Reconciling Coulomb Dissociation and Radiative Capture Measurements

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We calculate the energy spectrum for ^{8}B dissociation on a Pb target to all orders in the Coulomb and nuclear fields, and show that the slope of $S_{17}(E)$ obtained in previous analyses of Coulomb dissociation data is too steep, due to deficiencies in the conventional first-order analysis that was used. With a more complete theory that avoids the far-field approximation and includes $E2$, nuclear and dynamical projectile polarization, the disagreement between indirect and direct methods for determining the $S_{17}(E)$ slope and the extrapolated $S_{17}(0)$ values is reduced significantly.

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The indirect Coulomb dissociation (CD) method for determining radiative capture cross sections (or, equivalently, astrophysical S factors) involves measuring the dissociation cross section for the inverse reaction in the predominantly Coulomb field of a high-Z target nucleus. This method is important in nuclear astrophysics since it can be used to determine rates for reactions that are difficult or impossible to measure directly, including some that are critical to stellar nucleosynthesis. New radioactive beam facilities such as the proposed Rare Isotope Accelerator will open new opportunities for this type of measurement. However, substantial theoretical analysis is required to infer the capture cross section from the measured dissociation cross section, and hence it is extremely important to test our understanding of the method by comparison to precise direct measurements.

The best case at present for testing the accuracy of the CD method is the ^{7}Be$(p,\gamma)^8$B capture reaction. This is a crucial reaction in the chain that produces high-energy neutrinos in the Sun and has been a challenge to nuclear physics for many years. Fortunately, high precision capture cross sections [1] have recently become available, permitting a CD test to higher precision than possible with other reaction data. A comparison between CD and direct results in this case shows evidence for systematic differences between the two techniques (see Figs. 18–20 of [1]). In the CD analyses, small previously neglected corrections must be considered when accuracies of 10% or better are needed. Here we show that an improved treatment of $^8\mathrm{B} \rightarrow {}^7\mathrm{Be} + p$ CD reduces the systematic discrepancy and brings that method into better agreement with the direct measurements.

Two major differences between CD and direct determinations of the ^{7}Be$(p,\gamma)^8$B S factor (here labeled S_{17}) were identified in [1]. First, the zero-energy extrapolated $S_{17}(0)$ values inferred from CD measurements [2–7] are, on average, about 10% lower than the mean of modern direct measurements (see [1,8] for the most recent measurements). Second, the S_{17} values extracted from CD data have a significantly steeper slope as a function of E_{rel}, the relative energy of the proton and the ^{7}Be fragment, than the direct results. We show that these differences are due in part to the manner in which the CD experiments were analyzed and can be reduced considerably by using more precise Coulomb breakup theory.

In most cases the ^{8}B CD experiments have been analyzed assuming first-order $E1$ transitions into the $^7\mathrm{Be} + p$ continuum calculated in the far-field (FF) approximation, which assumes no overlap between projectile and target during the collision. In contrast, an unrestricted multipole expansion of the Coulomb interaction, such as in Eq. (2) of Ref. [9], does not involve this approximation. In previous work [9,10] we discussed the limitations of the first-order (FO) theory, the need for including $E2$ contributions, and the error associated with the FF approximation. Here we bring these corrections together and show their effect on the relative energy spectrum [11].

Our theory is based on the numerical solution of the time-dependent Schrödinger equation for relative motion of the proton and the ^{7}Be core, which is perturbed by the Coulomb and nuclear fields from a target nucleus [12,13]. The time dependence is generated by the projectile-target motion, which is approximated by a classical Coulomb trajectory. The proton-core Hamiltonian is adjusted to reproduced the ^{8}B proton separation energy of 137 keV [13]. While a two-body model of ^{8}B may not provide a perfect description of the S-factor energy dependence and absolute value, it simplifies the dynamical calculations considerably and helps us gain insight into the accuracy of approximations that are commonly made in the analysis of CD experiments.

To see how these approximations affect the relative energy (E_{rel}) spectrum, we take as an example the measurements performed at RIKEN (Institute of Physical and Chemical Research in Japan) on a Pb target at 51.9 MeV/n [2,3]. The first-order contributions to the relative energy spectrum, calculated at an impact parameter $b = 20$ fm, are shown in Fig. 1(a). The unrestricted first-order $E1$ and

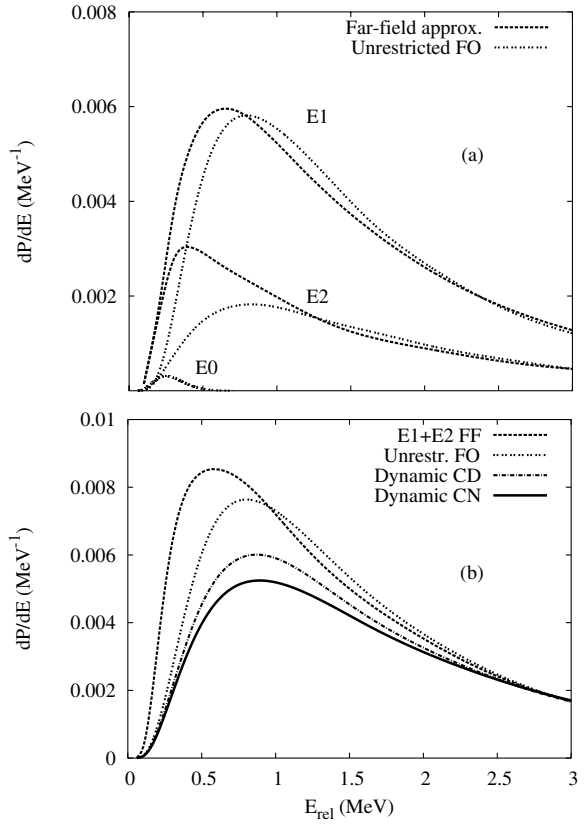

FIG. 1. Decay energy spectra for ^{8}B on ^{208}Pb at 51.9 MeV/n and impact parameter $b = 20$ fm. The FF approximation for $E1$ and $E2$ transitions is compared to the unrestricted first-order $E0$, $E1$, and $E2$ calculations in (a). The summed spectra are compared to dynamic calculations in (b).

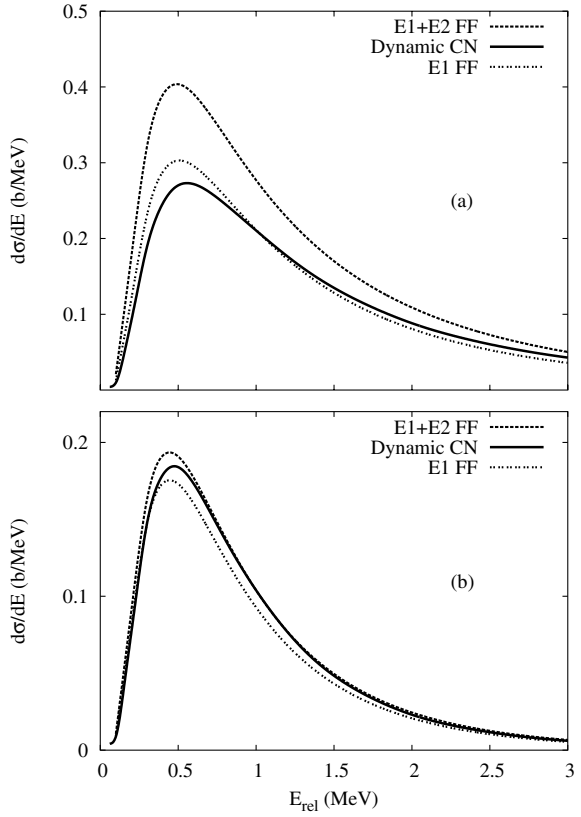

FIG. 2. Decay energy spectrum obtained in the full dynamic calculation at 51.9 MeV/n (solid curve) is compared to the results of first-order $E1$ and $E1 + E2$ transitions in the far-field approximation. The minimum impact parameter is $b_{\min} = 12$ fm in (a) and 30 fm in (b).

$E2$ spectra are seen to be suppressed compared to the FF approximation at small relative energies. Moreover, the unrestricted treatment produces a small $E0$ component.

The effects of other approximations are shown in Fig. 1(b). The two top curves combine the multipole components shown in Fig. 1(a) for the FF approximation and the unrestricted first-order calculation, respectively. As shown in Fig. 1(b) the spectrum is strongly reduced by an unrestricted treatment of the first-order Coulomb matrix elements. The next lower curve (dynamic CD) shows the results obtained by solving the time-dependent Schrödinger equation, i.e., using all orders of perturbation theory. Here the probability reduction with respect to the first-order calculations is due mainly to dynamic polarization as discussed in [10]. The bottom curve (labeled dynamic CN) includes both Coulomb (C) and nuclear (N) contributions in the dynamic calculations. Both the real and imaginary parts of the proton-target nuclear potential (taken from Ref. [14]) act to reduce the dissociation probability even further.

We show in Fig. 2(a) the energy-differential cross section at 59 MeV/n calculated by integrating the probability distribution over impact parameter, with a cutoff at $b_{\min} = 12$ fm, the strong absorption radius taken from a conventional parametrization [15]. We see the full theory is reduced significantly at low relative energies, as expected from our discussion above. At $E_{\rm rel} \approx 1$ MeV, the full theory is close to the first-order $E1$ FF approximation due to an accidental cancellation.

Several of the experiments avoid the need for information about the smaller impact parameters by measuring the distribution in θ_8, the scattering angle of the ^{8}B center of mass, and using it to emphasize the larger impact parameters. The RIKEN analysis [2,3] made a fit to the angular distribution. The experiments at the National Superconducting Cyclotron Laboratory (NSCL) [5] and at the Gesellschaft für Schwerionenforschung (GSI) in Germany [7] imposed a cutoff in θ_8 to select large-impact parameters. Such analyses impose an additional demand on theory because of diffraction effects in the projectile-target scattering. Several experimental analyses used distorted-wave calculations to avoid the classical approximation; however, Bertulani [16] estimated the effects of diffraction to be small for the ^{8}B breakup reactions discussed here. We have made our own estimate using the semiclassical theory for the scattering phase shifts (see, e.g., [15]). Omitting the details, we find that for scattering angles corresponding to large-impact parameter cutoffs of

the NSCL and GSI experiments (30 fm) the amplitude of the diffractive oscillations are only a few percent and the angle-integrated cross section is within 1% of the value obtained with classical trajectories. Thus the computation by the distorted-wave theory is unnecessary under the conditions of those experiments.

As an example of a θ_8 angular distribution, we show in Fig. 3 the measurements for one of the energy bins of the RIKEN experiment [2]. The theoretical distributions have been transformed by the experimental acceptance filter. The full dynamic calculation was scaled by a factor of 1.2 in order to minimize the χ^2 fit to the data. For the first-order FF approximation curves, with and without $E2$, the scaling factor was set to 1.0, which gives the best fit in the $E1$ FF approximation. These first-order FF approximation curves show a different falloff at large θ_8, which was taken as evidence that the $E2$ contribution is very small. The full calculation (solid line) has a better shape compared to the data at larger angles, but the best χ^2 (≈ 10 per point) is similar in the $E1$ FF approximation. The large value of the χ^2 is mainly caused by the poor fit at forward angles.

Here we discuss in more detail the controversial issue of the $E2$ strength magnitude. The $E2$ strength was first extracted from measurements of the asymmetry in the longitudinal momentum distributions of ^{7}Be fragments [17] using the first-order FF approximation, and the $E2$ matrix elements had to be quenched by a factor of 0.7 to reproduce the observed asymmetry. We know now that the FF approximation is one reason the $E2$ strength had to be quenched in the first-order calculation. This is evident from the work of Mortimer *et al.* [18], who showed that a distorted-wave Born approximation calculation, which was based on the unrestricted Coulomb form factors and essentially the same structure model of ^{8}B as used here and also in [17], did not require any adjustment of the $E2$ strength to fit the data. However, their calculation to all orders required a scaling factor of 1.6, implying a very large $E2$ strength. In any case, it is clear that ^{8}B does have a substantial $E2$ strength. The model we and Mortimer *et al.* use may actually be a lower limit.

Restricting the measurement to small scattering angles would greatly reduce the sensitivity to theoretical approximations. The calculated relative energy distribution with such a gate is shown in Fig. 2(b), obtained by integrating over angles $\theta_8 < 2.8°$, corresponding to $b \geq 30$ fm. The good agreement at low relative energies between the $E1$ FF approximation and the full (dynamic CN) calculation is due to an accidental cancellation between the $E2$ contribution and the correction to the FF approximation. This agreement disappears at higher relative energies, where the full calculation is seen to approach the first-order $E1 + E2$ FF result.

A characteristic feature of the full calculation is that the energy spectrum is suppressed at low relative energies when compared to the first-order FF approximation. It may even be suppressed compared to the first-order $E1$ FF approximation as seen in Fig. 2(a). In order to estimate corrections to that approximation, which is commonly used in data analyses, we present in Fig. 4 the ratio of energy spectra obtained in the full calculation and in the first-order $E1$ FF approximation for three choices of $b_{\min}$. As can be seen, the ratio is less than 1 at low $E_{\rm rel}$ and larger than 1 at high $E_{\rm rel}$.

A simple way to correct the S_{17} factors that were extracted from measurements using the first-order $E1$ FF approximation, $S_{17}^{E1}(E)$, is to divide by the appropriate ratio shown in Fig. 4,

$$S_{17}^{\rm Cor}(E) \approx S_{17}^{E1}(E) \times \frac{[d\sigma/dE]_{E1}}{[d\sigma/dE]_{\rm Full}}.$$

According to Fig. 4 one would then obtain a larger S_{17} at low relative energies and a smaller S_{17} at high energies, and hence this will reduce the slope anomaly identified in [1]. In fact, the ratios shown in Fig. 4 for $b_{\min} = 12$ and 20 fm

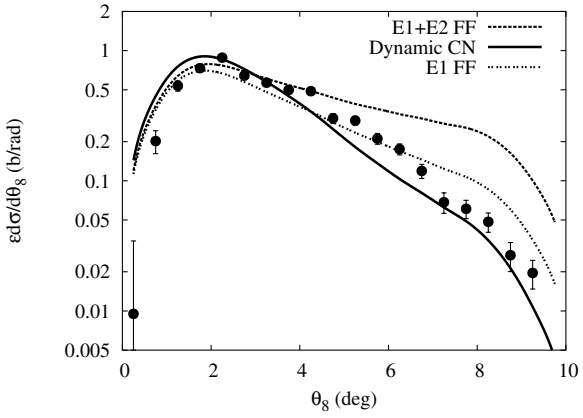

FIG. 3. Measured and calculated angular distributions of the ^{7}Be + p center-of-mass system for a 0.5–0.75 MeV relative energy cut. The calculated distributions have been filtered by the experimental acceptance of the RIKEN experiment [2]. The dynamic calculation has been multiplied by a factor of 1.2.

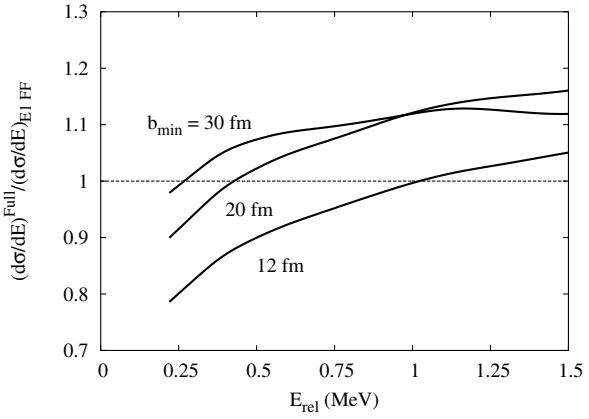

FIG. 4. Calculated ratio of relative energy spectra at 51.9 MeV/n obtained in the full calculation and in the first-order $E1$ FF approximation, for the three indicated values of the minimum impact parameter.

TABLE I. Indirect measurements of S_{17}. CT indicates classical trajectories assumed. See text for other symbols.

Laboratory	Beam energy (MeV/n)	Theoretical approximations	$\theta_{\rm cut}$	$b_{\rm min}$ (fm)
RIKEN [2,3]	51.9	$E1$, FF, FO	none	
NSCL [4,5]	83	FF, CT	1.8°	30
GSI [6]	254	$E1$, FF, FO	none	
GSI [7]	254	$E1$, FF, FO	0.62°	30

imply slope corrections similar in magnitude to the 0.25 MeV^{-1} average slope difference between CD and direct results shown in Fig. 19 of [1]. We note that correcting the CD slope problem, as outlined above, will tend to increase the extrapolated $S_{17}(0)$ values.

We now comment in more detail on the various ^{8}B CD experiments (see Table I for details). For the RIKEN experiment [2,3], we find a significant increase in S_{17} at low energies (a rough estimate is the 1.2 normalization factor discussed above) and a small reduction at higher $E_{\rm rel} \approx 1.375$ MeV (not shown), with the result that the corrected $S_{17}(E)$ slope and the $S_{17}(0)$ value are both in much better agreement with the direct results.

The NSCL analysis [4,5] employed a small θ_8 cutoff corresponding to $b_{\rm min} = 30$ fm and used the first-order FF approximation including a 5% $E2$ contribution. We find that the full calculation is essentially identical to the first-order $E1$ FF approximation at low $E_{\rm rel}$; thus the extracted value of $S_{17}(0)$ should be increased by about 5%. The assumed 5% $E2$ component [4,5] was extracted from the asymmetry of the measured longitudinal momentum distributions of ^{7}Be fragments. We argued above that the $E2$ strength required to fit the data in the far-field approximation is too small, because the far-field approximation is inaccurate. A more realistic $E2$ strength is at least twice as large. This implies that the correction for $E2$ contributions to the relative energy spectrum at high relative energies [where the $E1 + E2$ FF approximation is quite reasonable, as shown in Fig. 2(b)] should be at least 10%, instead of the 5% assumed in the original analysis. Thus S_{17} at high $E_{\rm rel}$ should be reduced by at least 5%. The resulting $S_{17}(E)$ slope agrees significantly better with the direct mean.

Of the two GSI experiments [6,7], we analyze only the Schümann *et al.* [7] measurement, which had a much improved θ_8 resolution and a $b_{\rm min} = 30$ fm cutoff. They used first-order theory in their analysis, which is acceptable since the high beam velocity suppresses dynamic polarization effects. There is still a significant correction at $b = 30$ fm due to the $E2$ contribution and the unrestricted treatment of the Coulomb field (see discussion of the NSCL experiment above), as these effects do not depend much on the beam energy. Our estimated corrections to the theoretical cross section for $E_{\rm rel} < 0.5$ MeV are +5% for the $E2$ component and −5% by replacing the FF approximation by the full Coulomb calculation. Thus there is a fortuitous cancellation, and the extracted $S_{17}(0)$ based on the low relative energy data is not affected. However, at higher $E_{\rm rel}$ the first-order $E1 + E2$ FF approximation becomes reasonable at the forward angles considered in the experiment, and the extracted $S_{17}(E)$ should be reduced by about 5%. This reduces the $S_{17}(E)$ slope somewhat, leaving it high compared to the direct mean.

In conclusion, our improved analysis brings the indirect CD method for determining S_{17} into better accord with the direct method, reducing the difference in the extracted $S_{17}(E)$ slopes. We also showed that the disagreement between extracted values of $S_{17}(0)$ would be reduced by using the full theory in the RIKEN and NSCL experimental analyses (this is probably true also for the earlier GSI experiment [6]). However, the NSCL and the latest GSI $S_{17}(0)$ values remain significantly low compared to the direct $S_{17}(0)$ mean value.

We note that precise corrections for the effects discussed here require detailed analyses that properly include the effects of experimental acceptance and are beyond the scope of this work.

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