Realistic Calculation of the ^{3}He + p (hep) Astrophysical Factor

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The astrophysical factor for the proton weak capture on ^{3}He is calculated with correlated hyperspherical harmonic wave functions corresponding to a realistic Hamiltonian consisting of the Argonne v_{18} two-nucleon and Urbana-IX three-nucleon interactions. The nuclear weak current has vector and axial-vector components with one- and many-body terms. All possible transitions connecting any of the $p\,^3$He S- and P-wave channels to ^{4}He are considered. The S factor at a $p\,^3$He center-of-mass energy of 10 keV is predicted to be 10.1×10^{-20} keV b, a factor of $\simeq 4.5$ larger than the value adopted in the standard solar model. The P-wave transitions are found to contribute about 40% of the calculated S factor.

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Recently, there has been a revival of interest in the reaction ^{3}He$(p, e^+ \nu_e)^4$He [1]. This interest has been spurred by the Super-Kamiokande Collaboration measurements of the energy spectrum of electrons recoiling from scattering with solar neutrinos [2]. At energies larger than 14 MeV, more recoil electrons have been observed than expected relative to standard-solar-model (SSM) predictions [3], reduced by a factor of $\simeq 0.5$ to fit the lower-energy bins. The hep process, as the proton weak capture on ^{3}He is known, is the only source of solar neutrinos with energies larger than 15 MeV—their end-point energy is about 19 MeV. This fact has naturally led to questions about the reliability of the currently accepted value for the astrophysical factor at zero energy, 2.3×10^{-20} keV b [4]. In particular, Bahcall and Krastev [1] have shown that a large enhancement, by a factor in the range 20–30, of this value would essentially fit the observed excess [2] of recoiling electrons.

The theoretical description of the hep process, as well as that of the neutron and proton radiative captures on deuteron and ^{3}He, constitute a challenging problem from the standpoint of nuclear few-body theory. Its difficulty can be appreciated by comparing the measured values for the cross section of thermal neutron radiative capture on ^{1}H, ^{2}H, and ^{3}He. Their respective cross sections are 334.2 ± 0.5 mb [5], 0.508 ± 0.015 mb [6], and 0.055 ± 0.003 mb [7]. Thus, in going from $A = 2$ to $A = 4$ the cross section has dropped by almost 4 orders of magnitude. These processes are induced by magnetic-dipole transitions between the initial two-cluster state in relative S wave and the final bound state. In fact, the inhibition of the $A = 3$ and $A = 4$ captures has been understood for a long time [8]: the ^{3}H and ^{4}He states are approximate eigenstates of the magnetic dipole operator $\boldsymbol{\mu}$, and consequently matrix elements of μ_z between nd ($n\,^3$He) and ^{3}H (^{4}He) vanish (approximately) due to orthogonality. This orthogonality argument fails in the case of the deuteron, since then μ_z can connect the large S-wave component of the deuteron to an isospin $T = 1$ $^1S_0$ np state.

This quasiorthogonality, while again invalid in the case of the proton weak capture on protons [9], is also responsible for inhibiting the hep process. Both of these reactions are induced by the Gamow-Teller operator, which differs from the (leading) isovector spin part of the magnetic dipole operator essentially by an isospin rotation. As a result, the hep weak capture and nd, pd, and $n\,^3$He radiative captures are extremely sensitive to (i) D-state admixtures generated by tensor interactions and (ii) many-body terms in the electroweak current operator. For example, many-body current contributions provide, respectively, 50% and over 90% of the calculated pd [10] and $n\,^3$He [4,11] cross sections at very low energies.

In this respect, the hep weak capture is a particularly delicate reaction for two additional reasons: firstly, and most importantly, the one- and many-body current contributions are comparable in magnitude, but of opposite sign [4,12]; secondly, many-body axial currents, specifically those arising from excitation of Δ isobars which give the dominant contribution, are model dependent [12]. This destructive interference between one- and many-body currents also occurs in the $n\,^3$He ("hen") radiative capture [4,11], with the difference that there the leading components of the many-body currents are model independent, and give a much larger contribution than that associated with the one-body current.

The cancellation in the hep process between the one- and two-body matrix elements has the effect of enhancing the importance of P-wave capture channels. Indeed, one of the results of this work is that these channels give about 40% of the S-factor calculated value. That the hep process could proceed as easily through P- as S-wave capture was not sufficiently appreciated [13] in all earlier studies of this reaction we are aware of, with the exception of Ref. [1], in which Horowitz suggested, on the basis of a very simple one-body reaction model, that the $^3P_0$ channel may be important.

Most of the earlier studies [7,13,14] attempted to relate the matrix element of the axial current occurring in the hep capture to that of the electromagnetic current in the hen capture, exploiting (approximate) isospin symmetry. This approach led, however, to S-factor values ranging from 3.7 to 57, in units of 10^{-20} keV b. In an attempt to reduce the uncertainties in the predicted values for both the radiative and weak capture rates, *ab initio* microscopic calculations of these reactions were performed in the early 1990s [4,11,12], using variational wave functions corresponding to a realistic Hamiltonian, and a nuclear electroweak current consisting of one- and many-body components. These studies showed that inferring the hep S-factor from the measured hen cross section can be misleading, because of different initial-state interactions in the $n\,^3$He and $p\,^3$He channels, and because of the large contributions associated with the two-body components of the electroweak current operator, and their destructive interference with the one-body current contributions.

The significant progress made in the past few years in the modeling of two- and three-nucleon interactions and the nuclear weak current, and the description of the bound and continuum four-nucleon wave functions, have prompted us to reexamine the hep reaction. In this paper, we briefly summarize the salient points in the calculation, and report our results for the S factor in the energy range 0–10 keV. An exhaustive account of this study [15], however, will be published elsewhere.

The cross section for the ^{3}He$(p, e^+\nu_e)^4$He reaction at a c.m. energy E is written as

$$\sigma(E) = \int 2\pi\delta\left(\Delta m + E - \frac{q^2}{2m_4} - E_e - E_\nu\right)\frac{1}{v_{\rm rel}} \times \frac{1}{4}\sum_{s_e s_\nu}\sum_{s_1 s_3} |\langle f|H_W|i\rangle|^2 \frac{d\mathbf{p}_e}{(2\pi)^3}\frac{d\mathbf{p}_\nu}{(2\pi)^3}, \quad (1)$$

where $\Delta m = m + m_3 - m_4 = 19.29$ MeV (m, m_3, and m_4 are the proton, ^{3}He, and ^{4}He rest masses, respectively), $v_{\rm rel}$ is the $p\,^3$He relative velocity, and the transition amplitude is given by

$$\langle f|H_W|i\rangle = \frac{G_V}{\sqrt{2}}\, l^\sigma \langle -\mathbf{q};^4\text{He}|\, j_\sigma^\dagger(\mathbf{q})|\mathbf{p}; p\,^3\text{He}\rangle. \quad (2)$$

Here G_V is the Fermi constant, $\mathbf{q} = \mathbf{p}_e + \mathbf{p}_\nu$, $|\mathbf{p}; p\,^3\text{He}\rangle$ and $|-\mathbf{q}; ^4\text{He}\rangle$ represent, respectively, the $p\,^3$He scattering state with relative momentum $\mathbf{p}$ and ^{4}He bound state recoiling with momentum $-\mathbf{q}$, l_σ is the leptonic weak current, $l_\sigma = \overline{u}_\nu\gamma_\sigma(1-\gamma_5)v_e$ (the lepton spinors are normalized as $v_e^\dagger v_e = u_\nu^\dagger u_\nu = 1$), and $j^\sigma(\mathbf{q})$ is the nuclear weak current, $j^\sigma(\mathbf{q}) = [\rho(\mathbf{q}), \mathbf{j}(\mathbf{q})]$. The dependence of the amplitude upon the spin projections of the leptons, proton, and ^{3}He has been omitted for ease of presentation. Since the energies of interest are of the order of 10 keV or less—the Gamow peak energy is 10.7 keV for the hep reaction—it is convenient to expand the $p\,^3$He scattering state into partial waves, and perform a multipole decomposition of the nuclear weak charge, $\rho(\mathbf{q})$, and current, $\mathbf{j}(\mathbf{q})$, operators. Standard manipulations lead to [15,16]

$$\frac{1}{4}\sum_{s_e s_\nu}\sum_{s_1 s_3}|\langle f|H_W|i\rangle|^2 = (2\pi)^2 G_V^2 L_{\sigma\tau}N^{\sigma\tau}, \quad (3)$$

where the lepton tensor $L^{\sigma\tau}$ is written in terms of electron and neutrino four velocities as $L^{\sigma\tau} = v_e^\sigma v_\nu^\tau + v_\nu^\sigma v_e^\tau - g^{\sigma\tau} v_e \cdot v_\nu + i\epsilon^{\sigma\alpha\tau\beta}v_{e,\alpha}v_{\nu,\beta}$, while the nuclear tensor is defined as

$$N^{\sigma\tau} \equiv \sum_{s_1 s_3} W^\sigma(\mathbf{q}; s_1 s_3) W^{\tau *}(\mathbf{q}; s_1 s_3), \quad (4)$$

with

$$W^{\sigma=0,3}(\mathbf{q}; s_1 s_3) = \sum_{LSJ} X_0^{LSJ}(\hat{\mathbf{q}}; s_1 s_3) T_J^{LSJ}(q), \quad (5)$$

$$W^{\sigma=\lambda}(\mathbf{q}; s_1 s_3) = -\frac{1}{\sqrt{2}}\sum_{LSJ} X_{-\lambda}^{LSJ}(\hat{\mathbf{q}}; s_1 s_3) \times [\lambda M_J^{LSJ}(q) + E_J^{LSJ}(q)], \quad (6)$$

where $\lambda = \pm 1$ denotes spherical components. The functions $X_{\lambda=0,\pm1}$ depend upon the direction $\hat{\mathbf{q}}$, and the proton and ^{3}He spin projections s_1 and s_3 [15] (note that the quantization axis for the hadronic states is taken along $\hat{\mathbf{p}}$, the direction of the $p\,^3$He relative momentum), while $T_J^{LSJ} = C_J^{LSJ}$ or L_J^{LSJ} for $\sigma = 0$ or 3. The quantities C_J^{LSJ}, L_J^{LSJ}, E_J^{LSJ}, and M_J^{LSJ} are the reduced matrix elements (RMEs) of the Coulomb ($C_{\ell\ell_z}$), longitudinal ($L_{\ell\ell_z}$), transverse electric ($E_{\ell\ell_z}$), and transverse magnetic ($M_{\ell\ell_z}$) multipole operators between the initial $p\,^3$He state with orbital angular momentum L, channel spin S ($S = 0, 1$), and total angular momentum J, and final ^{4}He state. The present study includes S- and P-wave capture channels, i.e., the $^1S_0$, $^3S_1$, $^3P_0$, $^1P_1$, $^3P_1$, and $^3P_2$ states, and retains all contributing multipoles (with their full momentum-transfer dependence) connecting these states to the $J^\pi = 0^+$ ground state of ^{4}He.

The bound- and scattering-state wave functions are obtained variationally with the correlated-hyperspherical-harmonics (CHH) method, developed in Refs. [17,18]. The nuclear Hamiltonian consists of the Argonne v_{18} two-nucleon [19] and Urbana-IX three-nucleon [20] interactions. This realistic Hamiltonian, denoted as AV18/UIX, reproduces the experimental binding energies and charge radii of the trinucleons and ^{4}He in "exact" Green's function Monte Carlo (GFMC) calculations [21]. The binding energy of ^{4}He calculated with the CHH method [15,17] is within 1% of that obtained with the GFMC method. The accuracy of the CHH method to calculate scattering states has been successfully verified in the case of three-nucleon systems, by comparing results for a variety of Nd scattering observables obtained by a number of groups using different techniques [22]. Studies along similar lines [23] to assess the accuracy of the CHH solutions for the four-nucleon continuum have already begun.

The CHH predictions [18] for the $n\,^3$H total elastic cross section and coherent scattering length have been found to be in excellent agreement with the corresponding experimental values. The $n\,^3$H cross section is known over a rather wide energy range, and its extrapolation to zero energy is not problematic [24]. The situation is different for the $p\,^3$He channel, for which the singlet and triplet scattering lengths a_s and a_t have been determined from effective range extrapolations of data taken above 1 MeV, and are therefore somewhat uncertain, $a_s = 10.8 \pm 2.6$ fm [25] and $a_t = 8.1 \pm 0.5$ fm [25] or 10.2 ± 1.5 fm [14]. Nevertheless, the CHH results are close to the experimental values above: the AV18/UIX Hamiltonian predicts [18] $a_s = 11.5$ fm and $a_t = 9.13$ fm. At low energies (below 4 MeV) $p\,^3$He elastic scattering proceeds mostly through S- and P-wave channels, and the CHH predictions, based on the AV18/UIX model, for the differential cross section [26] are in good agreement with the experimental data.

The nuclear weak current has vector and axial-vector parts, with corresponding one- and many-body components. The one-body components have the standard expressions obtained from a nonrelativistic reduction of the covariant single-nucleon vector and axial-vector currents, including terms proportional to $1/m^2$. The two-body weak vector currents are constructed from the isovector two-body electromagnetic currents in accordance with the conserved-vector-current hypothesis, and consist [15] of "model-independent" and "model-dependent" terms. The model-independent terms are obtained from the nucleon-nucleon interaction, and by construction satisfy current conservation with it, whereas the model-dependent terms are, by definition, purely transverse, and therefore are not constrained by the continuity equation. These latter currents can be varied, in principle, for a fixed nucleon-nucleon interaction. The π- and ρ-meson-exchange contributions to the weak vector charge operator [15] have also been retained in this work.

The leading many-body terms in the axial current due to Δ-isobar excitation are treated nonperturbatively in the transition-correlation-operator (TCO) scheme, originally developed in Ref. [4] and further extended in Ref. [27]. In the TCO scheme—essentially, a scaled-down approach to a full $N + \Delta$ coupled-channel treatment—the Δ degrees of freedom are explicitly included in the nuclear wave functions. The axial charge operator includes, in addition to Δ-excitation terms (which, however, are found to be unimportant [15]), the long-range pion-exchange term [28], required by low-energy theorems and the partially conserved axial-current relation, as well as the (expected) leading short-range terms constructed from the central and spin-orbit components of the nucleon-nucleon interaction, following a prescription due to Riska and collaborators [29].

The largest model dependence is in the $N\Delta$-transition axial coupling constant g_A^*. In the quark model, it is related to the axial coupling constant of the nucleon by the relation $g_A^* = (6\sqrt{2}/5)g_A$. However, given the uncertainties inherent to quark-model predictions, a more reliable estimate for g_A^* is obtained by determining its value phenomenologically in the following way. It is well established by now [9] that the one-body axial current leads to a $\simeq 4\%$ underprediction of the measured Gamow-Teller matrix element in tritium β decay. Since the contributions of the $N\Delta$ axial currents are found to be numerically dominant, this 4% discrepancy can then be used to determine g_A^*. While this procedure is model dependent, its actual model dependence is in fact very weak, as has been shown in Ref. [9].

The calculation proceeds in two steps [15]: first, the matrix elements of $\rho(\mathbf{q})$ and $\mathbf{j}(\mathbf{q})$ between the initial $p\,^3$He $LSJJ_z$ states and final ^{4}He are calculated with Monte Carlo integration techniques; second, the contributing RMEs are extracted from these matrix elements, and the cross section is calculated by performing the integrations over the electron and neutrino momenta in Eq. (1) numerically, using Gauss points.

The results for the S factor, defined as $S(E) = E\sigma(E)\exp(4\pi\alpha/v_{\mathrm{rel}})$ (α is the fine-structure constant), at $p\,^3$He c.m. energies of 0, 5, and 10 keV are reported in Table I. In the table, the column labeled S includes both the $^1S_0$ and $^3S_1$ channel contributions, although the former are at the level of a few parts in 10^3. The energy dependence is rather weak: the value at 10 keV is only about 4% larger than that at 0 keV. The P-wave capture states are found to be important, contributing about 40% of the calculated S factor. However, the contributions from D-wave channels are expected to be very small. We have verified explicitly that they are indeed small in $^3D_1$ capture. The many-body axial currents associated with Δ excitation play a crucial role in the (dominant) $^3S_1$ capture, where they reduce the S factor by more than a factor of 4. Thus the destructive interference between the one- and many-body current contributions, first obtained in Ref. [12], is confirmed in this paper. The (suppressed) one-body contribution comes mostly from transitions involving the D-state components of the ^{3}He and ^{4}He wave functions, while the many-body contributions are predominantly due to transitions connecting the S state in ^{3}He to the D state in ^{4}He, or vice versa.

TABLE I. The *hep* S factor, in units of 10^{-20} keV b, calculated with CHH wave functions corresponding to the AV18/UIX Hamiltonian model, at $p\,^3$He c.m. energies $E = 0$, 5, and 10 keV. The rows labeled "one-body" and "full" list the contributions obtained by retaining the one-body only and both one- and many-body terms in the nuclear weak current. The contributions due to the S-wave channels only and S- and P-wave channels are listed separately. The Monte Carlo statistical error is at the 5% level on the total S factor.

E (keV)	0		5		10	
	S	$S+P$	S	$S+P$	S	$S+P$
one-body	26.4	29.0	25.9	28.7	26.2	29.3
full	6.39	9.64	6.21	9.70	6.37	10.1

It is important to stress the differences between the present and all previous studies. Apart from ignoring, or at least underestimating, the contribution due to P waves, the latter only considered the long-wavelength form of the weak multipole operators, namely, their $q = 0$ limit. In $^3P_0$ capture, for example, only the C_0 multipole, associated with the weak axial charge, survives in this limit, and the corresponding S factor is calculated to be 2.2×10^{-20} keV b, including two-body contributions. However, when the transition induced by the longitudinal component of the axial current (via the L_0 multipole, which vanishes at $q = 0$) is also taken into account, the S factor becomes 0.82×10^{-20} keV b, because of a destructive interference between the C_0 and L_0 RMEs. Thus use of the long-wavelength approximation in the calculation of the *hep* S factor leads to inaccurate results.

Finally, besides the differences listed above, the present calculation also improves that of Ref. [4] in a number of other important respects: firstly, it uses accurate CHH wave functions, corresponding to the latest generation of realistic interactions. Secondly, the model for the nuclear weak current has been extended to include the axial charge as well as the vector charge and current operators. Thirdly, the one-body operators now take into account the $1/m^2$ relativistic corrections, which were previously neglected. In $^3S_1$ capture, for example, these terms increase by 25% the dominant (but suppressed) L_1 and E_1 RMEs calculated with the (lowest order) Gamow-Teller operator. These improvements in the treatment of the one-body axial current also indirectly affect the contributions of the Δ-excitation currents [15], because of the procedure used to determine the coupling constant g_A^*.

In conclusion, we have carried out a realistic calculation of the *hep* reaction, predicting a value for the S factor $\simeq$4.5 times larger than that used in the SSM. This enhancement, while very significant, is smaller than that required by fits to the Super-Kamiokande data. Although the present result is inherently model dependent, it is unlikely, as argued in Ref. [15], that the model dependence be so large as to accommodate a drastic increase in the prediction obtained here.

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