Nuclear Many-body dynamics beyond the limits of stability

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Beyond the Traditional Shell Model

Structure theory

- **relevant valence space**
- **symmetries**
- **fundamental and empirical parameters**
- **Large scale many-body numerical methods**

Reactions

- **relevant reaction channels**
- **reaction calculation (kinematics)**
- **symmetries: unitarity**
- **decay chain couplings**
- **New reaction many-body numerical methods**

Basic Theory

$|1\rangle$ - set of "internal" A-nucleon many-body states (P-space)
$|c; E\rangle$ set of "external" many-body continuum states (Q-space)
Solve problem:

$$H|\Psi\rangle = E|\Psi\rangle$$

where

$$|\Psi\rangle = \sum_{1} x_1 |1\rangle + \sum_{c} \int dE' \, \chi^c(E')|c; E'\rangle$$

For structure physics solve for internal coefficients x_1

$$\sum_2 \left[\underbrace{\langle 1|H|2\rangle + \sum_c \int dE' \frac{\langle 1|H|c; E'\rangle\langle c; E'|H|2\rangle}{E - E' + i0}}_{\mathcal{H}_{12}(E)} - \delta_{12} E \right] x_2 = 0$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

$\langle 1|H|2\rangle$ Usual shell-model Hamiltonian involving intrinsic states

$$\langle 1|H|2\rangle = H^{\circ}_{12} + V_{12}$$

$A^c_1(E') = \langle 1|H|c;E'\rangle$ decay amplitude

$$\sum_c \int dE' \frac{A^c_1 A^{c*}_2}{E-E'} = \underbrace{\sum_{c\,(\text{all})} P\int dE' \frac{A^c_1 A^{c*}_2}{E-E'}}_{\Delta(E)} - i\pi \underbrace{\sum_{c\,(\text{open})} A^c_1 A^{c*}_2}_{W(E)/2}$$

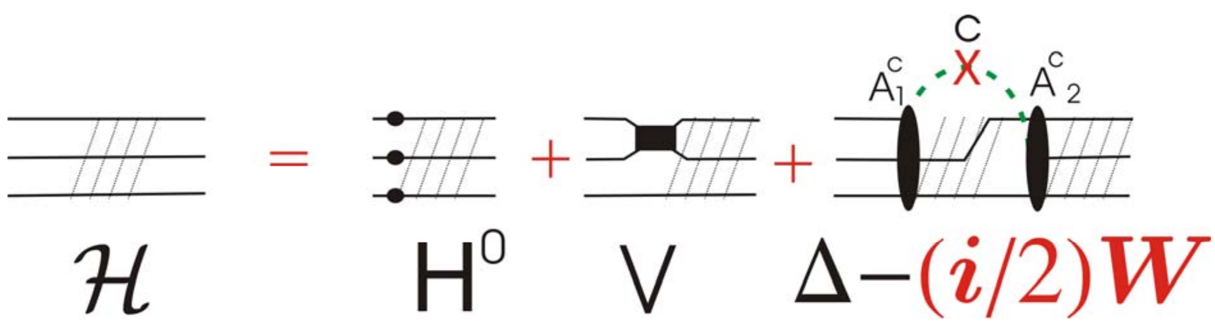
$$\mathcal{H}(E) = H^{\circ} + V + \Delta(E) - \frac{i}{2}W(E)$$

H° s.p energies

Δ interaction via continuum

V residual inteaction

W non-Hermitian - decay



Continuum Shell Model Hamiltonian

- PHP – Internal, one-body + two-body
  - "Original" shell model, adjusted, tested… (USD and others)
  - Exact shell model for bound states E<threshold
- QHQ - External, one-body (presently)
  - kinetic energy + long range interaction, (plane waves or Bessel functions, Coulomb functions)
- QHP – Cross space, one-body + two-body
  - Responsible for coupling of spaces

One-body continuum state $$|c;E\rangle_N = c_j^\dagger(\epsilon_j)|\alpha;N-1\rangle$$

Two-body continuum state $$|c;E\rangle_N = c_j^\dagger(\epsilon_j)c_{j'}^\dagger(\epsilon_j')|\alpha;N-2\rangle$$

One-body decay

- Continuum channel $$|c;E\rangle_N = c_j^\dagger(\epsilon_j)|\alpha;N-1\rangle$$
  - State α in N-1 nucleon daughter
  - Particle in continuum state j
  - Energy $E=E_\alpha+\varepsilon_j$

- Transition Amplitude

s.p. decay amplitude

$$A_1^c(E_\alpha+\epsilon_j) = a^j(\epsilon_j)\ \langle\alpha;N-1|b_j|1;N\rangle$$

Shell model s.p. transition

Important:
W-not a single particle operator

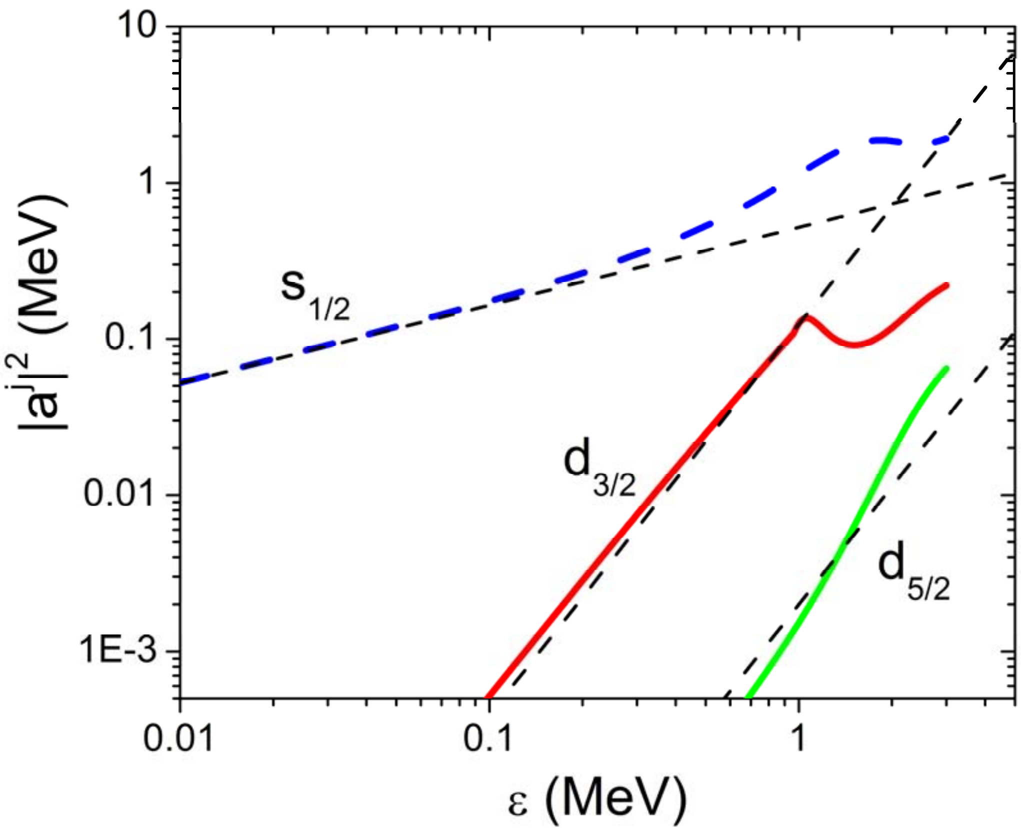
Single particle decay-Woods-Saxon potential calculation

$$a^j(\epsilon) = \langle 0|c_j(\epsilon)Vb_j^\dagger|0\rangle = \sqrt{\frac{2\mu}{\pi k}}\int_0^\infty dr F_l(r)\,V(r)\,u_l(r)$$

One-body decay

realistic one-body potential

Scattering calculation using Woods-Saxon with size parameters adjusted for ^{16}O

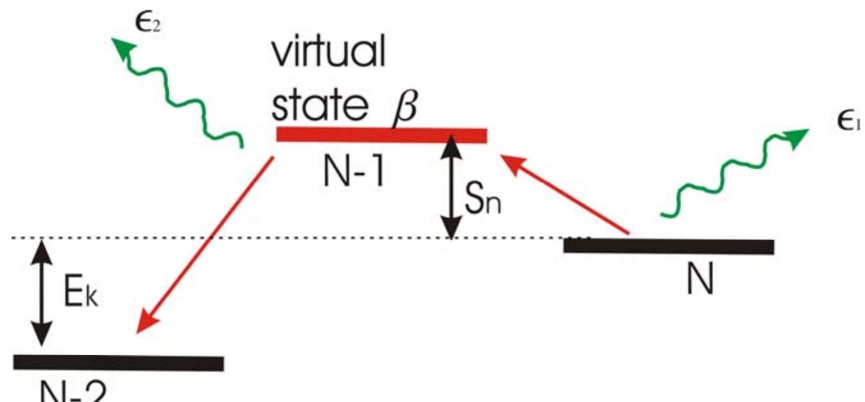


$$|a^{(j)}(\epsilon)|^2 \sim \epsilon^{l+1/2}$$

Two-body decay: sequential

Mediated by s.p. part of QHP

$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{s.p.} | 1; N \rangle$$



Virtual N-1 state from previous solution

$$|\beta; N-1\rangle = \underbrace{\sum_1 |\beta_1|1; N-1\rangle}_{\text{bound nucleus}} + \underbrace{\sum_c \int dE \chi^c(E) |c, E\rangle}_{\text{nucleon cloud}}$$

$$A \sim \sum_\beta \frac{a^{j_1}(\epsilon_1) a^{j_2}(\epsilon_2)}{E - E_\beta}$$

$$\langle 1 | W(E) | 2 \rangle_{E_k \to 0} = \delta_{12} \sum_\alpha E_k \frac{\gamma_{j_1}(E_k) \gamma_{j_2}(E_k)}{4 S_n^2} |\langle 1| \left(p^{(j_1 j_2)}\right)^\dagger |\alpha\rangle|^2$$

sum over daughter states — decay amplitude sequential one-body — Shell model pair removal amplitude

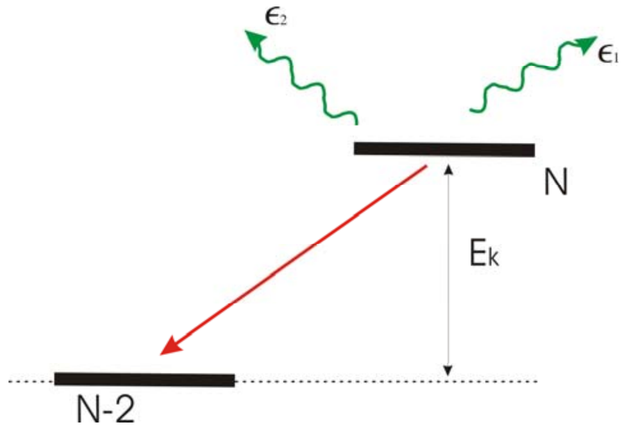
$$E_k = \epsilon_1 + \epsilon_2$$

Width at low energy

$$\Gamma(E_k) \sim W(E_k) \sim E_k^{l_1 + l_2 + 2}$$

Two-body decay: Direct

Mediated by 2-body part of QHP



$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{\mathrm{2body}} | 1; N \rangle$$

$$A^c_1(E) = a^{(j_1 j_2)}(\epsilon_1, \epsilon_2)\ \langle \alpha; N-2 | p_L^{(j_1 j_2)} | 1; N \rangle\ ,$$

two-body decay amplitude

Shell model pair removal amplitude

V(r,r') – effective pair potential

$$a^{(j_1 j_2)}(\epsilon_1 \epsilon_2) = \frac{2\mu}{\pi\sqrt{k_1 k_2}} \int_0^\infty dr dr' F_{j_1}(r) F_{j_2}(r')\, V(r,r')\, u_{j_1}(r) u_{j_2}(r')$$

Width at low energy $\Gamma(E_k) \sim W(E_k) \sim E_k^{l_1+l_2+2}$

Non-Hermitian eigenvalue problem | **Reaction calculation**

Traditional Shell Model

Hermitian part of interaction

$$H = H_{\mathcal{PP}} + \Delta$$

Unperturbed Green's function

$$G(E) = \frac{1}{E - H_{\mathcal{PP}} - \Delta(E)}.$$

Inclusion of decay channels

Full effective Hamiltonian

$$\mathcal{H}(E) = H_{\mathcal{PP}} + \Delta(E) - i\pi \mathbf{A}\mathbf{A}^\dagger$$

Full Internal propagator

$$\mathcal{G}(E) = \frac{1}{E - \mathcal{H}}$$

Results

- Matrix diagonalization
- Hermitian eigenvalues below thresholds
- Non-Hermitian eigenvalues above threshold
    - resonances and widths (need definition)

- R-matrix $R = \mathbf{A}^\dagger G \mathbf{A}$
- Dyson Equation

$$\mathcal{G} = G - (i/2) G W \mathcal{G}$$

- Transition matrix and cross section

$$T = \frac{R}{1 + i\pi R}, \qquad S = \frac{1 - i\pi R}{1 + i\pi R}$$

Complex Energy eigenvalue problem

Eigenvalue problem $$\mathcal{H}(E)|\alpha\rangle = E|\alpha\rangle$$

has only complex (E>threshold) roots but E is real?

Definitions of resonance

- Gamow: poles of scattering matrix $\mathcal{H}(\mathcal{E})|\alpha\rangle = \mathcal{E}|\alpha\rangle$

  Eigenvalue problem with regular w.f. inside outgoing outside boundary condition $\rightarrow$ discrete resonant states + complex energies $\mathcal{E}$

$$E_{\text{res}} = Re(\mathcal{E}) \qquad \Gamma_{\text{res}} = -2Im(\mathcal{E})$$

- Breit-Wigner: Find roots on real axis

$$Re\left[\mathcal{H}(E_{\text{res}})\right] = E_{\text{res}} \qquad \Gamma_{\text{res}} = -2Im\left[\mathcal{H}(E_{\text{res}})\right]$$

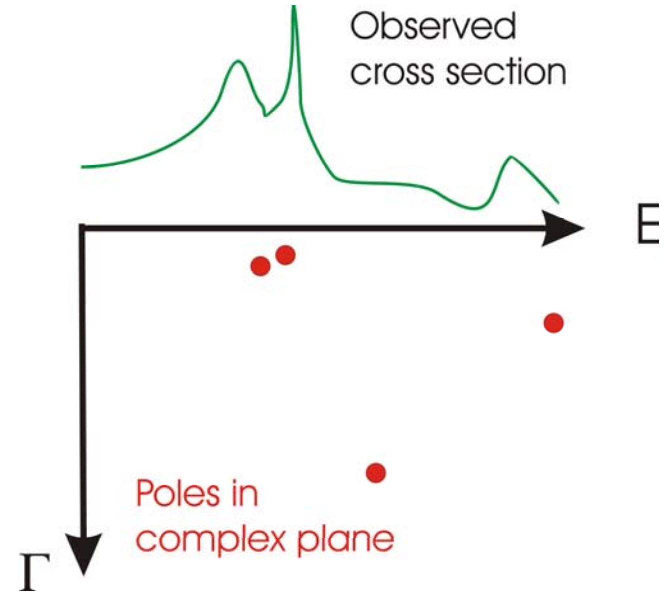
- Cross section peaks and lifetimes

$$\left.\frac{d^2\delta_l(E)}{dE^2}\right|_{E=E_{\text{res}}} = 0 \qquad \frac{2}{\Gamma_{\text{res}}} = \left.\frac{d\delta_l(E)}{dE}\right|_{E=E_{\text{res}}}$$

Interpretation of complex energies

- For isolated narrow resonances all definitions agree
- Real Situation
  - Many-body complexity
  - High density of states
  - Large decay widths
- Result:
  - Overlapping, interference, width redistribution
  - Resonance and width are definition dependent
  - Non-exponential decay
- Solution: Cross section is a true observable

  (S-matrix)



$$S^{ab}(E) = s_a^{1/2} \left\{ \delta^{ab} - \sum_{12} A_1^{a*} \left(\frac{1}{E - \mathcal{H}} \right)_{12} A_2^b \right\} s_b^{1/2}.$$

Non-Hermitian eigenvalue problem **Reaction calculation**

Traditional Shell Model

Hermitian part of interaction

$$H = H_{\mathcal{P}\mathcal{P}} + \Delta$$

Unperturbed Green's function

$$G(E) = \frac{1}{E - H_{\mathcal{P}\mathcal{P}} - \Delta}.$$

Inclusion of decay channels

Full effective Hamiltonian

$$\mathcal{H}(E) = H_{\mathcal{P}\mathcal{P}} + \Delta(E) - i\pi \mathbf{A}\mathbf{A}^\dagger$$

Full Internal propagator

$$\mathcal{G}(E) = \frac{1}{E - \mathcal{H}(E)}$$

Results

- Matrix diagonalization
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- R-matrix $R(E) = \mathbf{A}^\dagger G \mathbf{A}$
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Calculation Details, Propagator-Strength Function

$$G(E)|\lambda\rangle = \frac{1}{E-H} = -i\int_0^\infty dt\,\exp(iEt)\exp(-iHt)|\lambda\rangle$$

- Scale Hamiltonian so that eigenvalues are in [-1 1]
- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2-\delta_{n0})\, J_n(t)\, T_n(H)$$

- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle, \quad \text{and } |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

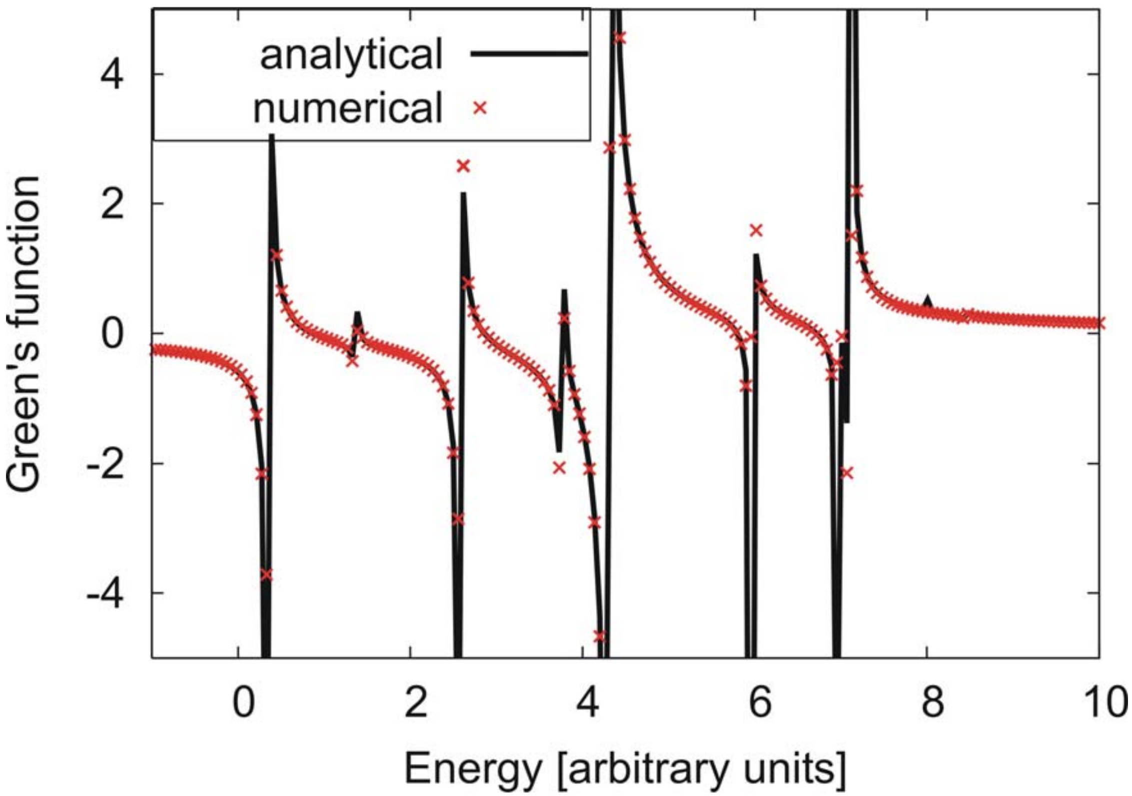
$$\langle\tilde{\lambda}|\lambda_{m+n}\rangle = 2\langle\tilde{\lambda}_m|\lambda_n\rangle - \langle\tilde{\lambda}|\lambda_{n-m}\rangle\,, \quad n \geq m$$

- Use FFT to find return to energy representation

T. Ikegami and S. Iwata, J. of Comp. Chem. **23** (2002) 310-318

Green's function calculation

- Advantages of the method
- -No need for full diagonalization or inversion at different E
- -Only matrix-vector multiplications
- -Numerical stability*



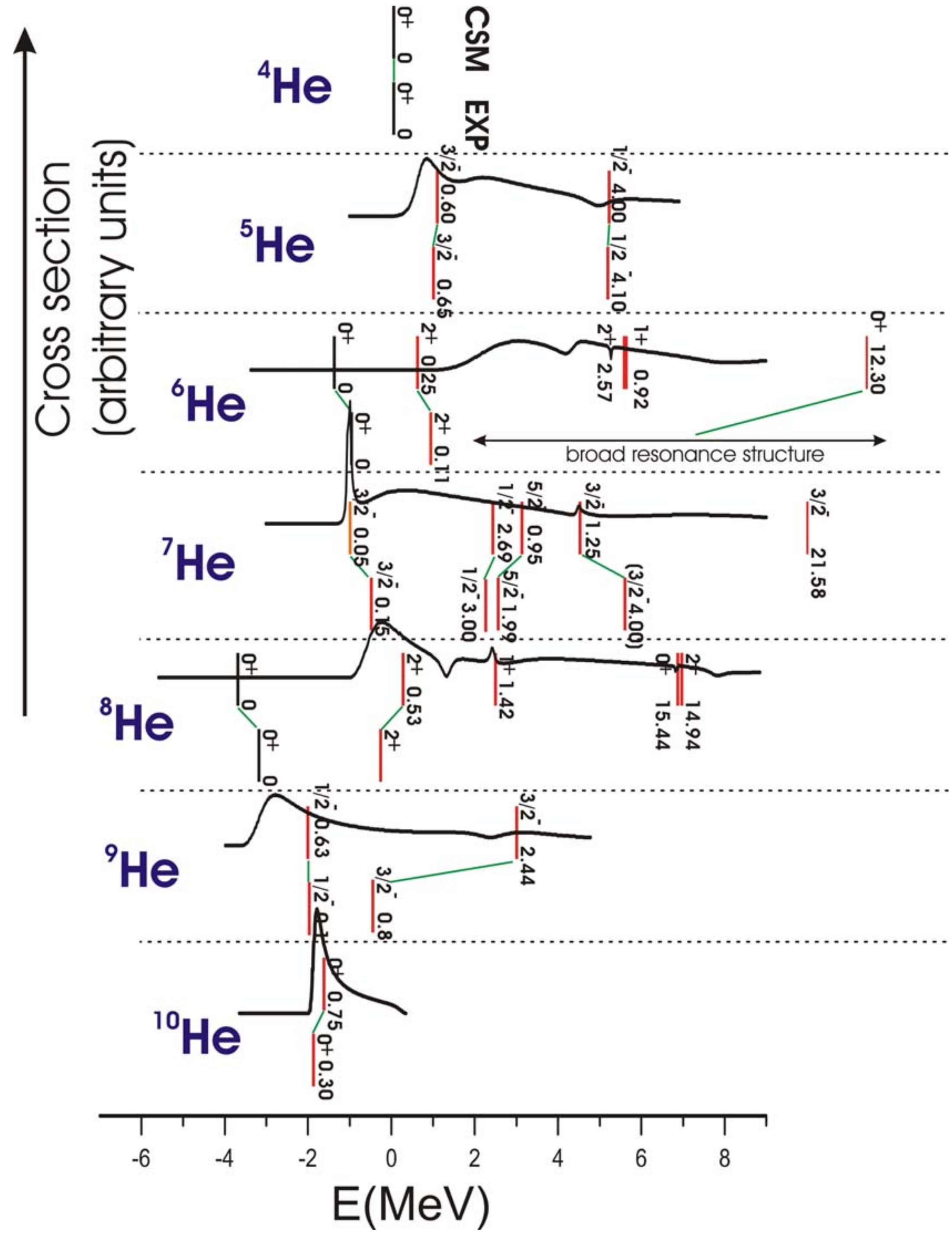
*W.Press, S. Teukolsky, W. Vetterling, B. Flannery, Numerical Recipes in C++ the art of scientific computing, Cambrige University Press, 2002

Realistic Shell Model Example

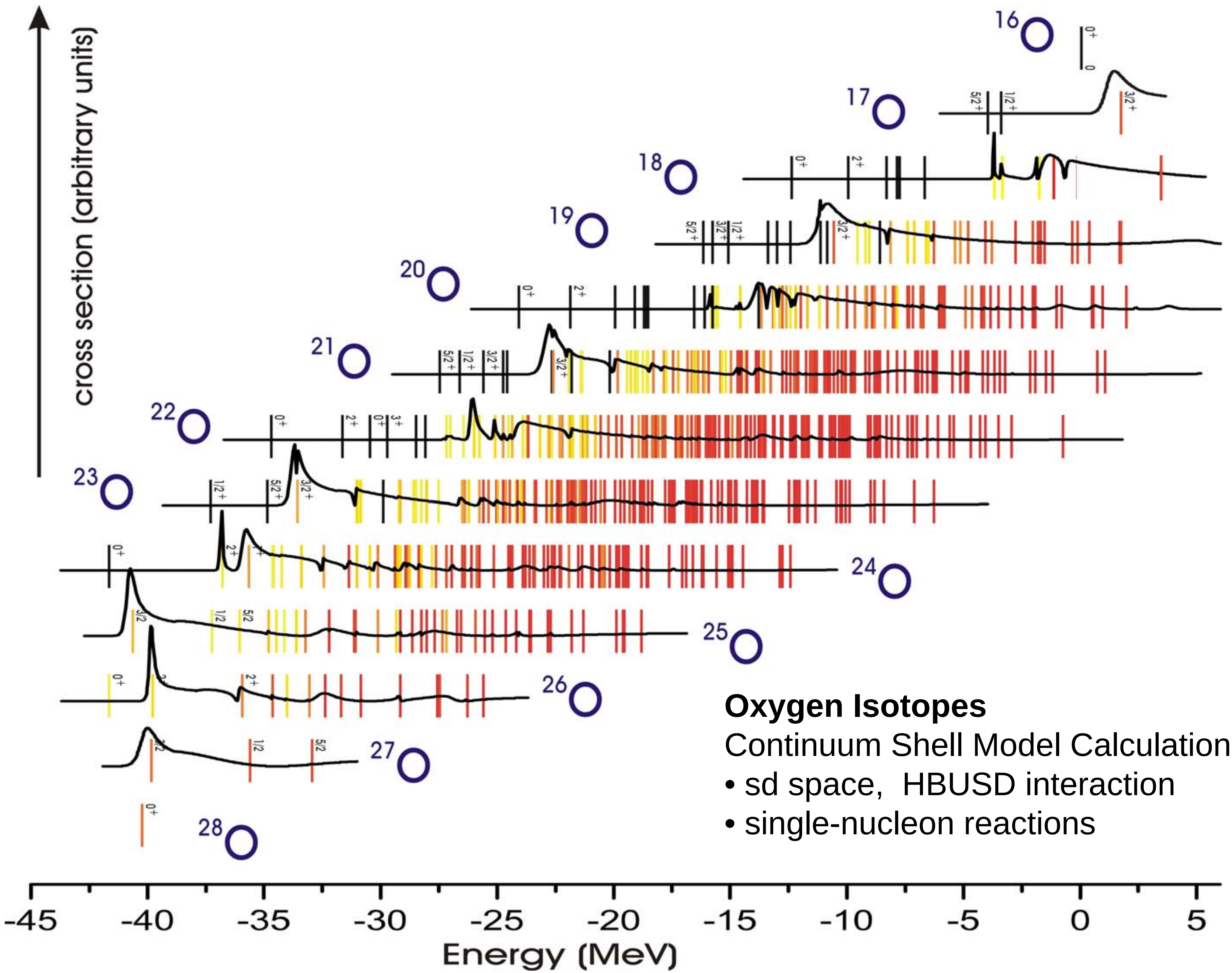
- Interaction
  - PHP – Shell model Hamiltonian, USD interaction
  - Assume that USD includes Hermitian Δ
  - QHQ and QHP one-body Woods-Saxon potential
  - QHP two-body phenomenological parameterization
- Solution
  - From ^{4}He up to ^{10}He
  - From ^{16}O up to ^{28}O
  - Given guess energy E for state α, W(E) is constructed by considering all open channels.
  - Non-Hermitian Hamiltonian is solved for iteratively for new E (Breit-Wigner resonance condition)

Continuum Shell Model He isotopes

•Cross section and structure within the same formalism
•Reaction l=1 polarized elastic channel



References
[1] A. Volya and V. Zelevinsky, Phys. Rev. Lett 94 (2005) 052501.
[2] A. Volya and V. Zelevinsky, Phys. Rev. C 67 (2003) 54322



Oxygen Isotopes

Continuum Shell Model Calculation

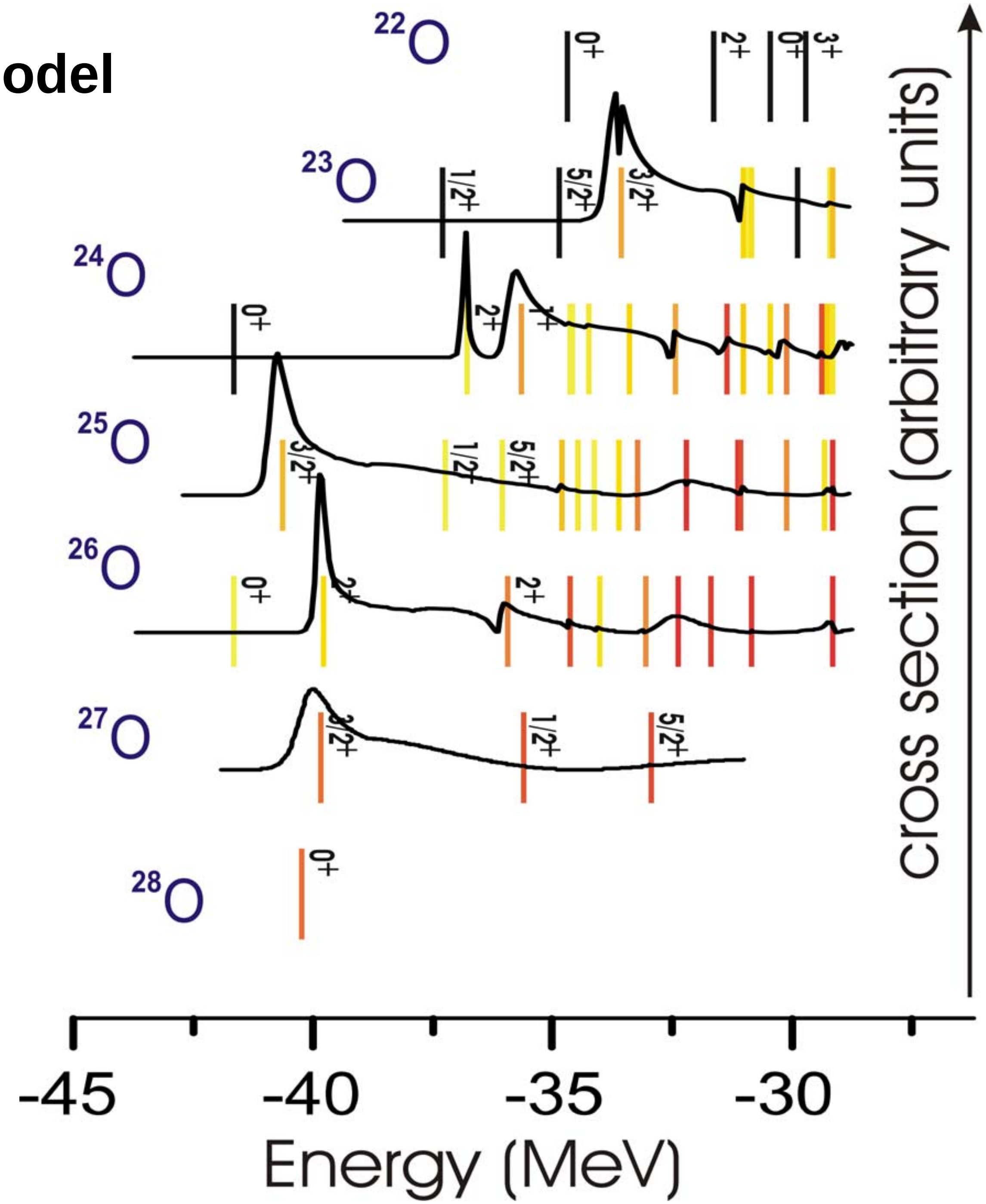
- sd space, HBUSD interaction
- single-nucleon reactions

CSM oxygen results

A	j	mode	EXP	Q	Γ	theory	E	Q	Γ
17	$3/2^+$	$\gamma\, n$	5.085	0.941	96	WS	4.5	1.0	122
18	2^+	$\gamma\, \alpha\, n$	8.213	0.169	1 ± 0.8	USD	9.465	1.242	200
18	1^+	$\alpha\, n$	8.817	0.773	70 ± 12	USD	10.823	2.600	85
18	4^+	$\gamma\, \alpha\, n$	8.955	0.911	43 ± 3	USD	8.750	0.526	28
19	$5/2^+$	n	5.148	1.191	3.4 ± 1	USD	5.011	1.121	5.1
19	$9/2^+$		5.384	1.427	~ 0	USD	5.175	1.282	0
19	$3/2^+$	n	5.54	1.58	320	USD	5.529	1.636	290
19	$7/2^+$	n	6.466	2.509	small	USD	6.880	0.808*	63
24	2^+	n	?	?	?	HBUSD	4.850	0.489	18
26	0+	$2n$	0	?	?	HBUSD	0	0.021	0.02
28	0+	$2n$	0	?	?	HBUSD	0	0.345	14

Lowest resonant states in the chain of oxygen isotopes. The experimental data on the left, (EXP) - energy of the state (MeV), Q - energy above threshold (MeV), γ - width (keV), are compared to the theoretical results on the right. The decay mode in the second column indicates the decay branches assumed by experimentalists (NNDC)

Continuum Shell Model
Oxygen Isotopes



Narrow resonance-spectroscopic factor approximation for sequential decay

SM spectroscopic factors

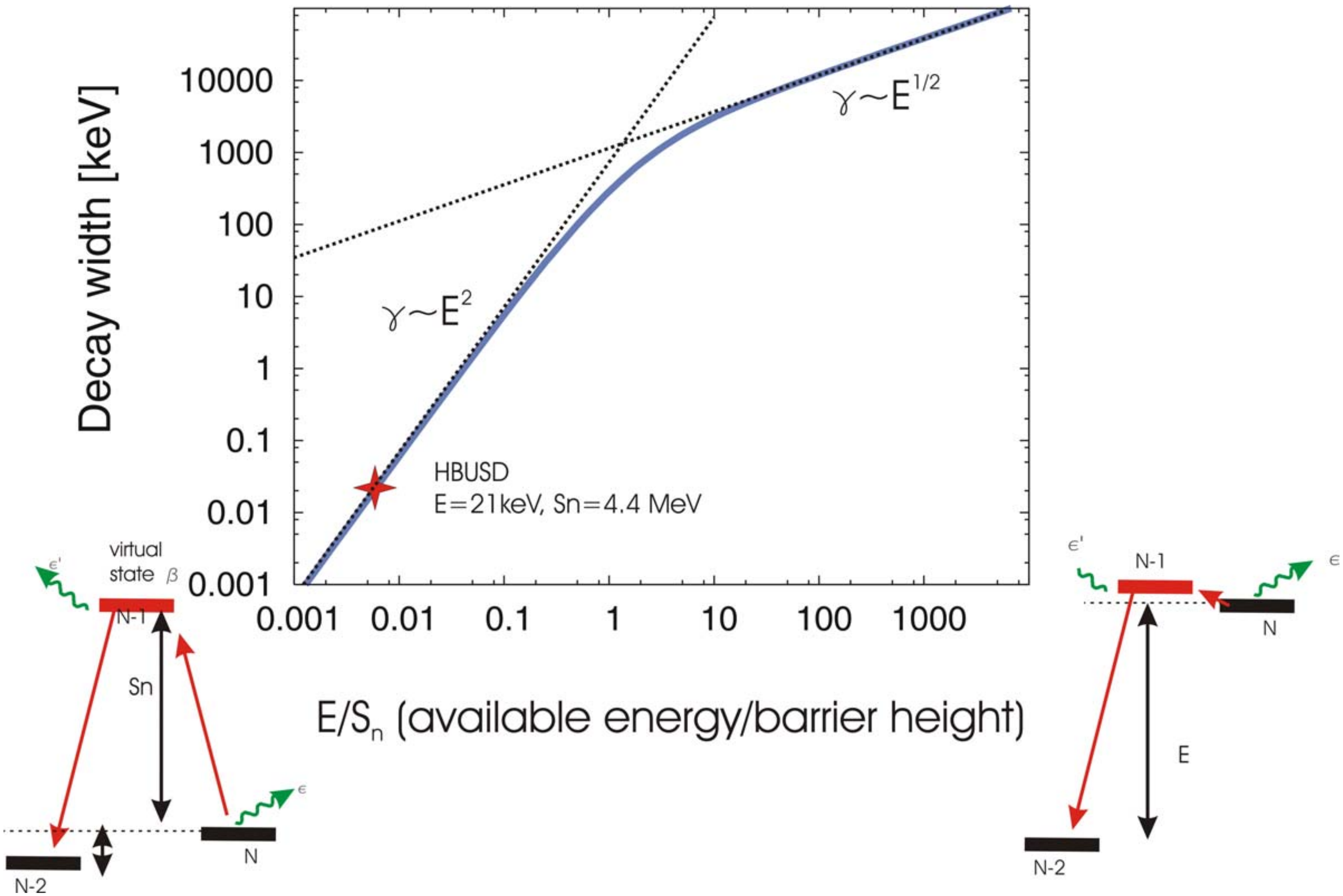
$$\Gamma = \frac{1}{2\pi} \int d\epsilon d\epsilon' \, \delta(E_t - \epsilon - \epsilon') \sum_{\beta\beta' j j'} \gamma_j(\epsilon)\gamma_{j'}(\epsilon') \left(\frac{\Upsilon^{\mathrm{d}}}{(S_\beta + \epsilon)^2} + \frac{\Upsilon^{\mathrm{x}}}{(S_\beta + \epsilon')(S'_\beta + \epsilon)} \right)$$

phase-space volume integral

single particle decay widths

energy denominator

Kinematics of sequential 2-body s-wave decay. Example: $^{26}O \rightarrow 2n + ^{24}O$



Interplay of collectivities

Definitions

n - labels particle-hole state

ε_n – excitation energy of state n

d_n - dipole operator

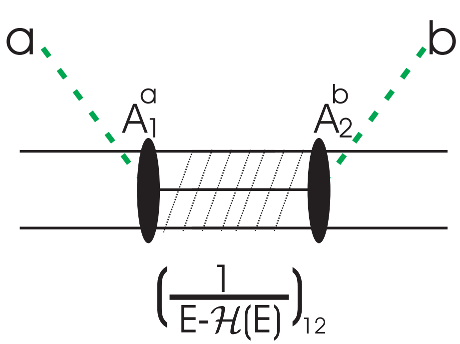
A_n – decay amplitude of n

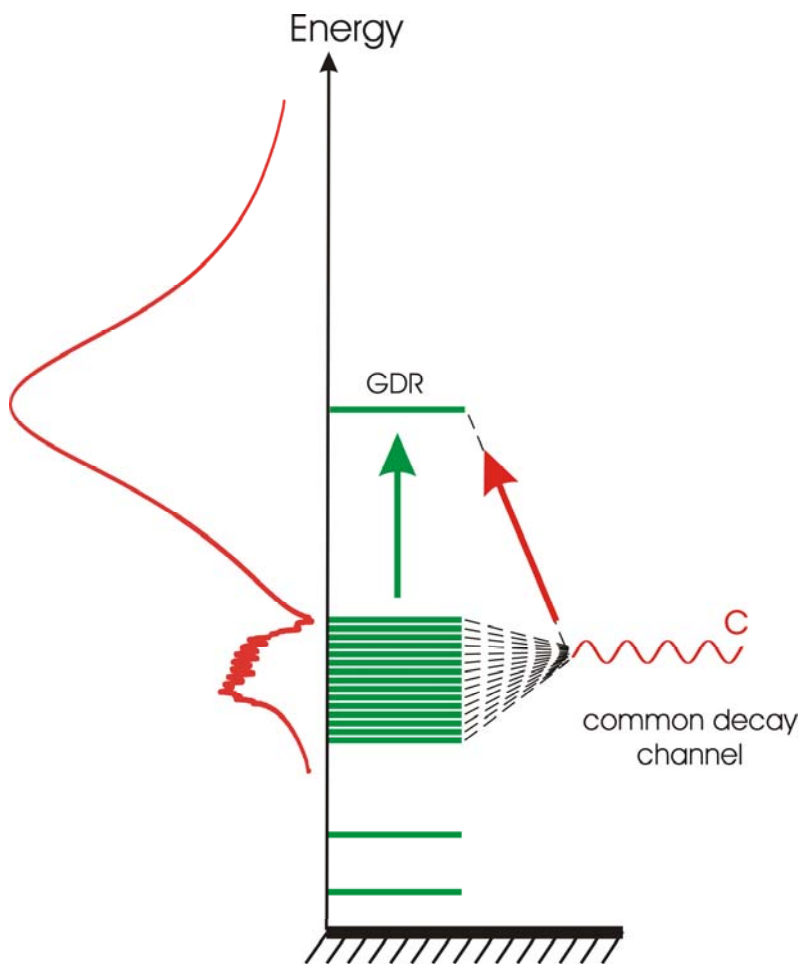
Model Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

Driving GDR externally

(doing scattering)



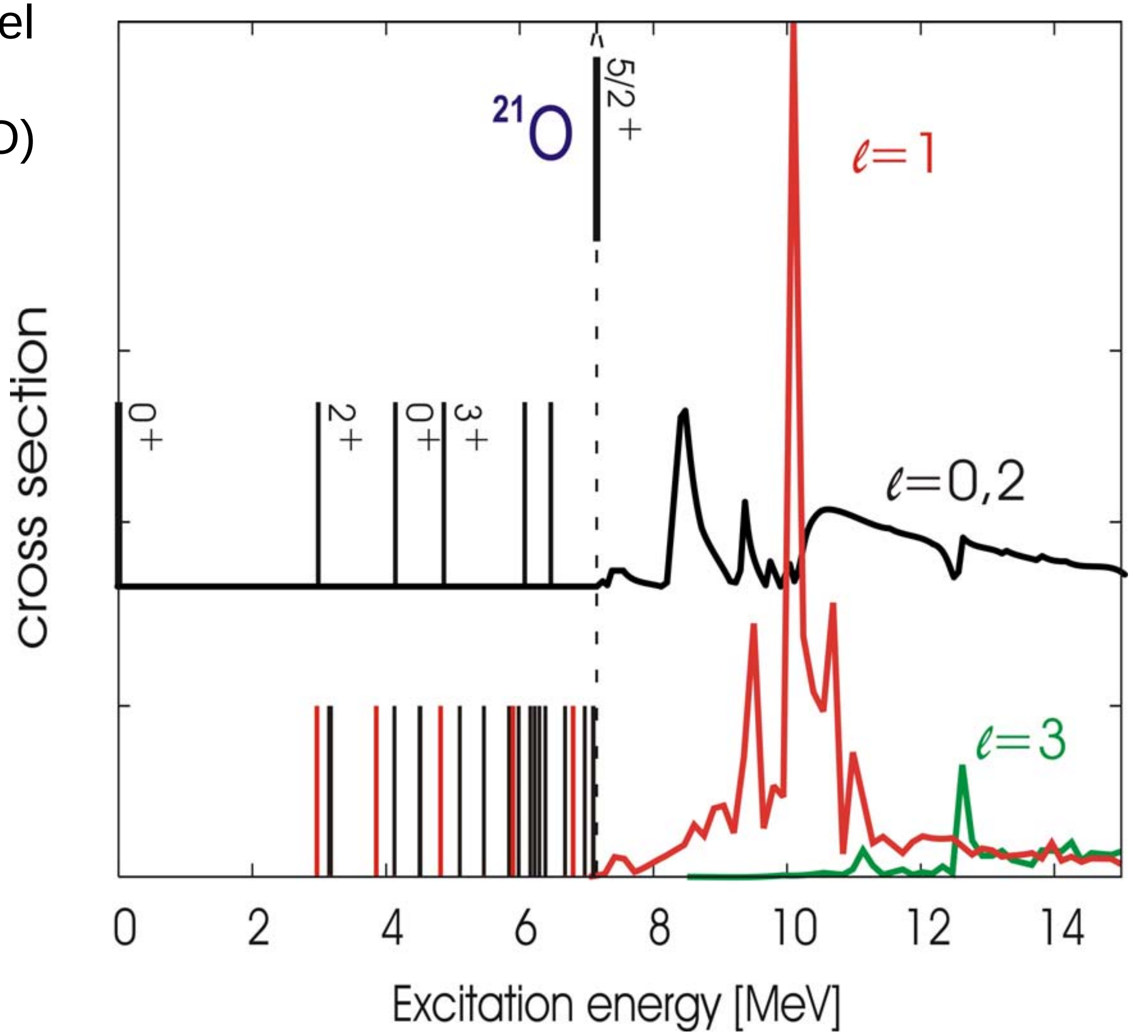


Everything depends on
angle between multi dimensional vectors
A and d

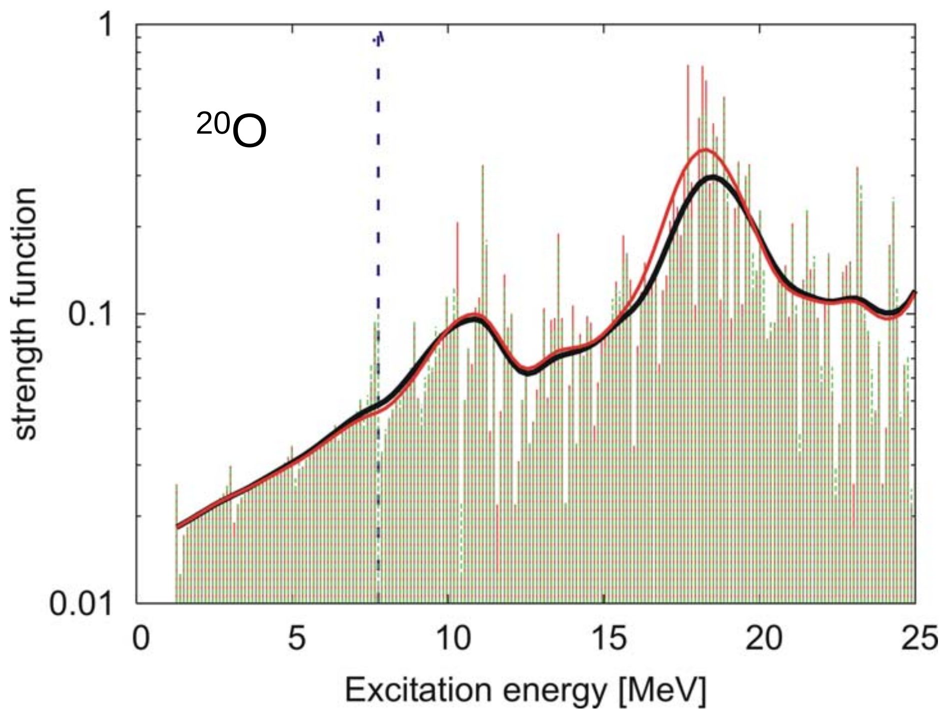
States and cross section in ^{22}O

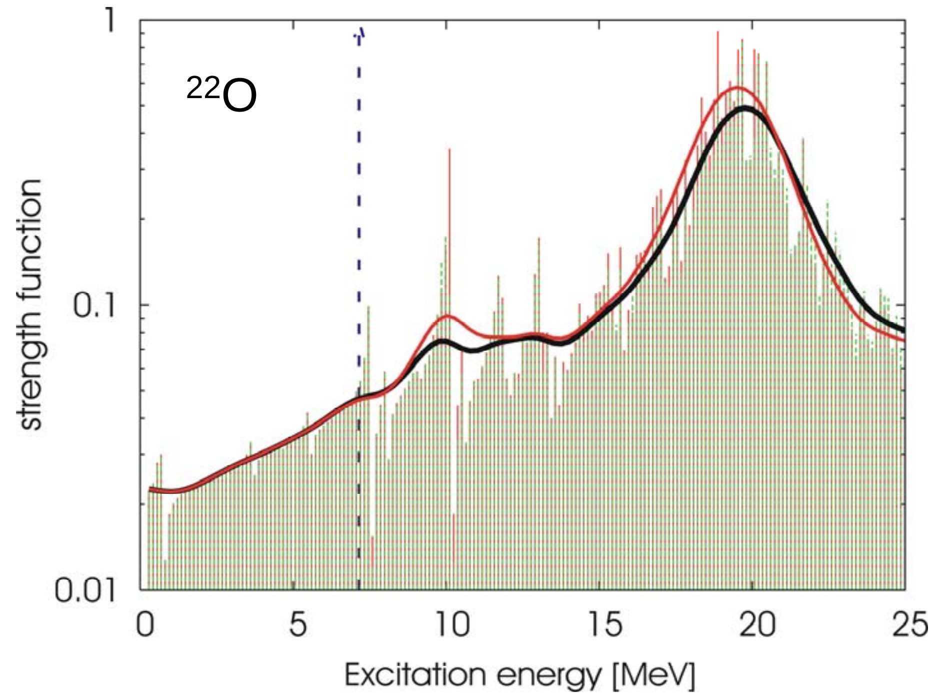
Parameters of the model

- Internal shell s-p-sd-pf shell model with WBP interaction
- Decay channels g.s. of ^{19}O(or ^{21}O) + neutron decay from fp
- EM channel: E1 strength from ^{20}O(or ^{22}O)



Isovector Dipole strength in Oxygen





Summary:

- New theoretical techniques
- First applications and success stories
- Interplay of internal and external, generic features
- Technical problems and solutions
- Toward realistic applications

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Recent publications:

•nucl-th/0509051
•Phys. Rev. Lett. 94, (2005) 052501
•Phys. Lett. B590, (2004) 45
•Phys. Rev. C **67**, (2003) 054322
•J. Opt. B **5**, (2003) S450