_{xA(\tau)}(E_{xA})}{\left(E_{xA} - E_{R_\tau}\right)^2 + \frac{\Gamma_\tau^2(E_{xA})}{4}}. \end{aligned} \tag{12}$$

The DWBA amplitude $M^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})$ remains practically constant on the interval of a few hundreds keV. The same is true for the factor $\mathrm{e}^{2\pi\eta_{xA}}\Gamma_{xA(\tau)}(E_{xA})$. The renormalization of the TH cross section normalizes it to the experimental astrophysical factor at resonance energy without affecting its energy dependence.

For two interfering resonances we need to consider the two-level, two-channel case. This requires the half-off-energy-shell (HOES) R matrix formalism. Here we address this formalism for a simple case when the distances between two resonances are significantly larger than their total widths. Then the OES reaction amplitude in the R matrix formalism is given by the sum of the amplitudes of each resonance (see equation (XII,5.15) [8]). The corresponding expression for the HOES reaction amplitude can be obtained by the replacement of the resonance partial widths in the entry channel of the binary reaction $x + A \to b + B$ by the corresponding generalized form factors $M_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_a)$, $\tau = 1, 2$. Thus the triple TH cross section in the presence of two interfering resonances in the subsystem $F = x + A = b + B$ is given by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E_{bB}\,\mathrm{d}\Omega_{\mathbf{k}_{yF}}\,\mathrm{d}\Omega_{\mathbf{k}_{bB}}} = \lambda_3 \left| \sum_{\tau=1,2} \frac{\Gamma_{bB(\tau)}^{1/2}(E_{bB}) M_\tau^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})}{E_{xA} - E_{R_\tau} + \mathrm{i}\frac{\Gamma_\tau(E_{xA})}{2}} \right|^2. \tag{13}$$

We assume that $E_{R_1} < E_{R_2}$. The goal of the THM is to determine the energy dependence of the astrophysical factor at the astrophysically relevant energies. The ratio $M_{21}^{DW} = M_2^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})\big/M_1^{DW}(\mathbf{k}_{yF}, \mathbf{k}_{aA})$ is practically constant in the interval of a few hundred keV, $E_{xA} \leqslant E_{R_1}$. Normalizing the TH cross section to the OES S factor at $E = E_{R_1}$, where the contribution from the second resonance can be neglected, gives the astrophysical factor

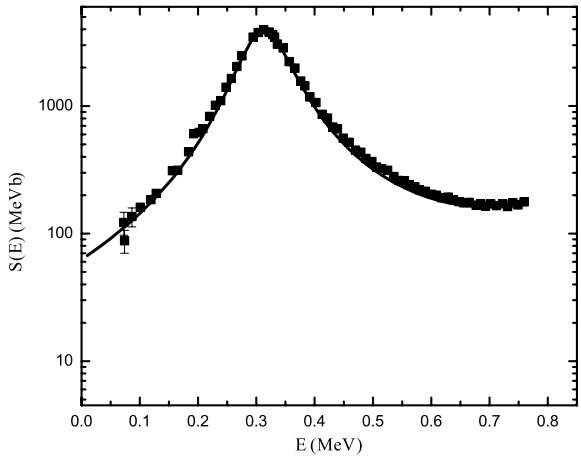

Figure 1. Comparison of the calculated astrophysical factor $S^{TH}(E)$ for $^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$ (solid line), where $E \equiv E_{xA}$, with the direct data [9–11].

determined from the TH reaction

$$S^{TH}(E_{xA}) = \frac{\pi\, \mathrm{e}^{2\pi\eta_{xA}}}{2\mu_{xA}}\Gamma_{xA(1)}(E_{xA})\left|\left[\frac{\Gamma^{1/2}_{bB(1)}(E_{bB})}{E_{xA}-E_{R_1}+\mathrm{i}\frac{\Gamma_1(E_{xA})}{2}}+\frac{\Gamma^{1/2}_{bB(2)}(E_{bB})M^{DW}_{21}}{E_{xA}-E_{R_2}+\mathrm{i}\frac{\Gamma_\tau(E_{xA})}{2}}\right]\right|^2. \tag{14}$$

This astrophysical factor is to be compared with the OES astrophysical factor determined from direct measurements

$$S(E_{xA}) = \frac{\pi\, \mathrm{e}^{2\pi\eta_{xA}}}{2\mu_{xA}}\Gamma_{xA(1)}(E_{xA})\left|\left[\frac{\Gamma^{1/2}_{bB(1)}(E_{bB})}{E_{xA}-E_{R_1}+\mathrm{i}\frac{\Gamma_1(E_{xA})}{2}}+\frac{\Gamma^{1/2}_{bB(2)}(E_{bB})\gamma_{(xA)21}}{E_{xA}-E_{R_2}+\mathrm{i}\frac{\Gamma_\tau(E_{xA})}{2}}\right]\right|^2. \tag{15}$$

Here, $\gamma_{(xA)21} = \gamma_{(xA)2}/\gamma_{(xA)1} = \Gamma^{1/2}_{xA(2)}(E_{xA})/\Gamma^{1/2}_{xA(1)}(E_{xA})$ and $\gamma_{(xA)\tau}$ is the reduced width for the τth resonance in the channel $x + A$. Each amplitude $M^{DW}_2(\mathbf{k}_{yF}, \mathbf{k}_a)$ is complex, but the ratio M^{DW}_{21} may have a small imaginary part. The normalization of the TH S factor to the OES one at resonance energy plays a crucial role in the TH method. After such a normalization, we need to know only the ratio of the DWBA amplitudes to calculate $S^{TH}(E_{xA})$. This ratio can be approximated by the ratio of the corresponding amplitudes calculated in a plane wave approximation, because a simple plane wave approximation gives similar angular and energy dependence to the DWBA but fails to reproduce the absolute value. It explains why a simple plane wave approximation worked so well in the previous TH analyses (see [4] and references therein). Note that in the plane wave approximation $M^{DW}_{21} = W_{(xA)2}(p_{xA})/W_{(xA)1}(p_{xA})$, where the form factor $W_{(xA)\tau}(p_{xA}) = \langle j_0(p_{xA}r)|\langle V_{xA}(r)\rangle_{xA}|I^{F_\tau}_{xA}(r)\rangle$, $I^{F_\tau}_{xA} = \langle\varphi_A|\Phi_\tau\rangle$, $j_0(p_{xA}r)$ is the s-wave Bessel function, $\langle V_{xA}(r)\rangle_{xA} = \langle\varphi_A|V_{xA}|\varphi_A\rangle$. If $M^{DW}_{21} \approx \gamma_{(xA)21}$, the astrophysical factor $S^{TH}(E_{xA})$ reproduces the OES S factor $S(E_{xA})$ at energies $E_{xA} \leqslant E_{R_1}$. In figure 1 the astrophysical factor $S^{TH}(E_{xA})$ for $^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$ calculated using equation (14) for the TH reaction $^{15}\mathrm{N}(d,n\alpha)^{12}\mathrm{C}$ is compared with the experimental $S(E_{xA})$ obtained from direct measurements. There are two 1^- interfering resonances at $E_{R_1} = 312$ keV and $E_{R_2} = 962$ keV. M^{DW}_{21} has been calculated taking into account the d–^{15}N Coulomb interaction in the initial state of the TH reaction and plane wave approximation was used for n–^{16}O at energy $E_{d^{15}\mathrm{N}} = 4.20$ MeV. We find that $M^{DW}_{21} \approx 1.13$

while $\gamma_{(xA)21} = 1.1 \pm 0.1$. It explains why the calculated $S^{TH}(E_{xA})$ shown in figure 1 is in an excellent agreement with the direct data.

Acknowledgments

We presented the expression for the resonant S factor determined from the TH reaction taking into account the off-energy-shell effects within the HOES R matrix formalism and justified a simple plane wave approximation. Validating this makes it clear why the TH method is such a powerful indirect technique for nuclear astrophysics. This work was supported in part by the US DOE under grant no. DE-FG02-93ER40773.

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