Electroweak excitation of Hybrid (gluonic excitations)

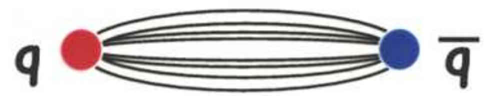
in Lattice-Lite$_{TM}$ QCD

(i.e. Isgur Paton flux tube simulation).

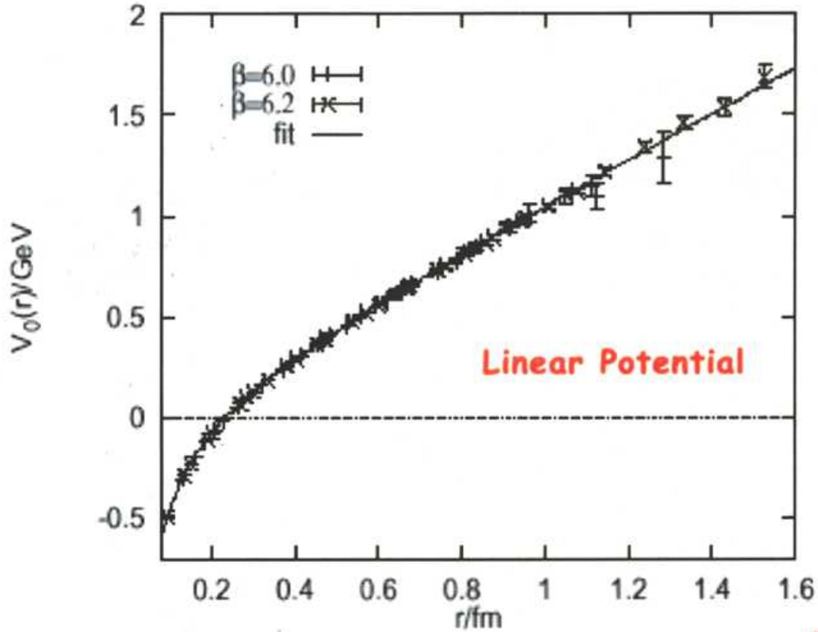
FEC
Jo Dudek

- First ever calculation of production rate
- Stimulate experiment (some are large!)
- Inspire Lattice to do it properly.

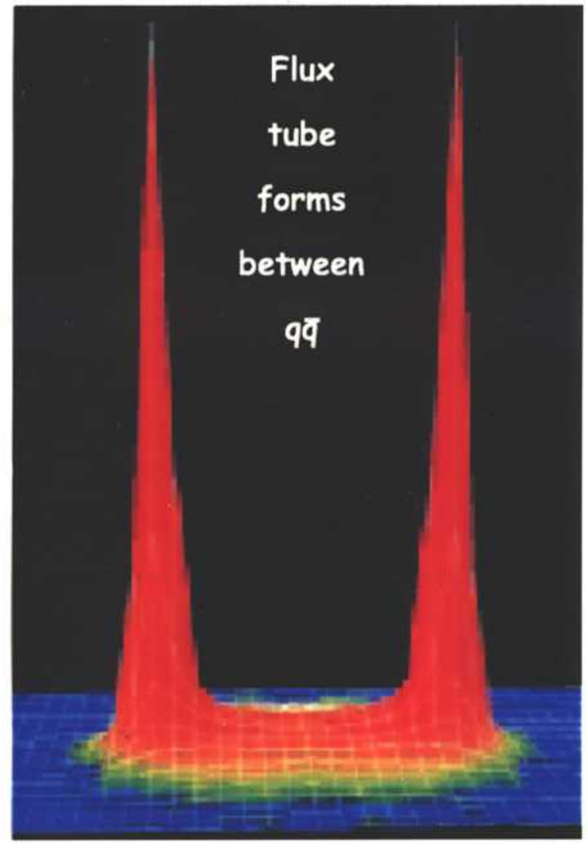
Lattice QCD

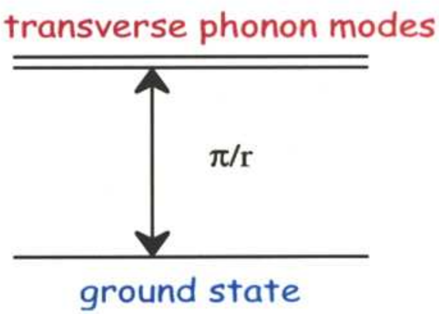


Flux tubes realized



from Bali et al



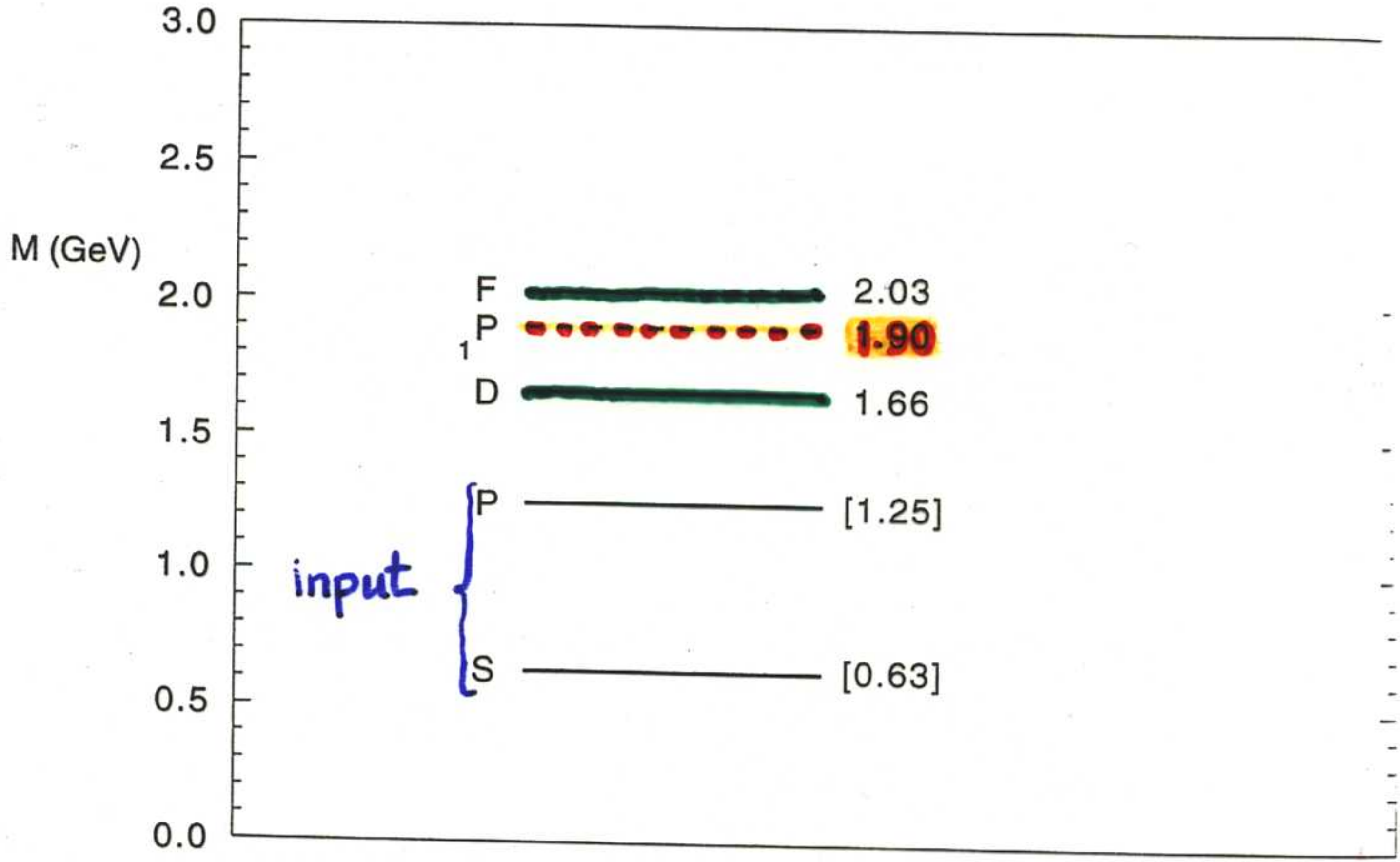


1. Confinement arises from flux tubes.
2. The excitation of these flux tubes leads to a new spectrum of mesons.

HYBRID MASS

Flux tube spectrum

Barnes Close Swanson
PRD52 5242 (95)

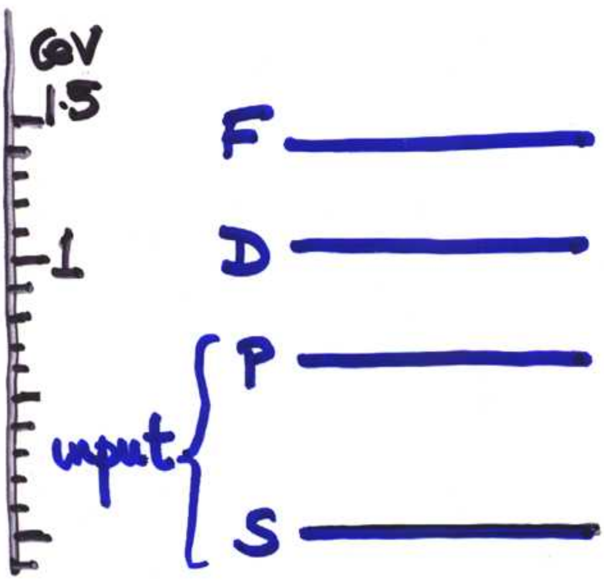


$m_b = ba$

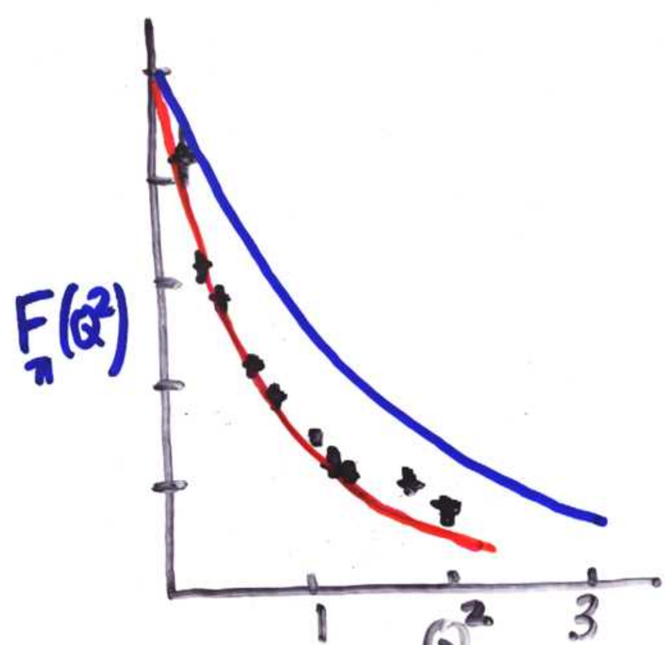
$R = (N+1)\,a$

$b = 1\,\mathrm{GeV/fm}$

$V = bR$



all flavours
$u\bar{d}$, $c\bar{c}$
$Q\bar{q}$



D, B Isgur Wise

$\xi(\omega) = 1 - \rho^2 (v.v' - 1)$

$\boxed{\rho^2_{D,B} > \rho^2_{(Q\bar{q})} \quad 20-50\%}$

$\boxed{\langle r^2 \rangle_\pi \simeq 2 * \langle r^2 \rangle_{q\bar{q}}}$

ad hoc renorm.

scales $\sim \left(\frac{m_d}{}\right)^2$

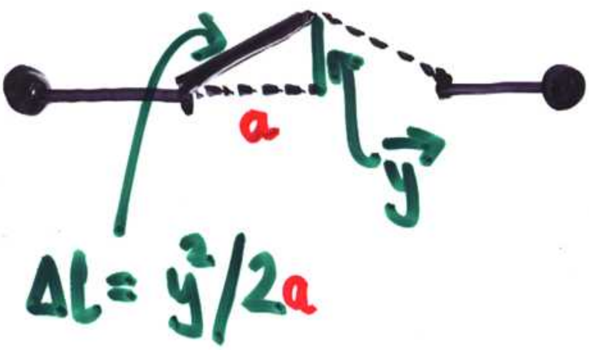
N beads in $\hat{z}$

$R = (N+1)a$

$b = 1\,\mathrm{GeV/fm}$

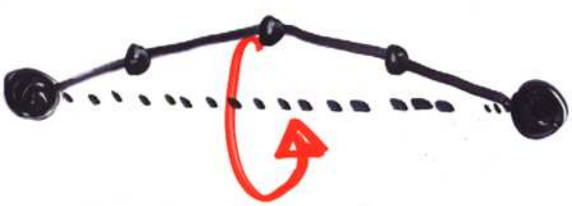
$$\boxed{V = bR}$$

Transverse oscillation of beads



$$V = bR + \underbrace{\frac{b}{2a}\sum_{n}^{N}\left(\vec{y}_{n+1} - \vec{y}_n\right)^2}_{\text{Fourier Modes}}$$

1st excited state



$$\Delta E \simeq \pi/R$$

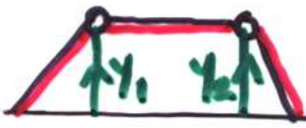
Fourier Coefficients $\vec{A}_p = \sum_{n=1}^{N} \vec{y}_n \sqrt{\frac{2}{N+1}} \sin(k_p z_n)$

$k_p = p\frac{\pi}{R} = \frac{p\pi}{(N+1)a}$ $\qquad z_n = na$

eg $N=1$ $\qquad A_1 = y_1 \sin\frac{\pi}{2} = y_1$

eg $N=2$ $\qquad A_1 = \sqrt{\frac{2}{3}}\, y_1 \sin\frac{\pi}{3} + y_2 \sqrt{\frac{2}{3}} \sin\frac{2\pi}{3}$

$\Rightarrow \frac{1}{\sqrt{2}}(y_1 + y_2)$



$y_1 = y_2$ $\qquad A_2 = 0 \quad A_1 = \frac{y}{\sqrt{2}}$

$A_2 \Rightarrow \frac{1}{\sqrt{2}}(y_1 - y_2)$

$y_1 = -y_2$ $\quad A_1 = 0 \quad A_2 = \frac{y}{\sqrt{2}}$

longer string
higher energy

$|\vec{y}_{1 \cdots N}\rangle$ are transverse e-states

$\Delta l = y^2/2a$

$$V = bR + \frac{b}{2a} \sum_{n=0}^{N} \left(\vec{y}_{n+1} - \vec{y}_n\right)^2$$

Transverse oscill^n of links

$b = 1\,\text{GeV}/\text{fm}$

$R = (N+1)a$

N links in $\hat{z}$

Some technology

$$\boxed{\vec{y}_n = \sqrt{\frac{2}{N+1}} \sum_p \vec{A}_p \sin\left(\frac{p\pi}{N+1}\, n\right)}$$

and $$\boxed{V = bR + \frac{b}{2a}\sum_{n=0}^{N} \underbrace{\left(\vec{y}_{n+1} - \vec{y}_n\right)^2}}$$

$$\frac{2}{N+1}\sum_p A_p^2 \sin^2\left(\frac{p\pi}{N+1}\right) \sum_n \underbrace{4\cos^2\left(\left(n+\tfrac{1}{2}\right)\frac{p\pi}{N+1}\right)}_{2(N+1)}$$

$$\boxed{\frac{b}{2a}\sum\left(y_{n+1}-y_n\right)^2 \Rightarrow \frac{ba}{2}\sum_p A_p^2 \underbrace{\left(\frac{2}{a}\sin\left(\frac{p\pi}{2(N+1)}\right)\right)^2}_{\omega_p^2}}$$

$$\omega_p = \frac{2}{a}\sin\frac{p\pi}{2(N+1)}$$

$R=(N+1)a$

Fix R $\quad a \to 0$

$N+1 \to \infty$

$$\frac{2}{a}\cdot\frac{p\pi}{2(N+1)}$$

$$\boxed{\omega_p = p\pi/R}$$

$N=1 \simeq N=\infty$

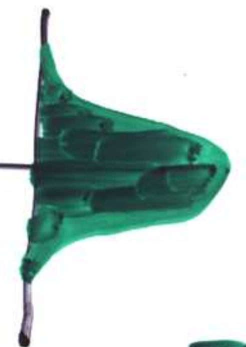
$\omega_p = \frac{2\sqrt{2}}{R}$

$N=\infty \qquad \frac{\pi}{R}$

$V = bR$

$$bR \neq \sum_n \frac{b}{2a} \Delta y_n^2$$

$$\chi_0(y) = \left(\frac{2b^2}{\pi}\right)^{1/4} e^{-\frac{by^2}{\sqrt{2}}}$$



Isgur 99:

- String Tension $\longleftrightarrow$ Transverse $\langle y_T \rangle$
- Dynamical d.o.f.
- Beyond naive const. q. model
  - affects hadron f.f.

$\Rightarrow$ (1) Isgur insight + how solves the ρ^2 problem

(2) FEC+ Dudek application $\Rightarrow$ hybrid excit^n.

Isgur PR D60, 114016 (99)

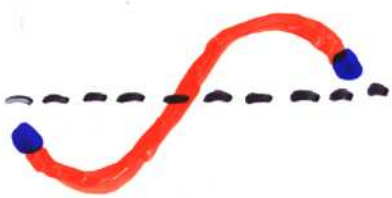
$$\vec r_{QT} = \frac{-br}{\pi p m_Q}\sqrt{\frac{2}{N+1}}\;\vec a_p$$

$$\vec r_{qT} = \boxed{(-)^p}\,\frac{br}{\pi p m_q}\sqrt{\frac{2}{N+1}}\;\vec a_p$$

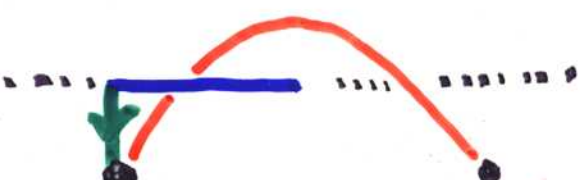
p = odd

qQ balance tube

p = even



qQ mutually counterbalance



$$\vec r_Q = \left[\frac{1}{2}\vec r\,;\;\; \frac{br}{\pi m_Q}\sqrt{\frac{2}{N+1}}\sum_p \frac{\vec a_p}{p}\right]$$

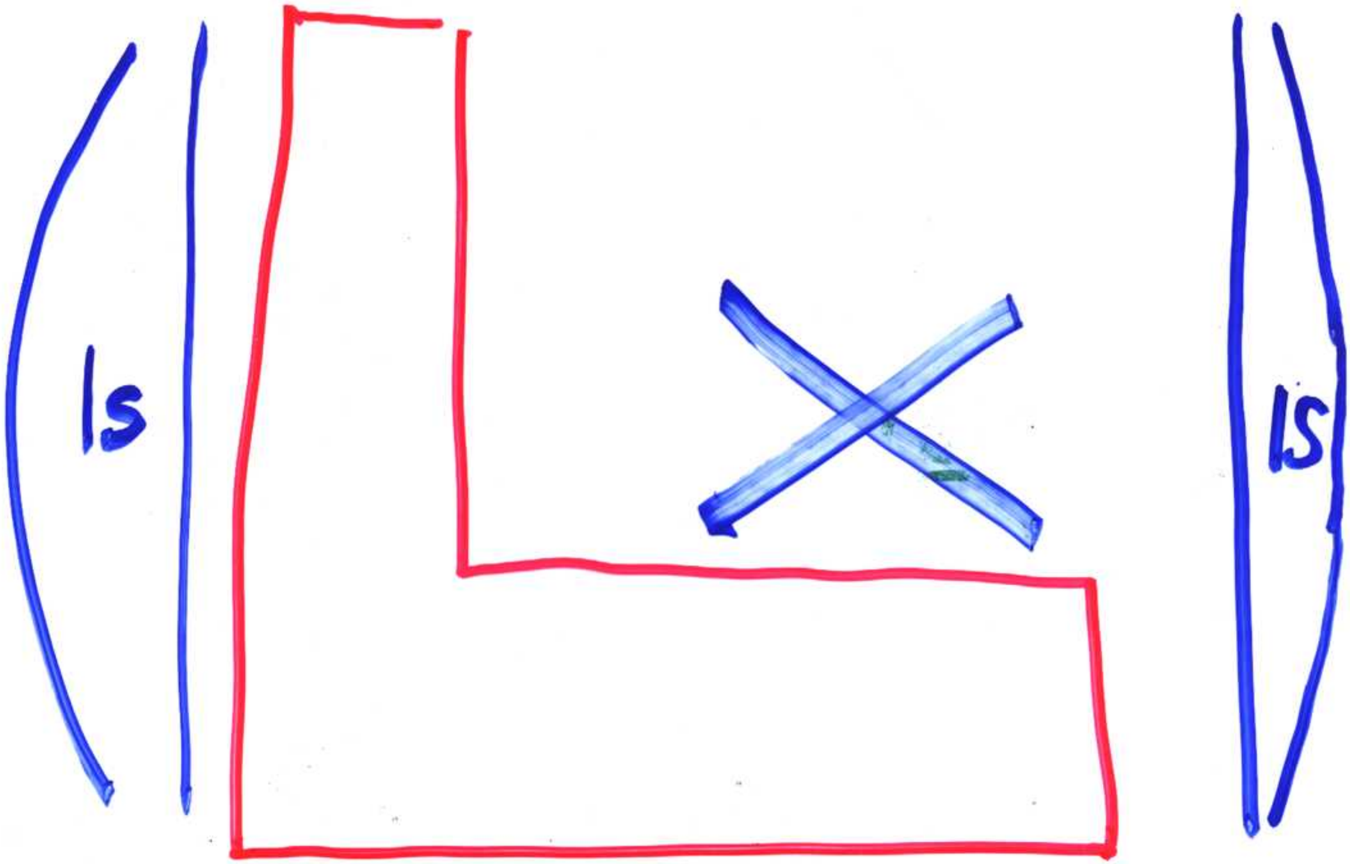
$$e^{i\vec{q}\cdot\vec{r}_Q} = e^{i\vec{q}\cdot\left[\frac{1}{2}\vec{r}\;;\;\frac{b_c}{\pi m_0}\sqrt{\frac{2}{N+1}}\sum_p \frac{\vec{x}_p}{p}\right]}$$

$$1$$

$$-i\vec{q}\cdot\frac{\vec{r}}{2}$$

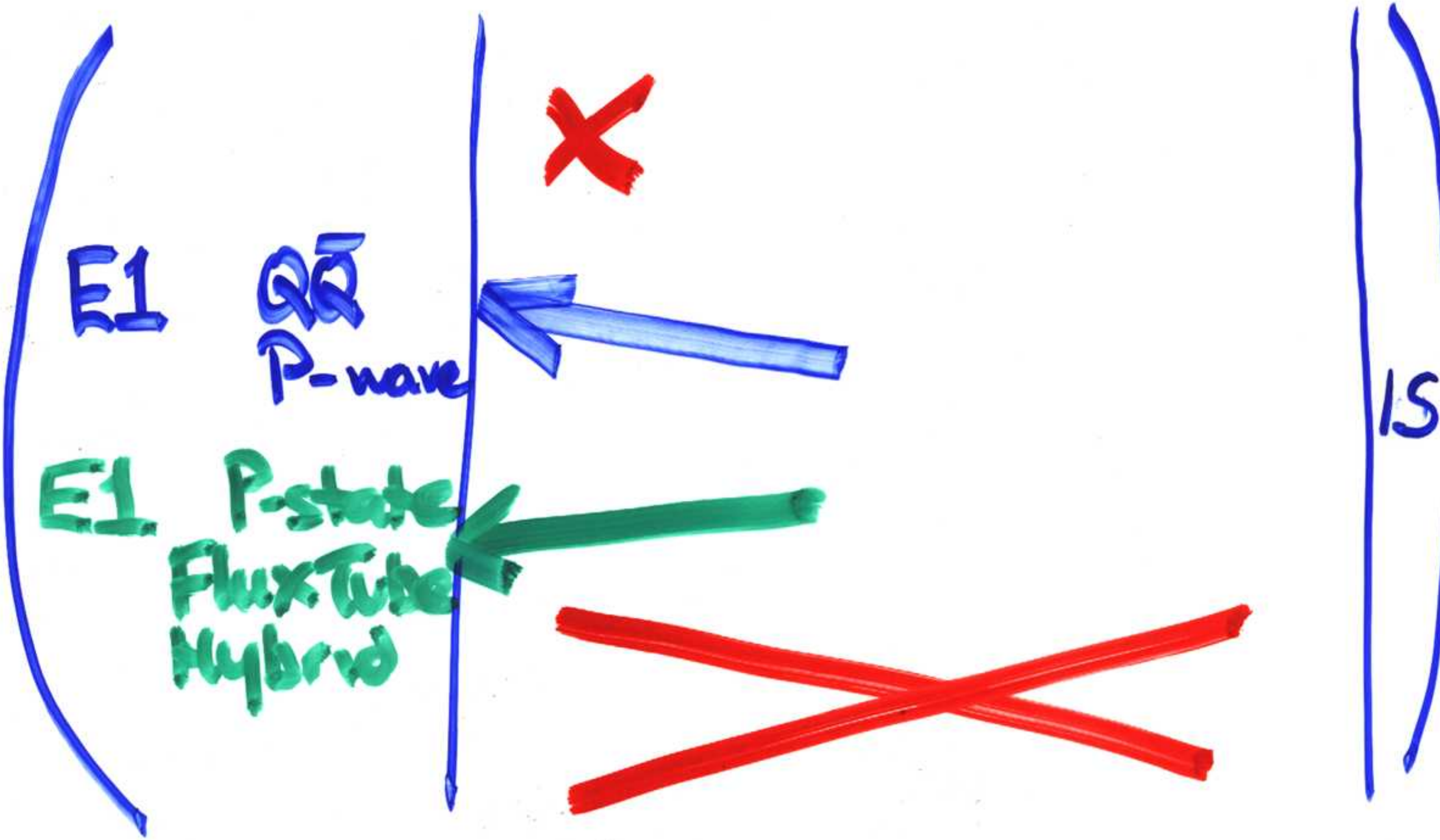
$$-i\vec{q}_i\vec{a}_i\left(\frac{\Delta r}{\pi m_a}\right)\sqrt{\frac{2}{N+1}}$$

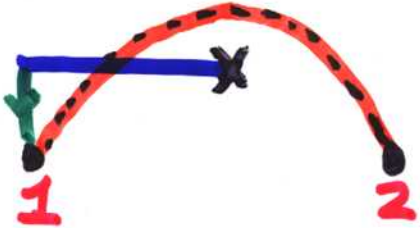
$$-\frac{q^2}{24}\langle r^2\rangle\left(1+\frac{8b}{\pi^3 m_a}\sum_p\frac{1}{p^3}\right)$$



$m_q \Rightarrow 50\%$

$qQ \Rightarrow 13\%.$


E1 $Q\bar{Q}$
P-wave
E1 P-state
FluxTube
Hybrid
1S



$\pi\ 0^{-+} \to 1^{+-}$ expt 0.24 MeV

$\pi\ 0^{-+} \to 1_H^{+(--)}$

$$\boxed{\frac{M(1_Q^{+-})}{M(1_H^{+-})} = \frac{\omega\langle r\rangle(e_1-e_2)}{\omega_H\langle r\rangle(e_1+e_2)\sqrt{\dfrac{8b}{\pi^3 m_Q^2}}}}$$

E1 TRANSITIONS

$Q\bar{Q}$

$e_1 - e_2$

$\therefore Q = 0$ ✓ e.g. $\chi_{c\bar{c}} \xleftrightarrow{\gamma} \psi_{c\bar{c}}$

Hybrid

$e_1 + e_2$

$\therefore Q = 0$ ✗

$\chi^{H}_{c\bar{c}} \not\rightarrow \gamma \psi_{c\bar{c}}$

$\psi^{H}_{c\bar{c}} \not\rightarrow \gamma \chi_{c\bar{c}}$

$(\psi'_{3.7} \neq \psi_H)$

Good News

$\Gamma(\pi^+ \xrightarrow{\gamma} b_1^+)$ $(e_1 - e_2 = 1/3)$ $\Rightarrow$ 0.24 MeV

$\Gamma(\pi^+ \longrightarrow a^+_{1(H)})$ $(e_1 + e_2 = 1)$ $\Rightarrow$ 2 ± 1 MeV

BIG 

 WHERE POSSIBLE

*1

Recap Isgur

$$\langle r_Q^2 \rangle = \left\langle \frac{r}{2} \right\rangle^2 \left[1 + \frac{8b}{\pi^3 m_Q^2} \sum_p \frac{1}{p^3} \right]$$

$$\langle r^2 \rangle \simeq \sum_n \langle 0^- | \vec{r} \,|\, \begin{matrix} 1^+ \\ 1^+ \end{matrix} \times \begin{matrix} 1^+ \\ 1^+ \end{matrix} | \vec{r} | 0^- \rangle$$

Hybrid "price" relative to conventional

$$\left(\frac{2}{\pi}\right)^3 \left(\frac{\sqrt{b}}{m_Q}\right)^2 \simeq \frac{1}{4}\left(\frac{0.4\,\mathrm{GeV}}{m_Q}\right)^2$$

c: 3%

s: 15% (hints ✓. $B \to \psi K_H$

u,d: 50% (approx begin to fail)

Dipole Sum Rule

$$\sum_n \langle i | \underset{\vec{y}}{\vec{r}} | n \rangle \langle n | \underset{\vec{y}}{\vec{r}} | i \rangle \longleftrightarrow \langle i | \underset{y^2}{r^2} | i \rangle$$

Saturated $n=1$ $\quad (Q\bar{Q})_P \quad (\text{Hybrid})_{n=1}$

C. Algebra Generalisation: Cabibbo Radicatti Sum Rule

$$\int d\nu\, \sigma(\gamma\pi \to R) = \frac{1}{6}\langle r^2 \rangle$$

$$\Rightarrow \int d\nu\, \sigma(\gamma\pi^{\pm} \to R^{\pm}_{H}) \simeq \int d\nu\, \sigma(\gamma\pi \to R_Q)$$

Recap $\langle r^2 \rangle = 1 + \frac{86}{\pi^3 m_Q^2}$!!

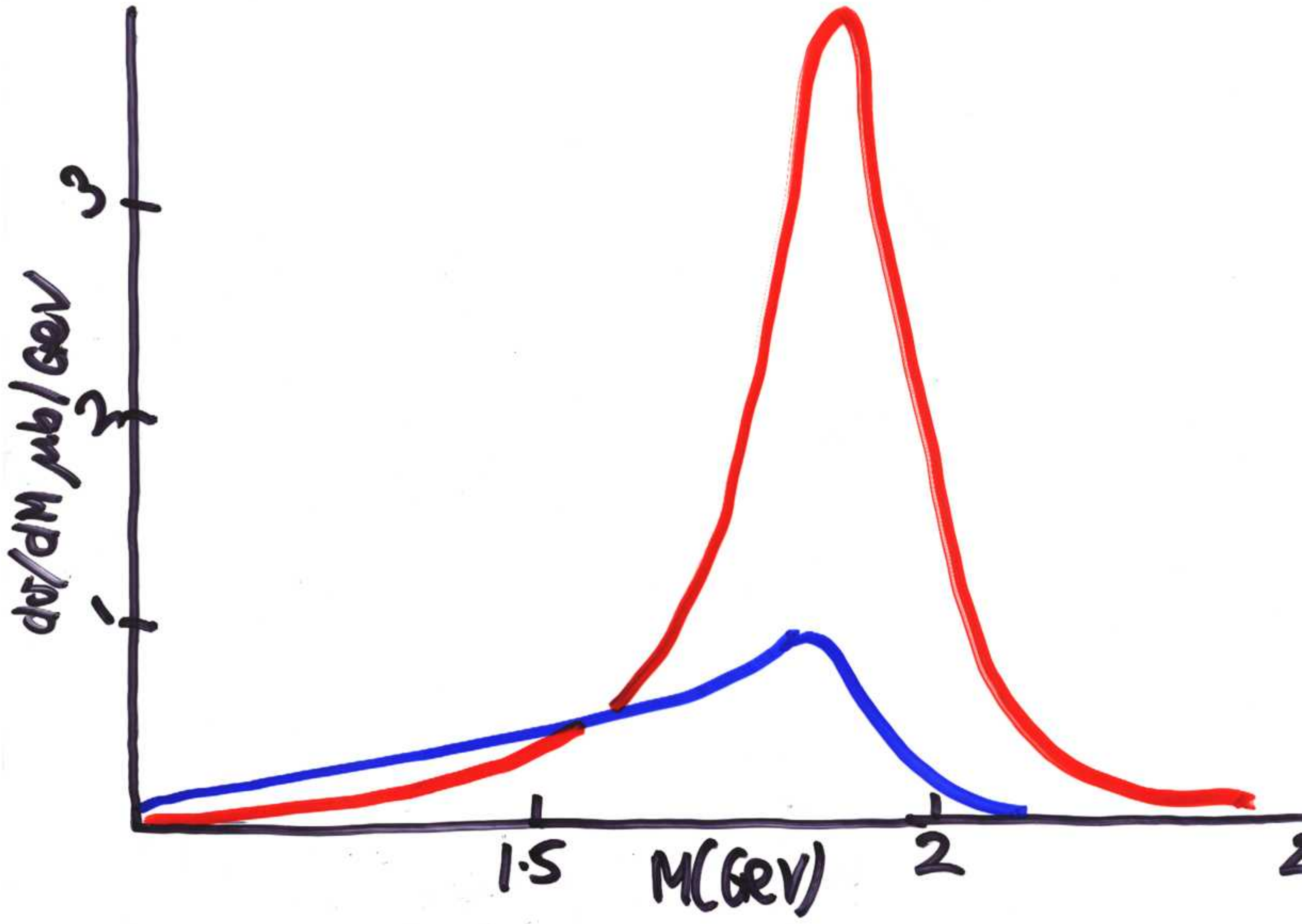
Photoprod of $a_{1H}^{+}(1900)$ at JLab.

Afanasev

Close Dudek $\Gamma(a_{1H} \to \pi\gamma)$

$E_\gamma = 5$ GeV

$E_\gamma = 9$ GeV



CLAS: 1/sec

GlueX: 14/sec

" upgrd: 140/sec

γ flux 10^{+7}/s

10^{8}/s

γ	M		FluxTube	
E1	$\pi\ 0^{-+} \to$	1^{+-}	1^{++}	CEX
	$\rho\ 1^{--} \to$	$0^{++}\ 1^{++}\ 2^{++}$	$0^{+-}\ 1^{+-}\ 2^{+-}$	CEX
	$b_1\ 1^{+-} \to$	$0^{-+}\ 1^{-+}\ 2^{-+}$	$0^{-+}\ 1^{-+}\ 2^{-+}$ ✓ but b_1 ??	
	$f_2/a_2\ 2^{++} \to$	1^{--}..	1^{-+}	CEX

$\vec{r} \Rightarrow \vec{r} \quad \vec{y}$

$\vec{p} \Rightarrow \nabla_r \quad \nabla_y$

$\Delta S_{q\bar{q}} = 0$

$\boxed{\omega \vec{\epsilon} \cdot \vec{r} \qquad E1 \qquad E1}$

$\frac{\vec{q} \times \vec{\epsilon}}{m} \cdot \vec{r} \times \vec{p} \qquad \cancel{M1}_{orbital} \qquad M1_y$

$\Delta S_{q\bar{q}} = 1$

$\frac{\vec{q} \times \vec{\epsilon}}{m} \cdot \vec{\sigma} \, (i \vec{q} \cdot \vec{r}) \qquad M1_{Spin} \qquad M1_{Spin}(qr)$

$\omega \frac{\vec{p} \times \vec{\epsilon} \cdot \vec{\sigma}}{m^2} \qquad E1', M2 \qquad E1', ..$

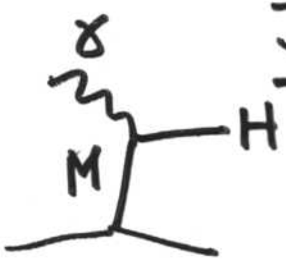
spin orbit

but $O(m^{-3})$

conventional
$Q\bar{Q}$
excitations

flux tube
hybrid
excitations

J^{PC} of Hybrid in J^{em}



$\vec{\epsilon}\cdot\vec{r}$ 　 E1

$\vec{r}\times\vec{p}\cdot\vec{q}\times\vec{\epsilon}/m$ 　 $M1_{orb}$ 　 $\Delta S=0$

$\vec{\sigma}\cdot\vec{q}\times\vec{\epsilon}\,(i\vec{q}\cdot\vec{r})/m$ 　 $M1_{spin}$

$\omega\,\vec{\sigma}\cdot\vec{p}\times\vec{\epsilon}/m^2$ 　 spin-orbit

$\left.\begin{array}{l}M1_{spin}\\ \text{spin-orbit}\end{array}\right\}\ \Delta S=1$

$b_1 \to \pi\gamma$ 0.23

$a_1^H \to \pi\gamma$ 2.1(0.9)

$b_J^H \to \rho\gamma$ 2.3(0.8)

$\pi_1 \to a_2\gamma$ 0.1

γ

M1 orbital

$$\vec{r} \times \vec{p} \cdot \vec{q} \times \vec{e}/m$$

$$\underbrace{\vec{y} \times \vec{p} \qquad \vec{r} \times \vec{p}_y}_{}$$

$\hat{y}$ relative ⊖ tend to cancel like b –

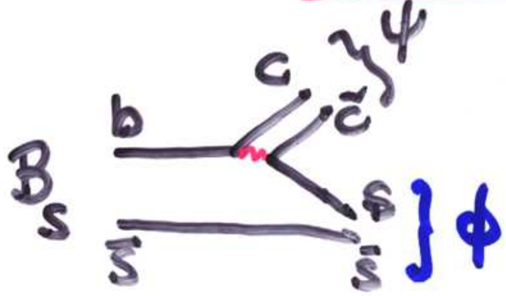
1 GeV/fm $\longleftrightarrow$ $(0.4\,\text{GeV})^2$

$\Rightarrow$ 0 for u, d

$\Rightarrow$ OK for $c\bar{c}$ (but $O(v^2/c^2)$)

BUT MODEL DODGY – DON'T UNDERSTAND B
$\therefore$ M1 I DON'T YET TRUST. B IN IT.

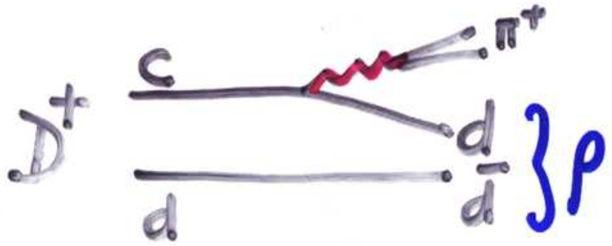
B and D decays



$\hat{\phi}(1^{-+})$ if < 2.1 GeV

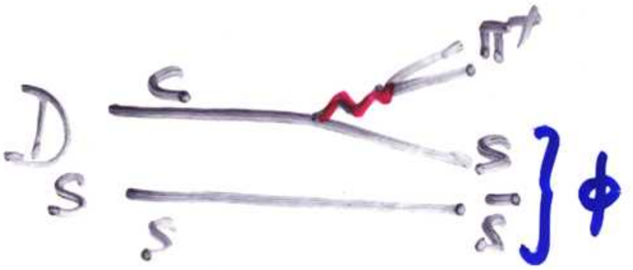
$D_H < 4.5$ GeV Rate $= 3-15\%$ of πD^{**}

$\frac{g_A}{g_V}$ distinction w.r.t. D^*



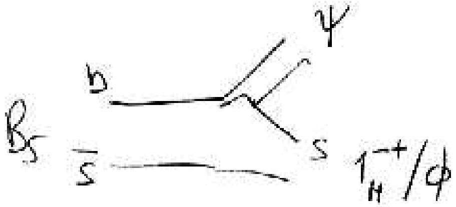
$\hat{\rho}(1^{-+}) \simeq \frac{1}{2} *$ Cab Supressed
$*$ phase space

? 1600 MeV has 125 MeV available



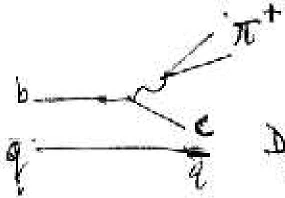
$\hat{\phi}(1^{-+}) \simeq \frac{1}{3} *$ Cab. leading
$*$ phase space

? An $I=0$ partner to 1600 MeV



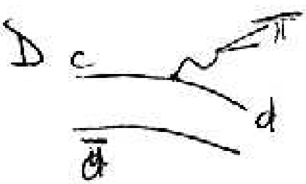
ie $1^{-+}_{s\bar{s}} < 2.1$ GeV

CDF → 2 events in assoc. with $B_s - \bar{B}_s$ mixing



$D_H \lesssim 5$ GeV. Rate = 3% to 15% × πD^{**}

$\frac{g_A}{g_V}$ distinction ?



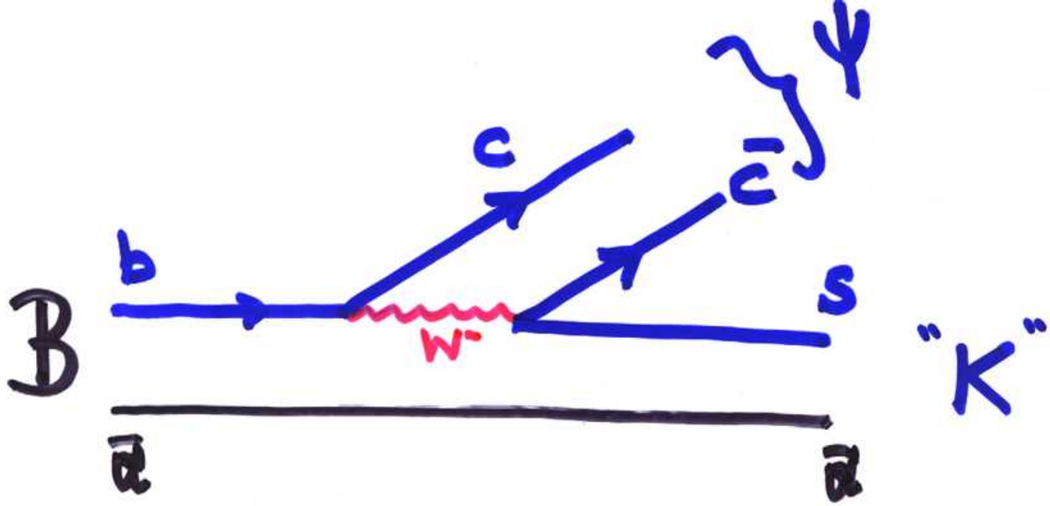
$\simeq \frac{1}{2} \times$ Cab. suppressed × phase space

1860
140

1800 MeV
(125 MeV available)

$\simeq \frac{1}{3} \times$ Cab lead × phase space

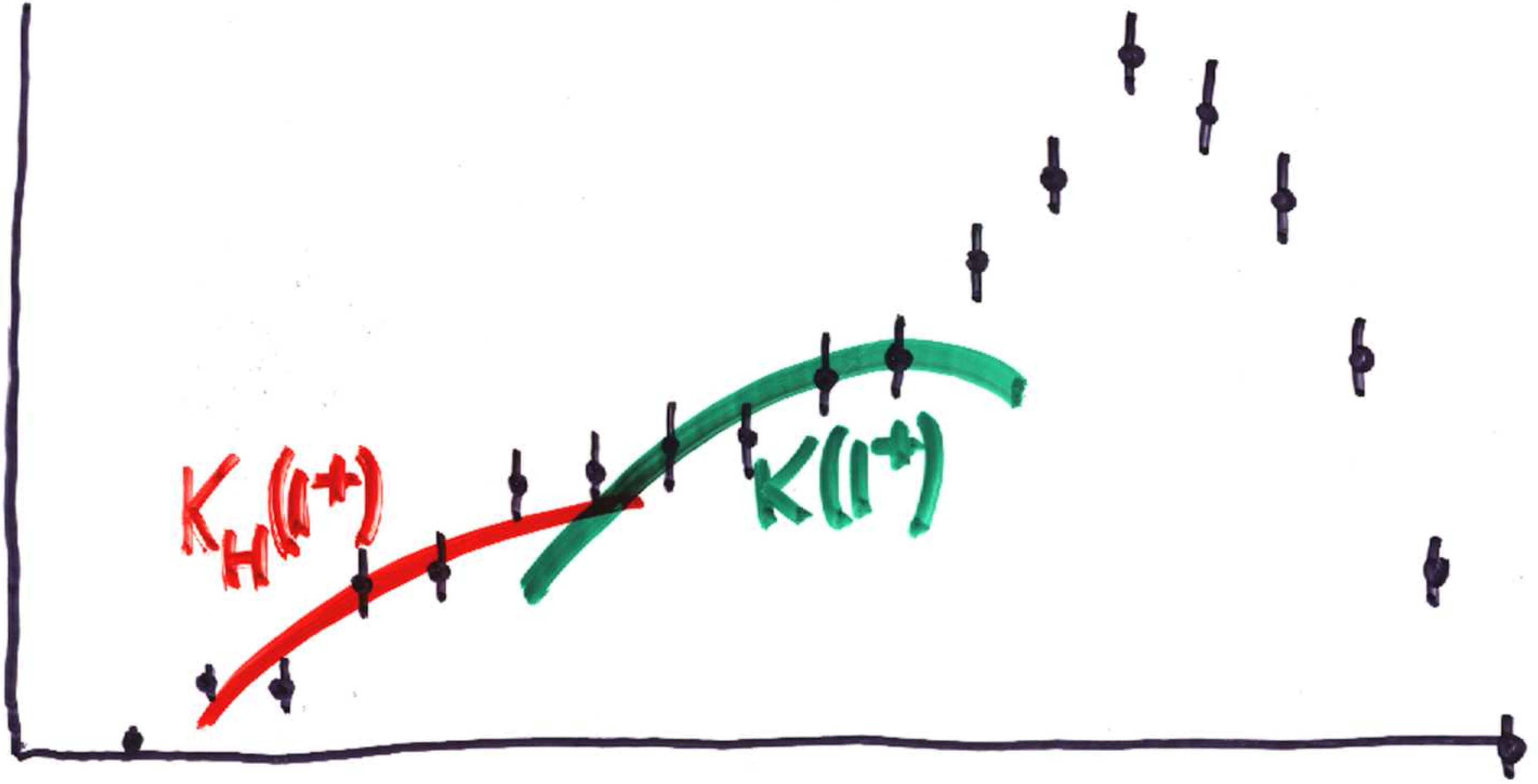
2.2

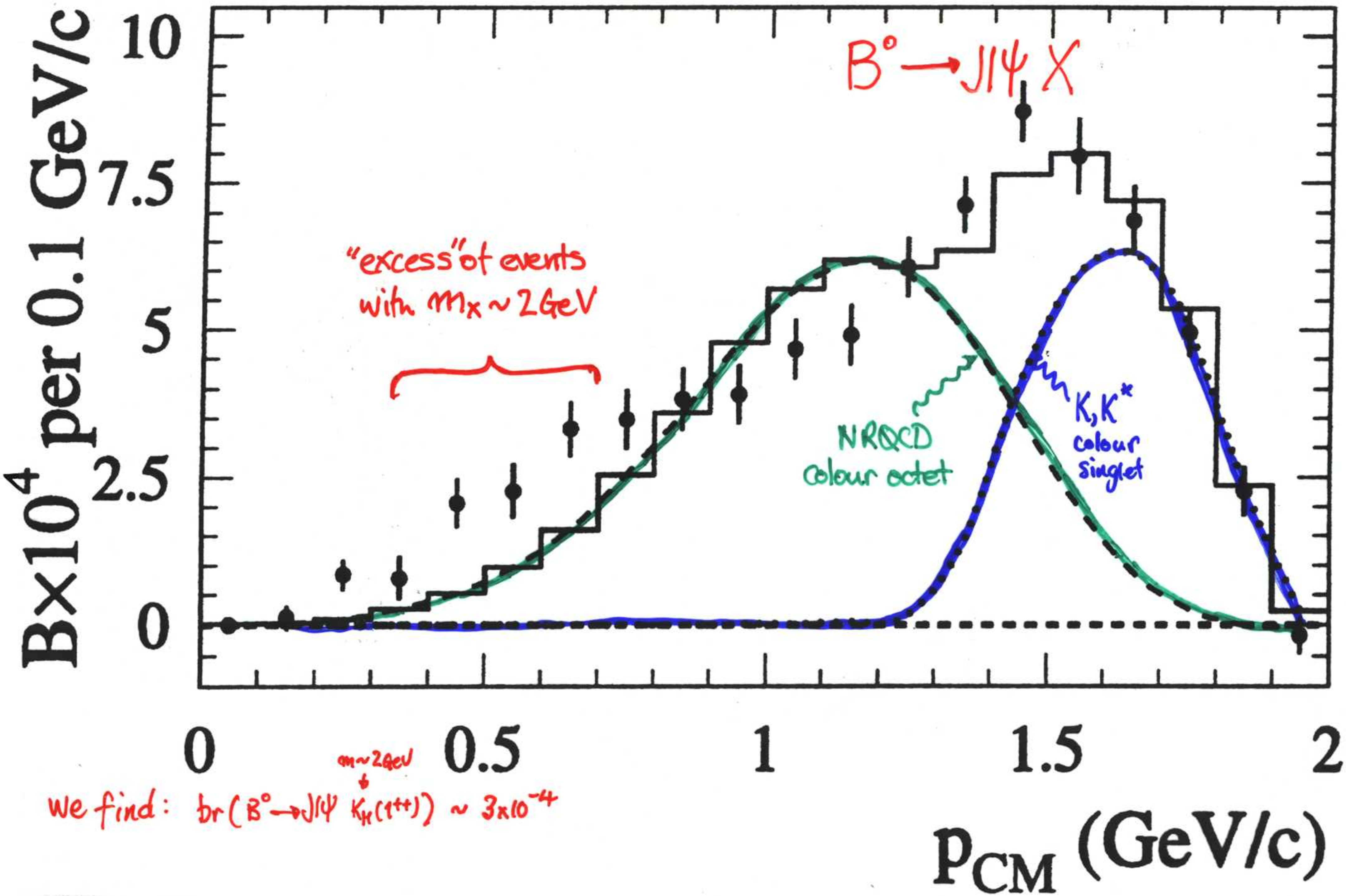


b.r. $B \to \psi K$, K^* $\simeq 10^{-3}$

$\to \psi \, K(1^+)$ $\quad 10^{-3}$

$\to \psi \, K_H(1^+)$ $\quad (1-4) * 10^{-4}$


$K_H(\mathbb{I}^+)$
$K(\mathbb{I}^+)$

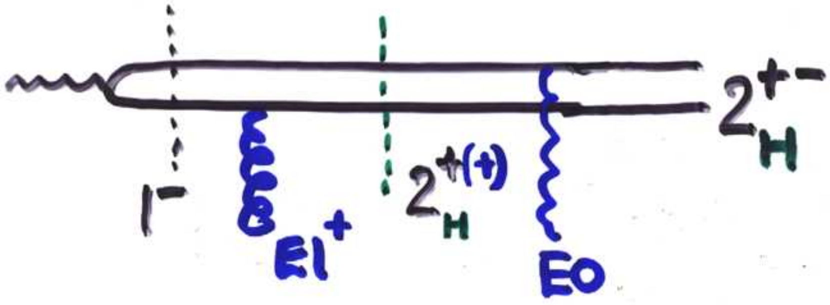


$B^0 \to J/\psi\, X$
"excess" of events
with $m_X \sim 2$ GeV
NRQCD
colour octet
K, K*
colour singlet
$B \times 10^4$ per 0.1 GeV/c
p_{CM} (GeV/c)
we find: $br(B^0 \to J/\psi\, \overset{m \sim 2\,GeV}{K_H(1^{++})}) \sim 3 \times 10^{-4}$

Application to other currents.

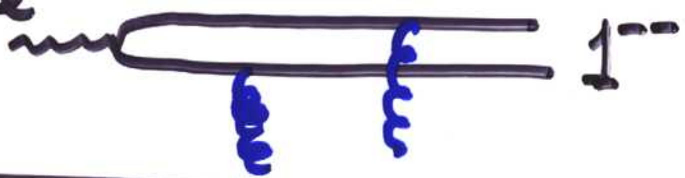
Gluons

Photoprod^n. Diffractive $\gamma \to 2^{+-}_H$ exotic



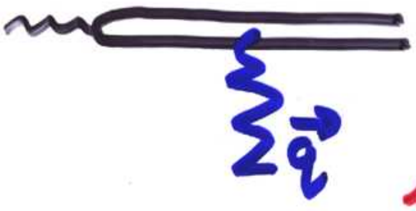
e.g. $\psi_{2(H)}$

compare



e.g. ψ

Diff^n. Pomeron



$\rho(S=1) \oplus \rho_H(S=0)$

$1 + i\vec{\sigma}\cdot\vec{p}\times\vec{q}/m_q^2$ SCNCX

Heavy Flavour

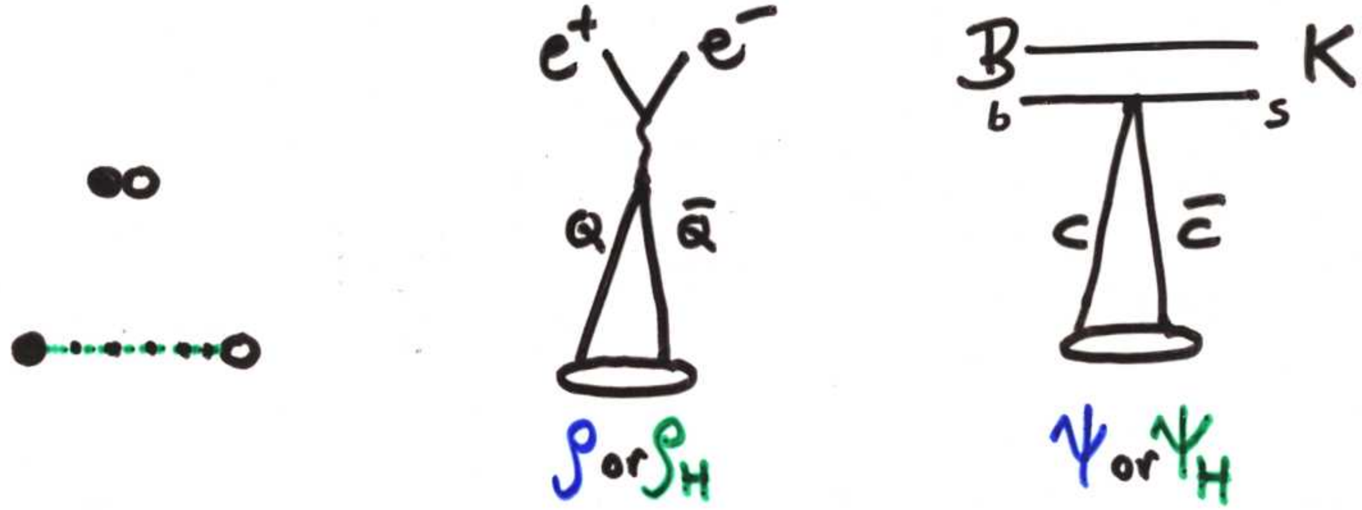
$$\left.\begin{array}{l} \tilde{\Gamma}(B \to D_H) \simeq 0.15\,\tilde{\Gamma}(B \to D^*) \\ \tilde{\Gamma}(B \to \hat{\pi}_H) \simeq 0.5\,\tilde{\Gamma}(B \to \rho) \end{array}\right\} \quad \left.\frac{g_A}{g_V}\right|_H < \left.\frac{g_A}{g_V}\right|_{D^*}$$

$\Gamma(D \to 1^{-+})$ needs $1^{-+} < 1.8$ GeV

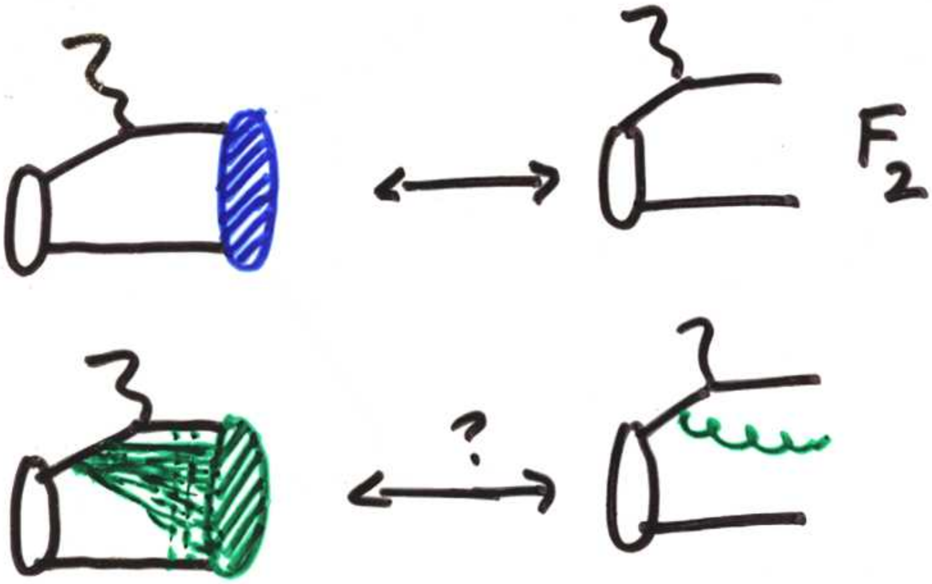
$\Gamma(D_s \to 1^{-+}(s\bar{s}))$ needs $1^{-+} < 2.1$ GeV
Cabibbo favoured

QUESTIONS

- How does fluxtube grow?



- Quark Hadron Duality



$O(\alpha_s)$?
g jets ?
What relation?
How formulate?

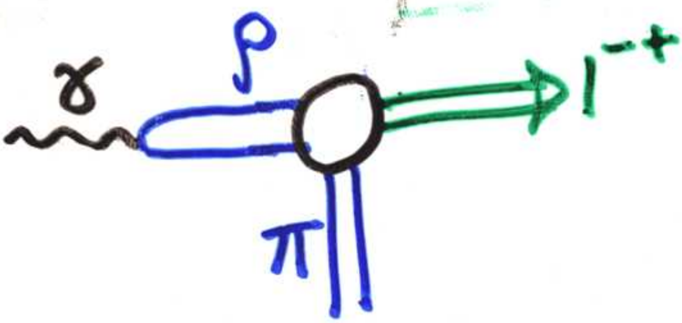
- Sum Rules

$$\langle r^2\rangle_{\substack{Q\\ FT}} \sim \int d\nu\, \sigma_{E1}\left(\gamma M \to \substack{M^*\\ \text{Hybrd}}\right)$$

Cabibbo Radicati

B,D → (ℓν).... ?
duality

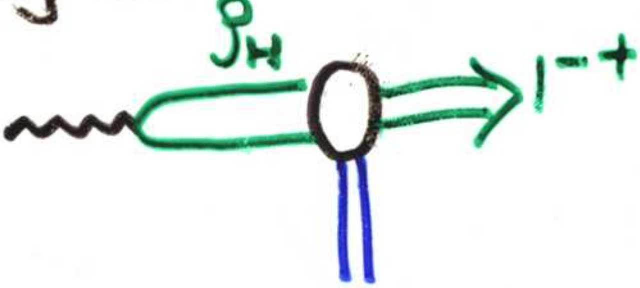
Why VDM is dangerous.



$\Rightarrow$ Afanasev Page estimate

Thy	✓	×
Expt	✓	✓

Why not?



Thy	?	✓
Expt	✓	?

? How big is

$\gamma e \rightarrow \rho_H$

"How does Flux Tube grow?"