Two-photon exchange: hadronic picture

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Outline

- Introduction: Rosenbluth vs polarization measurements of G_E and G_M of nucleon
  puzzle: different results extracted for G_E/G_M
- Hadronic model of two-photon exchange (TPEX)
- Results for unpolarized and polarized cross sections e+p! e+p (real part of TPEX)
- Resonance contribution (Δ) to elastic scattering
- TPEX in Δ production
- Low Q^2
  - Parity violating asymmetry A_{PV}
  - Proton radii (rms and Zemach)

Rosenbluth separation method

One-photon exchange cross section

$$d\sigma_0 = A\left(G_M^2 + \frac{\epsilon}{\tau}G_E^2\right)$$

$$\tau \equiv \frac{Q^2}{4M^2} \qquad \frac{1}{\varepsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

- ε is virtual photon polarization
  - Forward scattering ε! 1
  - Backward scattering ε ! 0
- Extract G_E and G_M from linear ε dependence
- G_E suppressed as Q^2 increases
  - Sensitive to small corrections linear in ε

Polarization transfer method

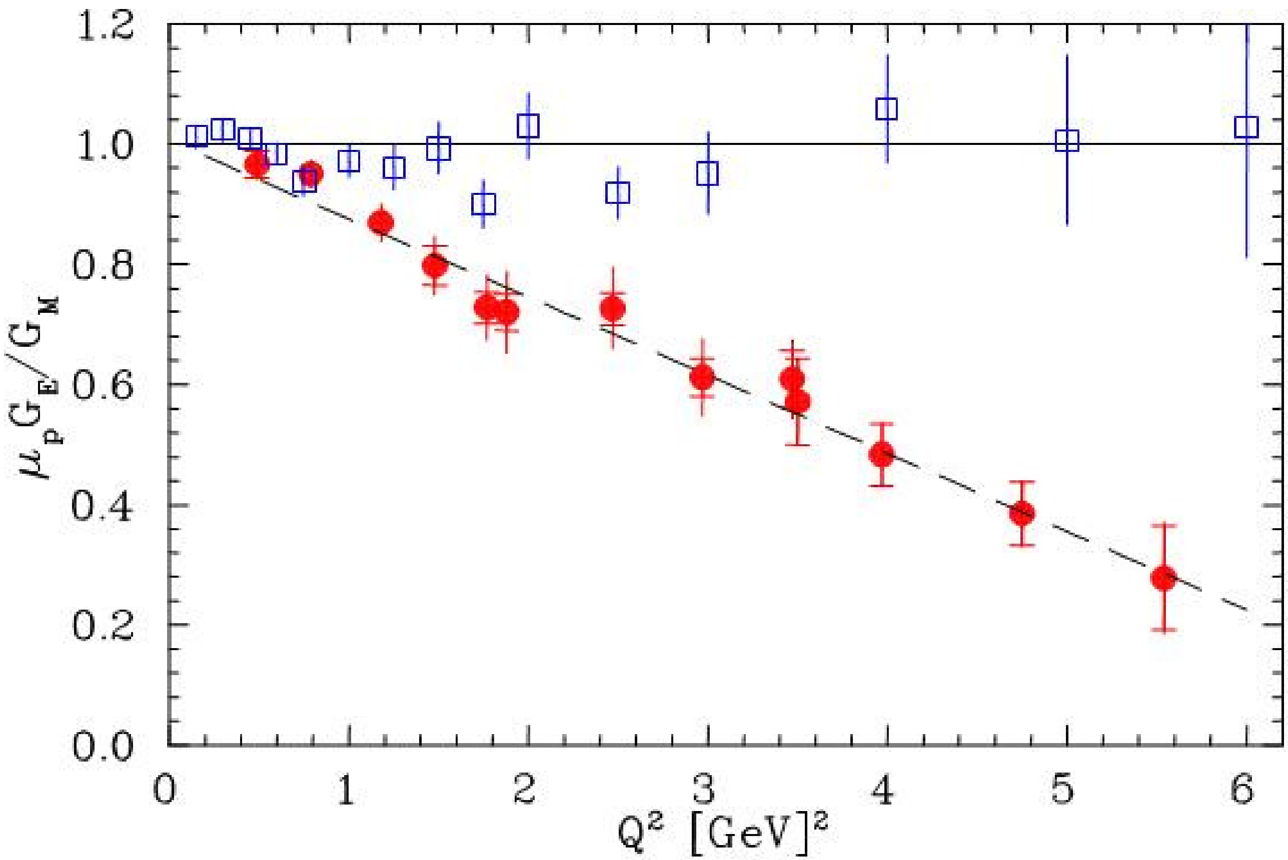
$$\vec{e} + p \rightarrow e + \vec{p}$$

- Look at ratio of transverse (P_T) to longitudinal (P_L) components of recoil proton polarization using a longitudinally polarized electron beam
- Doesn't depend on absolute normalization
- Ratio relatively insensitive to radiative corrections (e.g. bremsstrahlung corrections cancel)

$$\frac{P_T}{P_L} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\frac{G_E}{G_M}$$

in one-photon exchange approximation

Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



SLAC

Rosenbluth data

JLab

Polarization data

Speculation: radiative corrections

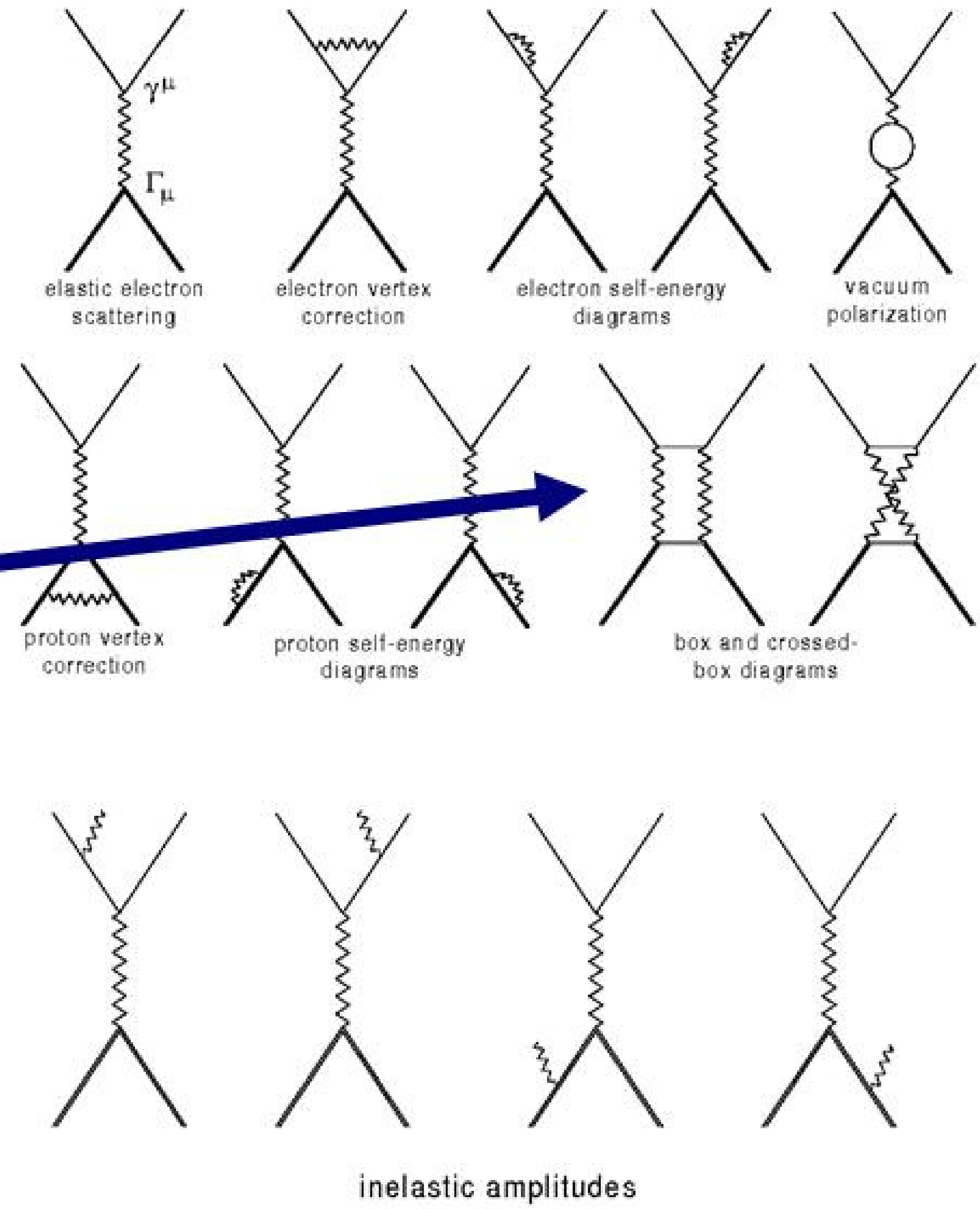
Missing effect is

- approximately linear in ε
- not strongly Q^2 dependent

Two-photon exchange

Bremsstrahlung

- SuperRosenbluth (detect proton)



Comments on radiative corrections

- Radiative corrections different depending on whether electron or proton is detected.

  well understood

- Soft bremsstrahlung

  involves long-wavelength photons

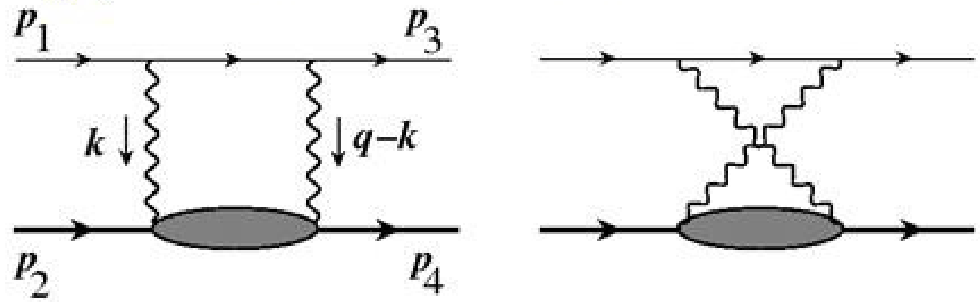
  independent of hadronic structure

- Box diagrams (TPEX $M_{\gamma\gamma}$) involve photons of all wavelengths

  long wavelength (soft photon) part is included in radiative correction (IR divergence is cancelled with electron proton bremsstrahlung interference)

  also independent of hadronic structure (by construction)

Hadronic approach: N, Δ, … intermediate states



- Tsai (1961), Mo & Tsai (1968)
  - box diagram calculated using only nucleon intermediate state and using k! 0 or k! q in both numerator and denominator (calculate 3-point function) ! gives correct IR divergent terms
  - Used in standard analyses for radiative corrections
  - No form factor in loop integral, so no hadronic dependence by construction
- Maximon & Tjon (2000)
  - same as above, but make the above approximation only in numerator (calculate 4-point function)

$\delta_{IR} \sim \ln(Q^2/\lambda^2)$ λ **dependence cancelled by bremsstrahlung**

- Blunden, Melnitchouk, Tjon (2003)
  - further improvement by keeping the full numerator + use on-shell nucleon form factors in loop integral
  - loop integrals evaluated analytically
  - consider only the additional terms not in standard treatment of Mo/Tsai

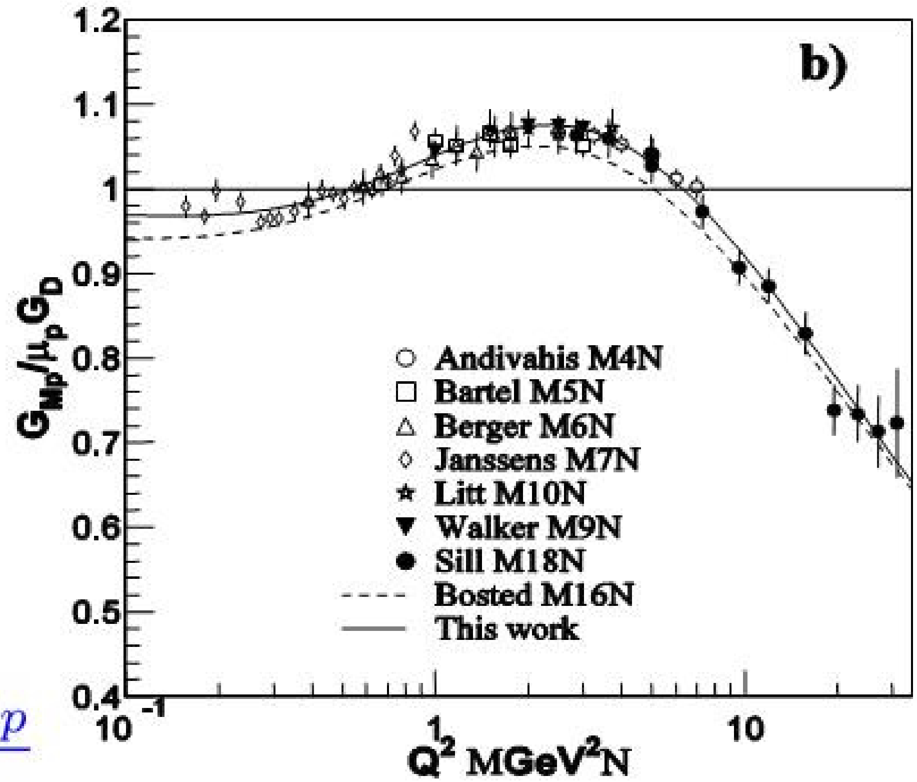
$$\Delta = \delta_{full} - \delta_{IR}\,(\text{MTsai}) \qquad \text{(IR finite)}$$
$$\sim Q^2\,(a + b\ln(Q^2) + c\,Q^2 + \dots)$$

  - used simple monopole form factors
  - right ε dependence, explains about $\frac{1}{2}$ the discrepancy

- Chen et al. (2004, 2005)
  - Partonic model using GPD's
  - shows high mass continuum can also contribute significantly

Model form factors used as input in calculation

magnetic proton form factor
Brash et al. (2002)



electric proton form factor :

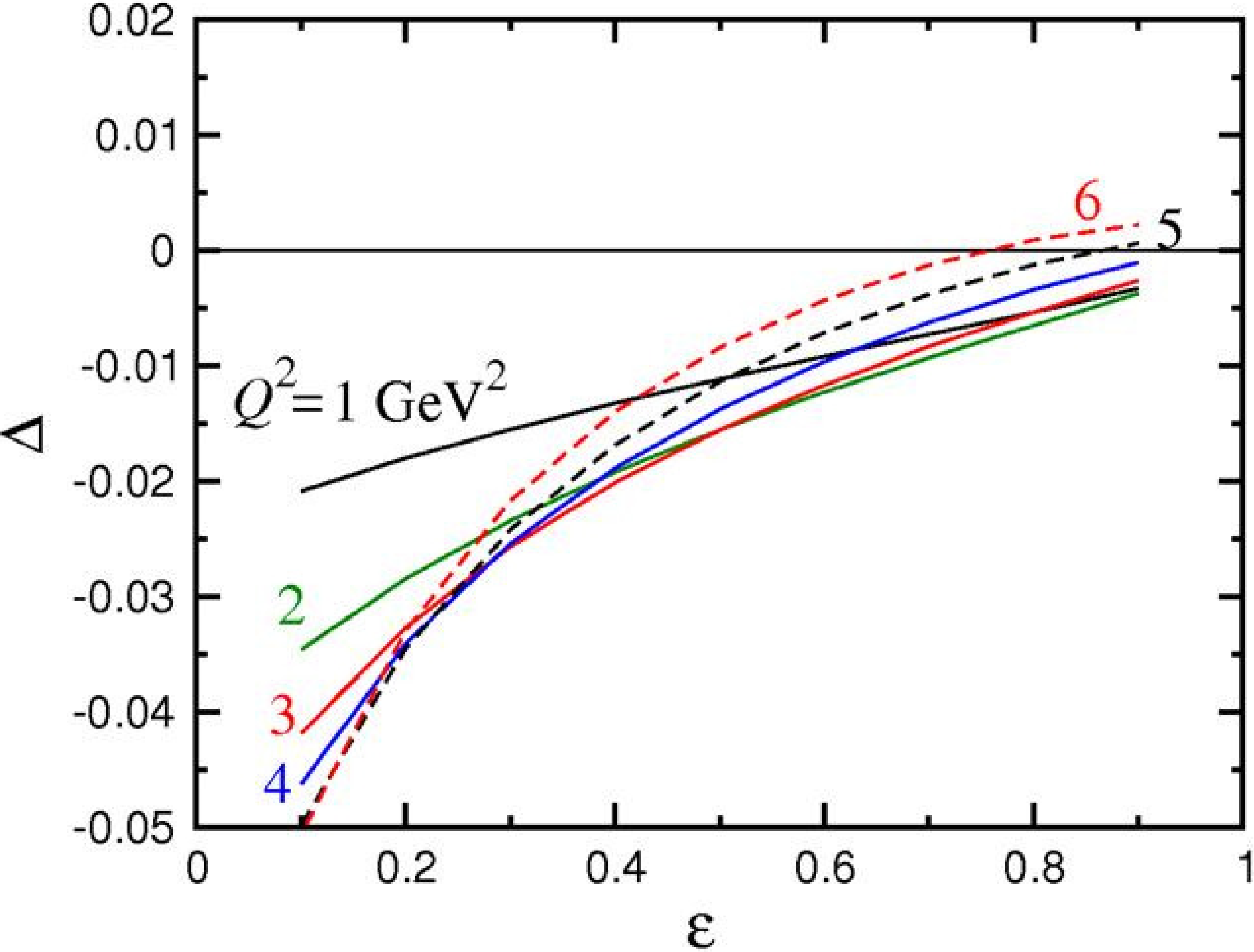
G_E/G_M of proton fixed from polarization data
Gayou et al. (2002)

$$G_{E\,p} = \left(1 - 0.13(Q^2 - 0.04)\right) \frac{G_{M\,p}}{\mu_p}$$

Parametrize as sum of monopoles
! maintains analytic form of result

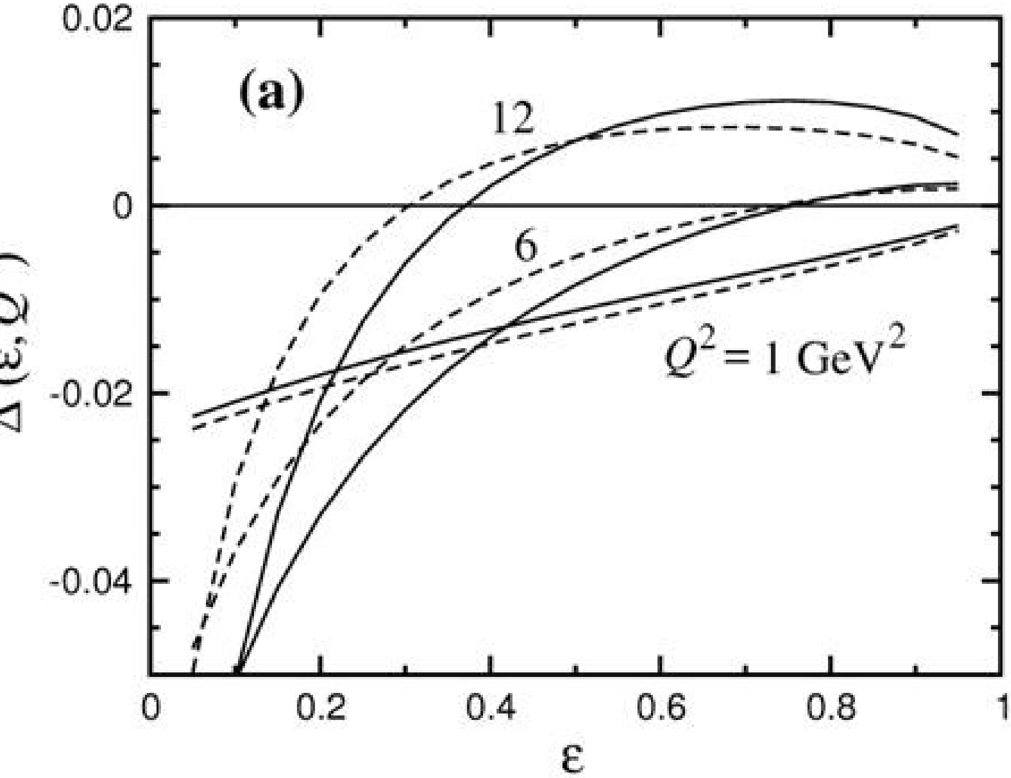
Numerical results not terribly sensitive to model for G_E, or to details of G_M

Corrections to unpolarized cross sections for Q^2=1 to 6 GeV^2

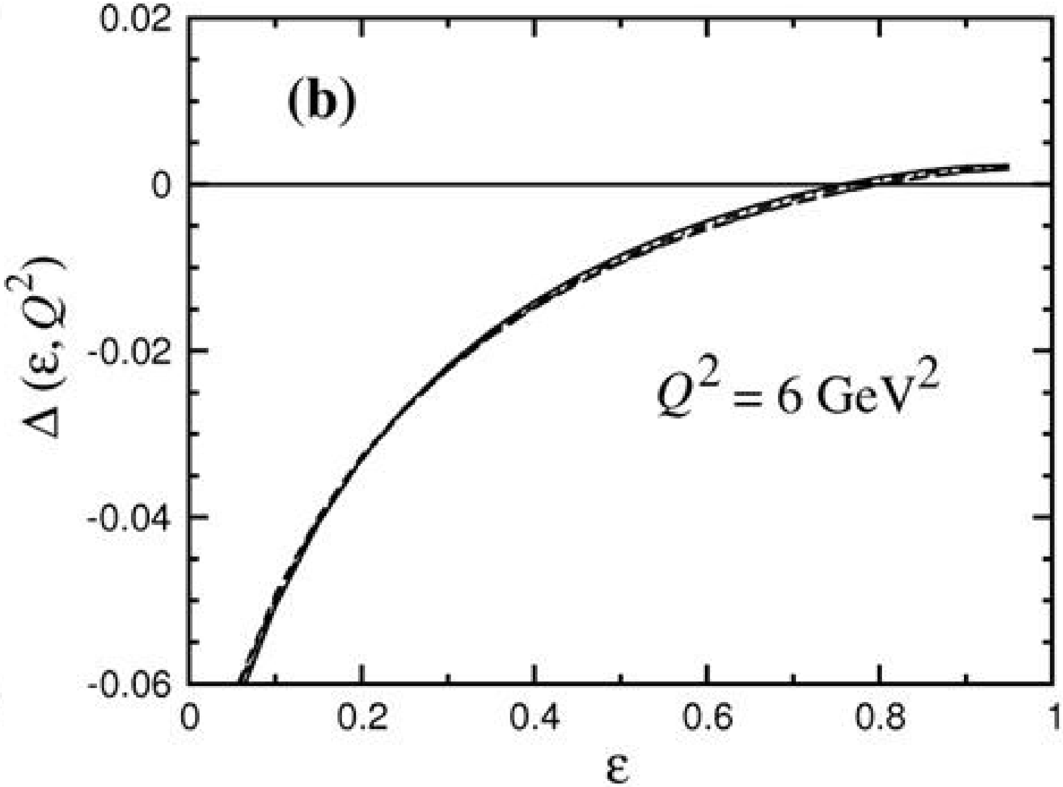


- Effect largest at small ε (backward angles)
- No effect as ε! 1
  ε ~ 1 - $Q^2/(2 E^2)$
- Nonlinearity grows with Q^2
- JLAB E05-017 (Arrington) will set limits on nonlinearity

Dependence on form factor in loop integrals



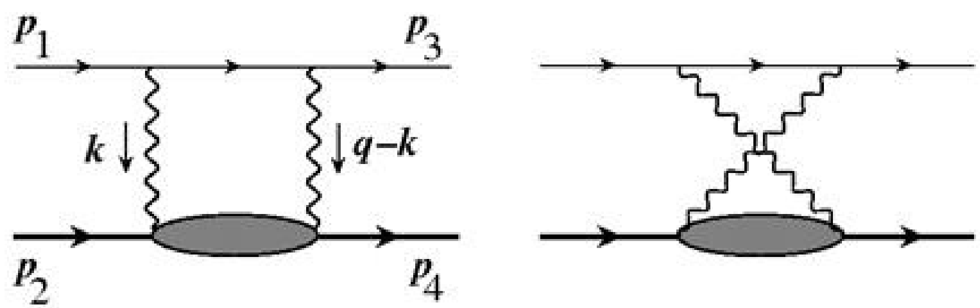
Realistic (solid) vs. dipole (dashed)



Effect of LT and PT values for G_E/G_M on TPEX correction

! Mostly sensitive to G_M

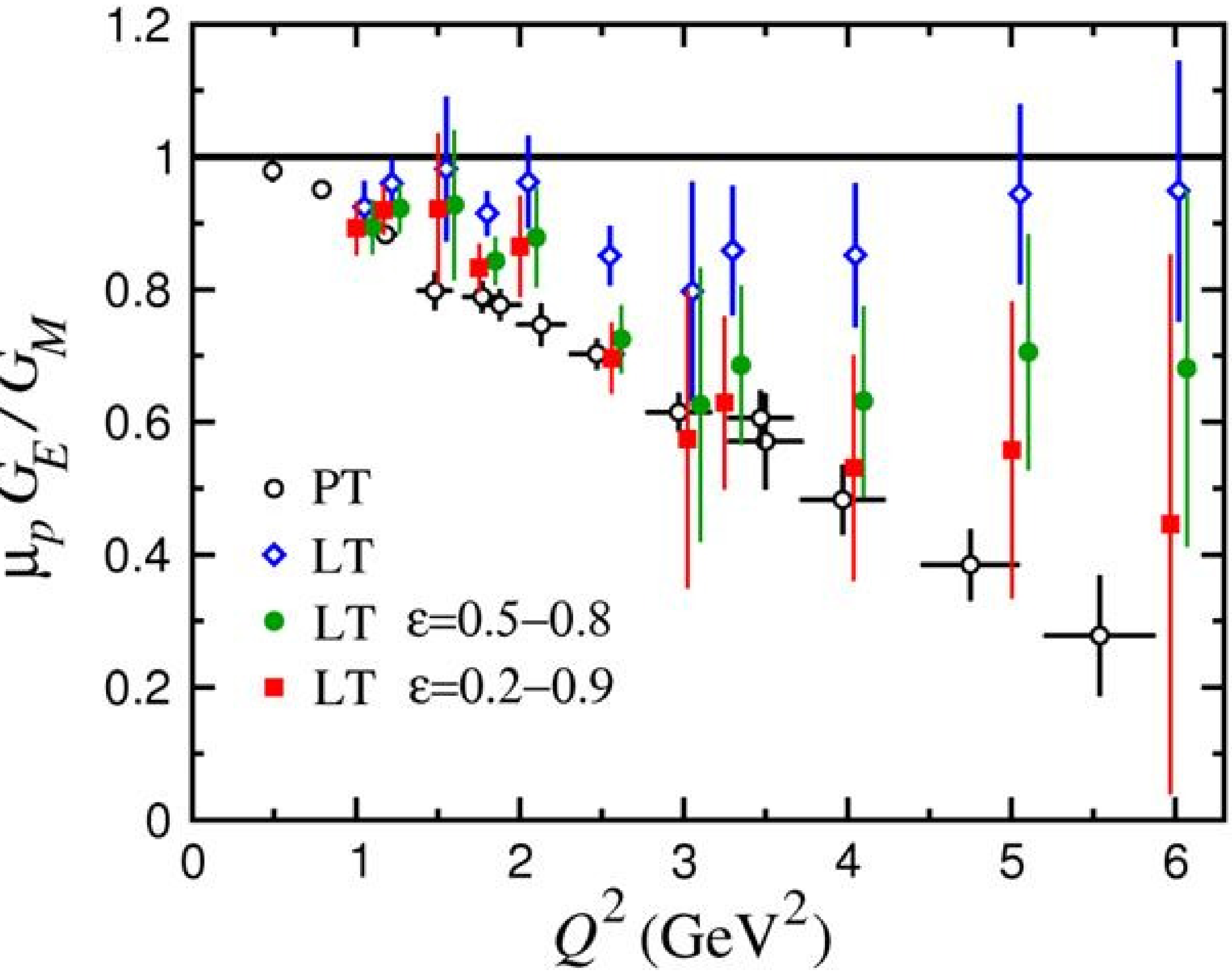
Crossing symmetry



Crossed box from box by $p_1 ! -p_3$

$$
\begin{aligned}
f^{2\gamma}(s,t) &= f^{2\gamma}_{box}(s,t) + f^{2\gamma}_{xbox}(s,t) \\
f^{2\gamma}_{xbox}(s,t) &= -f^{2\gamma}_{box}(u,t)|_{u=2M-t-s} \\
f^{2\gamma}(s,t) &= -f^{2\gamma}(2M^2 - t - s, t)
\end{aligned}
$$

Effect on ratio R

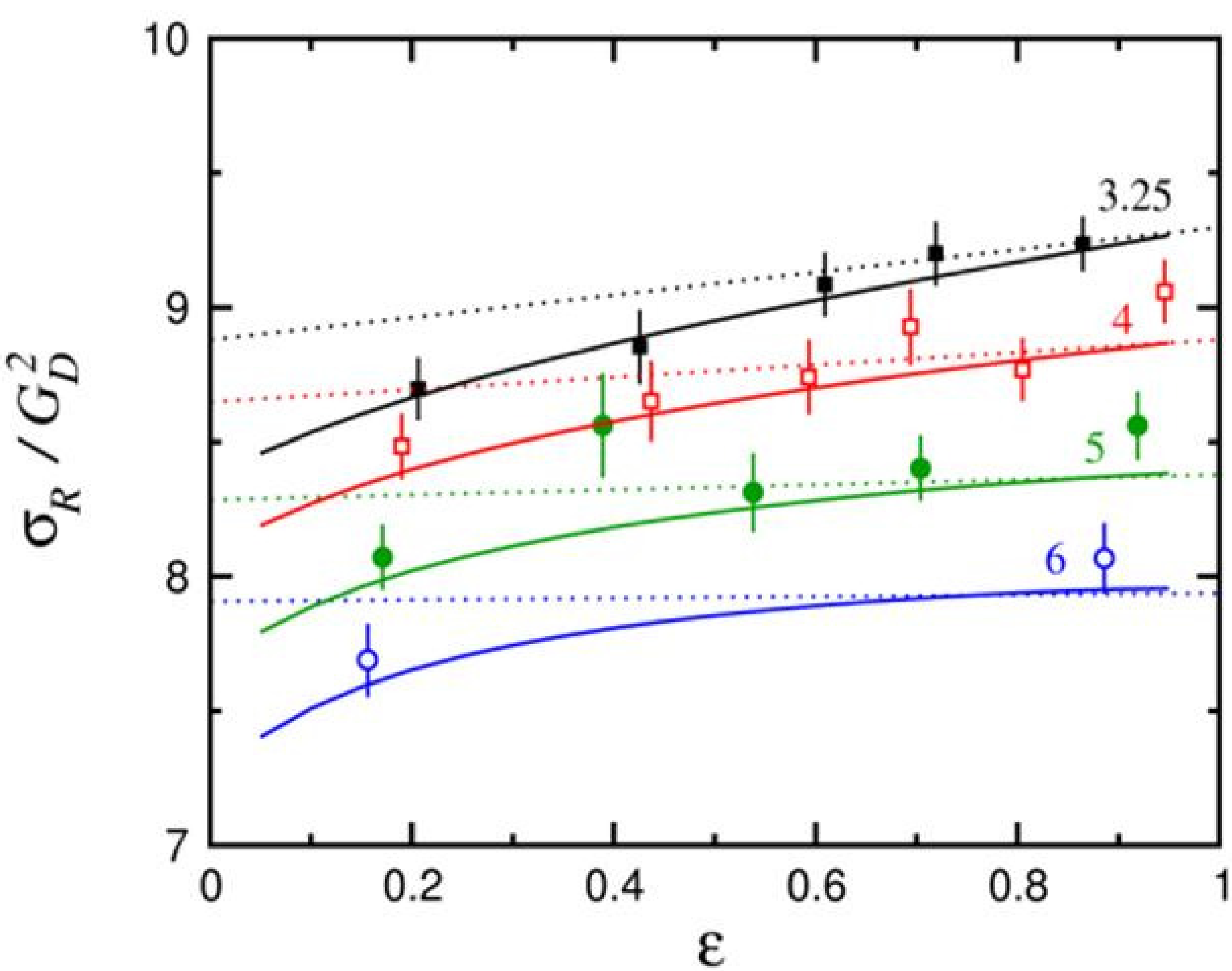


NOT a refit of data

Simple model: correct Rosenbluth data assuming TPEX correction is linear in ε over a certain range

Effect on SLAC reduced cross sections at different Q^2

(normalized to dipole G_D^2)

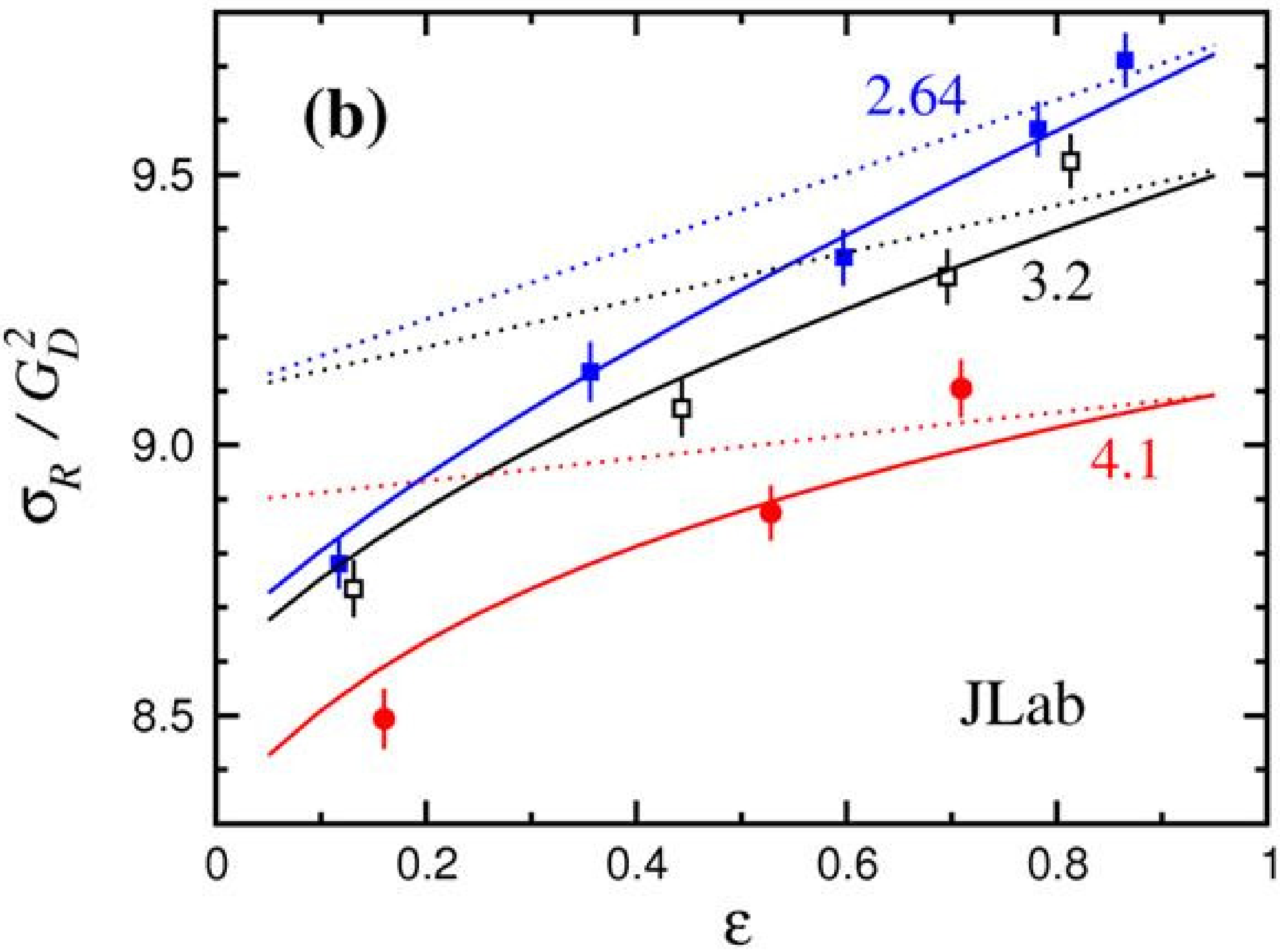


Nonlinearity in ε is displayed here

JLAB proposals to measure nonlinearity

For effect in SuperRosenbluth expt. see upcoming talk by Arrington

SuperRosenbluth (JLAB) data

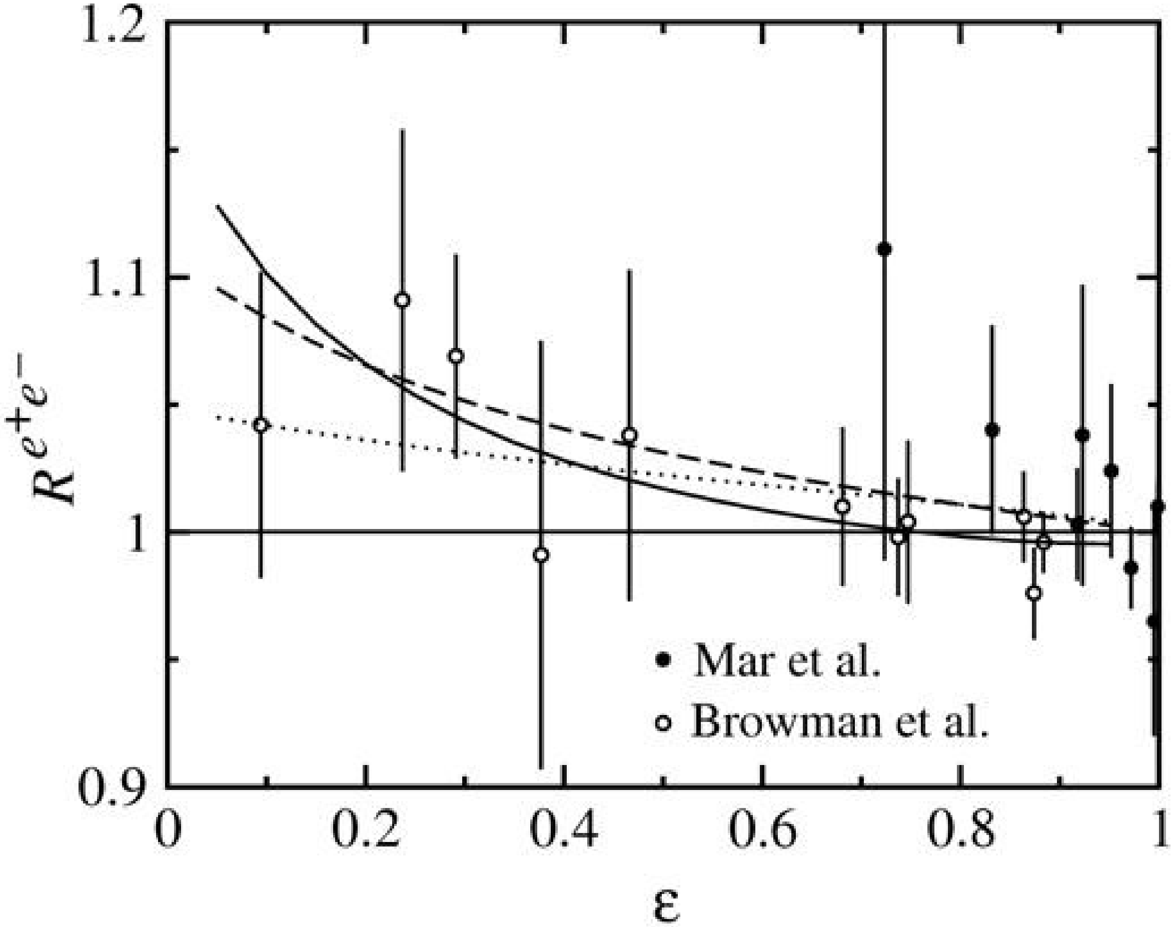


Curves shifted by

+1.0%	2.64
+2.1%	3.20
+3.0%	4.10

(Effect on determination of G_M)

Effect on ratio of e^+p to e^-p cross sections (SLAC, Q^2 from 0.01 to 5 GeV^2)



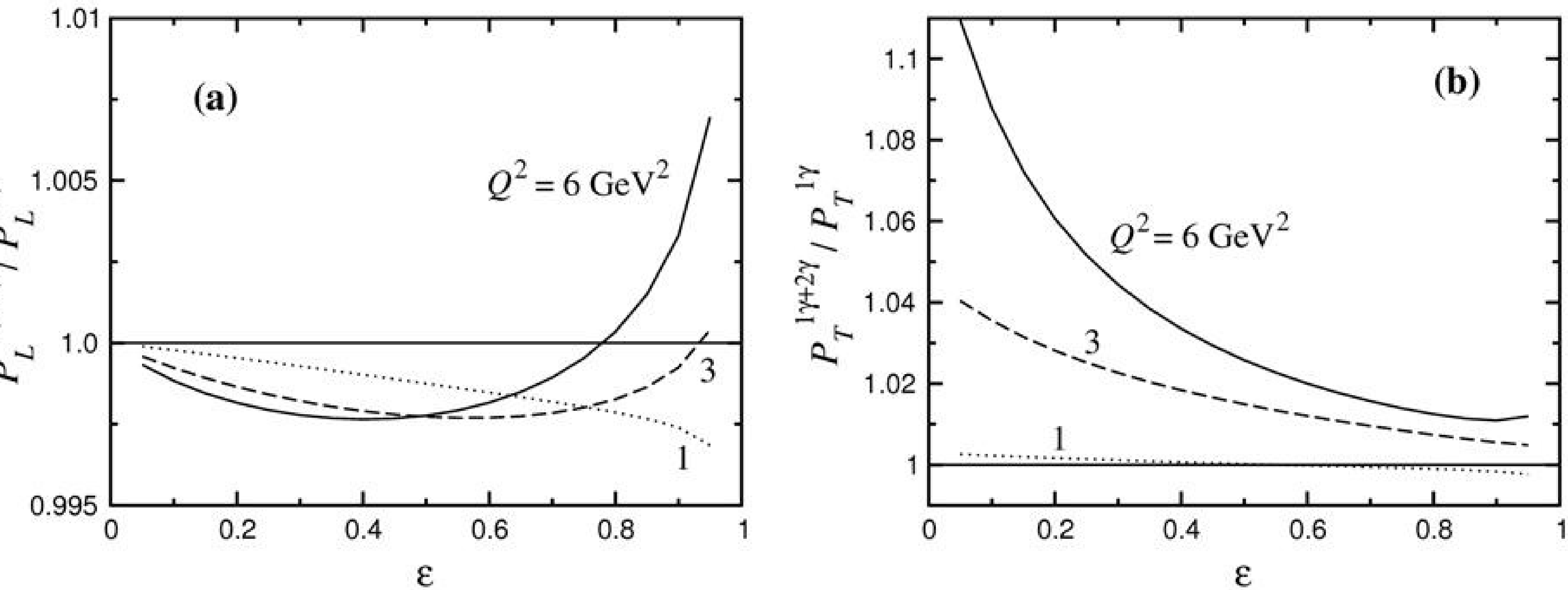
- M_{Born} opposite sign for e^+p vs. e^-p, so enhancement instead of suppression as ε! 0

- R(e^+p/e^-p) ¼ 1-2Δ

- Curves are elastic results for Q^2=1, 3, 6 GeV^2

$\vec{e} + p \rightarrow e + \vec{p}$

Corrections to P_L and P_T at Q^2=1, 3, and 6 GeV^2



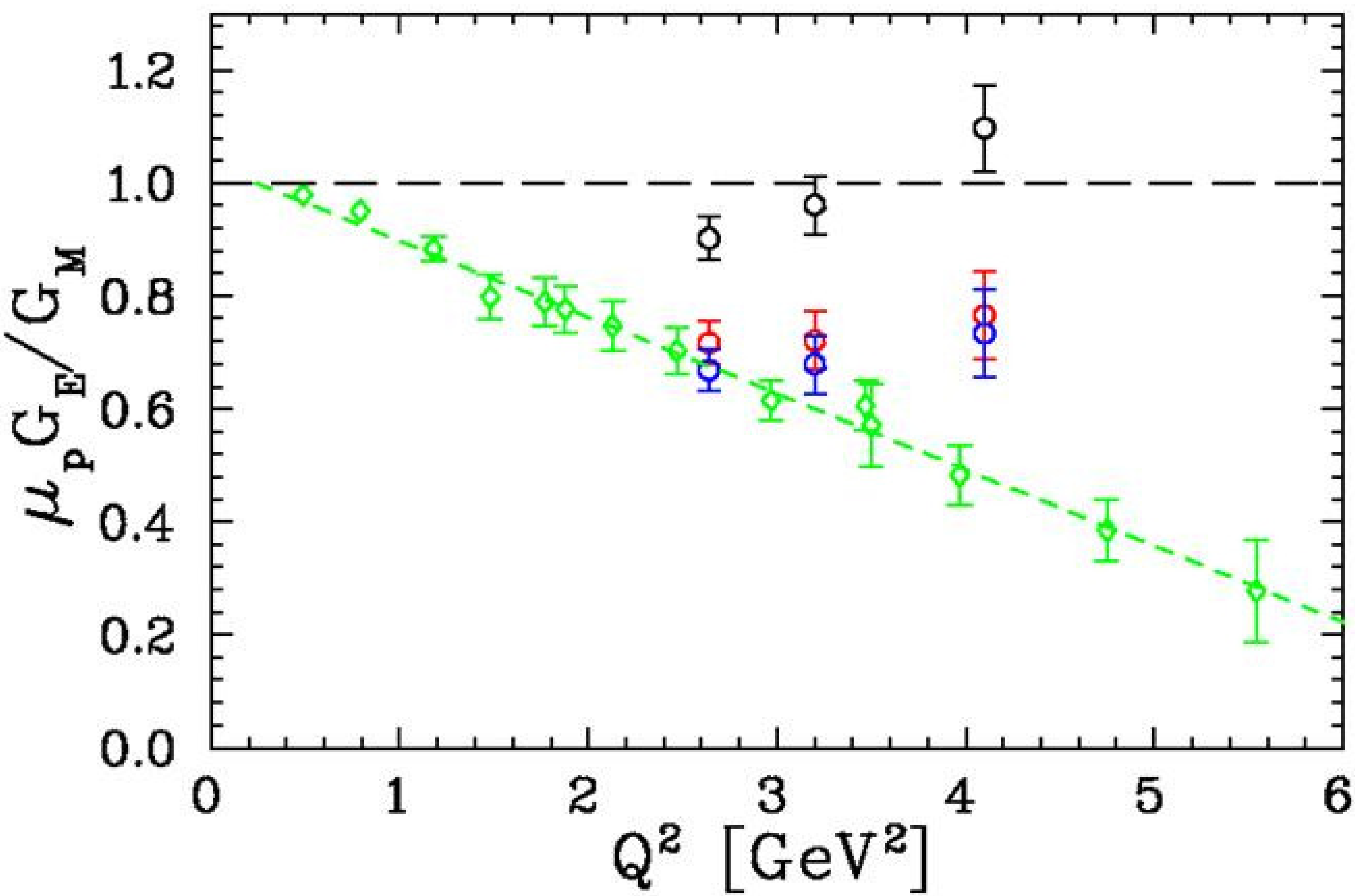
P_T/P_L will show some variation with ε, esp. at low ε
JLab data taken at ε~0.7

JLAB expt (Gilman) will measure P_T/P_L at low ε

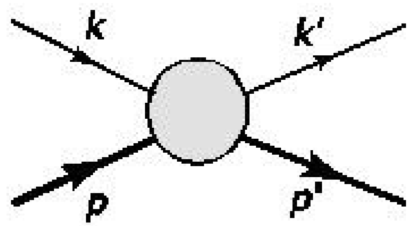
GPD calculation predicts suppression of P_T/P_L

Other effects: Coulomb distortion (Arrington & Sick)

Small, but not negligible. Should be put in.



Phenomenology: *Generalized* form factors



$$P \equiv \frac{p+p'}{2}, \quad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$Q^2 = -(p-p')^2$$
$$\nu = K \cdot P = (s-u)/4$$

In limit m$_e$! 0 (helicity conservation) general amplitude can be put in form

$$T = (\gamma_\mu)^{(e)} \left(\tilde{F}_1 \gamma^\mu + i \frac{\tilde{F}_2}{2M} \sigma^{\mu\nu} q_\nu + \frac{F_3}{M^2} \gamma \cdot K P^\mu \right)^{(p)}$$

Generalized (complex) form factors

$$\tilde{F}_1(\nu, Q^2) = F_1(Q^2) + \delta F_1$$
$$\tilde{F}_2(\nu, Q^2) = F_2(Q^2) + \delta F_2$$

$$\tilde{G}_M = \tilde{F}_1 + \tilde{F}_2$$
$$\tilde{G}_E = \tilde{F}_1 - \tau \tilde{F}_2$$
$$Y_2 = \frac{\nu}{M^2} \frac{F_3}{G_M}$$

Observables including two-photon exchange

Real parts of two-photon amplitudes

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\left\{\epsilon\left(\frac{\delta G_E}{G_E}\right)G_E^2 + \tau\left(\frac{\delta G_M}{G_M}\right)G_M^2 + \epsilon Y_2(\tau G_M^2 + G_M G_E)\right\}}{\epsilon G_E^2 + \tau G_M^2}$$

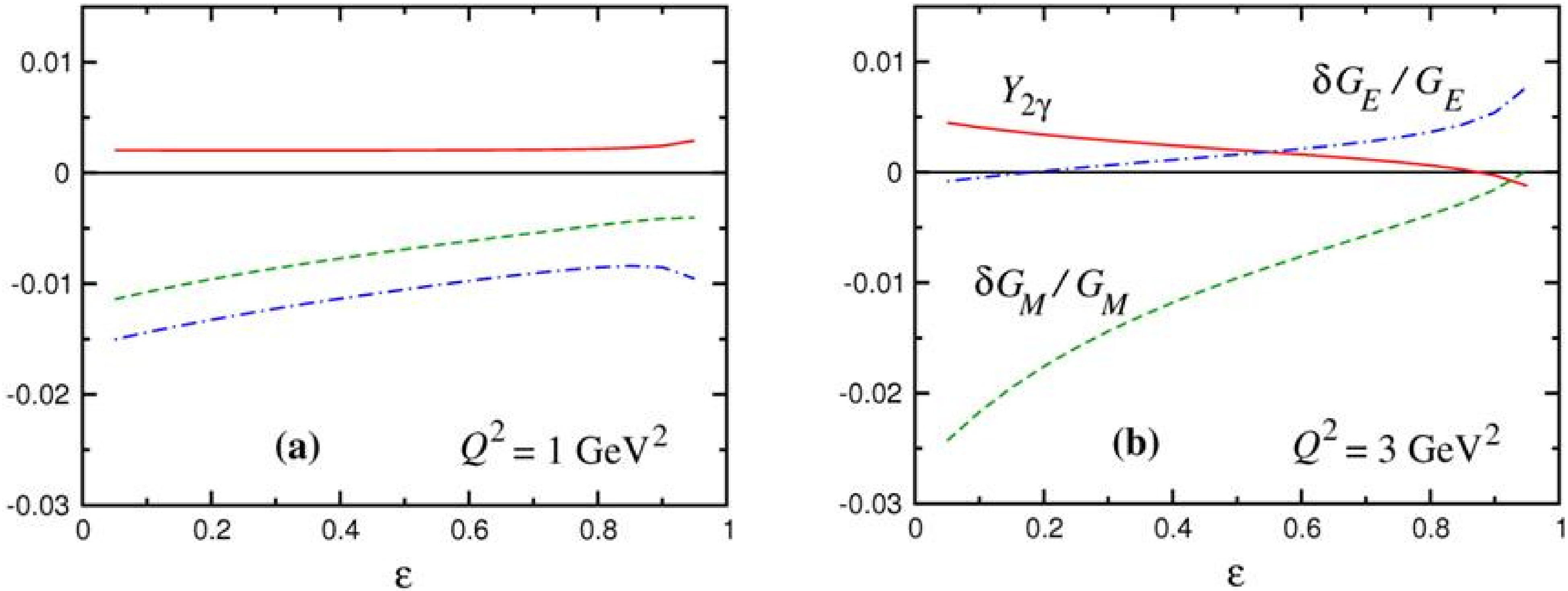
$$\frac{\delta P_L}{P_L} = 2\left(\frac{\delta G_M}{G_M}\right) + 2\frac{\epsilon}{1+\epsilon}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

$$\frac{\delta P_T}{P_T} = \left(\frac{\delta G_M}{G_M}\right) + \left(\frac{\delta G_E}{G_E}\right) + \frac{G_M}{G_E}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

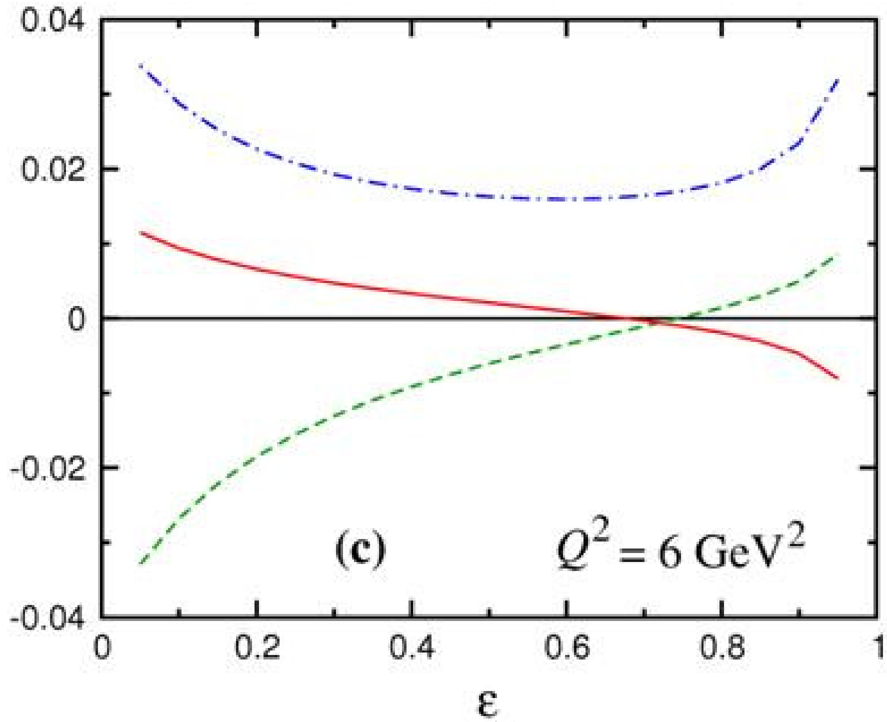
Caution needed about assumptions (generalized FF's are not observables)

- Parametrization of amplitude NOT unique

  Axial parametrization: A_3 $(\gamma_\mu\gamma_5)^{(e)}$ $(\gamma^\mu\gamma_5)^{(p)}$ instead of F_3 (or Y_2) term shifts some F_3 into δF_1 (and hence into δG_E and δG_M)



Real part of elastic results



Attempt to constrain parameters by looking at all data (nonlinearity, e^+/e^- ratio, P_L, P_T): Arrington

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\left\{\epsilon\left(\frac{\delta G_E}{G_E}\right)G_E^2 + \tau\left(\frac{\delta G_M}{G_M}\right)G_M^2 + \epsilon Y_2(\tau G_M^2 + G_M G_E)\right\}}{\epsilon G_E^2 + \tau G_M^2}$$

$$\frac{\delta P_L}{P_L} = 2\left(\frac{\delta G_M}{G_M}\right) + 2\frac{\epsilon}{1+\epsilon}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

$$\frac{\delta P_T}{P_T} = \left(\frac{\delta G_M}{G_M}\right) + \left(\frac{\delta G_E}{G_E}\right) + \frac{G_M}{G_E}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

Example: Consider ε! 1   δσ! 0 independent of τ

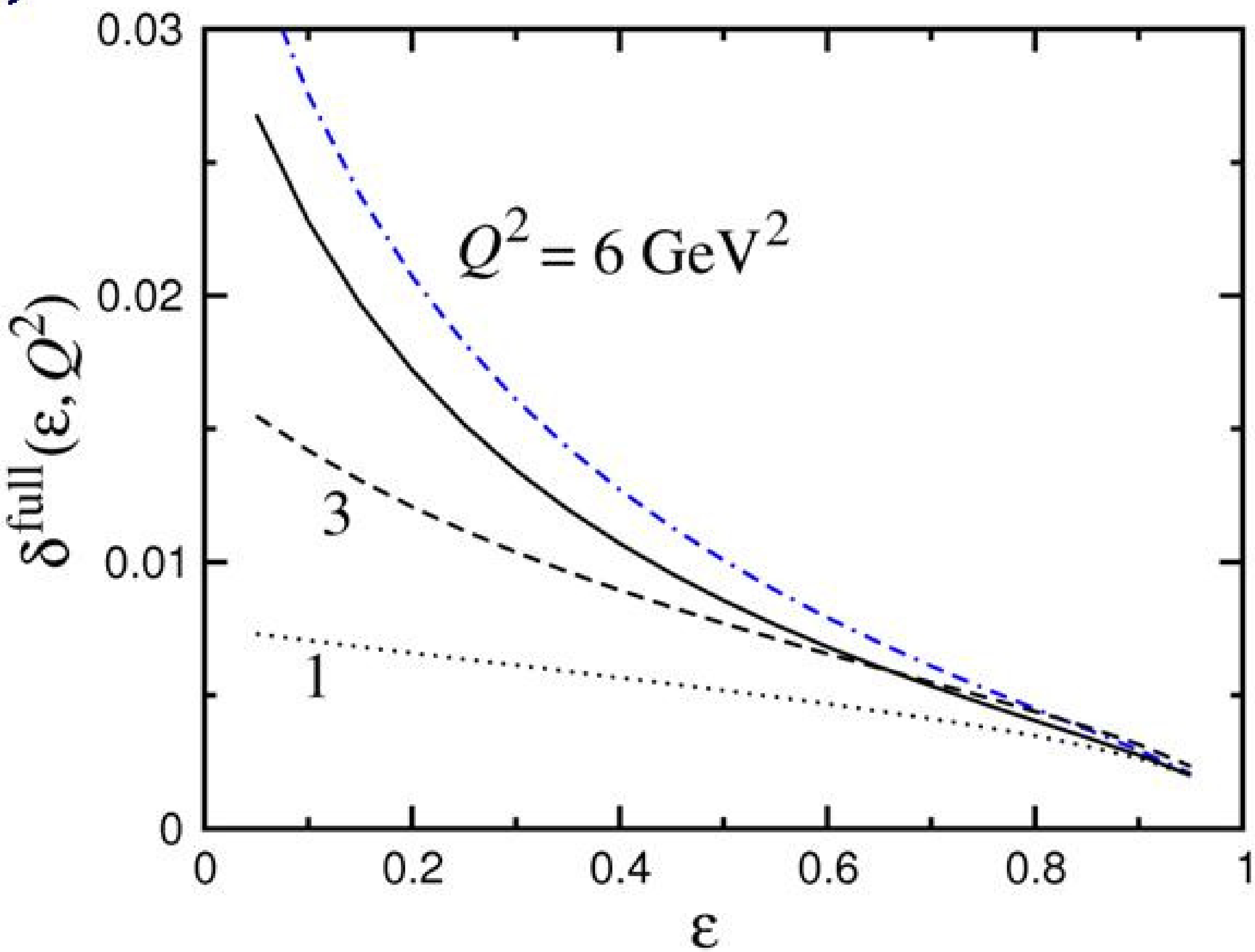
So $\delta G_E = \delta G_M$ and $Y_2 = -(\delta G_M/G_M)$

Therefore $(\delta P_L/P_L) = -Y_2$; $(\delta P_T/P_T) = -2Y_2$

Important for Parity violating asymmetry at forward angles (e.g. Qweak)

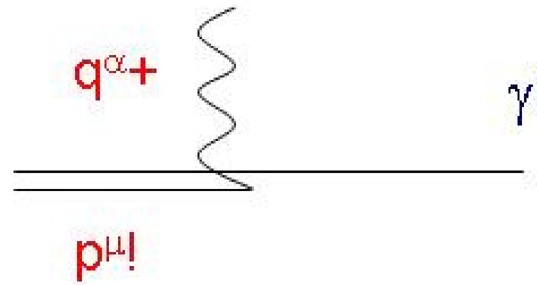
Neutron

- No infrared divergences
- Positive and about 2-3 times smaller than proton (dominance of magnetic form factor?)
- Some model dependence due to choice of form factors (blue curve)



Resonance (Δ) contribution:

$\gamma(q^\alpha)$ + $\Delta(p^\mu)$! N



γNΔ vertex

- Lorentz covariant form
- Spin ½ decoupled
- Obeys gauge symmetries

$$p_\mu \Gamma^{\alpha\mu}(p,q) = 0$$
$$q_\alpha \Gamma^{\alpha\mu}(p,q) = 0$$

$$\Gamma^{\alpha\mu}_{\gamma\Delta\to N}(p,q) = \frac{ieF_\Delta(q^2)}{2M_\Delta^2}\{g_1(g^{\alpha\mu}\not{p}\not{q} - p^\alpha\gamma^\mu\not{q} - \gamma^\alpha\gamma^\mu p\cdot q + \gamma^\alpha\not{p}q^\mu) + g_2(p^\alpha q^\mu - g^{\alpha\mu}p\cdot q) + (g_3/M_\Delta)\left(q^2(p^\alpha\gamma^\mu - g^{\alpha\mu}\not{p}\right) + q^\alpha(q^\mu\not{p} - \gamma^\mu p\cdot q)\}\gamma_5 T_3$$

3 coupling constants g_1, g_2, and g_3

At Δ pole:

g_1	magnetic
(g_2-g_1)	electric
g_3	Coulomb

Take dipole FF $F_\Delta(q^2) = 1/(1-q^2/\Lambda_\Delta^2)^2$ with Λ_Δ ¼ 0.84 GeV

No infrared divergences (since $M_\Delta > M_N$)

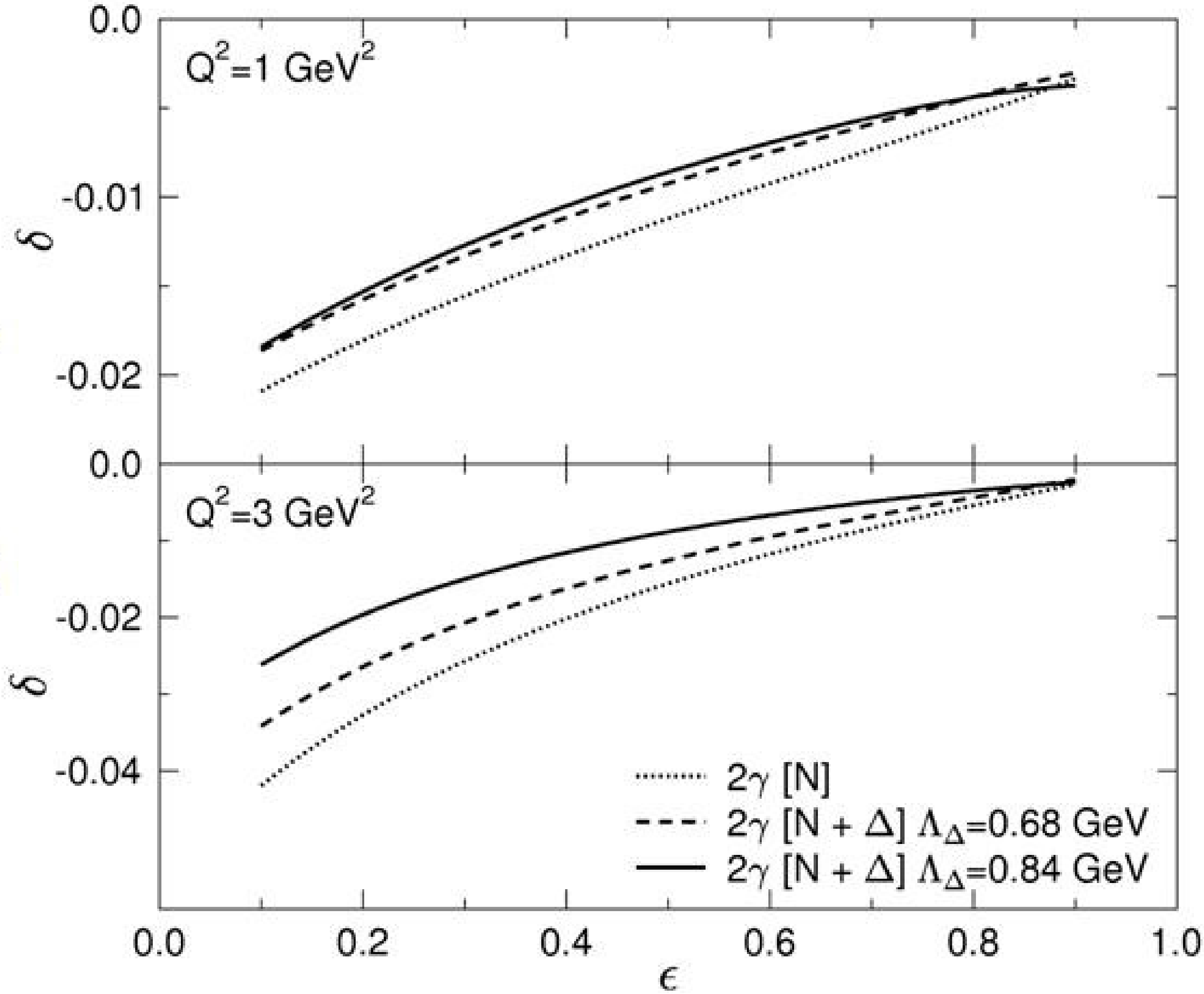
The $\gamma N\Delta$ vertex was used in Dressed K-matrix model (Kondratyuk and Scholten) to describe pion photoproduction, πN scattering, Compton scattering at low to medium energies

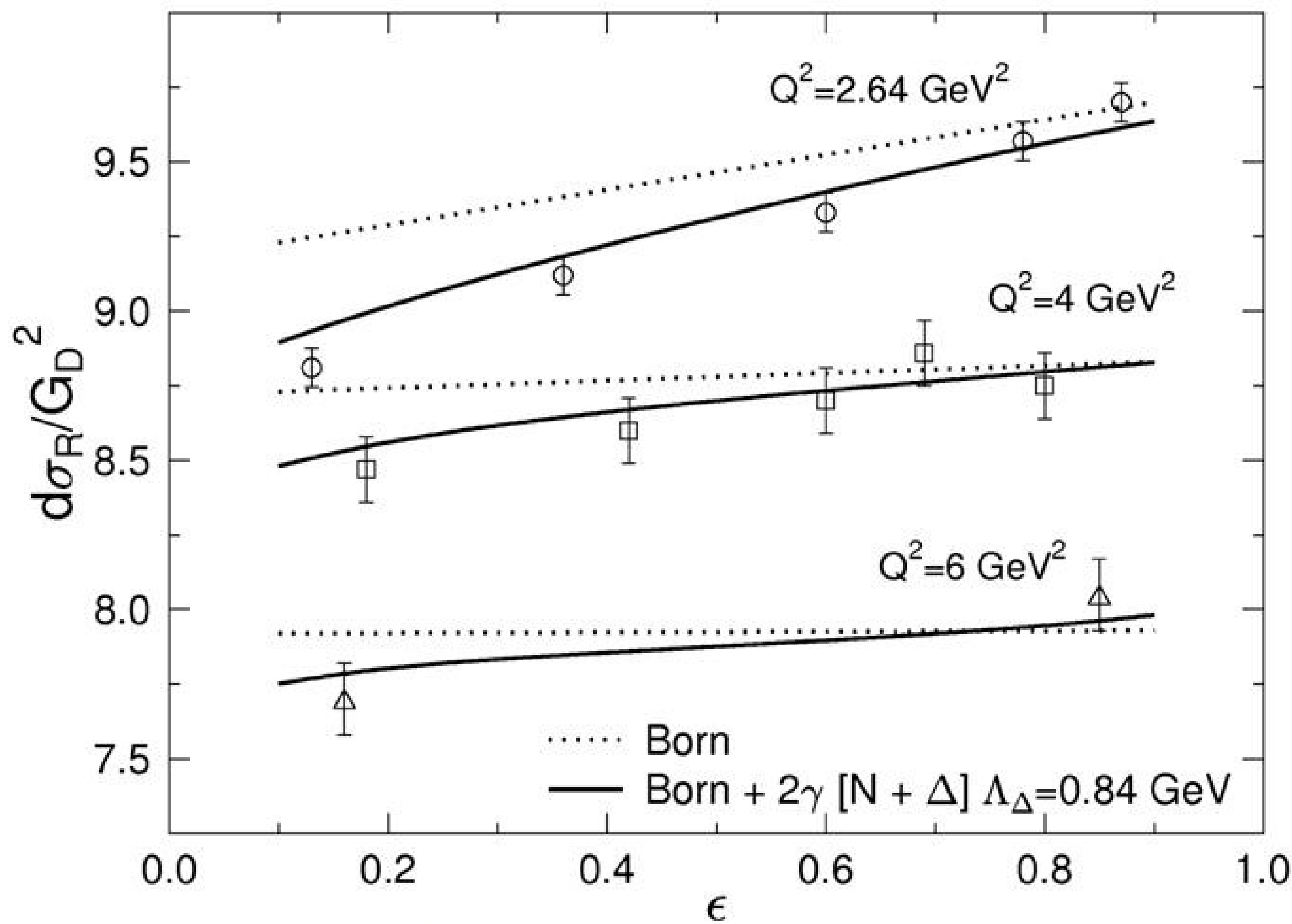
g_1 and g_2 taken from fits to E2/M1 ratio

Coulomb contribution ~ $(g_3)^2$ and is small, independent of sign

Box and crossed box obey crossing symmetry under p_1 ! – p_3 (s! u), just as for elastic nucleon contribution.

- Smaller than nucleon contribution for reasonable range of parameters
- Becomes more important as Q^2 increases
- Partially cancels the nucleon only contribution at backward angles
- Reduces nonlinear ε dependence somewhat

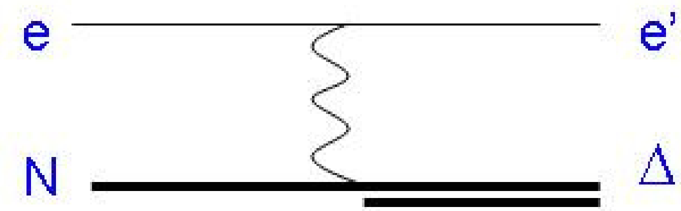




Effect on SLAC reduced cross sections

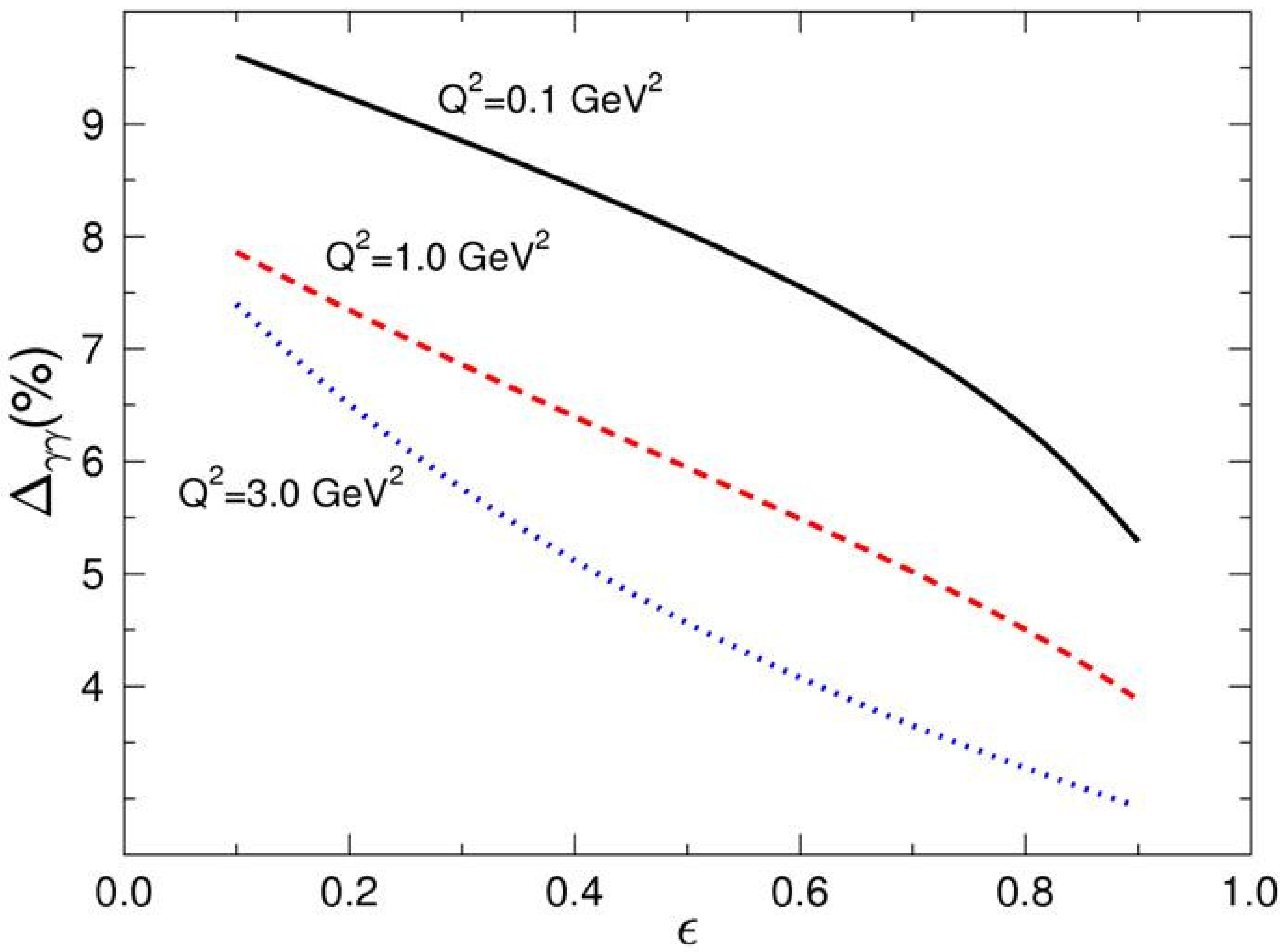
Δ production:

$e + N ! e' + \Delta$



- Compute TPEX relative to Born cross section σ_{Born}
- Remove universal IR divergence $-\delta_{IR}\, \sigma_{Born}$

- JLAB experiment looking for nonlinearities in ε over $Q^2 \sim 3\ GeV^2$
- see experimental talk by V. Tvaskis & theory talk by V. Pascalutsa this afternoon



Preliminary results for TPEX correction to Born Δ production
No strong nonlinearities evident, but effect is large

Effect on Parity-violating asymmetry in elastic e+p

$$\mathcal{A}_{PV} = \frac{2\Re\left\{M_\gamma^\dagger M_Z\right\}}{|M_\gamma|^2}$$

Weak radiative corrections interfere with M_γ

Electromagnetic radiative corrections interfere with M_Z

Afanasev and Carlson used generalized form factors to analyze effect on A (GPD model)

$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{A_E + A_M + A_A + A'_M + A'_A}{\sigma_R}$$

A_M' and A_A' are new terms

What is effect at low Q^2 (e.g. G0, Qweak, SAMPLE)?

Qweak   At low Q^2, forward angles (ε! 1)

$$\mathcal{A}_{PV} \approx -\frac{G_F Q^2}{e^2\sqrt{2}}\left(A + BQ^2\right)$$

A=(1 - 4 $\sin^2\theta_W$) independent of hadron structure

B=hadronic correction

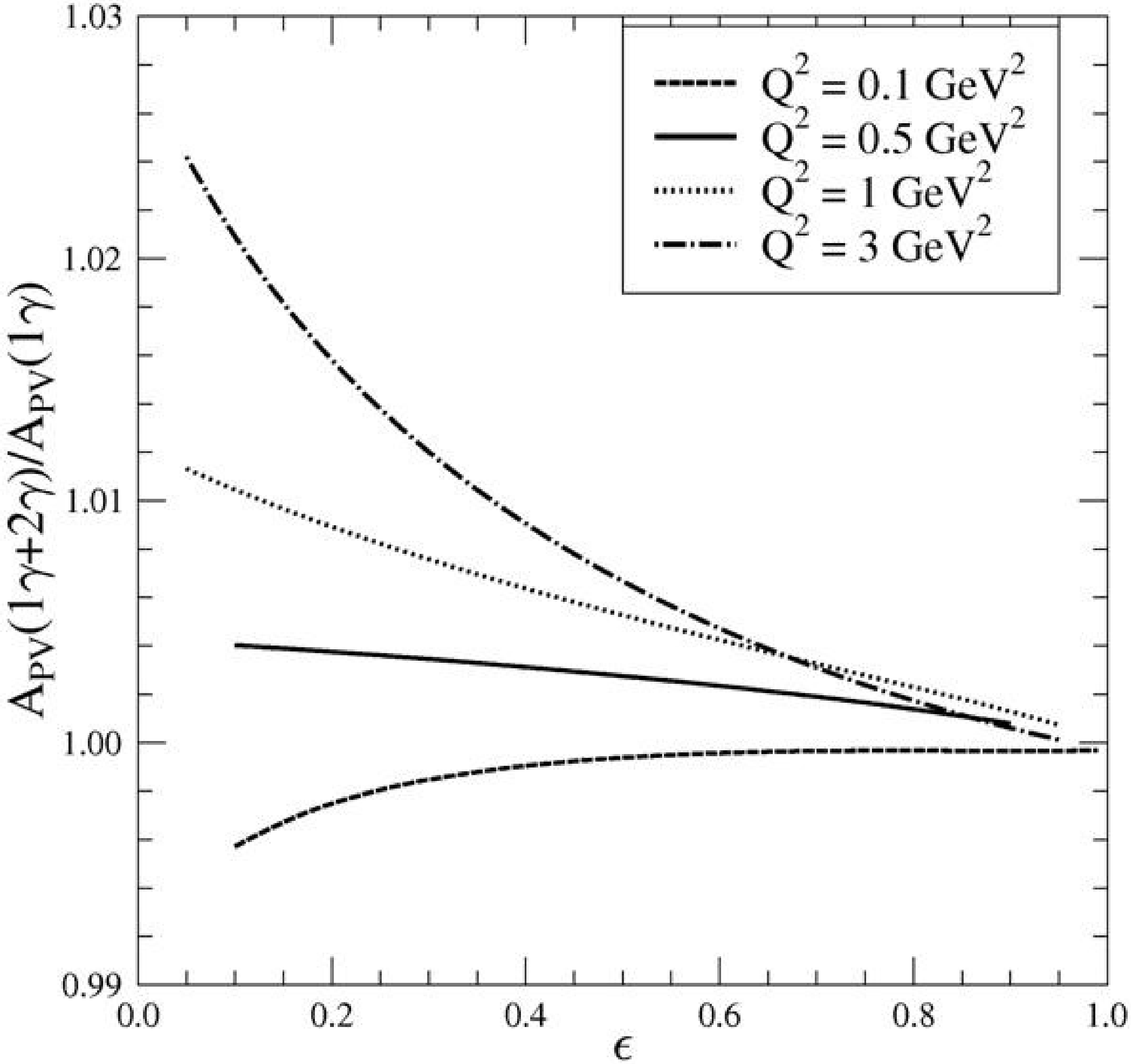
Qweak aims for a 2% measurement of A_{PV}

Though not obvious at first glance, A_M' and A_A' are of order Q^2

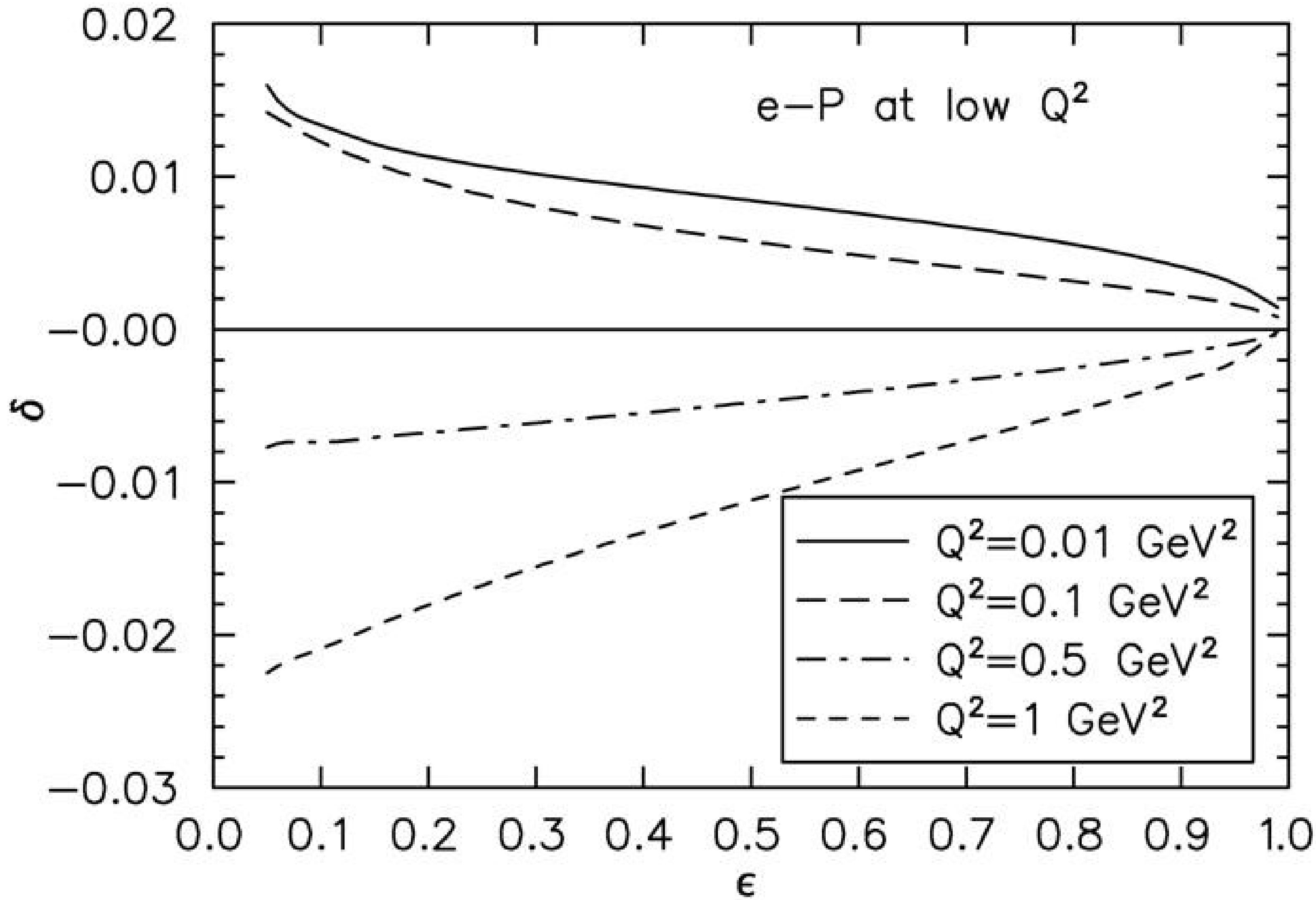
Our corrections to A vanish as ε! 1

At Qweak kinematics, TPEX correction is -0.05%

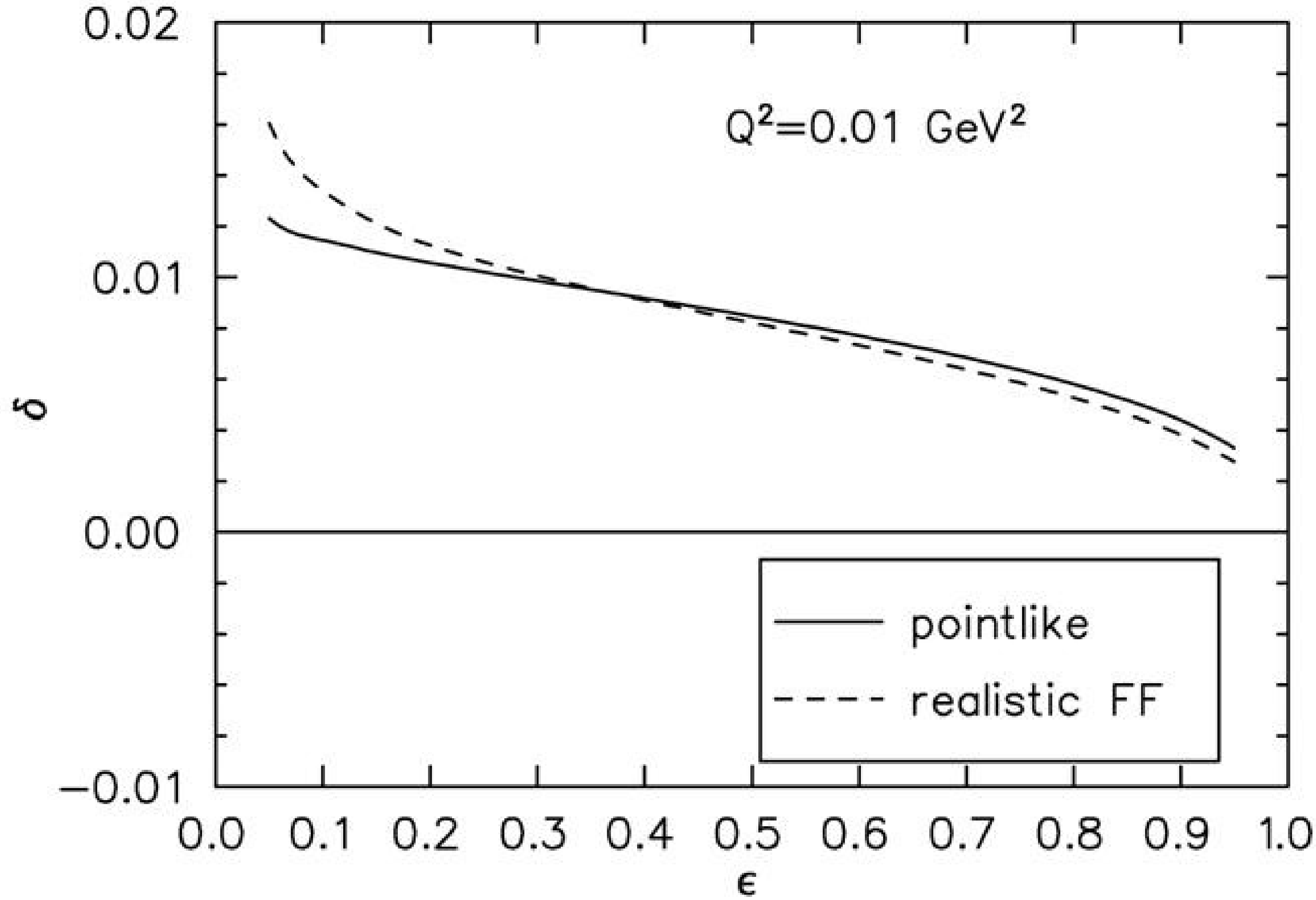
A_{PV} vs. ε for $Q^2 = 0.1, 0.5, 1.0, 3.0$ GeV^2



proton correction at low Q^2



proton correction at Q^2=0.01 GeV^2



- Essentially independent of mass (same for muon, quarks)
- At high Q^2, G_M dominates the loop integral
- At low Q^2, G_E dominates
- neutron correction vanishes at low Q^2 (pointlike neutron)

Proton radii and TPEX

H atom

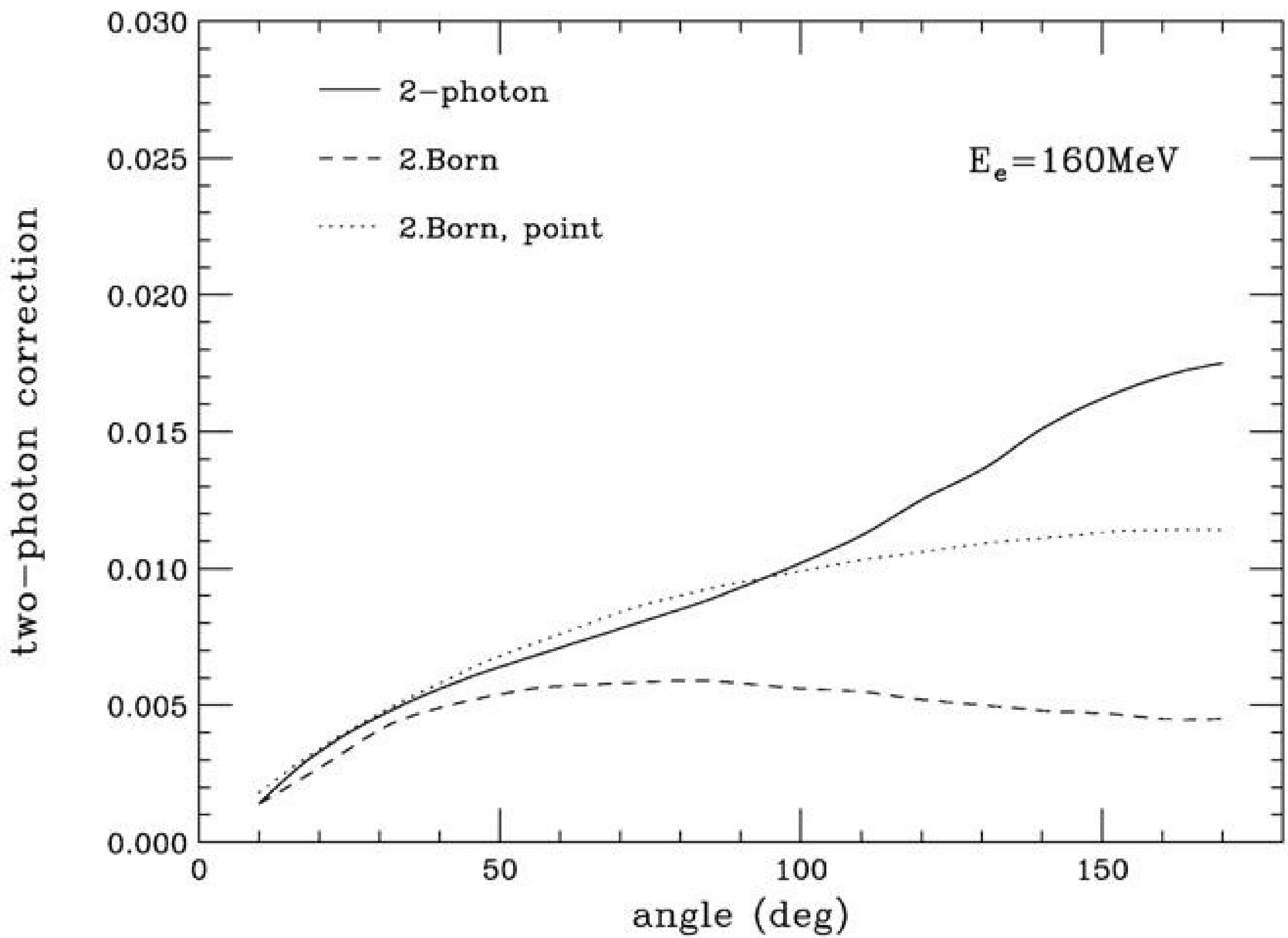
- 2p-1s transition energy known to 14 digits
- 1s hyperfine structure interval (HFS) known to 12 digits

Tests of QED now depend on accuracy of proton finite-size corrections, as determined through e-p scattering

2p-1s: needs charge rms radius $r_{rms} = 0.895 \pm 0.018$ fm
! calculated and expt. energies agree

1s HFS: needs Zemach moment (convolution of charge and magnetization densities)
$r_2 = 1.086 \pm 0.012$ fm
! calculated and expt. energies disagree by 3.6 ppm
Partly explained by nuclear polarization (1.6 ppm)

Interpreted using OPE + Coulomb distortion (soft photons).
What is role of TPEX?


two-photon correction
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0
50
100
150
angle (deg)
2-photon
2.Born
2.Born, point
E_e=160MeV

Analysis

- Reanalyze world e-p cross section data up to 4 fm^{-1} including contribution of TPEX effects

$r_{rms} = 0.897 \pm 0.018$ fm (from 0.895 ± 0.018 fm)

$r_2 = 1.091 \pm 0.012$ fm (from 1.086 ± 0.012 fm)

change in r_2 is small (40% of error bar), but in **wrong direction** to explain HFS discrepancy

! discrepancy cannot be attributed to TPEX

Outlook

Theory

- Connect real and imaginary parts of TPEX amplitude
  - more work needs to be done on hadronic models
- Look at sensitivity to off-shell form factors (preliminary work indicates probably not a large effect)

Experiment

- Lots of good proposals, but beamtime at JLab is scarce

Collaborators: Melnitchouk, Tjon + Kondratyuk (Δ) + Scholte (A_{PV}) + Sick (proton radii)