Dzuba, Flambaum, Sushkov 1989 } accuracy
Blundell, Johnson, Sapirstein 1990, 1992 } 1%
difference ~0.1 %
Boulder experiment 1997 0.35%

Precise calculation of parity nonconservation in cesium and test of the standard model

Vladimir Dzuba, Victor Flambaum, Jacinda Ginges

We have calculated the $6s - 7s$ parity nonconserving (PNC) E1 transition amplitude, E_{PNC}, in cesium. We have used an improved all-order technique in the calculation of the correlations and have included all significant contributions to E_{PNC}. Our final value

$$E_{PNC} = 0.904\Big(1 \pm 0.5\%\Big) \times 10^{-11} iea_B(-Q_W/N)$$

has half the uncertainty claimed in old calculations used for the interpretation of Cs PNC experiments. The resulting nuclear weak charge Q_W for Cs deviates by about 2σ from the value predicted by the standard model. Radiative corrections
→ agreement with standard model!

C.S. Wood, S.C. Bennet, D. Cho, B.P. Masterson, J.L. Roberts, C.E. Tanner, and C.E. Wieman

Science 275, 1759 (1997)

Discovery of anapole moment and very accurate measurement of Cs weak charge.

Q_w $\pm$ 0.35%

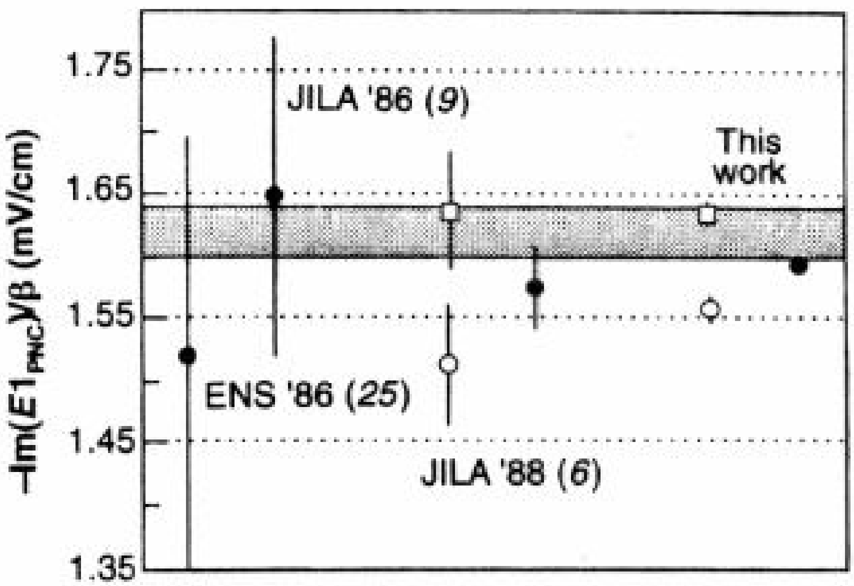


Fig. 4. Historical comparison of cesium PNC results. The squares are values for the 4-3 transition, the open circles are the 3-4 transition, and the solid circles are averages over the hyperfine transitions. The band is the standard-model prediction for the average, including radiative corrections. The $\pm 1\sigma$ width shown is dominated by the uncertainty of the atomic structure.

New very accurate data on the probabilities of electromagnetic transitions in Cs: theory predictions are accurate to 0.1–0.3% !

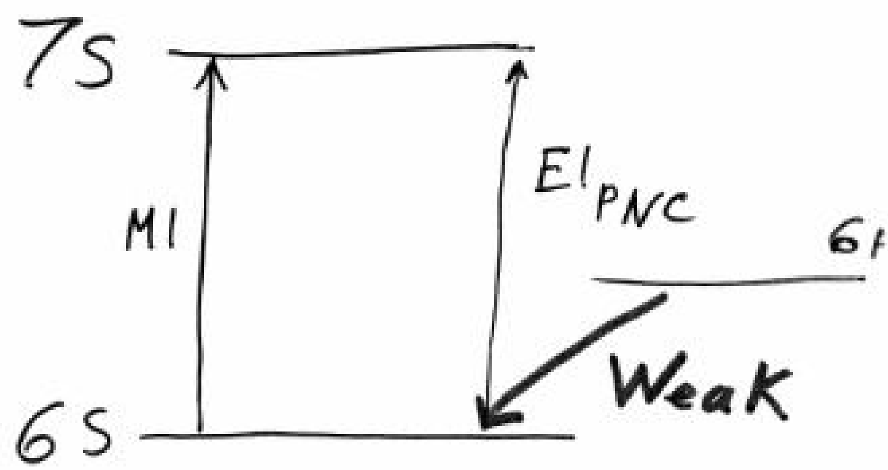
Bennet, Wieman: root mean squre error of theory 0.4% (instead of 1%). $\longrightarrow$

New physics beyond Standard Model, 2.5 σ

Cs Bouchiat 1974

- 6S - electron

Cs$^+$ closed shells (Xe - like)



Parity non-conserving (PNC) E1-amplitude

$$E1_{PNC} = \sum_{np} \frac{\langle 6S | \mathbf{W} | np \rangle}{E_{6S} - E_{np}} \langle np | E1 | 7S \rangle \; + \; \langle 7S | W$$

Interference M1 and E_{PNC} $\rightarrow$ PNC effects

photon circular polarization $P = 2 \frac{E_{PNC}}{M1}$

Electric field $\mathcal{E} \rightarrow$ Stark amplitude

$E_S = \beta \cdot \mathcal{E}$

Measured $\frac{E_{PNC}}{E_S} = \frac{E_{PNC}}{\beta \cdot \mathcal{E}}$

Method [a]

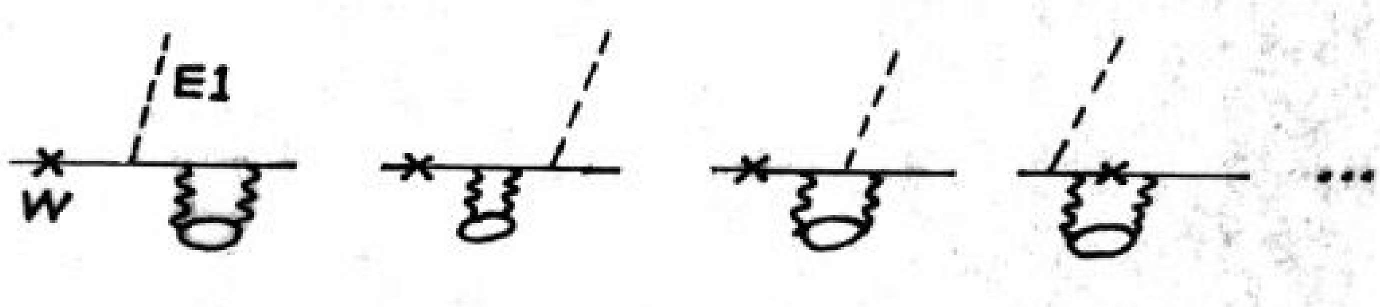
1. Zero approximation: **Relativistic Hartree-Fock** method, Hamiltonian H_{RHF} is used to generate zero-approximation energy levels, electron orbitals and Green's functions which are needed to apply Feynman diagram technique.

2. We took into account direct and exchange polarization of the atomic core by the external electric field of the photon (in E1-transition) and the weak nuclear potential using the **Time-Dependent Hartree-Fock** method (summation of the "RPA with exchange" chain of diagrams).

3. Many-body perturbation theory to calculate correlation corrections to TDHF results. Perturbation
$V = H\text{-}H_{RHF}$
(exact Hamiltonian H) - (Hartree-Fock Hamiltonian H_{RHF})

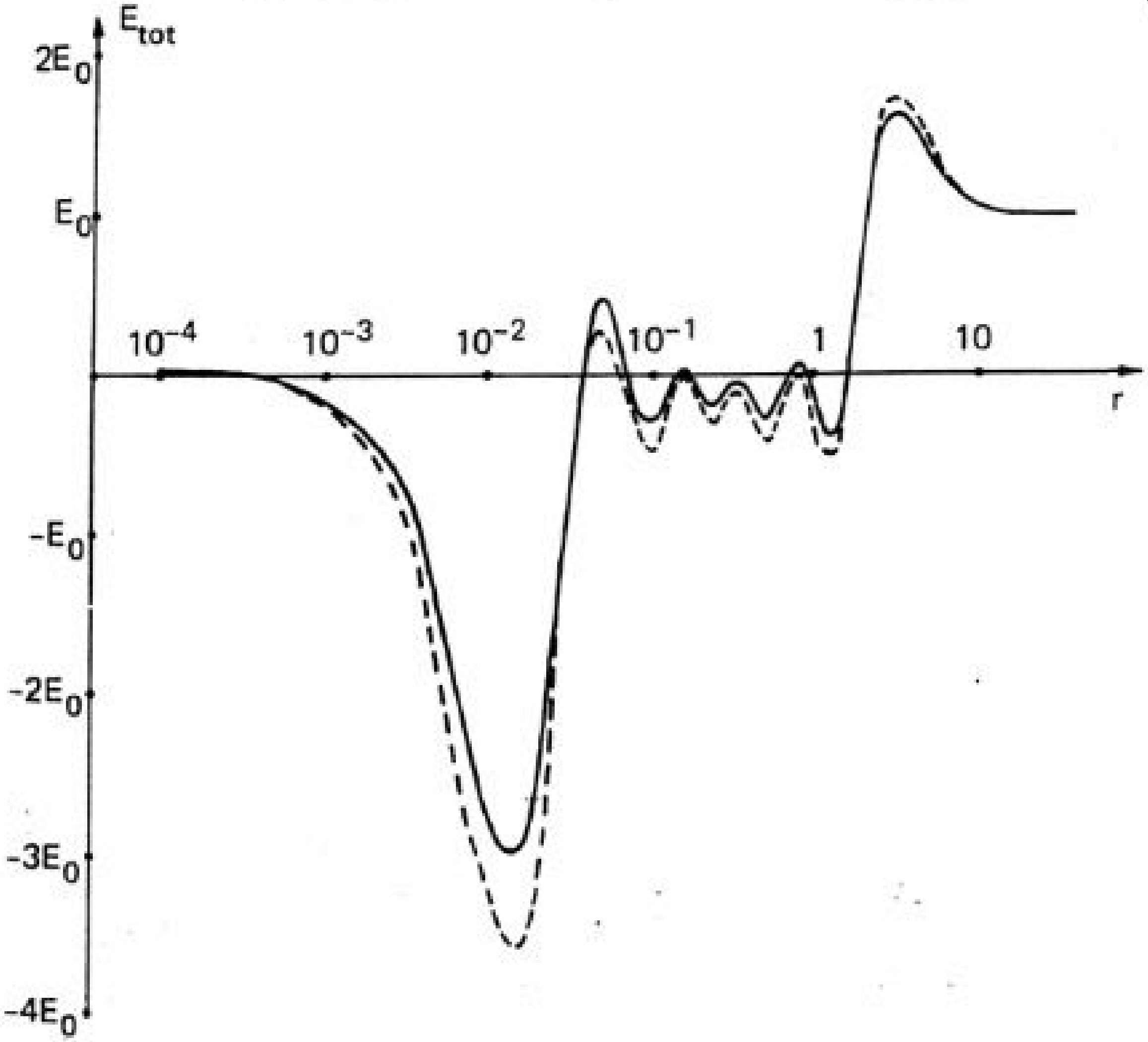
Parameter $\frac{Q_{ct}}{E_{core}} \lesssim \frac{1}{10}$

Calculated **second-order correlation corrections**



and 3 series of dominating higher-order diagrams:

1. **Screening of the electron-electron interaction**. This is a collective phenomenon and so the corresponding chain of diagrams is enhanced by a factor approximately equal to the number of electrons in the external closed subshell (the 5p electrons in Cs).



Plot of electric field $E_t = E_0 + \langle E_e \rangle$ in Tl^+ on the z axis. The distance in atomic units is shown in logarithmic scale. The solid curve corresponds to $\omega = 0$, the dotted curve to $\omega = 0.207\ Ry/\hbar$

2. **Hole-particle interaction**.
This effect is enhanced by the large zero-multipolarity diagonal matrix elements of the Coulomb interaction.

3. **Iterations of the self-energy operator ("correlation potential")**.
This chain of diagrams is enhanced by the small denominator, which is the energy for the excitation of an external electron (in comparison with the excitation energy of a core electron).


$+ \ldots$

Strong Coulomb field QED radiative corrections to E_{PNC}

$\sim \alpha \cdot (Z\alpha)$ Sushkov 2001

vacuum polarization (Uehling potential)

Johnson, Bednyakov, Soff 2002 Num
Milstein, Sushkov 2002 Ana.
Dzuba, Flambaum, Ginges 2002 Num
Kuchiev, Flambaum 2002 Analy

0.4 %

self-energy and vertex

Dzuba, Flambaum, Ginges 2002 $\sim -0.65\%$

Kuchiev, Flambaum 2002 $-0.73(20)\%$

-0.6% $\rightarrow -0.9(1)$

Kuchiev 2002 $\alpha(Z\alpha)$

Milstein, Sushkov, Terekhov (2002, 2003) -0.85%

$\alpha(Z\alpha)$, $\alpha(Z\alpha)^2 \ln m_e R_N$

Kuchiev, Flambaum 2003 $-0.9(1)\%$

Sapirstein, Pachucki, Veitia, Cheng 2003 -0.82%

$\langle 2s|W|2p \rangle$

weak only

Shabaev, Pachucki, Tupitsyn, Yerokhin (2005) -0.27%

Total E_{PNC}

Flambaum, Ginges 2005

Total E_{PNC} including many-body effects -0.32%

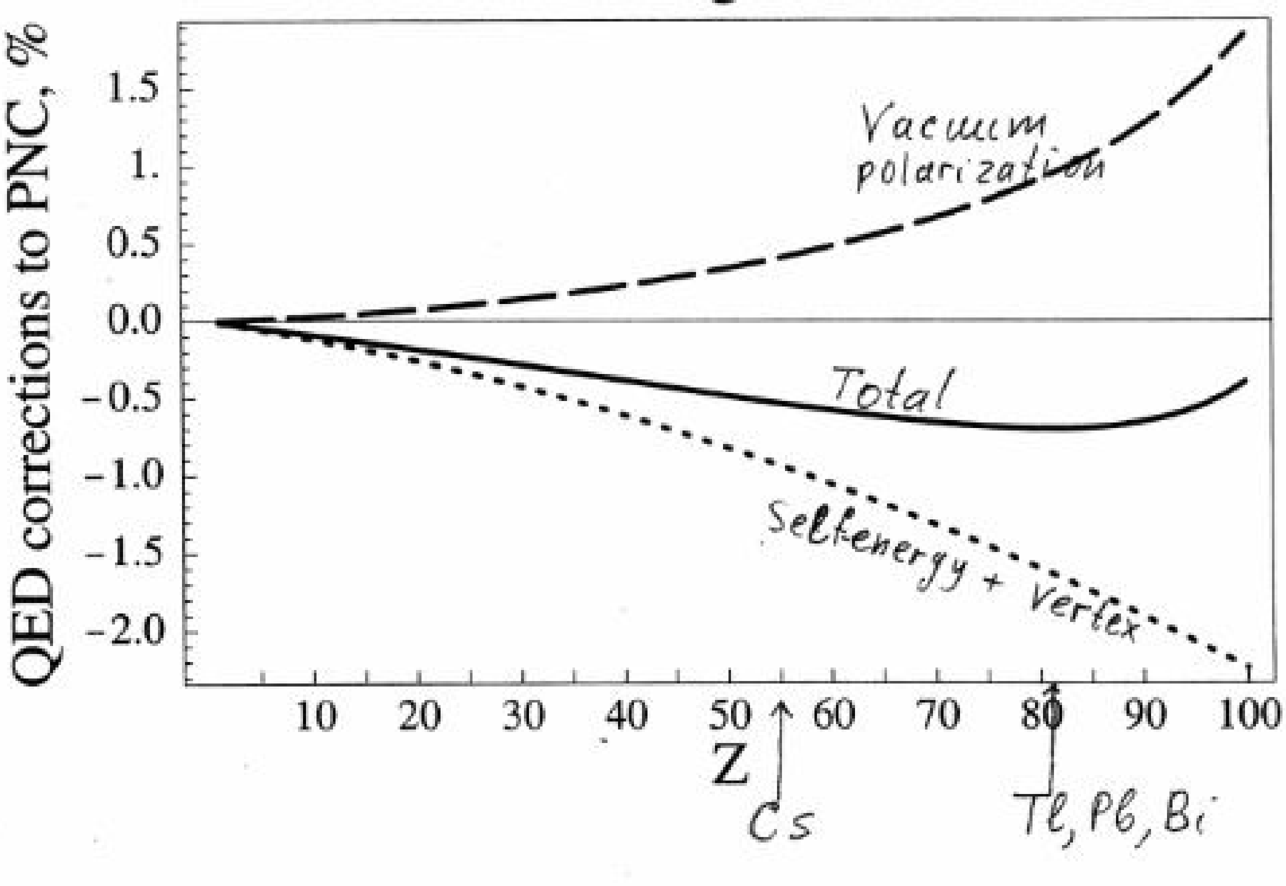
Radiative corrections to E_{PNC} due to strong Coulomb field.

$$\frac{\delta E_{PNC}}{E_{PNC}} = \alpha \cdot Z\alpha \cdot f(Z\alpha)$$

All-orders in $Z\alpha$

Kuchiev, Flambaum

Fig.8



Radiative corrections to E_{PNC} due to strong Coulomb field

Flambaum, Ginges 2005

$$E_{PNC} = \sum_{np} \frac{\langle 6s|W|np\rangle \langle np|E1|7s\rangle}{E_{6s} - E_{np}} + \ldots$$

$$\frac{\delta E_{PNC}}{E_{PNC}} = \alpha \cdot f(Z\alpha)$$ Cs: $Z\alpha = 0.4$

$$\frac{\delta W}{W} \rightarrow -0.41\%$$

$$\frac{\delta(\text{Energy})}{\text{Energy}} \rightarrow -0.33\%$$

$$\frac{\delta E1}{E1} \rightarrow 0.42\%$$

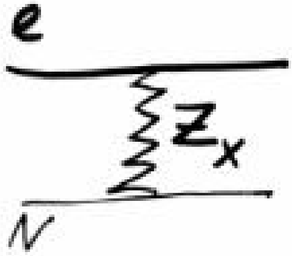
$$\frac{\delta E_{PNC}}{E_{PNC}} = -0.32\%$$

Including many-body effects!

Cs PNC: $Q_W = -72.66(29)_{exp}(36)_{Th}$

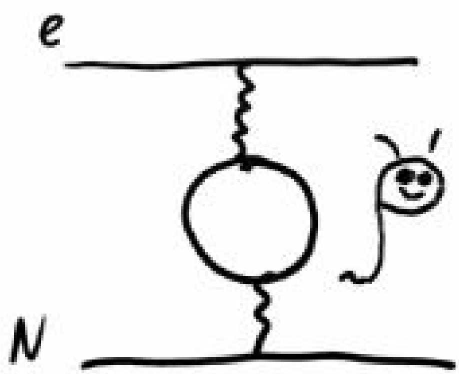
SO(10): $\Delta Q_W^{new} \equiv Q_W - Q_W^{SM} = 0.45(48)$

$$\Delta Q_W^{tree} \simeq 0.4\,(2N+Z)\left(\frac{M_W}{M_{Z_x}}\right)^2$$



$$M_{Z_x} > 750 \ \text{GeV}$$

Tevatron $\quad M_{Z_x} > 600$ GeV



Radiative corrections from new particles

$$\Delta Q_W^{oblique} = -0.800\, S$$

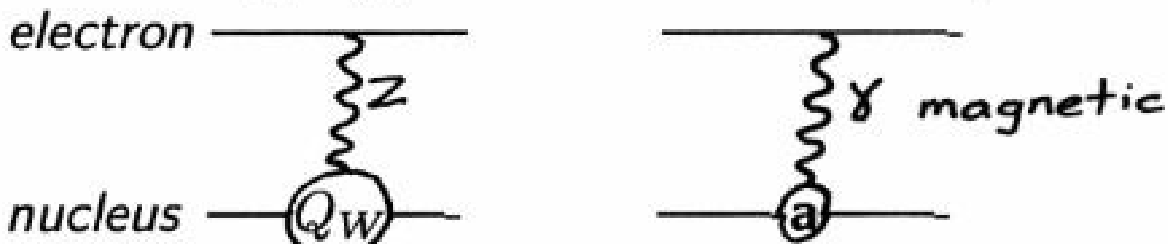
$$S = -0.56(60)$$

Limits on leptoquarks, composite fermions, ...

Nuclear Anapole Moment and Tests of the Standard Model

There are two sources of parity nonconservation (PNC) in atoms— electron-nucleus weak interaction and magnetic interaction of electron with nuclear anapole moment.

Weak charge Q_W Nuclear anapole **a**



Weak charge Q_W and nuclear anapole can be measured in one experiment.

Magnetic interaction between atomic electrons and nuclear anapole moment is "parity violating hyperfine interaction".

PNC E1 transition amplitude $= (.....)Q_W + (...)a\mathbf{I}\cdot\mathbf{j}$

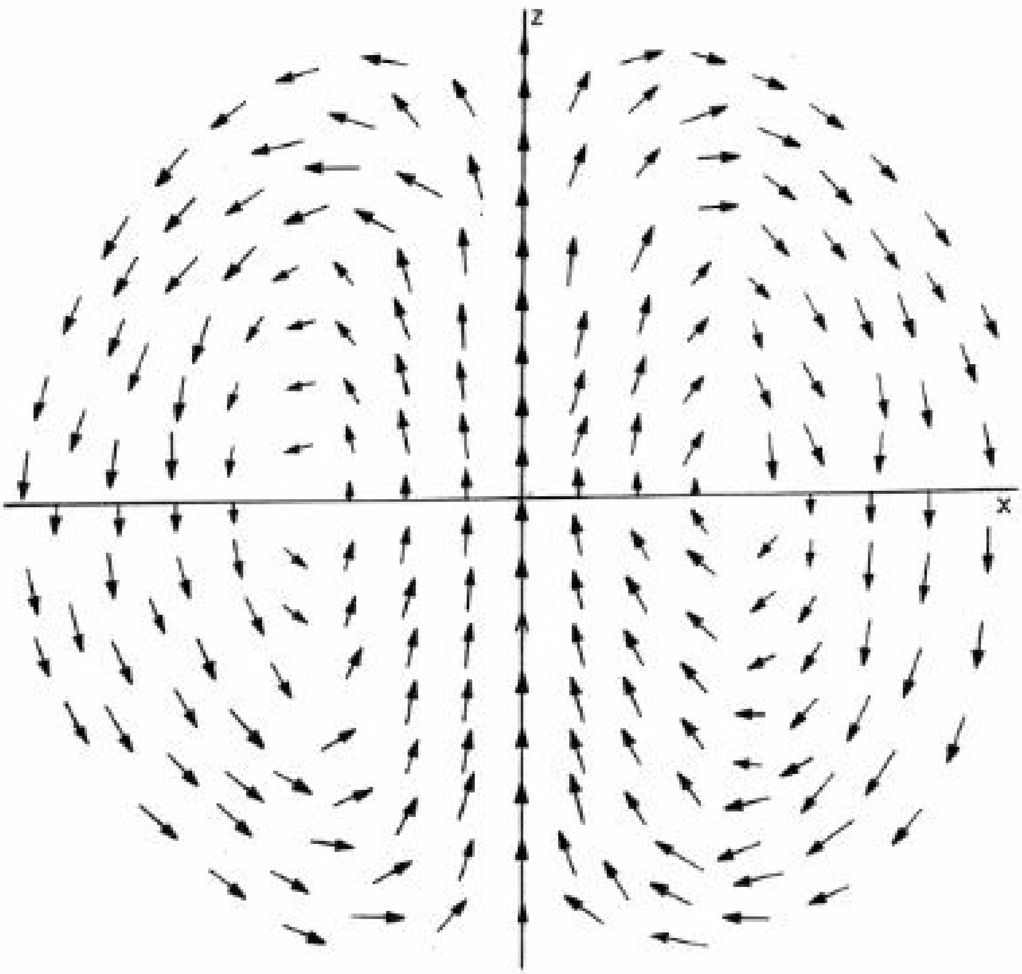
$\mathbf{I}$ is nuclear spin , $\mathbf{j}$ is electron angular momentum.

Anapole makes PNC effects for different hyperfine components different.

PNC effect: P+ΔP, ΔP= P(3-4) - P(4-3)

$\vec{F} = \vec{I} + \vec{J}$ Cs: F = 3 or F = 4 $= \frac{4}{3}$ 7s

Toroidal electromagnetic currents produced by weak interaction; this means that they will produce an anapole moment.



A cross-section (in the x-z plane) of the current distribution due to the spin helix (the anapole moment points along the z direction).

THEORY

Zeldovich 1957 , Vaks 1957: Parity violation → A particle should have one more formfactor, anapole moment, in addition to electric and magnetic formfactors.

Flambaum, Khriplovich 1980. First theory of nuclear anapole moment, proposal to measure nuclear anapole moment in atomic experiments.

Flambaum, Khriplovich, Sushkov 1984. Analytical formula for nuclear anapole moment (tested by numerical calculations).

$A^{2/3}$ enhancement of nuclear anapole which makes its contribution larger than that of electron-nucleon $\mathbf{I}$-dependent weak interaction.

Accurate calculations of nuclear anapole moments:

Haxton, Henley, Musolf 1989

Bouchiat, Piketty 1991, 199 2

Flambaum, Hanhart 1993

Dmitriev, Khriplovich, Telitsin 1994

Dmitriev, Telitsin 1997

Haxton, Ramsey-Musolf, [redacted] Liu 2001, 2002

Auerbach, Brown 1999

Dmitriev, Telitsin 1999

Experimental limits on nuclear anapole moment

Value of g_p from experiments (theory $g_p \sim 4.5$)

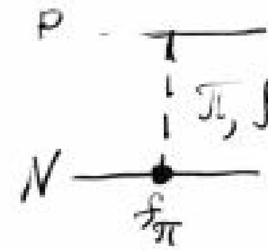
Paris 1984, Cs	$\lvert g_p \rvert < 1000$
Boulder 1986, Cs	$g_p = -20 \pm 20$
Boulder 1988, Cs	$g_p = 10 \pm 5$
Seattle 1997, Tl	$g_p = -2 \pm 3$
Boulder 1997, Cs	$g_p = 6 \pm 1$

g_p is the strenth of proton - nucleus weak parity non-conserving interaction

The Strength of the Parity Violating Nuclear Forces Derived From the Anapole Measurement

Calculation $\kappa_a = 0.06 g_p$
Experiment $\kappa_a = 0.364(62)$.
Anapole measurement gives strength constant for the parity violating interaction of an unpaired proton with the nuclear core (in units of Fermi constant G_F)
$g_p = 6 \pm 1$(exp.).

The proton-nucleus constant g_p can be expressed in terms of the meson-nucleon parity nonconserving interaction constants:



$$g_p = 8.0 \times 10^4 \left[70.4 f_\pi - 19.5 h_\rho^0\right], + \dots$$

The ρ constant is known well. However there is uncertainty about the value of f_π. Anapole measurement gives

$$f_\pi \equiv h_\pi^1 = [7 \pm 2 \text{ (exp.)}] \times 10^{-7}.$$

Some other estimates:
$|f_\pi| < 1.3 \times 10^{-7}$ from a ^{18}F PNC measurement
$f_\pi \equiv h_\pi^1 = 5\text{–}6 \times 10^{-7}$ QCD sum rule calculations
$f_\pi = 4.6 \times 10^{-7}$ DDH "best" value.

CP violation

• observed in 1964 in K^0 decay Christensen, Cronin, Fitch, Turlay

• incorporated into Standard Model (SM) via Kobayashi-Maskawa mechanism

• observed recently in B^0 system Belle BaBar

$\rightarrow$ confirmation of SM ? ?

However... CP-violation in SM does not explain the matter-antimatter asymmetry in the Universe

$\rightarrow$ must be some other source of CP violation

• CPT theorem

$\rightarrow$ CP-violation accompanied by T-violation
– electric dipole moments (EDM)

Electric dipole moments

- violate P and T

$$\mathbf{d} = e\mathbf{r} \propto \mathbf{J}$$

$$\mathbf{r} \to -\mathbf{r} \qquad -e\mathbf{r} \propto \mathbf{J}$$

$$t \to -t \qquad e\mathbf{r} \propto -\mathbf{J}$$

- SM gives values for EDMs that are negligibly small

- EDMs are very sensitive to theories of CP violation beyond the SM!

e.g. electron EDM

Theory	d_e (e cm)
Standard model	$< 10^{-38}$
Supersymmetric	$10^{-27} - 10^{-28}$
Multi-Higgs	$10^{-27} - 10^{-28}$
Left-right symmetric	$10^{-27} - 10^{-28}$

Best limit (90% confidence):

$|d_e| < 1.6 \times 10^{-27}\ e$ cm   Berkeley (2002)

Best limit on an atomic EDM

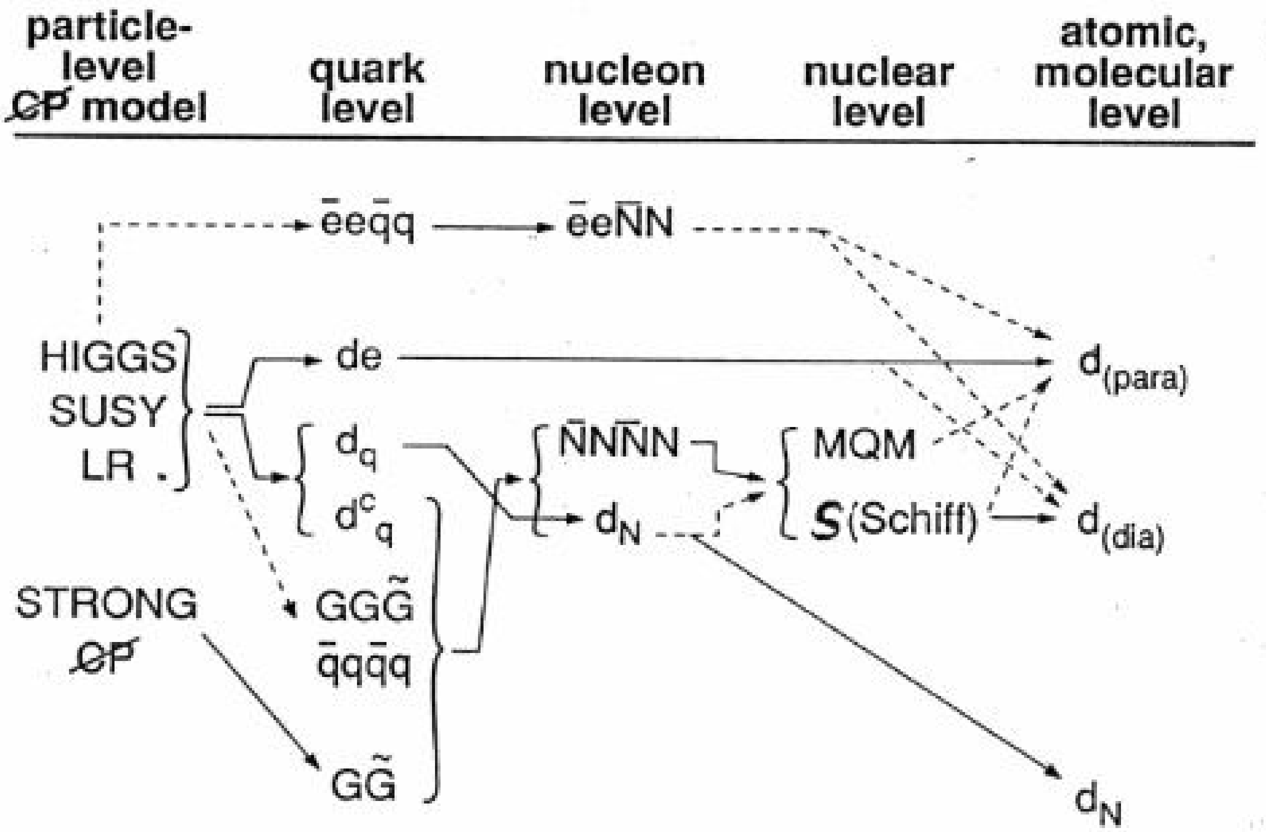
The best limit on an atomic EDM comes from experiments with ^{199}Hg (Seattle)

Romalis, Griffith, Jacobs, Fortson

$$d(^{199}\mathrm{Hg}) = -(1.06 \pm 0.49 \pm 0.40) \times 10^{-28}\ e\ \mathrm{cm}\ .$$

Hg has closed electron subshells (electron angular momentum $J = 0$). This makes experiments with Hg most sensitive to P, T-odd (parity and time reversal violating) interactions that originate from the nucleus.

What sorts of P, T-odd interactions induce atomic EDMs?



[Barr]

Fig. 1. A flow chart showing how CP violating effects at the particle physics level lead to atomic and molecular EDMs. Solid lines show the effects that are generally dominant. Dashed lines show less significant effects. In multi-Higgs, SUSY, and LR models it is d_e, d_q and d_q^c that are most important. d_e is the dominant effect in the EDMs of paramagnetic atoms (d_{para}). d_q^c contributes through the P and T odd nucleon–nucleon couplings (denoted $\bar{N}N\bar{N}N$) and thence through the Schiff moment (S) to give the dominant contribution to the EDM of diamagnetic atoms (d_{dia}). The neutron EDM comes predominantly from d_q and d_q^c.

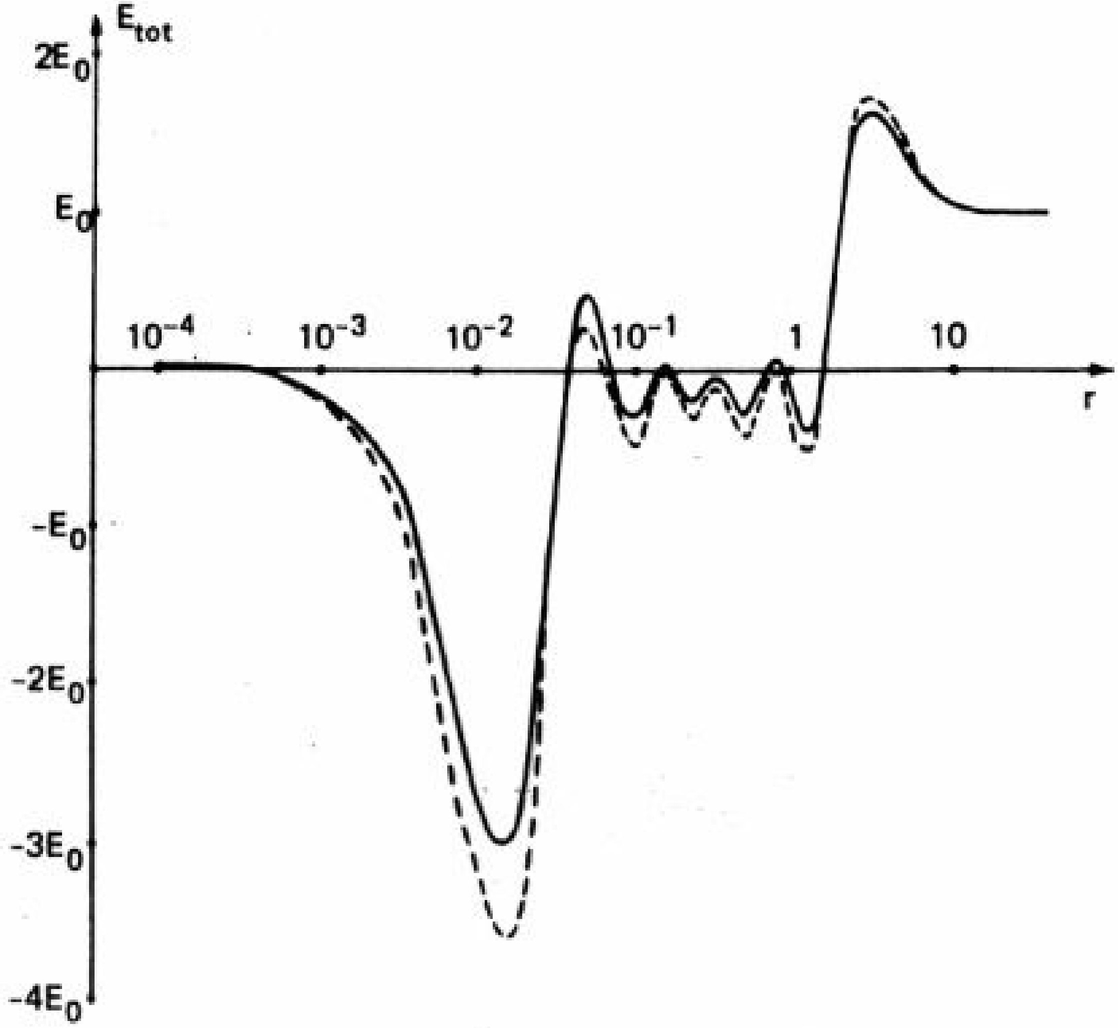


Fig. 2. Plot of electric field $E_t = E_0 + \langle E_e \rangle$ in Tl^+ on the z axis. The distance in atomic units is shown in logarithmic scale. The solid curve corresponds to $\omega = 0$, the dotted curve to $\omega = 0.207$ Ry$/\hbar$.

Screening of external electric field E_0 in atoms

$E(\infty) = E_0$, $E(0) = 0$

Nuclear Schiff moment

Nuclear electrostatic potential

$$\varphi(\mathbf{R}) = \int \frac{e\rho(\mathbf{r})}{|\mathbf{R}-\mathbf{r}|} d^3r + \underbrace{\frac{1}{Z}(\mathbf{d}\cdot\boldsymbol{\nabla}) \int \frac{\rho(\mathbf{r})}{|\mathbf{R}-\mathbf{r}|} d^3r}_{\text{electron screening term}}$$

$e\rho$ - nuclear charge density $\qquad \int \rho d^3r = Z$

$\mathbf{d} = e\int \rho \mathbf{r} d^3r \equiv e\langle \mathbf{r} \rangle$ - nuclear EDM

Nuclear EDM is screened by atomic electrons

$\rightarrow$ Schiff moment is lowest-order surviving P, T-odd nuclear electric moment

Previously, calculations performed for:

Point-like nucleus

$$\varphi^{(1)}(\mathbf{R}) = 4\pi \mathbf{S} \cdot \nabla \delta(\mathbf{R})$$

$\mathbf{S} = \frac{e}{10}[\langle r^2 \mathbf{r} \rangle - \frac{5}{3Z}\langle r^2 \rangle \langle \mathbf{r} \rangle] \ = \ S\mathbf{I}/I$

S Schiff moment

$\langle r^2 \rangle \equiv \int \rho r^2 d^3 r$

This expression is not suitable for relativistic atomic calculations

$\langle s | - e\varphi | p \rangle = 4\pi e \mathbf{S} \cdot (\boldsymbol{\nabla} \psi_s^\dagger \psi_p)_{R=0}$

$= \mathrm{constant}$ for nonrelativistic wave functions

$\to \infty$ for relativistic wave functions

$\psi_s \sim R^{-Z^2\alpha^2/2} \to \infty$ as $R \to 0$

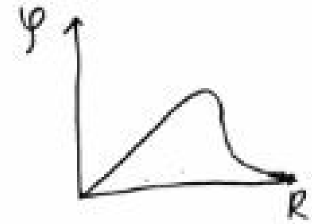
TABLE OF LIMITS

We have obtained a more suitable expression for $\varphi^{(1)}(\mathbf{R})$:

$$\varphi^{(1)}(\mathbf{R}) = -\frac{3\mathbf{S}\cdot\mathbf{R}}{B}\rho(R) \qquad < R_N - \delta$$

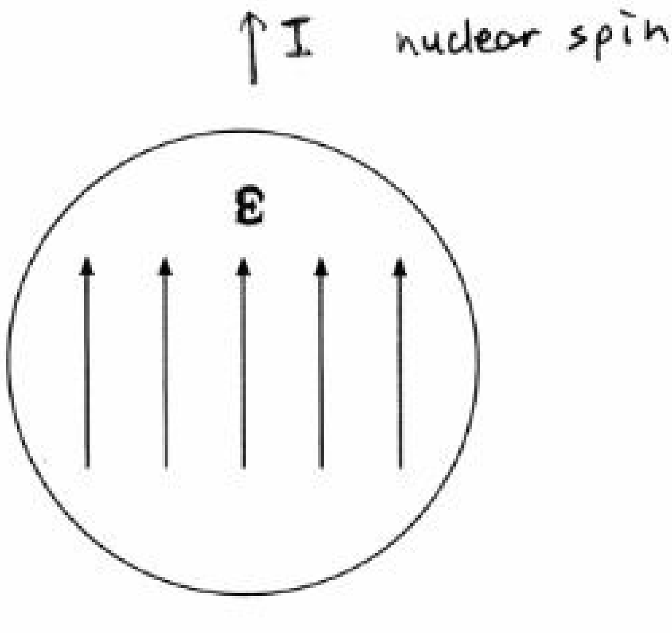
$$= 0 \qquad > R_N + \delta$$

$B = \int \rho(R) R^4 dR \approx R_N^5/5$



For Hg, this agrees with $\varphi^{(1)}$ induced by L to about 10%

$\Rightarrow$ electric field distribution $\varepsilon = -\nabla\varphi^{(1)} \propto \mathbf{I}$ corresponding to the Schiff moment:



Electric field induced by T, P-odd nuclear forces which influence proton charge density.

Final result: Dzuba, Flambaum, Ginges, Kozlov

$$d(^{199}\mathrm{Hg}) = -2.8 \times 10^{-17}\Big(\frac{S}{e\ \mathrm{fm}^3}\Big)e\ \mathrm{cm}$$

Accuracy $\approx$ 20%

c.f. old estimate [Flambaum, Khriplovich, Sushkov (1985)]

$$d(^{199}\mathrm{Hg}) = -4 \times 10^{-17}\Big(\frac{S}{e\ \mathrm{fm}^3}\Big)e\ \mathrm{cm}$$

$\Rightarrow$ more conservative limit on P,T-violating parameters

Nuclear calculation

$S =$

S/L obtained from nuclear calculations

Schiff moment/ LDM generated mainly by the P, T-odd nucleon-nucleon interaction

$$\hat{W}_{ab} = \frac{G}{\sqrt{2}}\frac{1}{2m}[(\eta_{ab}\boldsymbol{\sigma}_a - \eta_{ba}\boldsymbol{\sigma}_b)\cdot\boldsymbol{\nabla}_a\delta(\mathbf{r}_a - \mathbf{r}_b)$$
$$+\eta'_{ab}\,[\boldsymbol{\sigma}_a \times \boldsymbol{\sigma}_b]\cdot\{(\mathbf{p}_a - \mathbf{p}_b), \delta(\mathbf{r}_a - \mathbf{r}_b)\}]$$

In Hg, external nucleon is neutron

$\Rightarrow$ P, T-odd interaction (η_{np}) of external neutron with core polarizes charge density

Numerical calculation [Flambaum, Khriplovich, Sushkov (1985)]

^{199}Hg $$S = -1.4 \times 10^{-8}\eta_{np} e\ \mathrm{fm}^3$$

We performed relativistic corrections (analytically) to S using "giant resonance approach" [Flambaum, Khriplovich, Sushkov (1986)]

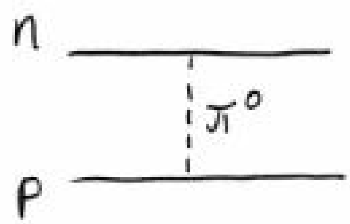
$\Rightarrow$ "Local dipole moment" $L = S(1 - 0.3Z^2\alpha^2) \approx 0.8S$

Best limit on an atomic EDM [Seattle (2001)]:

$$\mathrm{d}(^{199}\mathrm{Hg}) = -(1.06 \pm 0.49 \pm 0.40) \times 10^{-28}\ e\ \mathrm{cm}$$

Induced primarily by the nuclear Schiff moment S

$$d = 4 \cdot 10^{-25}\ e \cdot cm \cdot \eta_{np}$$



$$\frac{G}{\sqrt{2}}\,\eta_{np} \sim \frac{g\,\bar{g}^{\,0}_{\pi NN}}{m_\pi^2}$$

$$\bar{g}_{\pi NN} \sim \bar{\theta}$$

TABLES

TABLE I. Limits on P,T-violating parameters in the hadronic sector extracted from ^{199}Hg compared with the best limits from other experiments. We omit the signs of the central points. Errors are experimental. Some relevant theoretical works are presented in the last column.

P,T-violating term	Value	System	Exp.	Theory
neutron EDM d_n	$(17 \pm 8 \pm 6) \times 10^{-26}$ e cm	^{199}Hg	[1]	[2,3]
	$(1.9 \pm 5.4) \times 10^{-26}$ e cm	neutron	[4]	
	$(2.6 \pm 4.0 \pm 1.6) \times 10^{-26}$ e cm	neutron	[5]	
proton EDM d_p	$(1.7 \pm 0.8 \pm 0.6) \times 10^{-24}$ e cm	^{199}Hg	[1]	[2,3,6]
	$(17 \pm 28) \times 10^{-24}$ e cm	TlF	[7]	[2,8]
$\eta_{np} i \frac{G}{\sqrt{2}} \bar{p}p\bar{n}i\gamma_5 n$	$\eta_{np} = (2.7 \pm 1.3 \pm 1.0) \times 10^{-4}$	^{199}Hg	[1]	[9]
$\bar{g}^0_{\pi NN}$	$(3.0 \pm 1.4 \pm 1.1) \times 10^{-12}$	^{199}Hg	[1]	[3]
$G\tilde{G}$ QCD phase $\bar{\theta}$	$(1.1 \pm 0.5 \pm 0.4) \times 10^{-10}$	^{199}Hg	[1]	[10,3]
	$(1.6 \pm 4.5) \times 10^{-10}$	neutron	[4]	[11]
	$(2.2 \pm 3.3 \pm 1.3) \times 10^{-10}$	neutron	[5]	[11]
CEDMs $\tilde{d}$ and	$e(\tilde{d}_d - \tilde{d}_u) = (1.5 \pm 0.7 \pm 0.6) \times 10^{-26}$ e cm	^{199}Hg	[1]	[12]
EDMs d of quarks	$e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.3d_d - 0.3d_u$			
	$= (3.5 \pm 9.8) \times 10^{-26}$ e cm	neutron	[4]	[13]
	$= (4.7 \pm 7.3 \pm 2.9) \times 10^{-26}$ e cm	neutron	[5]	[13]

Limits on η_{np} and d_n $\rightarrow$

$\rightarrow$ Weinberg model of CP-violation closed

Supersymmetric models – close values!

and where to now?

- Experiments with Hg continue
- EDM experiments with Rn and Ra are in preparation

Rn	Ann Arbor	, TRIUMF
Ra	Los Alamos / Duke Groningen, Argonne, ...	?

These atoms have deformed nuclei

Schiff moments are enhanced $\sim 10^3$ in nuclei with static [Auerbach, Flambaum, Spevak] or even soft [Engel, Friar, Hayes] octupole deformation
Flambaum, Zelevinsky : S(soft) ≈ S(static)

There is also an electronic enhancement

$$\Rightarrow d(\mathrm{Rn}),\ d(\mathrm{Ra}) \sim 1000 \times d(\mathrm{Hg})$$

There is a huge electronic enhancement for Ra in state $^3D_2$ (due to very close $\sim 5\ \mathrm{cm}^{-1}$ states of opposite parity),

$$d(\mathrm{Ra}) \sim 10^5 \times d(\mathrm{Hg})$$

[Flambaum (1999)], [Dzuba, Flambaum, Ginges (2000)]

Violation of fundamental SYMMETRIES (P, T) in atoms and test of Standard Model.

1. Parity violation in Cs –
no deviation from Standard Model !

2. Nuclear anapole moment –
parity violating magnetic moment.

3. Schiff moment – electric
moment violating P and T.

4. Huge enhancement of P, T
- violation in odd (unstable)
isotopes of Ra, Rn, Fr.
Electric dipole moments – up to 10^5.
Anapole - up to 10^3.
Weak charge – up to 10^2.

5. Supersymmetric models of
CP – violation – about to
be confirmed or eliminated
by atomic EDM measurements

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VIOLATIONS OF FUNDAMENTAL SYMMETRIES IN ATOMS AND TESTS OF UNIFICATION THEORIES OF ELEMENTARY PARTICLES

J.S.M. GINGES, V.V. FLAMBAUM

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