JLab, May 2003

Hybrid Mesons in the Flux Tube Model

- flux tubes
- hybrids
- checks:
  - comparison to lattice
  - adiabatic + small oscillation
- IKP decay model
- extensions:
  - charge radii
  - surface mixing
  - spin dependence
  - spin dependence II
  - vector decay model
  - other applications
- conclusions

Jefferson Lab

Eric Swanson

construct a ladder from the debris of nuclear explosions to the beauty of Babylon

PHYSICAL REVIEW B VOLUME 47, NUMBER 6 1 FEBRUARY 1993-II

Excitation spectrum of Heisenberg spin ladders

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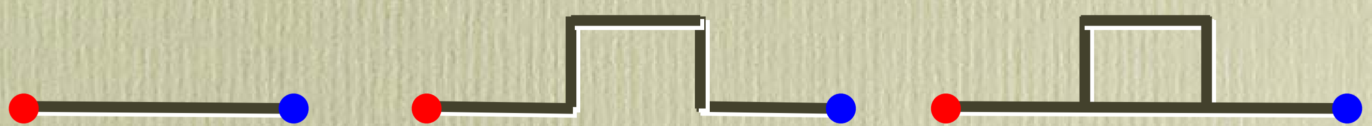
Flux Tubes

Isgur & Paton, PRD31, 2910 (85)

strong coupling Hamiltonian lattice gauge theory

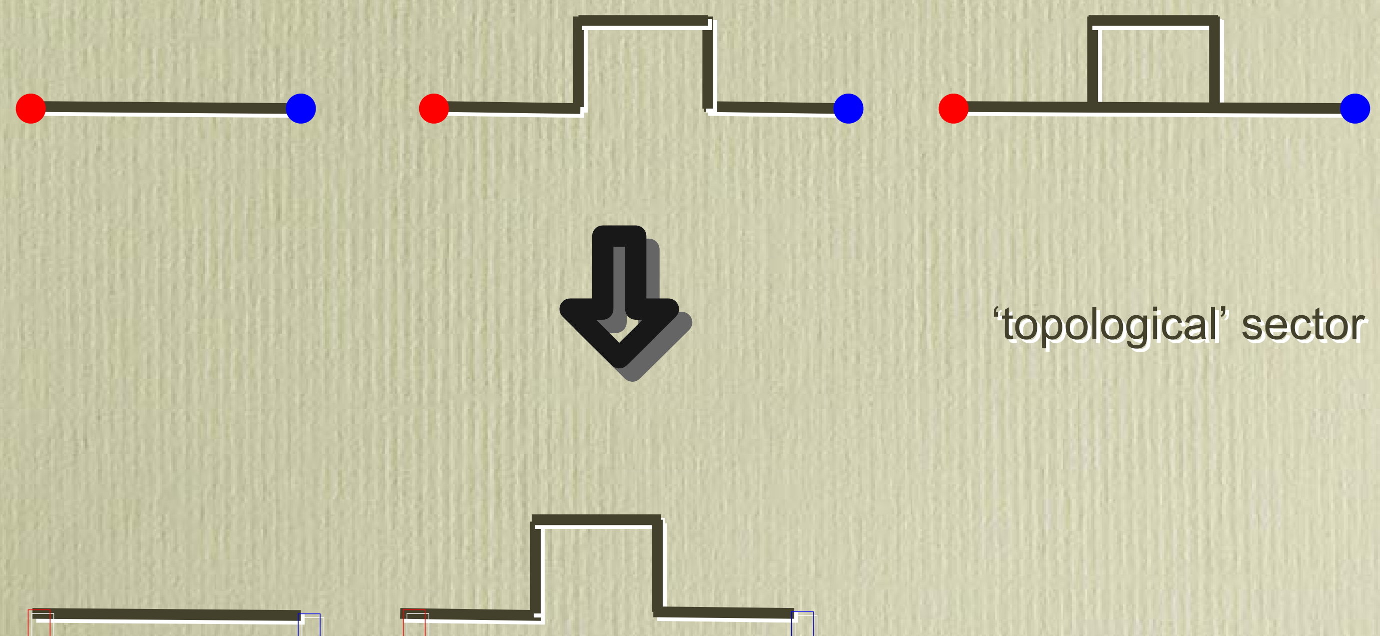
$$H = \frac{g^2}{2a}\sum_{l} E_l^a E_l^a + \sum_{n} m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu} \bar{\psi}_n \gamma_\mu U_{n\mu} \psi_{n+\mu} + \frac{1}{ag^2}\sum_{p} \mathrm{tr}(N - U_p - U_p^\dagger)$$



Isgur & Paton, PRD31, 2910 (85)

strong coupling Hamiltonian lattice gauge theory

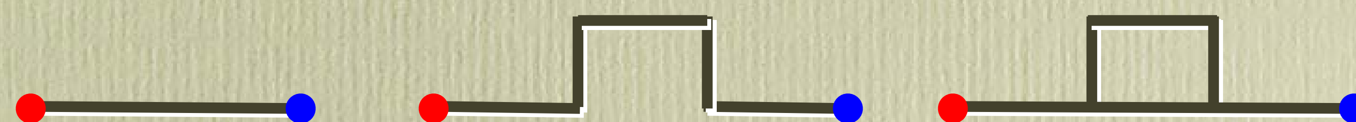
$$H = \frac{g^2}{2a}\sum_{\ell} E_\ell^a E_\ell^a + \sum_{n} m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{\langle nm\rangle} \psi_n^\dagger \alpha_{nm} U_{nm}\psi_m + \frac{1}{ag^2}\sum_{p} \mathrm{tr}(N - U_p - U_p^\dagger)$$



Isgur & Paton, PRD31, 2910 (85)

strong coupling Hamiltonian lattice gauge theory

$$H = \frac{g^2}{2a}\sum_l E_l^a E_l^a + \sum_x m \psi_x^\dagger \psi_x + \frac{1}{a}\sum_{x,\mu} \psi_x^\dagger \alpha_\mu U_\mu(x) \psi_{x+\mu} + \frac{1}{ag^2}\sum_p \mathrm{tr}(N - U_p - U_p^\dagger)$$



adiabatic

small oscillation

nonrelativistic beads

$m_b = b\, a$

string Hamiltonian

$$H = \hbar_0 R + \sum_{n}\left[\frac{p_n^2}{2m} + \frac{k}{2a}(y_n - y_{n+1})^2\right]$$

$$s_{m\lambda} = \sum_{n}^{N} y_n(\lambda)\sqrt{\frac{2}{N+1}}\sin\frac{nm\pi}{N+1} \qquad y_n(\lambda) = \sum_{m} s_{m\lambda}\sqrt{\frac{2}{N+1}}\sin\frac{nm\pi}{N+1}$$

$$H = \hbar_0 R + \sum_{m\lambda}\left[\frac{p_{m\lambda}^2}{2m} + \frac{m}{2}\omega_m^2 s_{m\lambda}^2\right] \qquad \omega_m = \frac{2}{a}\sin\frac{\pi m}{2(N+1)}$$

$$\omega_m \approx \frac{\pi m}{R}$$

$$a_{m\lambda} = \sqrt{\frac{m\omega_m}{2}}\, s_{m\lambda} + i\frac{p_{m\lambda}}{\sqrt{m\omega_m}}$$

$$H = \hbar_0 R + \sum_{m\lambda}\omega_m\left(a^\dagger_{m\lambda}a_{m\lambda} + \frac{1}{2}\right)$$

$$H = \hbar_0 R + \left(\frac{2}{\pi a^2}R - \frac{1}{a} - \frac{\pi}{12R}\right) + \sum_{m\lambda}\omega_m a^\dagger_{m\lambda}a_{m\lambda}$$

Hybrids

Hybrid State

$$|LM_L;s;\Lambda\{n_m \ldots n_{m'}\}\rangle \propto \int d\Omega\, D^{L*}_{M_L,\Lambda}(\phi,\theta,-\phi) \prod_m |n_m \ldots\rangle\, |n_m \ldots\rangle\, |0\rangle$$

$$\Lambda = \sum_m (n_m \ldots n_m)$$

$$E = E_0 + N\frac{\pi}{R}$$

$$N = \sum_{m=1} m(n_m + \ldots n_m)$$

Hybrid Quantum Numbers

$$P|LM_L;SM_S;\Lambda\{n_m \ldots n_m\}\rangle = (-1)^{L+\Lambda+\ldots}|LM_L;SM_S;-\Lambda\{n_m \ldots n_m\}\rangle$$

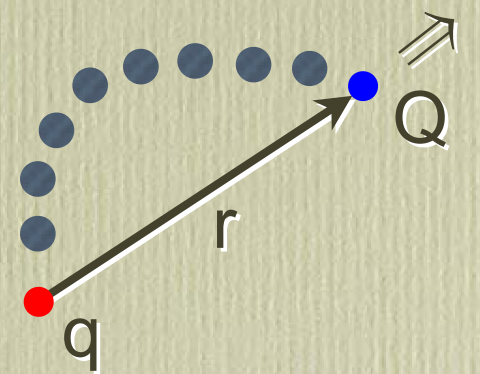
$$C|LM_L;SM_S;\Lambda\{n_m \ldots n_m\}\rangle = (-1)^{L+S+\Lambda+N}|LM_L;SM_S;-\Lambda\{n_m \ldots n_m\}\rangle$$

Λ + ζ (-Λ)

Adiabatic Surface Quantum Numbers

the diatomic molecule

$\Lambda = \Sigma, \Pi, \Delta, \ldots$



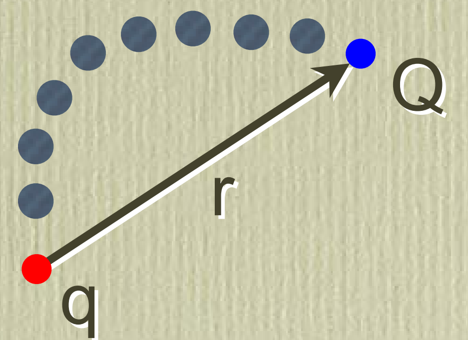
projection of the string angular momentum

onto the qq axis

Adiabatic Surface Quantum Numbers

the diatomic molecule

$\eta = u/g$

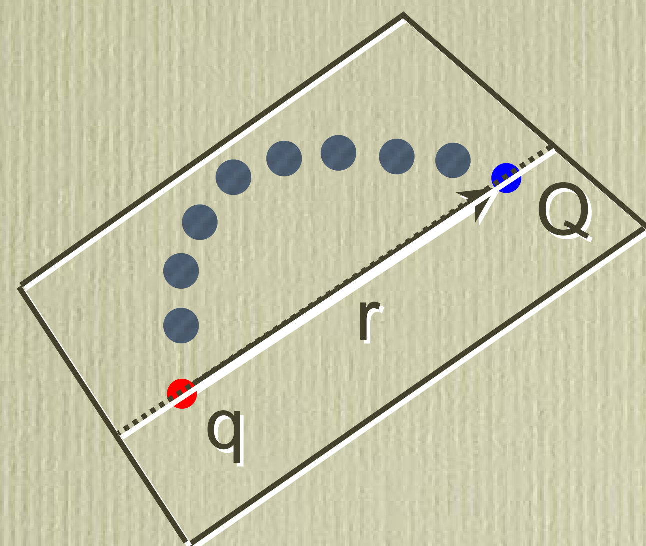


PC acting on the glue

Adiabatic Surface Quantum Numbers

the diatomic molecule

$Y = +/-$



reflection in a plane containing the $q\bar{q}$ axis

ex: $\Lambda^{Y}_{\eta} = \Sigma^{+}_{g}$ ground state

Hybrid Quantum Numbers (one m=1 phonon)

	L	S	J^{PC}
$\zeta = +$	1	0	1^{++}
	1	1	$(2,1,0)^{+-}$
	2	0	2^{--}
	2	1	$(3,2,1)^{-+}$
$\zeta = -$	1	0	1^{--}
	1	1	$(2,1,0)^{-+}$
	2	0	2^{++}
	2	1	$(3,2,1)^{+-}$

FTM Model Hamiltonian

[illegible]

I&P Hybrid Masses

flavour	m	m’
I=1	1.67	1.9
I=0	1.67	1.9
$s\bar{s}$	1.91	2.1
$c\bar{c}$	4.19	4.3
$b\bar{b}$	10.79	10.8

$2^{\pm\pm}, 1^{\pm\pm}, 0^{\pm\pm}, 1^{\pm\pm}$

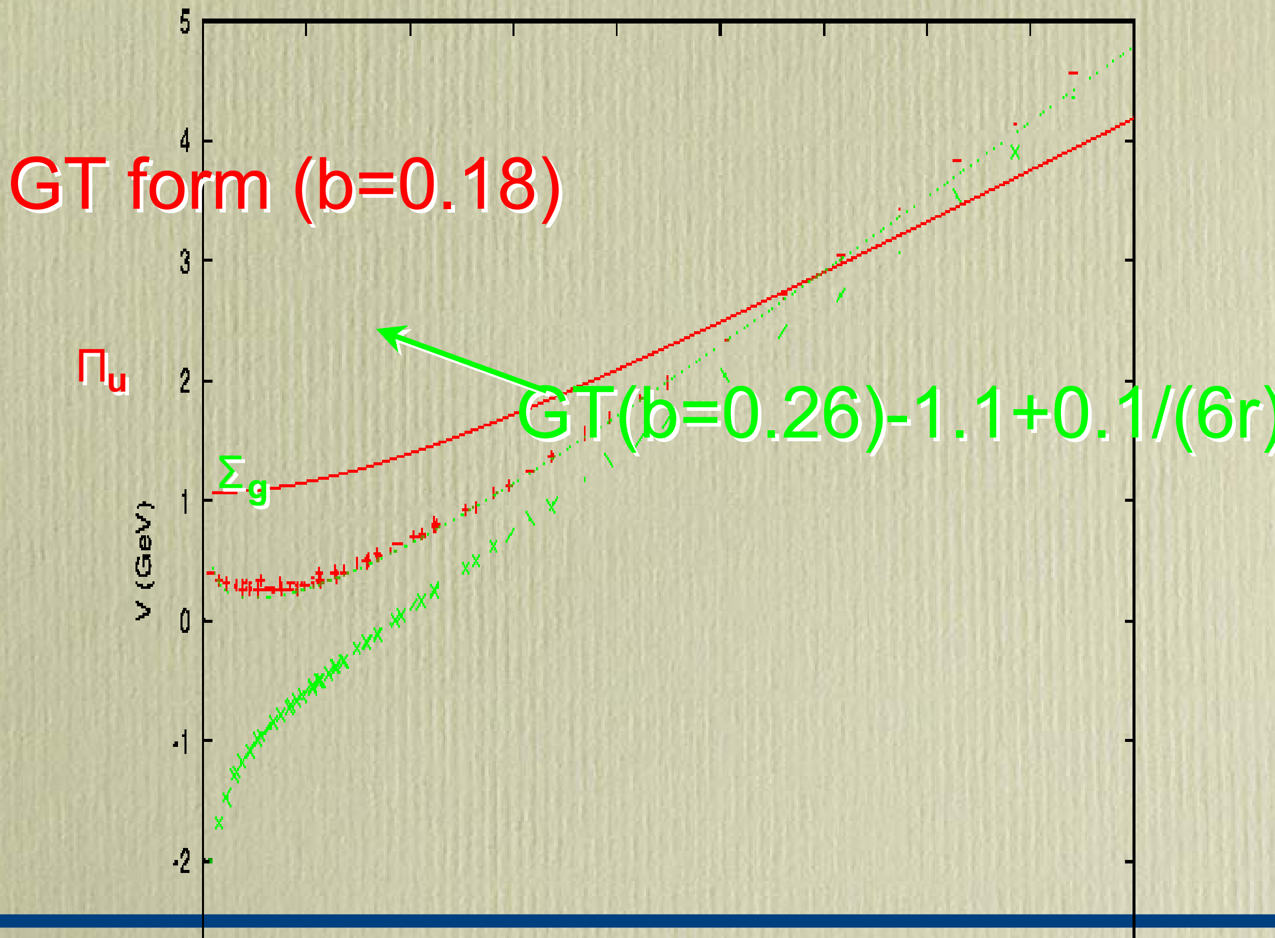
Aside: Giles and Tye, PRL **37**, 1175 (1976)

Coupled quarks to a relativistic 2d sheet... the “Quark Confining String Model”.

“The presence of vibrational levels gives ... extra states in quantum mechanics. ... that are absent in the charmonium model.”

[illegible] GT

[illegible] J.F. Arvis, PLB127, 106 (83); Luescher



GT also computed finite-mass corrections, and spin-orbit splittings.

16 APPLICATION OF THE QUARK-C

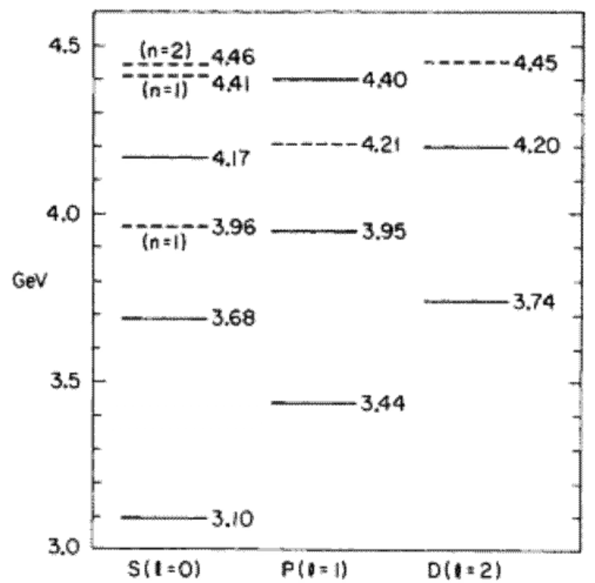
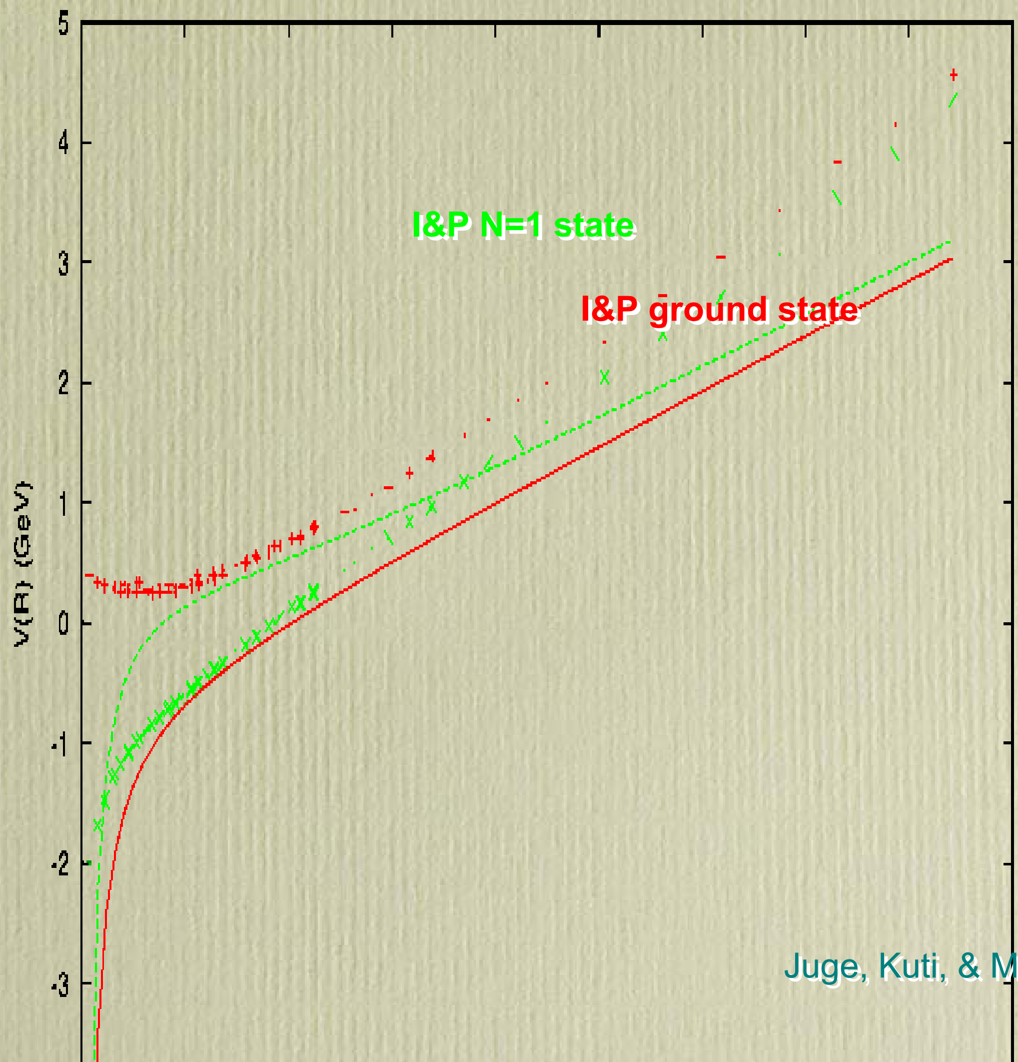


FIG. 4. The nonrelativistic spectroscopy of the charm string. $\psi(3.10)$ and $\psi(3.68)$ are fitted to obtain $M = 1.154$ GeV and $k = 0.21$ GeV^2. The dashed lines are the vibrational levels absent in the charmonium model. Levels with $E > 4.5$ GeV or $l > 2$ are not shown.

Putting together Eqs. (5.1), (5.2), (5.34), (5.39)

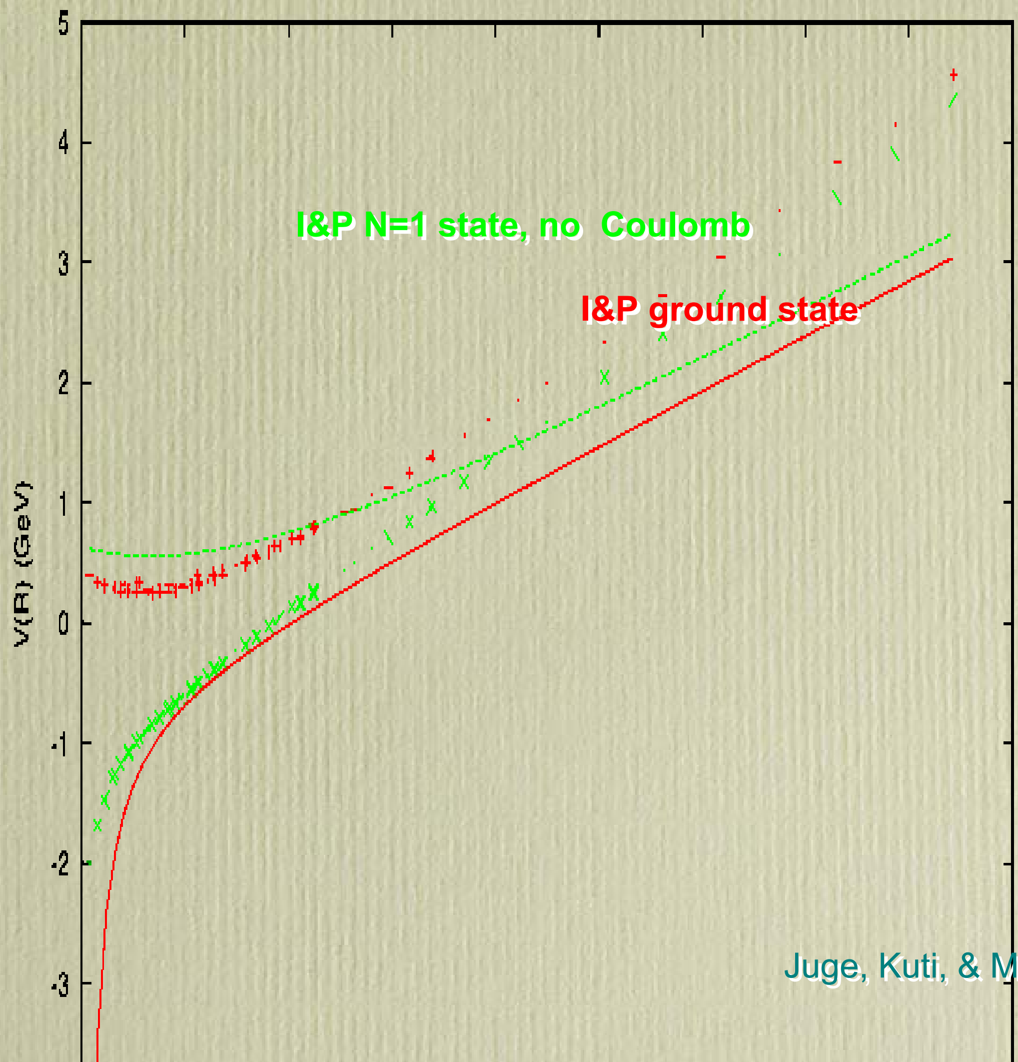
Checks

Comparison to the lattice



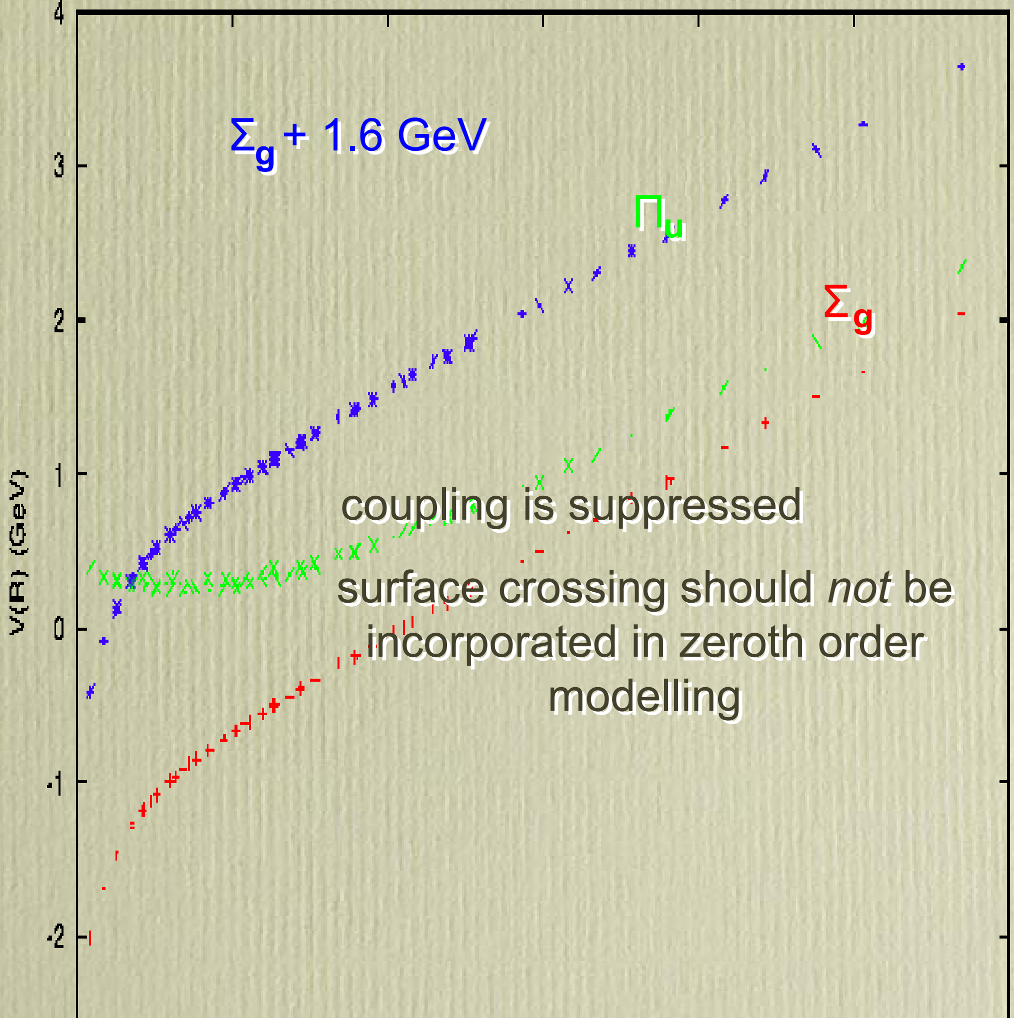
Juge, Kuti, & Morningstar

Comparison to the lattice



Juge, Kuti, & Morningstar

Should the Coulomb potential be there?



coupling is suppressed

surface crossing should *not* be incorporated in zeroth order modelling

Born-Oppenheimer + lattice potentials

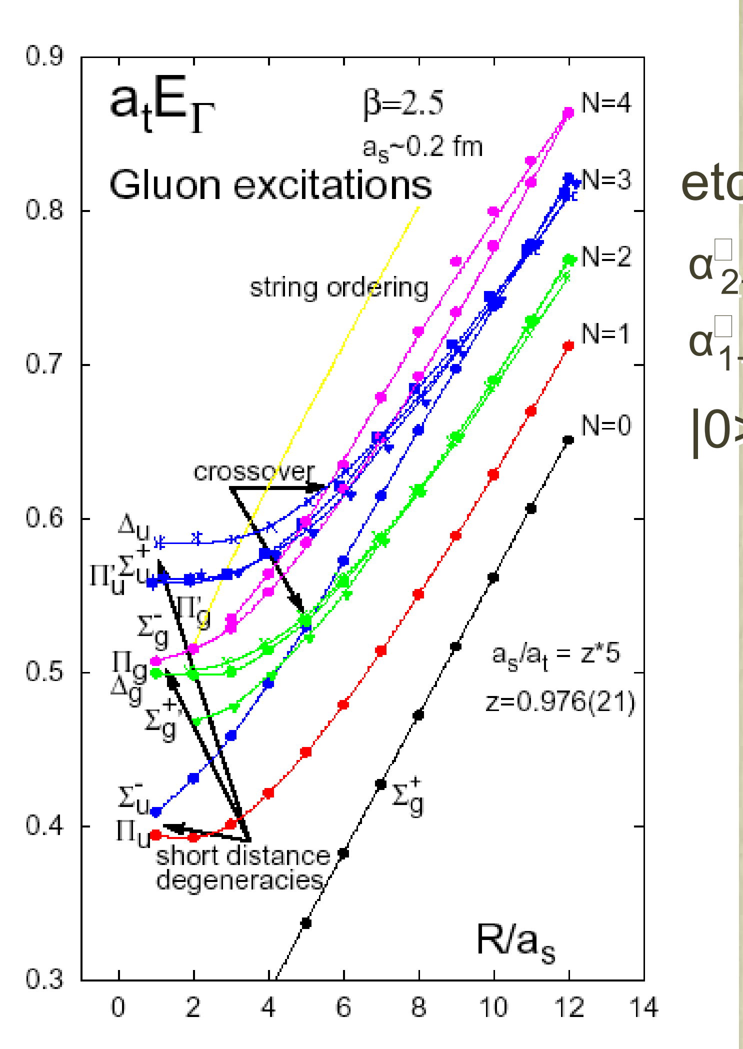
$$H_{tot} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L(L+1) - 2\Lambda^2 + \cdots}{2\mu r^2} + V_L$$

2

flavour	m	m’	lat
I=1	1.67	1.9	1.85
I=0	1.67	1.9	1.85
$s\bar{s}$	1.91	2.1	2.07
$c\bar{c}$	4.19	4.3	4.34
$b\bar{b}$	10.79	10.8	10.85

K. Waidelich

strings and flux tubes



etc

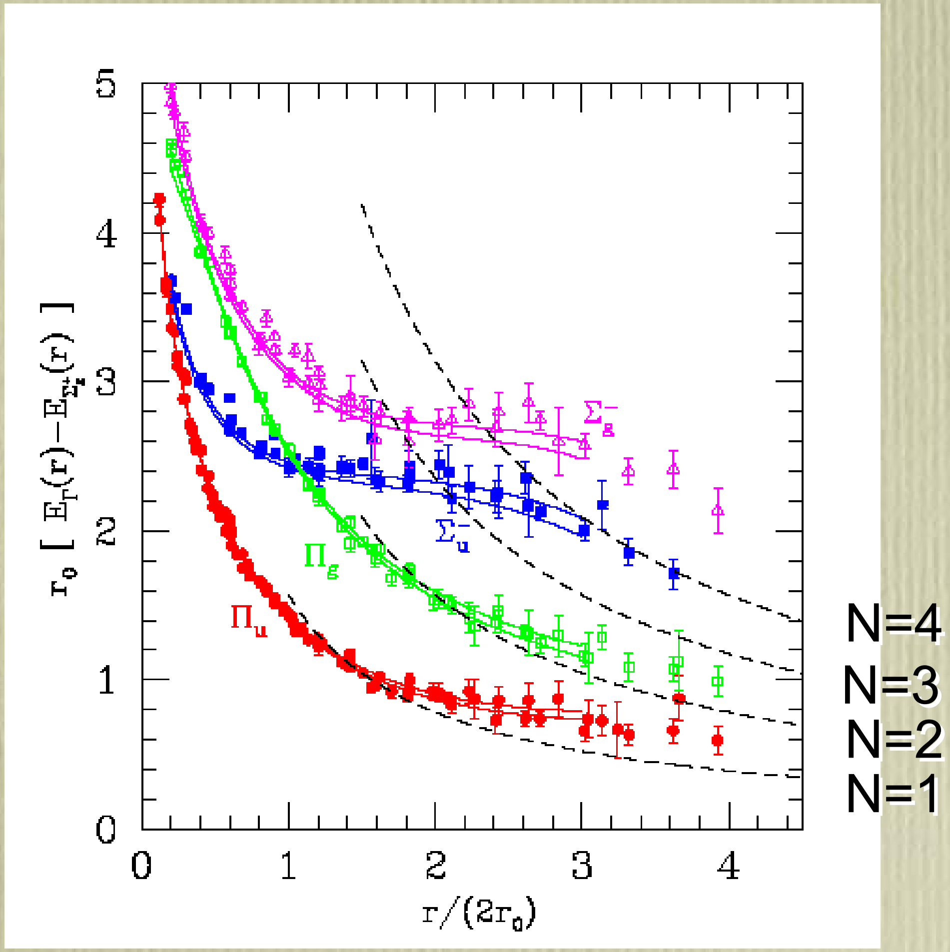
$\alpha^{\dagger}_{2+}|0\rangle$ $\alpha^{\dagger}_{2-}|0\rangle$; $\alpha^{\dagger}_{1+}\alpha^{\dagger}_{1+}|0\rangle$ $\alpha^{\dagger}_{1-}\alpha^{\dagger}_{1-}|0\rangle$; $\alpha^{\dagger}_{1+}\alpha^{\dagger}_{1-}|0$

$\alpha^{\dagger}_{1+}|0\rangle$ $\alpha^{\dagger}_{1-}|0\rangle$

$|0\rangle$

Juge, Kuti, & Morningstar

strings and flux tubes



Juge, Kuti, & Morningstar

Adiabatic and small oscillation approximations

Barnes, Close, & ES, PRD52, 5242 (95)

m_b = 0.2 GeV

linear potential

Adiabatic limit (check small osc)

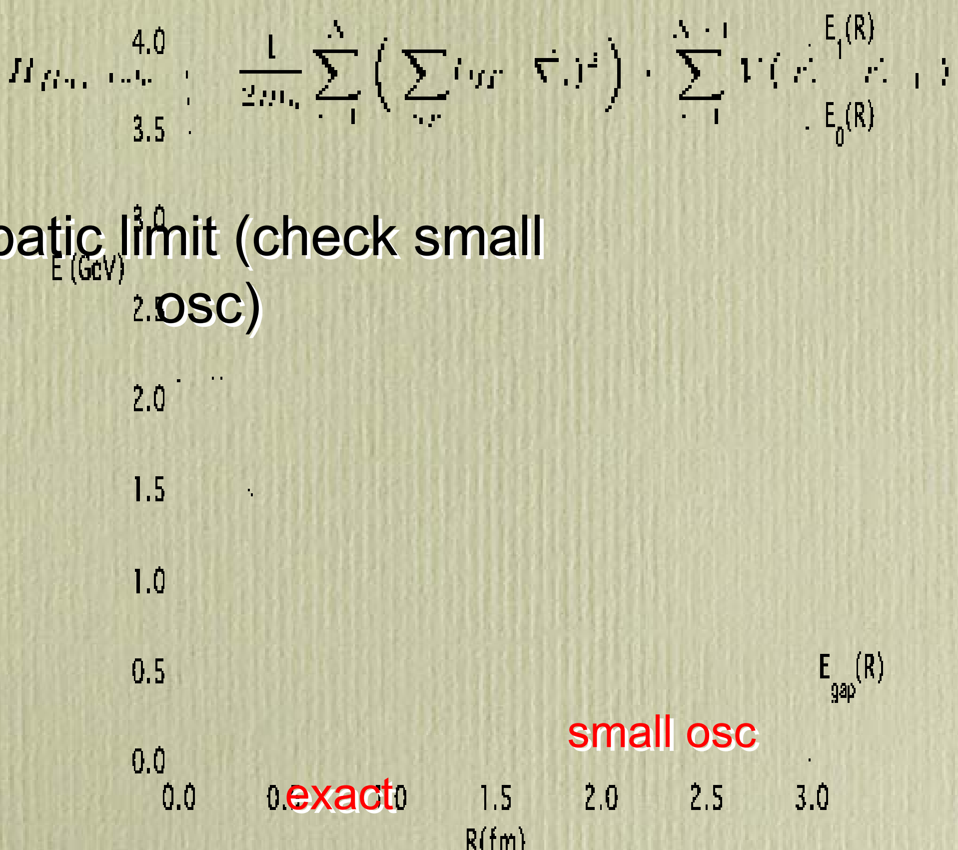
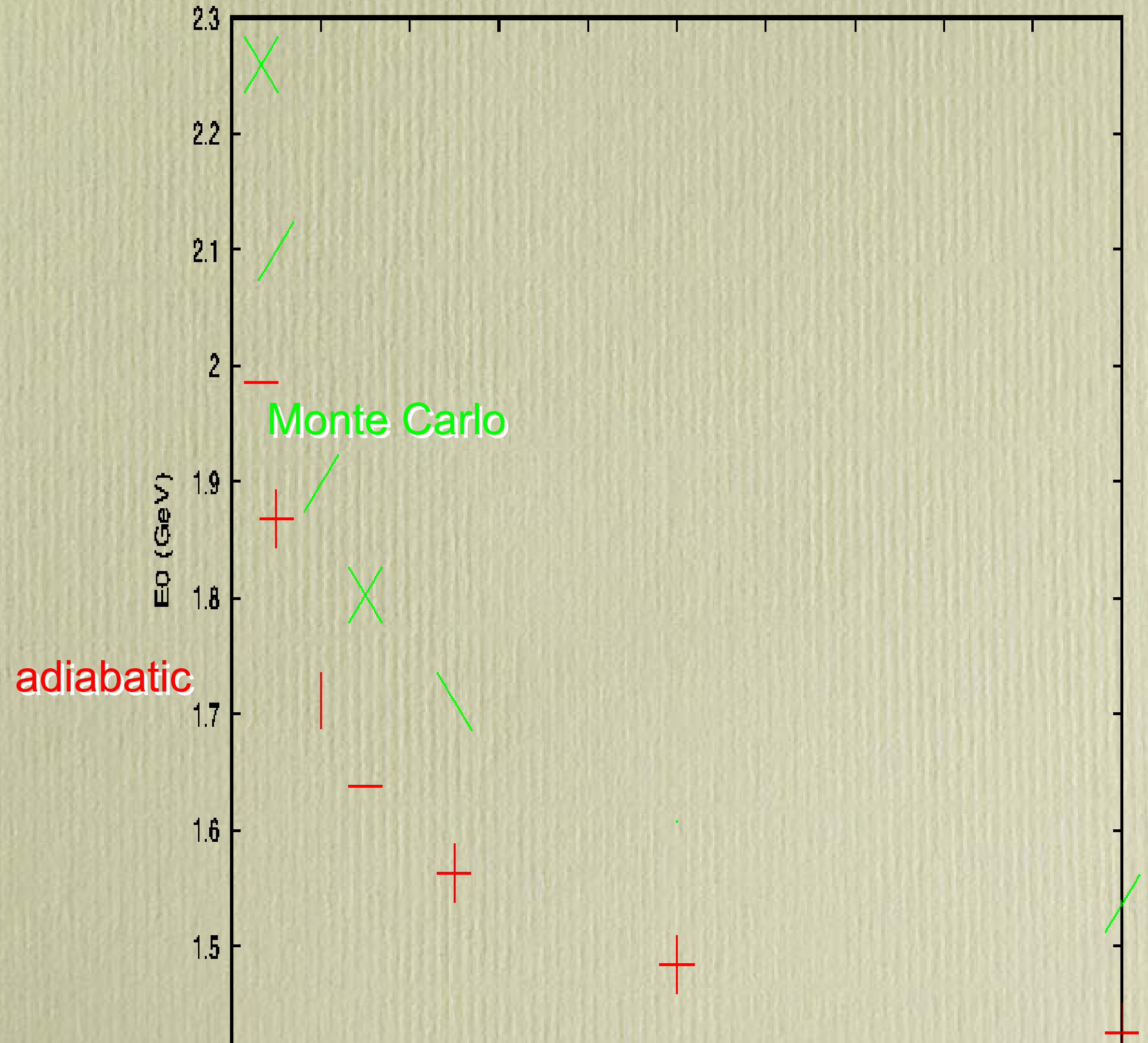


Fig.1. Ground state and first hybrid adiabatic potentials and their difference, for N=
Solid lines are exact and dashed lines are the small oscillation approximation.
String tension a=1.0GeV/fm, bead mass m_b=0.2GeV.

Adiabatic approximation (ground state meson)



Adiabatic approximation (hybrid gap)

1.5

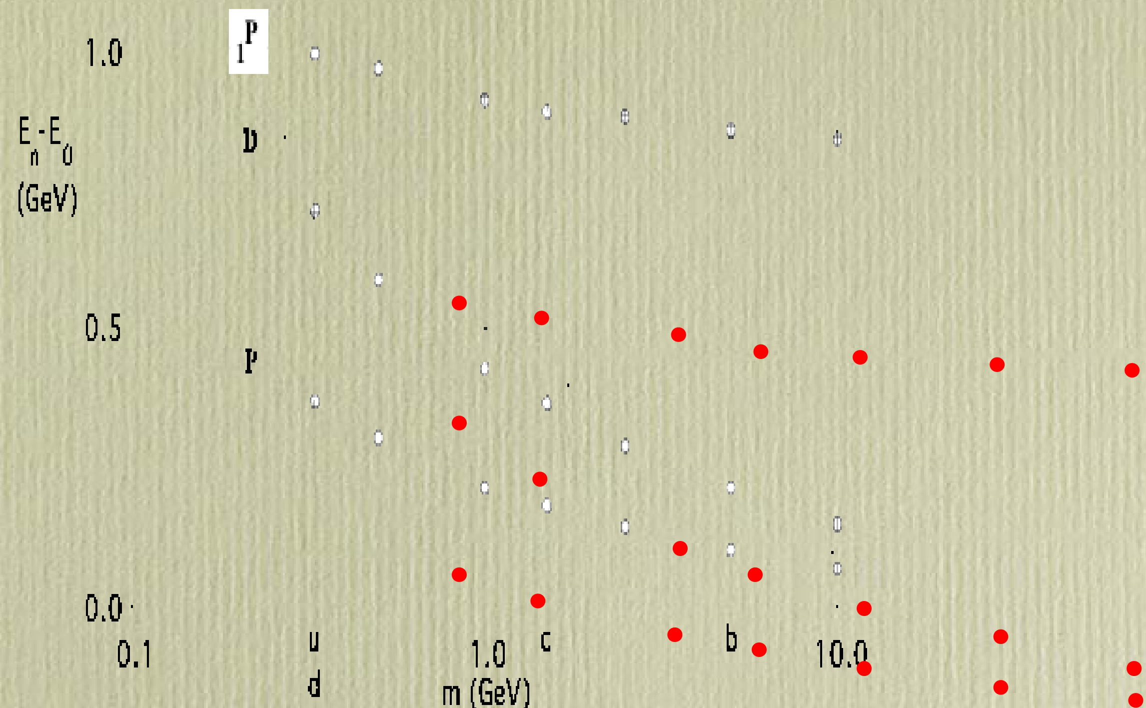


Fig.4. Energies of the lightest L=1,2 $q\bar{q}$ and $_\Lambda L = {}_1P$ hybrid states relative to $E_0 = E_S$ for N=1. Lines show the adiabatic approximation and the points are Monte Carlo, M=0 (open) and M=L (plus), m_b=0.2GeV, a=1.0GeV/fm, α_s=0.

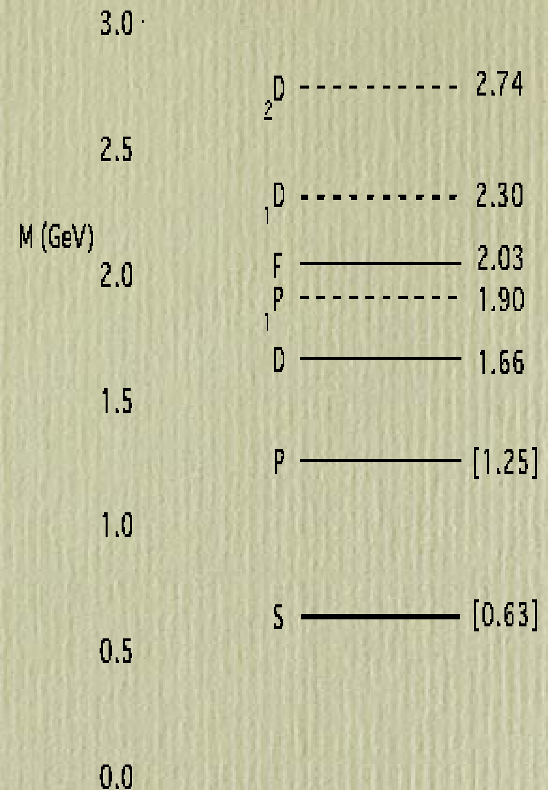


Fig.5. The lightest L=0-3 $q\bar{q}$ (q=u,d) and $_\Lambda L = {}_1P$, ${}_1D$ and ${}_2D$ hybrid masses from Monte Carlo with physical parameters, m_q=0.33GeV, m_b=0.2GeV, a=1.0GeV/fm, α_s^{ft}=1.3. Square brackets denote masses used as input.

IKP decay model

quark creation operator

[illegible]

[illegible]

[illegible]

[illegible]

$^3S_1$ $^3P_0$

flux tube overlaps

Kokoski & Isgur, PRD35, 907 (87)

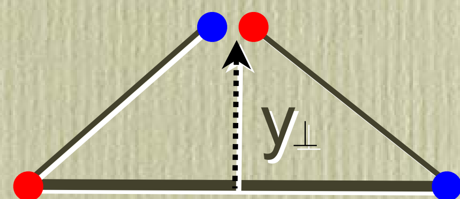
meson decay

$$\langle\{0...0\}bd;\{0...0\}bd|O|\{0...0\}bd\rangle \sim \langle bd;bd|\,{}^3P_0|bd\rangle \cdot$$

$$\langle\{0...0\};\{0...0\}|\{0...0\}\rangle$$

$$\downarrow$$

$$e^{-f\,b\,y_\perp^2}$$



hybrid decay

Isgur, Kokoski, & Paton, PRL54, 869 (85)

$$\langle\{0...0\}bd;\{0...0\}bd|O|\{1,0...0\}bd\rangle \sim \langle bd;bd|\,{}^3P_0|bd\rangle \cdot$$

$$\langle\{0...0\};\{0...0\}|\{1,0...0\}\rangle$$

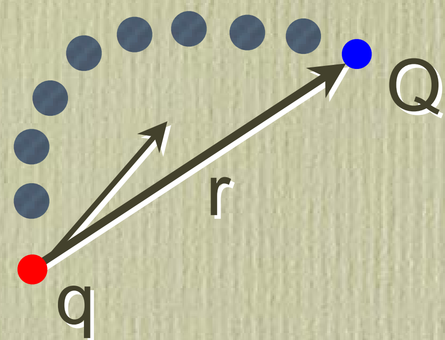
$$\downarrow$$

$$y_\perp e^{-f\,b\,y_\perp^2}$$

Extensions

A. Charge Radii

Isgur, PRD60,114016 (99)



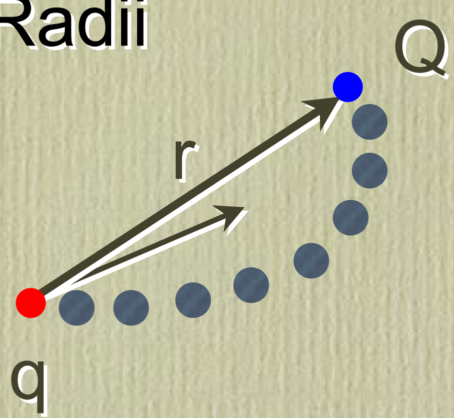
flux tube zero point motion induces transverse oscillation in the quarks → larger charge radius

[illegible]

increased elastic form factor slope → flux lost to hybrid production

A. Charge Radii

Isgur, PRD60,114016 (99)



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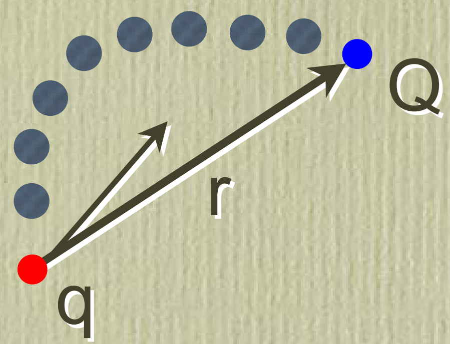

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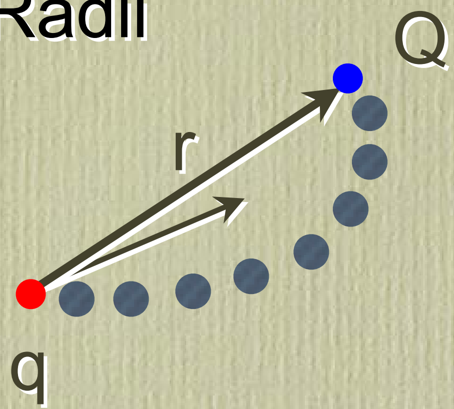
A. Charge Radii

Isgur, PRD60,114016 (99)



flux tube zero point motion induces transverse oscillation in the quarks → larger charge radius

[illegible]

increased elastic form factor slope → flux lost to hybrid production

B. Adiabatic Surface Mixing

Merlin & Paton, J. Phys. G11, 439 (85)

adiabatic surface mixing is induced by terms neglected by I&P, eg:

$$H_1 \quad \frac{1}{2\mu r^2}\left(L^2_{\ldots} \quad L^2_{\ldots} \quad 2L_{\ldots}L_{\ldots} \quad 2L_{\ldots}L_{\ldots}\right)$$

resulting mass shifts were quoted in I&P

C. Spin-Orbit Force

Merlin & Paton, PRD35, 1668 (87)

[illegible]

obtain a small spin-orbit shift and conclude that
spin-orbit splittings are mostly due to Thomas precession

[illegible]

[illegible]

find $u\bar{u}g$ splittings of: -140 (2+-) -20 (2-+) 20 (1-+) 40 (0-+)
140 (1+-) 280 (0+-) 0 (1++) 0 (1--)

D. Spin-Orbit Force, II

Szczepaniak & ES, PRD55, 3987 (97)

map chromofields to phonon degrees of freedom

[illegible]

[illegible]

[illegible]

spin-dependence in the confinement potential

$$V_{conf} \to \epsilon + V_{SD} + \ldots$$

Γ	ϵ_Γ	V_1	V_2	V_3	V_4
scalar	S	$-S$	0	0	0
vector	V	0	V	$V'/r - V''$	$2\nabla^2 V$
pseudoscalar	0	0	0	$P'' - P'/r$	$\nabla^2 P$

Gromes

$$\begin{aligned} V_{SD} = & \left(\frac{\sigma_q \cdot L_q}{4m_q^2} - \frac{\sigma_{\bar{q}} \cdot L_{\bar{q}}}{4m_{\bar{q}}^2}\right)\left(\frac{1}{r}\frac{d\epsilon}{r} + \frac{2}{r}\frac{dV_1}{dr}\right) + \left(\frac{\sigma_{\bar{q}} \cdot L_q}{2m_q m_{\bar{q}}} - \frac{\sigma_q \cdot L_{\bar{q}}}{2m_q m_{\bar{q}}}\right)\left(\frac{1}{r}\frac{dV_2}{dr}\right) \\ + & \frac{1}{12 m_q m_{\bar{q}}}\left(3\sigma_q \cdot \hat{r}\sigma_{\bar{q}} \cdot \hat{r} - \sigma_q \cdot \sigma_{\bar{q}}\right) V_3(r) + \frac{1}{12 m_q m_{\bar{q}}} \sigma_q \cdot \sigma_{\bar{q}} V_4(r) \end{aligned}$$

Eichten & Feinberg

Ng, Pantaleone, & Tye

spin-dependence in the confinement potential

examine in Coulomb gauge via the Foldy-Wouthuysen transformation

$$H_{QCD} \to H_{FW} = \int dx \left(m_q h^\dagger(x)h(x) - m_{\bar{q}}\chi^\dagger(x)\chi(x)\right) + H_{YM} + \\ + V_C + H_1 + H_2 + \ldots$$

$$H_1 = \frac{1}{2m_q}\int dx h^\dagger(x)\left(D^2 - g\sigma\cdot B\right)h(x) - (h \to \chi; m_q \to m_{\bar{q}})$$

$$H_2 = \frac{1}{8m_q^2}\int dx h^\dagger(x) g\sigma\cdot[E, \times D]h(x) + (h \to \chi; m_q \to m_{\bar{q}})$$

$$D = i\nabla + gA$$

$$E^a = -\Pi^a + E^a_\ell$$

$$E^a_\ell = -\nabla A^a_0 - g\nabla\nabla^{-2} f^{abc} A^b \cdot \nabla A^c_0$$

$$A^a_0(x) = g\int dy V^{ab}(x, y; A)\rho^b(y)$$

spin-dependence in the confinement potential

perform Rayleigh-Schrödinger perturbation theory

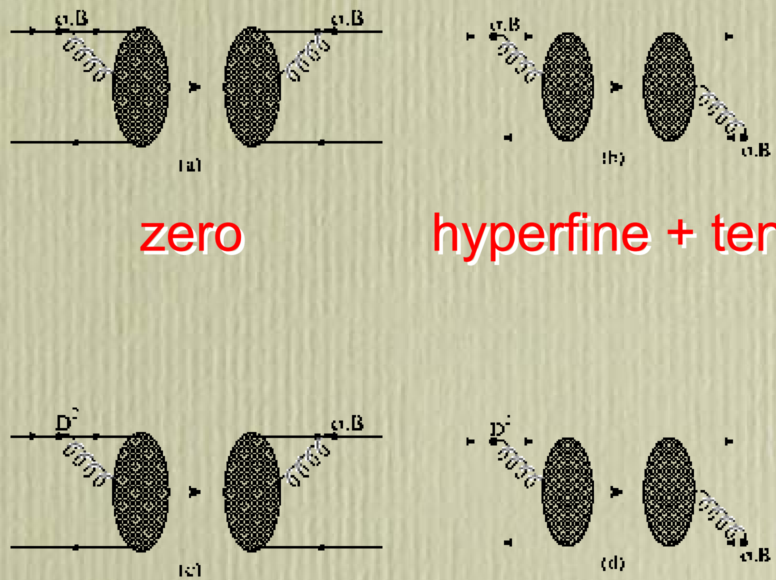
basis: $H_0|n_r; r_q r_{\bar{q}}\rangle = \epsilon_n(r)|n_r; r_q r_{\bar{q}}\rangle$

first order: $\delta\epsilon_n^{(1)}(r) = \langle n_r; r_q r_{\bar{q}}|H_2|n_r; r_q r_{\bar{q}}\rangle$

$$\langle n_r|\nabla^j_{r_q} g^2 T^a V^{ab}(r_q, r_{\bar{q}}; A)T^b|n_r\rangle = -\nabla^j_{r_q}\epsilon_n(r)$$

$$\delta\epsilon_n^{(1)} = \left(\frac{\sigma_q \cdot L_q}{4m_q^2} - \frac{\sigma_{\bar{q}} \cdot L_{\bar{q}}}{4m_{\bar{q}}^2}\right)\frac{1}{r}\frac{d\epsilon_n}{dr}$$

second order perturbation theory in H_I



zero hyperfine + tensor

V_1 V_2

evaluate matrix elements in the flux tube model: $V_1 = \sigma r$; $V_2 = O(1/N)$

Szczepaniak & ES, PRD56, 5692 (97)
Page, Szczepaniak & ES, PRD59, 014035 (99)

E. Vector Decay Model

use the same mapping to obtain $\bar{\psi}\ \alpha.A\ \psi$

[illegible]

[illegible]

Isgur & Paton, PRD31, 2910 (85)

F. Other

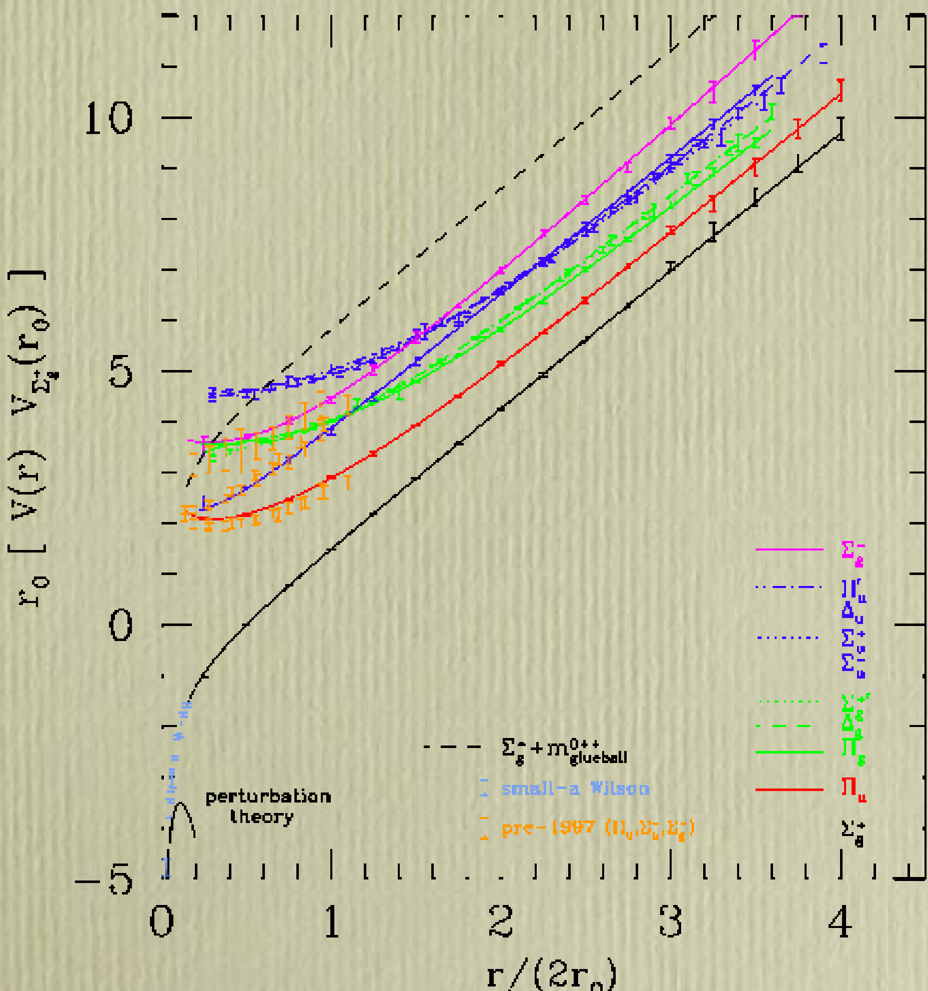
- glueballs (glue loops)
- baryons
- check Luescher term and adiabatic surfaces
- apply to glueloops in SU(2)
- apply to 2+1 U(1)
- improve semiclassical fragmentation formalism
- examine long range spin-spin and spin-orbit forces

Conclusions

- the FTM provides a compelling picture of strong QCD dynamics
- it is a picture only!
- many extensions and applications, explored and unexplored, exist

fin

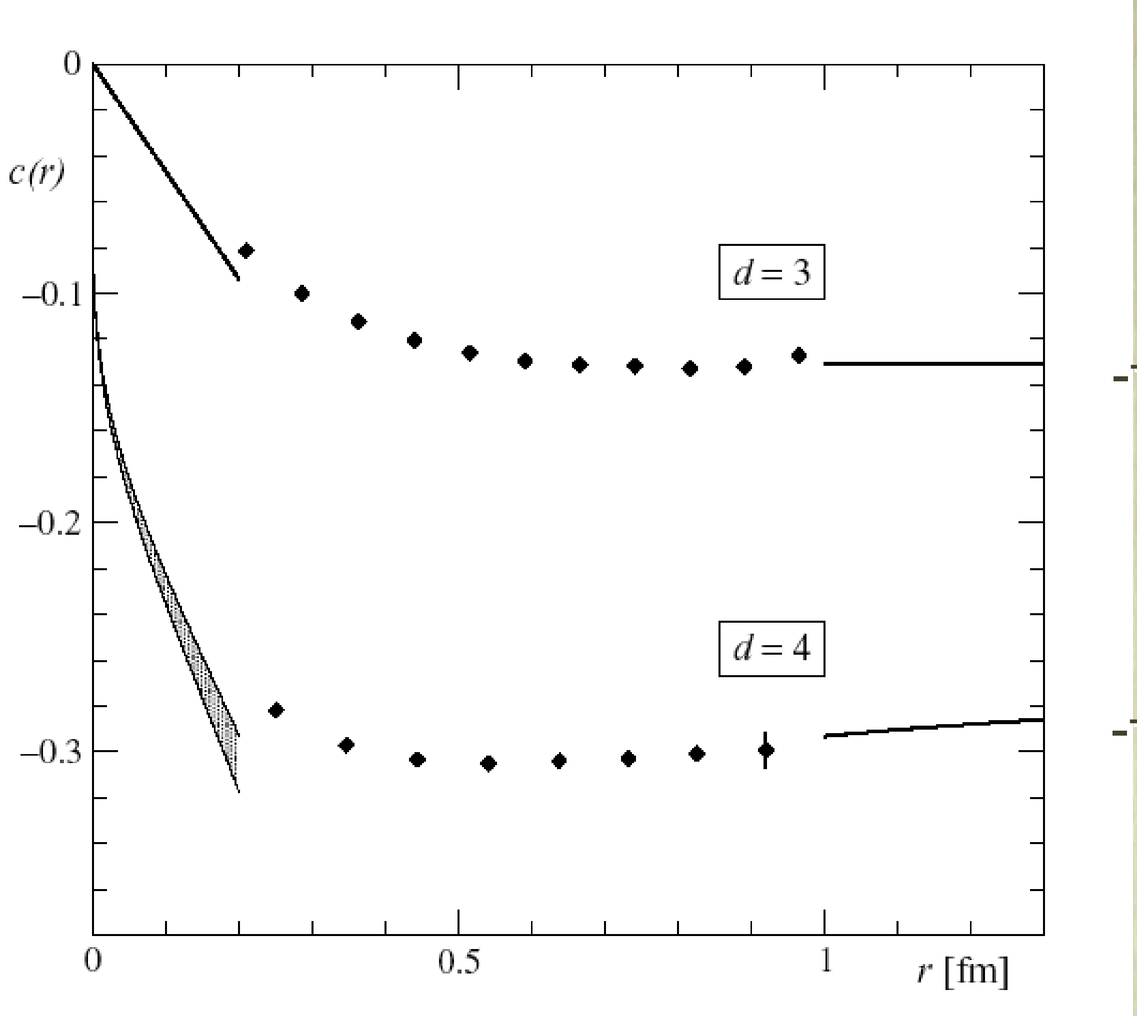
strings and flux tubes



Juge, Kuti, & Morningstar

strings and flux tubes

Lüscher & Weisz

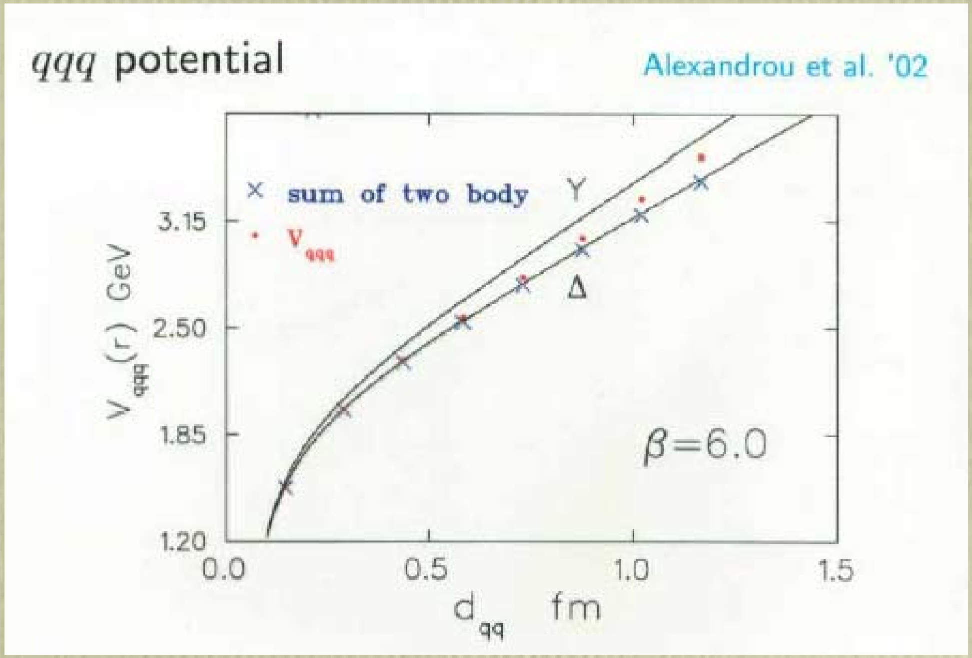


-π/24

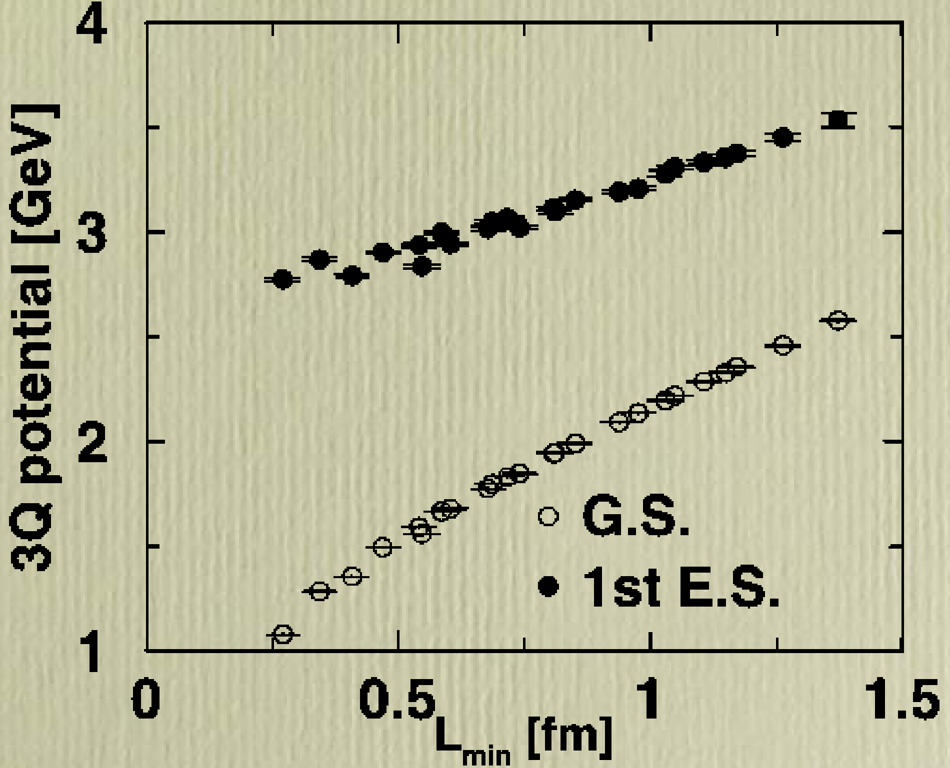
-π/12

see talk by Juge

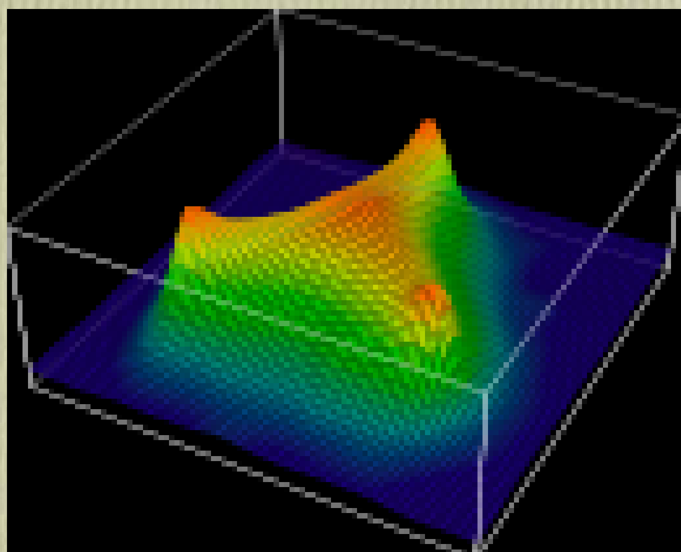
strings and flux tubes

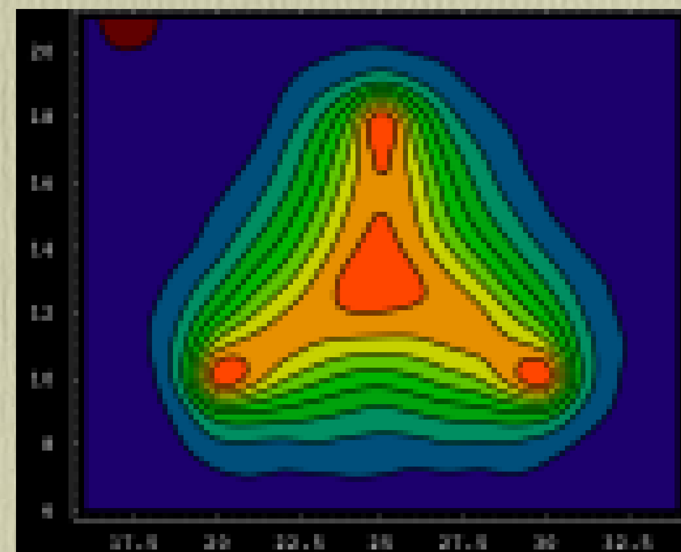


strings and flux tubes



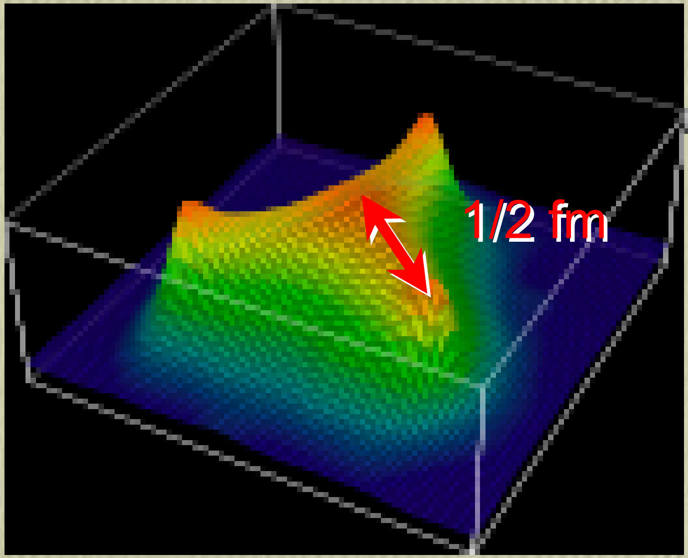
strings and flux tubes

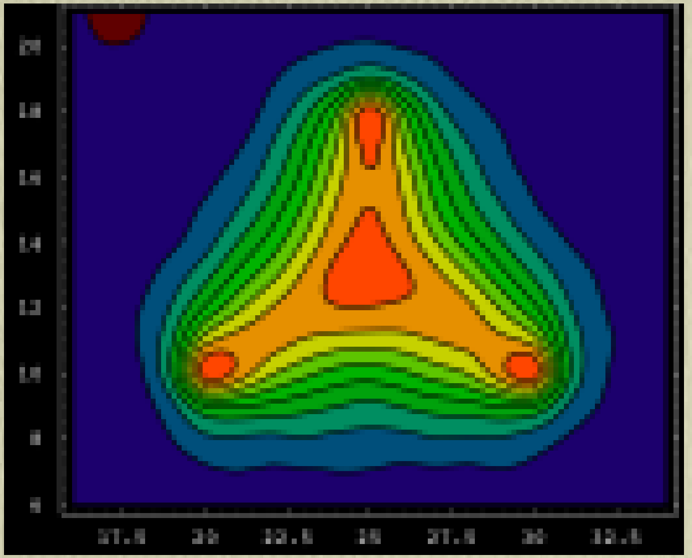




Ichie, Bornyakov, Schierholz, & Streuer, hep-lat/0212036

strings and flux tubes





Ichie, Bornyakov, Schierholz, & Streuer, hep-lat/0212036