Hybrid Mesons from Lattice QCD

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- Introduction
- Heavy quark hybrids — $Q\bar{Q}$ + glue
- Light quark hybrids — $q\bar{q}$ + glue
- Hybrid meson decays
- Outlook

What can (and can't) the lattice do?

\+ Vary N_f and masses of sea-quarks

\- so extrapolation may be needed to u, d quarks

\+ Determine masses (and energies if momentum $\neq 0$)

\- excited states with same quantum numbers much harder

\- care unstable particles.

\+ Determine hadronic transition strengths

\- only "on shell" where $E_{init} \approx E_{final}$

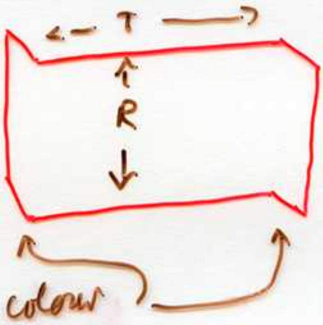
\+ No parameters or "phenomenology": first principles

\- so no "why?"

Heavy Quarks

- relativistic propagating quarks — OK to m_c
- non-relativistic quarks (effective theory, $a \neq 0$) (coefficients PT or NP...)
  NRQCD
- HQET ($1/m_Q$ expansion)
  leading order: static quarks.

Historically — 20th anniversary of first lattice hybrid calculation

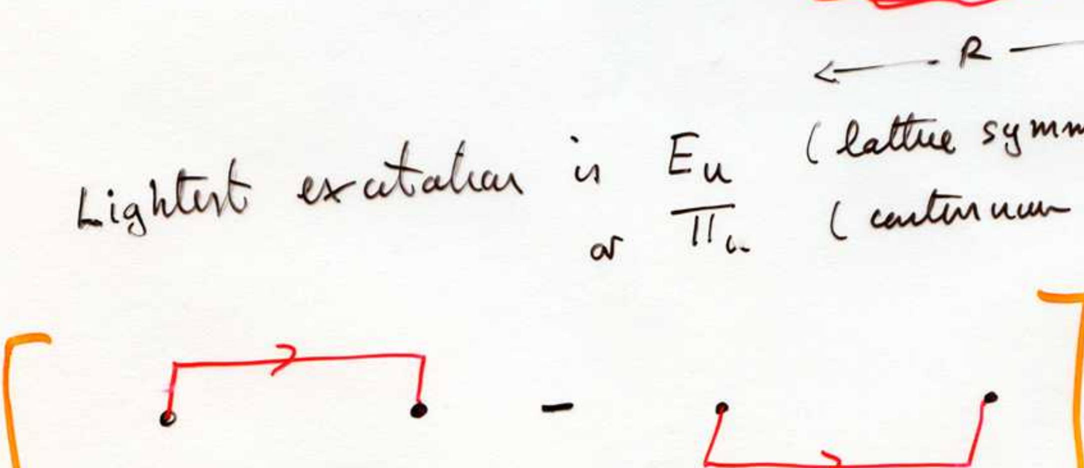


$\approx e^{-E_H(R)\,T}$

excited colour flux

classify states by J_z, end/end swap, etc (CP)

Lightest excitation is E_u or Π_u (lattice symmetry D_{4h}) (continuum $D_{\infty h}$)



2-dim.

These lattice results motivated the flux-tube model.

Griffiths CM Ralkow PRL

to symmetry (antisymmetry) by a rotation of 180° about ated to C, charge conjuga- mmetry (antisymmetry) un- inversion in the midpoint luon field). The relationship f D_{4h} to those of $D_{\infty h}$ is 1). (Note that B_1 and B_2 are wed J^{PC} values for the com- a quark and antiquark one tal angular momentum from k spins, must be at least as angular momentum along ws the lower lying possibili- n be reached with some

s section is discussed in texts, such as ref. [6].

it our calculations on a 8^4 ary conditions. For $R = a$, nd 125 paths respectively. were obtained from on the ations of the appropriate thermalised sequence of lat- neasured at $T = 0$, a and $2a$. rreducible representations ional technique was used to within each representation. ound by comparing $T = a$ g $T = 2a$ with $T = a$ were gives us confidence that igh to represent the rele- and so give a good deter- with time separations T ne from the comparison this is less sensitive to the aths.

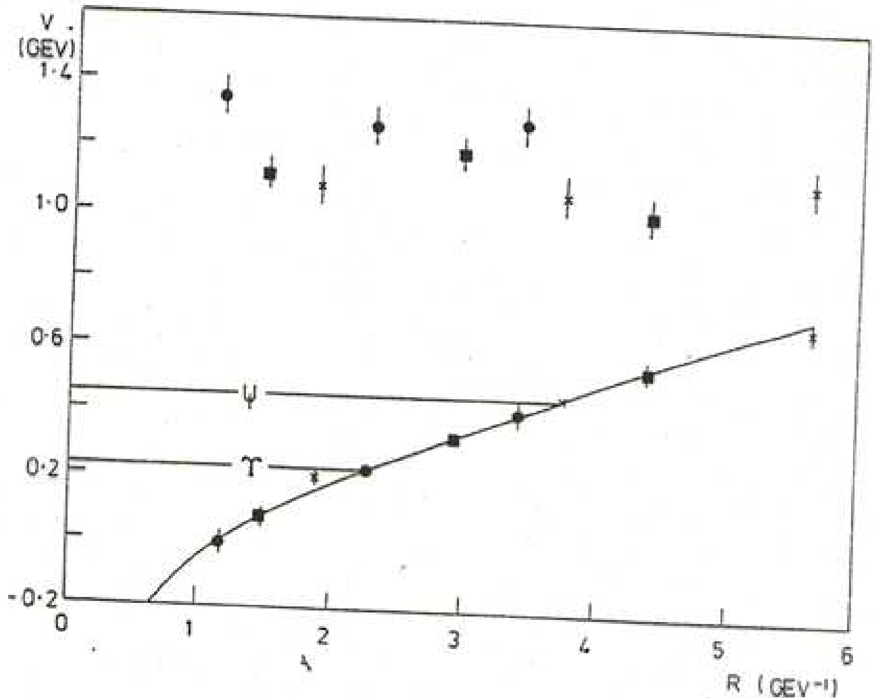
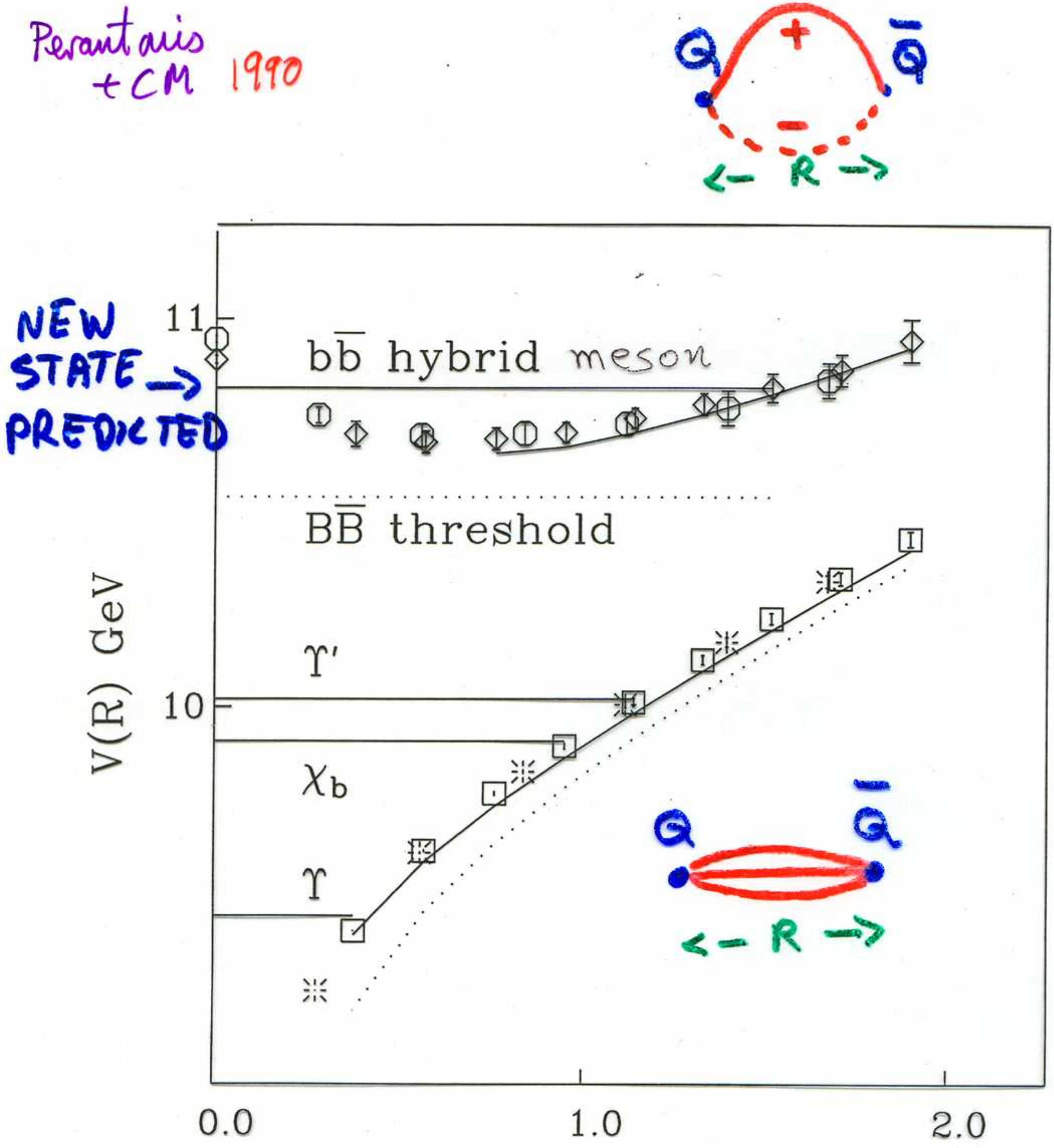


Fig. 2. The potential $V(R)$ for static colour sources in the fundamental representation of SU(2) at separation R. The ground state of the gluon field (A_{1g} representation) has a potential given by the curve (taken from a fit by Stack [2]). Our results at $R = a$, $2a$ and $3a$ for $\beta = 2.1$ (×), 2.2 (■) and 2.3 (●) confirm this curve. The energies of the ψ and Υ in such a potential are shown. For a gluon field with symmetry E_u the potential is shown by the higher lying points, which are consistent with an approximately constant potential. Λ_L is 5.2×10^{-3} GeV.

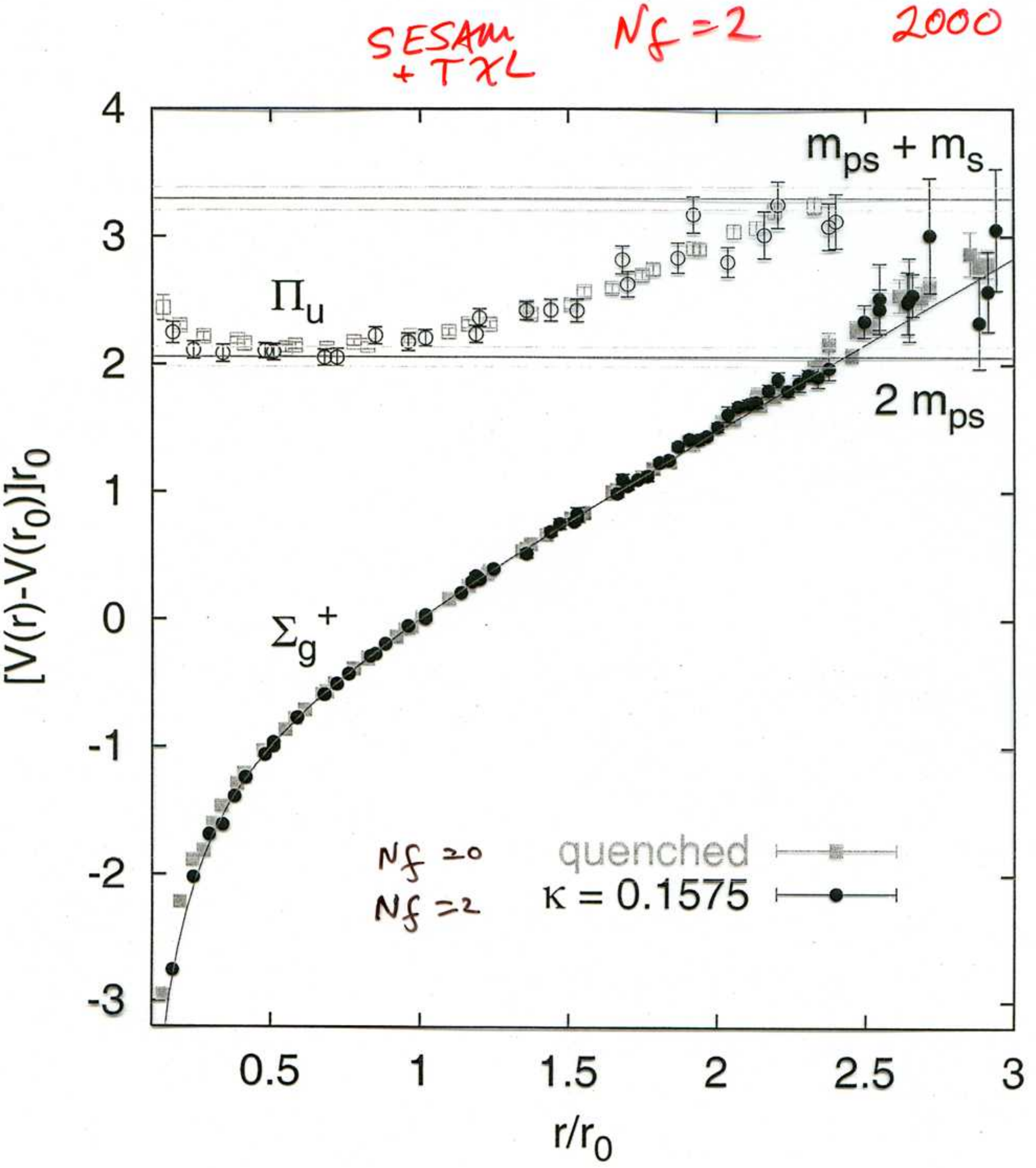
nals at $T = a$, $2a$, while the conventional method needs larger T separations, where the signal is smaller. Our results confirm those of Stack, with the interquark force decreasing slightly at larger R. The decreasing component can be described by a Coulomb force, and the asymptotic constant force is given by a string tension $\sqrt{K} \simeq 70\ \Lambda_L$, somewhat smaller than some determinations. Fig. 2 shows our points compared with Stack's best fit [2].

The potential with the gluon field in the group representation s is found from the ratio of eigenvalues.

$$V_s(R) - V_0(R) = a^{-1} \ln[\lambda_s(R)/\lambda_0(R)]\ ,$$



Perantonis + CM 1990
Q
$\bar{Q}$
+
−
<− R −>
NEW STATE PREDICTED
11
$b\bar{b}$ hybrid meson
$B\bar{B}$ threshold
V(R) GeV
Υ'
10
χ_b
Υ
0.0
1.0
2.0


SESAM + TXL
Nf =2
2000
m_ps + m_s
Π_u
2 m_ps
Σ_g^+
[V(r)-V(r_0)]r_0
r/r_0
Nf =0
quenched
Nf =2
κ = 0.1575

Summary of Heavy Quark Hybrid spectra.

Π_u is lowest excitation $L^{PC} = 1^{+-}$; 1^{-+}

$$\Rightarrow \text{mesons } J^{PC} = \begin{cases} 1^{--} & 0^{-+} & \boxed{1^{-+}} & 2^{-+} \\ 1^{++} & \boxed{0^{+-}} & 1^{+-} & \boxed{2^{+-}} \end{cases}$$

spin-exotic states predicted.

other states (eg 0^{-+}) will MIX with $Q\bar{Q}$ states.

(explored using NRQCD by Burch + Toussaint)

expected at 10.73 (7) GeV ← Quenched $N_f=0$ for $b\bar{b}$ + glue

11.02(18) GeV ← CPPACS $N_f=2$

Decays discussed later

$N_f=0$ case (Quenched) : impressive exploration of "excited string" spectra for a wide range of R and comparison with models. (Juge Kuti Morningstar)

Light Quark hybrid mesons

Focus on exotic-spin mesons $J^{PC} = 1^{-+}$ etc

Need non-local operators to create these states on a lattice eg

Not easy to explore non-spin-exotic hybrids (eg $\pi(1800)$) since same quantum numbers as π : so obscured on lattice.

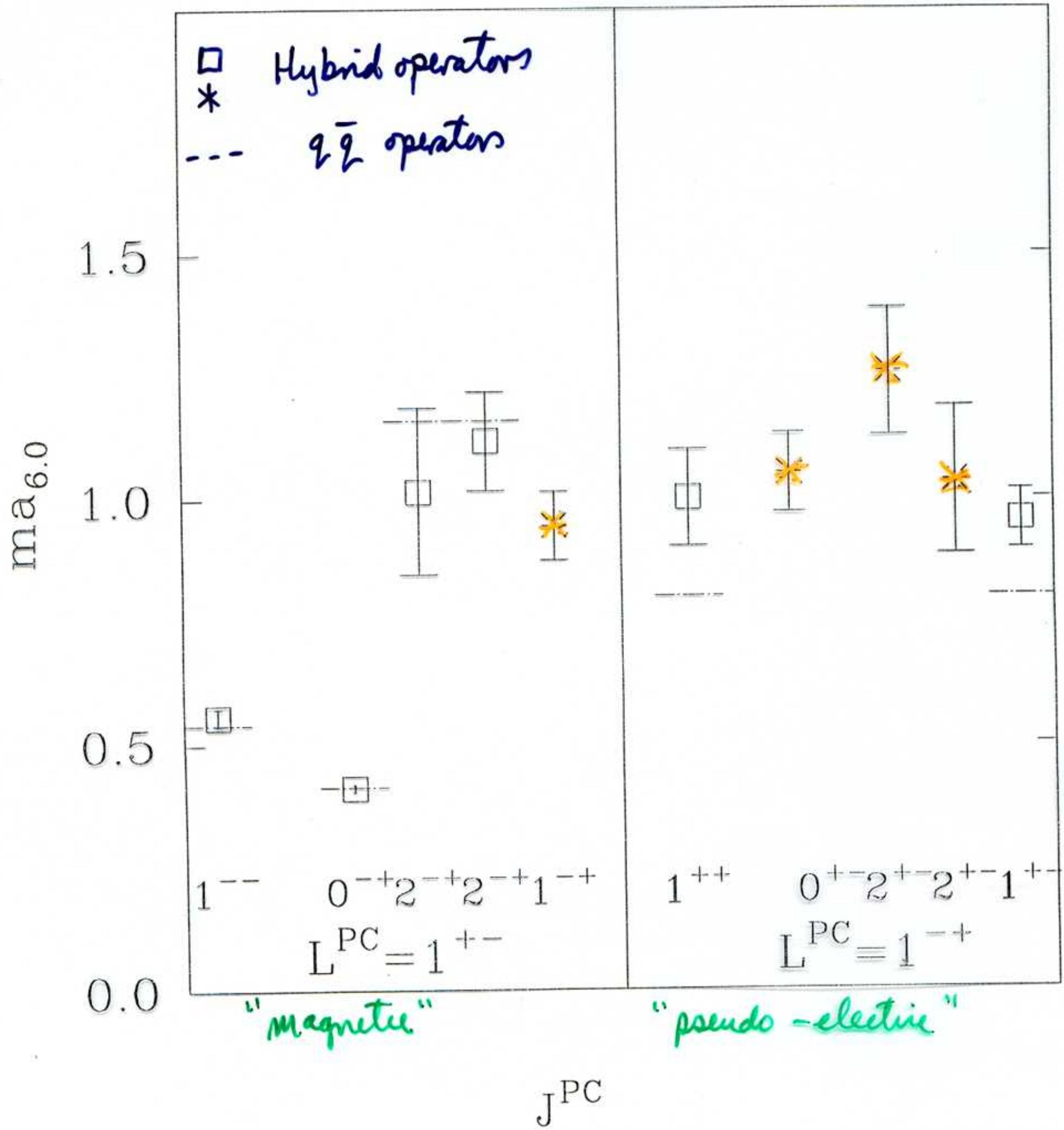
1^{-+} mass results

	$n\bar{n}$	$s\bar{s}$	$c\bar{c}$
UKQCD $N_f=0$	1.88 (20+)	2.00 (20)	4.39 (8,20)
MILC $N_f=0$	1.97 (9,30)	2.17 (8,10,10)	
SESAM $N_f=2$	1.90 (20)		
MILC $N_f=2+1$		2.10 (12)	

so good agreement among lattice groups
but $\neq$ expt 1.4 ($\eta\pi$) states.
1.6 ($\rho\pi$)

- Issue ($N_f \neq 0$) has open 2 body decay channels – which will look like discrete levels on a lattice $\left(p = \frac{2\pi n}{L}\right)$

UKQCD ($s\bar{s}$) 1996, 7
$N_f = 0$

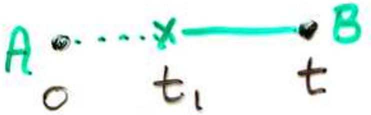


Lacock et al   PL B401, 308 (1997)

Hadronic Transitions (decays) from the lattice

The problem:

$$\langle 0 | O_A \cdots O_B^{\dagger} | 0 \rangle = C_{AB}(t)$$



$$\approx \sum_{t_1=0}^{t} c_A\, e^{-m_A t_1}\, x\, e^{-m_B(t-t_1)}\, c_B$$

$$\approx c_1 e^{-m_A t} + c_2 e^{-m_B t} \quad ✓$$

But also

$$+ \sum_{t_1=0}^{t} c_{A'}\, e^{-m_{A'} t_1}\, x'\, e^{-m_B(t-t_1)}\, c_B \approx c' e^{-m_B t} \quad ✗$$

$$+ \cdots$$

Not suppressed as $t \to$ BIG

The solution:

if $m_A = m_B$ term $c_A c_B\, x\, t\, e^{-m_B t}$

dominates excited state contributions

(also admixtures of B in O_A etc)

SO ON-SHELL (ENERGY CONSERVING) TRANSITIONS ACCESSIBLE.

NEED

$$(m_A - m_B)\, t \ll 5$$

$$x\, t \ll 1$$

$$(m' - m)\, t \gg 1$$

ON SHELL

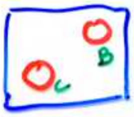
"WEAK"

excited state removal

More rigorous (but MUCH more computing)
Lüscher approach: only uses energies on a lattice.

$A \longrightarrow \underbrace{\begin{matrix} B \\ C \end{matrix}}$

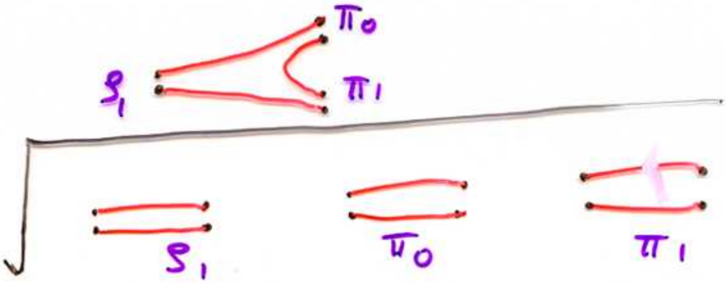
explore the BC spectrum for specific momentum for different lattice size L^3 : $\Longrightarrow$ phase shift BC-BC

Large L 

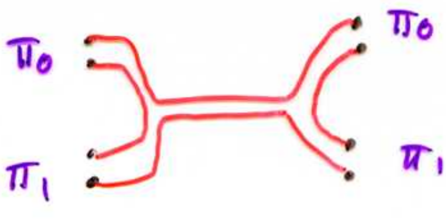
small L 

ILLUSTRATION $\qquad \rho \to \pi\pi$ R

$\rho_1 \to \pi_1 \pi_0 \qquad p = \frac{2\pi n}{L} \qquad n = 0, 1$

$xt =$ 

F

Cross checks • 

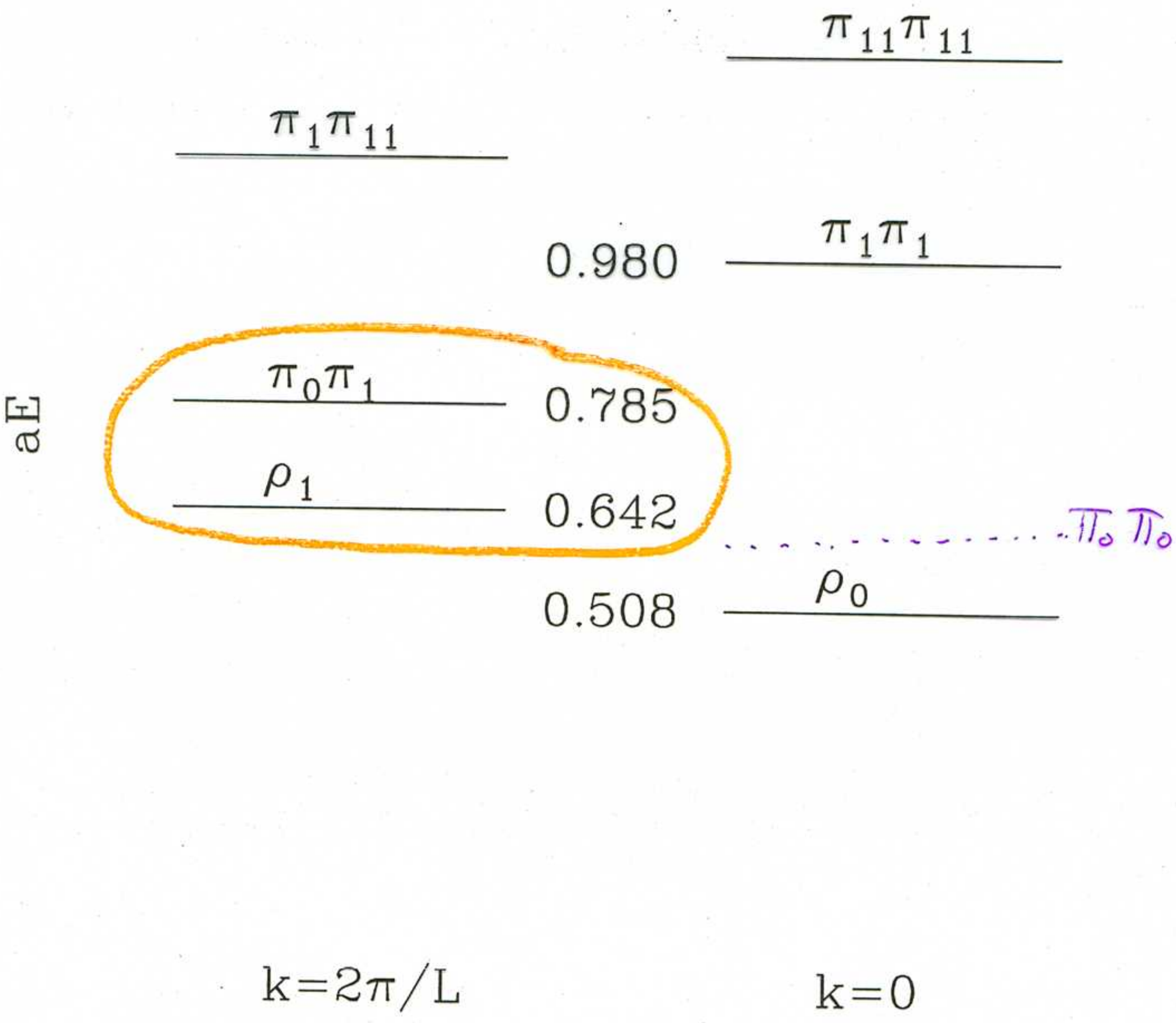
$\approx \frac{1}{2} x^2 t^2$

• Levelmixing pushes
$\rho \quad J_z$; P_z down
$P_{x,y}$ no effect. F

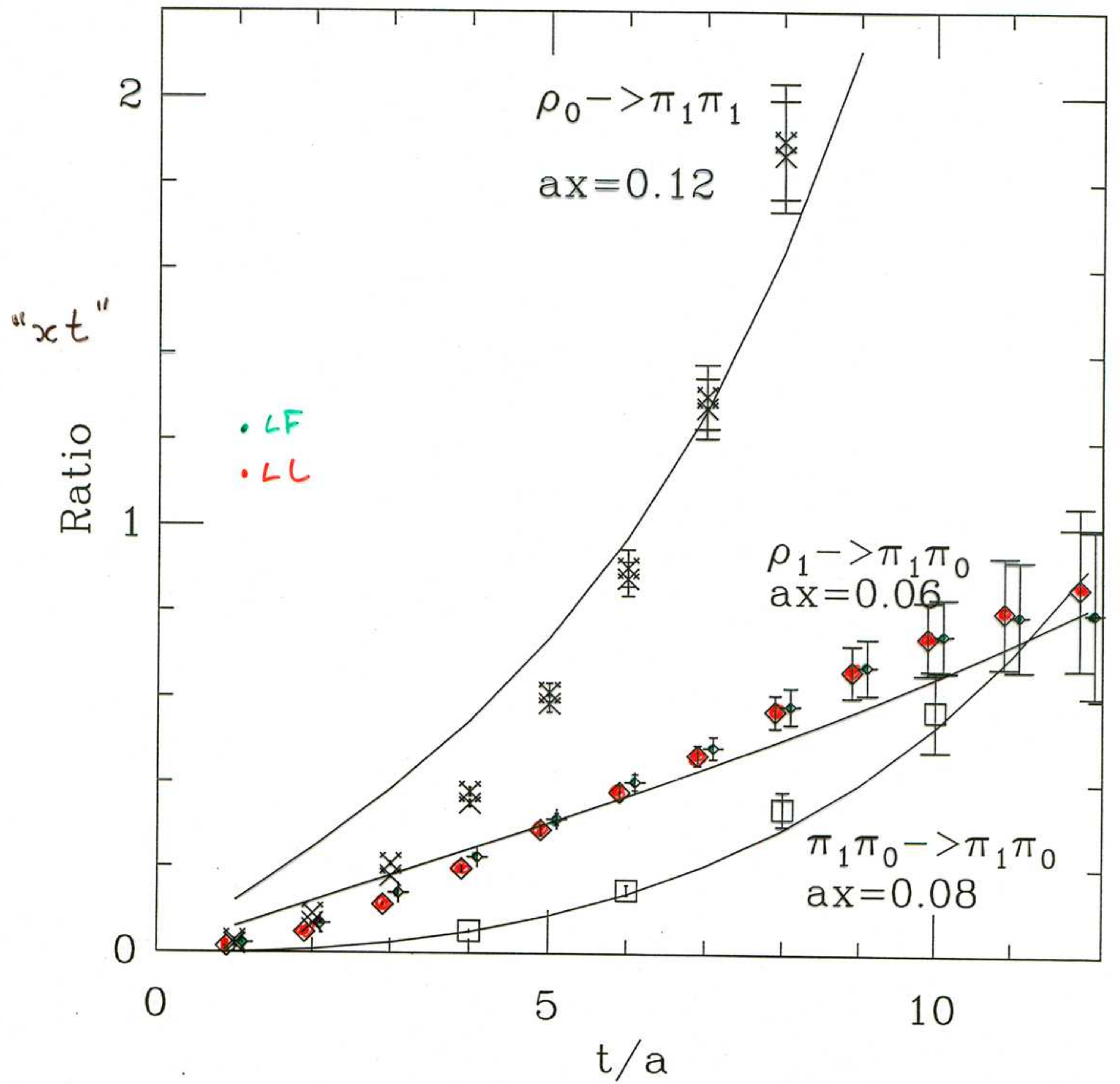
• [more data needed] observe $\pi_0 \pi_1$ energy shift versus L ($\sim 0.02(2)$ seen)
shift in $\frac{Ea}{\pi\pi}$

UKQCD
$N_f = 2$
$a \approx 0.11$ fm
$m_q \approx \frac{2}{3} m_s$

Lattice Spectrum



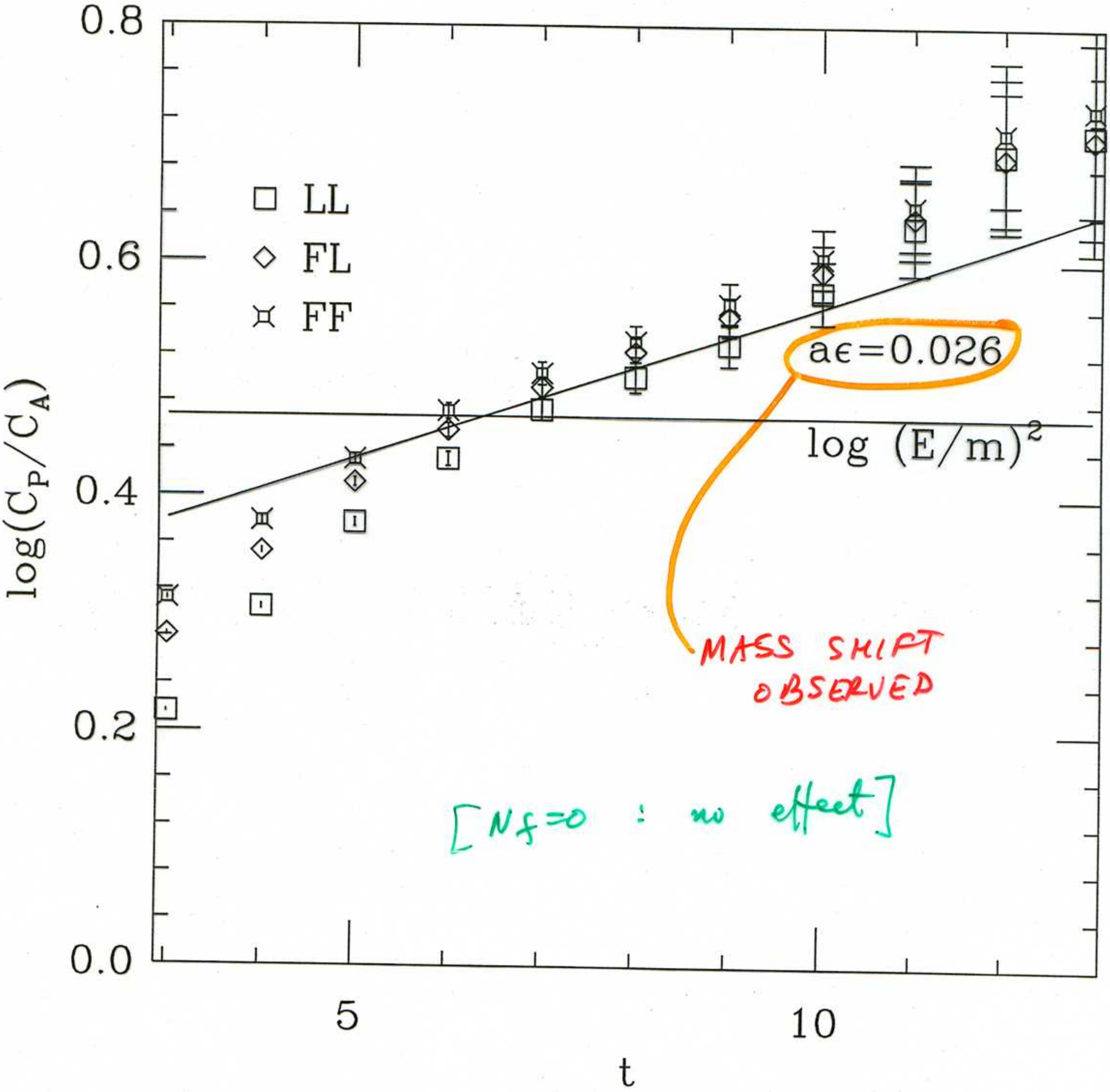
UKQCD Mcneile +CM



$\underline{P} = \frac{2\pi \underline{n}}{L}$

P : spin $\parallel^{el}$ momentum : MIXES $\pi_0 \pi_1$
A : spin $\perp$r momentum

UKQCD McNeile & CM



Summary $\langle \rho | \pi\pi \rangle$

$(\bar{g})^2 = \frac{TME}{k^3}$

$\bar{g}$ 1.40^{47}_{23} $\rho - \pi\pi$ "xt"

1.56^{21}_{13} $\rho_{\parallel}$ vs $\rho_{\perp}$

1.5 expt $\phi \to K\bar{K}$

No decay allowed on lattice! but on shell transition $\rho_1 - \pi_1 \pi_0$ gives $\bar{g}$.

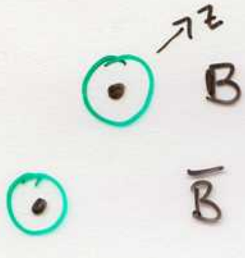
[checked $\rho_0 \to \pi_1 \pi_{-1}$ but bigger energy gaps so less "safe"]

Hybrid decay

In leading order HQET (static)

J_z and CP conserved

E_u has $J_z = 1$, CP = −



B has $J_z = 0$, CP = − or $J_z = 1$, CP = +

$q\bar{q}$ singlet $q\bar{q}$ triplet

$\bar{B}$

So decay to 2 S-wave B (or B^*) states **not** allowed.

Allowed: B^{**} (L=1) B or de-excitation ✓

↖ but ΔQ too big

χ_b

$J_z = 0$

CP = +

$q\bar{q}$ meson

$J_z = 1$

CP = − → η L=2

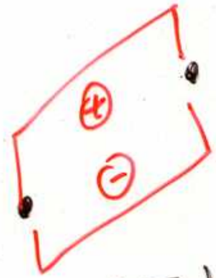
f_0 L=1

(CM)² + AP explored:

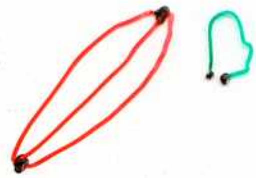
Estimate $\Gamma \approx 88$ MeV $H \to \chi_b f_0$ F

$\lesssim 2$ MeV $H \to \chi_b$ "η_0" F

BUT:
- wavefunction vs R F
- $m_q \to m_{u,d}$ (have $m_s \cdot \frac{2}{3}$)
- S (f_0) has $\pi\pi$ decay itself
- vary R
- $a \to 0$



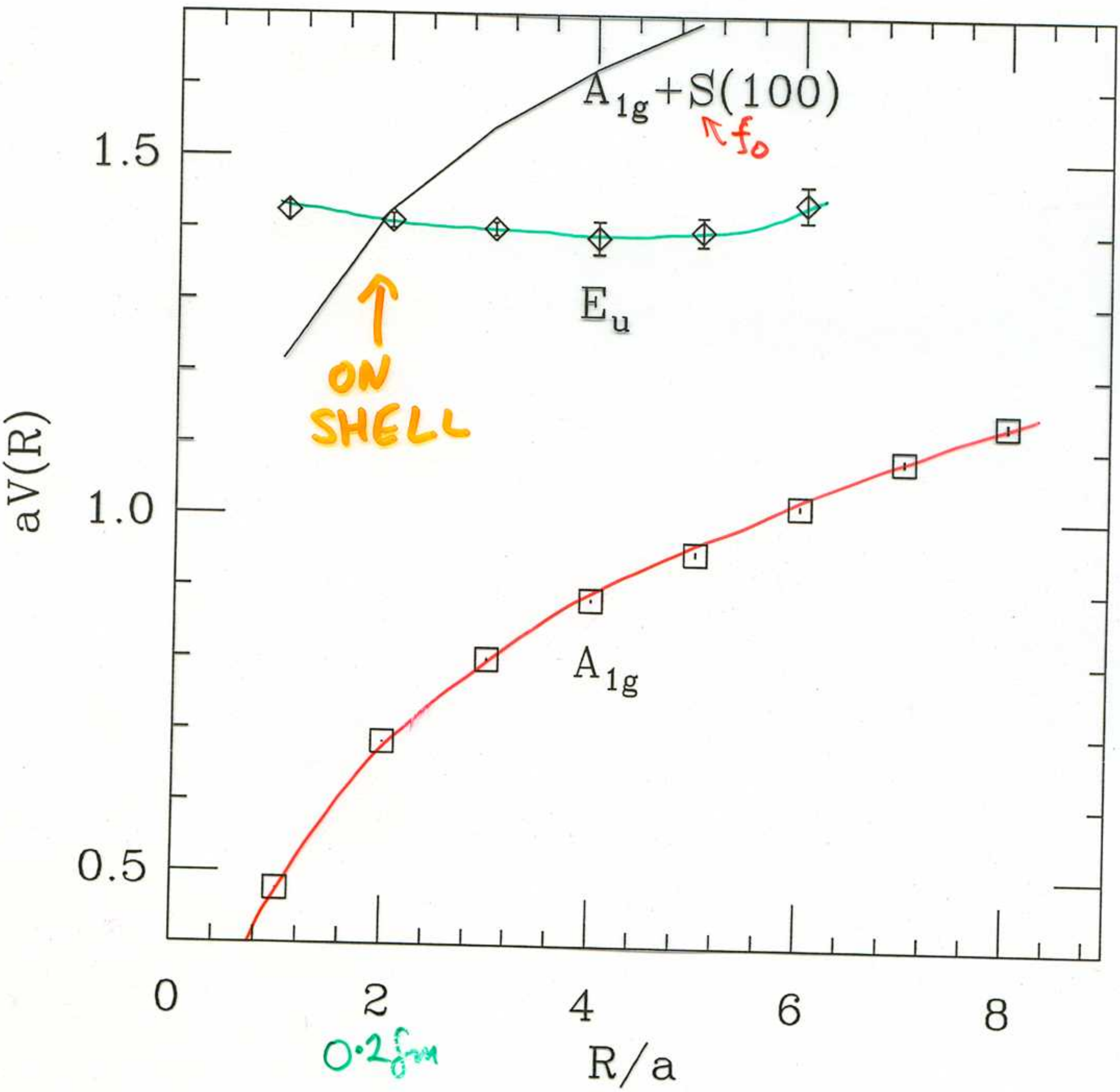
$E_u \, (\Pi_u)$

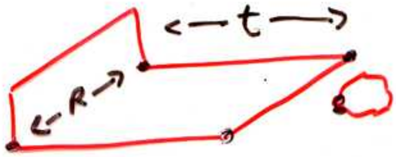


$A_{1g} \, (\Sigma_g^+)$ f_0

Transition conserves R, J_z, CP, "L".

UKQCD $N_f = 2$ $m_q \sim \frac{2}{3} m_s$

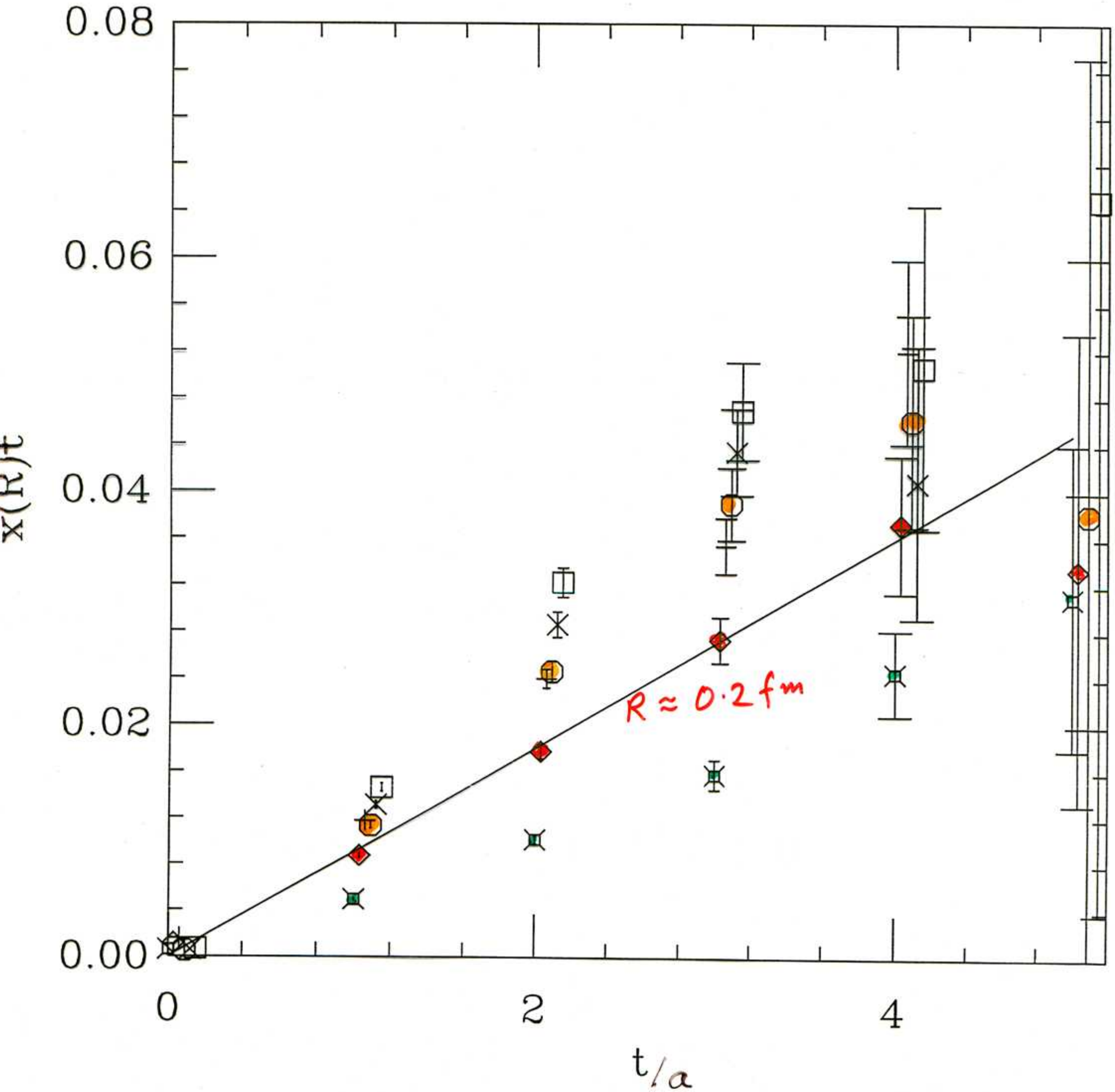


H  χ_b S

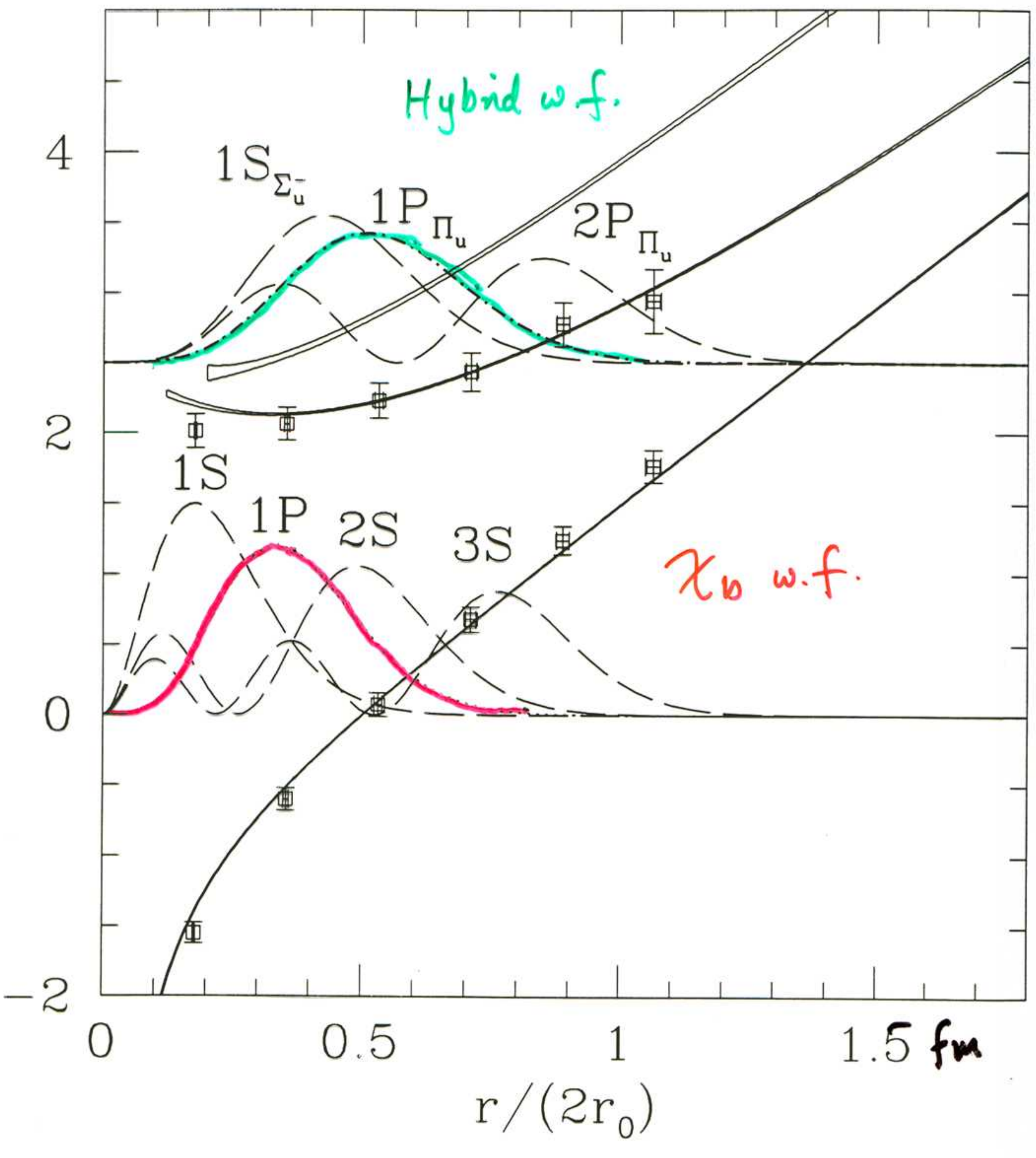
$\nwarrow 0^{++}$ $k = (1,0,0)\frac{2\pi}{L}$

$$xt = \frac{H \to \chi_b \; S}{\sqrt{H-H \quad \chi_b - \chi_b \quad S-S}}$$

$S \equiv f_0$



LA JOLLA, JKM


Hybrid w.f.
$1S_{\Sigma_u^-}$
$1P_{\Pi_u}$
$2P_{\Pi_u}$
1S
1P
2S
3S
χ_b w.f.
$r_0[V-V(r=r_0)]$
4
2
0
−2
0
0.5
1
1.5 fm
$r/(2r_0)$

Prospects for future

Underway

$a_0 \to \eta\pi \quad K\bar{K}$

$f_0 \to \pi\pi \quad K\bar{K}$

$B^* \to B\pi$

$B_{scalar} \to B\pi$

- fate of the scalar glueball

$\left[D^s_{scalar} \to \begin{matrix} DK \\ D^s\pi \end{matrix} \right]$

Feasible $\quad H(1^{-+}) \to \begin{matrix} \eta\pi \\ \rho\pi \end{matrix}$

Care : disconnected (flavour singlet) contributions more noisy on lattice (f_0, η)

Electromagnetic transitions are feasible (easier than hadronic transitions)