Hadron masses from FLIC fermions in lattice QCD

Anthony G. Williams

[for the CSSM Lattice Collaboration]

Sundance Bilson-Thompson, Frederic Bonnet, John Hedditch, Ben Lasscock, Frank X. Lee*, Derek Leinweber, Wally Melnitchouk**, Anthony G. Williams, James Zanotti and Jianbo Zhang

at the Gluonic Excitations Workshop, JLab, 14-16 May, 2003

CSSM (University of Adelaide), Jefferson Lab**, George Washington University*

Outline

- Introduction to lattice QCD
- Observables from the lattice
- FLIC fermion action
- Scaling analysis for FLIC fermions
- Ground state hadrons
- Excited baryons
- Spin 3/2 nucleon and delta baryons
- Hybrid mesons
- Conclusions and outlook

Go to References

Some references

- FLIC fermions and hadron masses:
  - Introduction of FLIC fermions: J. M. Zanotti *et al.* [CSSM Lattice Collaboration], Phys. Rev. D 65, 074507 (2002) [arXiv:hep-lat/0110216].
  - Excited baryons: W. Melnitchouk *et al.*, [CSSM Lattice Collaboration], to appear in Phys. Rev. D arXiv:hep-lat/0202022.
  - Spin 3/2 baryons: J. M. Zanotti, *et al.*, [CSSM Lattice collaboration], arXiv:hep-lat/0304001.
  - Hybrid mesons from the FLIC action, J.N. Hedditch *et al.*, [CSSM Lattice Collaboration], in preparation.

Go to Outline

Some references (contd.)

- Other applications of FLIC fermions:
  - Accelerated overlap fermions: W. Kamleh, *et al.*, [CSSM Lattice Collaboration], Phys. Rev. D 66, 014501 (2002) [arXiv:hep-lat/0112041].
  - Baryon electromagnetic form factors: J. M. Zanotti *et al.*, [CSSM Lattice Collaboration], in preparation.

Go to Outline

Introduction to lattice QCD

- Complete solution of QCD $\equiv$ knowing all possible Minkowski space Green's functions of the theory.
- Implies for every possible combination of quark and gluon operators, $O[\hat{A}, \hat{\bar{q}}, \hat{q}]$, we need to know

$$\langle\Omega|\hat{T}\left(O[\hat{A}, \hat{\bar{q}}, \hat{q}]\right)|\Omega\rangle = \frac{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, O[A, \bar{q}, q]\exp(iS[A, \bar{q}, q])}{\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, \exp(iS[A, \bar{q}, q])}$$

$$= \frac{\int \mathcal{D}A\, \det\left[S_F^{-1}[A]\right]\, O[A, S_F[A]]\exp(iS[A])}{\int \mathcal{D}A\, \det\left[S_F^{-1}[A]\right]\, \exp(iS[A])},$$

- Note that $S[A]$ is the pure gluon (i.e., pure gauge) action.
- $|\Omega\rangle \equiv$ nonperturbative vacuum, $\hat{T} \equiv$ time-ordering operator, $S_F([A]; x, y) \equiv$ quark propagator in gluon field, A.

Go to Outline

Introduction to lattice QCD (contd.)

- $O[A, S_F[A]] \equiv \{O[A, \bar{q}, q]$ with every possible pairwise contraction of $\bar{q}$ and q replaced by the propagator $S_F([A]; x, y)\}$
- To do Monte Carlo estimates of functional integrations we need to work in Euclidean space, where all quantities are now Euclidean. So we need to know

$$\langle \Omega | \hat{T} \left(O[\hat{A}, \hat{\bar{q}}, \hat{q}] \right) | \Omega \rangle \equiv \frac{\int \mathcal{D}A \mathcal{D}\bar{q} \mathcal{D}q \, O[A, \bar{q}, q] \exp(-S[A, \bar{q}, q])}{\int \mathcal{D}A \mathcal{D}\bar{q} \mathcal{D}q \, \exp(-S[A, \bar{q}, q])}$$

$$= \frac{\int \mathcal{D}A \, O[A, S_F[A]] \det \left[S_F^{-1}[A] \right] \exp(-S[A])}{\int \mathcal{D}A \, \det \left[S_F^{-1}[A] \right] \exp(-S[A])}$$

Go to Outline

Introduction to lattice QCD (contd.)

- There is one factor of $\det[S_F^f[A]]$ for each quark flavor f, i.e., we use the notation $\det[S_F^{-1}[A]] \equiv \Pi_f \det[S_F^f[A]]$.
- Can study many observables from Euclidean space.
- We will only sample gauge inequivalent A's and hence for observables (i.e., for color singlet $O[\hat{A}, \hat{\bar{q}}, \hat{q}]$) $\Rightarrow$ won't have to bother with gauge fixing!

Go to Outline

Introduction to lattice QCD (contd.)

- The lattice approximates infinite Euclidean space by a four-dimensional discrete space-time lattice, where
  - $a \equiv$ lattice spacing, (typically $0.1 \sim 0.2$ fm).
  - N_s and N_t are number of lattice sites in space and time directions respectively.
  - $L_s = N_s a$ and $L_t = N_t a$ are physical length of lattice in space and time directions respectively.
  - $V = {L_s}^3 \times L_t \equiv$ physical lattice volume; ${N_s}^3 \times N_t \equiv$ lattice volume in lattice units.
- Introduce links, $U_\mu(x) \equiv U(x, x + a\mu) \in SU(3)$, between sites in the Cartesian directions $\mu = 1, \cdots, 4$.

Go to Outline

Introduction to lattice QCD (contd.)

- Links replace gluon fields: $A_\mu(x) \equiv \sum_{a=1}^{8} A_\mu^a(x)(\lambda^a/2)$.
- Links are parallel transport operators $U_\mu(x) = \hat{P} \exp\left(ig_s \int_x^{x+a\mu} dx' \cdot A(x')\right) \in SU(3)$, where $\hat{P} \equiv$ path ordering.
- We can express the gauge field in terms of finite differences of links, i.e., we can always express $A_\mu(x)$ as $A_\mu([U], x)$.
- We can generate an ensemble of gauge field configurations, $\{U_1, \cdots, U_{N_{\rm cf}}\}$ weighted with the probability distribution

$$P[U] \equiv \frac{(\Pi_f \det[S_F^f[U]]) \exp(-S[U])}{\int \mathcal{D}U \, (\Pi_f \det[S_F^f[U]]) \exp(-S[U])} .$$

Go to Outline

Introduction to lattice QCD (contd.)

- Since $\exp(-S[U]) \geq 0$ then provided $(\Pi_f \det[S_F^f[U]]) \geq 0$ we will have a well-defined probability distribution $P[U]$, i.e.,
  - $0 \leq P[U] \leq 1$
  - $\int \mathcal{D}U \, P[U] = 1$
- For chiral lattice fermions (e.g., overlap, domain-wall, staggered, etc.) we have $\det[S_F^f[U]]) \geq 0$ and all is well.
- For non-chiral lattice fermions (e.g., Wilson, clover, FLIC, etc.) at small enough quark masses we will always encounter some configurations U with an anomolously large determinant ($\not{D}$ e-value close to $-m$). These are called *exceptional configurations* $\Longrightarrow$ lower limit for quark mass.

Go to Outline

Introduction to lattice QCD (contd.)

- Will never have two gauge equivalent configurations in a finite ensemble (since number of gauge-inequivalent configurations is infinite) $\Rightarrow$ *no gauge fixing needed for color singlet quantities, e.g., physical observables.*
- We frequently approximate $P[U] \propto \exp(-S[U])$, which omits the fermion determinant and is equivalent to omitting all quark loops $\longrightarrow$ called the *quenched approximation* - undesirable and becoming less necessary as computers get more powerful.

Go to Outline

Introduction to lattice QCD (contd.)

- Hence we can now evaluate the Euclidean Green's function for any color-singlet $O[\cdots]$ by simply taking its *ensemble average*

$$\langle\Omega|\hat{T}\left(O[\hat{A},\hat{\bar{q}},\hat{q}]\right)|\Omega\rangle \equiv \langle O[U,S_F[U]]\rangle$$

$$= \frac{\int \mathcal{D}U\, O[U,S_F[U]]\,\det\left[S_F^{-1}[U]\right]\,\exp(-S[U])}{\int \mathcal{D}U\,\det\left[S_F^{-1}[U]\right]\,\exp(-S[U])}$$

$$= \lim_{V\to\infty}\lim_{a\to 0}\lim_{N_{\rm cf}\to\infty}\frac{\sum_{i=1}^{N_{\rm cf}} O[U_i,S_F[U_i]]}{\sum_{i=1}^{N_{\rm cf}}}$$

$$= \lim_{V\to\infty}\lim_{a\to 0}\lim_{N_{\rm cf}\to\infty}\frac{1}{N_{\rm cf}}\sum_{i=1}^{N_{\rm cf}} O[U_i,S_F[U_i]]$$

Go to Outline

Observables from the lattice

- We move from Minkowski space $\longrightarrow$ *Euclidean space* by the analytic continuation: $t \longrightarrow -it_E$ or in a different notation $x^0 \longrightarrow -ix_4$.
- Thus the Minkowski-space evolution operator, becomes the Euclidean-space version: $\exp(-i\hat{H}t) \longrightarrow \exp(-\hat{H}t_E)$.
- Note that replacing t_E with $\beta \equiv 1/kT$ and taking the trace gives the *partition function* of statistical mechanics:

  $$Z(\beta) \equiv \mathrm{tr}[\exp(-\beta\hat{H})] = \sum_n \exp(-\beta E_n).$$

  This is why statistical methods are so useful in lattice studies.

Go to Outline

Observables from the lattice (contd.)

- To extract information about observables from the lattice we need to use so-called *interpolating fields*.
- Consider ordinary Quantum Mechanics in the presence of some conserved charge operator $\hat{Q}$. Since $[\hat{H}, \hat{Q}] = 0$ we have:
  - $\hat{H}|E_n^q\rangle = E_n|E_n^q\rangle$ and $\hat{Q}|E_n^q\rangle = q|E_n^q\rangle$, where E_n and q are the energy and charge e-values respectively.
  - $\Rightarrow$ Hilbert space is divided up into charge sectors labelled by q and for *any* state $|\chi^q\rangle$ in the q charge sector:
    - $|\chi^q\rangle = \sum_n c_n|E_n^q\rangle$ for some set of coeffs $\{c_1, c_2, \cdots\}$
    - $\hat{Q}|\chi^q\rangle = q|\chi^q\rangle$.

Go to Outline

Observables from the lattice (contd.)

- Define $|\Omega\rangle \equiv$ ground state (i.e., vacuum) $\Longrightarrow$
  $\hat{H}|\Omega\rangle = \hat{Q}|\Omega\rangle = 0$
- Define the Schrödinger picture operators $\hat{\bar{\chi}}^q$ and $\hat{\chi}^q$ such that $\langle\chi^q| = \langle\Omega|\hat{\chi}^q$ and $|\chi^q\rangle = \hat{\bar{\chi}}^q|\Omega\rangle$.
- In Euclidean space the Heisenberg picture operators are:
  - $\hat{\bar{\chi}}^q(t_E) \equiv \exp(+\hat{H}t_E)\,\hat{\bar{\chi}}^q\,\exp(-\hat{H}t_E)$
  - $\hat{\chi}^q(t_E) \equiv \exp(+\hat{H}t_E)\,\hat{\chi}^q\,\exp(-\hat{H}t_E)$.
- Then we can define the *correlation function*:

$$
\begin{aligned}
G(t_E) &\equiv \langle\Omega|\hat{\chi}^q(t_E)\hat{\bar{\chi}}^q(0)|\Omega\rangle = \langle\chi^q|\exp(-\hat{H}t_E)|\chi^q\rangle \\
&= \sum_{n=0} |c_n|^2 \exp(-E_n^q t_E)
\end{aligned}
$$

Go to Outline

Observables from the lattice (contd.)

- As $t_E \to \infty$ have $\exp(-E^q_{n+1} t_E)/\exp(-E^q_n t_E) \to 0$ for $E^q_{n+1} > E^q_n$.
- Hence for large t_E can extract first few energies in the q charge sector, e.g., $E^q_0 = \lim_{t_E \to \infty}(1/t_E)\ln G(t_E)$, *etc.*
- Generalization to quantum field theory is straightforward.
- $\hat{\bar{\chi}}^q(t_E)$ and $\hat{\chi}^q(t_E)$ are called *interpolating field* operators.
- In QCD energy eigenstates are hadrons $\Longrightarrow$ are energy (E) *and* 3-momentum ($\vec{p}$) eigenstates $\Longrightarrow$ select 3-momentum using FT over spatial location $\Longrightarrow$ $\hat{\chi}^q \to \hat{\chi}^q(\vec{p})$.

Go to Outline

Observables from the lattice (contd.)

- To extract lowest ground state and lowest excited state masses for hadrons with quantum number q, need to study large t_E behavior of:

$$\begin{aligned} G(t_E) &\equiv \langle\Omega|\hat{\chi}^q(t_E,\vec{p}=0)\,\hat{\bar{\chi}}^q(0,\vec{p}=0)|\Omega\rangle \\ &= \langle\chi^q,\vec{p}=0|\exp(-\hat{H}t_E)|\chi^q,\vec{p}=0\rangle \\ &= \sum_{n=0}|c_n|^2\exp(-M_n^q\,t_E) \end{aligned}$$

- An *effective mass plot* is a plot of $M_{\rm eff}(t_E) \equiv -\ln[G(t_E+1)/G(t_E)]$ vs t_E.
- Clearly for t_E large we have $M_{\rm eff}(t_E) \to M_0^q \equiv$ (lowest mass hadron with quantum munbers q).

Go to Outline

Observables from the lattice (contd.)

- Similarly, we can extract hadron form factors associated with a current $V_\mu(x)$ by studying the correlation functions $\langle\Omega|\hat{\chi}^q(t'_E,\vec{p})\,\hat{V}_\mu(t_E,\vec{p})\,\hat{\bar{\chi}}^q(0,\vec{p}=0)|\Omega\rangle$ for $(t'_E - t_E)$ and t_E both large.

Go to Outline

Determining the lattice spacing

- Use the Static Quark Potential

$$V(\mathbf{r}) = V_0 + \sigma r - e\left[\frac{1}{\mathbf{r}}\right] + l\left(\left[\frac{1}{\mathbf{r}}\right] - \frac{1}{r}\right)$$

where $\sqrt{\sigma} = 440$MeV and $1/\mathbf{r}$ denotes the tree-level lattice Coulomb term (used to compensate for hypercubic artifacts)

$$\left[\frac{1}{\mathbf{r}}\right] \equiv 4\pi \int \frac{d^3\tilde{\mathrm{k}}}{(2\pi)^3} \cos(\vec{k}\cdot\mathbf{r}) D_{00}(0,\vec{k}),$$

and where $D_{00}(k)$ is the time-time component of the gluon propagator, [note: $D_{00}(0,\vec{k})$ is indep. of gauge parameter].

- In the continuum limit $[1/\mathbf{r}] \to 1/r$.

Go to Outline

Computing resources

- Calculations carried out on the Orion computer cluster:
  - 40 Enterprise E420R Sun nodes - each node has 4 Ultrasparc II 450 MHz processors - each processor has 1 GB RAM and 4 MB L2 cache
  - All 40 nodes have both Myrinet and fast ethernet
- Orion has a total peak theoretical speed of 144 Gflops and with 160 GBytes of RAM and 640 MBytes of cache. Linpack benchmark is 110 Gflops.
- Photograph of Orion.
- Rear and front photographs of the our 1.2 Teraflop IBM 1350 cluster Hydra - 129 dual Pentium 4's with Myrinet.

Go to Outline

FLIC fermion action

- The continuum Dirac operator, $\not{D} = \gamma^{\mu}(\partial_{\mu} + i\, g\, A_{\mu})$, is discretized by:
  - Replacing the derivative with a discrete difference, and
  - Including gauge links which
    - Encode the gluon field, A_{μ}, and
    - Maintain gauge invariance.

$$\bar{\psi}\,\not{D}\psi = \frac{1}{2a}\bar{\psi}(x)\sum_{\mu}\gamma_{\mu}\left[U_{\mu}(x)\psi(x+\hat{\mu}) - U_{\mu}^{\dagger}(x-\hat{\mu})\psi(x-\hat{\mu})\right].$$

- The continuum Dirac action is recovered in the limit $a \to 0$ by Taylor expanding the U_{μ} and $\psi(a+\hat{\mu})$ in powers of the lattice spacing a.

Go to Outline

FLIC fermion action (contd.)

- Hence we arrive at the simplest, naive lattice fermion action,

$$
\begin{aligned}
S_N &= m_q \sum_x \bar{\psi}(x)\psi(x) \\
&+ \frac{1}{2a} \sum_x \bar{\psi}(x)\gamma_\mu \left[U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu}) \right].
\end{aligned}
$$

Go to Outline

FLIC fermion action (contd.)

- While preserving chiral symmetry, encounters the fermion - doubling problem (i.e., it gives rise to $2^d = 16$ flavours rather than one).
- This doubling problem is demonstrated by the inverse of the free field propagator (obtained by taking the fourier transform of the action with all $U_\mu = 1$).

$$S^{-1}(p) = m_q + \frac{i}{a} \sum_\mu \gamma_\mu \sin p_\mu a$$

which has 16 zeros within the Brillouin cell in the limit $m_q \longrightarrow 0$, i.e.,
$p_\mu = (0,0,0,0), (\pi/a,0,0,0), (\pi/a,\pi/a,0,0), \cdots$.

Go to Outline

FLIC fermion action (contd.)

- Wilson introduced an irrelevant (energy) dimension-five operator (the so-called Wilson term) to fix this problem, $M_W = m_0 + \sum_\mu \left(\gamma_\mu \nabla_\mu - \frac{1}{2} r a \Delta_\mu\right)$, where $\nabla_\mu \psi(x) = \frac{1}{2a}[U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})]$ and $\Delta_\mu \psi(x) = \frac{1}{a^2}[U_\mu(x)\psi(x+\hat{\mu}) + U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu}) - 2\psi(x)]$.
- The Wilson action written in terms of the links $U_\mu(x)$ is

$$
\begin{aligned}
S_W &= \left(m_q + \frac{4r}{a}\right) \sum_x \bar{\psi}(x)\psi(x) + \frac{1}{2a} \sum_{x,\mu} \bar{\psi}(x) \\
&\times \left[(\gamma_\mu - r) U_\mu(x) \psi(x+\hat{\mu}) - (\gamma_\mu + r) U_\mu^\dagger(x-\hat{\mu}) \psi(x-\hat{\mu}) \right],
\end{aligned}
$$

but has large $\mathcal{O}(a)$ errors and “bad scaling”.

Go to Outline

Improvement of lattice actions

- The quality of a lattice action is measured by how rapidly it approaches the continuum limit as $a \to 0$, i.e., by how well it *scales*.
- The scaling properties of any action at finite a can be *improved* by introducing any number of irrelevant operators of increasing dimension which vanish in the continuum limit. One needs to choose appropriate *improvement coefficients* for these.
- Choosing the value of these coefficients so as to cancel error terms in the classical action to some $\mathcal{O}(a)$ is reffered to as *tree-level improvement*. It does *not* remove any $\mathcal{O}(ga)$ errors.

Go to Outline

Improvement (contd.)

- Fine-tuning of these coefficients (using the Schrödinger functional method) minimize the errors in nonperturbative calculations of observables is referred to as *nonperturbative improvement*. It tries to minimize errors to all orders in g.
- Replacing all occurences of links $U_\mu(x)$ with “mean-field improved” links $U_\mu(x)/u_0$ is called *mean-field improvement*. Here u_0 is the called the “mean link” and is usually defined as $u_0 = \left(\frac{1}{3}\mathcal{R}e\,\mathrm{tr}\langle U_{\mathrm{sq}}\rangle\right)^{1/4}$. This is cheaper and easier than fine-tuning of coefficients and it reduces ga errors, however is not as good in general as nonperturbative improvement.

Go to Outline

FLIC fermion action (contd.)

- Can improve the poorly-scaling Wilson fermion action by adding the so-called *Clover term*.
- The *Clover* or *Sheikholeslami-Wohlert (SW) action* introduces an additional irrelevant dimension-five operator to remove $\mathcal{O}(a)$ errors:

$$S_{SW} = S_W - \frac{iaC_{SW}r}{4}\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x)\,,$$

where C_{SW} is the clover coefficient.

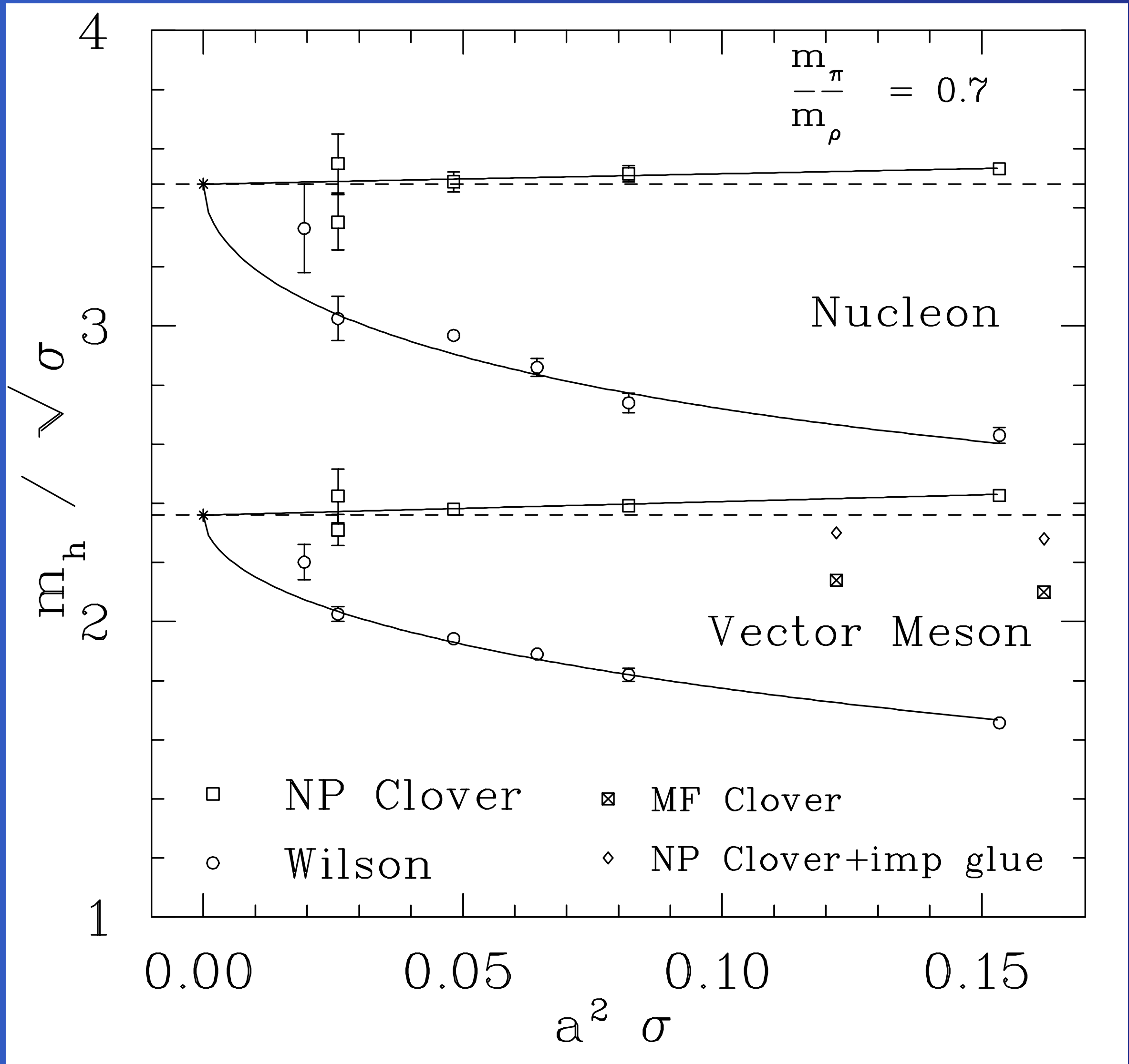
Go to Outline

FLIC fermion action (contd.)

- The difficulty lies in determining the precise renormalization of C_{SW} in the interacting theory:
  - $C_{SW} = 1$ at tree-level.
  - $C_{SW} = \frac{1}{u_0^3}$ with mean-field improvement,
  - Non-perturbative $\mathcal{O}(a)$ improvement (ALPHA Collaboration) - nonperturbatively tune C_{SW}.
- The NP-Improved Clover action displays excellent scaling.

Go to Outline

Clover Scaling Edwards, Heller, Klassen, PRL 80:3448-3451, 199



NP clover shows excellent scaling

Go to Outline

FLIC fermion action (contd.)

- The clover action has an exceptional configuration problem:
  - The Clover action is not a chiral symmetric action and so suffers from the exceptional configuration problem, i.e., the quark propagator encounters singular behaviour as
    - the quark mass becomes light,
    - as the lattice spacing becomes large.
  - Happens because chiral symmetry breaking in the action shifts e-values of the Dirac operator that would be zero modes in the continuum into the negative e-value region.
- Light-quarks are expensive since need fine lattices.
- The single plaquette-based $F_{\mu\nu}$ used has large $\mathcal{O}(a^2)$ errors.

Go to Outline

FLIC fermion action (contd.)

- The use of Fat-Link Fermion Actions was pioneered by Tom DeGrand, Anna Hasenfratz *et al.*
- Fat links are created by averaging or *smearing* the links in the action with their nearest neighbours in a gauge-equivariant manner.
- A link is replaced with a sum of
  - $(1-\alpha)$ of the original link, and
  - $\alpha/6$ times its six neighbouring "staples"
- The Smeared Link is projected back to $SU(3)$ colour.
- The process is repeated n times (n_{ape} sweeps).
- This process of making fat links is called APE smearing.

Go to Outline

FLIC fermion action (contd.)

- Benefits of use of APE-smeared links in fermion actions:
  - the nonperturbative renormalisation of improvement coefficients such as C_{SW} is reduced $\Longrightarrow$ mean-field improvement is sufficient.
  - the exceptional configuration problem is reduced because the gluon link configurations U are smoother.
- Difficulties:
  - gluon structure/interactions at the scale of the cutoff are smoothed away $\Longrightarrow$ lose short-distance gluon and quark interactions.

Go to Outline

FLIC fermion action (contd.)

- A useful solution to these problems is to work with *two sets of links* in the fermion action:
  - The relevant dimension-four operators are constructed with untouched Monte-Carlo generated links.
  - The irrelevant operators are constructed with fat links.
- Advantages of *Fat-Link Irrelevant (FLI)* fermion actions:
  - retain all relevant short-distance interactions
  - mean-field improvement sufficient - mean-field values of improvement coefficients adequate
  - reduced exceptional configuration problem (c.f., non fat-link version)

Go to Outline

FLIC fermion action (contd.)

- Applying these FLI principles to the simple Wilson action gives the *Mean-field improved Fat-Link Irrelevant Wilson (FLIW)* fermion action:

$$
\begin{aligned}
S_W^{FL} &= \left(m_q + \frac{4r}{a}\right) \sum_x \bar\psi(x)\psi(x) \\
&+ \frac{1}{2a} \sum_{x,\mu} \bar\psi(x) \Big[\gamma_\mu \left(\frac{U_\mu(x)}{u_0} \psi(x+\hat\mu) - \frac{U_\mu^\dagger(x-\hat\mu)}{u_0} \psi(x-\hat\mu) \right) \\
&- r \left(\frac{U_\mu^{FL}(x)}{u_0^{FL}} \psi(x+\hat\mu) + \frac{U_\mu^{FL\dagger}(x-\hat\mu)}{u_0^{FL}} \psi(x-\hat\mu) \right) \Big]
\end{aligned}
$$

Go to Outline

FLIC fermion action (contd.)

- Applying the FLI principles to the Clover/SW action finally brings us to the *Mean-field improved Fat-Link Irrelevant Clover (FLIC)* action

$$S_{SW}^{FL} = S_W^{FL} - \frac{iaC_{SW}r}{4(u_0^{FL})^4}\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x)\,. \qquad (1)$$

  where the FLI principles tell us that $F_{\mu\nu}$ is to be constructed using fat-links, since it is an irrelevant operator.

- The advantages of the addition of the Clover term remain and so the FLIC fermion action is expected to lead to better scaling than the FLIW action.

Go to Outline

FLIC fermion action (contd.)

- The FLI principles can be applied to any fermion action, e.g., using a FLIC kernel in the overlap fermion action gave rise to a lower density of low-lying e-values and an increased overlap fermion performance. See, e.g.,
  - Accelerated overlap fermions: W. Kamleh, *et al.*, [CSSM Lattice Collaboration], Phys. Rev. D 66, 014501 (2002) [arXiv:hep-lat/0112041].

Go to Outline

FLIC fermion action (contd.)

- Fat-link mean-field improvement parameter $u_0^{FL} \to 1$.

n	u_0^{FL}	$(u_0^{FL})^4$
0	0.889	0.624
4	0.997	0.986
12	0.999	0.997

- After four sweeps. a mean-field improved estimate of coefficients is sufficient.
- Highly improved actions with many irrelevant operators (e.g., D234) can be handled with confidence.
- Can use improved definitions of $F_{\mu\nu}$ using up to u_0^8.

Go to Outline

FLIC fermion action (contd.)

- Following calculations were performed using a mean-field improved, plaquette + rectangle, gauge action on a $16^3 \times 32$ lattice at $\beta = 4.60\,(\beta = 6/g^2)$, with lattice spacing $a = 0.122(1)$ fm.
- Fixed boundary condition in time direction, ie. $U_t(\vec{x}, nt) = 0 \qquad \forall \vec{x}$.
- The source was created at a space-time location of $(x, y, z, t) = (1, 1, 1, 3)$.
- Gauge-invariant gaussian smearing was applied at the source to increase the overlap of the interpolating operators with the ground states.

Go to Outline

FLIC fermion action (contd.)

- Use $\mathcal{O}(a^4)$ definition of $F_{\mu\nu}$, see Bilson-Thompson *et al.*, [CSSM Lattice Collaboration], hep-lat/0203008.
- Only *quenched* calculations for FLIC so far. Lattices used:

β	a(fm)	$L^3 \times T$	Length(fm)
4.38	0.165	$12^3 \times 24$	1.980
4.60	0.122	$12^3 \times 24$	1.464
4.60	0.122	$16^3 \times 32$	1.952
4.80	0.093	$16^3 \times 32$	1.488

- Results here are for 16×32, $\beta = 4.60$, and ≤ 400 configs.

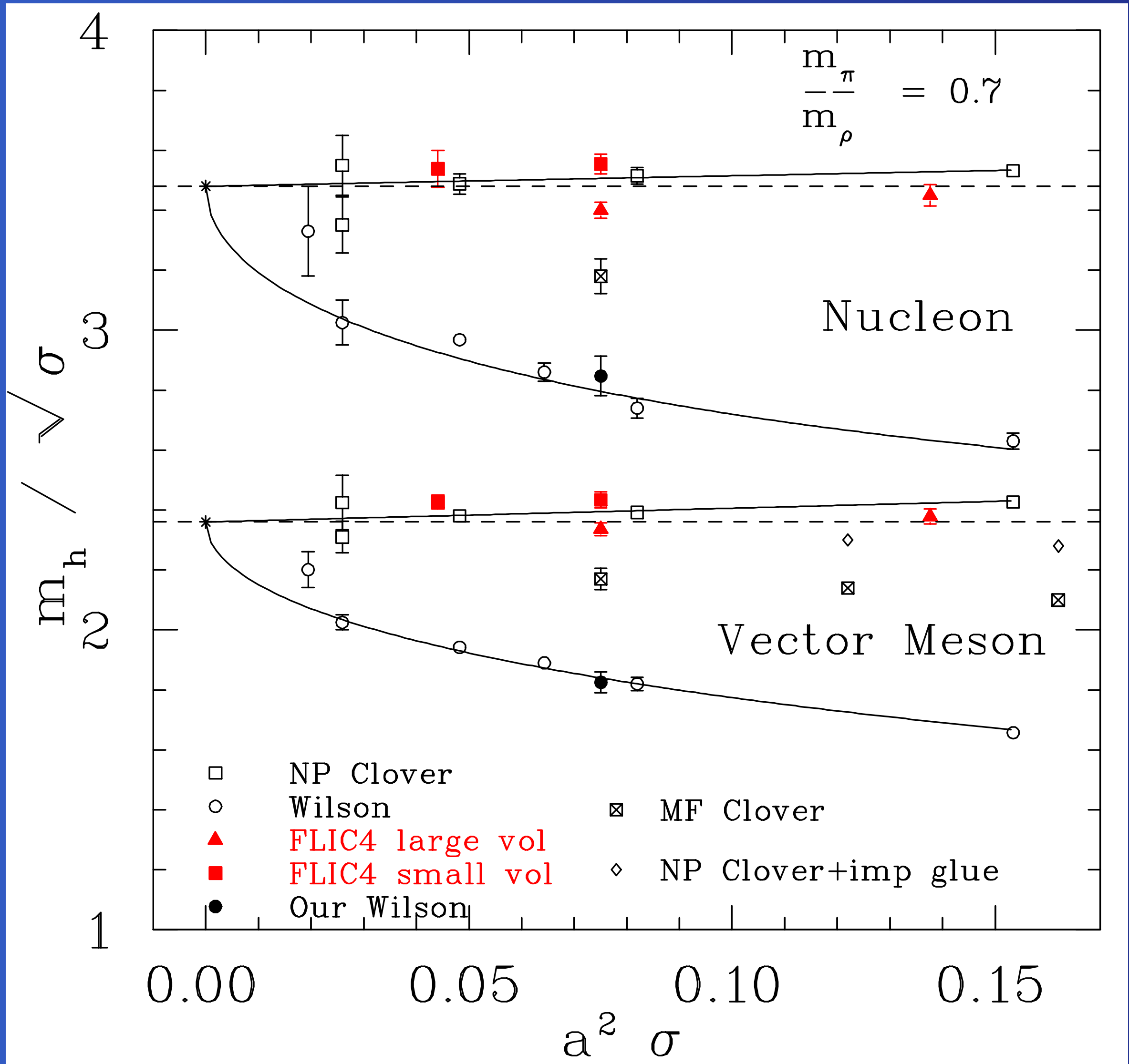
Go to Outline

FLIC fermion action (contd.)

- 5 quark masses: $\kappa = 0.1260, 0.1266, 0.1273, 0.1279, 0.1286$, which corresponds to approximately 193, 163, 129, 100, and 66 MeV respectively.

Go to Outline

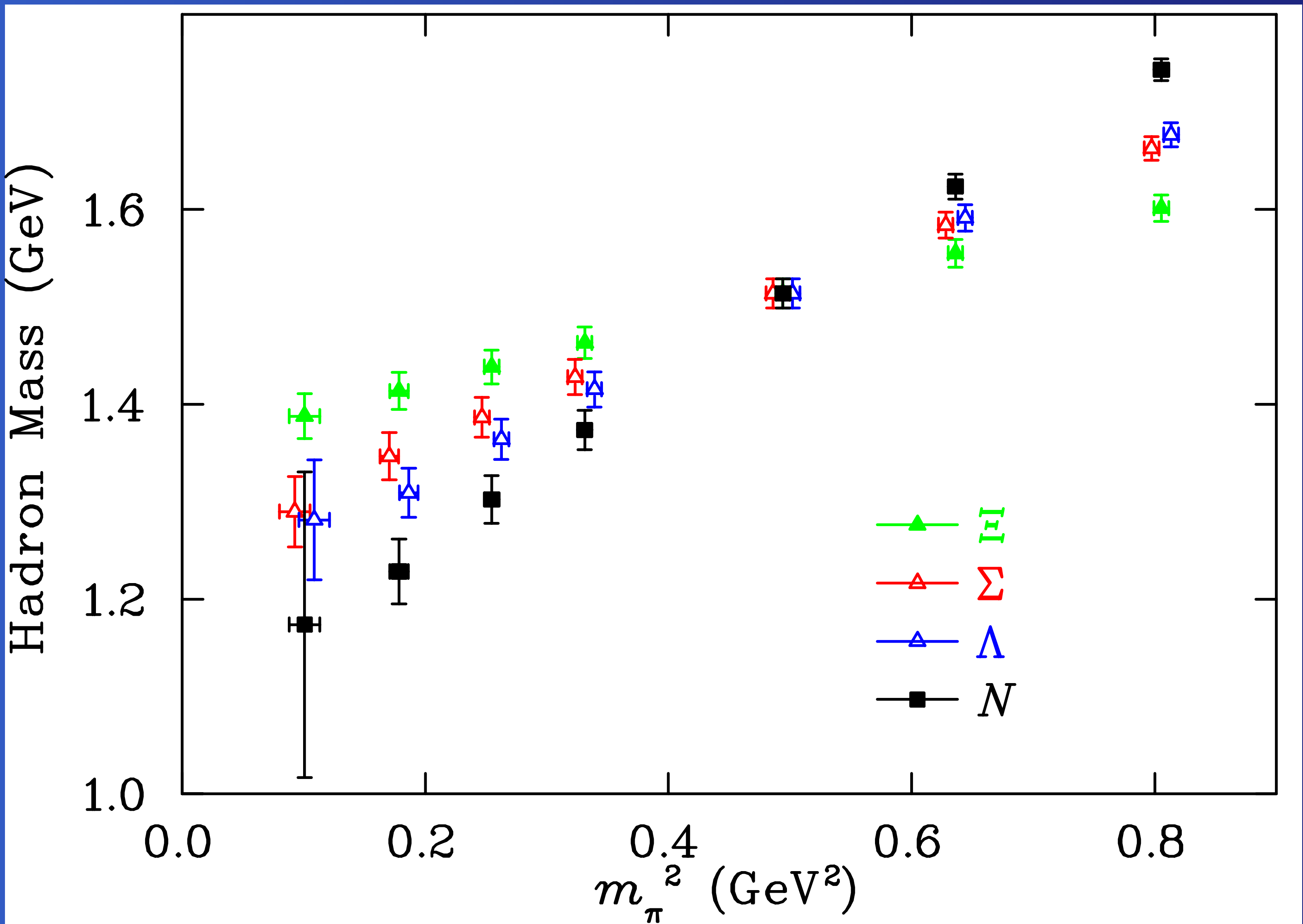
FLIC fermion action scaling



- FLIC action shows excellent scaling.
- Small lattice shows fi nite volume effects.

Go to Outline

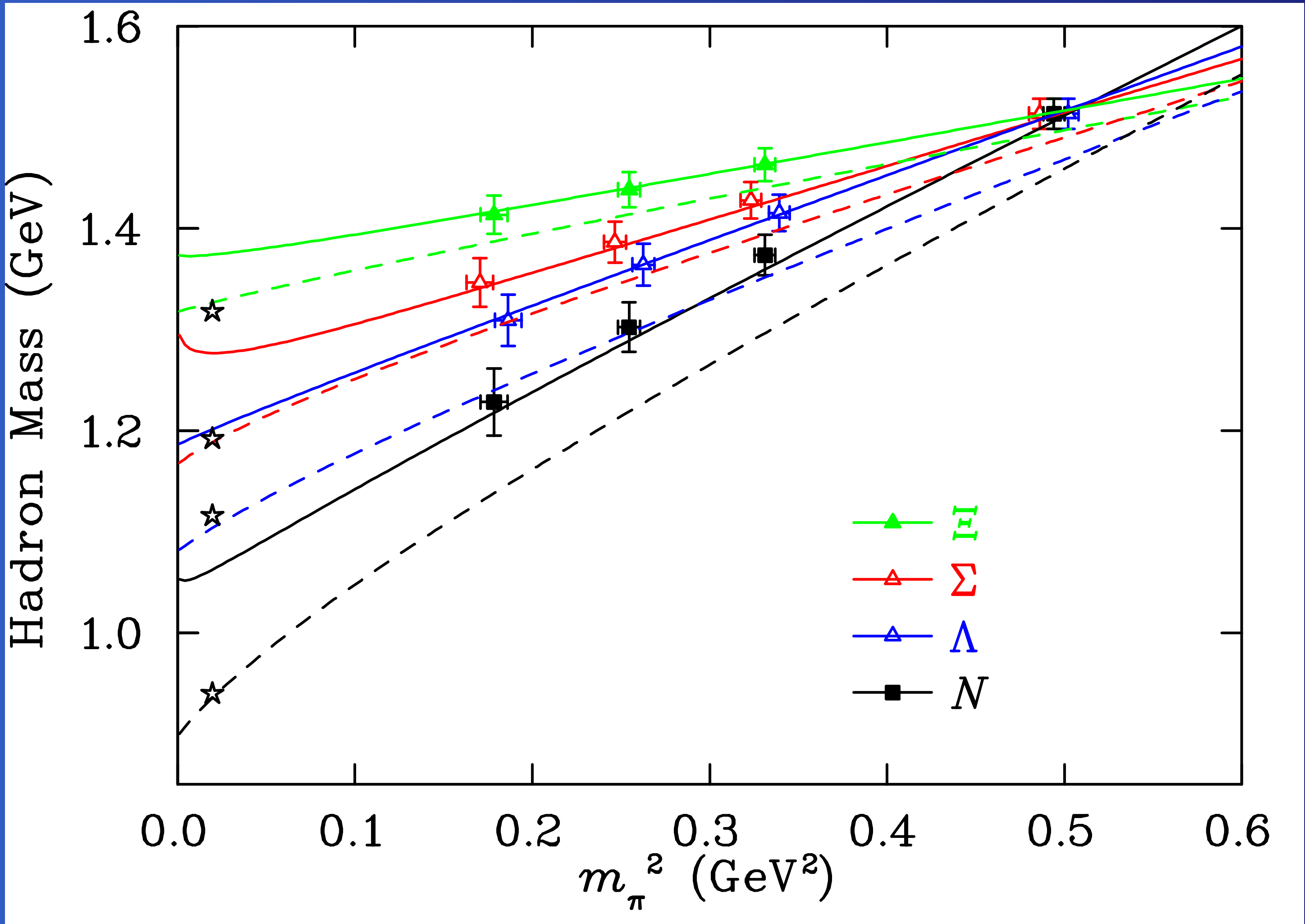
Octet Baryons with Light Quarks



Ground state baryon masses

Go to Outline

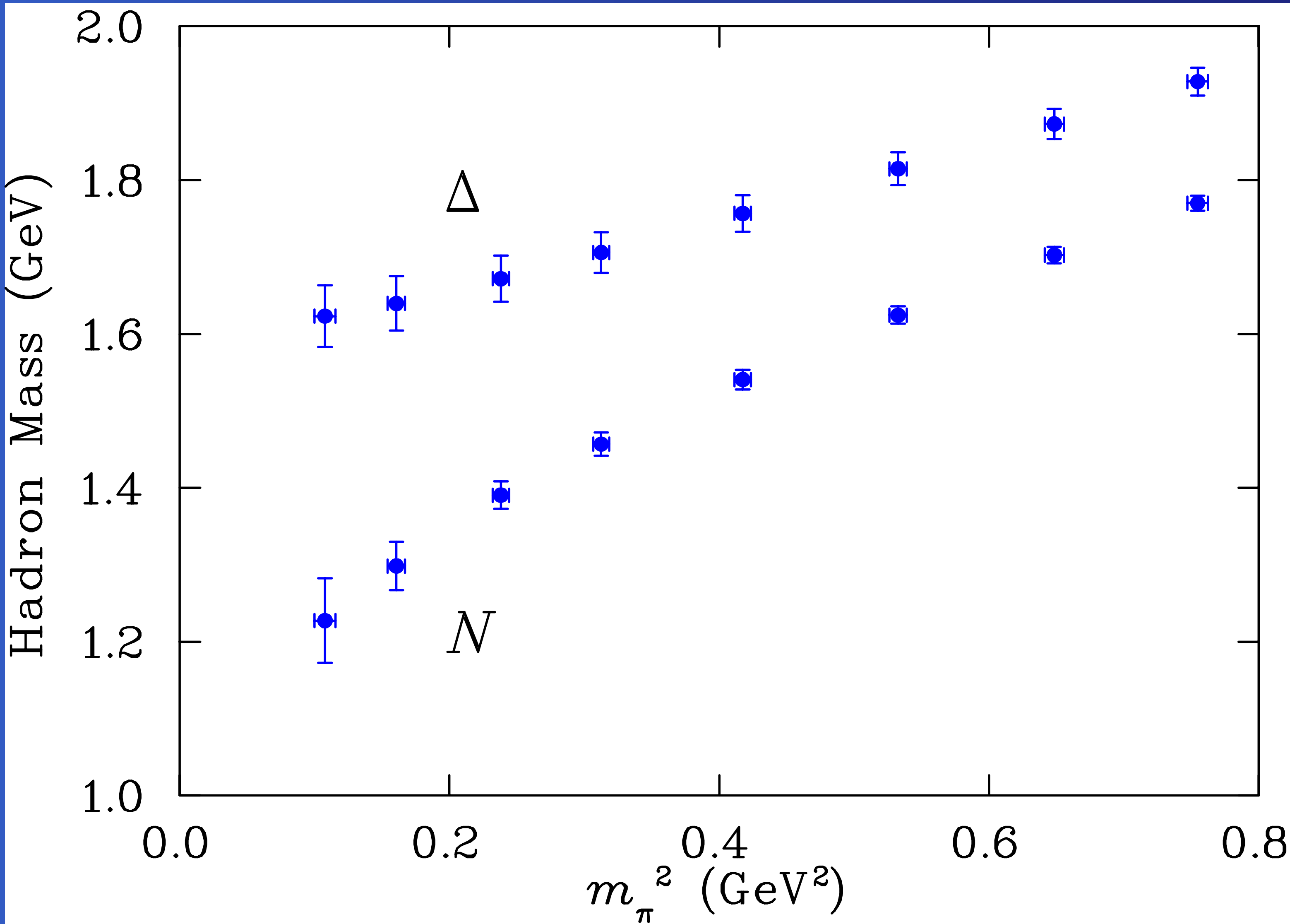
Is current data of interest?



Chiral extrapol by R. Young, *etal.* shown for illustrative purposes

Go to Outline

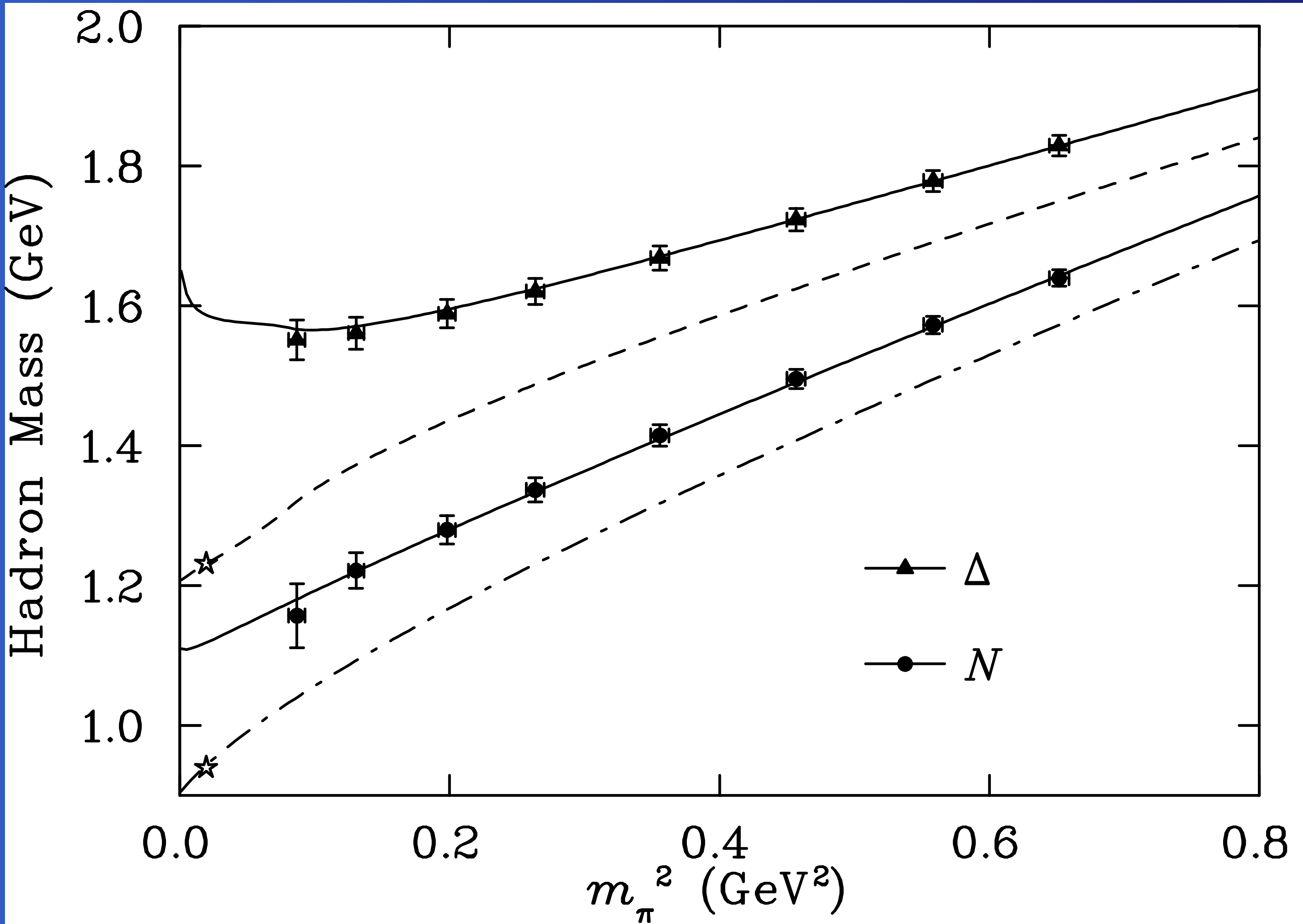
Nucleon and Delta



Nucleon and Delta masses

Go to Outline

Sample chiral extrap (incl unquench)



Chiral extrapol by R. Young, *etal.* shown for illustrative purposes

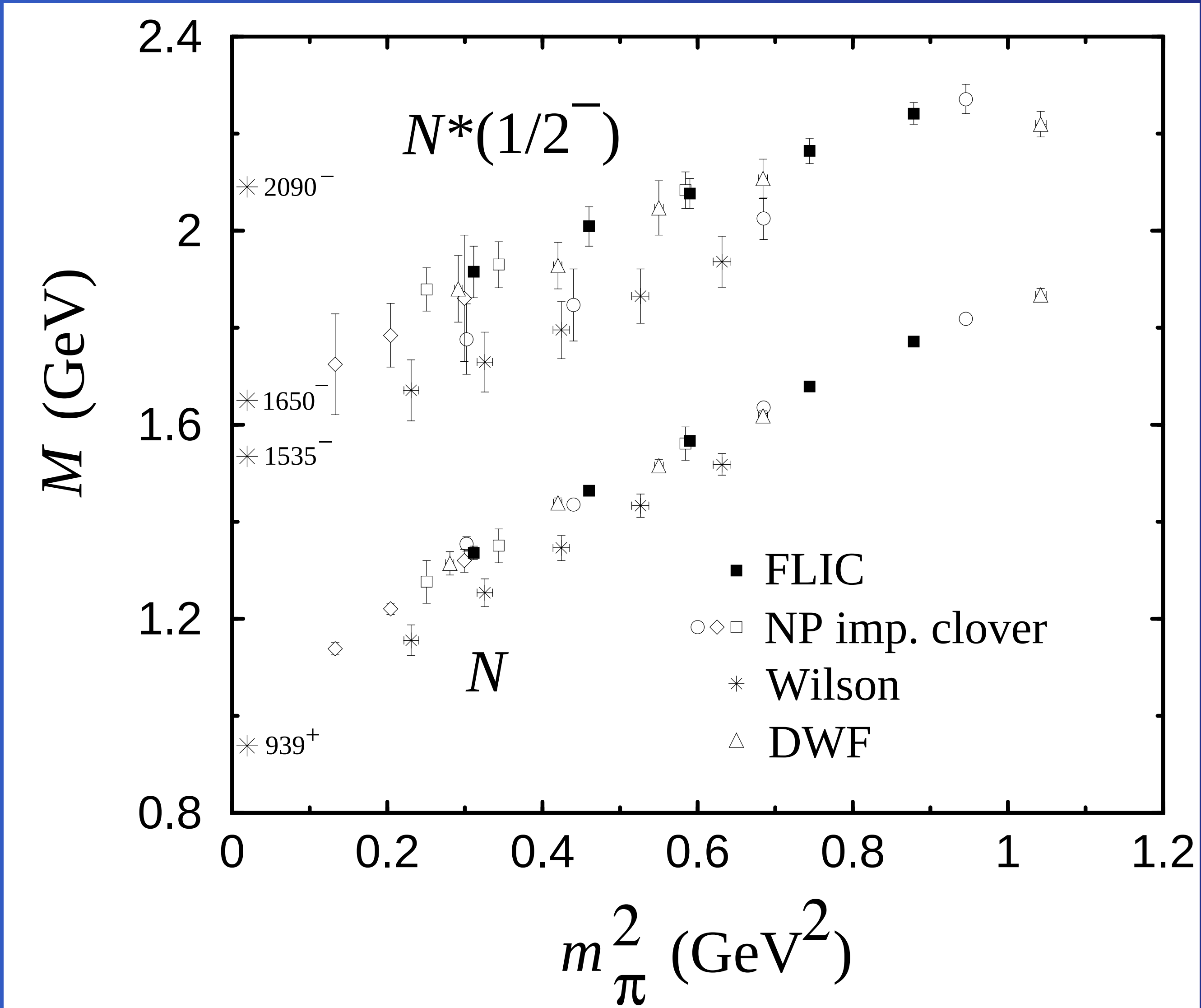
Go to Outline

Excited baryons

- In order to study excited states we need to perform a *parity projection*, since in general a given interpolating accesses both positive and negative parity states.
- Parity projection for $\vec{p} = 0$ is straightforward and given by $\Gamma_\pm \equiv (1/2)[1 \pm \gamma_4]$.

Go to Outline

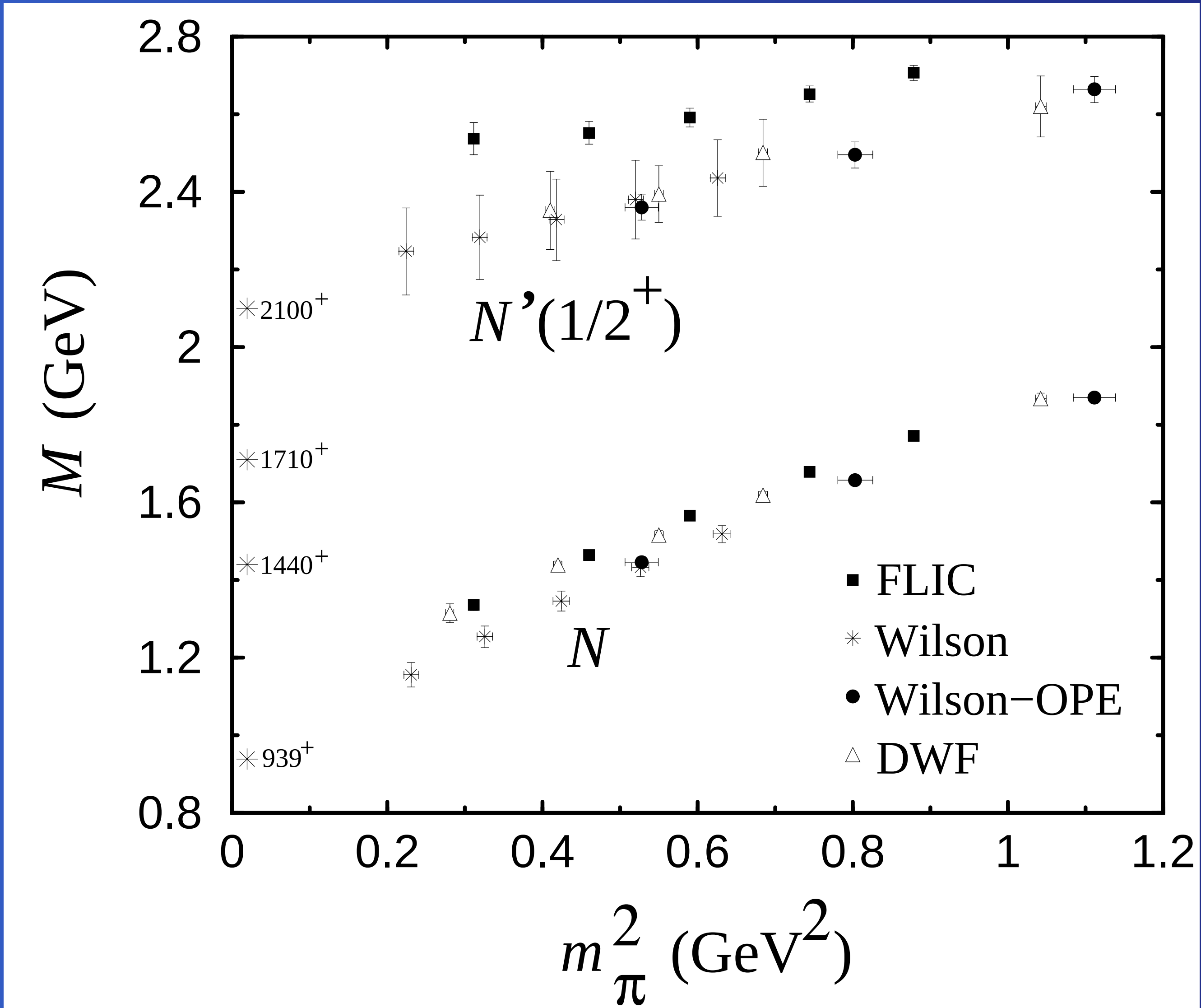
Excited baryons (contd.)



- Masses of nucleon (N) and lowest $J^P = \frac{1}{2}^-$ excitation ("N^*").
- FLIC and Wilson results [CSSM] c.f. DWF [Sasaki *et al.*] and NP improv clover [Richards *et al.*].
- Empirical masses indicated by asterisks.

Go to Outline

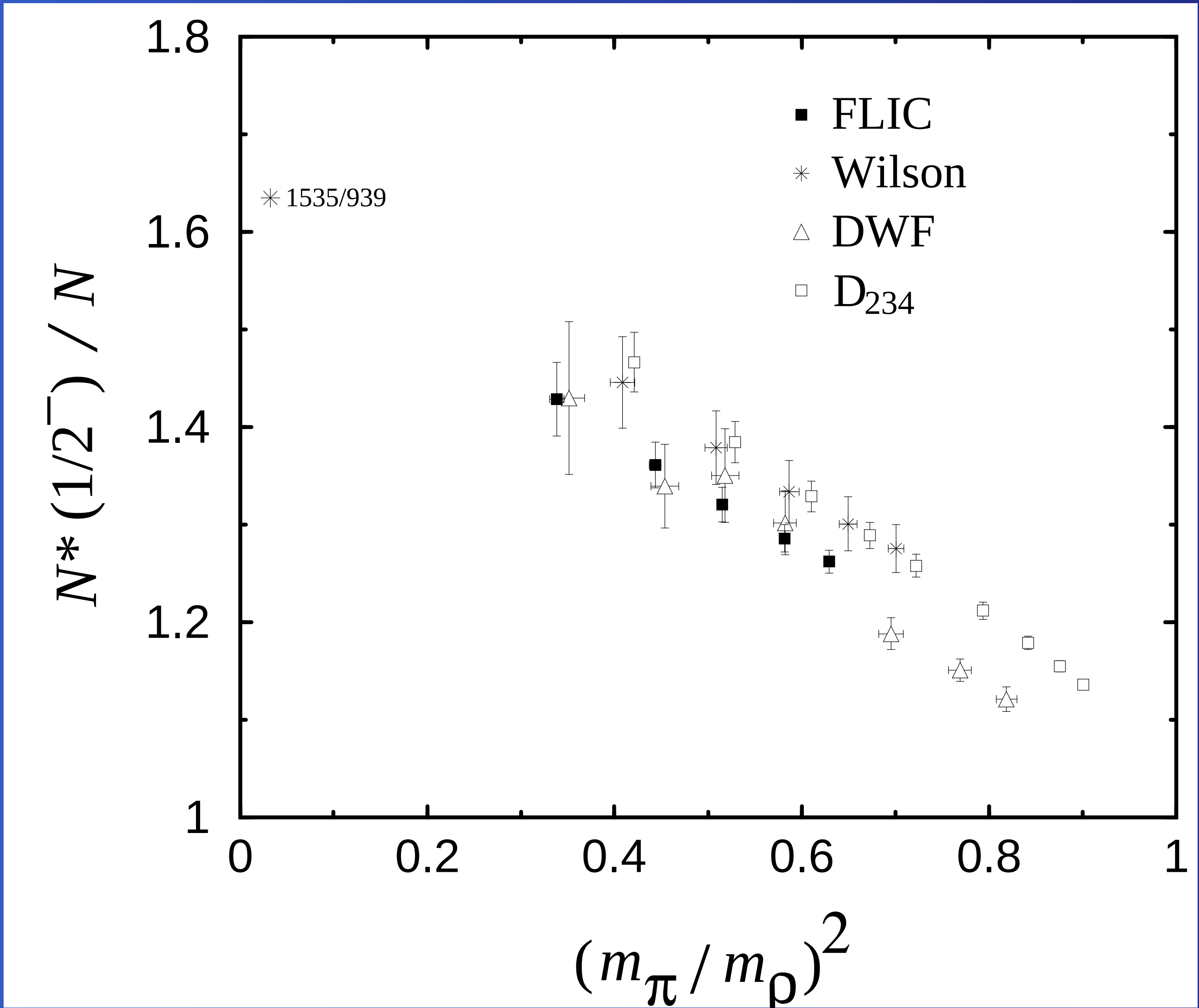
Excited baryons (contd.)



- Masses of nucleon, and lowest $J^P = \frac{1}{2}^+$ excitation ("N'").
- FLIC and Wilson results [CSSM] c.f. DWF [Sasaki *et al.*] and Wilson-OPE [Leinweber].
- Empirical masses indicated by asterisks.

Go to Outline

Excited baryons (contd.)



- Ratio of the lowest $N^*(\frac{1}{2}^-)$ and nucleon masses.
- FLIC and Wilson results [CSSM] compared c.f. D_{234} [Lee] and DWF [Sasaki *et al.*].
- Empirical $N^*(1535)/N$ mass ratio is denoted by asterisk.

Go to Outline

Spin 3/2 Nucleon and Delta baryons

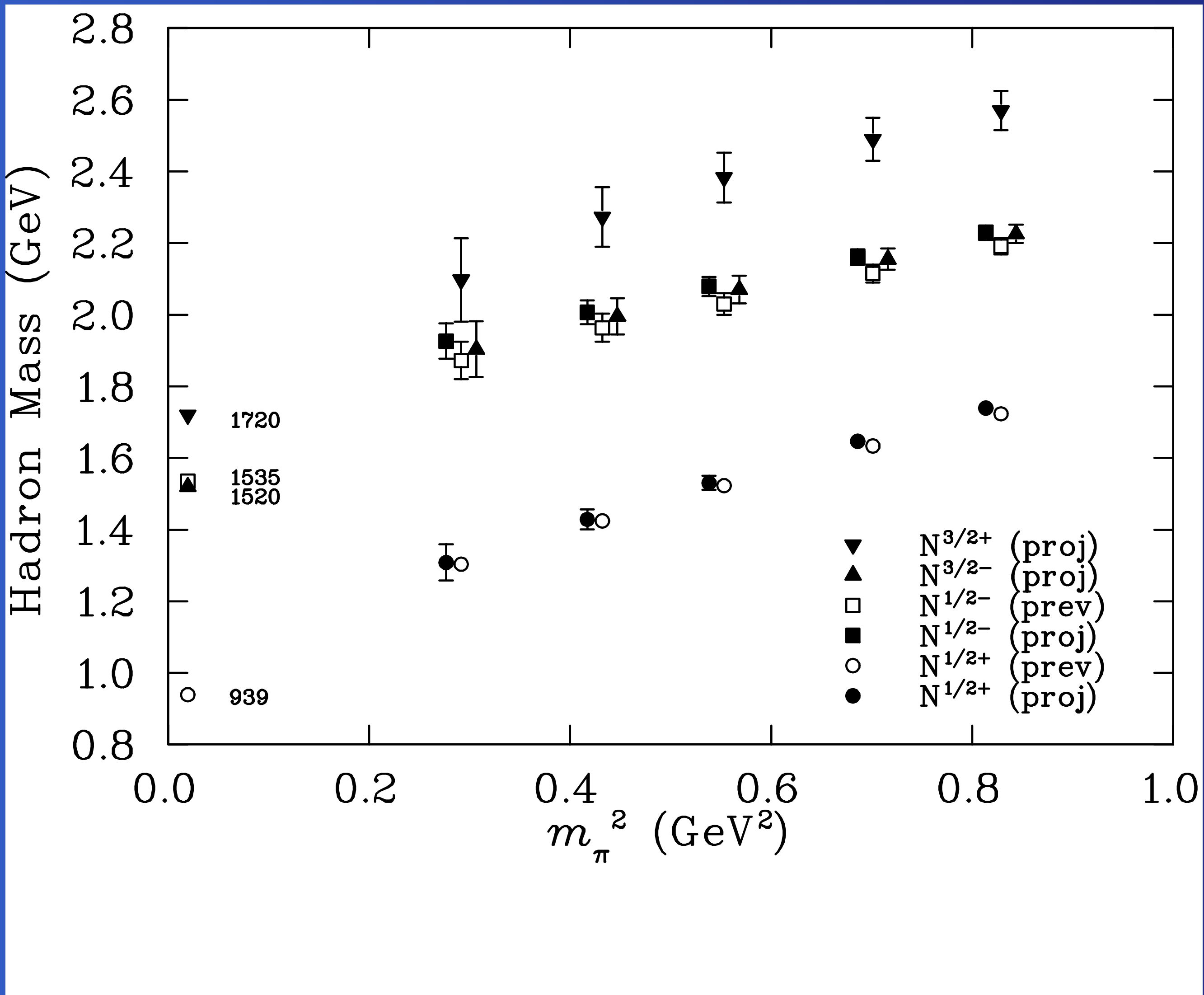
- Use the correlation function $G_{\mu\nu}(t, \vec{p}; \Gamma) = \mathrm{tr}_{\mathrm{sp}} \{\Gamma \mathcal{G}_{\mu\nu}(t, \vec{p})\}$, where $\mathcal{G}_{\mu\nu}^{\alpha\beta}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 | \, T \left(\chi_\mu^\alpha(x) \, \overline{\chi}_\nu^\beta(0) \right) |0\rangle$, and where χ_μ^α is a spin-$\frac{3}{2}$ interpolating field, Γ is a matrix in Dirac space with α, β Dirac indices, and μ, ν Lorentz indices.
- An interpolating field operator for the isospin-$\frac{1}{2}$, spin-$\frac{3}{2}$, (charge $+1$) state is (similarly for isospin 3/2)

$$\chi_\mu^N = \epsilon^{abc} \left(u^{Ta}(x) \, C\gamma_5\gamma^\nu \, d^b(x) \right) \left(g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu \right) \gamma_5 u^c(x) \, .$$

- Requires both spin (to spin 1/2 or 3/2) and parity projection (to $+$ or $-$).
- Results shown are for 392 configs of our $\beta = 4.60$ lattice.

Go to Outline

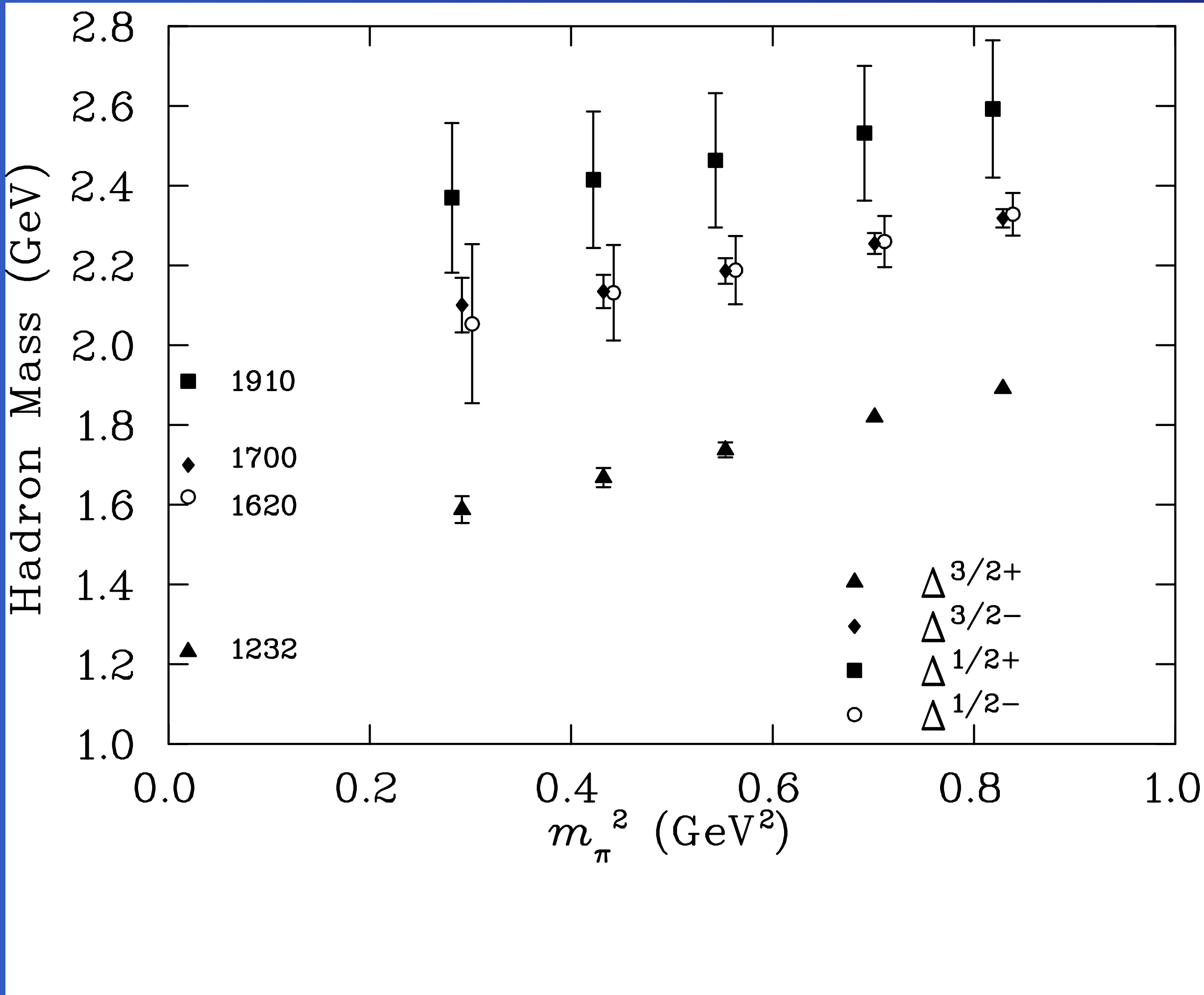
Spin 3/2 (contd.)



- Masses of $N\frac{3}{2}^-$, $N\frac{3}{2}^+$, $N\frac{1}{2}^+$, and $N\frac{1}{2}^-$ states.
- C.f. direct calcn of $N\frac{1}{2}^+$ and $N\frac{1}{2}^-$ from spin-1/2.
- Empirical masses are shown on left at physical m_π.

Go to Outline

Spin 3/2 (contd.)



- Masses of $\Delta\frac{3}{2}^{\pm}$ and $\Delta\frac{1}{2}^{\pm}$ resonances.
- Empirical masses are shown on left at physical m_π.

Go to Outline

Hybrid mesons

- Standard interpolating fields for mesons are of the form

$$\chi(x) = \sum_a \overline{q}^a(x)\Gamma q^a(x)\,,$$

where Γ is a combination of gamma-matrices chosen to provide the quantum numbers desired, and a denotes the colour of the quark.

- One cannot reproduce all possible mesonic quantum numbers with such fields, i.e., one cannot produce 'exotics' such as s-wave mesons with $J^{PC} = \{0^{+-}, 0^{--}, 1^{-+}, etc\}$.

Go to Outline

Hybrid mesons

- Can generalize interpolating fields to include the gauge field, i.e.,

$$\chi(x) = \sum_{a,b} \overline{q}^a(x)\Gamma\mathcal{G}^{ab}(x)q^b(x)\,,$$

  where the new term $\mathcal{G}$ is a gauge functional and both Γ and $\mathcal{G}$ determine the meson quantum number, e.g., $\mathcal{G} = E$ or B.

- Could also generalise to non-local interpolating fields, e.g.,

$$\chi(x) = \sum_{y}\sum_{a,b} \overline{q}^a(x)\Gamma\mathcal{G}^{ab}(x,y)q^b(y)\,.$$

  But more expensive to calculate $\Longrightarrow$ for now restrict ourselves to *local* interpolating fields.

Go to Outline

$J = 0, 1$ interpolators with E and B

0^{++}	0^{+-}	0^{-+}	0^{--}
$\bar{q}^a q^a$	$\bar{q}^a \gamma_4 q^a$	$\bar{q}^a \gamma_5 q^a$	$-i\bar{q}^a \gamma_5 \gamma_j E_j^{ab} q^b$
$-i\bar{q}^a \gamma_j E_j^{ab} q^b$	$\bar{q}^a \gamma_5 \gamma_j B_j^{ab} q^b$	$\bar{q}^a \gamma_5 \gamma_4 q^a$	
$-\bar{q}^a \gamma_j \gamma_4 \gamma_5 B_j^{ab} q^b$		$-\bar{q}^a \gamma_j B_j^{ab} q^b$	
$-\bar{q}^a \gamma_j \gamma_4 E_j^{ab} q^b$		$-\bar{q}^a \gamma_4 \gamma_j B_j^{ab} q^b$	

1^{++}	1^{+-}	1^{-+}	1^{--}
$-i\bar{q}^a \gamma_5 \gamma_j q^a$	$-i\bar{q}^a \gamma_5 \gamma_4 \gamma_j q^a$	$\bar{q}^a \gamma_4 E_j^{ab} q^b$	$-i\bar{q}^a \gamma_j q^a$
$i\bar{q}^a \gamma_4 B_j^{ab} q^b$	$i\bar{q}^a B_j^{ab} q^b$	$-\epsilon_{jkl} \bar{q}^a \gamma_k B_l^{ab} q^b$	$\bar{q}^a E_j^{ab} q^b$
$i\epsilon_{jkl} \bar{q}^a \gamma_k E_l^{ab} q^b$	$\bar{q}^a \gamma_5 E_j^{ab} q^b$	$\epsilon_{jkl} \bar{q}^a \gamma_4 \gamma_k B_l^{ab} q^b$	$-i\bar{q}^a \gamma_5 B_j^{ab} q^b$
$i\epsilon_{jkl} \bar{q}^a \gamma_k \gamma_4 E_l^{ab} q^b$	$\bar{q}^a \gamma_5 \gamma_4 E_j^{ab} q^b$	$-i\epsilon_{jkl} \bar{q}^a \gamma_5 \gamma_4 \gamma_k E_l^{ab} q^b$	$i\bar{q}^a \gamma_4 \gamma_5 B_j^{ab} q^b$

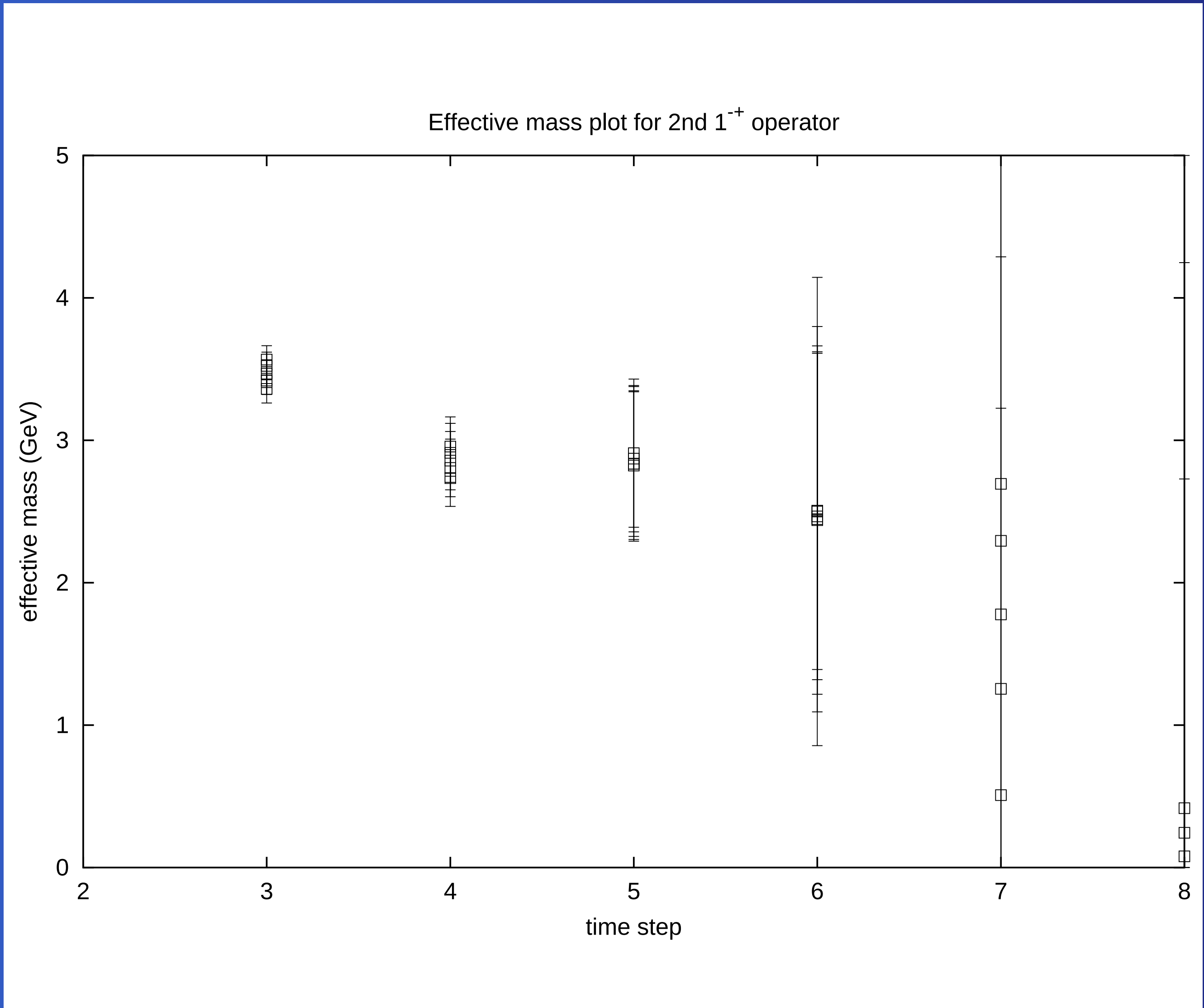
Go to Outline

Conventional mesons

Name	J^{PC}	Operator	Mass(GeV) $\kappa = 0.1260$	$\kappa = 0.1279$	$\kappa = 0.1286$	T_{fit}
$\pi(140)$	0^{-+}	$\bar{q}^a \gamma_5 q^a$	$0.762 \pm .006$	$0.670 \pm .006$	$0.544 \pm .008$	$6-7$
		$\bar{q}^a \gamma_5 \gamma_4 q^a$	$0.751 \pm .006$	$0.660 \pm .006$	$0.536 \pm .007$	$6-7$
		$-\bar{q}^a \gamma_j B_j^{ab} q^b$	$0.805 \pm .103$	$0.701 \pm .109$	$0.573 \pm .117$	$6-7$
		$-\bar{q}^a \gamma_4 \gamma_j B_j^{ab} q^b$	$0.820 \pm .052$	$0.722 \pm .055$	$0.584 \pm .060$	$6-7$
$a_0(1450)$	0^{++}	$\bar{q}^a q^a$	$1.458 \pm .047$	$1.457 \pm .067$	$1.517 \pm .082$	$3-4$
$a_1(1260)$	1^{++}	$-i\bar{q}^a \gamma_5 \gamma_j q^a$	$1.567 \pm .016$	$1.527 \pm .017$	$1.483 \pm .021$	$3-4$
$b_1(1238)$	1^{+-}	$-i\bar{q}^a \gamma_5 \gamma_4 \gamma_j q^a$	$1.580 \pm .025$	$1.540 \pm .027$	$1.501 \pm .033$	$3-7$
		$i\bar{q}^a B_j^{ab} q^b$	$2.518 \pm .281$	$2.481 \pm .280$	$2.498 \pm .307$	$2-3$
$\rho(770)$	1^{--}	$-i\bar{q}^a \gamma_j q^a$	$1.063 \pm .012$	$1.009 \pm .014$	$0.947 \pm .017$	$6-7$
		$-i\bar{q}^a \gamma_5 B_j^{ab} q^b$	$1.116 \pm .204$	$1.032 \pm .225$	$0.919 \pm .272$	$4-5$
		$i\bar{q}^a \gamma_4 \gamma_5 B_j^{ab} q^b$	$1.138 \pm .126$	$1.067 \pm .129$	$0.978 \pm .138$	$4-5$

Go to Outline

Recent (yesterday) result for 1^{-+} mass



- 1^{-+} effective mass plot from $i\bar{q}^a B_j^{ab} q^b$ interpolator.
- Based on small ensemble of 104 so far

Go to Outline

1^{-+} Meson

Name	J^{PC}	Operator	Mass(GeV) $\kappa = 0.1273$	$\kappa = 0.1279$	$\kappa = 0.1286$	$\mathrm{T}_{\mathrm{fit}}$
π	0^{-+}	$\bar{q}^a \gamma_5 q^a$	$0.762 \pm .006$	$0.670 \pm .006$	$0.544 \pm .008$	$6 - 7$
		$\bar{q}^a \gamma_5 \gamma_4 q^a$	$0.751 \pm .006$	$0.660 \pm .006$	$0.536 \pm .007$	$6 - 7$
$Exotic$	1^{-+}	$-\epsilon_{jkl} \bar{q}^a \gamma_k B_l^{ab} q^b$	$2.777 \pm .549$	$2.774 \pm .552$	$2.802 \pm .579$	$2 - 3$

- The 1^{-+} result was only obtained in the last few days. Hopefully with more configs and better statistics signals from other interpolating fields will result.

Go to Outline

Conclusions and outlook

- Had a brief introduction to lattice QCD and improvement of operators
- Derived the form of FLIC and explained the FLI principles, which is generalizable to other actions.
- FLIC shows excellent scaling and reuced exceptional configurations problems.
- Ground state hadron masses obtained even down to relatively low pion masses of 346 MeV.
- Results obtained for excited spin-1/2 baryons [not time to show all of these].

Go to Outline

Conclusions and outlook (contd.)

- FLIC results for spin 3/2 N and Δ baryons.
- Recovered conventional meson masses with a variety of interpolating fields containing color E and B fields.
- First preliminary result for the exotic 1^{-+} meson mass - hopefully others to follow.

Go to Outline