Lecture (2)
Heavy-Ion Collisions:
Experiments, Models and Phenomenology

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Phenomenology

As a philosophical movement:

From Wikipedia:

There are several assumptions behind phenomenology that help explain its foundations:

1. **Phenomenologists reject the concept of objective research.** They prefer grouping assumptions through a process called phenomenological epoché.
2. They believe that analyzing daily human behavior can provide one with a greater understanding of nature.
3. They assert that persons should be explored. This is because persons can be understood through the unique ways they reflect the society they live in.
4. **Phenomenologists prefer to gather "capta", or conscious experience, rather than traditional data.**
5. **They consider phenomenology to be oriented toward discovery, and therefore they research using methods that are far less restrictive than in other sciences.**

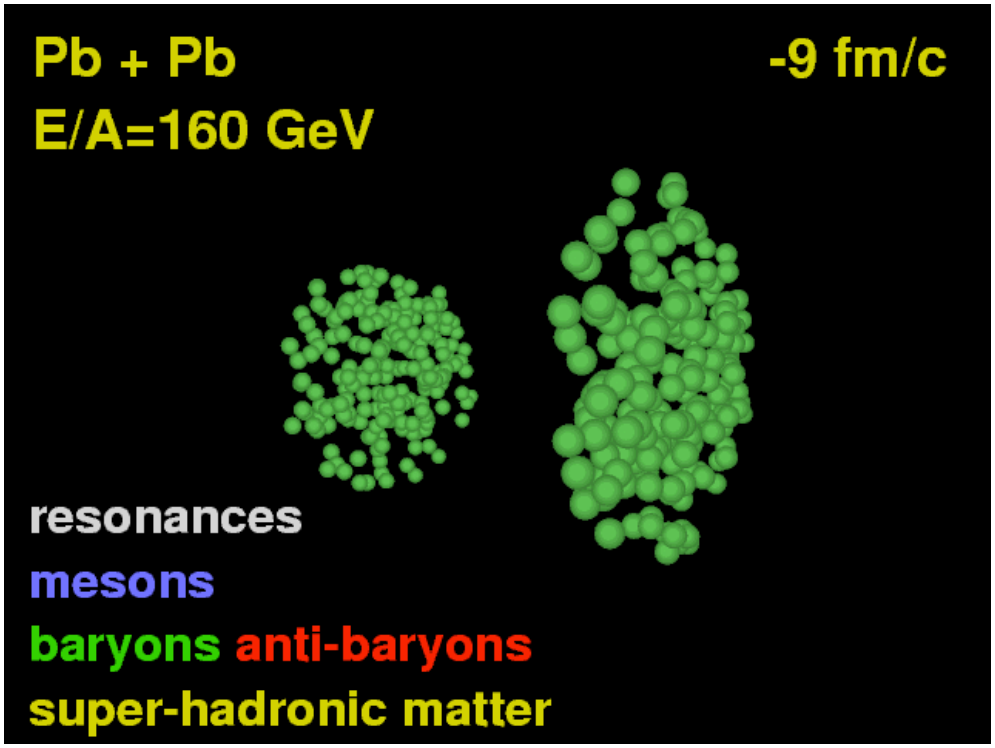
To a physicist:

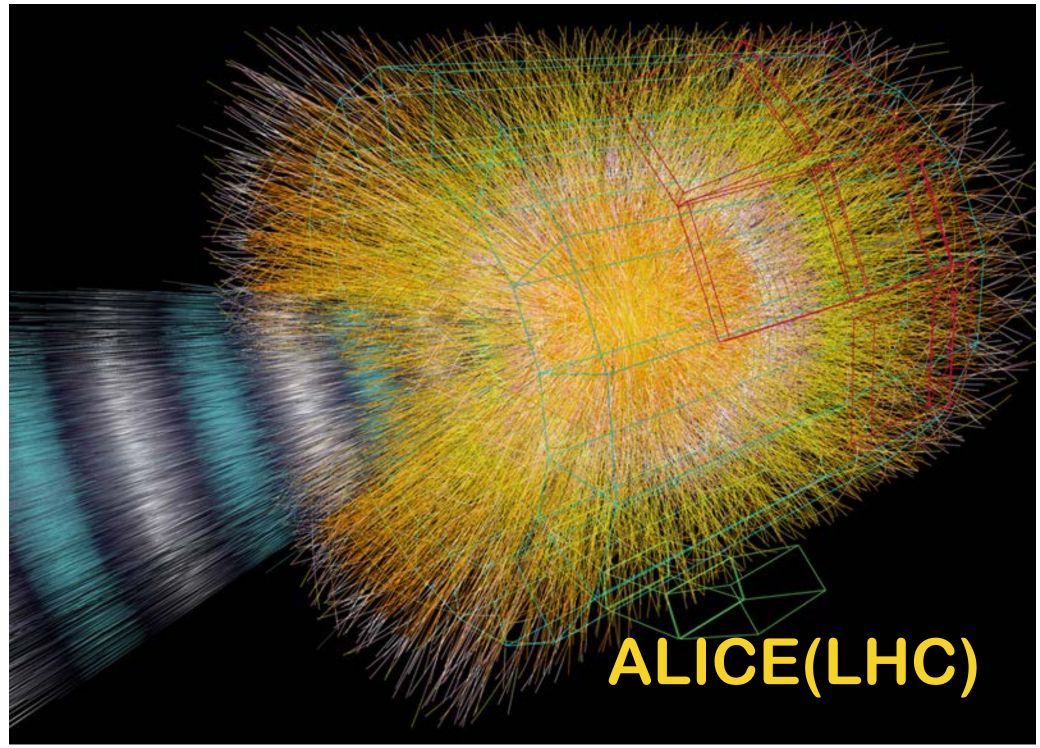
- **Experiment (momenta and IDs of tracks)**

  ➜ Evolution of ε, P, ν, ρ…

- **Can be heuristic or semi-quantitative**

Facilities





AGS(11A GeV), SPS(160A GeV), RHIC(100A+100A GeV), LHC(1.4A+1.4A TeV)

Facility	Operating	Equiv. pp c.o.m. Energy	Temperature
AGS at BNL	1990s	$\lesssim$ 6 GeV	$\lesssim$ 160 MeV
SPS at CERN	1990s —	$\lesssim$ 20 GeV	$\lesssim$ 200 MeV
RHIC at BNL	2000 —	$\lesssim$ 200 GeV	$\lesssim$ 300 MeV
LHC at CERN	2010 —	$\lesssim$ 15 TeV	$\lesssim$ 400 MeV

SPS: Briefly visits QGP
RHIC and LHC: Well into QGP

ASIDE: 3 kinds of rapidity

1. "y", the rapidity, is a measure of velocity along beam axis
   — rapidities add, just like Newtonian velocities
2. "η", the pseudo-rapidity is approximation to "y"
   — depends on θ, angle relative to beam axis
   — for massless particles $y = \eta$
3. "η_s" Is the spatial rapidity
   — measure of position along z axis (Bjorken coordinates)

ASIDE: 3 kinds of rapidity

Consider v along z axis

$$\gamma^2 - \gamma^2 v^2 = \frac{1}{1-v^2} - \frac{v^2}{1-v^2} = 1$$

$$u_0 = \gamma, \quad u_z = \gamma v, \quad u_0^2 - u_z^2 = 1$$

$$u^\alpha = (\gamma, \gamma v) = (\cosh y, \sinh y)$$

$$u_B^\alpha = (\gamma_B, \gamma_B v_B) = (\cosh y_B, \sinh y_B)$$

$$u'^\alpha = (u_0\gamma_B + u_z\gamma_B v_B, u_z\gamma_B + u_0\gamma_B v_B)$$

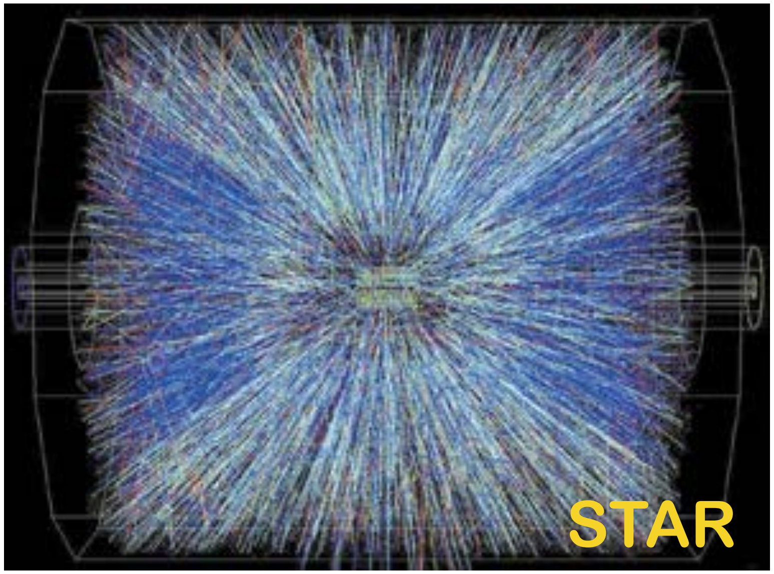
$$= (\cosh y \cosh y_B + \sinh y \sinh y_B, \sinh y \cosh y_B, \sinh y_B \cosh y) = (\cosh(y + y_B), \sinh(y + y_B))$$

Rapidity defined as:

$$y = \sinh^{-1}(\gamma v) = \tanh^{-1}(v) = \frac{1}{2}\ln\left[\frac{1+v}{1-v}\right]$$

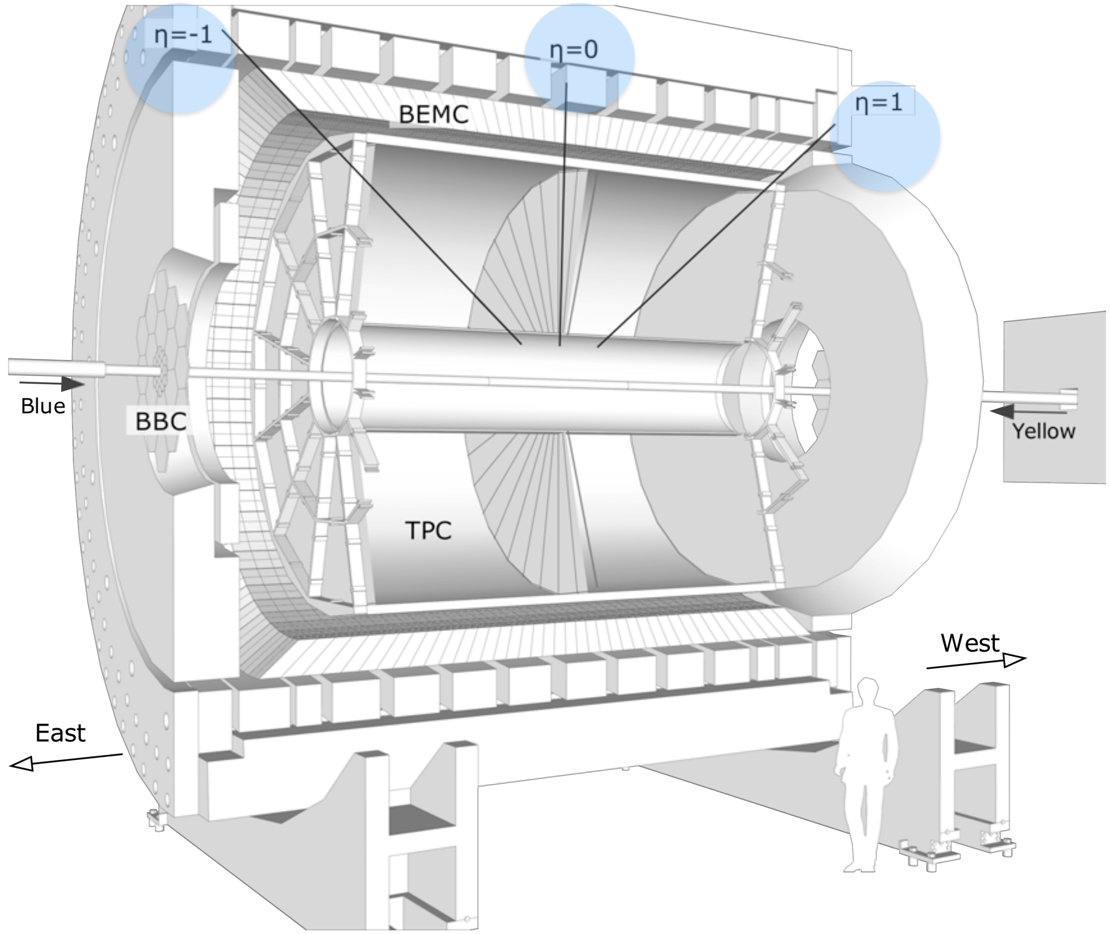
RHIC beams: ± 5.4 units of y
LHC beams: ± 9.5 units of y

Experiments measure mid-rapidity



$$y = \frac{1}{2}\ln\left[\frac{1+v_z}{1-v_z}\right]$$

$$\eta = \frac{1}{2}\ln\left[\frac{1+\cos\theta}{1-\cos\theta}\right]$$



STAR/ALICE measure best for -1 < η < 1

3 Modeling Stages

1. Pre-equilibrium, $\tau \lesssim 0.5$ fm/c

   — no good quasi-particles, off-shell
   — flux tubes or classical Yang-Mills field
   — parametric

2. Hydrodynamics (T$\gtrsim$160 MeV, $1 \lesssim \tau \lesssim 5$ fm/c)

   — QGP

3. Hadron simulation (T$\lesssim$160 MeV)
   — hadrons struggle to maintain chemical/kinetic equilibrium

4. Superimposed on 1 - 3:
   — Femtoscopic correlations, jets, heavy-flavor dynamics
   — correlations…

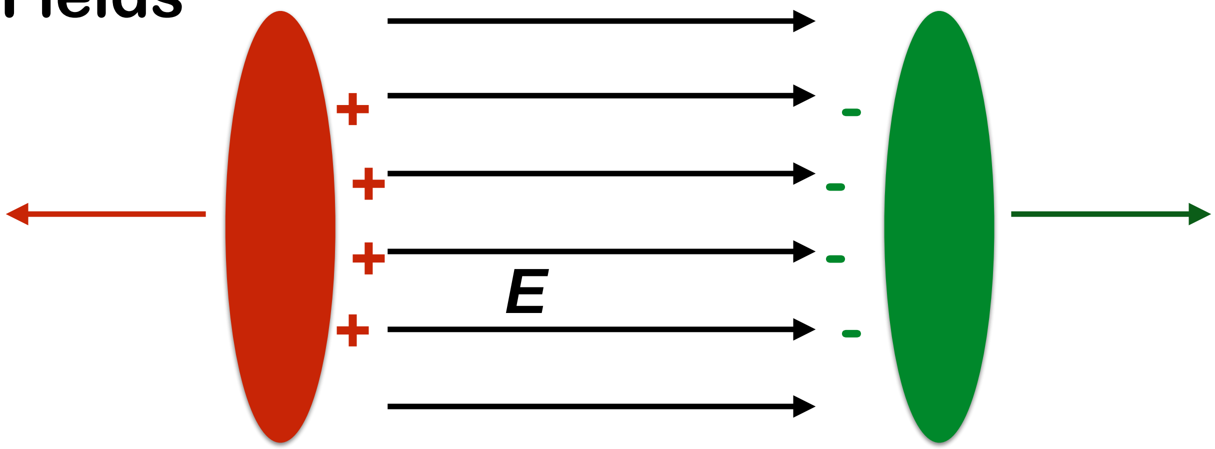
Pre-equilibrium

Two kinds of energy deposition:

1. Partonic Scattering

2. Fields

$$\text{Energy} \sim \frac{A}{2}|\vec{E}|^2(2c\tau)$$



Energy increases over time (negative pressure)

$$T_{00} = \frac{E^2}{2}, \quad T_{xx} = T_{yy} = \frac{E^2}{2}, \quad T_{zz} = -\frac{E^2}{2}$$

QCD similar, but with 8 interacting fields (color-glass condensate)

Hydrodynamics and the QGP

1. Justified because of strong interaction and nearly all light quasi-particles
2. Eq. of state can come from lattice
3. Must account for viscosity

Israel-Stewart equations (several variants)

$$\partial_t \pi_{ij} = -\frac{1}{\tau_{IS}} \left(\pi_{ij} - \pi_{ij}^{(\mathrm{NS})} \right) + \cdots$$

Arbitrary initial anisotropy of SE tensor

Parameters are viscosity and relaxation times

Why is hydro valid?

Hydro is based on:

a) energy-momentum conservation
b) profiles are smooth on scale of system
c) T_{ij} is not too far from equilibrium
d) different species don't flow differently

Should work for QGP

$$\partial_t \pi_{ij} = -\frac{1}{\tau_{IS}} \left(\pi_{ij} - \pi_{ij}^{(\mathrm{NS})} \right) + \cdots$$

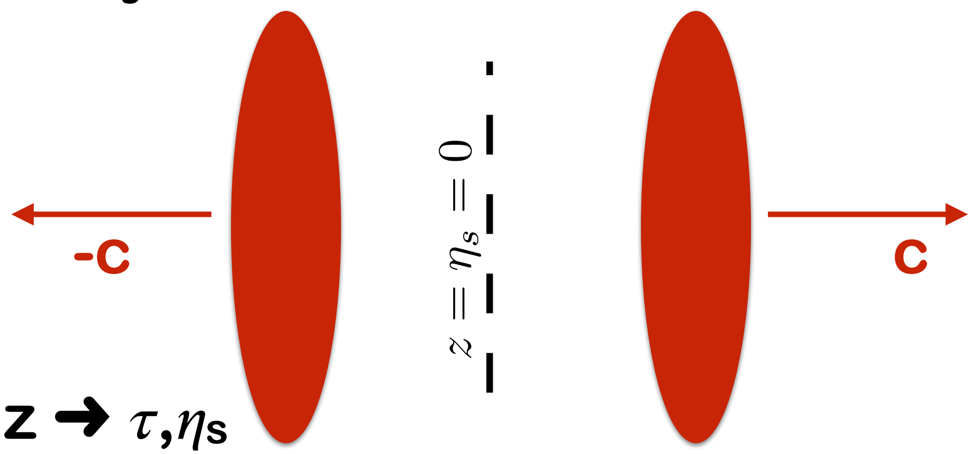
a) Israel-Stewart is flexible
b) dominated by light degrees of freedom
— not true for hadron gas

Two-dimensional reduction of hydro

"Translational" invariance along beam axis

Small boosts along beam axis don't change physics

Bjorken coordinates



$$v_z = z/t$$

$$\tau = t\sqrt{1 - v_z^2} = \sqrt{t^2 - z^2}$$

$$\eta_s = \tanh^{-1}(z/t)$$

t,z ➜ τ,η_s

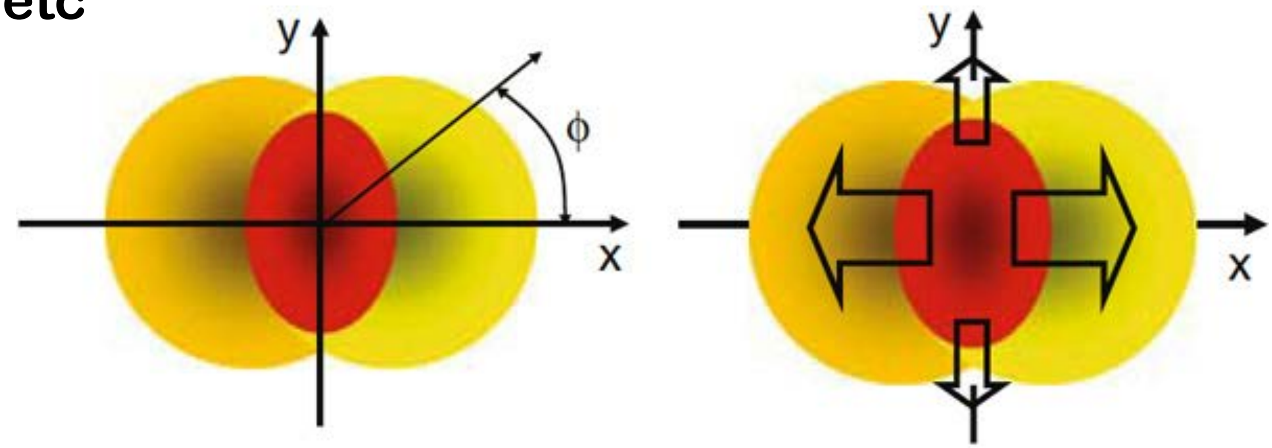
$\varepsilon(\tau)$ — Nothing depends on η_s, no longitudinal acceleration

Hydro becomes effectively 2-D

Doesn't apply for lower RHIC energies

Collective Flow

1. Hallmark of hydrodynamic behavior
2. Reduced by viscosity
3. Radial, elliptic, etc



Radial flow

Non-relativistically,

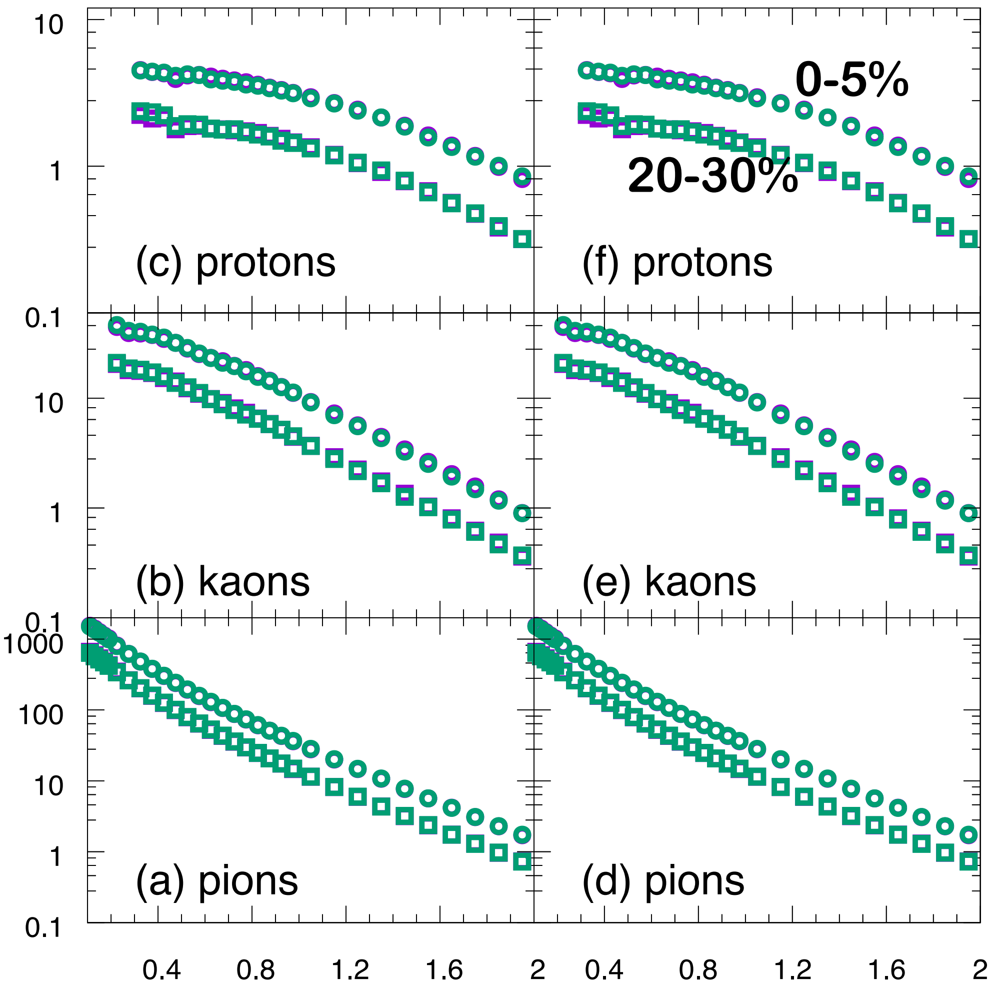
$$\left\langle \frac{p_x^2}{2m} + \frac{p_y^2}{2m} \right\rangle = T + \frac{m}{2} v_{\rm coll}^2$$

Spectra hotter for protons than pions

More pressure — more flow

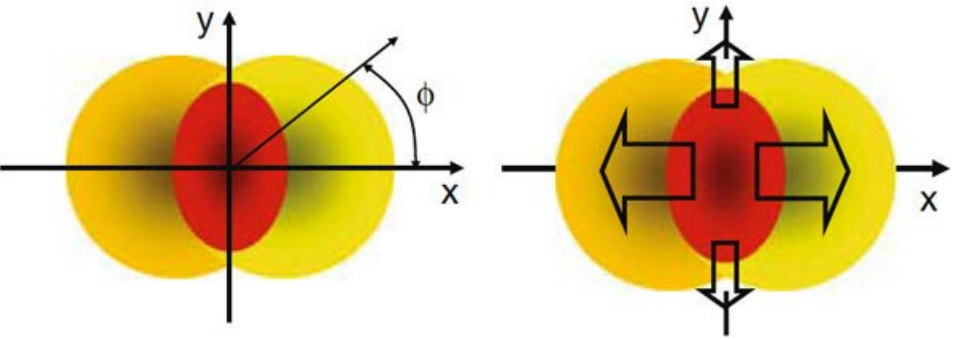
Flow velocities ~ 0.7c

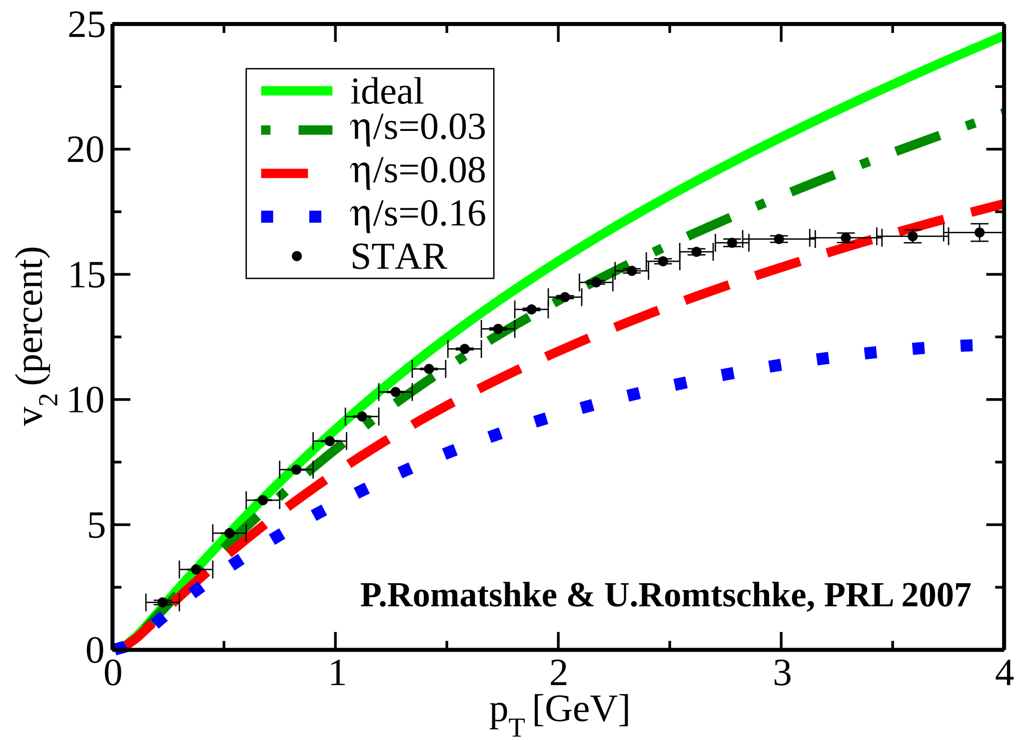
ALICE SPECTRA



Elliptic Flow

$$v_2 \equiv \langle \cos 2\phi \rangle$$



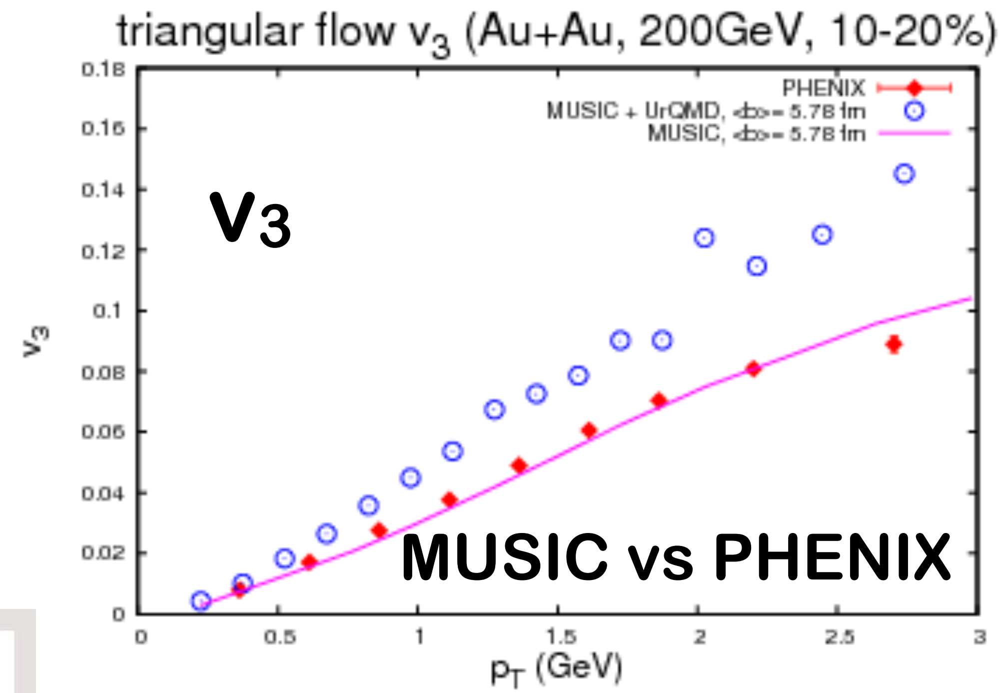


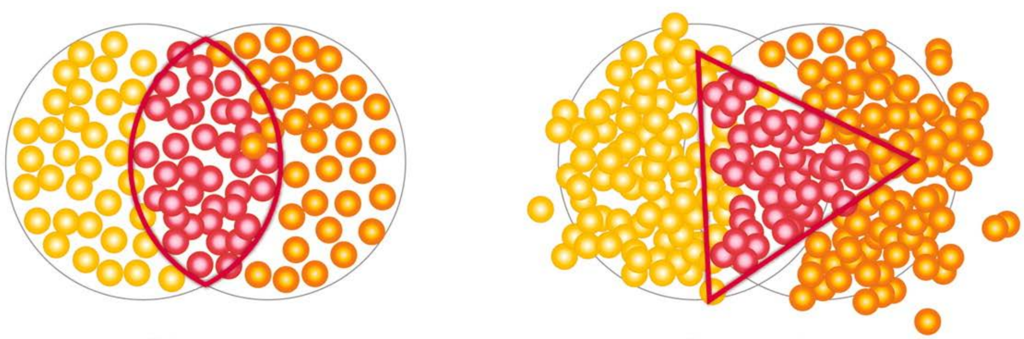
Suggests low viscosity (close to uncertainty limit)

P.Danielewicz and M.Gyulassy, PRD(1985)

Higher Moments of Flow

$$v_n \equiv \langle \cos(n\phi) \rangle$$

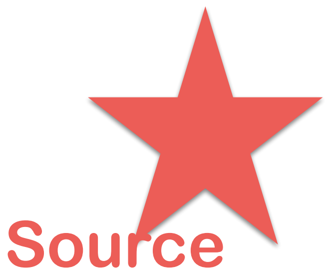


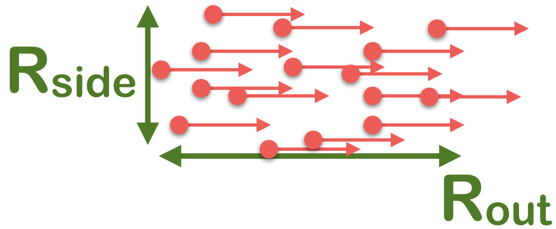


Elliptic flow   Triangular flow

Reflects on lumpiness of initial conditions

Femtoscopic Correlations





$$f(\vec{p}, \vec{r}, t)$$

Fix *p,t*

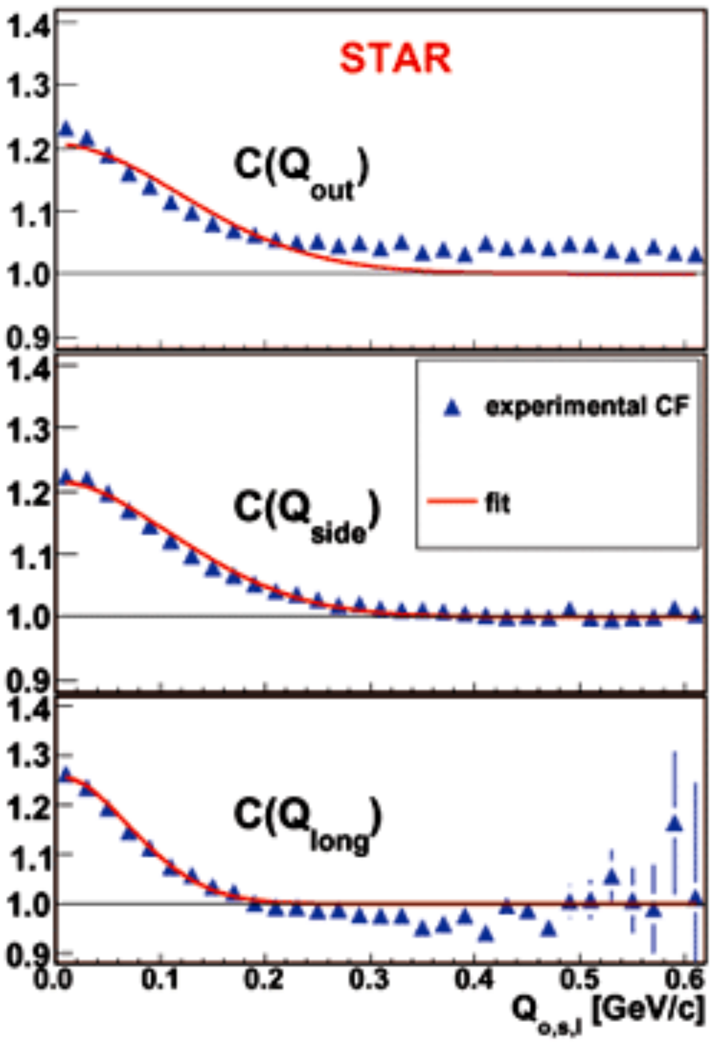
$$P_2(\boldsymbol{p}_a, \boldsymbol{p}_b) = P_1(\boldsymbol{p}_a)P_1(\boldsymbol{p}_b) + \frac{1}{(2\pi\hbar)^6}\int d^3r_a d^3r_b \; f(\bar{\boldsymbol{p}}, \boldsymbol{r}_a, t) f(\bar{\boldsymbol{p}}, \boldsymbol{r}_b, t)\left\{|\phi(\boldsymbol{q}, \boldsymbol{r}_a - \boldsymbol{r}_b)|^2 - 1\right\}$$

$$C(\boldsymbol{p}_a, \boldsymbol{p}_b) = \frac{P_2(\boldsymbol{p}_a, \boldsymbol{p}_b)}{P_1(\boldsymbol{p}_a)P_1(\boldsymbol{p}_b)}$$

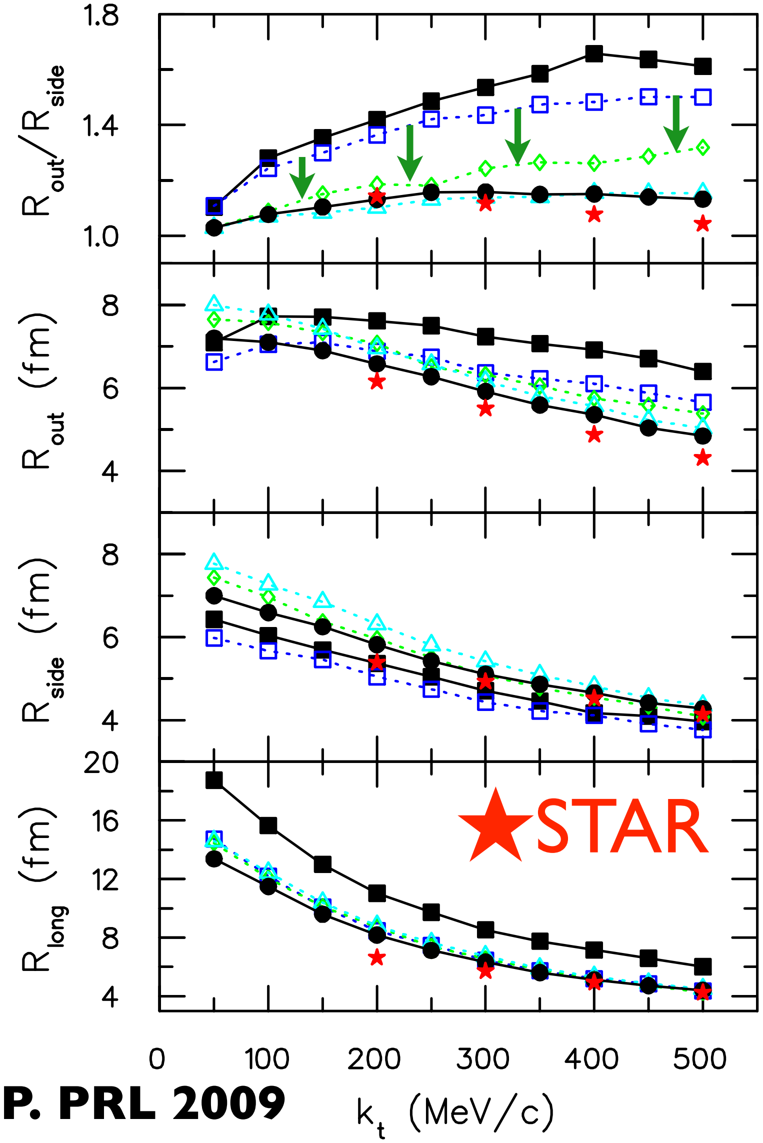
Low pressure: R_{out} >> R_{side} and R_{long} is large

High pressure: R_{out} ~ R_{side} and R_{long} is small

Femtoscopic Correlations



- Stiffer EoS
  (blue to green)



S.P. PRL 2009

Six dimensions $C(p_a,p_b)$ analyzed

- $R_{out/long/side}$ as functions of p_t, y, φ
- Directions of ellipse
- non-Gaussian details of source
- Source sizes for pions, kaons, protons, Lambdas
- Relative offset for different species, e.g. πp, Kp, $K\pi$

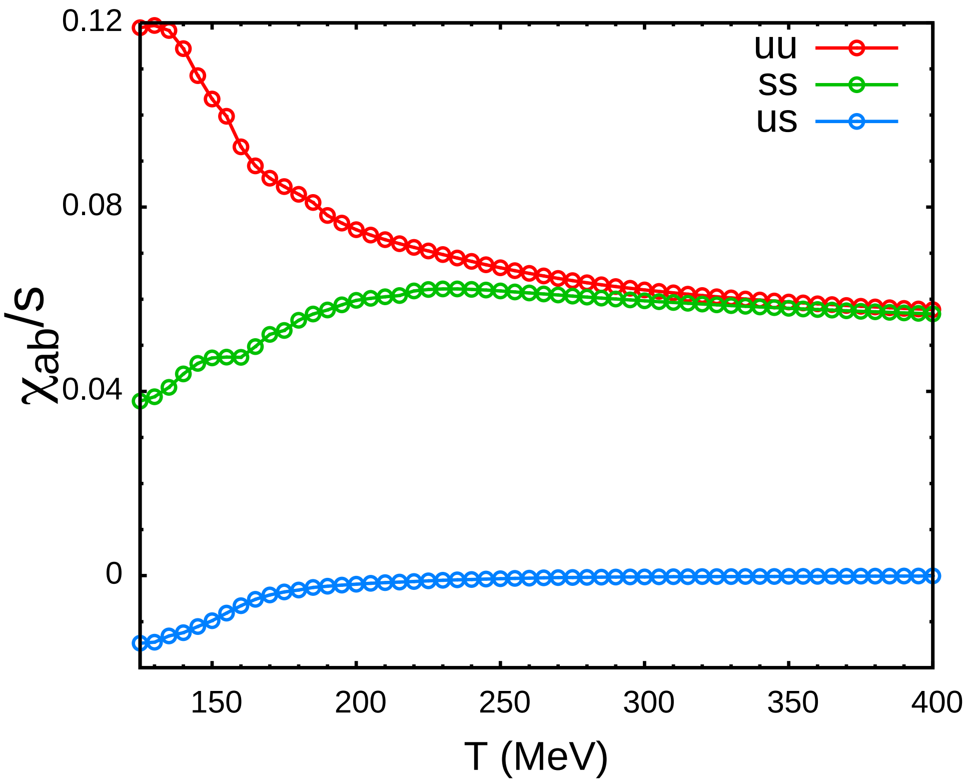
  — At low energy, correlations with d,t,α,Li,C…

All consistent with large collective flow!

Correlations from Charge Conservation — Balance Functions

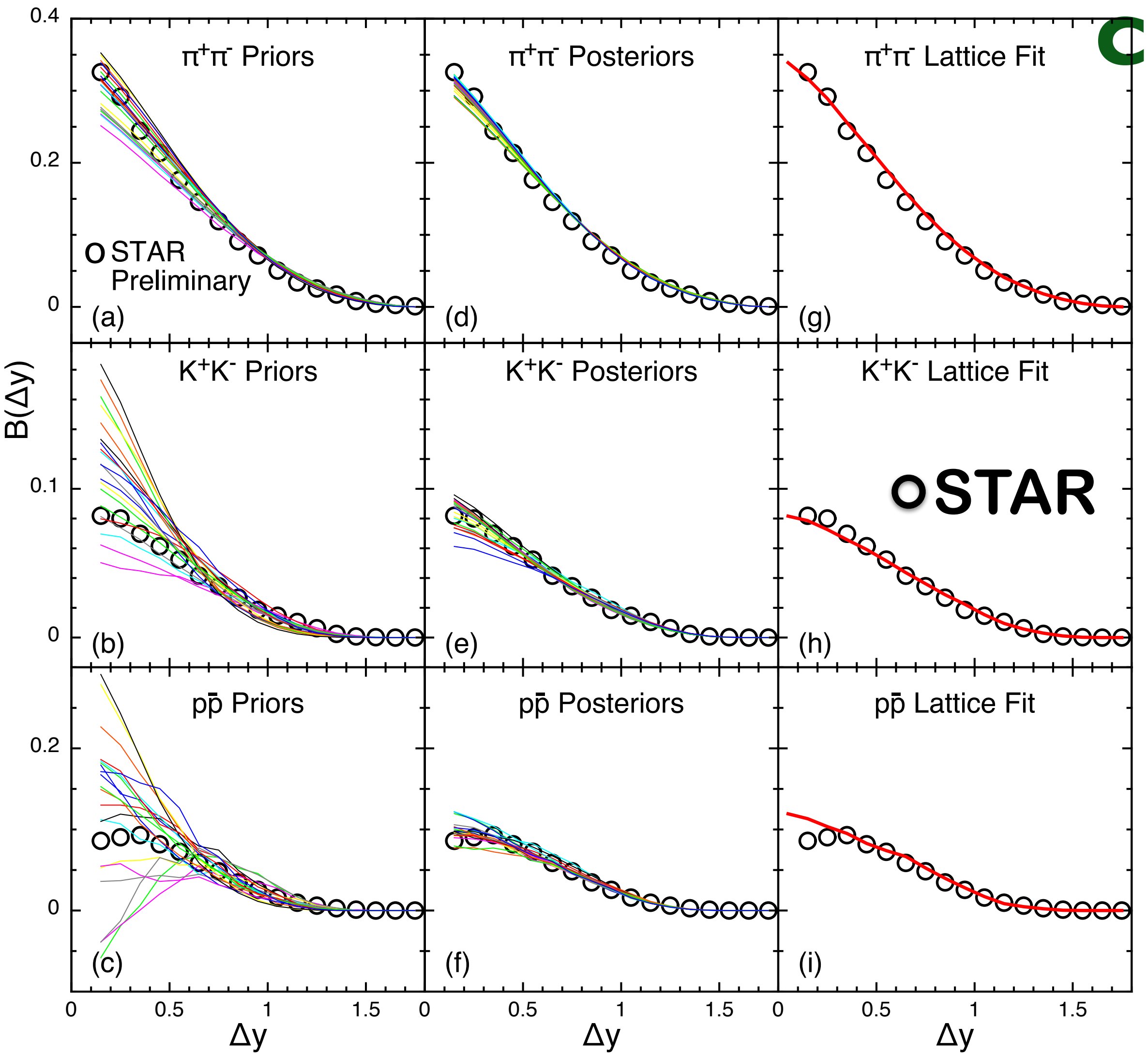
$$B(p_b|p_a) = \frac{P_{+-}(p_a,p_b) - P_{++}(p_a,p_b)}{2P_+(p_a)} + \frac{P_{-+}(p_a,p_b) - P_{--}(p_a,p_b)}{2P_-(p_a)}$$

- Integrates to unity
- Early production broader BFs
- Larger diffusion broader BFs
- Can be indexed on species
  - — strangeness/baryons made early, electric charge made late
  - — Narrow ππ BFs, broad pp and KK BFs



Charge balance functions verify chemistry

S.P., W.McCormack and C.Ratti, PRC 2015

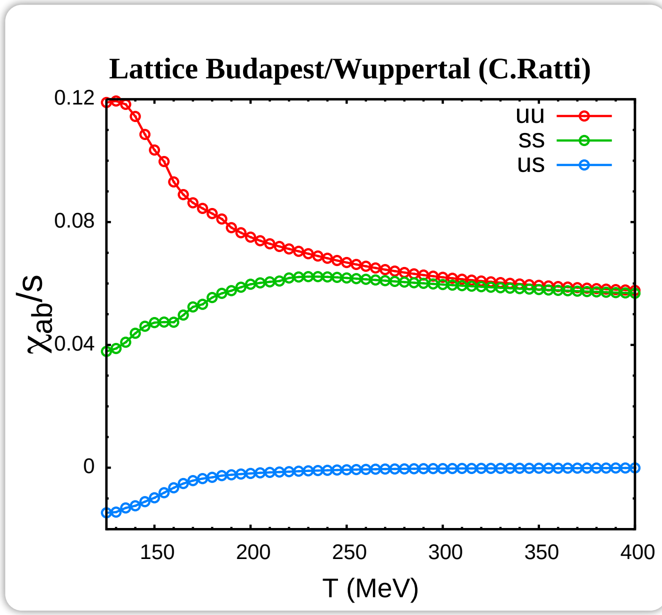


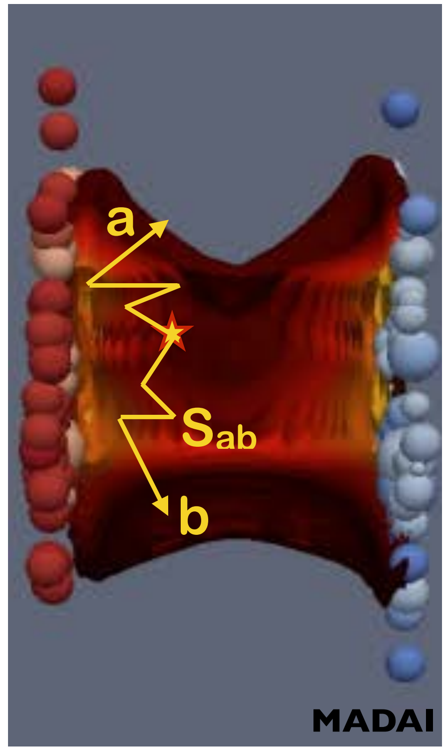
ππ narrower than KK

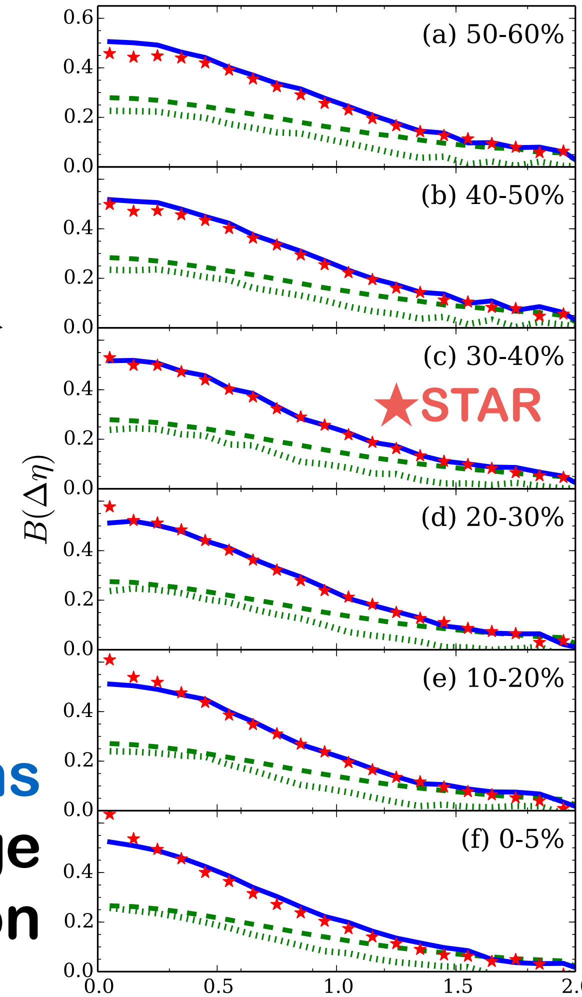
KK narrower than pp

Matches expectations
if susceptibility/chemistry
equilibrated

IV. Phenomenology
Susceptibility







$$C_{ab}(t,\mathbf{r}_1,\mathbf{r}_2) = \langle \delta\rho_a(t,\mathbf{r}_1)\delta\rho_b(t,\mathbf{r}_2)\rangle$$

$$\partial_t C_{ab} + \nabla_1\cdot(\mathbf{v}_1 C_{ab}) + \nabla_2\cdot(\mathbf{v}_2 C_{ab}) - D\nabla_1^2 C_{ab} - D\nabla_2^2 C_{ab} = S_{ab}(t,\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

$$S_{ab}(t,\mathbf{r}) = -s\frac{D}{Dt}\frac{\chi_{ab}(t,\mathbf{r})}{s}$$

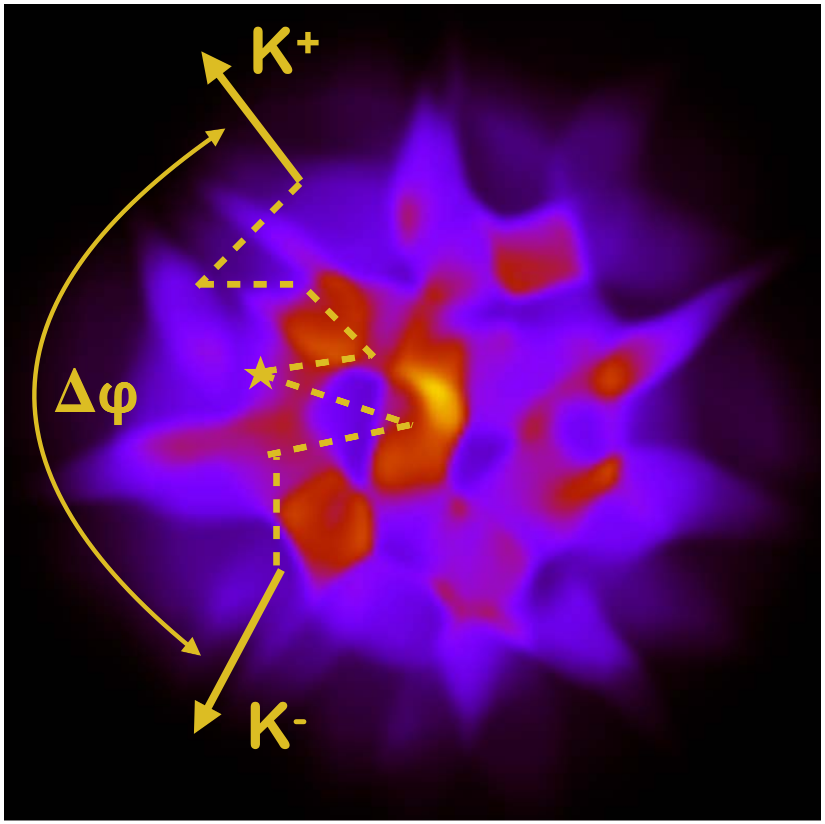
Charge-balance correlations

Early production of charge

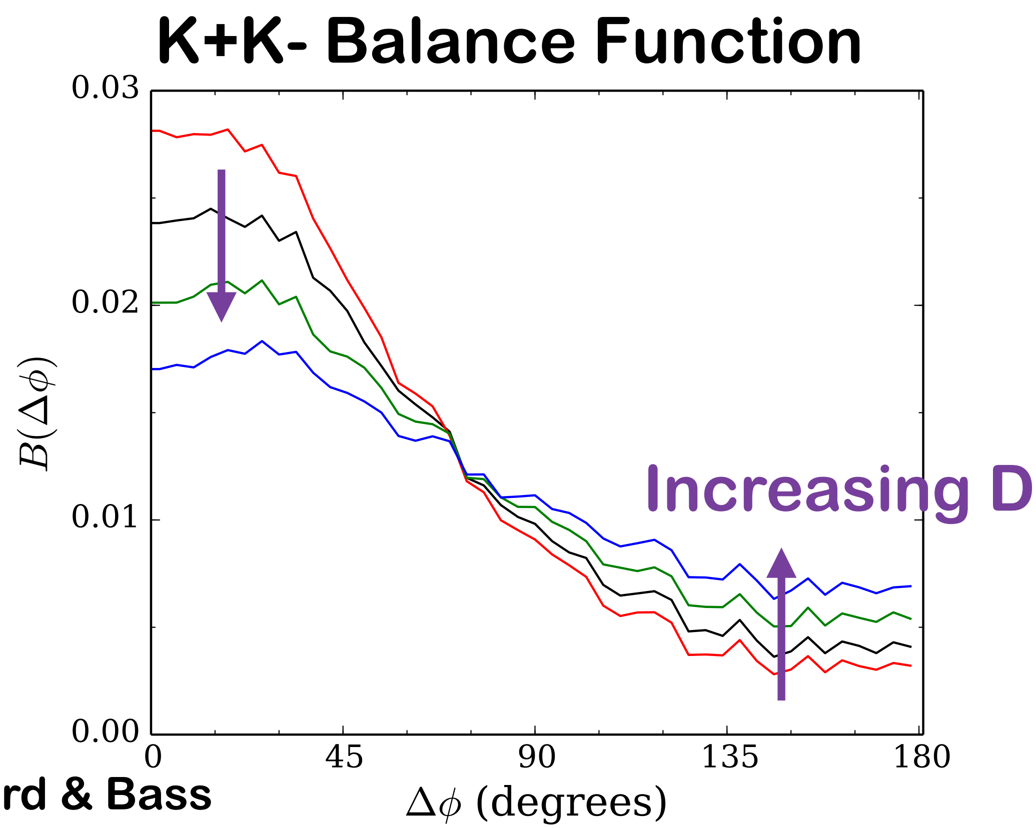
➜ broader correlation

S.P. and C.Plumberg, PRC(2019) $\Delta\eta$

Phenomenology — Diffusivity



Strangeness made early
∴ kaon separation determined by diffusivity



Similar work already done for charm, Bernhard & Bass

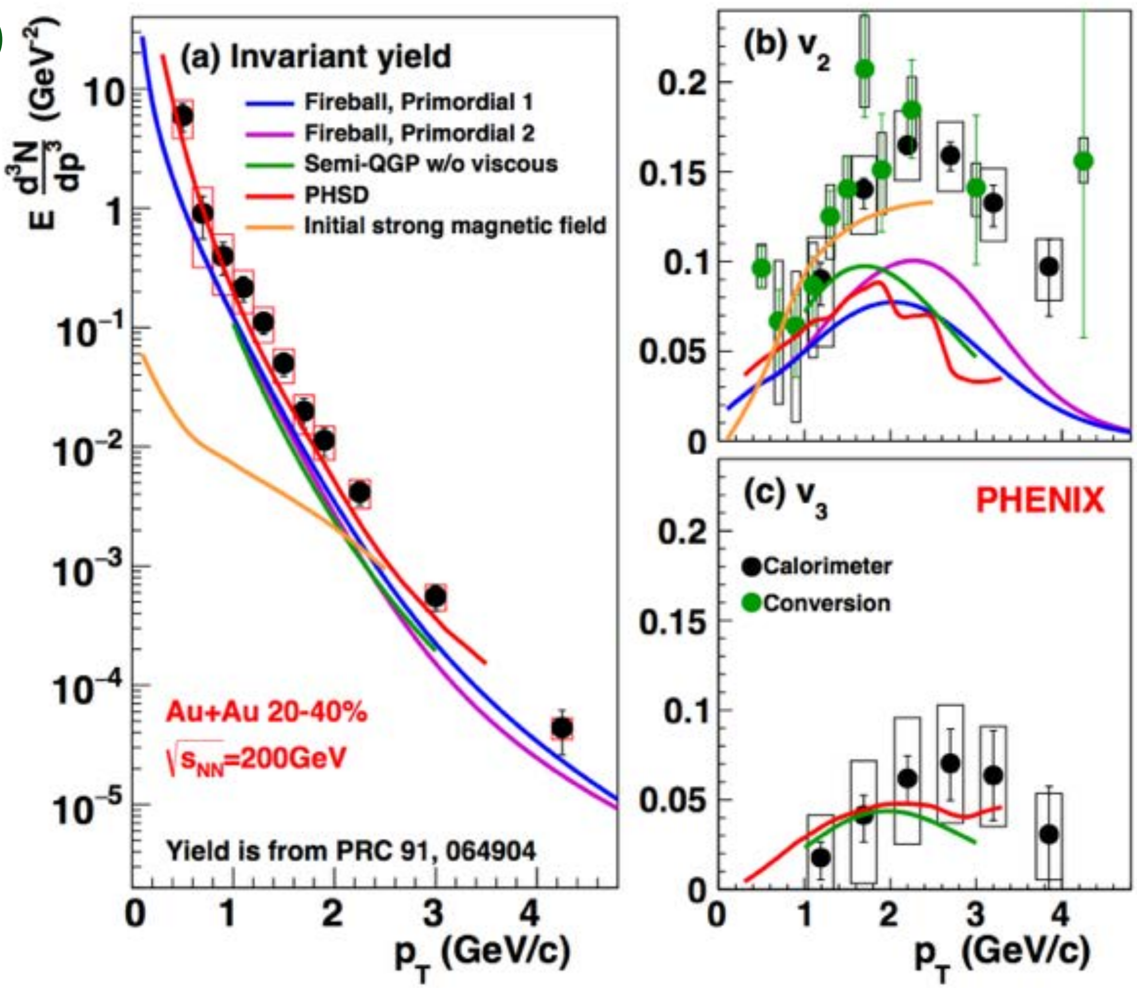
Electromagnetic Signals
Penetrating Probes

- Photon has ~90% chance of traversing fireball
- Direct photons
  - Must subtract contribution from meson decays (π_0)
- Dileptons
  - Function of invariant mass

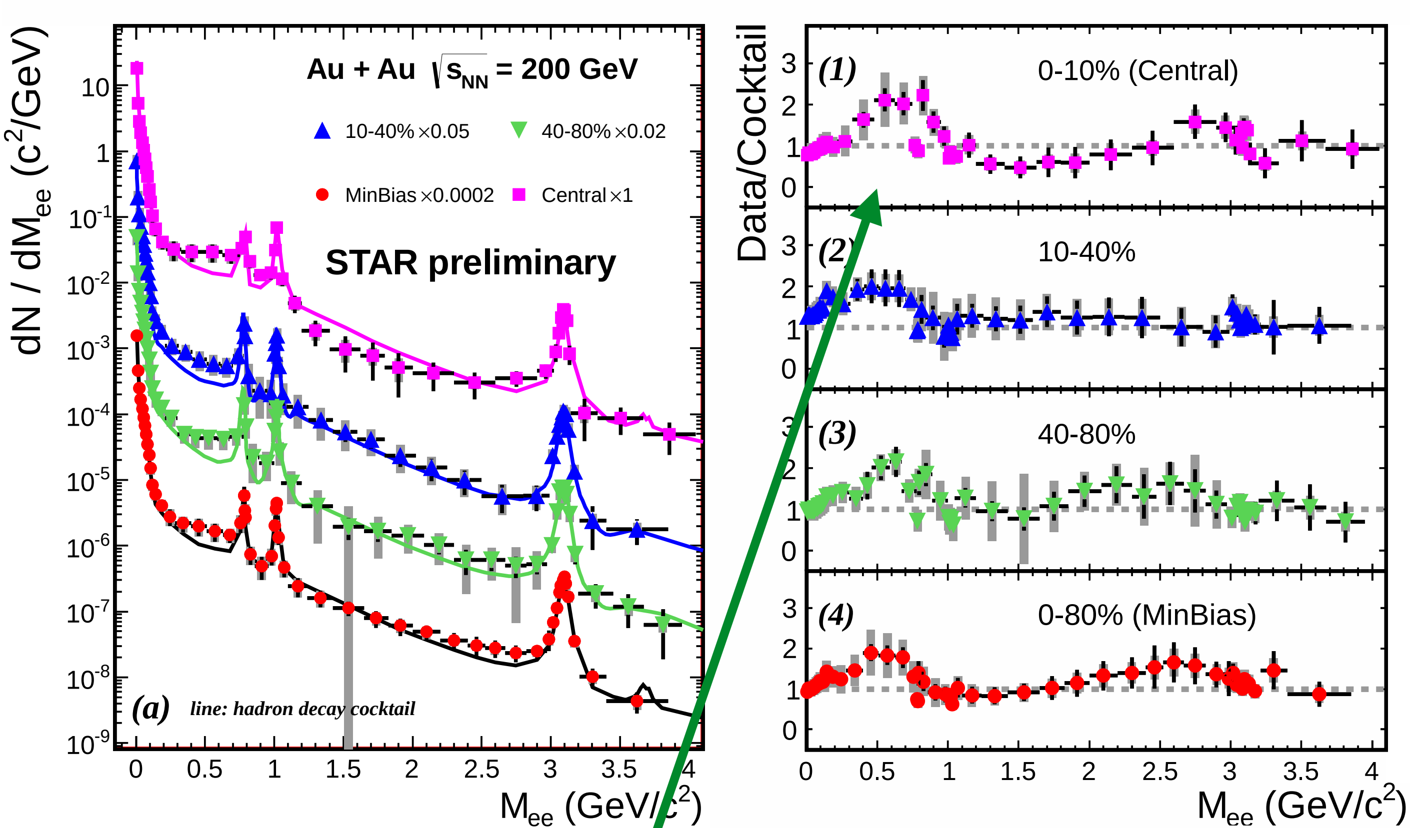
Direct Photons
(not from hadron decays)

PRC94, 064901 (2016)

Puzzling:
- Yield seems high
- High flow
- From hadronization?



Dileptons



ρ peak not 5x pp!!

Doesn't exist in QGP

Excess at 500 MeV

Meson masses

Hadronization? Shifted ρ?

Beam Energy Scan at RHIC: 2019-2021

- Energies from 7.7 GeV up (to 200)
- Less T, significantly more ρ_B
- Search for phase transition
  — correlations and fluctuations
- Difficult to model:
  — 3D
  — Larger corona
  — EoS depends on baryon density
  — Hadron simulation needs mean fields
  — Stopping 4-dimensional
  — Phase separation/critical phenomena dynamics difficult

Modeling Phase Dynamics

- **Need gradient terms and thermal noise to**
  a) generate critical correlations
  b) generate surface energies
  c) finite-size droplets

Ideas from:
Stephanov (hydro+),
Steinheimer, Young,
Kapusta, S.P.,...

$$\epsilon_\kappa = \epsilon + \frac{1}{2}\rho\nabla^2\rho,$$

$$s = \bar{s}(\epsilon_\kappa, \rho),$$

$$\beta = \bar{\beta}(\epsilon_\kappa, \rho),$$

S.P. PRC 2018

$$\alpha = \bar{\alpha}(\epsilon_\kappa, \rho) + \frac{\bar{\beta}\kappa}{2}\nabla^2\rho + \frac{\kappa}{2}\nabla^2(\bar{\beta}\rho),$$

$$M_i = -\frac{\kappa}{2}\rho(\partial_j\rho)(\partial_i v_j + \partial_j v_i) - \frac{\kappa}{2}\rho^2\partial_i\boldsymbol{\nabla}\cdot\boldsymbol{v} + \frac{\kappa}{2}\rho(\partial_i\rho)\boldsymbol{\nabla}\cdot\boldsymbol{v}.,$$

$$T_{ij} = \bar{P}\delta_{ij} - \kappa\left[\rho\nabla^2\rho + \frac{1}{2}(\boldsymbol{\nabla}\rho)^2\right]\delta_{ij} + \kappa(\partial_i\rho)(\partial_j\rho),$$

Open questions & puzzles
(soft physics)

How is flow generated in small systems?
Why so many direct photons? (hadronization?)
What is the source of soft dileptons?
Can we infer EoS for $B \neq 0$? (beam energy scan)
Can we model signals of critical point or phase separation?
— if so, could signals be observable?

Jets and heavy flavor: Coming soon to a theatre near you