Constraints on Proton Structure from Precision Atomic-Physics Measurements

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Introduction

- consider difference between the well-known hyperfine splittings (hfs) in hydrogen and muonium.
- correct for magnetic moment and reduced mass effects.
- the large QED contributions for a pointlike nucleus essentially cancel.
- difference then due solely to proton structure.
- this provides a sum rule that constrains a particular combination of proton form factors and structure functions.

Acknowledgments

- published as S.J. Brodsky, C.E. Carlson, JRH, and D.S Hwang, PRL **94**, 022001 (2005); 169902(E) (2005).
- useful remarks in A.V. Volotka et al., Eur. Phys. J. D **33**, 23 (2005).
- see also comments in J.L Friar and I. Sick, PRL **95**, 049101 (2005) and the reply in PRL **95**, 049102 (2005).
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Outline

- introduction
- sum-rule derivation
- evaluation
- interpretation
- conclusions

Hyperfine splittings

- Fermi energy: $E_F^N = \frac{8\alpha^3}{3\pi} \frac{\mu_B \mu_N m_e^3}{(1+m_e/m_N)^3}$,
  with $N = \mu^+$ or p, $\mu_B = \frac{e}{2m_e}$, and $\mu_N = (1+\kappa_N)\frac{e}{2m_N}$
- muonium
  $E_{\rm hfs}(e^-\mu^+) = (1 + \Delta_{\rm QED} + \Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\rm weak}^\mu) E_F^\mu$
- hydrogen
  $E_{\rm hfs}(e^-p) = (1+\Delta_{\rm QED}+\Delta_R^p+\Delta_S+\Delta_{hvp}^p+\Delta_{\mu vp}^p+\Delta_{\rm weak}^p) E_F^p$
- construct ratio rescaled by μ_N and reduced masses

$$\Delta_{\rm hfs} \equiv \frac{E_{\rm hfs}(e^-p)}{E_{\rm hfs}(e^-\mu^+)} \frac{\mu_\mu}{\mu_p} \frac{(1+m_e/m_p)^3}{(1+m_e/m_u)^3} - 1$$

$$= \frac{1+\Delta_{\rm QED}+\Delta_R^p+\Delta_S+\Delta_{hvp}^p+\Delta_{\mu vp}^p+\Delta_{\rm weak}^p}{1+\Delta_{\rm QED}+\Delta_R^\mu+\Delta_{hvp}^\mu+\Delta_{\rm weak}^\mu} - 1$$

The atomic side

- $$\Delta_S = \Delta_{\mathrm{hfs}} + \Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\mathrm{weak}}^\mu - (\Delta_R^p + \Delta_{hvp}^p + \Delta_{\mu vp}^p + \Delta_{\mathrm{weak}}^p) + \Delta_{\mathrm{hfs}}(\Delta_{\mathrm{QED}} + \Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\mathrm{weak}}^\mu)$$
- the leading Δ_{QED} cancel.
- the remaining Δ_{QED} can be replaced by the lowest-order approximation, $\alpha/2\pi$.

The hadronic side

- $\Delta_S = \Delta_Z + \Delta_{\rm pol}$
- Zemach contribution: $\Delta_Z = -2\alpha m_e (1 + \delta_Z^{\rm rad}) \langle r \rangle_Z$, with $\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa_p} - 1 \right]$
$$= \int d^3r d^3r' |\vec{r} - \vec{r}'| \rho_E(\vec{r}) \rho_M(\vec{r}')$$
- polarization contribution: $\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi m_p (1+\kappa_p)} (\Delta_1 + \Delta_2)$, with

$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2} \left\{ \frac{9}{4} F_2^2(Q^2) - 4 m_p \int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2} \beta_1\left(\frac{\nu^2}{Q^2}\right) g_1(\nu, Q^2) \right\},$$
$$\Delta_2 = -12 m_p \int_0^\infty \frac{dQ^2}{Q^2} \int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2} \beta_2\left(\frac{\nu^2}{Q^2}\right) g_2(\nu, Q^2),$$
$$\nu_{\rm th} = m_\pi + (m_\pi^2 + Q^2)/2m_p,$$
$$\beta_1(\theta) = 3\theta - 2\theta^2 - 2(2-\theta)\sqrt{\theta(\theta+1)},$$
$$\beta_2(\theta) = 1 + 2\theta - 2\sqrt{\theta(\theta+1)}$$

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Radiative correction to Zemach radius

- $\delta_Z^{\rm rad}$ estimated by Bodwin & Yennie, PRD **37**, 498 (1988).
- Karshenboim, PLA **225**, 97 (1997) calculated analytically for dipole form factors:
  $\delta_Z^{\rm rad} = (\alpha/3\pi)\left[2\ln(\Lambda^2/m_e^2) - 4111/420\right]$.
- with $\Lambda^2 = 0.71$ GeV^2, this yields $\delta_Z^{\rm rad} = 0.0153$.

Recoil corrections in muonium

- muonium: $\Delta_R^\mu = \Delta_{\rm rel}^\mu + \Delta_{\rm rad}^\mu$.
- relativistic recoil
  [Bodwin & Yennie, PRD **37**, 498 (1988)]:

$$\Delta_{\rm rel}^\mu = \frac{1}{1+\kappa_\mu}\left[\frac{-3\alpha}{\pi}\frac{m_e m_\mu}{m_\mu^2-m_e^2}\ln\frac{m_\mu}{m_e} + \alpha^2\frac{m_e}{m_\mu}\left(2\ln\frac{1}{2\alpha} - 6\ln 2 + \frac{65}{18}\right)\right]$$

- radiative recoil [Kinoshita, hep-ph/9808351;
  Eides et al., hep-ph/0412372]:

$$\Delta_{\rm rad}^\mu =$$

$$\frac{1}{1+\kappa_\mu}\left[\frac{\alpha^2}{\pi^2}\frac{m_e}{m_\mu}\left(-2\ln^2\frac{m_\mu}{m_e} + \frac{13}{12}\ln\frac{m_\mu}{m_e} + \frac{21}{2}\zeta(3) + \zeta(2) + \frac{35}{9}\right)\right.$$

$$+\frac{\alpha^3}{\pi^3}\frac{m_e}{m_\mu}\left(-\frac{4}{3}\ln^3\frac{m_\mu}{m_e} + \frac{4}{3}\ln^2\frac{m_\mu}{m_e} - \left[4\pi^2\ln 2 + \frac{29}{12}\right]\ln\frac{m_\mu}{m_e} + 47.7213\right)$$

$$\left. +\alpha^2\left(\frac{m_e}{m_\mu}\right)^2\left(-6\ln 2 - \frac{13}{6}\right)\right]$$

Recoil corrections in hydrogen

- Bodwin & Yennie, PRD **37**, 498 (1988): $\Delta_R^p = -1.55$ ppm.
- finite-size corrections $\rightarrow +5.68(1)$ ppm.
- radiative recoil corrections [Karshenboim, PLA **225**, 97 (1997)] $\rightarrow 5.77(1)$ ppm.
- Volotka et al, EPJD **33**, 23 (2005):
  - re-evaluation of finite-size corrections $\rightarrow 5.86$ ppm.
  - forced G_M to reproduce their $\langle r \rangle_Z \rightarrow 6.01$ ppm.
- chose $\Delta_R^p = 5.86(15)$ ppm.

Atomic inputs

- S.G. Karshenboim, Can. J. Phys. **77**, 241 (1999):
  $E_{\rm hfs}(e^-p) = 1\ 420.405\ 751\ 766\ 7(9)$ MHz.
- W. Liu et al., PRL **82**, 711 (1999):
  $E_{\rm hfs}(e^-\mu^+) = 4\ 463.302\ 765(53)$ MHz.
- S. Eidelman et al., PLB **592**, 1 (2004):
  $m_p = 938.272\ 029(80)$ MeV, $m_\mu = 105.658\ 369(9)$ MeV,
  $m_e = 0.510\ 998\ 918(44)$ MeV, $\alpha^{-1} = 137.035\ 999\ 11(46)$.
- G.W. Bennett et al., PRL **92**, 161802 (2004):
  $\kappa_\mu = 0.001\ 165\ 920\ 8(6)$.
- P.J. Mohr and B.N. Taylor, RMP **77**, 1 (2005):
  $m_\mu/m_e = 206.768\ 2838(54)$, $m_p/m_e = 1836.152\ 672\ 61(85)$.
- P.J. Mohr, private communication:
  $\mu_\mu/\mu_p = 3.183\ 345\ 20(20)$, free of muonium hfs.

Cross-check with QED

- muonium:

$$\Delta_{\rm QED} = \frac{E_{\rm hfs}(e^-\mu^+)}{E_F^\mu} - 1 - (\Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\rm weak}^\mu)$$

$$= 1136.12(13) \text{ ppm.}$$

- hydrogen:

$$\Delta_{\rm QED} = \frac{E_{\rm hfs}(e^-p)}{E_F^p} - 1 - (\Delta_R^p + \Delta_S + \Delta_{hvp}^p + \Delta_{\mu vp}^p + \Delta_{\rm weak}^p)$$

- mix:

$$\Delta_{\rm QED} = \frac{E_{\rm hfs}(e^-p)}{E_F^p} - 1 - (\Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\rm weak}^\mu + \Delta_{\rm hfs}) - \Delta_{\rm hfs}(\frac{\alpha}{2\pi} + \Delta_R^\mu + \Delta_{hvp}^\mu + \Delta_{\rm weak}^\mu)$$

$$= 1136.09(14) \text{ ppm.}$$

- consistent with Dupays et al., PRA **68**, 052503 (2003) and Volotka et al., EPJD **33**, 23 (2005).

Evaluation of atomic side

- $\Delta_{\text{hfs}} = 145.51(4)$ ppm.
- $\Delta_R^\mu = -178.34$ ppm.
- more inputs [Volotka et al., EPJD **33**, 23 (2005)]:
  $\Delta^\mu_{hvp} = 0.05$ ppm, $\Delta^\mu_{\text{weak}} = -0.01$ ppm, $\Delta^p_{hvp} = 0.01$ ppm,
  $\Delta^p_{\mu vp} = 0.07$ ppm, $\Delta^p_{\text{weak}} = 0.06$ ppm.
- $\Delta_S = -38.62(16)$ ppm.

$\rightarrow$ constraint on G_E, G_M, g_1, and g_2 that is better than 1%.

Interpretation of hadronic side

- if use estimate of $\Delta_{\text{pol}} = 1.4(6)$ ppm by Faustov and Martynenko [EPJC **24**, 281 (2002)], then $\Delta_Z = -40.0(6)$ ppm and $\langle r \rangle_Z = 1.043(16)$ fm.
- Griffioen et al.: $\Delta_{\text{pol}} = 0.72(37)$ ppm $\rightarrow \Delta_Z = -39.3(4)$ ppm and $\langle r \rangle_Z = 1.024(16)$ fm.
- if use estimate of $\langle r \rangle_Z = 1.086(12)$ fm by Friar and Sick [PLB **579**, 285 (2004)], then $\Delta_{\text{pol}} = 3.05(49)$ ppm.

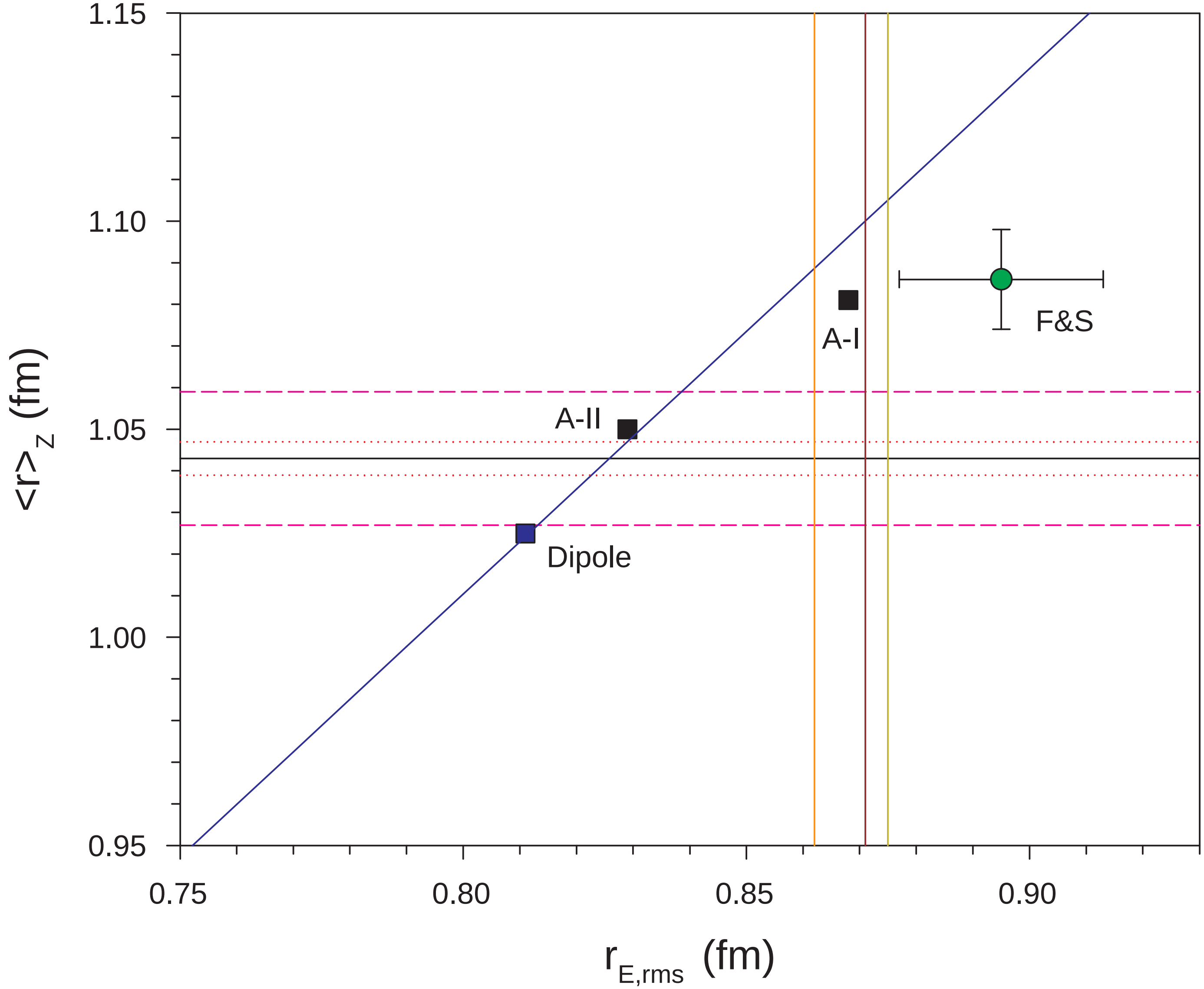
$\langle r \rangle_Z$ from form-factor models

- dipole $\rightarrow$ 1.025 fm
- fit to standard Rosenbluth separation
  [J. Arrington, PRC **69**, 022201(R) (2004), Table I]
  $G_E(Q^2)$, $G_M(Q^2)/(1+\kappa_p) = 1/(1 + p_2 Q^2 + p_4 Q^4 + \cdots)$
  $\rightarrow$ 1.081 fm
- fit constrained by polarization transfer data
  [Arrington, Table II] $\rightarrow$ 1.050 fm

Electric charge radius

- $r_{\mathrm{E,rms}} = \sqrt{-6 \frac{d}{dQ^2} G_E(Q^2)|_{Q^2=0}}$.
- obtain estimates from
  - a standard empirical fit: $0.862(12)$ fm
    [G.G. Simon et al., NPA **333**, 381 (1980)]
  - Lamb-shift measurements: $0.871(12)$ fm
    [K.Pachucki, PRA **63**, 042503 (2001);
    K. Pachucki and U.D. Jentschura, PRL **91**, 113005 (2003); updated by M. Eides, private communication]
  - a continued-fraction fit for G_E: $0.895(18)$ fm
    [I. Sick, PLB **576**, 62 (2003)]
  - the 2002 CODATA value: $0.8750(68)$ fm
    [P.J. Mohr and B.N. Taylor, RMP **77**, 1 (2005)]

Plot of $\langle r \rangle_Z$ vs $r_{\mathrm{E,rms}}$



Conclusions

- atomic physics provides a very precise constraint on proton structure, to better than 1%.
- the subtraction method removes uncertainties associated with pure QED contributions to hfs.
- the method could also be applied to Lamb shifts, to extract $r_{\mathrm{E,rms}}$ with less uncertainty.
- the interpretation of individual structure contributions requires more data and analysis, particularly for
  - g_1, g_2, and Δ_{pol}.
  - two-photon contributions to electron-proton scattering.