Meson Confinement and Gluonic Excitation

1. **QCD String and the TCV Potential**

2. **Analytic Semi – Classical Result**

3. **Heavy – Light Spectroscopy**

4. **Excited Glue Jump rope States**

5. **The $D_{s\,0}^{**}$ (2320) State (if time)**

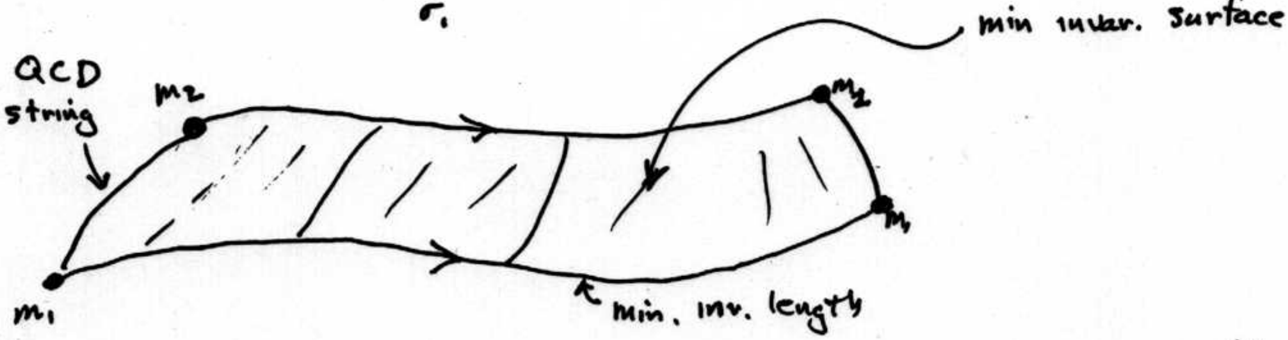
1. QCD String and the Time Component Vector Potential

$$\delta S = \delta \int dt \left[\mathcal{L}_q + \mathcal{L}_s \right] \cong 0$$

temporal gauge $\tau = t$

$$\mathcal{L}_q = -m_1\sqrt{1-v_1^2} - m_2\sqrt{1-v_2^2}$$

$$\mathcal{L}_s = -a\int_{\sigma_1}^{\sigma_2} d\sigma \sqrt{1-v_\perp^2(\sigma)}$$



$$\gamma_\perp = \left(1-v_\perp^2\right)^{-1/2}$$

Identity $\qquad m\gamma = W_r\gamma_\perp \qquad W_r = \sqrt{p_r^2+m^2}$

straight string $\qquad v_\perp(\sigma) =$ linear in σ

$$\left.\begin{aligned} v_\perp(\sigma_1) &= v_{\perp 1} \\ v_\perp(\sigma_2) &= v_{\perp 2} \end{aligned}\right\} \text{ quark perp. velocities}$$

Conserved Quantities

$P_\perp$

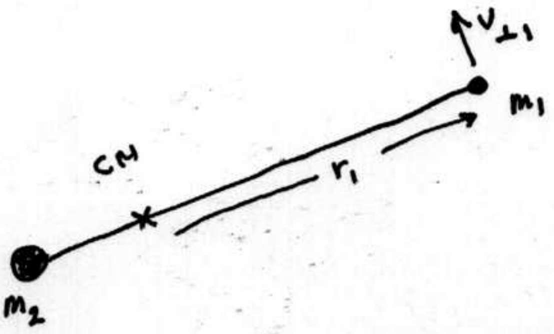
$$P_\perp \equiv 0 = W_{r_1}\gamma_{\perp 1} v_{\perp 1} + \frac{a r_1}{v_{\perp 1}}\left(1 - \gamma_{\perp 1}^{-1}\right) - (1 \to 2)$$

ANGULAR MOMENTUM

$$L = W_{r_1}\gamma_{\perp 1} v_{\perp 1} r_1 + \frac{a r_1^2}{2 v_{\perp 1}}\left(\frac{\sin^{-1} v_{\perp 1}}{v_{\perp 1}} - \gamma_{\perp 1}^{-1}\right) + (1 \to 2)$$

ENERGY

$$M = W_{r_1}\gamma_{\perp 1} + a r_1 \frac{\sin^{-1} v_{\perp 1}}{v_{\perp 1}} + (1 \to 2)$$



The Approximation

method indicated by a numerical example

take $\quad m_1 = m_2 = 0$

to simplify notation $\quad$ suppress $\perp$'s

$$v_\perp \to v \qquad \gamma_\perp \to \gamma$$

Define

$$S(v) \equiv \frac{\sin^{-1} v}{v} \quad ; \quad f(v) = \frac{1}{2v}\left(\frac{\sin^{-1} v}{v} - \sqrt{1-v^2} \right)$$

$P_\perp = 0$ $\quad$ trivially satisfied $\quad$ since $v_1 = v_2$

$$L = W_r \gamma v r + \frac{1}{2} a r^2 f$$

$$M \equiv E = 2 W_r \gamma + a r S$$

excitation energy

eliminate $\quad W_r \gamma = \frac{1}{2}\left(E - a r S \right)$

$$\frac{2L}{r} = v\left(E - a r S \right) + a r f$$

dimensionless variables

$$x = \frac{ar}{E} \qquad \beta = \frac{aL}{E^2}$$

$$\boxed{xv\left(1 - x\,S(v)\right) + x^2 f(v) = 2\beta}$$

$$S = \frac{\sin^{-1} v}{v} \qquad f = \frac{1}{2v}\left(S - \sqrt{1-v^2}\right)$$

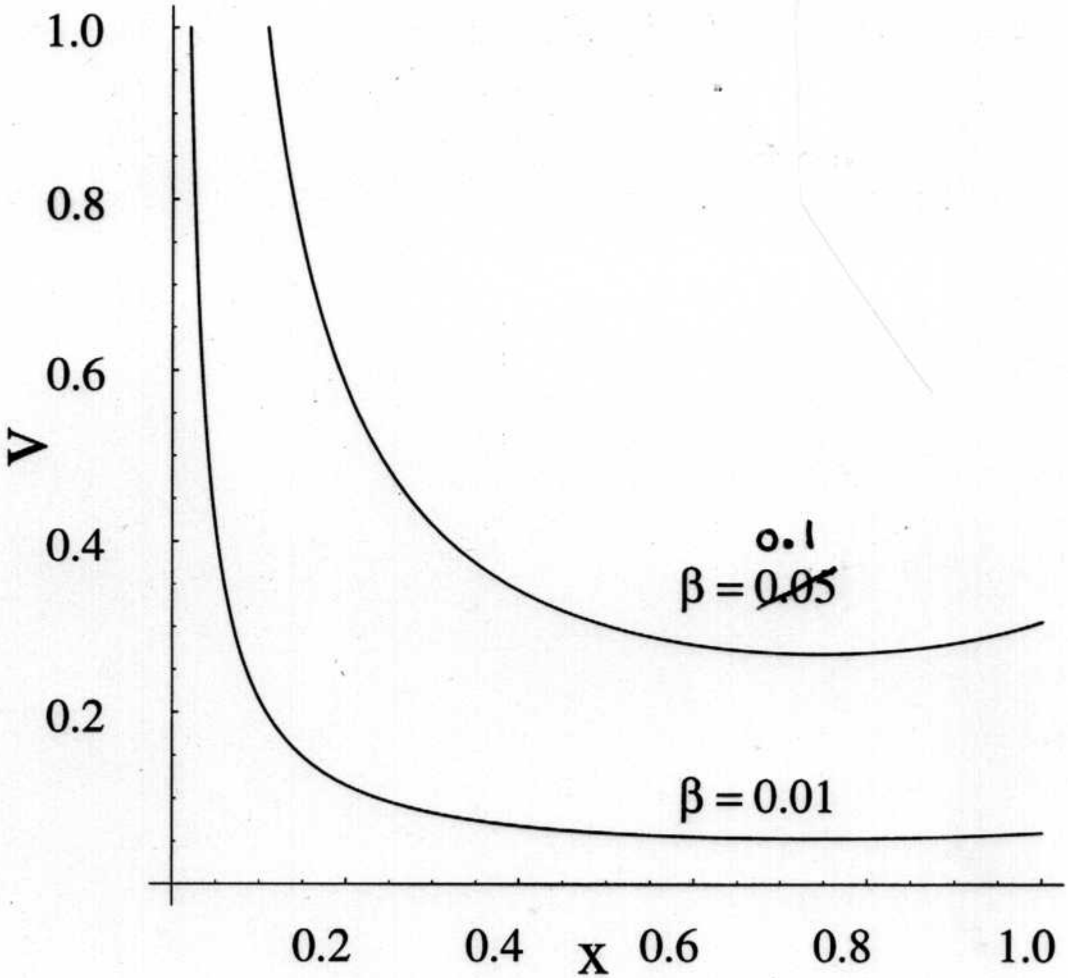
turning points $(v_\perp \equiv 1)$ at $\left[\text{ie } \dot{r} = 0 \Rightarrow v_\perp = 1 \quad \text{since } \dot{r}^2 + v_\perp^2 \equiv 1\right]$

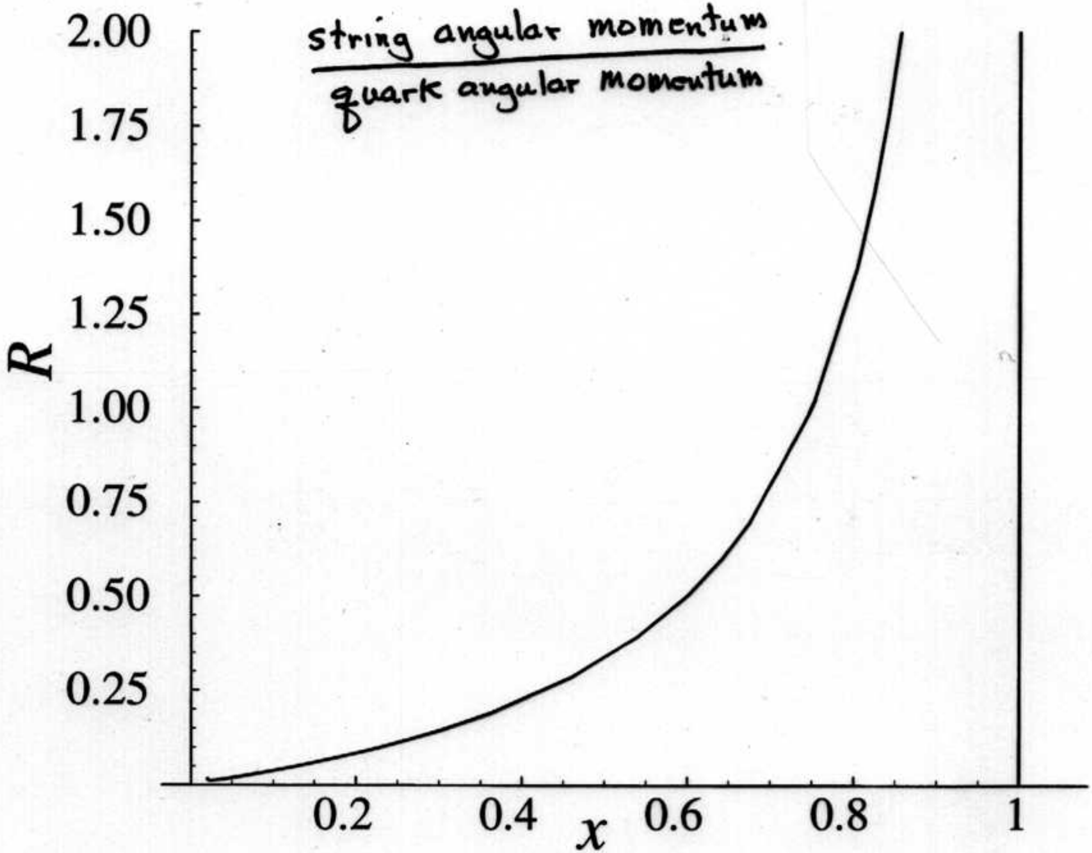
$$x_- \simeq \beta \qquad x_+ \simeq 1 - \beta \qquad \beta \ll 1$$

$$\beta \leq 1/\pi$$

$m_1 = m_2 = 0$

Exact solution





$\frac{\text{string angular momentum}}{\text{quark angular momentum}}$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
R
0.2
0.4
0.6
0.8
1
x

So the approximation is —

For $\beta \ll 1$ $(E^2 \gg aL)$ high radial excitation

we may assume $v \ll 1$ for the string for any r without changing the result

$$S(v) = \frac{\sin^{-1} v}{v} \longrightarrow 1 + O(v^2)$$

$$f(v) = \frac{1}{2v}\left(S - \sqrt{1-v^2}\right) \rightarrow \frac{1}{3}v + O(v^3)$$

works because the string L & E are small where v not small.

$$\frac{2L}{rv_1} = E + \frac{k}{r} - \frac{2}{3}ar$$

$$W_r \gamma_1 v_1 = \frac{L}{r}(1 - F)$$

$$F = \frac{ar/6}{E + \frac{k}{r} - \frac{2}{3}ar}$$

or

$$(\gamma v)^2 = \gamma^2 - 1 = \left[\frac{\frac{L}{r}(1-F)}{W_r} \right]^2$$

$$W_r \gamma_1 = \sqrt{W_r^2 + \frac{L^2}{r^2}(1-F)^2}$$

where $$F = \frac{\frac{1}{6} a r}{E + \frac{k}{r} - a r}$$ Similiar results for $m_1 \neq m_2$

<u>Note</u> $\frac{L^2}{r^2}$ large only at inner turning point

but F is small there !

<u>$\beta \ll 1$</u> (general case $m_1 \neq m_2$)

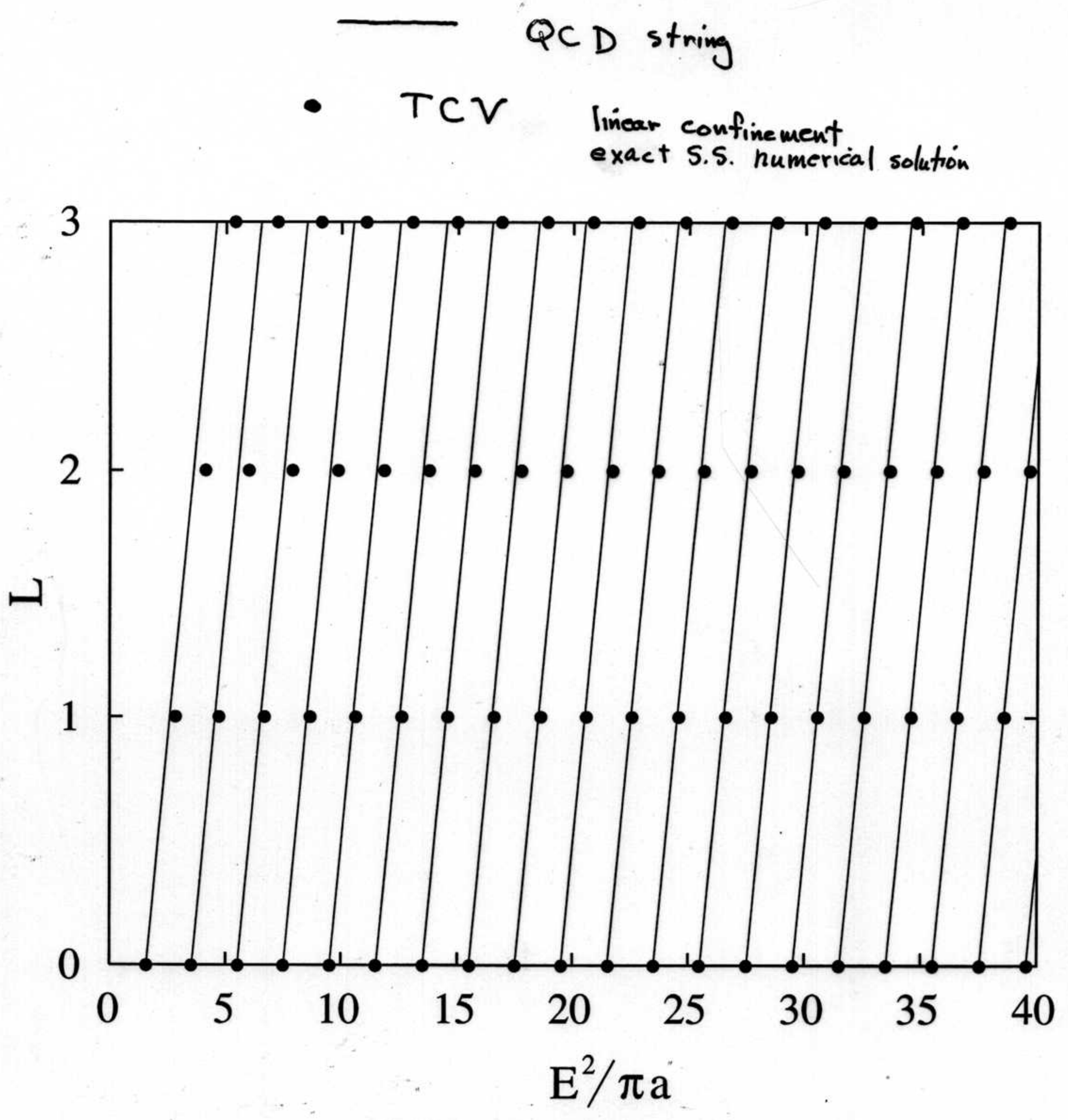
Relativistic Time Component Vector Spinless Salpeter Eq.

$$\boxed{H \simeq \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + a r - \frac{k}{r}}$$

$$p^2 = p_r^2 + L^2/r^2$$

Wave Equatión

$$H \psi = M \psi$$


QCD string
TCV
linear confinement
exact S.S. numerical solution
L
3
2
1
0
0
5
10
15
20
25
30
35
40
E²/πa

2. Semi – Classical Quantization
Analytic Results

consider Heavy-Light case

$$m_2 \to \infty \qquad m_1 \to 0 \qquad E \equiv M - m_2$$

TCV

$$\Big[\underbrace{\sqrt{p_r^2 + \frac{L^2}{r^2}} + ar - \frac{k}{r}}_{H} \Big] \psi = E \psi$$

$$p_r^2 = \left(E - ar + \frac{k}{r} \right)^2 - \frac{L^2}{r^2}$$

S.C. Quantization

$$\int_{r_-}^{r_+} dr \, p_r = \pi \left(n + \frac{1}{2} \right) \qquad n = 0, 1, \cdots$$

Dimensionless variables

$$x = \frac{ar}{E} \qquad \beta = \frac{aL}{E^2} \qquad \sigma = \frac{ak}{E^2}$$

$$\int_{x_-}^{x_+} \frac{dx}{x} \sqrt{Q} = \frac{\pi a}{E^2} \left(n + \frac{1}{2} \right)$$

$$Q = (x - u_-)(x - x_-)(x_+ - x)(u_+ - x)$$

This is a messy combination of elliptic integrals

$$\int_{x_-}^{x_+} \frac{dx}{x} \sqrt{Q}$$

because of $\frac{1}{x}$ most of integral near x_-

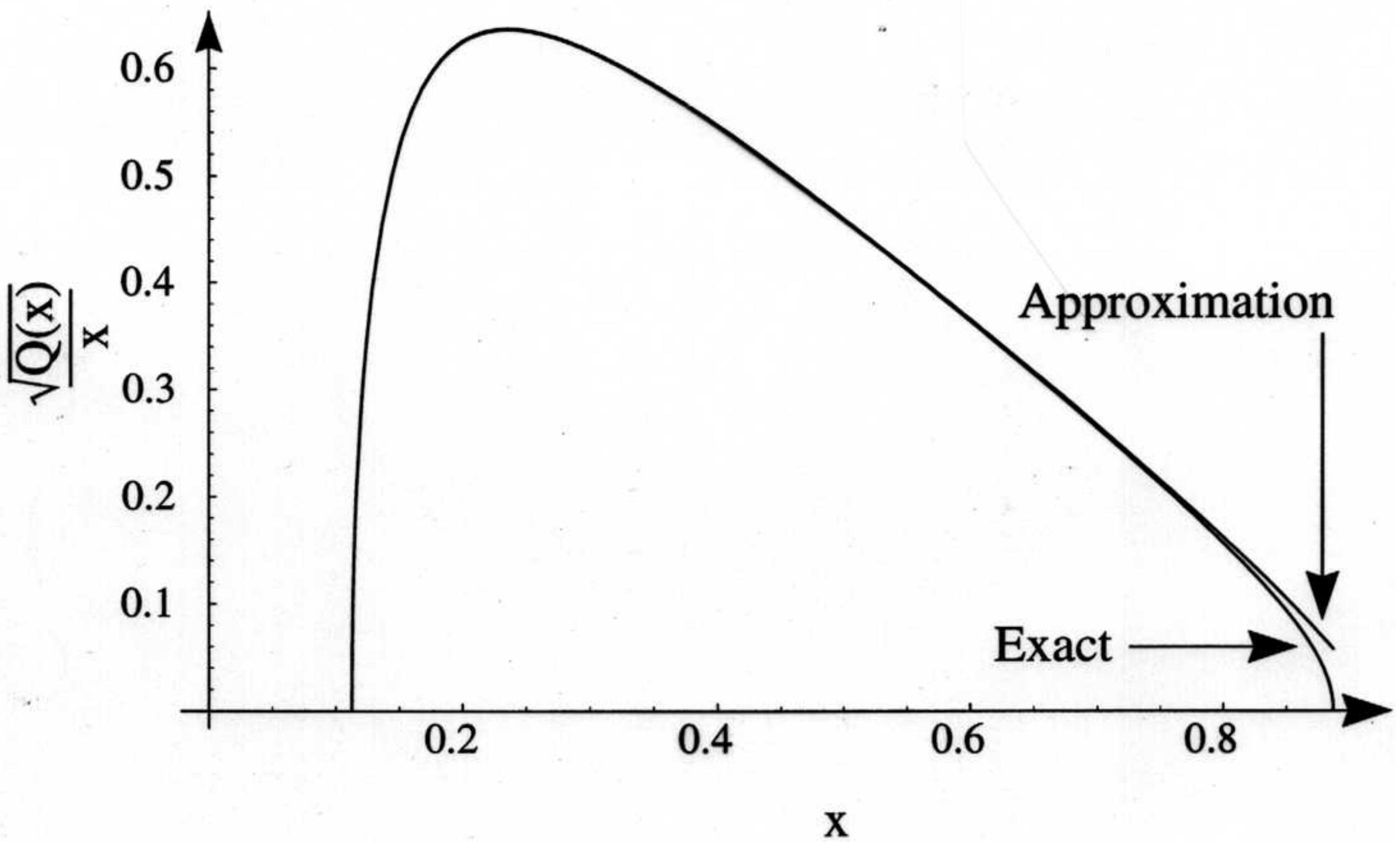
In radially excited regime

$$\beta \, , \sigma \ll 1$$

expanding in small β, σ the outer root terms

$$\sqrt{Q} \simeq \sqrt{(x-u_-)(x-x_-)} \quad (1+\sigma-x)$$

$\sigma = 0$, $\beta = 0.1$



Quantization is now in terms of elementary functions

$$\frac{\pi a}{E^2}\left(n+\frac{1}{2}\right) \simeq \frac{1}{2} + \sigma\left(1+\ln\frac{2}{\beta}\right) + \sqrt{\beta^2-\sigma^2}\,\sin^{-1}\left[\frac{\sqrt{\beta^2-\sigma^2}}{\beta}\right]$$

$n = 0, 1, 2, \cdots$ radial excitation

In terms of E, k, and $\underbrace{L \equiv \ell + \frac{1}{2}}_{\text{Langer correction}}$

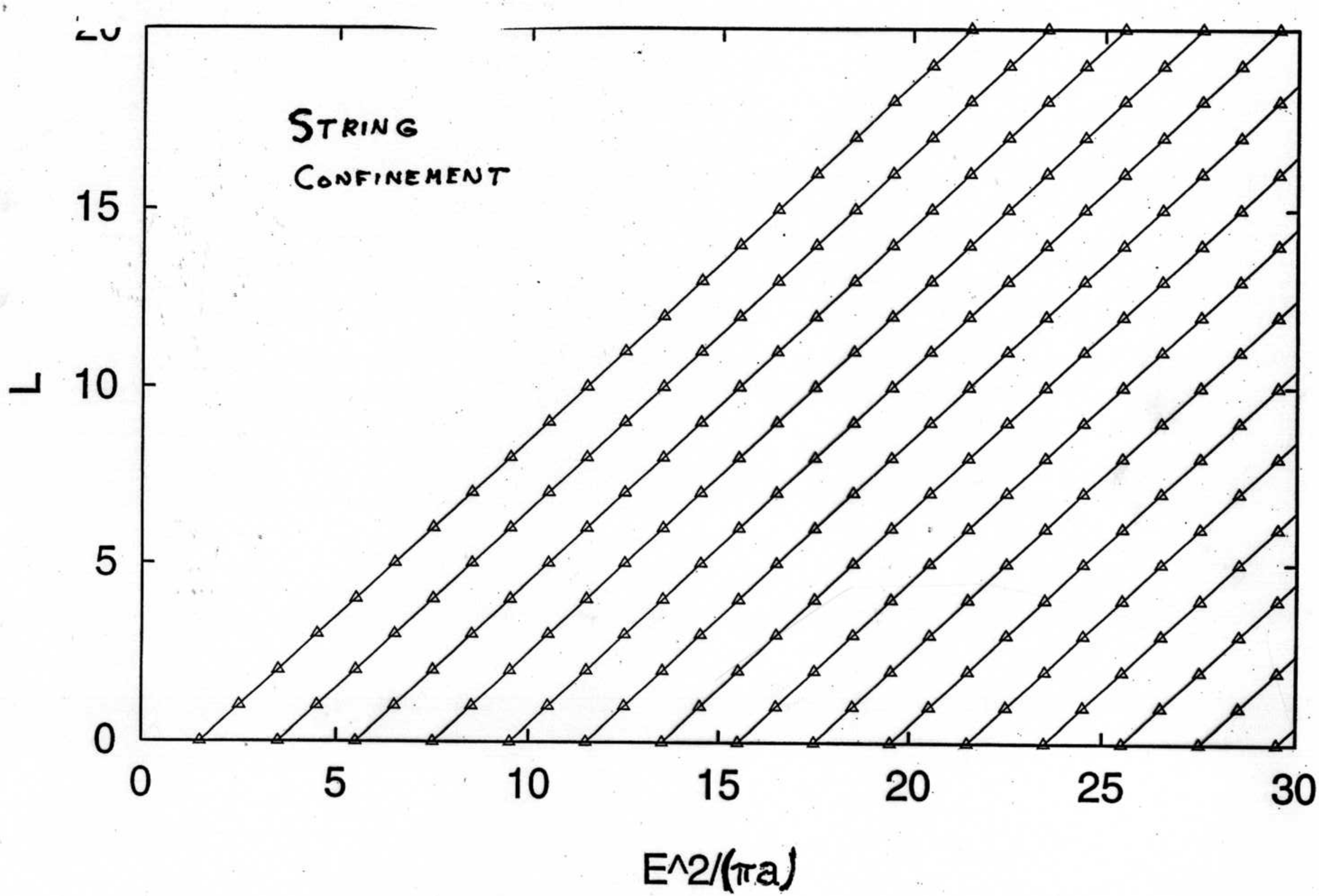
$$\boxed{\frac{E^2}{\pi a} = 2n+1 + \frac{2}{\pi}\sqrt{\left(\ell+\tfrac{1}{2}\right)^2 - k^2}\,\sin^{-1}\left[\frac{\sqrt{\left(\ell+\frac{1}{2}\right)^2-k^2}}{\ell+\frac{1}{2}}\right] - \frac{2}{\pi}k\left(1+\ln\frac{2E^2}{a\left(\ell+\frac{1}{2}\right)}\right)}$$

where H.L. meson mass M is

$$M = m_Q + E$$

Note if $\sigma = 0$

$$\frac{E^2}{\pi a} = \ell + 2n + 3/2$$



STRING
CONFINEMENT
L
0
5
10
15
20
0
5
10
15
20
25
30
E^2/(πa)

3. Heavy – Light Spectroscopy
Charm

3 parameters m_Q, a, k

1. Universal Regge slope $\alpha' = .9\ \mathrm{GeV}^{-2} \equiv \frac{1}{2\pi a}$

$$\boxed{a = 0.2\ \mathrm{GeV}^2}$$

p-wave s-wave spin av.

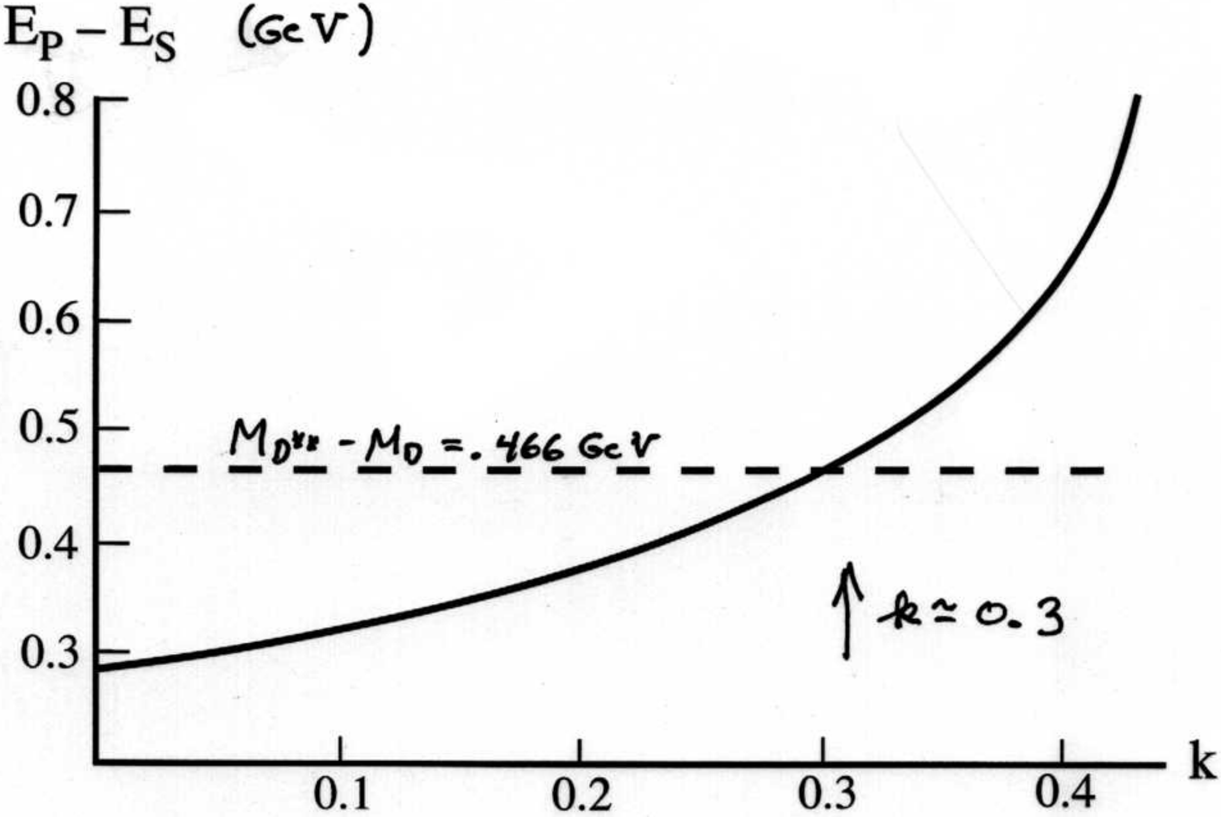
2. $M_{D^{**}} - M_D = 2440 - 1974 = 466\ \mathrm{MeV}$

$$\boxed{k = 0.3}$$

note $M_{D^{**}} - M_D = E_{\ell=1} - E_{\ell=0}$ m_c cancels

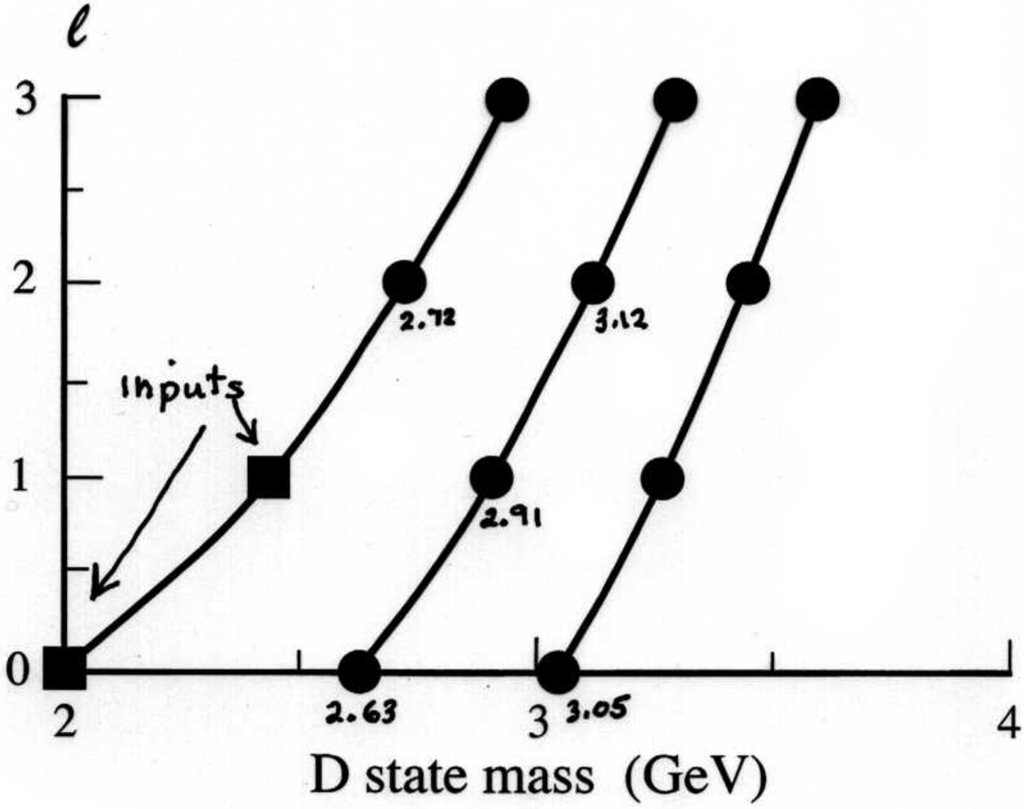
3. $m_c = M_D - E_{\ell=0} = 1420\ \mathrm{MeV}$

other states predicted

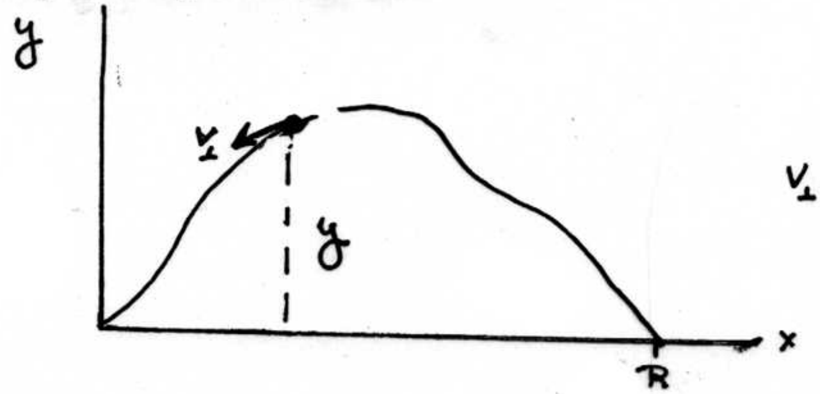


E_P – E_S (GeV)
0.8
0.7
0.6
0.5
0.4
0.3
M_D** - M_D = .466 GeV
k ≃ 0.3
0.1
0.2
0.3
0.4
k

D spectroscopy



4. Excited Glue
Jump Rope States



$v_\perp \equiv \omega\, y(x)$

Semi-classical approach

$$\mathcal{L} = -a \int ds \sqrt{1 - v_\perp^2(s)}$$

$$ds = dy \sqrt{1 + x'^2} \qquad x' = \frac{dx}{dy}$$

$$\mathcal{L} = -a \int dy \sqrt{1 + x'^2} \sqrt{1 - \omega^2 y^2}$$

Euler eq.

$$\frac{d}{dy}\left(\frac{\partial \mathcal{L}}{\partial x'}\right) = 0$$

$$\frac{x'}{\sqrt{1 + x'^2}} \sqrt{1 - \omega^2 y^2} = \beta \leftarrow \text{integ. constant}$$

solving for $\acute{x}$

$$\acute{x} = \frac{dx}{dy} = \frac{\beta}{\sqrt{1-\beta^2-\omega^2 y^2}}$$

$$x = \beta \int \frac{dy}{\sqrt{1-\beta^2-\omega^2 y^2}} = \frac{\beta}{\omega} \operatorname{sen}^{-1} \frac{\omega y}{\sqrt{1-\beta^2}}$$

note $\quad y=0 \Rightarrow x=0$

$$y = \frac{\sqrt{1-\beta^2}}{\omega} \sin \frac{\omega x}{\beta}$$

other end condition $\quad x=R \quad y=0 \quad \Rightarrow \boxed{\frac{\omega R}{\beta} = m\pi} \quad m=1,2,\cdots$

m = # half wave length.

<u>string shape</u>

$$y(x) = \frac{\sqrt{1-\beta^2}}{\omega} \operatorname{sen}\left(\frac{m\pi x}{R}\right)$$

<u>angular momentum</u> $\quad \Lambda = 1,2,\cdots$

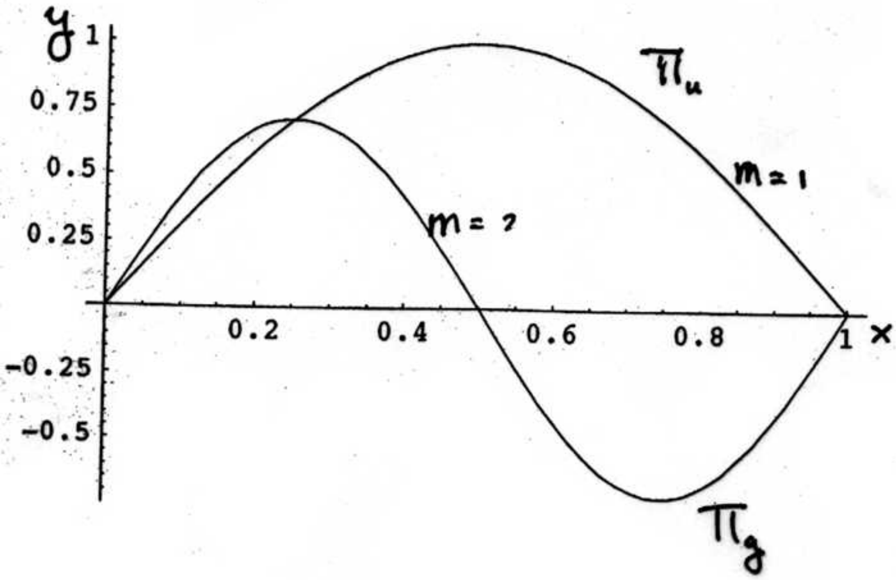
$$\Lambda = a \int dy \frac{\sqrt{1+\acute{x}^2}}{\sqrt{1-\omega^2 y^2}}\, y(\omega y) = \frac{m\pi a}{2} \frac{(1-\beta^2)}{\omega^2}$$

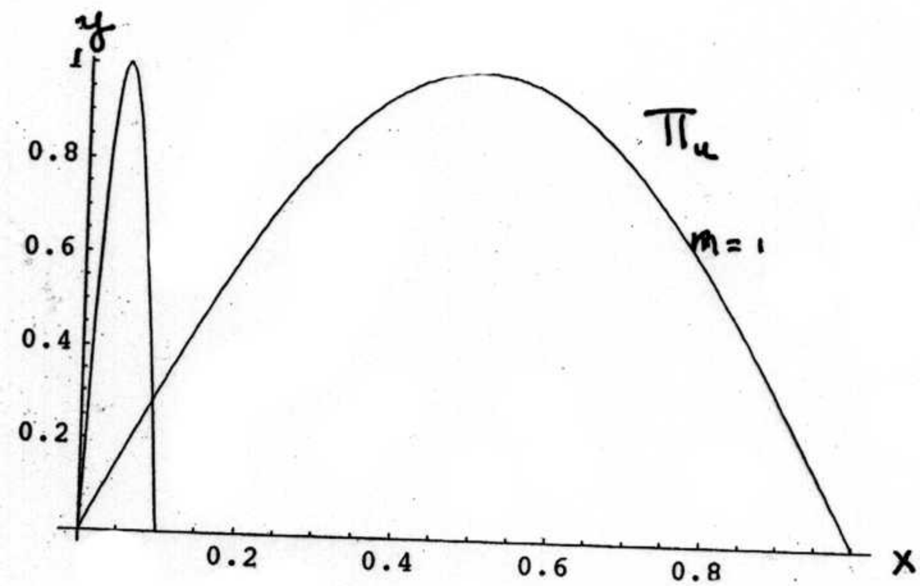
$$\frac{\sqrt{1-\beta^2}}{\omega} = \sqrt{\frac{2\Lambda}{m\pi a}}$$

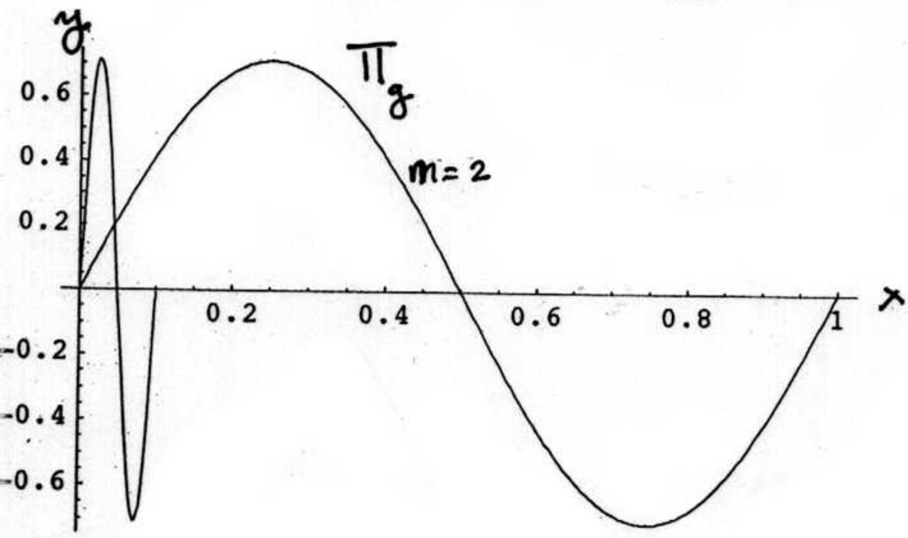
$$\boxed{y(x) = \sqrt{\frac{2\Lambda}{m\pi a}} \sin\left(\frac{m\pi x}{R}\right)} \quad \text{shape}$$

$$\Lambda = 1$$

$$V_{\perp}^{max} = \sqrt{\frac{1}{1+\frac{am R^2}{\pi \Lambda}}}$$







Energy

$$E = a\int dy\,\frac{\sqrt{1+\dot{x}^2}}{\sqrt{1-\omega^2 y^2}} = \frac{m\pi a}{\omega}$$

$$\underbrace{\frac{\omega}{\beta} = \frac{m\pi}{R} \qquad \frac{\sqrt{1-\beta^2}}{\omega} = \sqrt{\frac{2\Lambda}{m\pi a}}}_{\text{solving for } \omega \;\Rightarrow\; E}$$

$$\boxed{E = \sqrt{(aR)^2 + 2\pi a m\Lambda}}$$

From quantized string —

$$E = \sqrt{(aR)^2 + 2\pi a(N+c)}$$ normal ordering constant

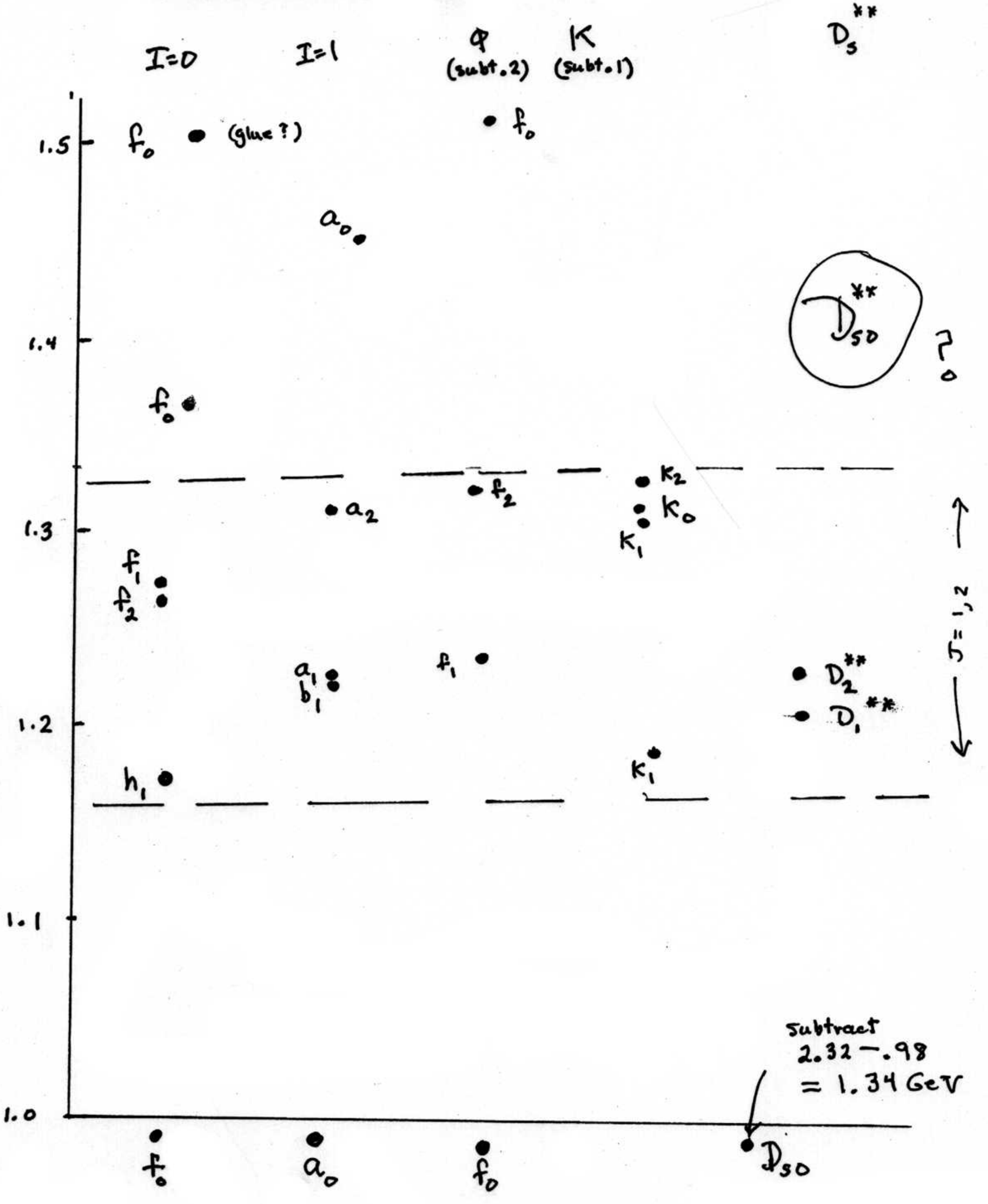
$$N = \sum_{m=1} m\,(n_m^+ + n_m^-) \qquad n_\pm \text{ occupancy of } m^{th} \pm \text{helicity states}$$

$$\Lambda = \sum_{m=1} n_m^+ - n_m^-$$

comparing — single mode of occupancy n_m^+

$$\left.\begin{aligned} \Lambda &= n_m^+ \\ N &= m\,n_m^+ \end{aligned}\right] \text{ so identify } m \text{ with } m$$

5. $D^{**}_{s\,0}$ (2320) State



A gold plated prediction

If $D_{s0}(2320)$ is DK bound state (~40 MeV binding)

then by heavy quark symmetry $b \leftrightarrow c$ quark

there must be

$\underline{\underline{B_{s0}(5740)}}$ bound state of BK

Summary

1. The QCD string / flux tube model reduces to a linear TCV potential model in the deep radial regime.

2. Using semi classical quantization we find an analytic form for the string spectroscopy with a short range interaction.

3. Prediction of the D – meson spectroscopy

4. Non perturbative approach to the jump rope excited glue states.