

Two-photon exchange in elastic e-p scattering: partonic picture

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Outline

Motivation

General scattering amplitude in elastic e-N scattering

Partonic calculation of two-photon exchange contribution at large Q^2

Result and other extensions

Summary

Motivation

Why we interested in two-photon physics

Starting from the electric and magnetic form factors (G_E & G_M)

which are defined by the electromagnetic current J^μ :

$$\langle N(p') | J^\mu(0) | N(p) \rangle = e \bar{u}(p') \left[F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu}(p' - p)_\nu}{2M} \right] u(p),$$

$$G_M(Q^2) = F_1 + F_2$$

$$G_E(Q^2) = F_1 - \tau F_2 = G_M - (1 + \tau) F_2$$

then the differential cross section for e-N scattering is given by:

$$\left(\frac{d\sigma}{d\Omega} \right)_{Born} = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \frac{1}{1 + \tau} \left[G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right],$$

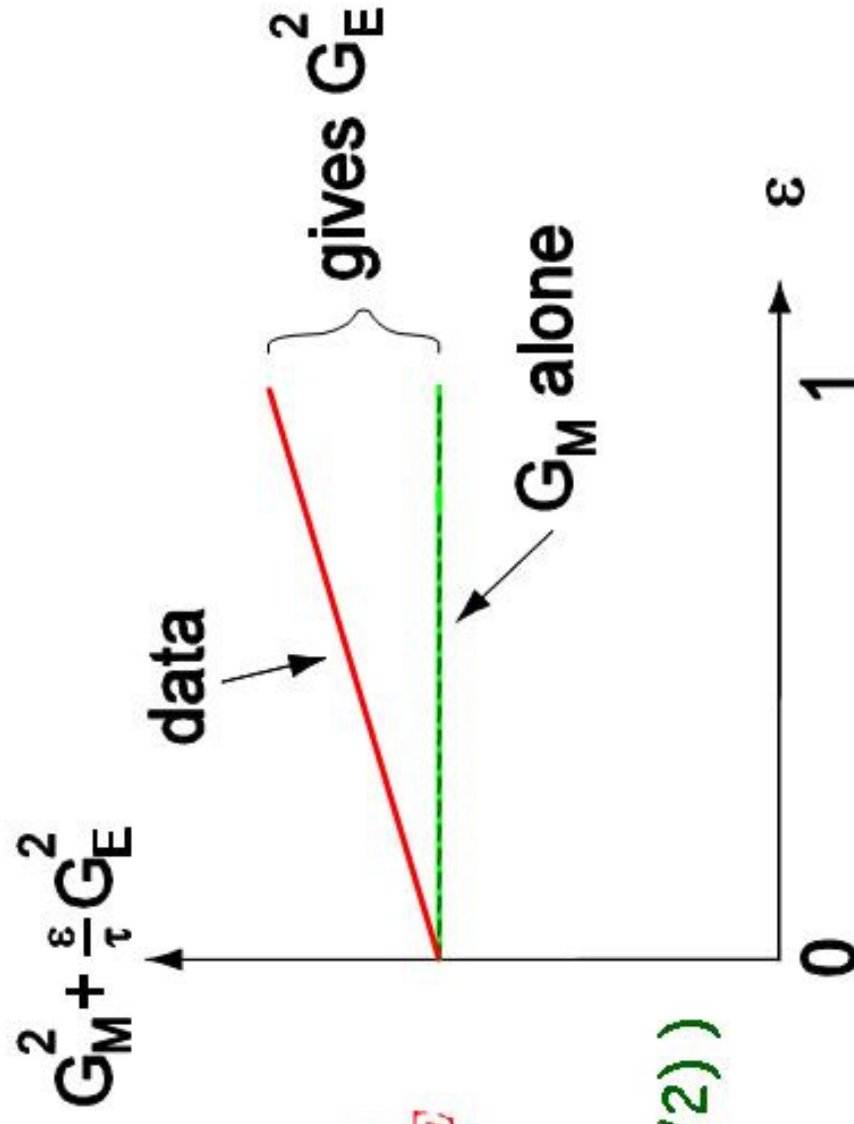
Rosenbluth separation method (LT)

reduced cross section:

$$\sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

at a fix Q^2 , vary ε

$$(1/\varepsilon = 1 + 2(1 + \tau) \tan^2(\theta_{lab}/2))$$

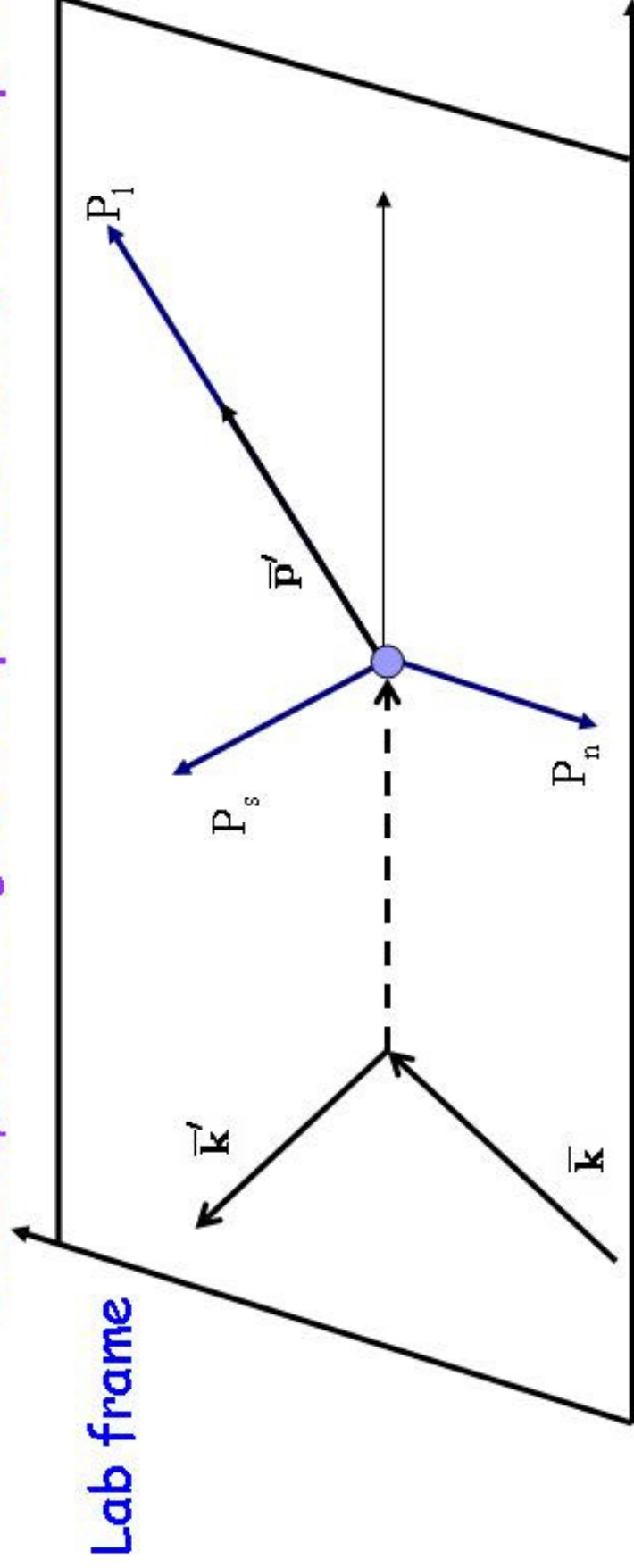


in the elastic e-p scattering, one gets:

$$G_M(Q^2)/\mu_p \cong G_E(Q^2) \cong \frac{1}{(1 + Q^2/0.71 \text{GeV}^2)^2} = G_D(Q^2)$$

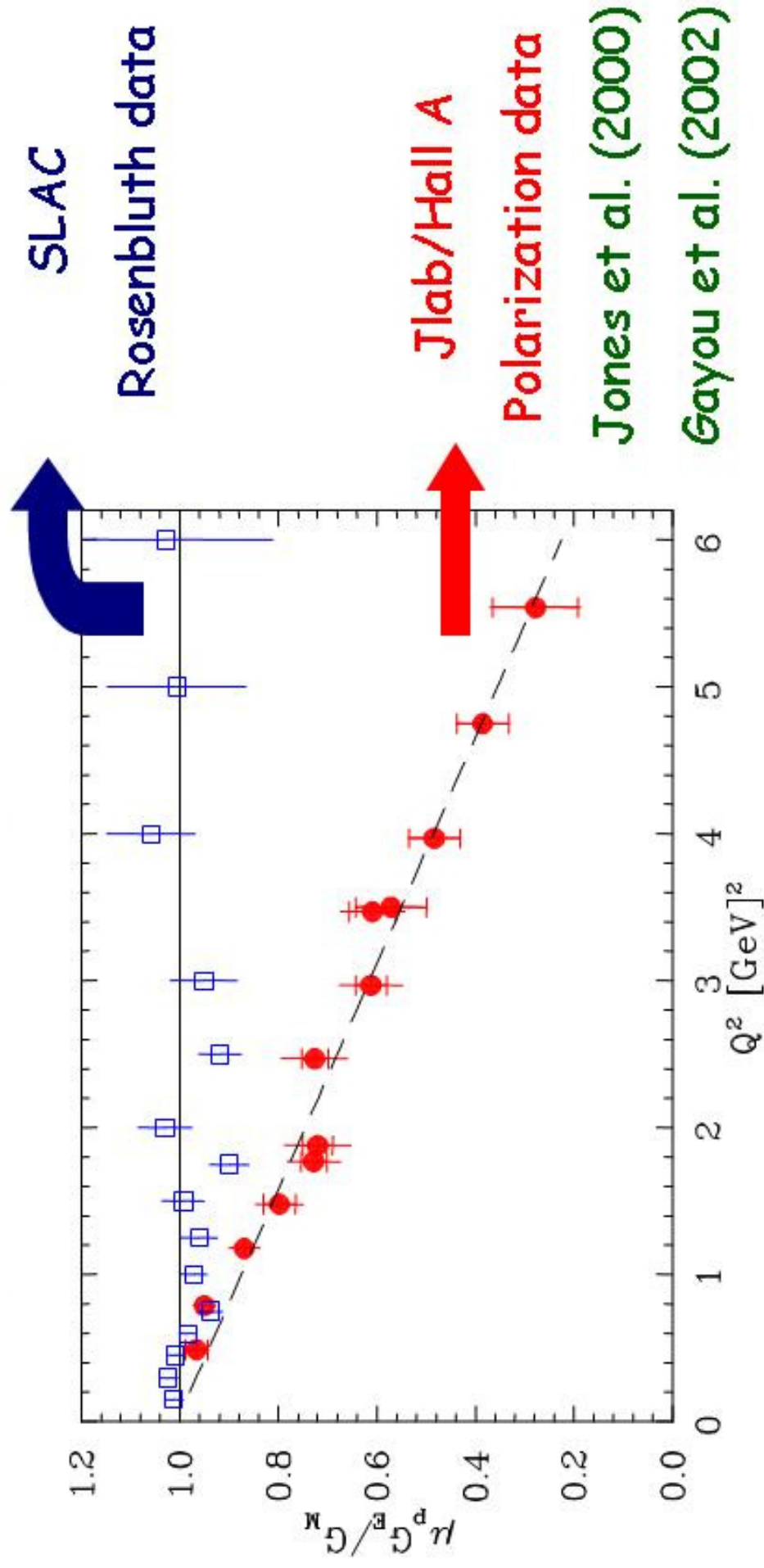
Polarization transfer method

→ sideway and longitudinal polarization for recoil proton
Polarized electron beam



$$\frac{P_s}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \frac{G_E}{G_M}$$

Two independent measurement of $R(G_E/G_M)$

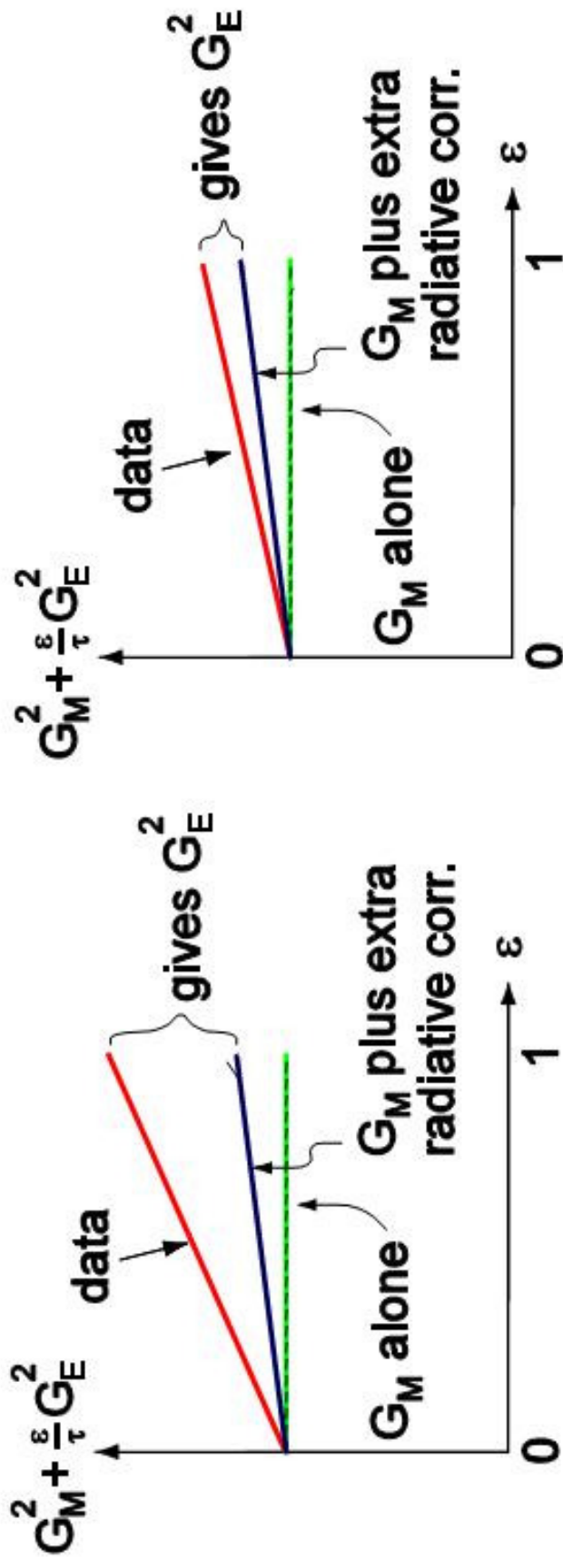


Two methods, two different results!

Speculation : missing radiative corrections

Speculation : there are radiative corrections to Rosenbluth experiments that are important and are not included

missing correction : linear in ϵ , not strongly Q^2 dependent



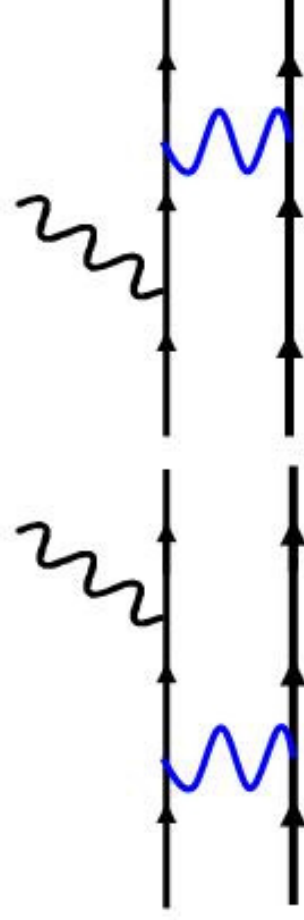
Low τ (Low Q^2)

High τ (High Q^2)

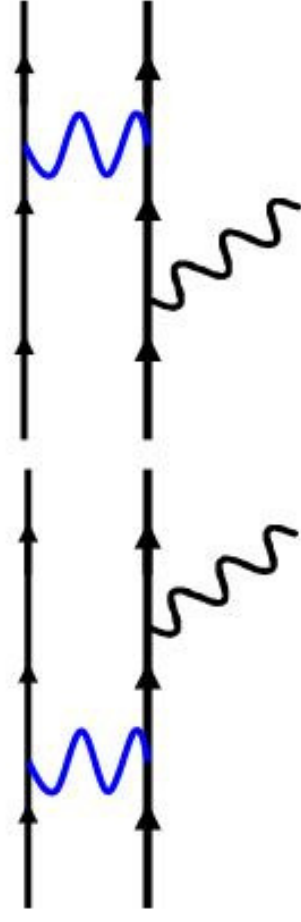
G_E term is proportionally smaller at large Q^2

effect is more visible at large Q^2

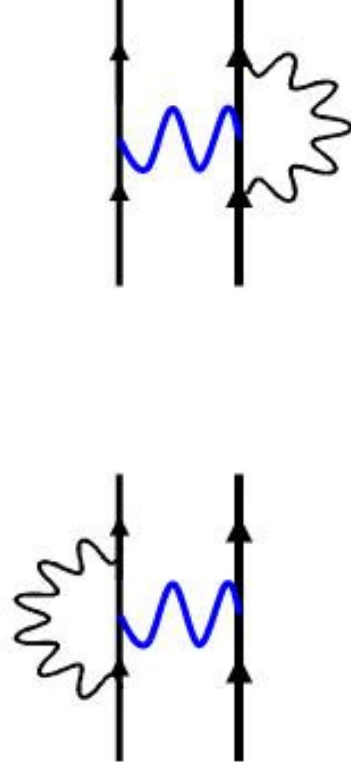
Radiative correction diagrams



bremsstrahlung



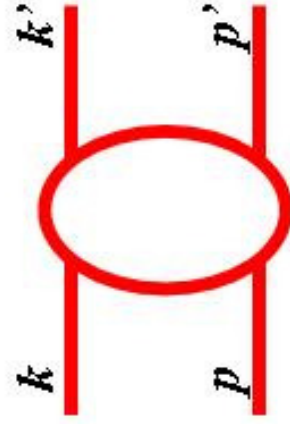
vertex corrections



2 photon exchange box diagrams



General scattering amplitude in elastic e-N scattering



$$I(k) + N(p) \rightarrow I(k') + N(p'),$$

$$P \equiv \frac{p+p'}{2}, K \equiv \frac{k+k'}{2}, s = (p+k)^2, u = (p-k)^2,$$

$$q = k - k' = p' - p, Q^2 = -q^2, \nu = (K \cdot P),$$

$$p^2 = p'^2 = M^2, k^2 = k'^2 = m_e^2 \ll M^2, s, u, Q^2.$$

For a theory respects Lorentz, parity and charge conjugation invariance, the elastic electron-nucleon scattering amplitude can be expanded in terms of **six independent Lorentz structure**, and one can separate the elastic electron-nucleon scattering amplitude into:

$$T = T^{non-flip} + T^{flip}$$

where

$$\begin{aligned}
 T_{h'\lambda'_N, h\lambda_N}^{non-flip} &= \frac{e^2}{Q^2} \bar{u}(k', h') \gamma_\mu u(k, h) \\
 &\times \bar{u}(p', \lambda'_N) \left(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \right) u(p, \lambda_N) \\
 T_{h'\lambda'_N, h\lambda_N}^{flip} &= \frac{e^2}{Q^2} \frac{m_l}{M} \left[\bar{u}(k', h') u(k, h) \cdot \bar{u}(p', \lambda'_N) \left(\tilde{F}_4 + \tilde{F}_5 \frac{\gamma \cdot K}{M} \right) u(p, \lambda_N) \right. \\
 &\quad \left. + \tilde{F}_6 \bar{u}(k', h') \gamma_5 u(k, h) \cdot \bar{u}(p', \lambda'_N) \gamma_5 u(p, \lambda_N) \right]
 \end{aligned}$$

In one-photon-exchange approximation, the phases and all the F_{3-6} terms vanished, they must originate from process involving at least the change of **two-photon**. Similarly, define

$$\begin{aligned}
 \tilde{G}_E &\equiv \tilde{G}_M - (1 + \tau) \tilde{F}_2 & \tilde{G}_M(\nu, Q^2) &= G_M(Q^2) + \delta \tilde{G}_M \\
 \tilde{G}_E(\nu, Q^2) &= G_E(Q^2) + \delta \tilde{G}_E & \tilde{F}_2(\nu, Q^2) &= F_2(Q^2) + \delta \tilde{F}_2 \\
 & & \tilde{F}_3(\nu, Q^2) &= 0 + \delta \tilde{F}_3
 \end{aligned}$$

Observables including two-photon exchange

Real parts of two-photon amplitudes

$$\begin{aligned}
 \uparrow \sigma_R &= G_M^2 \left(1 + 2 \frac{\mathcal{R}(\delta\tilde{G}_M)}{G_M} \right) \\
 &+ \varepsilon \left\{ \frac{1}{\tau} G_E^2 \left(1 + 2 \frac{\mathcal{R}(\delta\tilde{G}_E)}{G_E} \right) + 2G_M^2 \left(1 + \frac{1}{\tau} \frac{G_E}{G_M} \right) \frac{\nu}{M^2} \frac{\mathcal{R}(\tilde{F}_3)}{G_M} \right\} \\
 &+ \mathcal{O}(e^4) \qquad Y_{2\gamma}
 \end{aligned}$$

$$\begin{aligned}
 \uparrow \frac{P_s}{P_t} &= -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \left\{ \frac{G_E}{G_M} \left(1 - \frac{\mathcal{R}(\delta\tilde{G}_M)}{G_M} \right) + \frac{\mathcal{R}(\delta\tilde{G}_E)}{G_M} \right\} \\
 &+ \left(1 - \frac{2\varepsilon}{1+\varepsilon} \frac{G_E}{G_M} \right) \frac{\nu}{M^2} \frac{\mathcal{R}(\tilde{F}_3)}{G_M} \qquad Y_{2\gamma} \\
 &+ \mathcal{O}(e^4)
 \end{aligned}$$

Observables including two-photon exchange

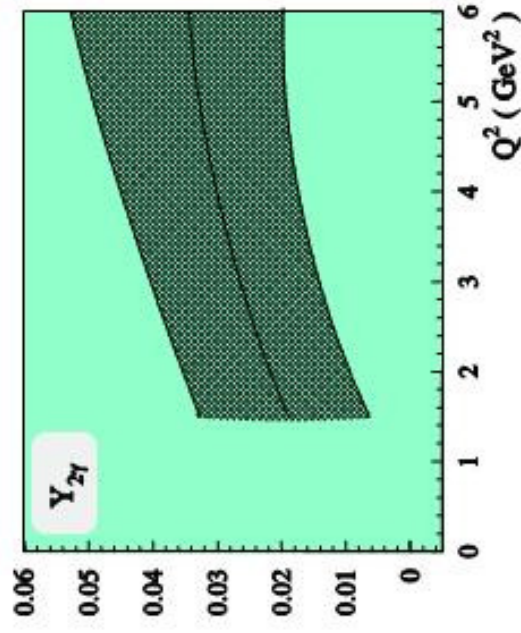
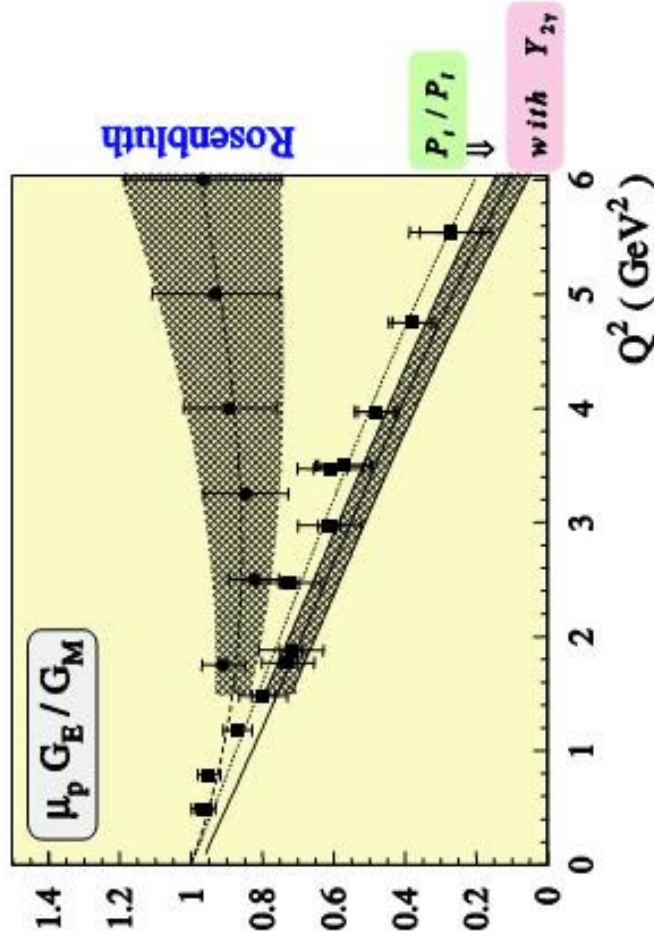
Real parts of two-photon amplitudes

$$(R_{\text{Rosenbluth}}^{\text{exp}})^2 \simeq \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2 \left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma}$$

effect is more visible at large $Q^2(\tau)$

$$(R_{\text{Polarization}}^{\text{exp}}) \simeq \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\varepsilon}{1+\varepsilon} \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma}$$

effect is small as $Y_{2\gamma}$ is small

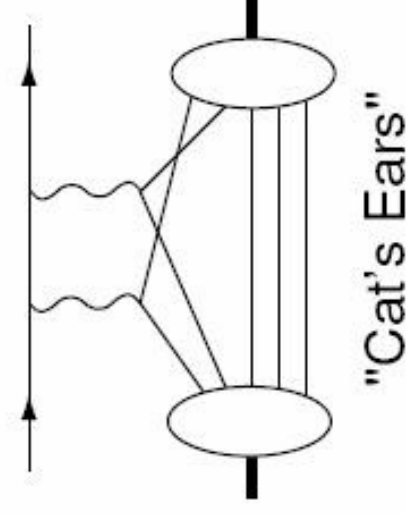
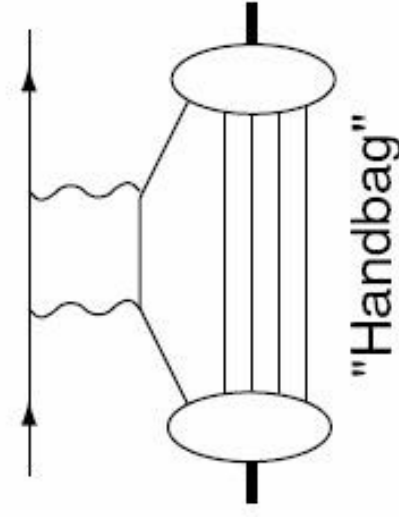


2-photon exchange is a
 candidate to explain the
 discrepancy between both
 experimental methods

P. Guichon and M.
 Vanderhaeghen,
 (2003)

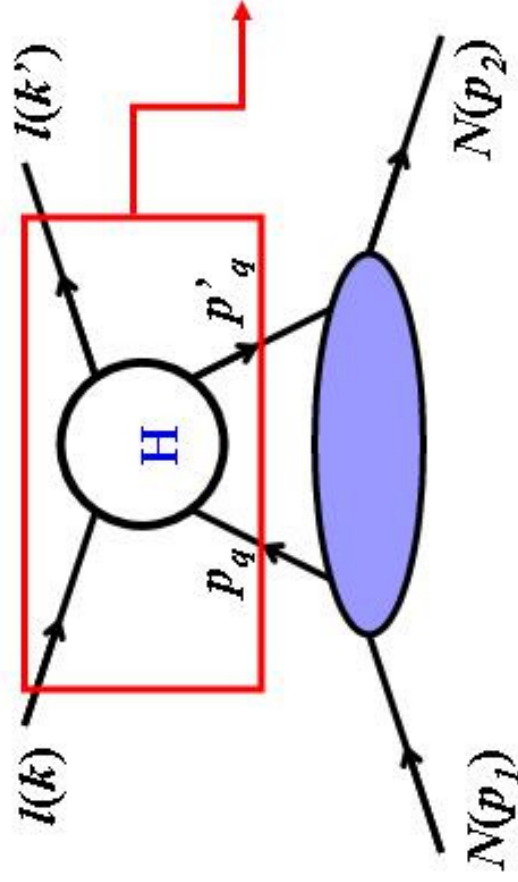
Partonic calculation of two-photon exchange contribution at large Q^2

To estimate δG_M , δF_2 , and F_3 at large Q^2 , we start from calculating the elastic e-q scattering with massless quarks.

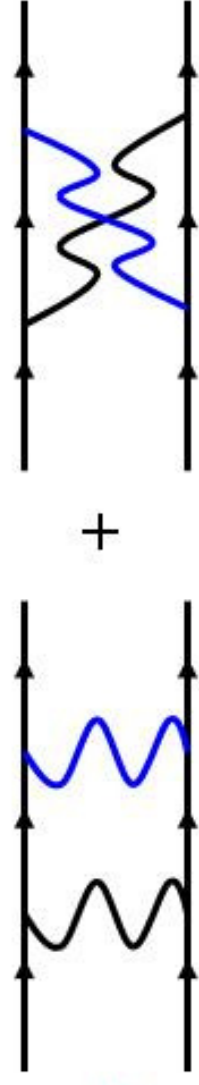


- Main contributions comes from "handbag diagrams" when both photons are hard at large Q^2 .
- "Cat's ears" diagrams is important for getting over all IR divergence correct.

hard scattering amplitude



$$l(k) + q(p_q) \rightarrow l(k') + q(p'_q)$$



S_{Direct} and S_{Cross}

$$H_{h,\lambda} = \frac{(ee_q)^2}{Q^2} \bar{u}(k', h) \gamma_\mu u(k, h) \cdot \bar{u}(p'_q, \lambda) \left(\tilde{f}_1 \gamma^\mu + \tilde{f}_3 \gamma \cdot K P_q^\mu \right) u(p_q, \lambda)$$

electron helicity

quark helicity

$$K \equiv (k + k')/2$$

$$P_q \equiv (p_q + p'_q)/2$$

kinematics for partonic subprocess :

$$\hat{s} \equiv (k + p_q)^2, \quad \hat{u} \equiv (k - p'_q)^2,$$

$$\hat{s} + \hat{u} = Q^2$$

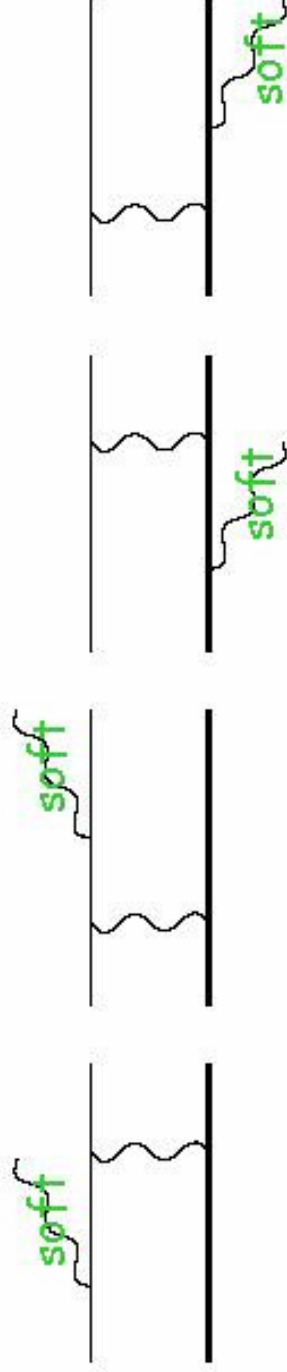
λ is infinitesimal photon mass

$$R(\tilde{f}_1) = \frac{e^2}{4\pi^2} \left\{ \ln \left(\frac{\lambda^2}{\sqrt{-\hat{s}u}} \right) \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{\pi^2}{2} \right.$$

$$\left. + \frac{1}{2} \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{Q^2}{4} \left[\frac{1}{\hat{u}} \ln^2 \left| \frac{\hat{s}}{Q^2} \right| - \frac{1}{\hat{s}} \ln^2 \left| \frac{\hat{u}}{Q^2} \right| - \frac{1}{\hat{s}} \pi^2 \right] \right\}$$

$$R(\tilde{f}_3) = \frac{e^2}{4\pi^2} \left\{ \hat{s} \ln \left| \frac{\hat{s}}{Q^2} \right| + \hat{u} \ln \left| \frac{\hat{u}}{Q^2} \right| + \frac{\hat{s} - \hat{u}}{2} \left[\frac{\hat{s}}{\hat{u}} \ln^2 \left| \frac{\hat{s}}{Q^2} \right| - \frac{\hat{u}}{\hat{s}} \ln^2 \left| \frac{\hat{u}}{Q^2} \right| - \frac{\hat{u}}{\hat{s}} \pi^2 \right] \right\}$$

Bremsstrahlung



$$\sum_{q_i} \left[\text{soft} + \text{soft} + \text{soft} \right] = \text{soft}$$

λ is infinitesimal photon mass

$$R(\tilde{f}_1) = \frac{e^2}{4\pi^2} \left\{ \ln \left(\frac{\lambda^2}{\sqrt{-\hat{s}u}} \right) \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{\pi^2}{2} \right.$$

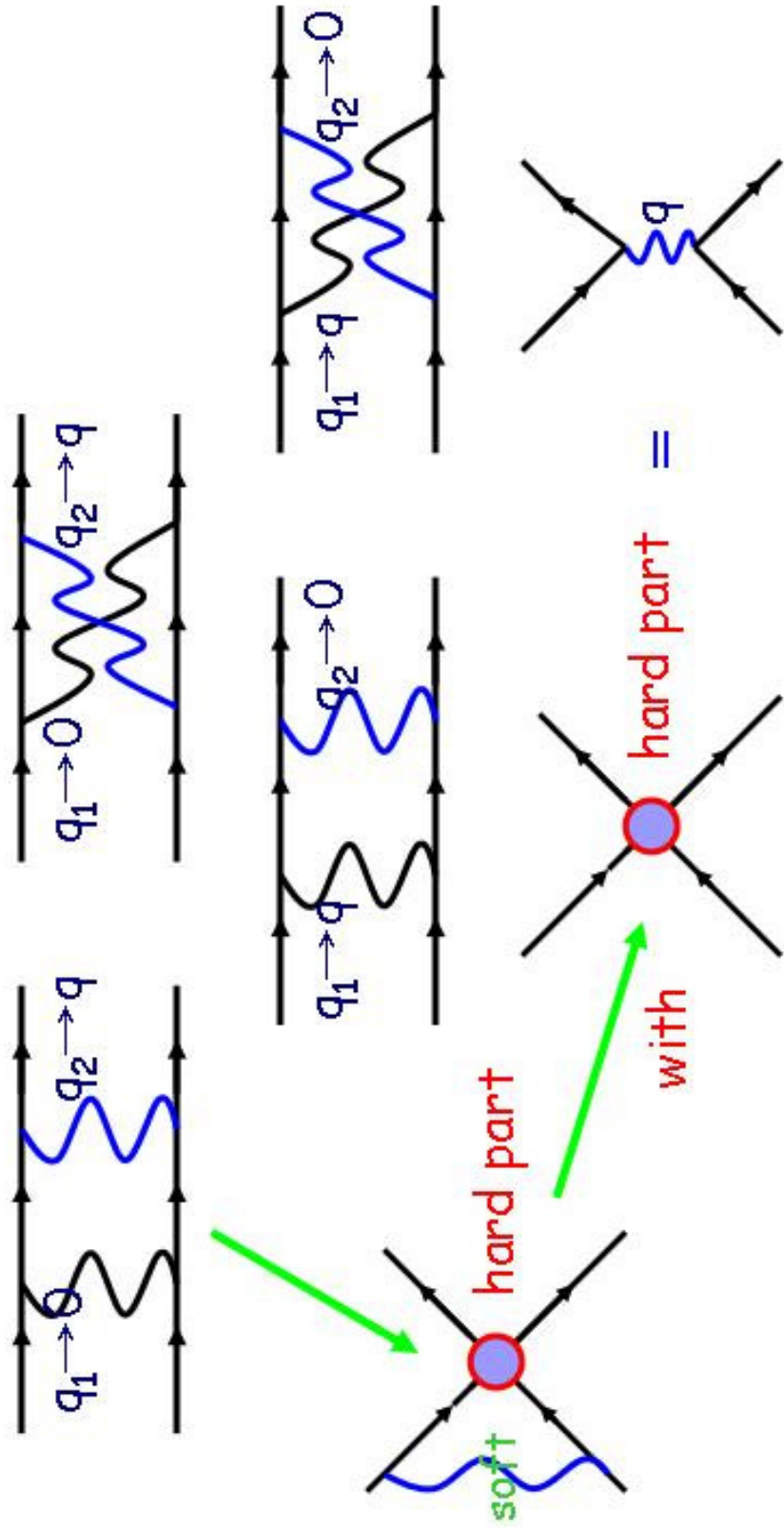
$$\left. + \frac{1}{2} \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{Q^2}{4} \left[\frac{1}{\hat{u}} \ln^2 \left| \frac{\hat{s}}{Q^2} \right| - \frac{1}{\hat{s}} \ln^2 \left| \frac{\hat{u}}{Q^2} \right| - \frac{1}{\hat{s}} \pi^2 \right] \right\}$$

$$R(\tilde{f}_3) = \frac{e^2}{4\pi^2} \left\{ \hat{s} \ln \left| \frac{\hat{s}}{Q^2} \right| + \hat{u} \ln \left| \frac{\hat{u}}{Q^2} \right| + \frac{\hat{s} - \hat{u}}{2} \left[\frac{\hat{s}}{\hat{u}} \ln^2 \left| \frac{\hat{s}}{Q^2} \right| - \frac{\hat{u}}{\hat{s}} \ln^2 \left| \frac{\hat{u}}{Q^2} \right| - \frac{\hat{u}}{\hat{s}} \pi^2 \right] \right\}$$

$$\sigma_R^{lab} = \sigma_{1\gamma}^{lab} \underbrace{(1 + \delta_{2\gamma, soft} + \delta_{brems}^{ep})}_{\text{Should be IR finite}}$$

Should be IR finite

Soft part of 2γ box diagram (at nucleon level)



$$\begin{aligned}
 M_{(a)}^{IR} &= e^4 \int \frac{d^4 k_1}{(2\pi)^4} \bar{u}(k') \gamma^\mu (\gamma \cdot k_1 + m_e) \gamma^\nu u(k) \bar{N}(p) \gamma_\mu (\gamma \cdot (p + q_1) + M) \gamma_\nu N(p) \Big|_{q_2 \sim q, q_1 = k - k_1} \\
 &= \frac{e^4}{q^2} \int \frac{d^4 k_1}{(2\pi)^4} \bar{u}(k') \gamma^\mu (\gamma \cdot (K + q/2) + m_e) \gamma^\nu u(k) \bar{N}(p) \gamma_\mu (\gamma \cdot (P - q/2) + M) \gamma_\nu N(p) \\
 &\quad \frac{[k_1^2 - m_e^2][k_1^2 - \lambda^2]}{[k_1^2 - m_e^2][q_1^2 - \lambda^2][(q_2)^2 - \lambda^2][(p + q_1)^2 - M^2]}
 \end{aligned}$$

soft part of electron-proton box

$$\delta_{2\gamma}^{soft} = \frac{e^2}{2\pi^2} \left\{ \ln \left(\frac{\lambda^2}{\sqrt{(s-M^2)|u-M^2|}} \right) \ln \left| \frac{s-M^2}{u-M^2} \right| \right. \\ \left. - L \left(\frac{s-M^2}{s} \right) - \frac{1}{2} \ln^2 \left(\frac{s-M^2}{s} \right) + \mathcal{R} \left[L \left(\frac{u-M^2}{u} \right) \right] + \frac{1}{2} \ln^2 \left(\frac{u-M^2}{u} \right) + \frac{\pi^2}{2} \right\}$$

where $L(z)$ is the spence function defined by: $L(z) \equiv - \int_0^z dt \frac{\ln(1-t)}{t}$

The sum of the soft part of handbag and cat-ears diagrams in quark level give the whole soft contribution of box diagram in nucleon level.

Now, we can separate the soft part from handbag calculation result.

$$\mathcal{R}(\tilde{f}_1^{soft}) = \frac{e^2}{4\pi^2} \left\{ \ln \left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}} \right) \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{\pi^2}{2} \right\}$$

$$\mathcal{R}(\tilde{f}_1^{hard}) = \frac{e^2}{4\pi^2} \left\{ \frac{1}{2} \ln \left| \frac{\hat{s}}{\hat{u}} \right| + \frac{Q^2}{4} \left[\frac{1}{\hat{u}} \ln^2 \left| \frac{\hat{s}}{Q^2} \right| - \frac{1}{\hat{s}} \ln^2 \left| \frac{\hat{u}}{Q^2} \right| - \frac{1}{\hat{s}} \pi^2 \right] \right\}$$

Soft part in nucleon level

bremsstrahlung contribution : Maximon, Tjon (2000)

$$\delta_{brems}^{ep} = \frac{e^2}{2\pi^2} \left\{ \ln \left(\frac{4(\Delta E)^2 E_e^2}{\lambda^2 y E_e'^2} \right) \ln \left(\frac{E_e}{E_e'} \right) + L \left(1 - \frac{1 E_e}{y E_e'} \right) - L \left(1 - \frac{1 E_e'}{y E_e} \right) \right\}$$

where

$$y \equiv (\sqrt{\tau} + \sqrt{1 + \tau})^2$$

$$\Delta E = E_e'^{el} - E_e'$$

$$\sigma_R^{lab} = \sigma_{1\gamma}^{lab} \underbrace{(1 + \delta_{2\gamma, soft} + \delta_{brems}^{ep})}_{\text{IR finite}}$$

IR finite

The maximum energy of the soft emission photon (ΔE) dependence on the sensitivity of the detector. $\Delta E \approx 1\% E_e'$, so the above formula gives correction factor $(1 + \pi \alpha) +$ terms of size 0.001

Hard part in nucleon level (GPDs)

$$\begin{aligned}
 T_{h,\lambda'_N\lambda_N}^{\text{hard}} &= \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} \left[H_{h,+ \frac{1}{2}}^{\text{hard}} + H_{h,- \frac{1}{2}}^{\text{hard}} \right] \\
 &\times \left[H^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \gamma \cdot n u(p, \lambda_N) + E^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \frac{i\sigma^{\mu\nu} n_\mu q_\nu}{2M} u(p, \lambda_N) \right] \\
 &+ \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} \left[H_{h,+ \frac{1}{2}}^{\text{hard}} - H_{h,- \frac{1}{2}}^{\text{hard}} \right] \text{sgn}(x) \tilde{H}^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \gamma \cdot n \gamma_5 u(p, \lambda_N)
 \end{aligned}$$

result for e-q scattering amplitude

work in frame $q^+ = 0$, n^μ is a Sudakov vector ($n^2 = 0$, $n \cdot P = 1$)

handbag amplitude depends on $\text{GPD}(x, \xi = 0, Q^2)$, $x = P_q^+ / P^+$

$$\delta\tilde{G}_M^{hard} = C$$

$$\delta\tilde{G}_E^{hard} = -\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}}B$$

$$\tilde{F}_3 = \frac{M^2}{\nu} \left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C)$$

A, B & C can be defined by GPD integrals

$$A \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q + E^q)$$

“magnetic” GPD

$$B \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q)$$

“electric” GPD

$$C \equiv \int_{-1}^1 \frac{dx}{x} \tilde{f}_1^{hard} \operatorname{sgn}(x) \sum_q e_q^2 \tilde{H}^q$$

“axial” GPD

Final inputs for GPDs

use **gaussian-valence model** : **Radyushkin (1998)**, **Diehl et al. (1999)**

$$H^q(x, 0, q^2) = q_v(x) \exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right)$$

$$\tilde{H}^q(x, 0, q^2) = \Delta q_v(x) \exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right) \quad \sigma = 0.8 \text{ GeV}^2$$

$$E^q(x, 0, q^2) = \frac{\kappa^q}{N^q} (1-x)^2 q_v(x) \exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right)$$

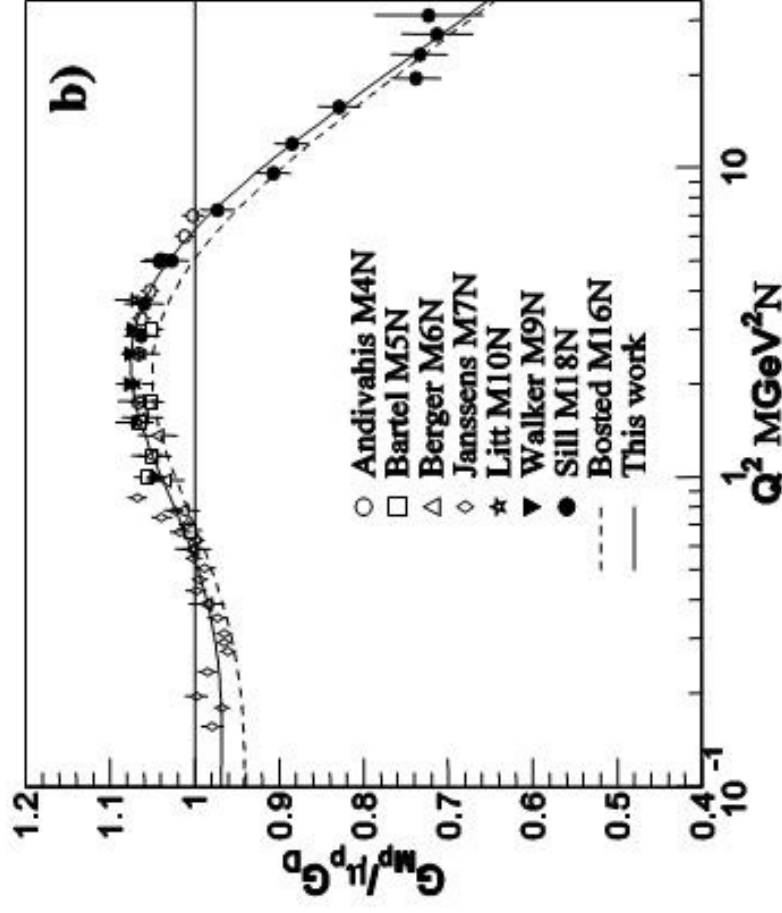
Forward parton distributions at $\mu^2 = 1 \text{ GeV}^2$

$$\begin{array}{l} \text{MRST2002 NNLO} \\ \text{Leader, Sidorov,} \\ \text{Stamenov (2002)} \end{array} \left\{ \begin{array}{l} u_v = 0.262 x^{-0.69} (1-x)^{3.50} (1 + 3.83 x^{0.5} + 37.65 x) \\ d_v = 0.061 x^{-0.65} (1-x)^{4.03} (1 + 49.05 x^{0.5} + 8.65 x) \\ \Delta u_v = 0.505 x^{-0.33} (1-x)^{3.428} (1 + 2.179 x^{0.5} + 14.57 x) \\ \Delta d_v = -0.0185 x^{-0.73} (1-x)^{3.864} (1 + 35.47 x^{0.5} + 28.97 x) \end{array} \right.$$

Fianl inputs for form factor G_M & $R(G_E/G_M)$

magnetic proton form factor

Brash et al. (2002)



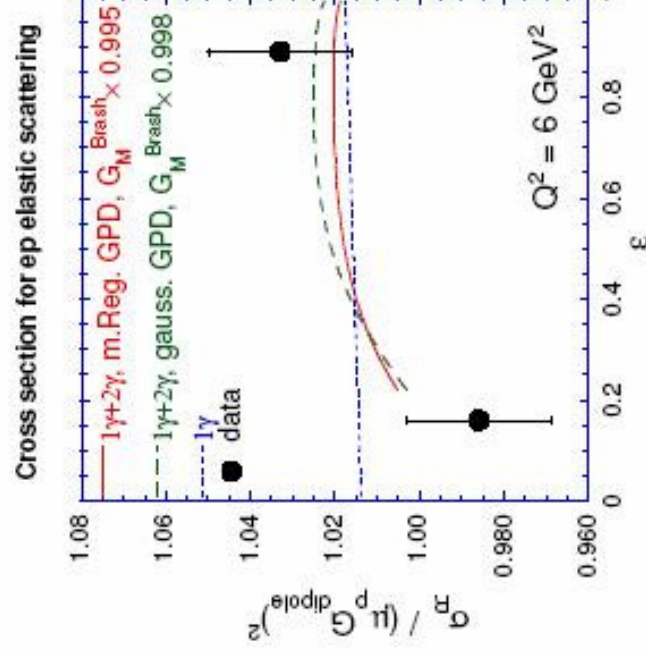
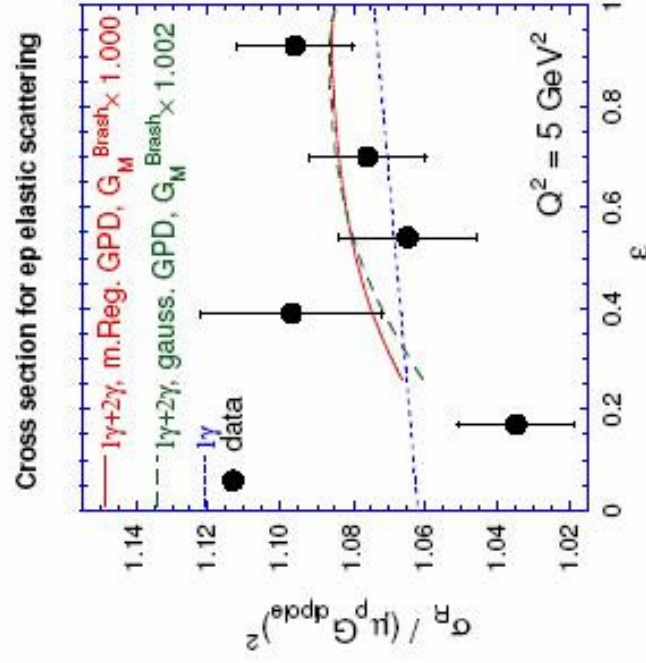
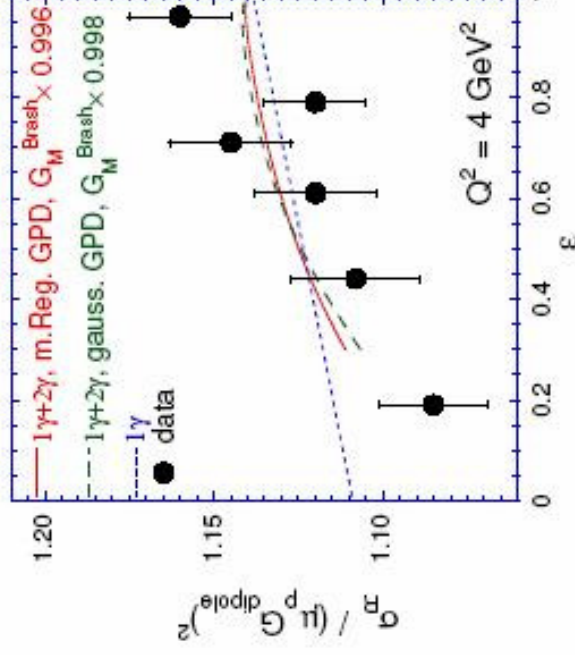
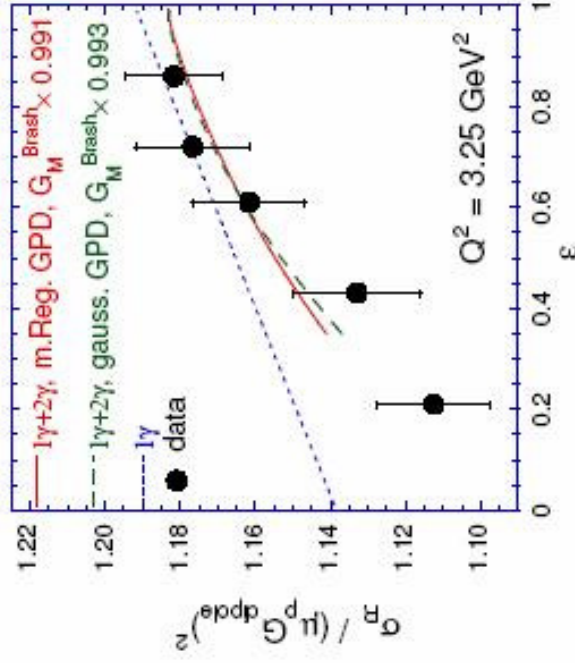
electric proton form factor :

G_E / G_M of proton fixed from polarization data [Gayou et al. \(2002\)](#)

$$G_{Ep} = (1 - 0.13(Q^2 - 0.04)) \frac{G_{Mp}}{\mu_p}$$

Result: cross section

with $G_D = 1/(1 + Q^2/0.71)^2$

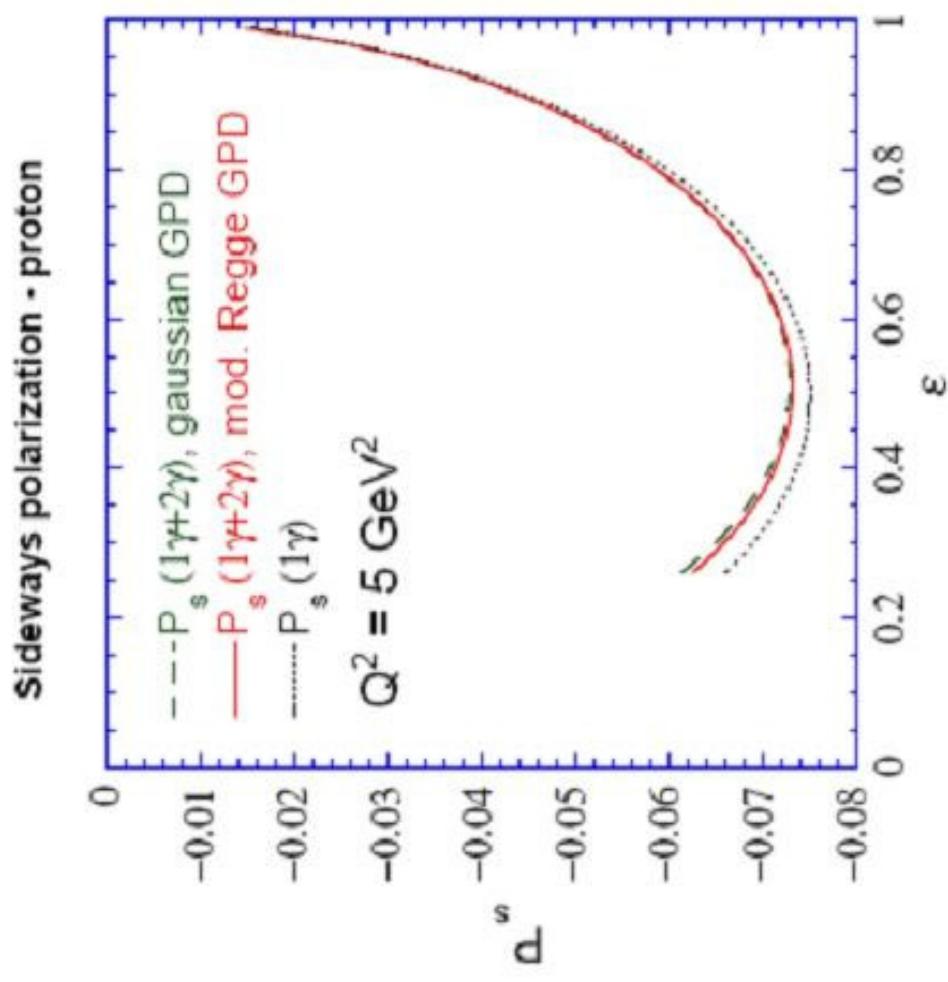
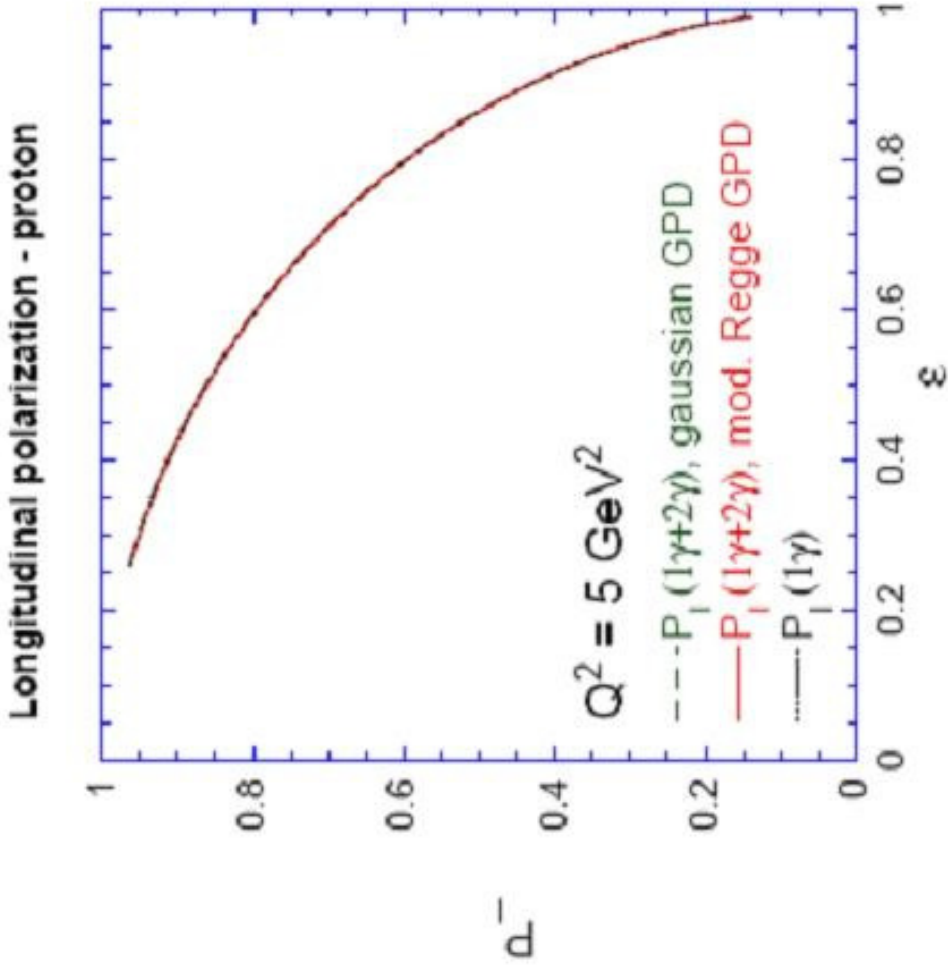


$S, -U, Q^2 > M^2$

Y.C. Chen et al.

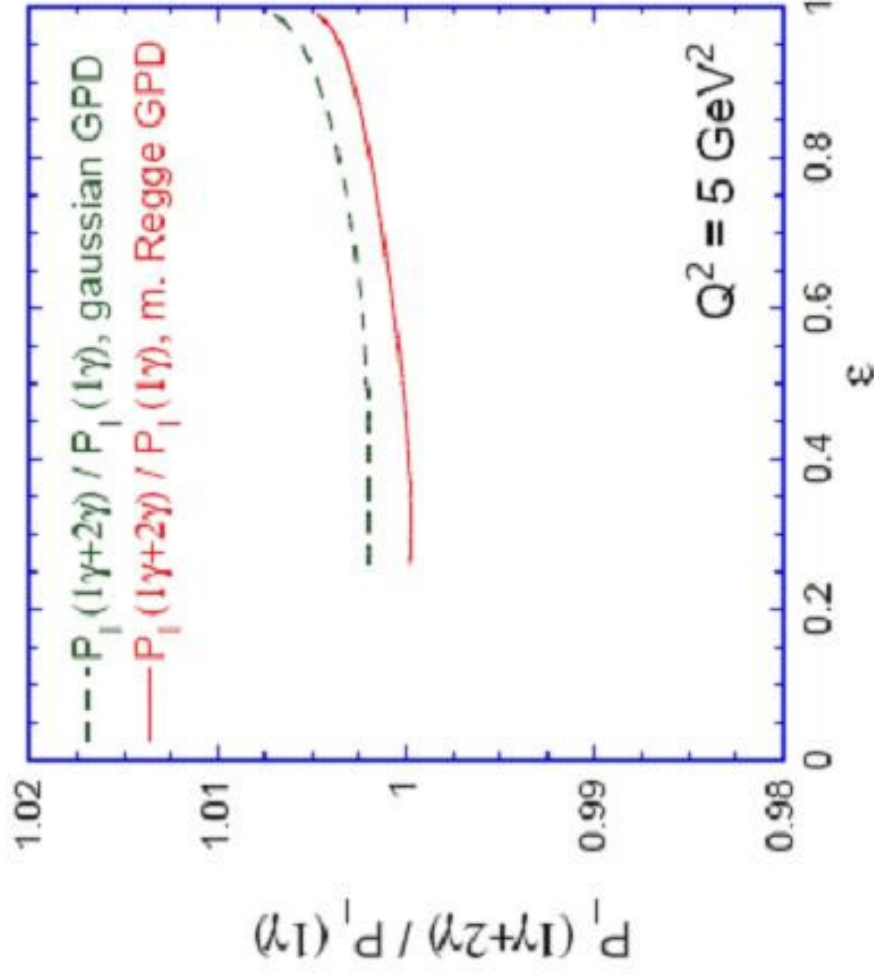
PRL(2004)

Result: polarization transfer observables

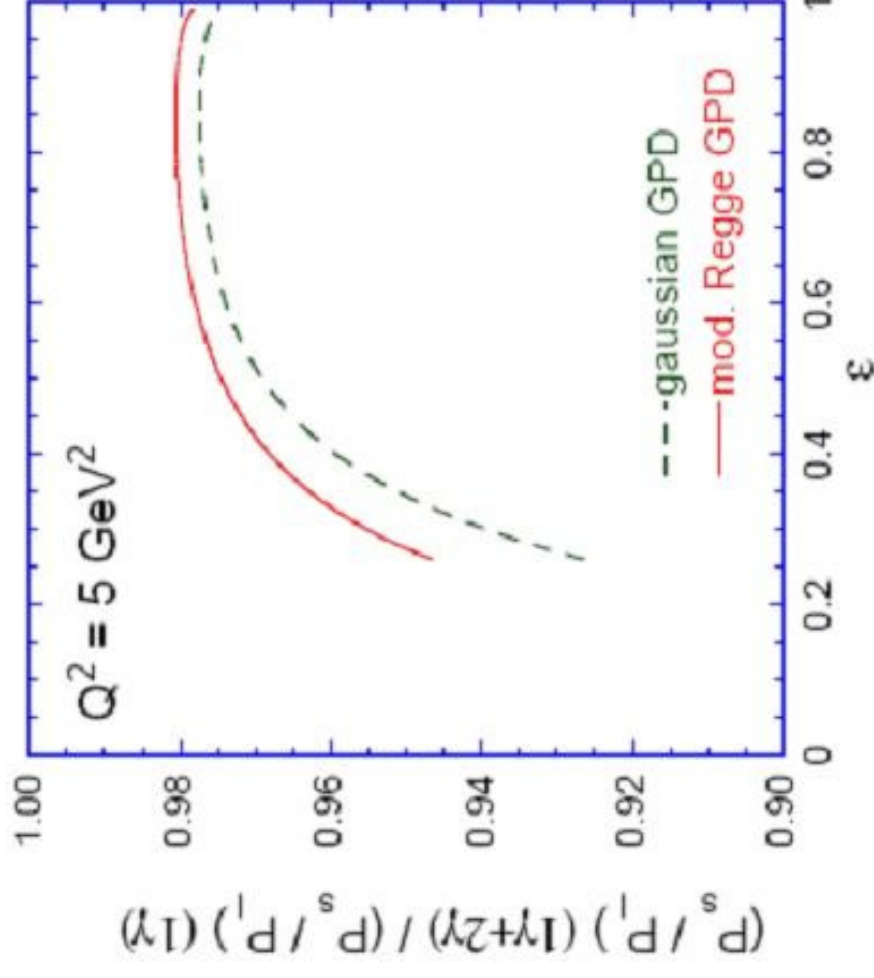


$s, -u, Q^2 > M^2$

2- γ corrections to long. polarization - proton

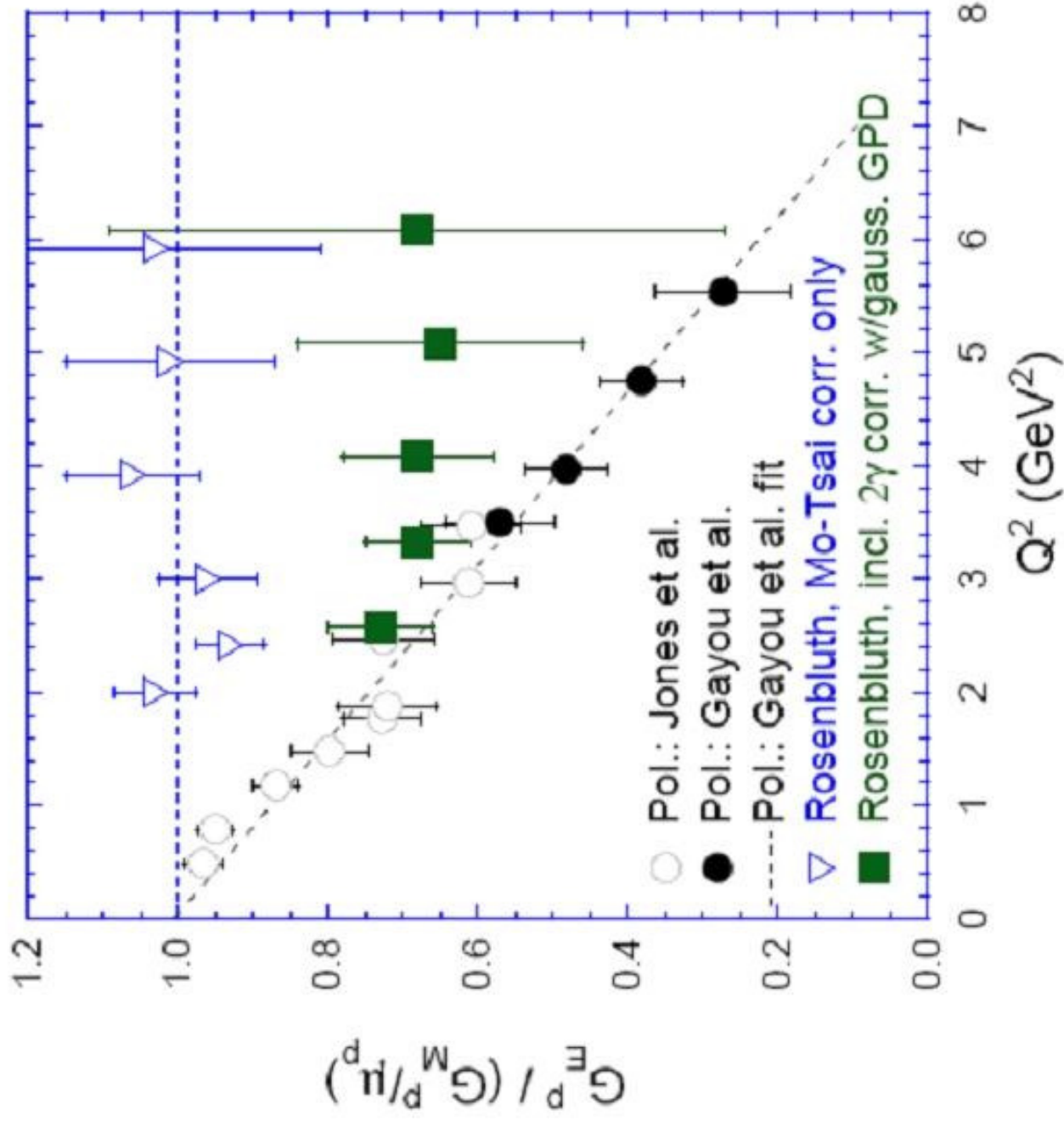


2- γ corrections to polarization ratio - proton



$s, -u, Q^2 > M^2$

Rosenbluth w/2- γ corrections vs. Polarization data



Other extension: Target normal spin asymmetry

→ general formula, of order e^2

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left\{ -G_M \mathcal{I} \left(\delta \tilde{G}_E + \frac{\nu}{M^2} \tilde{F}_3 \right) + G_E \mathcal{I} \left(\delta \tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon} \right) \frac{\nu}{M^2} \tilde{F}_3 \right) \right\}$$

→ involves the **imaginary part** of two-photon exchange amplitudes

Target normal spin asymmetry : partonic calculation

$$\left. \begin{aligned}
 & \mathcal{I}(\tilde{f}_1^{soft}) = -\frac{e^2}{4\pi} \ln\left(\frac{\lambda^2}{\hat{s}}\right) \\
 & \mathcal{I}(\tilde{f}_1^{hard}) = -\frac{e^2}{4\pi} \left\{ \frac{Q^2}{2\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + \frac{1}{2} \right\} \\
 & \mathcal{I}(\tilde{f}_3) = -\frac{e^2}{4\pi} \frac{1}{\hat{u}} \left\{ \frac{\hat{s}-\hat{u}}{\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + 1 \right\}
 \end{aligned} \right\}$$

two-photon
 electron-quark
 amplitude

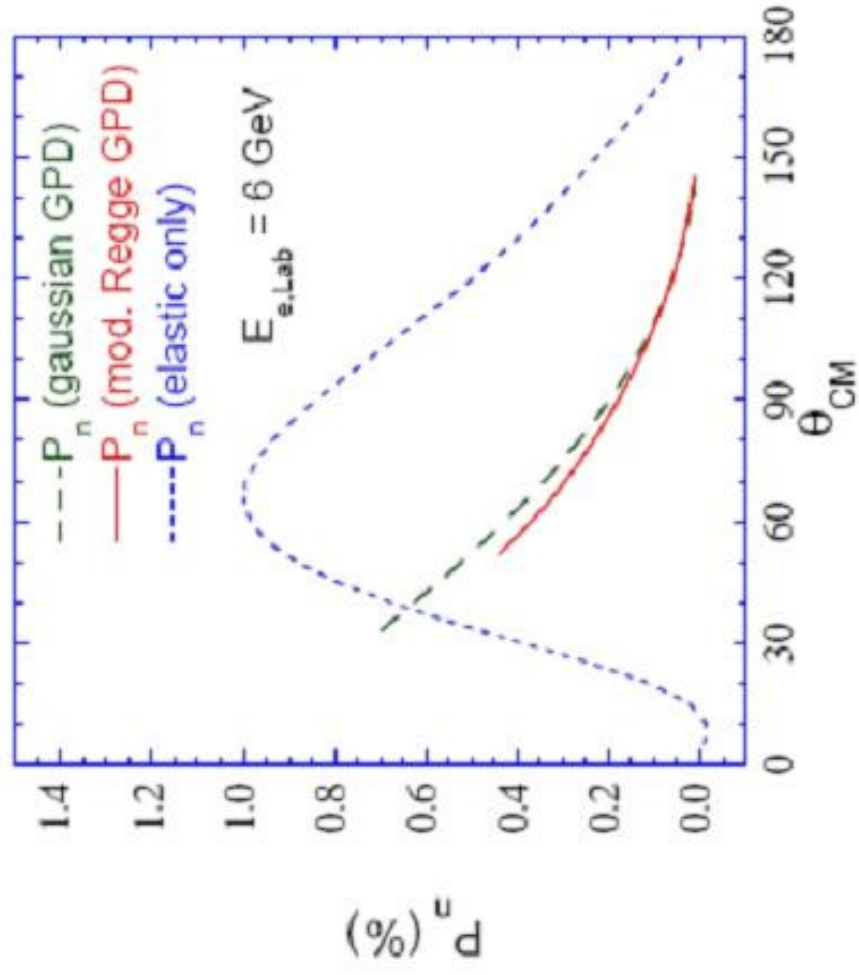
$$\rightarrow A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left\{ G_E \mathcal{I}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}} G_M \mathcal{I}(B) \right\}$$

$$A \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q + E^q) \quad \text{"magnetic" GPD}$$

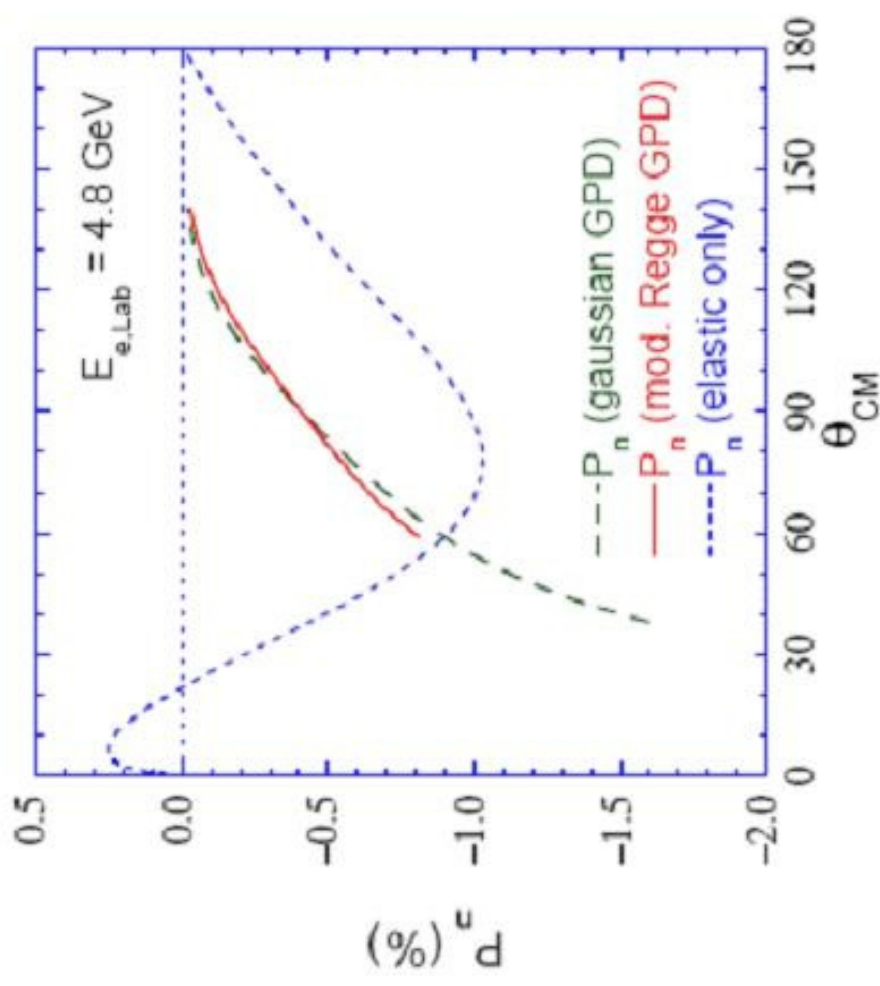
$$B \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q) \quad \text{"electric" GPD}$$

Target normal spin asymmetry : results

Normal Polarization or Analyzing Power - Proton



Normal Polarization or Analyzing Power - Neutron



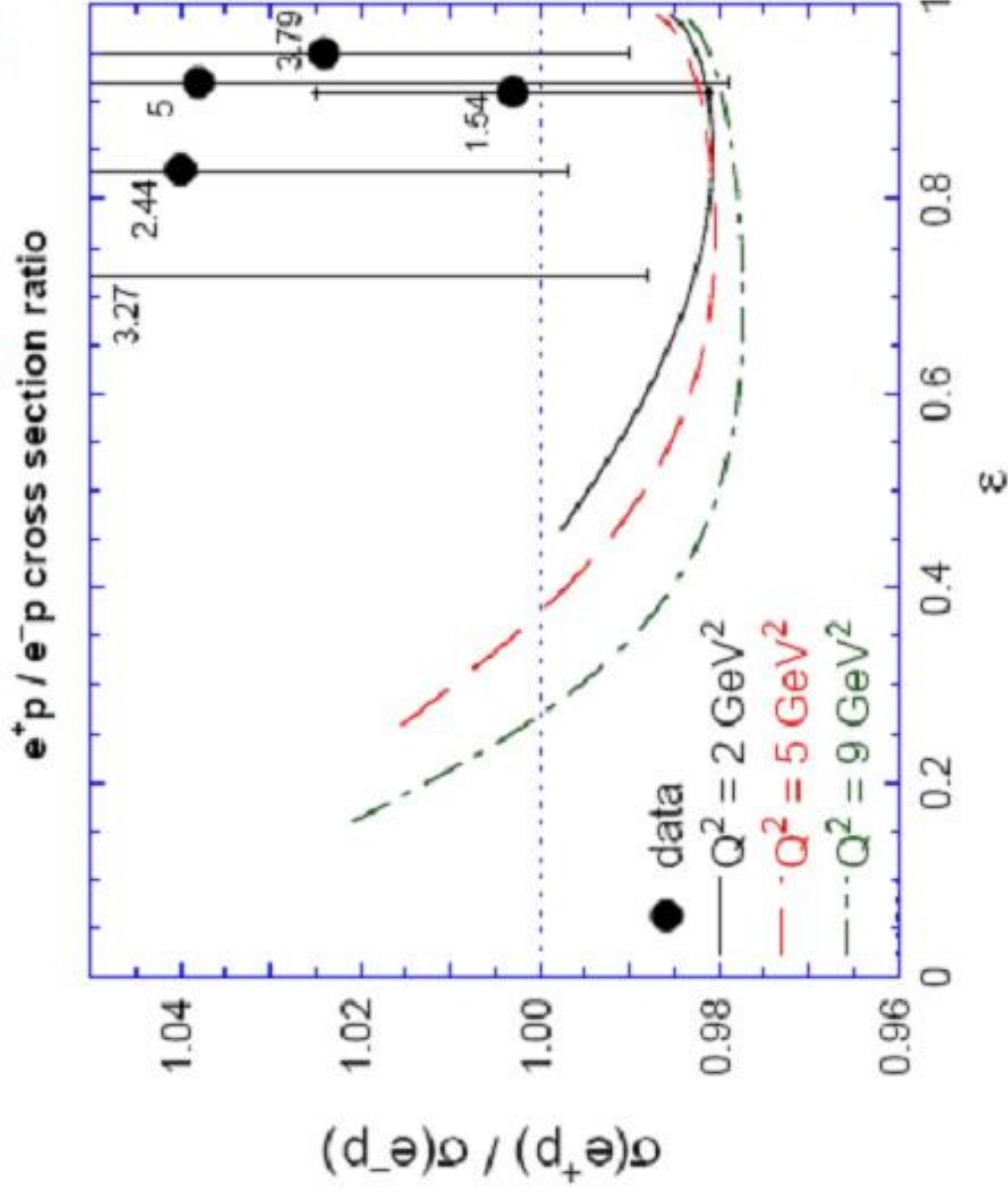
$Q^2 > 1 \text{ GeV}$

$Q^2 > 2 \text{ GeV}$

$-u > M^2$

Other extension: $e^+ p / e^- p$ Ratio

Direct test of real part of 2γ amplitude



data figure from
Arrington (2003)

Other extension: quark mass sensitivity

When quark mass does not set to zero:

$$\hat{s} \equiv \frac{(x+\eta)^2}{4x\eta} Q^2 + \frac{x+\eta}{x} m_q^2$$

$$\hat{u} \equiv -\frac{(x-\eta)^2}{4x\eta} Q^2 + \frac{x-\eta}{x} m_q^2$$

where

$$\eta \equiv \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} = \frac{s - u - 2\sqrt{M^4 - su}}{Q^2 + 4M^2}$$

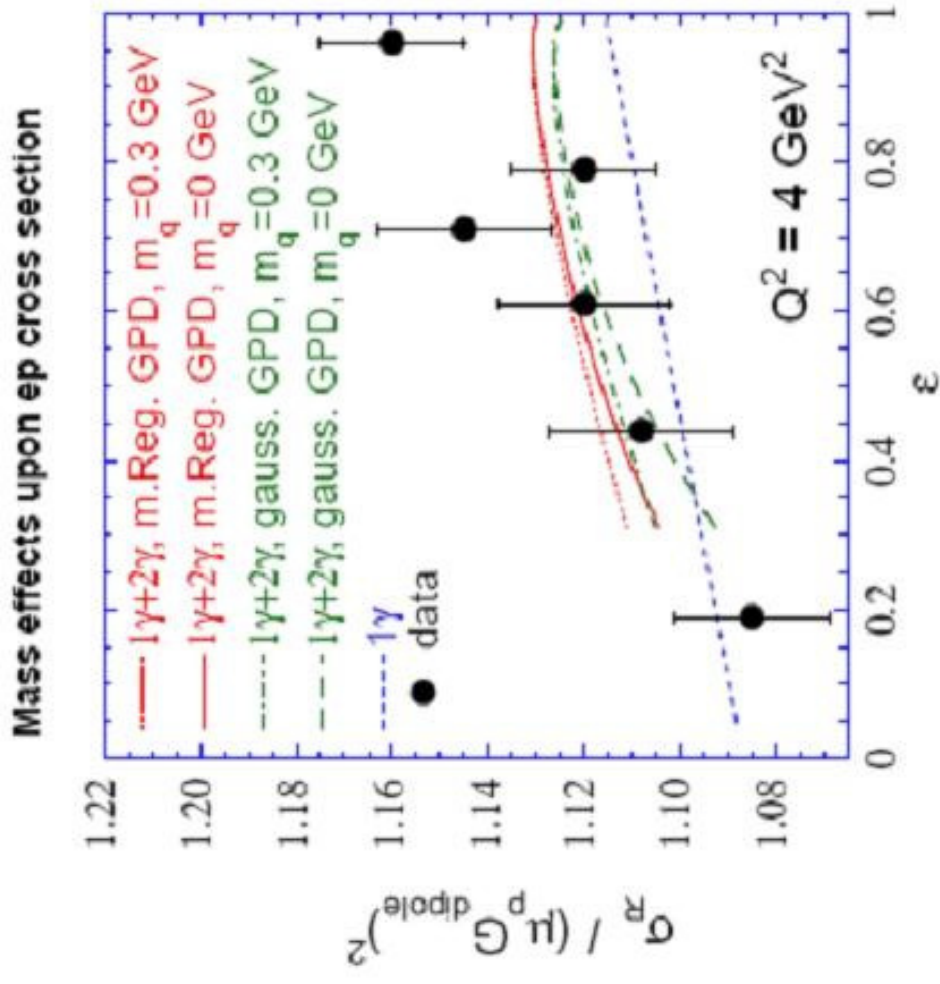
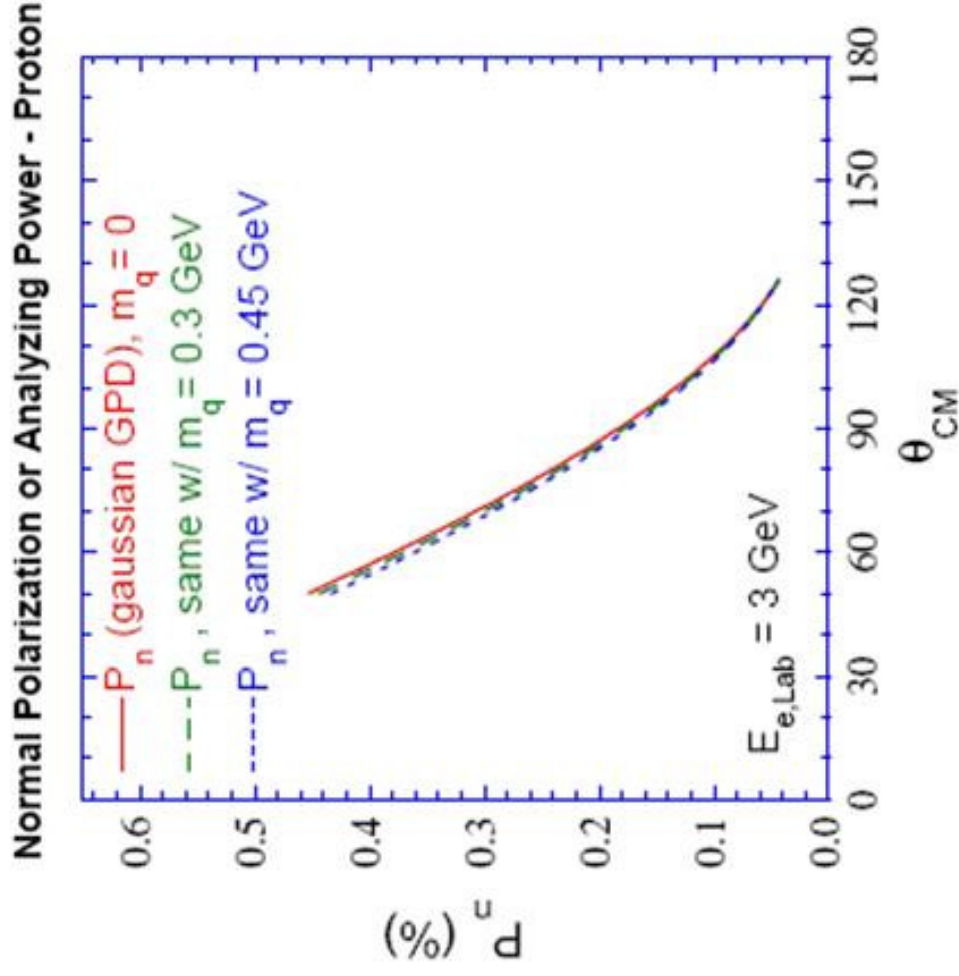
and then

$$\frac{1}{2} [H_{h,1/2}^{hard} + H_{h,-1/2}^{hard}] \equiv \frac{e^2}{Q^2} \left\{ [\hat{s} - \hat{u} - \frac{2m_q^2}{\hat{s} - m_q^2} Q^2] \tilde{f}_1^{hard} + (m_q^4 - \hat{s}\hat{u}) \tilde{f}_3 \right\}$$

Change GPD's integral



Quark mass sensitivity: result



Summary

- Develop the formalism to describe the elastic e-N scattering beyond one-photon exchange approximation.
- performed a partonic calculation of two-photon exchange contribution in GPDs.
- When taking the polarization transfer determinations of the form factors input, adding in the 2 photon correction, does reproduce the cross section data.

- The GPDs results also provide some useful cross-check such as recoil normal spin asymmetry (A_n) and e^+p/e^-p ratio. (see upcoming talks given by L. Weinstein & X. Jiang)
- Quark mass effect might be important at small values of ε , when u is close to zero.