Chiral Extrapolation of Lattice QCD Data

Anthony W. Thomas

CSSM

ADELAIDE UNIVERSITY AUSTRALIA

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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Outline

PROBLEM: Lack of computer power

$\Rightarrow$ need chiral extrapolation

χ PT : Radius of convergence TOO SMALL

RESPONSE: STOP?? **NO!**

Just impose a little physics insight !

$\Rightarrow$ MODEL INDEPENDENT RESULTS!

Then: 10 Teraflops (next generation) machines will allow 1% accuracy

Subtitle: How to make effective field theory more effective……

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Outline (cont.)

Implication for hybrids:

First examination of chiral extrapolation for $\pi_1(1600)$

For fun:

Recent investigation of the valence structure of the pion : using data of QCDSF

Problem of Chiral Extrapolation

♦ Currently limited to $m_q > 30\text{-}50$ MeV

Time to decrease m_π by factor of 2 : $2^7 \sim 100$

♦ NEED perhaps 500 Teraflops to get to 5 MeV !

Furthermore EFT implies ALL hadronic properties are

non-analytic functions of m_q ……..

HENCE: NO simple power series expansion about $m_q = 0$

: NO simple chiral extrapolation

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Formal Chiral Expansion

<u>Formal</u> expansion of Hadron mass:

$$M_N = c_0 + c_2\, m_\pi^2 + c_{LNA}\, m_\pi^3 + c_4\, m_\pi^4 + c_{NLNA}\, m_\pi^4 \ln m_\pi + c_6\, m_\pi^6 + \ldots..$$

Mass in chiral limit

No term linear in m_π
(in FULL QCD……
there is in QQCD)

First (hence "leading")
non-analytic term $\sim m_q^{3/2}$
(LNA)
Source: $N \to N\,\pi \to N$

c_{LNA} MODEL INDEPENDENT

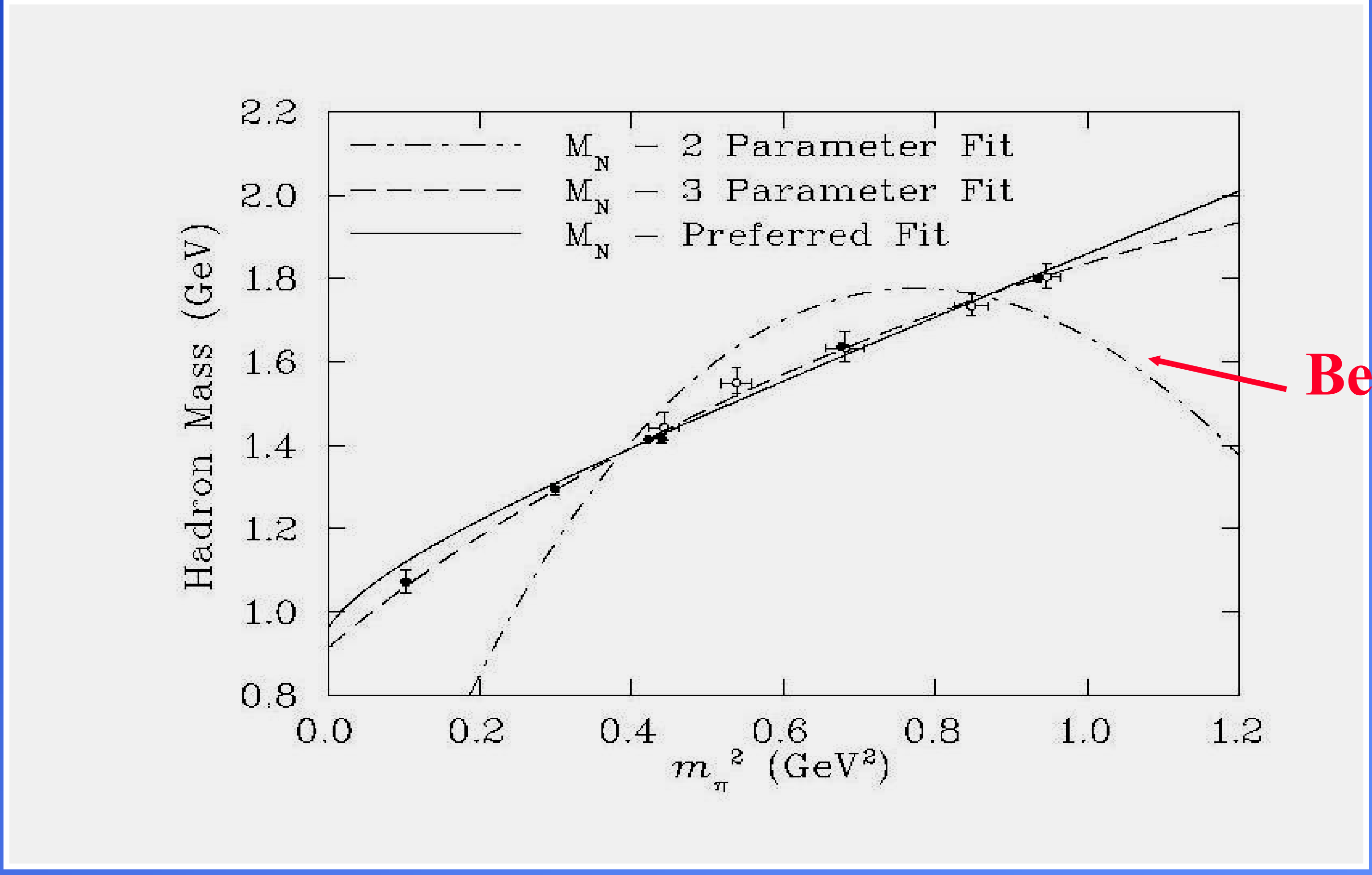
Another branch cut
from $N \to \Delta\,\pi \to N$
- higher order in m_π
- hence "next-to-leading" non-analytic (NLNA)

c_{NLNA} MODEL INEPENDENT

Convergence?

Relevance for Lattice data

Knowing χ PT , fit with: $\alpha + \beta\, m_\pi^2 + \gamma\, m_\pi^3$ (dashed curve)



Best fit with γ as in χ P T

Problem: γ = - 0.76 c.f. model independent value -5.6 !!

(From: Leinweber *et al.*, Phys. Rev., D61 (2000) 074502)

The Solution

There is another SCALE in the problem

- not natural in (e.g.) dim-regulated χPT

$\Lambda \sim 1$ / Size of Source of Goldstone Boson

$\sim 400 - 500$ MeV

IF Pion Compton wavelength is smaller than source…..

($m_\pi \geq 0.4 - 0.5$ GeV ; $m_q \geq 50$-60 MeV)

ALL hadron properties are smooth, slowly varying (with m_q)

and Constituent Quark like !

(Pion loops suppressed like $(\Lambda / m_\pi)^n$)

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WHERE EXPANSION FAILS: NEW, EFFECTIVE DEGREE OF FREEDOM TAKES OVER

Model Independent Non-Analytic Behaviour

When Pion Compton wavelength larger than pion source:

(i.e. $m_\pi < 0.4 - 0.5$ GeV : $m_q \leq 50$ MeV)

All hadron properties have rapid, model independent,

non-analytic behaviour as function of m_q :

m_H (Full QCD) $\sim m_q^{3/2}$; m_H (QQCD) $\sim m_q^{1/2}$;

$\langle r^2 \rangle_{ch} \sim \ln m_q$; $\mu_H \sim m_q^{1/2}$; $\langle r^2 \rangle_{mag} \sim 1 / m_q^{1/2}$

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Acceptable Extrapolation Procedure

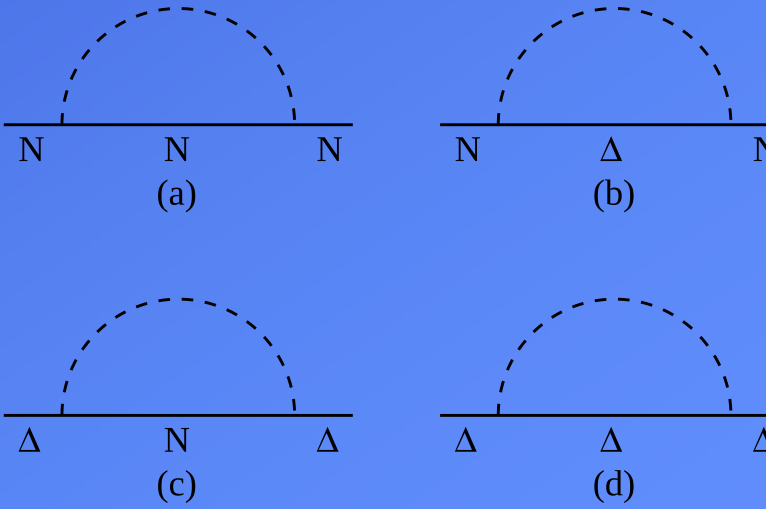
1. Must respect non-analytic behaviour of χ P T in region $m_\pi < 400 - 500$ MeV ($m_q \leq 50$ MeV)
   ………..with correct coefficients!

2. Must suppress chiral behaviour as inverse power of m_π in region $m_\pi > 400 - 500$ MeV

Extrapolation of Masses

At "large m_π" preserve observed linear (constituent-quark-<u>like</u>) behaviour: $M_H \sim m_\pi^2$

As $m_\pi \sim 0$: ensure LNA & NLNA behaviour:



(BUT must die as $(\Lambda / m_\pi)^2$ for $m_\pi > \Lambda$)

Hence use:

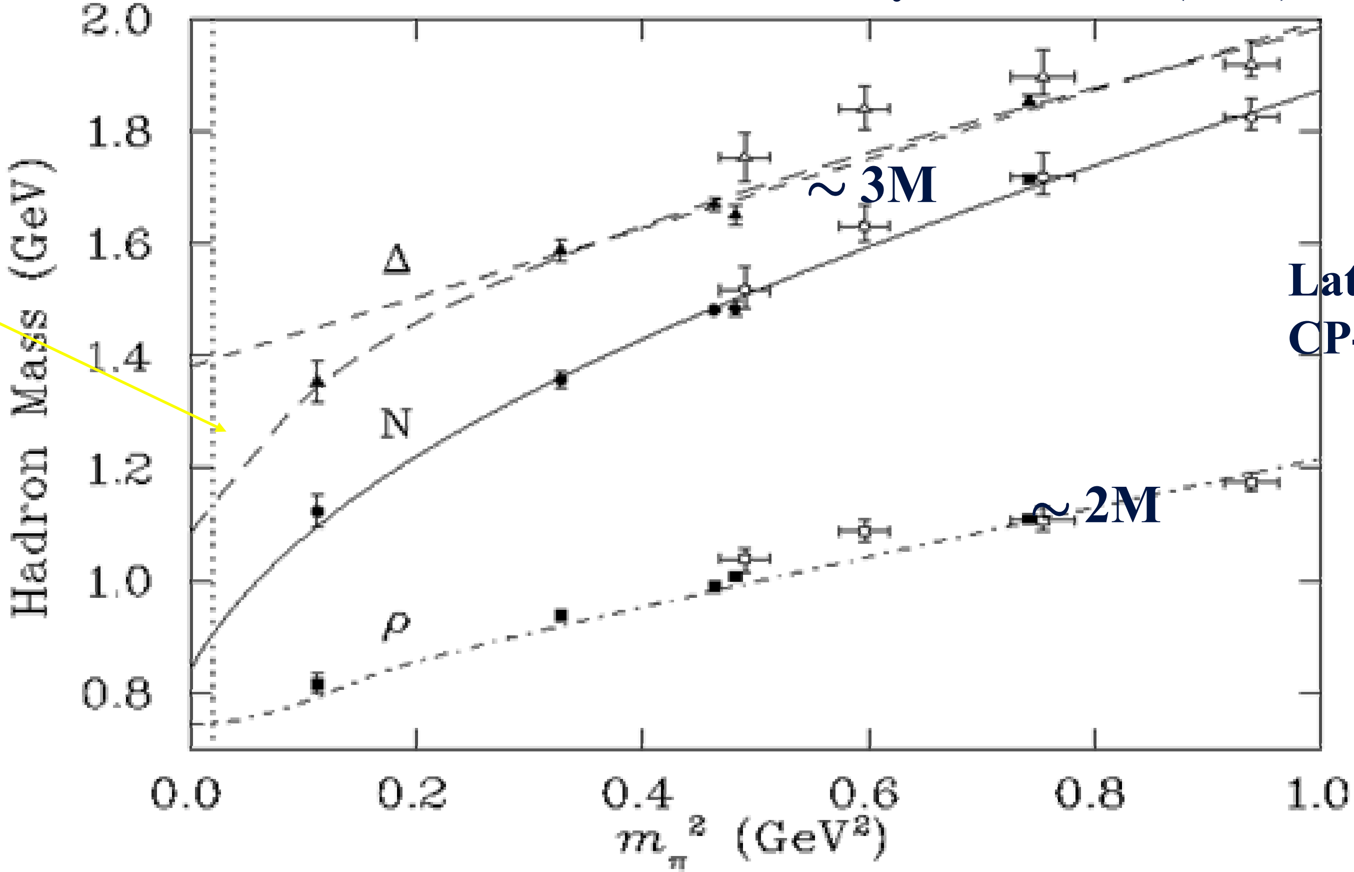
$$M_H = a_0 + a_2 m_\pi^2 + \sigma_{LNA}(m_\pi, \Lambda) + \sigma_{NLNA}(m_\pi, \Lambda)$$

- **Evaluate self-energies with form factor , "finite range regulator", FRR, with $\Lambda \sim$ 1/Size of Hadron**

Behaviour of Hadron Masses with m_π

From: Leinweber *et al.*, Phys. Rev., D61 (2000) 074502

Point of inflection at opening of N π channel



Lattice data from CP-PACS & UKQCD

BUT how <u>model dependent</u> is the extrapolation to the physical point?

Evaluation of Self-Energies: Finite Range Regulator

- Infrared behaviour is NOT affected by UV cut-off on Goldstone boson loops.

e.g. for $N \to N \pi \to N$ use sharp cut-off (SC), $\theta(\Lambda - k)$:

$$\sigma(\Lambda, m_\pi) = -3 g_A^2 / (16 \pi^2 f_\pi^2) \left[m_\pi^3 \arctan(\Lambda / m_\pi) + \Lambda^3/3 - \Lambda m_\pi^2 \right]$$

(/// to "Long Distance Regularization": Donoghue *et al*., Phys Rev D59 (1999) 036002)

- Coefficient of LNA term $\to \pi/2$ as $m_\pi \to 0$

- But coefficient gets rapidly smaller for $m_\pi \sim \Lambda$
  (hence hard to determine by naïve fitting procedure
  …….. <u>explains "γ problem"</u> discussed earlier)

- $\sigma \sim 1/m_\pi^2$ for $m_\pi > \Lambda$

A GAME : HOW MODEL DEPENDENT?

- Quantitative comparison of 6 different regulator schemes

- Ask over what range in m_π^2 is there consistency at 1% level?

- Use new CP-PACS data from >1 year dedicated running:
    - Iwasaki gluon action
    - perturbatively improved clover fermions
    - use data on two finest lattices (largest β): a = 0.09 & 0.13 fm
    - follow UKQCD (Phys. Rev. D60 ('99) 034507) in setting physical scale with Sommer scale, r_0 = 0.5 fm (because static quark potential insensitive to χ'al physics)
    - restricted to $m_\pi^2 > 0.3$ GeV^2

- Also use measured nucleon mass in study of model dependence

Young, Leinweber & Thomas, hep-lat/0212031

Regularization Schemes

{Fit parameters shown in black

DR: Naïve dimensional regularization

$$M_N = \underline{c_0 + c_2\, m_\pi^2} + c_{LNA}\, m_\pi^3 + \underline{c_4\, m_\pi^4} + c_{NLNA}\, m_\pi^4 \ln m_\pi + \underline{c_6\, m_\pi^6} + \ldots..$$

BP: Improved dim-reg.... for Δ-π loop use correct <u>branch point</u> at $m_\pi = \Delta$

$$m_\pi^4 \ln m_\pi \rightarrow (\Delta^2 - m_\pi^2)^{3/2} \ln(\Delta + m_\pi - [\Delta^2 - m_\pi^2]^{1/2}) - \Delta/2\, (2\Delta^2 - 3m_\pi^2) \ln m_\pi$$

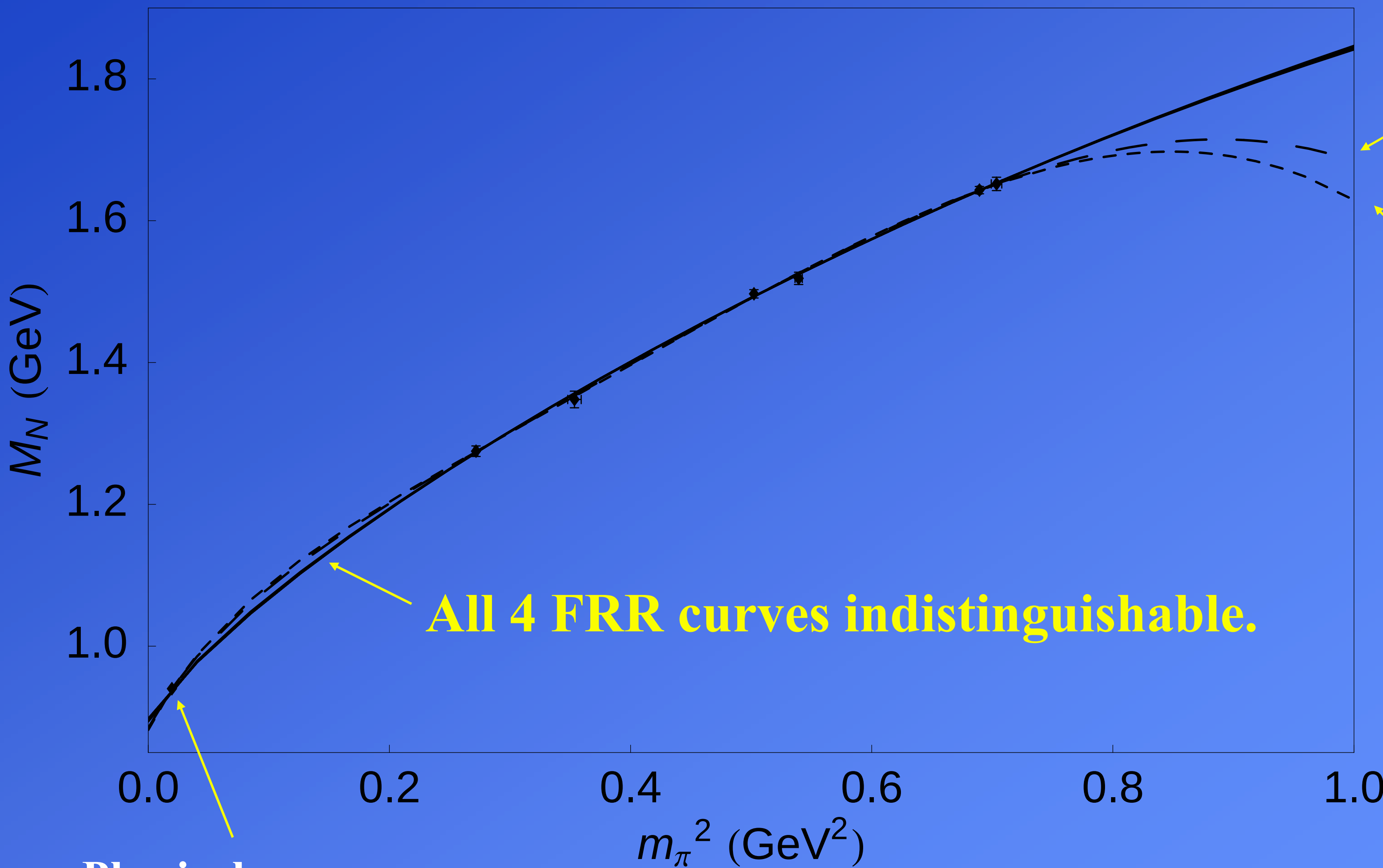
- for $m_\pi < \Delta$; ln → arctan for $m_\pi > \Delta$

FRR: <u>F</u>inite <u>R</u>ange <u>R</u>egulator – use MONopole, DIPole, GAUssian and Sharp Cutoff in π-N and π-Δ self-energy integrals:

$$M_N = \underline{a_0 + a_2\, m_\pi^2 + a_4\, m_\pi^4} + \sigma_{LNA}(m_\pi, \Lambda) + \sigma_{NLNA}(m_\pi, \Lambda)$$

? Is this more convergent with good choice of FRR ?

Constraint Curves



BP

DR

Dim-Reg curves wrong above last data point

Physical nucleon mass

CP-PACS data

Best fit (bare) parameters

Regulator	a_0	a_2	a_4	a_6	Λ (GeV)
DR	0.882	3.82	6.65	-4.24	-
BP	0.825	4.37	9.72	-2.77	-
SC	1.03	1.12	-0.292	-	0.418
MON	1.56	0.88	-0.204	-	0.496
DIP	1.20	0.97	-0.229	-	0.785
GAU	1.12	1.01	-0.247	-	0.616

N.B. Improved convergence of residual series is dramatic!

Finite Renormalization ⇒ Physical Low Energy Constants

- True low energy constants, $c_{0, 2, 4, 6}$ should not depend on either regularization or renormalization procedure

- To check we need formal expansion of σ_{LNA} and σ_{NLNA} * :

$$\sigma_{LNA}(m_\pi,\Lambda) = b_0 + b_2\, m_\pi^2 + c_{LNA}\, m_\pi^3 + b_4\, m_\pi^4 + \qquad + \ldots$$

$$\sigma_{NLNA}(m_\pi,\Lambda) = d_0 + d_2\, m_\pi^2 + \qquad d_4\, m_\pi^4 + c_{NLNA} m_\pi^4 \ln m_\pi^4 + \ldots$$

- Hence: $c_i \equiv a_i + b_i + d_i$ and even though each of a_i , b_i and d_i is scheme dependent, <u>c_i should be scheme independent</u>!

* Complete algebraic expressions for d_i , b_i given in hep-lat/0212031: for all regulators

Low Energy Parameters are Model Independent for FRR

Regulator	c_0	c_2	c_4
DR	0.882	3.82	6.65
BP	0.885	3.64	8.50
SC	0.894	3.09	13.5
MON	0.898	2.80	23.6
DIP	0.897	2.84	22.0
GAU	0.897	2.87	20.7

- Inaccurate higher order coefficients with DR & BP
- SC not as good as other FRR
- Note consistency of low energy parameters!

- Note terrible convergence properties of conventional series…..
  ($c_6 \sim -60$)

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Unbiased Assessment of Schemes

- Take each fit, in turn, as the "exact" data

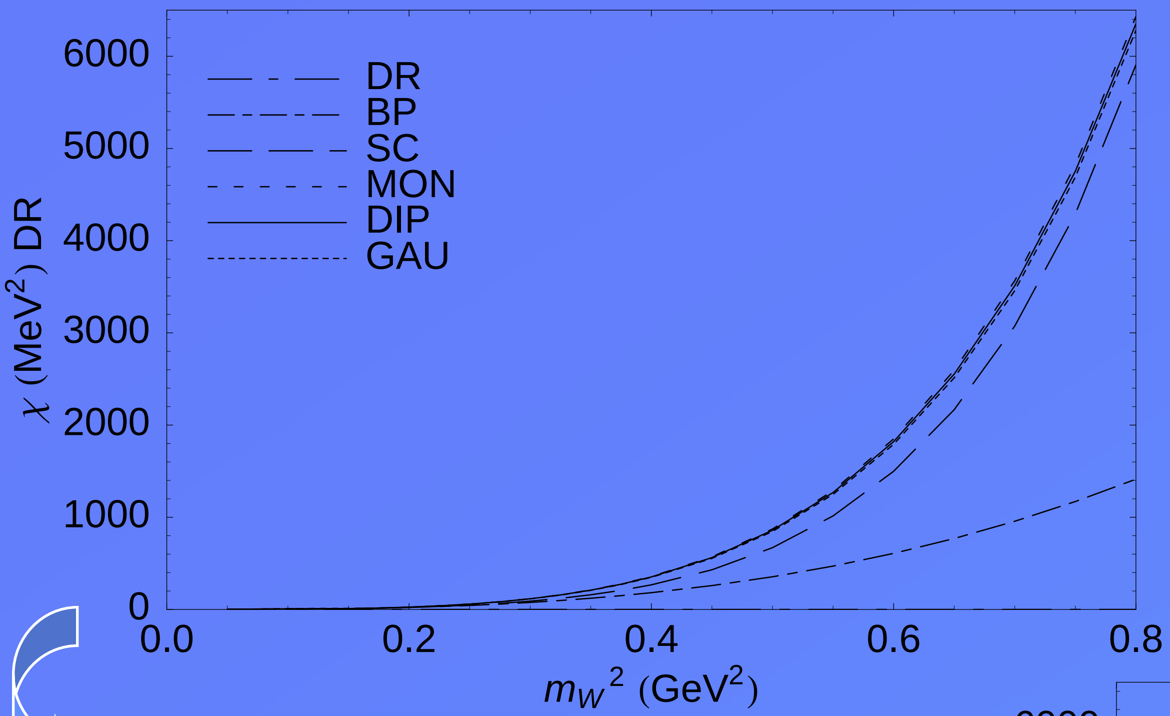
- Then see how well each of the other regulators can reproduce this "data"

- Define "goodness of fit", χ , as area between exact "data" and best fit for given regulator over <u>window</u> $m_\pi^2\ \varepsilon\ (0, m_W^2)$

e.g. with $m_W^2 = 0.5\ \text{GeV}^2$, $\chi = 700\ \text{MeV}^2$

$\Longrightarrow$ on average fit is within 1 MeV of "data"

Fits - 1

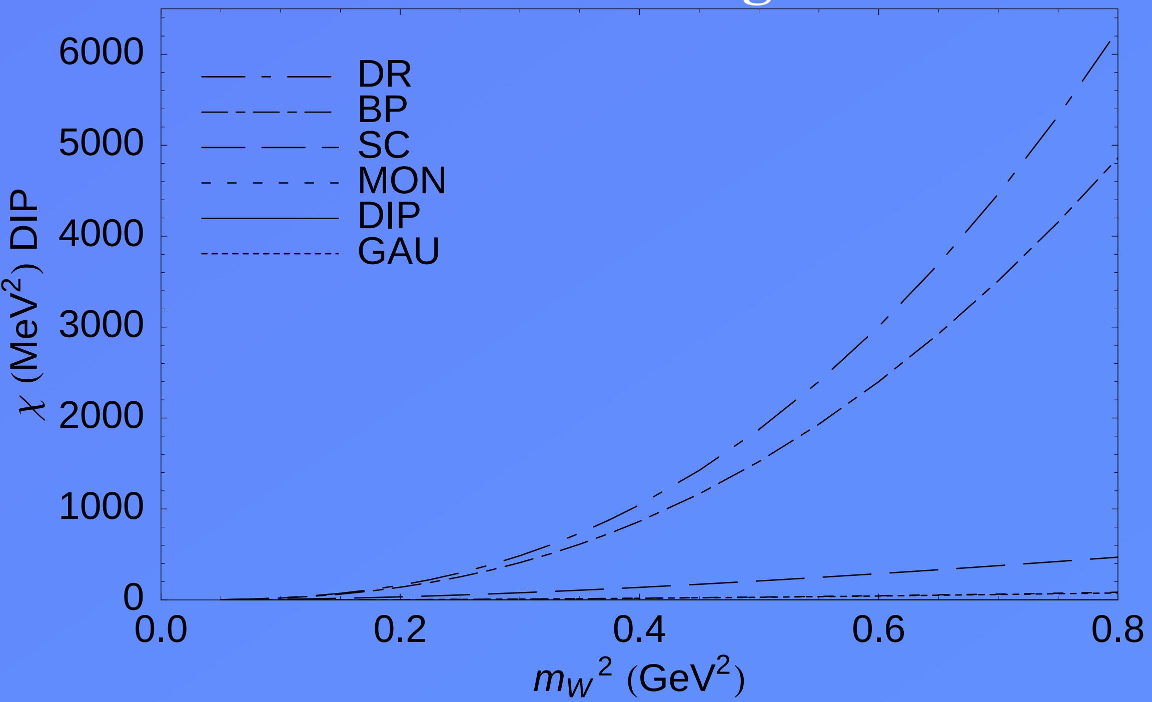


DIP constraint curve

- DR & BP (dim-reg) fail above 0.3 GeV²
- <u>all FRR</u> work over entire range!

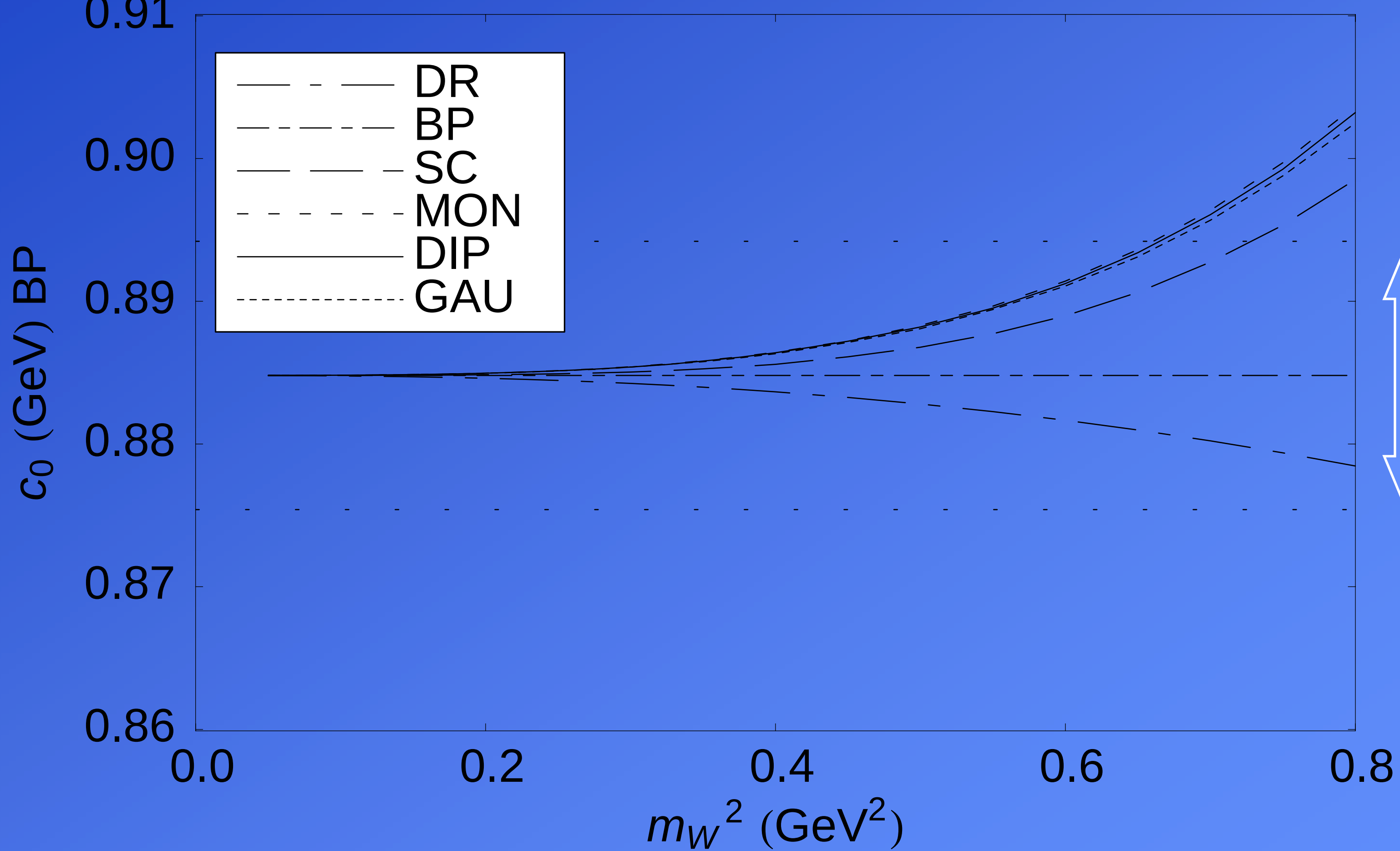
For DR constraint curve

- all regulators fit well up to 0.4 GeV²



Recovery of Chiral Coefficients

Recovery of c_0 from fit to BP "data" over window $(0, m_W^2)$

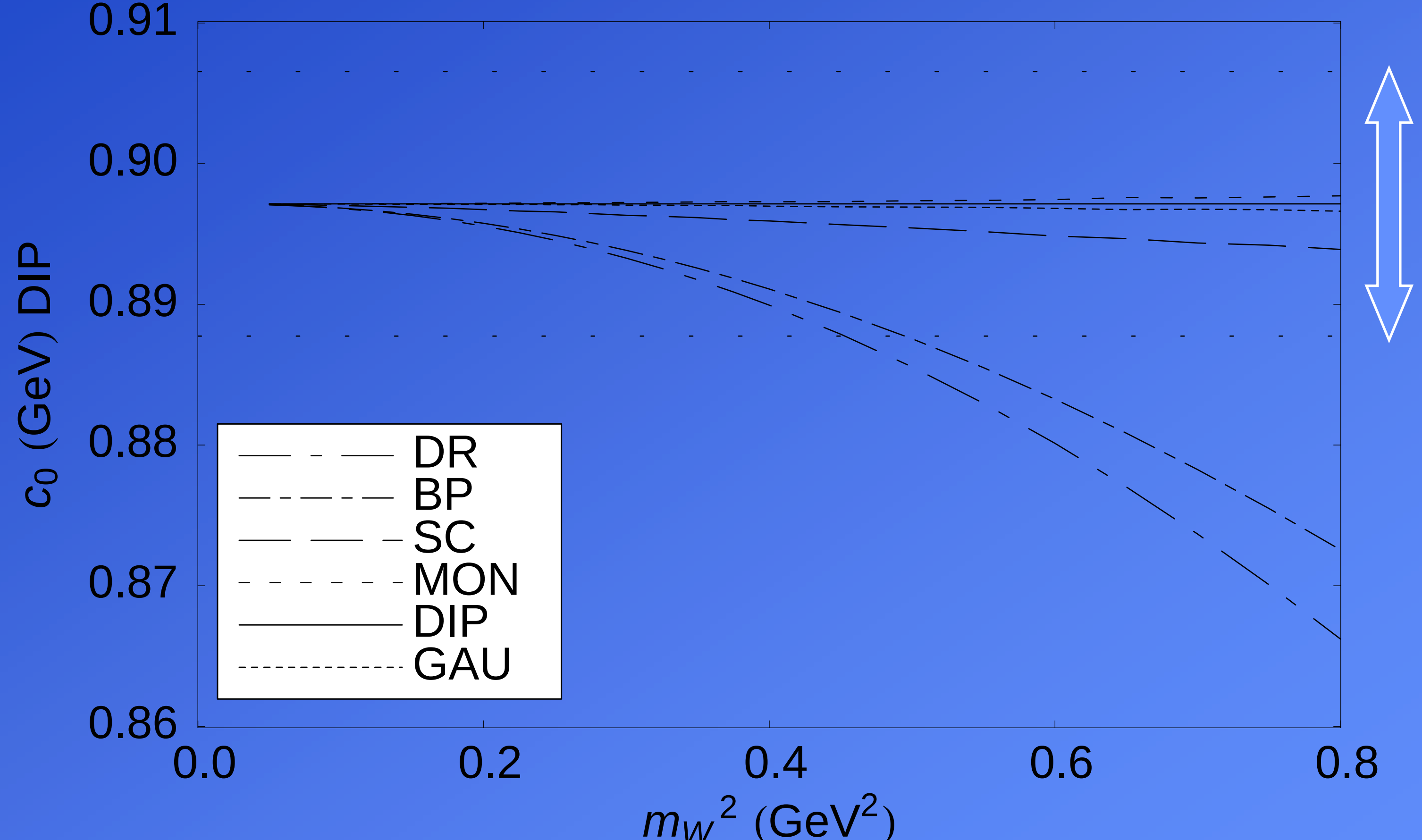


Region leading to 1% error in M_N

All regulators work up to 0.7 GeV^2 – where BP "data" wrong anyway

Recovery of Chiral Coefficients - 2

Recovery of c_0 from fit to DIP "data" over window $(0, m_W^2)$

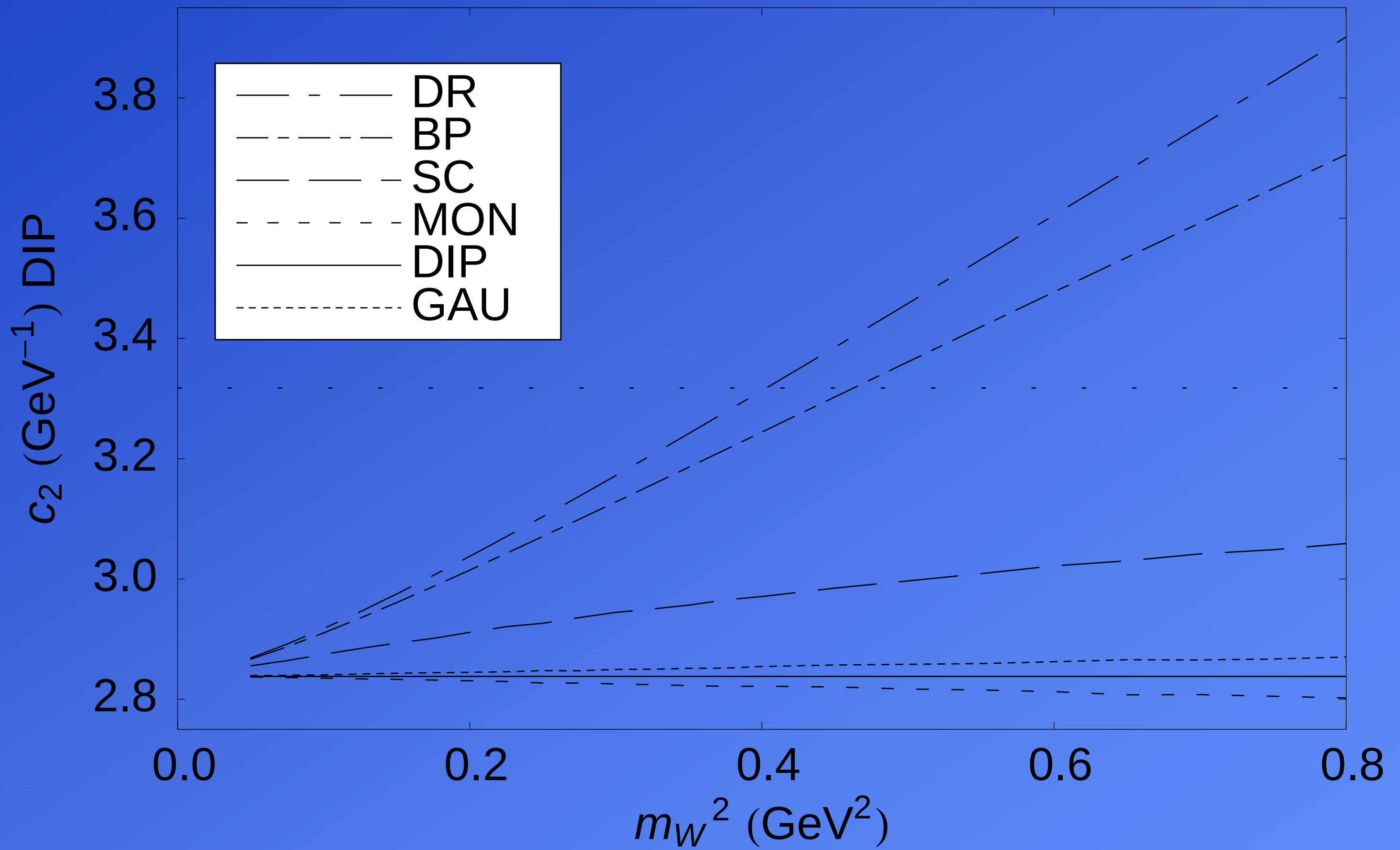


Region leading to 1% error in M_N

DR & BP fail above 0.4 GeV^2 : all FRR work over entire range!

Recovery of Chiral Coefficients - 3

Recovery of c_2 from fit to DIP "data" over window $(0,m_W^2)$



Region leading to 1% error in M_N

DR & BP fail above 0.4 GeV^2 : all FRR work over entire range!

Summary : Extraction of Low Energy Constants

C_0:

- For "data" based on DR &BP:
  all schemes give c_0 to <1% for window up to 0.7 GeV^2

- BUT dim-reg schemes FAIL at 1% level for FRR
  once window exceeds 0.5 GeV^2

C_2:

- Similar story for c_2:

  - for "data" based on DR & BP all schemes good enough to give M_N^{phys} to 1% for window up to 0.7 GeV^2

  - for DIP "data" DR & BP fail above 0.4 GeV^2 while all other FRR's work over entire range (and all except SC give c_2 itself to 1% !)

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Application to lattice extrapolation problem

How low do we need to go?

- Use DIP curve as the “lattice data”
- Take points at 0.05 GeV^2 intervals from 0.8 GeV^2 down to $m_L{}^2$
- Then, for each regulator, fit 4 free parameters
- Ask how low must $m_L{}^2$ be to determine $c_{0,2,4}$?

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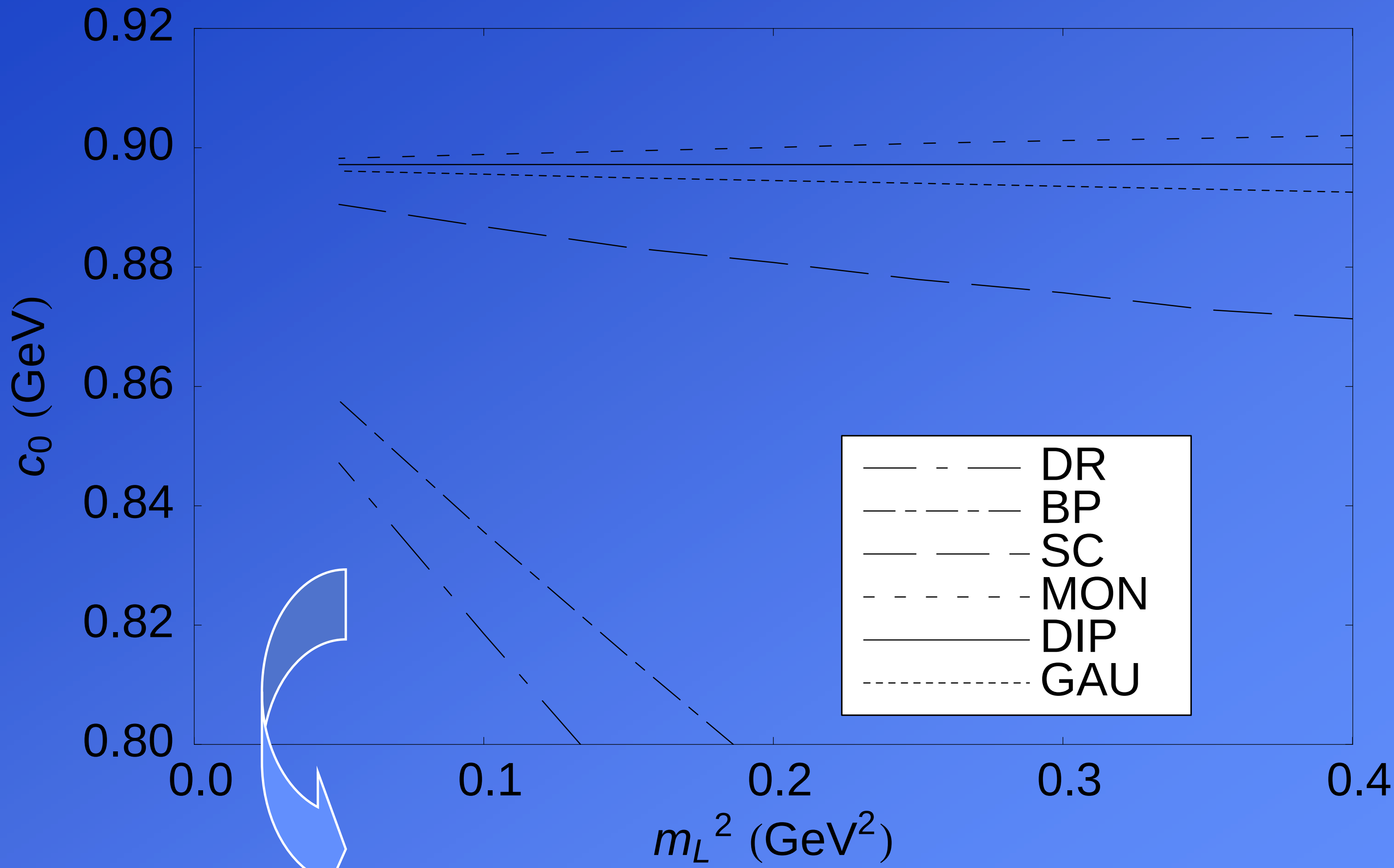
Finite Range Regulators Fix c_0 Accurately

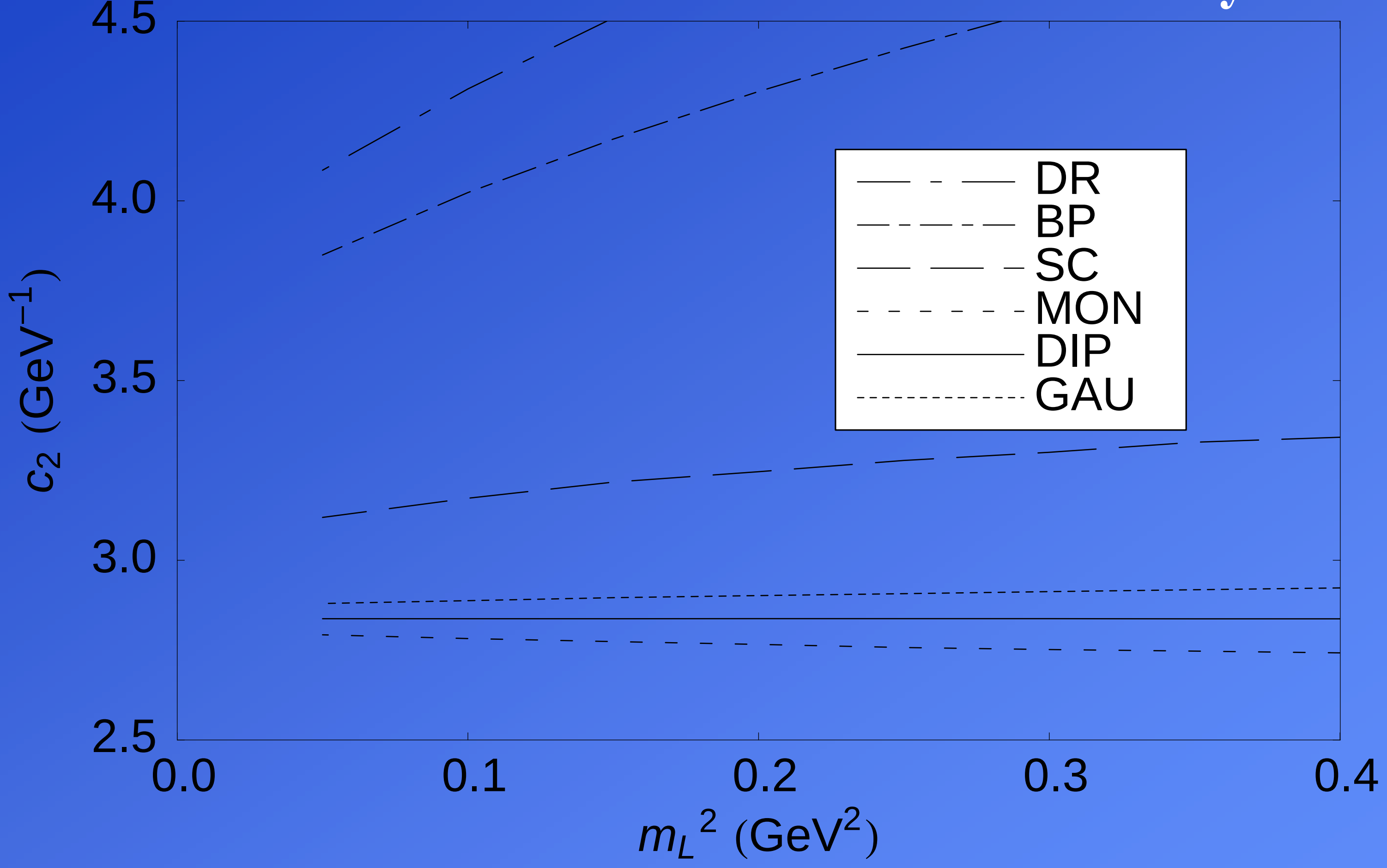


- All FRR work at 2% level; omitting SC they work to better than 1% for m_L^2 below 0.4 GeV^2 !

Dimensional regularization fails at 10% level !

FRR Permit Accurate Determination of c_2 Too

• DR & BP fail miserably

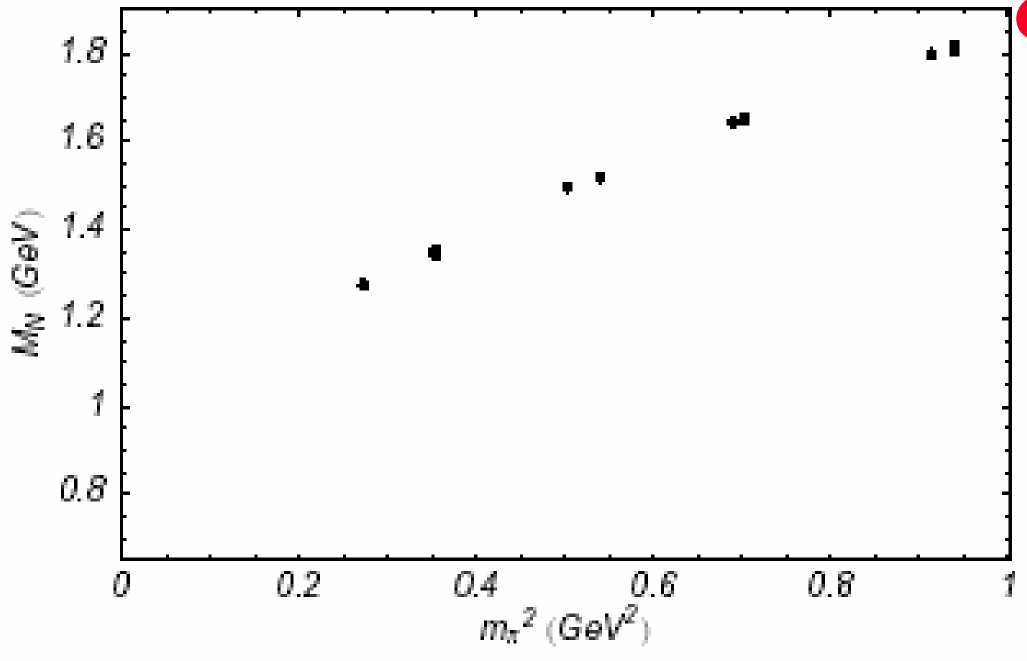


• SC not so good

• MON & GAU yield 3% accuracy even for $m_L^2 = 0.4$

Analysis of the Best Available Data

- Which curve is model independent?



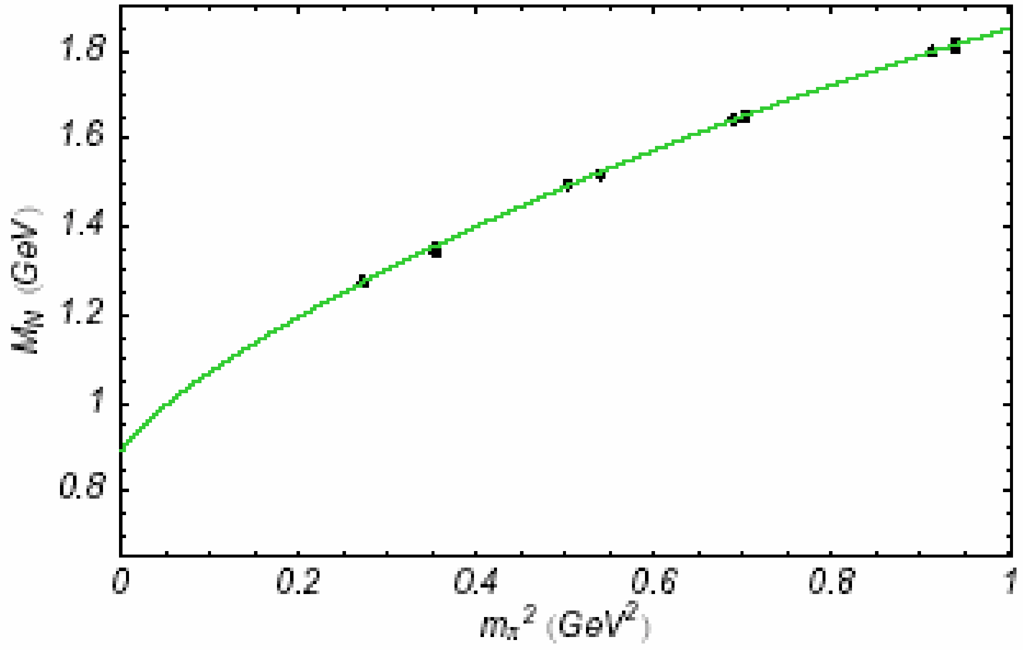
Data :
CP-PACS
Phys Rev D65 (2002)

Accuracy better than 1% !

Sufficient to determine 4 parameters ……..

Analysis of the Best Available Data

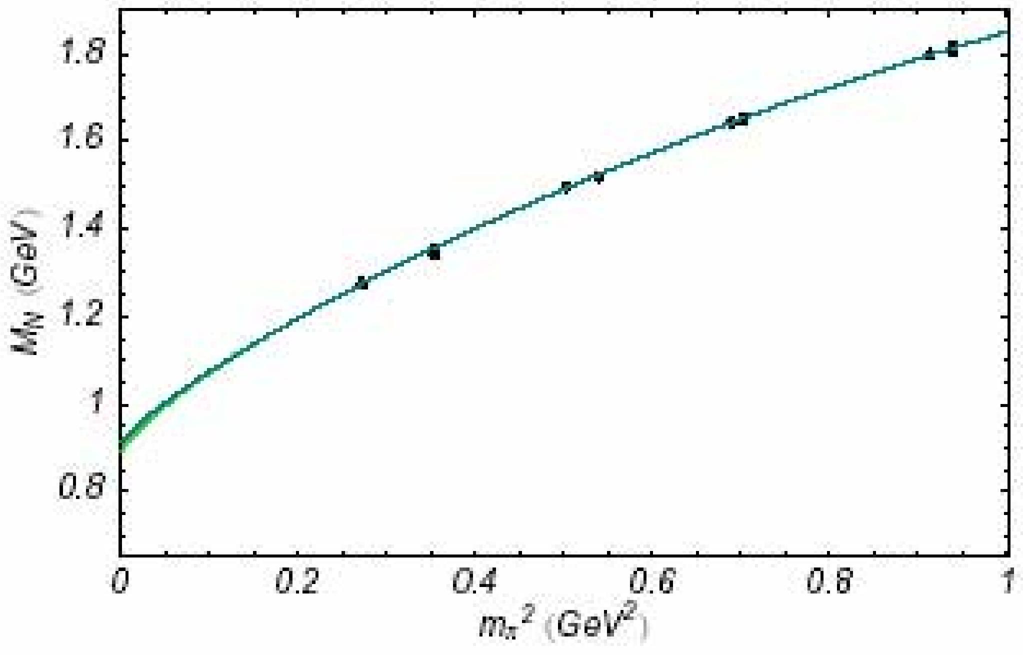
- Which curve is model independent?



- A: Sharp cut-off

Analysis of the Best Available Data

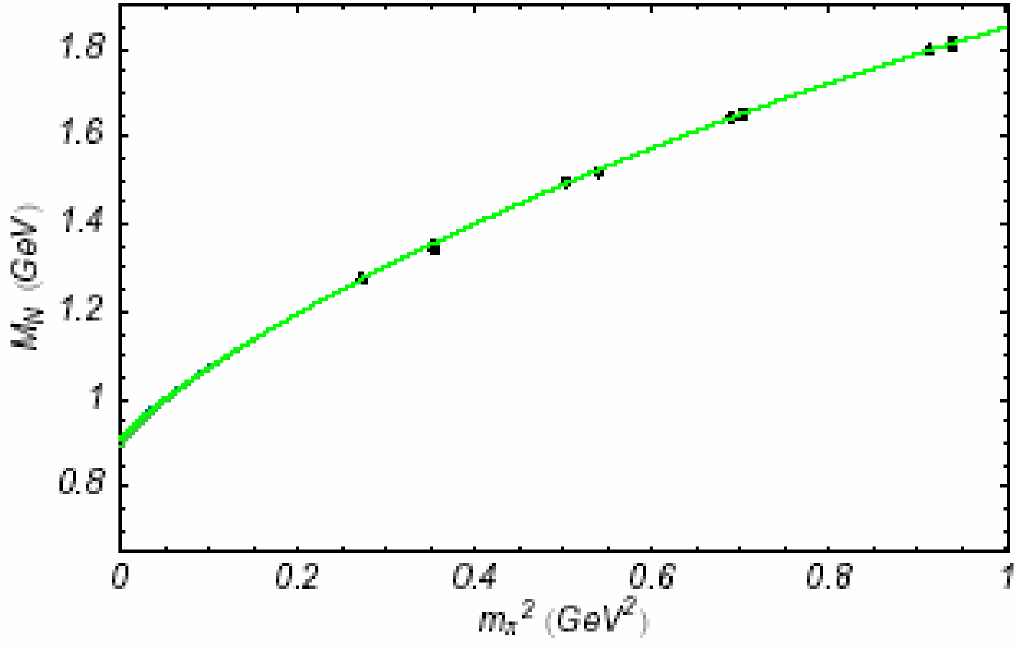
- Which curve is model independent?



- B: Monopole

Analysis of the Best Available Data

- Which curve is model independent?



- C: Dipole

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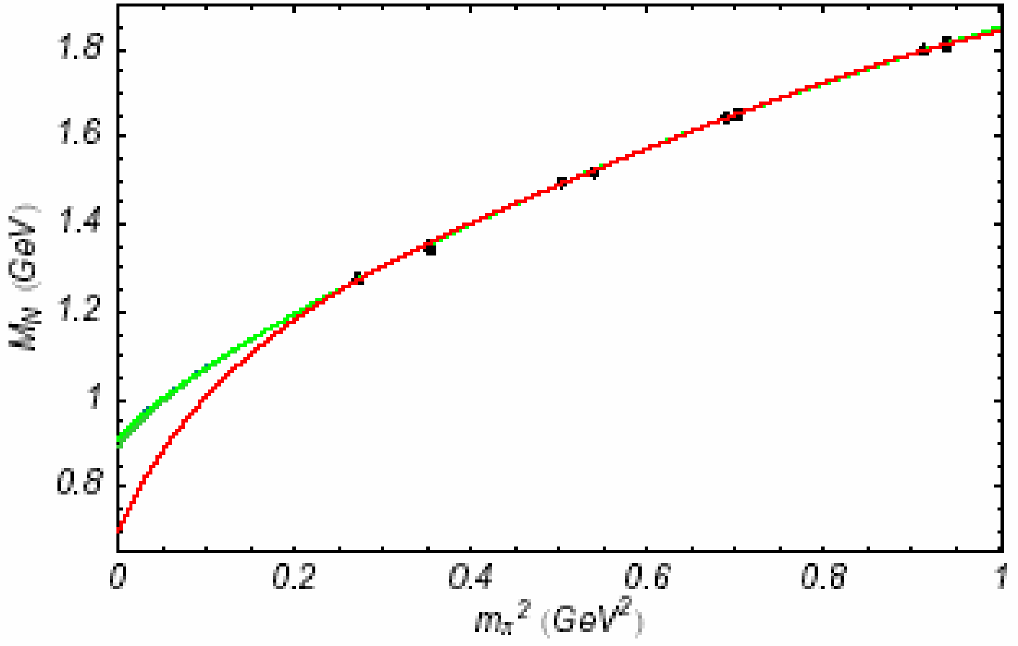
Analysis of the Best Available Data

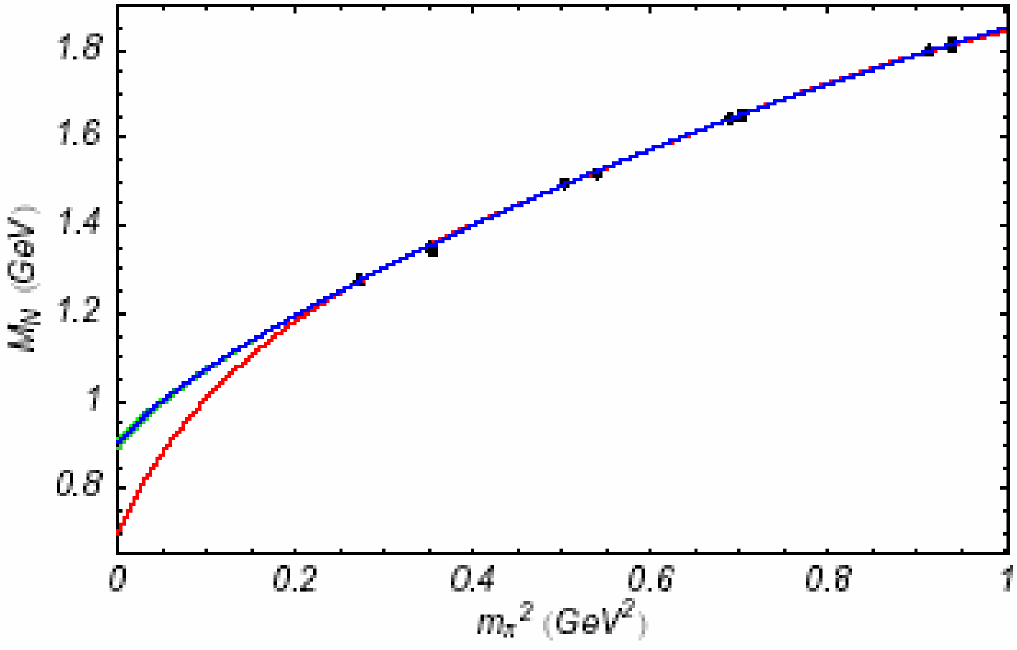
- Which curve is model independent?



- D: Dimensional Regularisation

Analysis of the Best Available Data

- Which curve is model independent?



- E: Gaussian

Scheme Dependent Fit Parameters

(Young, Leinweber & Thomas : hep-lat/0302020)

Regulator	a_0	a_2	a_4	a_6	Λ
D-R	0.700	5.10	3.87	-2.35	-
Sharp	1.06	1.06	- 0.249	-	0.440
Monopole	1.38	0.950	- 0.217	-	0.443
Dipole	1.15	0.998	- 0.227	-	0.732
Gaussian	1.11	1.02	- 0.234	-	0.593

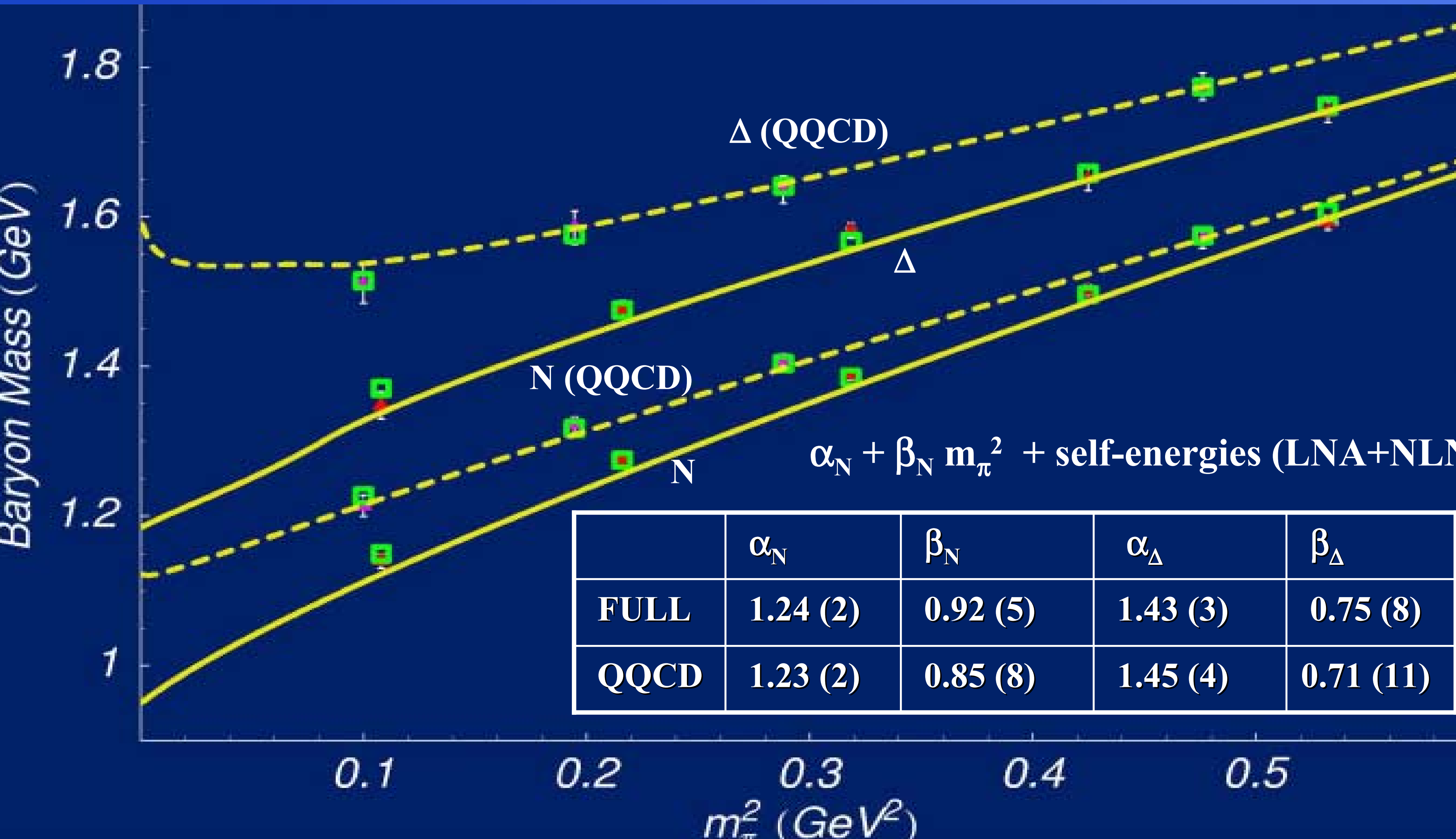
Output: Chiral Coefficients, Nucleon Mass and Sigma Commutator

(Young, Leinweber & Thomas : hep-lat/0302020)

Regulator	c_0	c_2	c_4	m_N (MeV)	σ_N (MeV)
DR	0.700	5.10	3.87	782	72.8
Sharp	0.892	3.16	12.9	939	40.9
Monopole	0.914	2.63	26.1	953	35.1
Dipole	0.910	2.72	23.4	951	36.1
Gaussian	0.906	2.79	21.4	948	36.9

CAUTION: Stat. error large ($\sim$ 0.07 GeV) + NO finite V corrections applied

- Lattice data (from MILC Collaboration) : red triangles
- Green boxes: fit evaluating σ's on <u>same finite grid as lattice</u>
- Lines are exact, continuum results



$\alpha_N + \beta_N m_\pi^2$ + self-energies (LNA+NLNA)

	α_N	β_N	α_Δ	β_Δ
FULL	1.24 (2)	0.92 (5)	1.43 (3)	0.75 (8)
QQCD	1.23 (2)	0.85 (8)	1.45 (4)	0.71 (11)

From: Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Hybrid Mesons

e.g. 1^{-+} π_1 meson :

BNL states somewhat below lattice estimates ($\sim$ 1.9 GeV)

Calculation of chiral correction to mass may be model dependent ?

BUT

1. Coupling to (non-exotic) decay channels not so important

2. Main corrections come from $\pi_1 \to \pi$ plus (isoscalar) exotic
   - estimate coupling using PCAC (c.f. $\rho \to \omega \pi$)
   - $\sigma_{LNA} \sim - 3\, m_\pi^3$

Effect should lower mass by ~ 100 MeV

Szczepaniak & Thomas, Phys Lett B526 (2002) 72

Decay Channels Explored

Decay channel	wave	PSS		IKP	
		$g_L^2(k(m_X))/4\pi$	Γ_L[MeV]	$g_L^2(k(m_X))/4\pi$	Γ_L[MeV]
$\eta\pi$	P	9.4×10^{-3}	$\mathcal{O}(10^{-2})$	7.3×10^{-3}	$\mathcal{O}(10^{-2})$
$\eta'\pi$	P	9.4×10^{-3}	$\mathcal{O}(10^{-2})$	7.3×10^{-3}	$\mathcal{O}(10^{-2})$
$\rho\pi$	P	8.32	$\mathcal{O}(10)$	5.95	$\mathcal{O}(10)$
$f_2(1270)\pi$	D	5.3	$\mathcal{O}(10^{-2})$	$\emptyset$	$\emptyset$
$f_1(1285)\pi$	S	1.6	$\mathcal{O}(10)$	1.9	$\mathcal{O}(10)$
	D	2.3	$\mathcal{O}(10^{-2})$	309.	$\mathcal{O}(1)$
$b_1(1235)\pi$	S	6.1	$\mathcal{O}(100)$	6.54	$\mathcal{O}(100)$
	D	37.9	$\mathcal{O}(1)$	324.	$\mathcal{O}(10)$
$\eta_u(1295)\pi$	P	37.6	$\mathcal{O}(10)$	21.25	$\mathcal{O}(10)$
$\rho(1450)\pi$	P	30.7	$\mathcal{O}(10^{-2})$	15.8	$\mathcal{O}(10^{-2})$

PSS: Page, Swanson & Szczepaniak, Phys Rev D59 (1999) 034016

IKP: Isgur, Kokoski, Paton, PRL 54 (1985) 869

Parton Distribution Function of the Pion

Leading twist operators:

$$O_q^{\alpha,\beta,\gamma\ldots\ldots} = i^n \, \bar{\psi}_q \, \gamma^\alpha D^\beta D^\gamma \ldots \psi_q \text{ - traces}$$

with <u>n+1 indices</u>, symmetrized on α, β, ……

Their hadronic matrix elements:

$$<\pi(p)| \, O_q^{\alpha,\beta,\gamma\ldots} \, |\pi(p)> = <x^n>_q \, p^\alpha \, p^\beta \, p^\gamma \ldots$$

define "moments" of PDF's:

$$<x^n> \equiv \int_{-1}^{+1} dx \, x^n \, q(x)$$

and under crossing: $q(x) \rightarrow - \bar{q}(-x)$

Leading non-analytic behaviour

Hence: $\langle x^n \rangle = \int_0^{+1} dx\, x^n \,(q(x) - (-)^n \bar{q}(x))$

and therefore n even $\Rightarrow$ $q - \bar{q}$: valence (non-singlet)

n odd $\Rightarrow$ $q + \bar{q}$: singlet

• singlet has NO chiral logarithm

• valence (Arndt & Savage: Nucl. Phys. A697 (2002) 429)

$$\langle x^n \rangle_q^{NS} = a_n \,[1 - (1 - \delta^{n0})/(4\pi f_\pi^2)^2 \; m_\pi^2 \ln m_\pi^2/(m_\pi^2 + \mu^2)]$$

How to fit to valence distribution ?

Have only two moments: n=0 is "trivial" plus n=2

IF we try to determine parameters of:

$$q_v(x) = A\, x^b\, (1 - x)^c$$

can clearly only find two $\Rightarrow$ $b \approx -0.9 + 0.2\, c$

(choose $\mu \in (0.4, 1.0)$ GeV – a little larger than for N)

e.g. <u>If</u> b = -0.5 (as Regge theory) c = 2 as in perturbative QCD!!

BUT would like to do better…..

Method II: use what we have learnt

Recall that at $m_\pi > 0.4$ GeV loops are suppressed

⇒ Treat ALL moments *in this mass region* as though valence

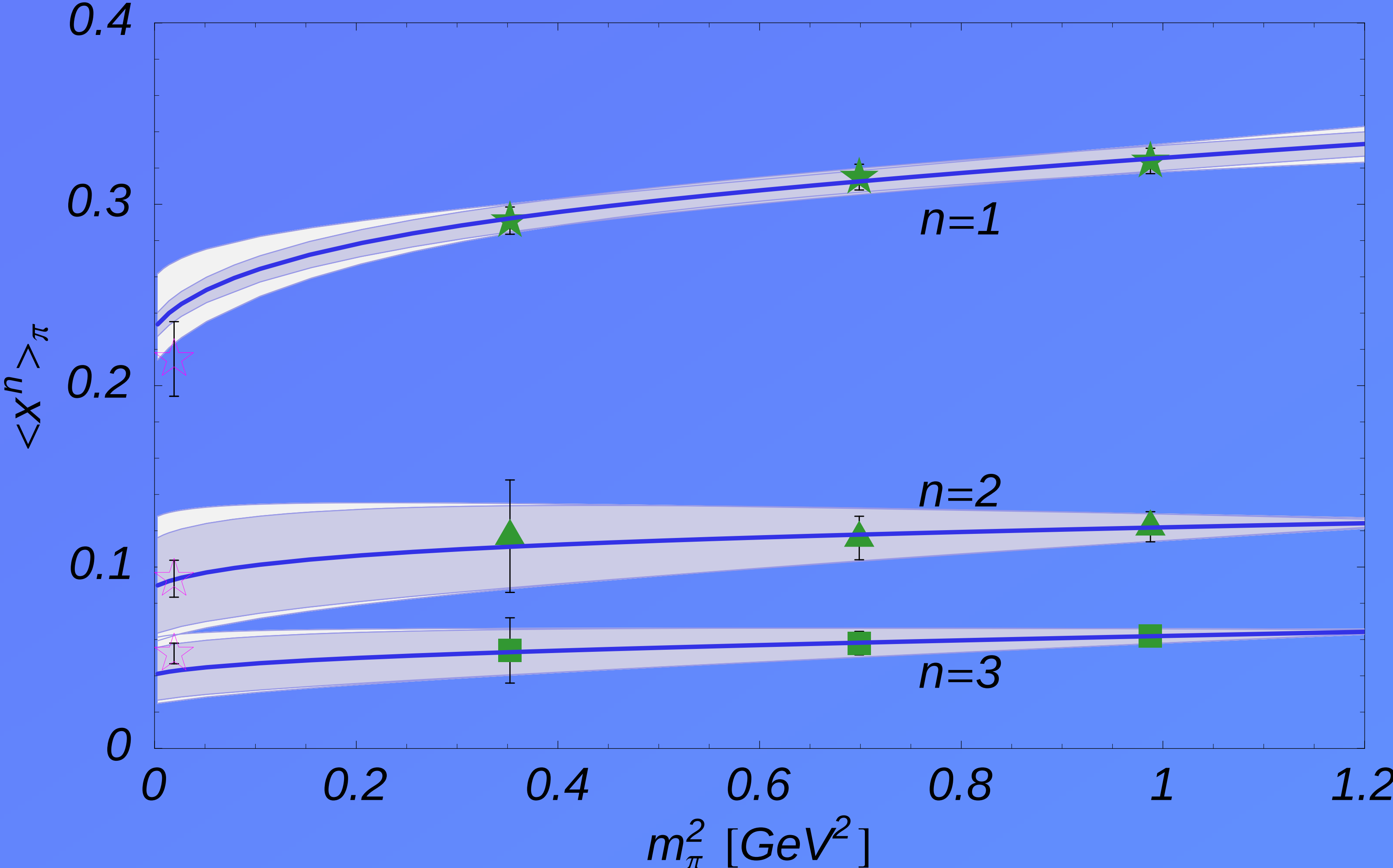
(i.e. sea is negligible)

Use linear fit with chiral logarithm (2 fit parameters)

	n=1	n=2	n=3
Chiral Extrapolation	0.24 (1)(2)	0.09(3)(1)	0.043(15)(3)
Experiment	0.21 (2)	0.09 (1)	0.052 (5)

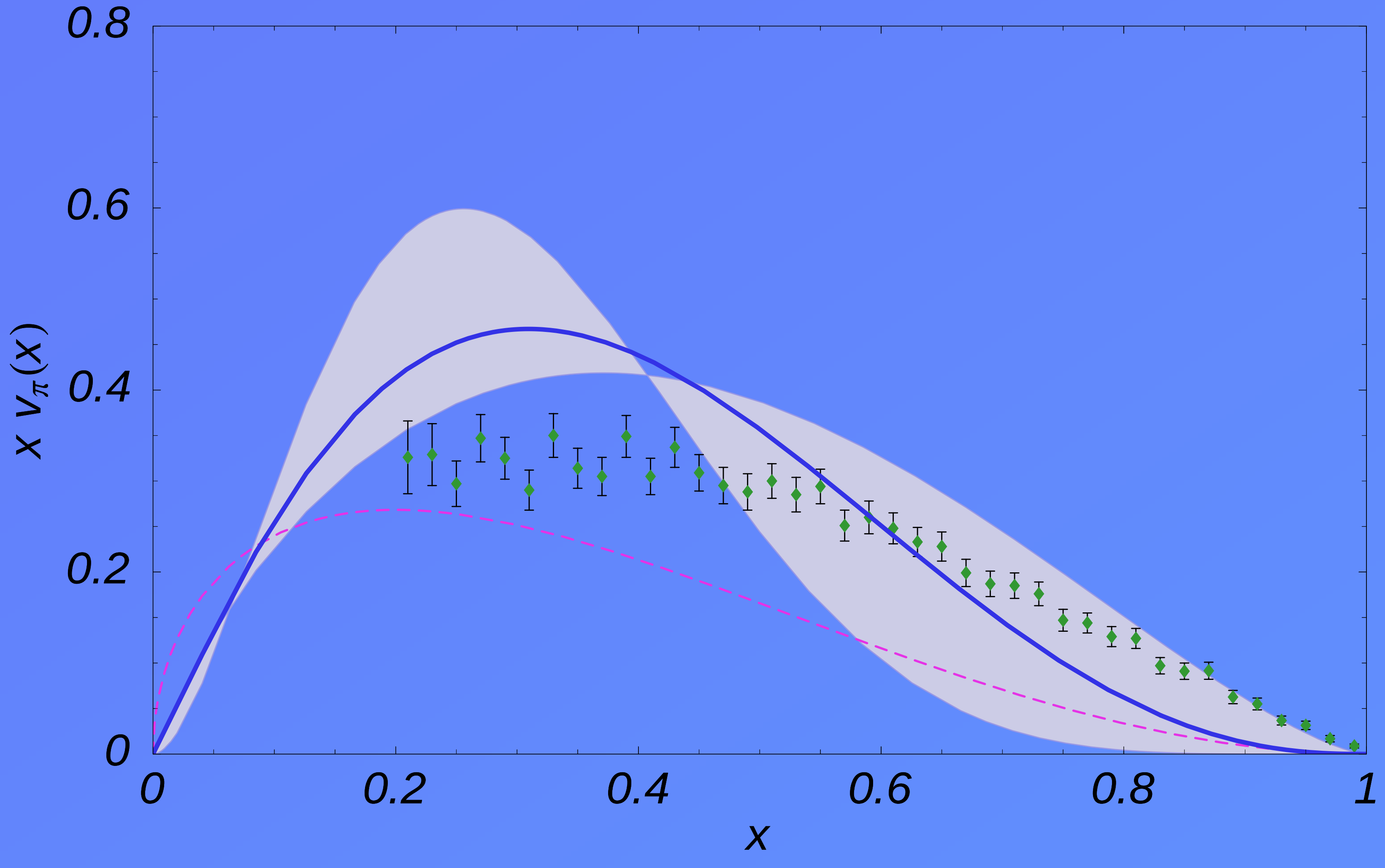
$$q_v(x) = 4.4\, x^{0.1\,(5)}\, (1 - x)^{2.5\,(1.5)}$$

Fit to lattice data treated as non-singlet



Data: QCDSF :Best *et al.*, Phys. Rev. D56 (1997) 2743

Valence distribution of pion: method II



Detmold, Melnitchouk & Thomas (hep-lat/0303015)

Method III: More phenomenological

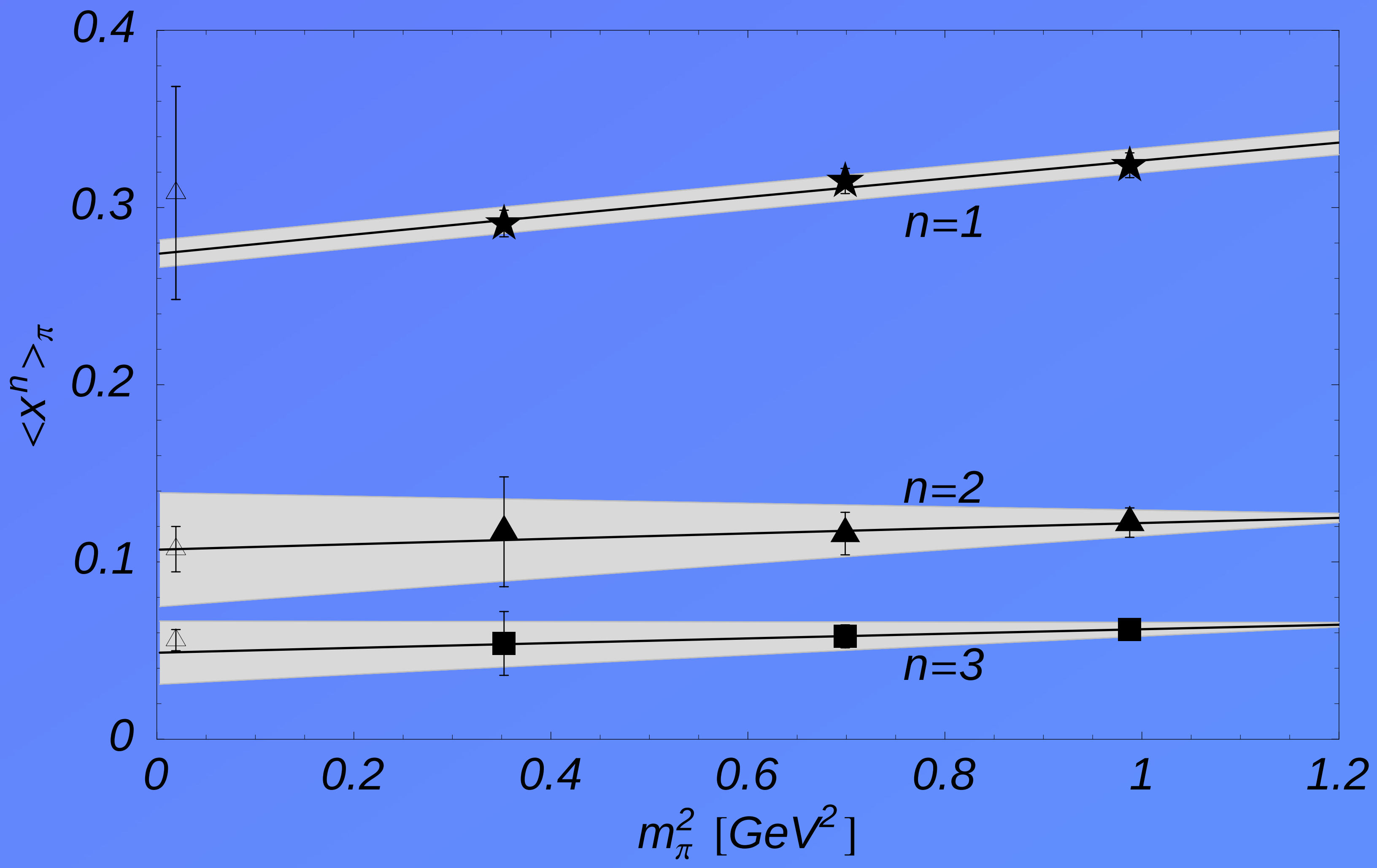
n-odd moments *are* singlet:
Extrapolate as purely linear function of m_π^2

• Then subtract phenomenological sea (SMRS or GRS)

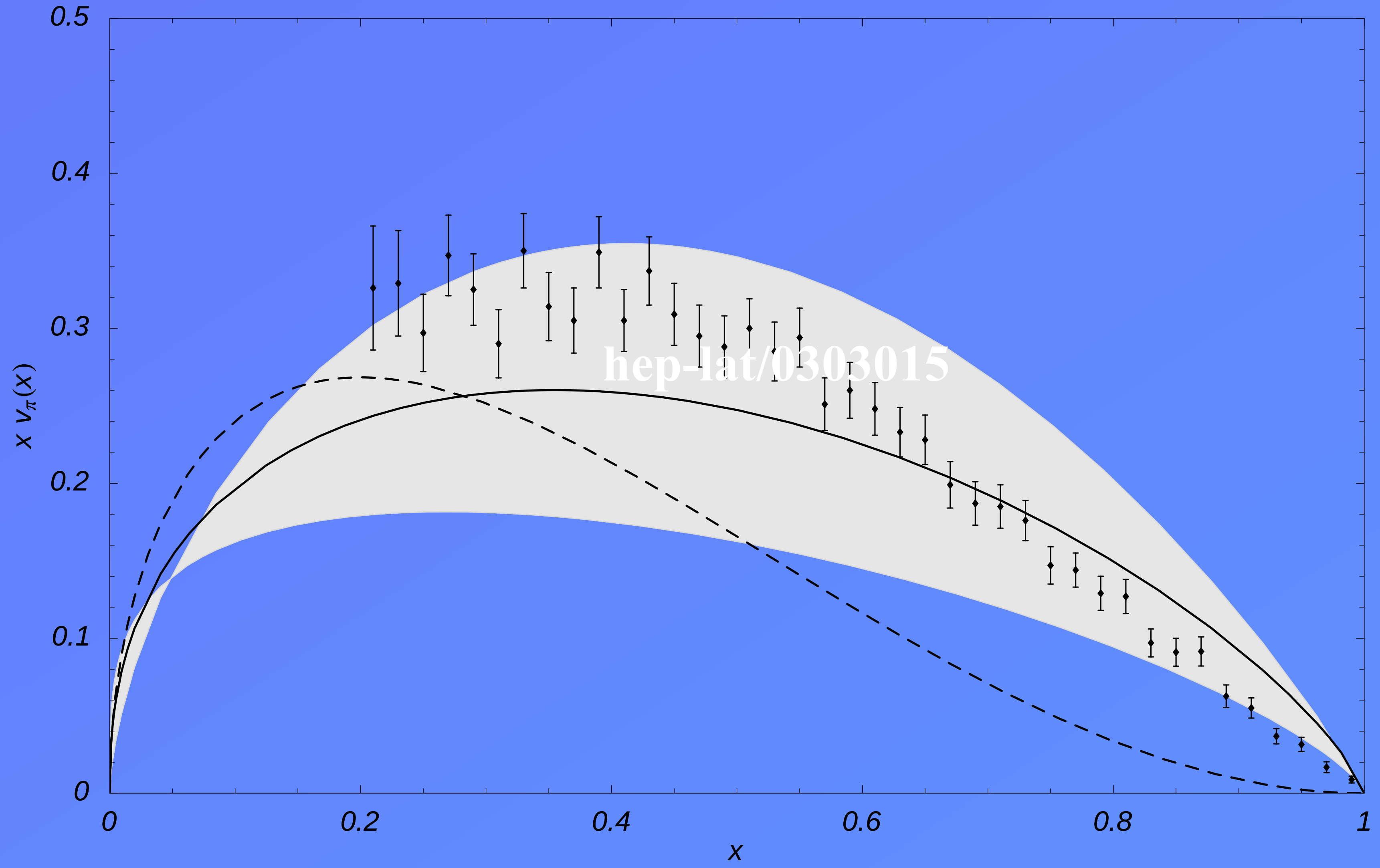
	n=1	n=2	n=3
Extrapolation	0.18 (6)	0.10 (3)	0.05 (2)
Experiment	0.21 (2)	0.09 (1)	0.052 (5)

$$q_v(x) = 0.6\, x^{-0.6 \pm 0.3} (1 - x)^{0.8 \pm 0.9}$$

Method III: Extrapolate as singlet



Method III: Valence reconstruction



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Summary : Structure of Pion

- Cannot yet decide for or against p-QCD prediction of large-x behaviour

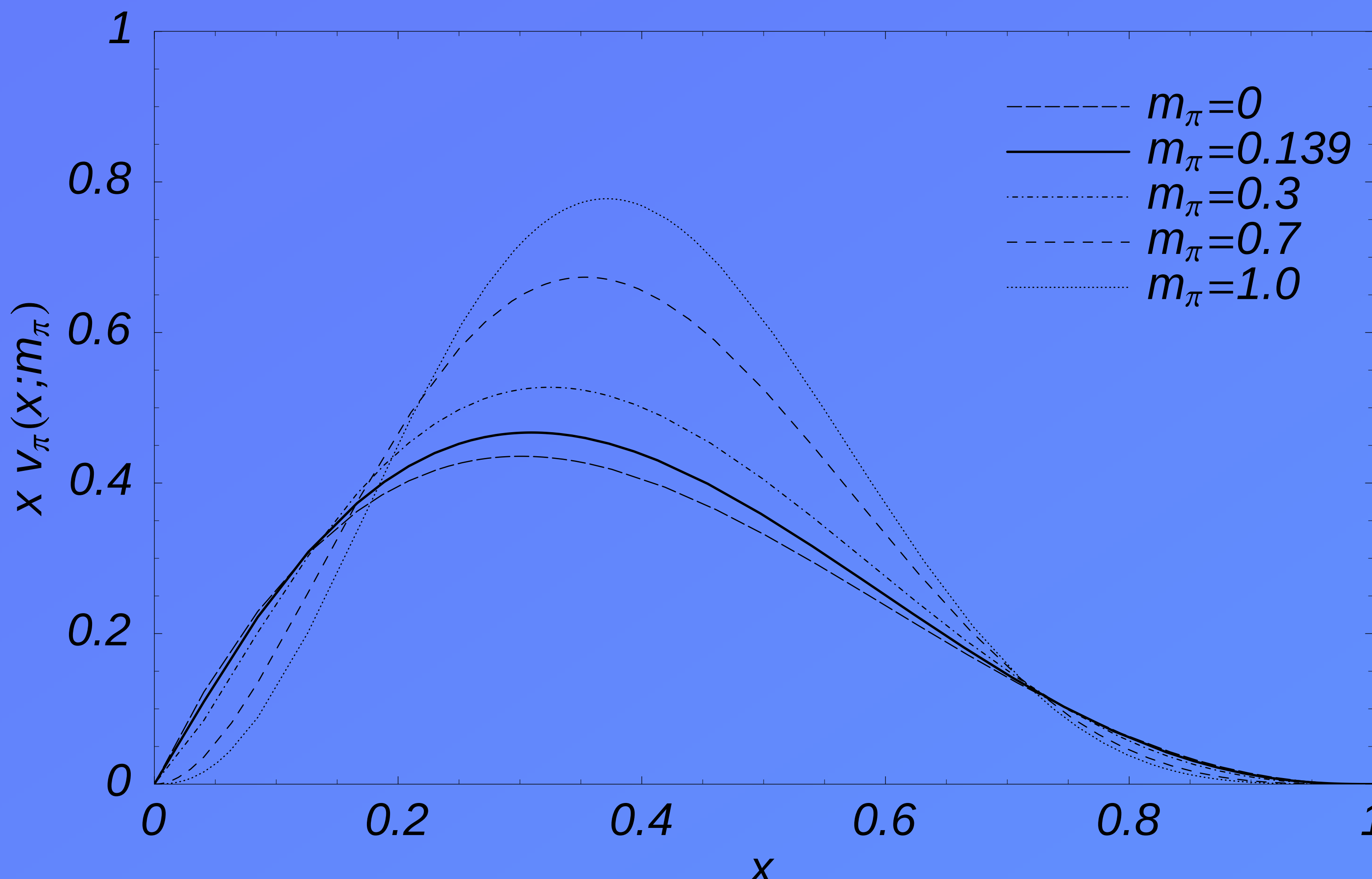
- BUT within relatively large errors PDF agrees with Drell - Yan data

- Increase in accuracy PLUS full-QCD in next few years

$\Rightarrow$ should give comparable accuracy to experiment!

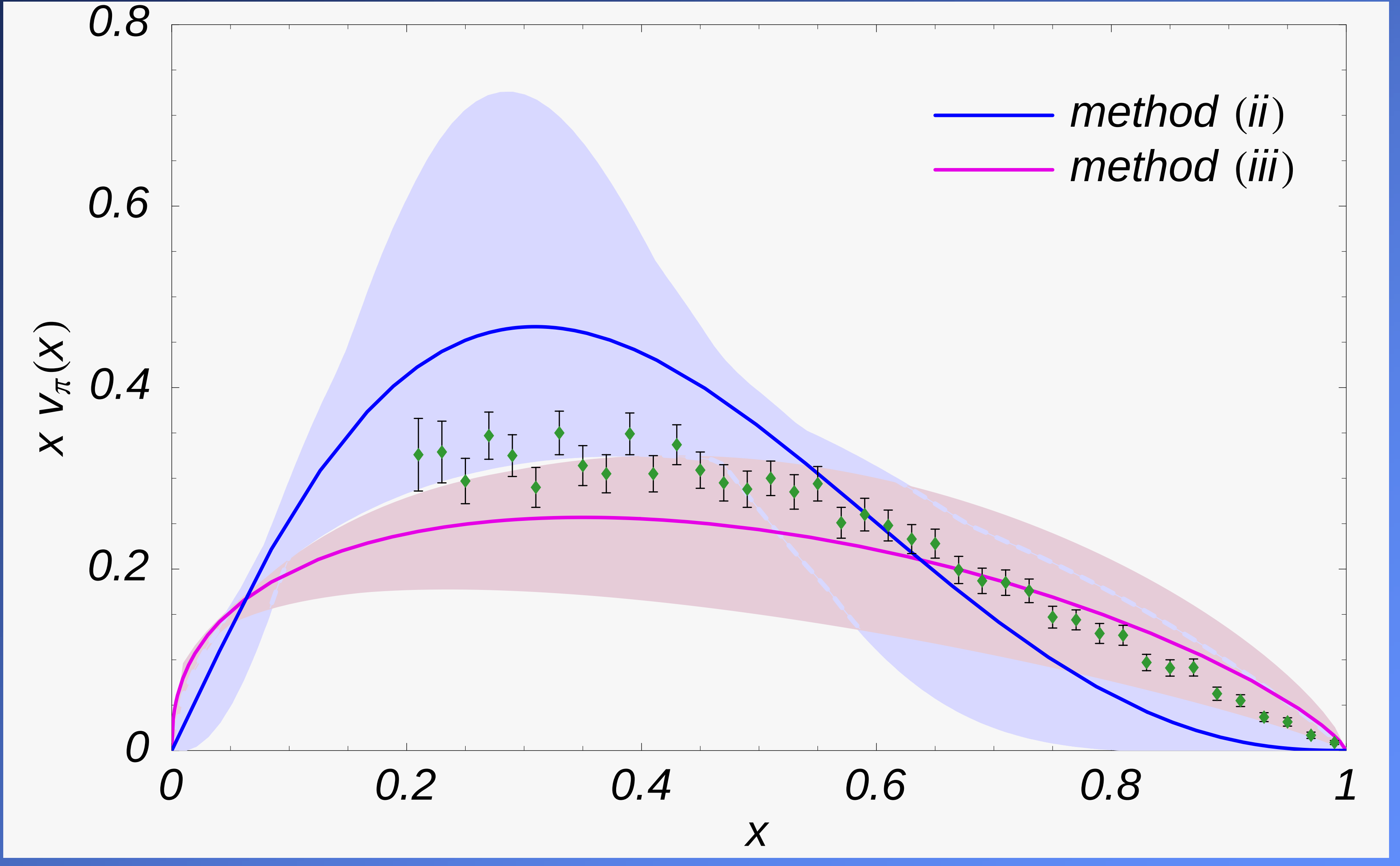
- Finally can study quark mass dependence of valence distribution of pion

Pion structure vs quark mass



Already at $m_\pi \sim 0.7$ ($m_q \sim 100$ MeV) : π starts to look like two valence quarks - **Detmold, Melnitchouk & Thomas (hep-lat/0303015)**

Comparison of two methods



Conclusions

- Study of hadron properties as function of m_q in lattice QCD is extremely valuable…..

  Has given major qualitative & quantitative advance in understanding !

- Inclusion of model independent constraints of chiral perturbation theory to get to physical quark mass is essential

- But radius of convergence of conventional χ P T is too small

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Conclusions - 2

- Nevertheless: Chiral extrapolation problem is solved !

- Use of FRR yields accurate, model independent:
  - physical nucleon mass
  - low energy constants

- Can use lattice QCD data over entire range m_π^2 up to 0.8 GeV^2

- Accurate data point needed at 0.1 GeV^2 in order to reduce statistical errors

Conclusions - 3

- Expect similar conclusion will apply to other cases where chiral extrapolation has been successfully applied
  - Octet magnetic moments
  - Octet charge radii
  - Moments of parton distribution functions

- Findings suggest new approach to quark models i.e. build Constituent Quark Model where chiral loops are suppressed and make chiral extrapolation to compare with experimental data

(see Cloet *et al*. , Phys Rev C65 (2002) 062201)

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Special Mentions…

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