Influence of protons on the capture of electrons by ^{7}Be in the Sun

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The capture of electrons by the nucleus ^{7}Be from the three-body initial state $p + e^- + {}^7$Be in the continuum is studied. On the basis of the expansion of the three-body continuum wave function in the small parameter $\epsilon \approx (m_e/m_p)^{1/2}$ [m_e (m_p) is the electron (proton) mass], the role of the protons on the electron capture is considered. The results are compared with the traditional treatment of the electron capture by the nucleus ^{7}Be. For stars with density and temperature like in the center of the Sun the studied mechanism can make a non-negligible contribution to the capture rate.

I. INTRODUCTION

The process of the electron capture by the nucleus ^{7}Be is important because it contributes to the low energy part of the spectrum of neutrinos radiated by the Sun. In addition, it is obvious that 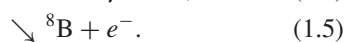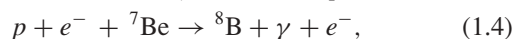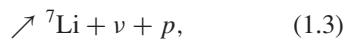the balance of the disappearance channels of ^{7}Be in the Sun regulates the amount of the nucleus ^{8}B that is the source of the high energy solar neutrinos. This is the main reason why this process has attracted considerable attention over many years [1–7].

Practically all the discussion so far of the electron capture in ^{7}Be is concentrated on considering the electron wave function in the vicinity of the nucleus and on the screening effects on it; the study of this capture in the plasma is described in Ref. [7].

In what follows we estimate the role of the process that is usually not included in the standard theory of the *pp* cycle in the Sun. Let us first note that in the standard theory of this cycle the destruction of the nucleus ^{7}Be takes place in the binary reactions

$$p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma, \qquad (1.1)$$
$$e^- + {}^7\text{Be} \rightarrow {}^7\text{Li} + \nu. \qquad (1.2)$$
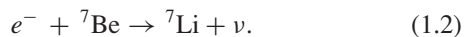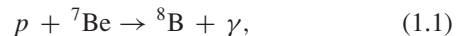
Because the nucleus ^{7}Be participates in both processes, instead of the binary reactions (1.1) and (1.2) we consider the contribution to the electron capture rate from the three-particle initial state $p + e^- + {}^7$Be. In this case the following reactions can take place:

$$p + e^- + {}^7\text{Be} \rightarrow \begin{cases} \nearrow {}^7\text{Li} + \nu + p, & (1.3) \\ {}^8\text{B} + \gamma + e^-, & (1.4) \\ \searrow {}^8\text{B} + e^-. & (1.5) \end{cases}$$

As was shown in Ref. [2], the screening corrections for the electrons in the continuum are rather small. Therefore, we consider in the initial state the bare Coulomb interaction in all two-body subsystems $e^- + p$, $p + {}^7$Be, and $e^- + {}^7$Be. In this case one can immediately realize that there is a qualitative

difference between the binary and ternary mechanisms[1] of the electron capture. Indeed, if one starts from the three-body initial state, then the processes (1.1) and (1.2) should be interdependent because the wave function of three charged particles cannot be presented as a product of pair wave functions, as is required by the binary processes (1.1) and (1.2).[2] Here we apply the method and results obtained in Ref. [9] where an alternative to the Born-Oppenheimer approach has been suggested for the calculations of wave functions in the continuum for three charged particles.

II. CALCULATIONS

In analogy with the approach developed in Ref. [9], the continuum wave function of three charged particles can be expanded in a small parameter, ϵ,

$$\epsilon = \left[\frac{Mm_e}{(M + m_p)(m_p + m_e)} \right]^{1/2} \approx \left[\frac{m_e}{m_p} \right]^{1/2}, \qquad (2.1)$$

where, in addition to the electron and proton mass, also M, the mass of the nucleus ^{7}Be, enters. The expansion of the wave function of the studied three-body system is then

$$\Psi(\vec{r}, \vec{R}) \approx \Psi_0(\vec{r}, \vec{R}) + \epsilon \Psi_1(\vec{r}, \vec{R}) + \cdots. \qquad (2.2)$$

Because $\epsilon \approx 0.0233$, one expects the effects of the second term on the right-hand side of Eq. (2.2) to be at the level of 2% in comparison with the first term.

It was found in Ref. [9] that in the limit $\epsilon \rightarrow 0$ the Jacobi coordinates $\vec{r}$ and $\vec{R}$ (see Fig. 1) separate. This means that the structure of the wave function $\Psi_0(\vec{r}, \vec{R})$ is

$$\Psi_0(\vec{r}, \vec{R}) = \Psi^C(\vec{R})\Psi^C(\vec{r}, Z = Z_1 + Z_2), \qquad (2.3)$$

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[1] We use the terms binary reaction and ternary reaction as synonyms for the reactions in the two- and three-component systems.

[2] There is only one exception corresponding to the case when all three particles are at very large distances between themselves [8], which is not applicable to the electron capture.
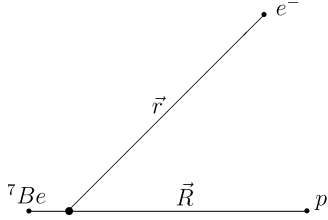
FIG. 1. The Jacobi coordinates for the $p + e^- + {}^7\mathrm{Be}$ system.

where $\Psi^C(\vec{R})$ is the Coulomb wave function describing the relative motion of the proton and the nucleus ${}^7\mathrm{Be}$, and $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$ is the Coulomb wave function that describes the motion of the electron in the field of the effective Coulomb potential of the charge $Z = Z_1 + Z_2$. The crucial point is that the wave function $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$ depends on the distance between the electron and the center of mass of the subsystem of heavy particles. It means that even if the distance between the electron and the nucleus ${}^7\mathrm{Be}$ is zero, as is required by the Hamiltonian of the weak interaction, the wave function $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$, defining the probability of the electron capture by the nucleus ${}^7\mathrm{Be}$, should be taken at a non-zero distance $|\vec{r}| = \beta|\vec{R}|$, where $\beta = 1/7$ is the ratio of the proton and ${}^7\mathrm{Be}$ masses.

It is clear that this phenomenon appears to be due to the electron movement in the Coulomb field of two charged particles with positive charges. Following the arguments presented above, we now consider two effects acting in opposite directions. On one hand, increasing the effective positive charge of the heavy particles system by one unit will enlarge the electron capture rate. On the other hand, using the Coulomb wave function at finite distances instead of the function taken at zero distance should damp the capture rate.

Taking into account that the nuclear matrix elements of the reactions (1.2) and (1.3) are the same, as a measure of influence of the third particle (the proton in this case) on the capture rate of electrons by the nuclei ${}^7\mathrm{Be}$ we introduce the ratio $\varsigma(R, T)$, which is the function of the distance R between the particles and the temperature T,

$$\varsigma(R, T) = \frac{\int_0^\infty \left|\Psi_E^C(\beta R, Z = 5)\right|^2 e^{-E/kT} dE}{\int_0^\infty F(Z = 4, \nu) e^{-E/kT} dE}. \quad (2.4)$$

Here the denominator on the right-hand side of the equation contains the quantity that enters the electron capture rate from the continuum for the reaction (1.2) [1]. The Fermi function $F(Z, \nu)$ is given by the equation

$$F(Z, \nu) = 2\pi\nu/(e^{2\pi\nu} - 1), \quad (2.5)$$

where the parameter ν is given by the equation $\nu = -Z\alpha m_e/p = -Z\alpha/v$, α is the fine structure constant, and $p = \sqrt{2m_e E}$ is the electron momentum. It is obtained by using the solution of the Dirac equation with the Coulomb potential [10].

An analogous integral in the numerator should reflect the effect of the Coulomb potential on the electron in the continuum for the reaction (1.3). For the wave function $\Psi_E^C(\vec{r}, Z)$, we use the Coulomb continuum wave function for

the state with zero angular momentum

$$\Psi_E^C(\rho) = \frac{F_0(\eta, \rho)}{\rho}, \quad (2.6)$$

where the function $F_0(\eta, \rho)$ satisfies the equation

$$\frac{d^2 F_0}{d\rho^2} + \left[1 - \frac{2\eta}{\rho}\right] F_0 = 0, \quad (2.7)$$

with $\rho = p\beta R$, and $\eta = -Z\alpha\sqrt{m_e/E} = -Z\alpha\sqrt{2}/v$ is the Sommerfeld parameter. The function $F_0(\eta, \rho)$ can be expressed in terms of the Kummer function M (see Ref. [11], Chap. 14) as

$$F_0(\eta, \rho) = C_0(\eta)\rho e^{-i\rho} M(1 - i\eta, 2, 2i\rho), \quad (2.8)$$

where $C_0^2(\eta) = 2\pi\eta/(e^{2\pi\eta} - 1)$.

Instead of the quantity $\varsigma(R, T)$, one can consider

$$\varsigma_C(R, T) = \frac{\int_0^\infty \left|\Psi_E^C(\beta R, Z = 5)\right|^2 e^{-E/kT} dE}{\int_0^\infty \left|\Psi_E^C(0, Z = 4)\right|^2 e^{-E/kT} dE}. \quad (2.9)$$

Using Eq. (2.7), one obtains

$$\left|\Psi_E^C(0, Z)\right|^2 = 2\pi\eta/(e^{2\pi\eta} - 1), \quad (2.10)$$

which is of the same form as the Fermi function (2.5); however, $\eta = \sqrt{2}\nu$.

One can find the following integral representation (see Ref. [11], Chap. 13),

$$M(1 - i\eta, 2, 2i\rho) = \frac{sh\pi\eta}{\pi\eta} \int_0^1 e^{2i\rho t} \left(\frac{1-t}{t}\right)^{it} dt, \quad (2.11)$$

for this function. In the numerical calculations of the integral over energy in Eqs. (2.4) and (2.9), we used this representation of the Kummer function for the energies $E > 0.1$ keV. In the interval $E < 0.1$ keV, the function under the integral in Eq. (2.11) strongly oscillates which makes the calculations difficult. Instead, we applied the program PFQ developed in Ref. [12]. Let us note that for the energies $E > 0.1$ keV the program PFQ and Eq. (2.11) provide the same results to a high degree of accuracy.

We also introduce the mean value $\langle\varsigma(R_0, T)\rangle$ of the function $\varsigma(R, T)$

$$\langle\varsigma(R_0, T)\rangle = N \int e^{-\frac{(R-R_0)^2}{2R_0^2}} \varsigma(R, T) d\vec{R}$$

$$= 4\pi R_0^3 N \int_0^\infty e^{-\frac{(x-1)^2}{2}} x^2 \varsigma(R_0 x, T) dx, \quad (2.12)$$

where

$$N^{-1} = \int e^{-\frac{(R-R_0)^2}{2R_0^2}} d\vec{R}$$

$$= \frac{4\pi R_0^3}{\sqrt{e}} \left[1 + \sqrt{2\pi e}\left(1 - \phi\left(-\frac{1}{\sqrt{2}}\right)\right)\right] \quad (2.13)$$

is the normalization constant. In Eq. (2.13), the function $\phi(y)$ is the error function (see Ref. [11], Chap. 7). It can be seen from Eq. (2.12) that the quantity $\langle\varsigma(R_0, T)\rangle$ depends on the mean

TABLE I. The mean values $\langle \varsigma(R_0, T) \rangle$ and $\langle \varsigma_C(R_0, T) \rangle$ for the electron capture by the nuclei ^{7}Be in the Sun.

$\langle \varsigma(R_0, T) \rangle$	0.0991	0.0991	0.0965	0.0913
$\langle \varsigma_C(R_0, T) \rangle$	0.0718	0.0717	0.0696	0.0658
$R_s/R_\odot$	0.007	0.03	0.06	0.1
kT (keV)	1.353	1.300	1.161	1.088
ρ_p (g/cm^3)	52.1	51.6	48.9	44.9
$R_0 \times 10^{-4}$ (fm)	3.179	3.188	3.250	3.340

distance R_0 between the particles defined by the density in the Sun and on the temperature T. The mean value $\langle \varsigma_C(R_0, T) \rangle$ is defined analogously by using $\varsigma_C(R, T)$. We checked the precision of numerical calculations of the quantities $\langle \varsigma(R_0, T) \rangle$ and $\langle \varsigma_C(R_0, T) \rangle$ by using Mathematica® and also independent numerical procedures. The results of these two independent ways of calculating agree within the required accuracy, which is 0.1%.

III. RESULTS AND DISCUSSION

The results of the calculations are presented in Table I and Figs. 2–4. In Fig. 2, the dependence of the mean value $\langle \varsigma(R_0, T) \rangle$ given in Eq. (2.12) on the temperature T and the value of the mean distance R_0 is shown. A weak dependence of $\langle \varsigma(R_0, T) \rangle$ on the temperature means, as it follows from Eq. (2.4), that the temperature dependence of the electron capture by ^{7}Be is almost the same for the ternary and binary reactions. Such behavior can be understood from the fact that in both cases only rarely is all the kinetic energy carried by one particle, which is the electron. On the other hand, the dependence of $\langle \varsigma(R_0, T) \rangle$ on the value of R_0 shows that the contribution to the capture rate of the ternary reaction is presumably suppressed in stars but it can be at the same level as the contribution to the capture rate for the binary reaction or even prevail over it at very high densities. This is natural because at short distances between the particles the factor of the larger effective charge acting on the electron will
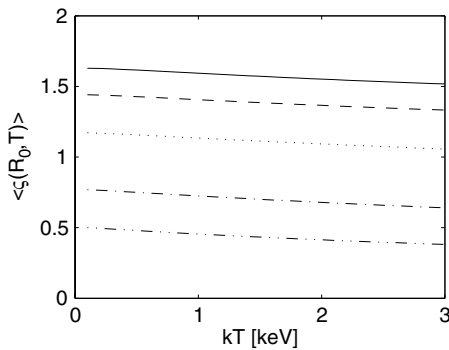

FIG. 3. The dependence of the mean value $\langle \varsigma(R_0, T) \rangle$ on the values of the mean distance R_0. Solid curve, $kT = 1.161$ keV, $\beta = 1/7$, and $Z = 5$; dashed and dotted curve, $kT = 1.161$ keV, $\beta = 4/7$, and $Z = 6$; dashed curve, $kT = 1.5$ keV, $\beta = 1/7$, and $Z = 5$. The dashed and dotted curve corresponds to analogous calculations for the nuclei ^{4}He.

dominate. The same conclusion can be drawn from Fig. 3. In this figure, the solid and dashed curves practically coincide. This again shows a very smooth dependence of $\langle \varsigma(R_0, T) \rangle$ on the temperature. Let us note that the values of R_0, considered in Fig. 2, correspond to rather dense stars. For example, the value of $R_0 = 10^4$ fm corresponds to the proton density $\rho_p = 1673$ g/cm^3, which is about 11 times larger than that in the center of the Sun. Let us further discuss the electron capture by ^{7}Be solely in the Sun.

In Table I, we show the influence of protons on the electron capture in the Sun in more detail. For the Standard Solar Model, we choose Model SSMBP2004 [13]. According to Fig. 6.1 of Ref. [14], the maximal intensity of the electron capture by the nuclei ^{7}Be takes place at the distance $R_s/R_\odot \approx 0.06$, where $R_\odot$ is the radius of the Sun, and it drops to one half at $R_s/R_\odot \approx 0.03$ and $R_s/R_\odot \approx 0.1$. Using the data on the temperature, the density, and the fraction of hydrogen in this area of the Sun, we obtain the mean values $\langle \varsigma(R_0, T) \rangle$ and $\langle \varsigma_C(R_0, T) \rangle$ presented in Table I. In the second column, we add the mean value calculated at $R_s/R_\odot = 0.007$ which is close to the very center of the Sun. It can be seen that the

FIG. 2. The dependence of the mean value $\langle \varsigma(R_0, T) \rangle$ on the temperature T and the value of the mean distance R_0. Solid line, $R_0 = 0.1 \times 10^4$ fm; dashed line, $R_0 = 0.25 \times 10^4$ fm; dotted line, $R_0 = 0.5 \times 10^4$ fm; dashed and dotted line, $R_0 = 1.0 \times 10^4$ fm; dashed and double dotted line, $R_0 = 1.5 \times 10^4$ fm.
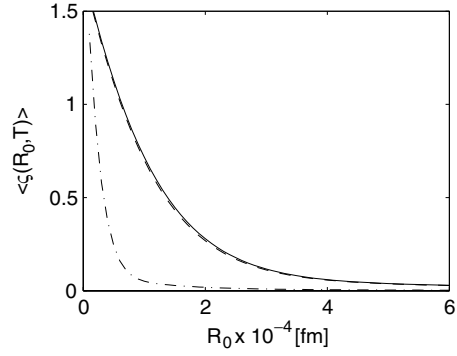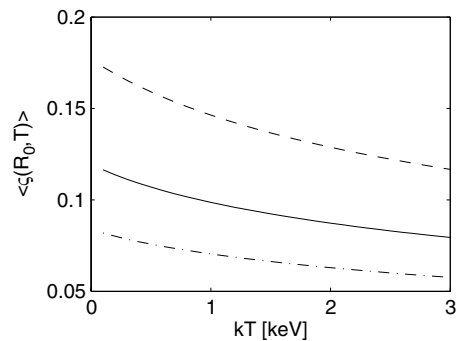

FIG. 4. The dependence of the mean value $\langle \varsigma(R_0, T) \rangle$ on the temperature T and the value of the mean distance R_0. Solid line, $R_0 = 3.25 \times 10^4$ fm; dashed line, $R_0 = 2.75 \times 10^4$ fm; dashed and dotted line, $R_0 = 3.75 \times 10^4$ fm.

change in the average quantities $\langle \varsigma(R_0, T) \rangle$ and $\langle \varsigma_C(R_0, T) \rangle$ is very smooth.

As can be seen from the first row of Table I and from Fig. 4, the contribution to the capture rate of the ternary reaction at the Sun is about 10% of the binary one.[3] This means that it could increase sensibly the burning out of the nuclei ^{7}Be in comparison with the binary reaction, thus decreasing the concentration of the nuclei ^{8}B that appear after the capture of protons by ^{7}Be.

Comparing the first and the second rows of Table I shows a difference of 3% between the values of $\langle \varsigma(R_0, T) \rangle$ and $\langle \varsigma_C(R_0, T) \rangle$. This variation arises from the difference between the relativistic and the nonrelativistic estimations of the electron wave function at zero distance for the binary reaction (1.2).

In Fig. 4 we show the variation of $\langle \varsigma(R_0, T) \rangle$ for the reaction (1.3) for larger intervals of T and R_0.

One can consider in analogy the influence of the nuclei ^{4}He on the electron capture by ^{7}Be in the Sun. In this case, $Z = 6$, $\beta = 4/7$, and at the radius $R_s/R_\odot = 0.06$ the mean

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[3]The next term in the expansion of Eq. (2.2) is expected to change it only by $\approx 2\%$.

distance between the nuclei ^{4}He is $R_0 = 5.34 \times 10^4$ fm. Then one obtains from Eq. (2.12) that $\langle \varsigma(R_0, T) \rangle = 0.0036$, which is about 27 times smaller than the analogous value of $\langle \varsigma(R_0, T) \rangle$ for the protons given in the fourth column of Table I. Evidently, this influence on the electron capture is negligible. This can also be seen from Fig. 3.

The main conclusion following from our calculations is that the three-body process due to the presence of the proton in the vicinity of the nucleus ^{7}Be results in the capture of the electron by an effective charge $Z = 5$ instead of $Z = 4$, which is qualitatively the new effect that cannot be simulated by introducing the Debye screening. This effect can increase the rate of the electron capture from the continuum by ^{7}Be in the Sun, which will reduce the concentration of the nuclei ^{8}B that appear after the capture of protons by ^{7}Be.

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