Heavy-Ion Collisions and the Quark-Gluon Plasma

Tuesday: QGP Properties — Idealized Theory
Wednesday(1): Heavy-Ion Collisions and Models
Wednesday(2): Bayesian Model/Data Analysis

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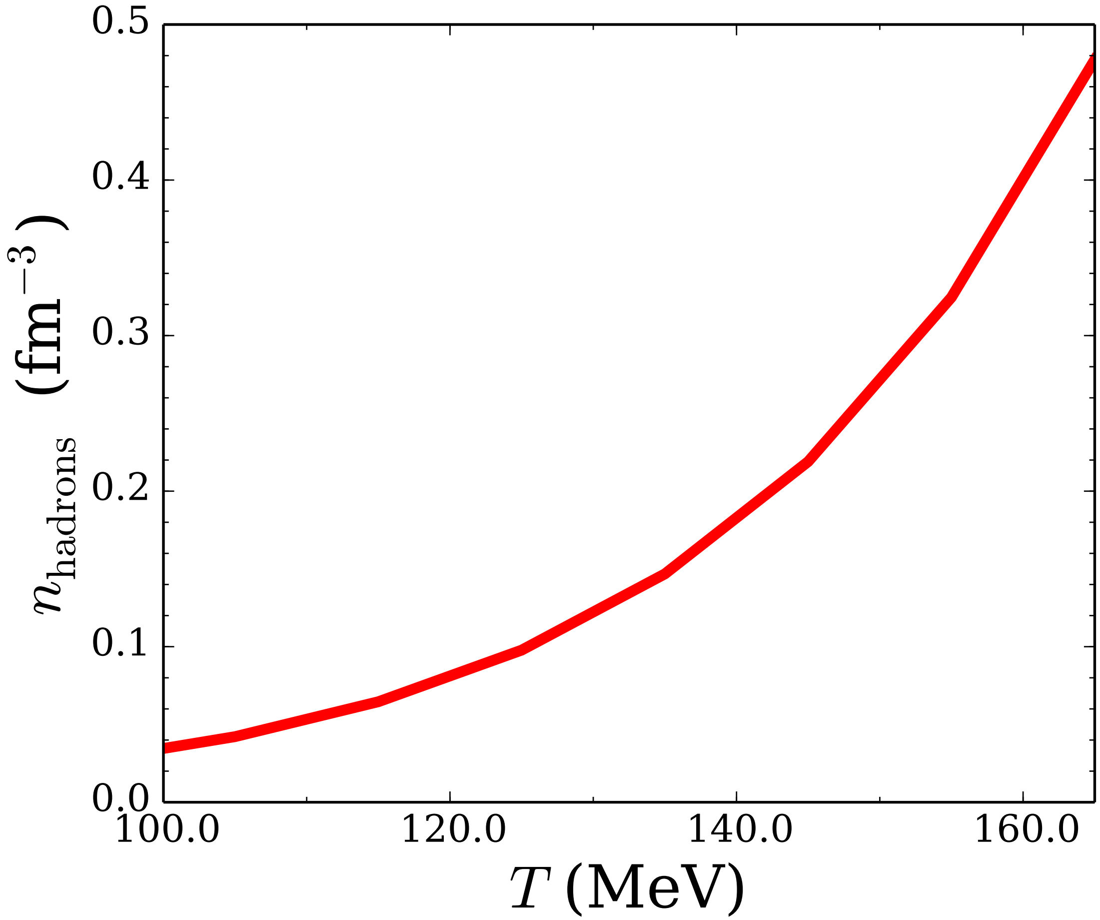
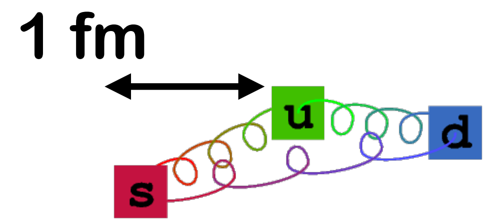
I. QGP Properties



T ≲150 MeV

Hadron Gas — Color Singlets

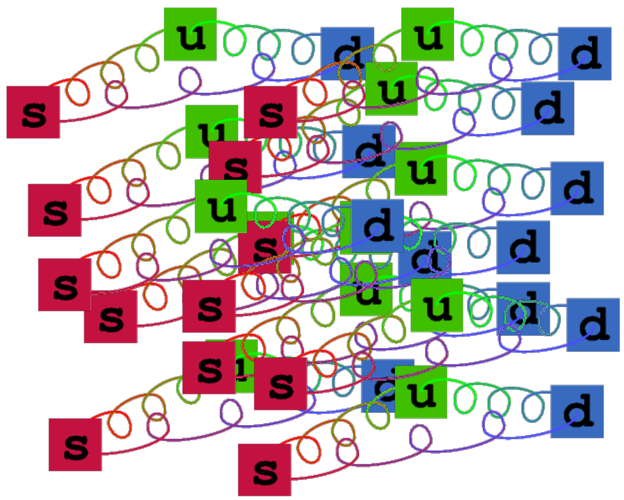


$$n_h = (2S_h + 1) \int \frac{d^3p}{(2\pi\hbar)^3} e^{-E(p)/T},$$

$$E = \sqrt{p^2 + m^2}$$

Like blackbody but with mass penalty

I. QGP Properties



Onset of QGP

$T \gtrsim T_c \simeq 160$ MeV

$\gtrsim$ 3x nuclear density

10,000 x T of sun interior

10^{30} J/cm^3

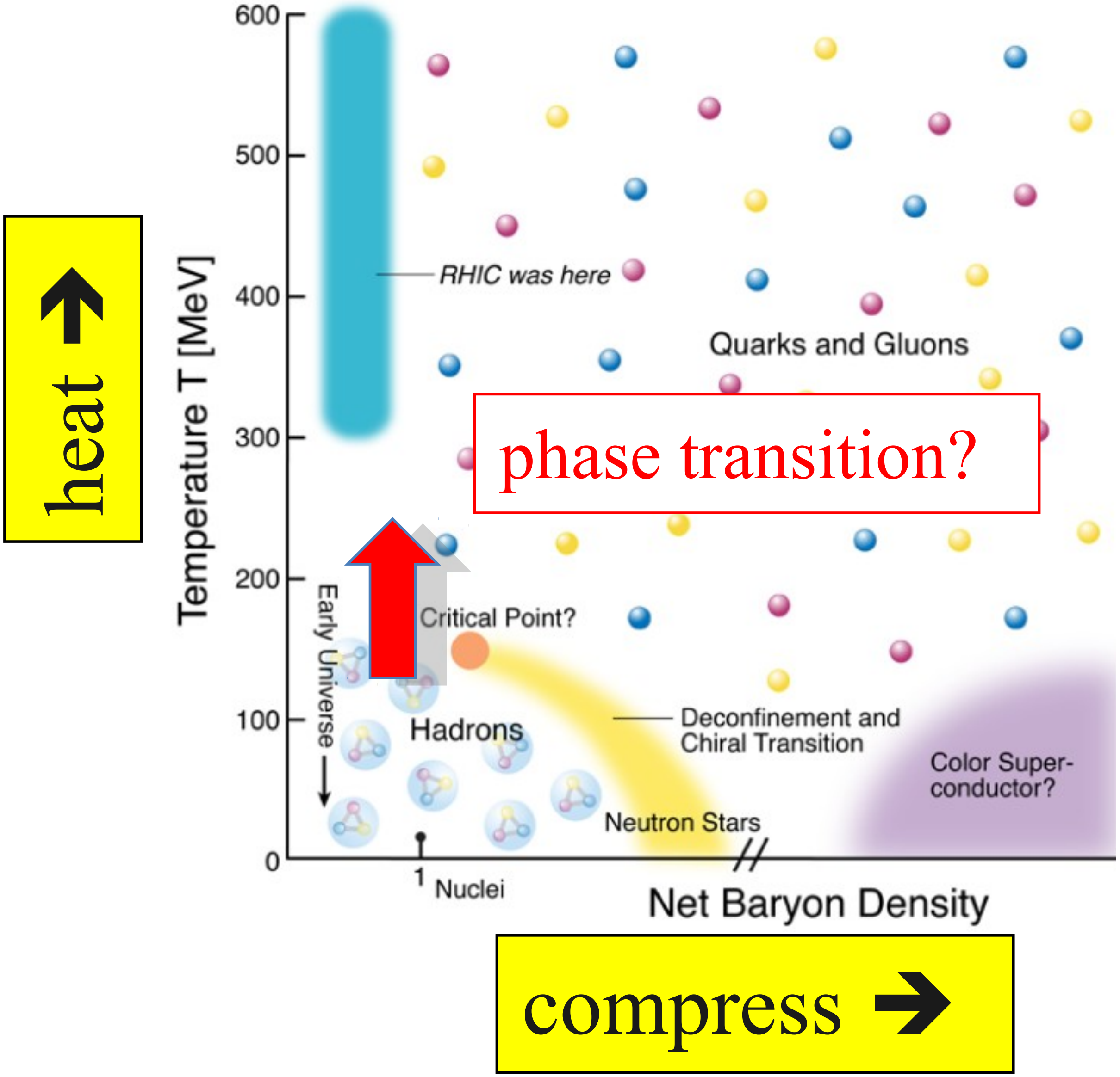
QGP Properties

Charge Rich!!!
52 colored degrees of freedom
16 gluons
36 *light* quarks:
up, down strange,
anti-up, anti-down, anti-strange
spin ↑, spin↓
red,green,blue

52 quasi-particles in ~ one thermal wavelength

Defining Properties of the QGP

1. **Eq. of state (B=0 & B$\neq$0)**
   P(n_B,ε) or P(μ,T) or c_s^2(n_B,ε)…
2. **Charge susceptibility and fluctuations**
   $\chi_{ab} = \langle \delta Q_a \delta Q_b \rangle / V$ **- describes chemistry**
3. **Quark-antiquark condensate** $\langle \bar{\psi} \psi \rangle$
   "Chiral symmetry'' restoration
4. **Viscosity — response to flow gradient**
   $\delta T_{ij} = -\eta \left[\partial_i v_j + \partial_j v_i - (2/3)\delta_{ij} \nabla \cdot \mathbf{v} \right] - \zeta \nabla \cdot \mathbf{v}$
5. **Diffusivity/Conductivity — response to density gradient**
   $\mathbf{j}_a = -D_{ab} \nabla \rho_b$
6. **Electromagnetic opacity and emissivity**
7. **QCD opacity (Jet quenching)**



Eq. of State

- possibly 1st order ??
- phase separation & critical point ??

Eq. of State:
Lattice Gauge Theory

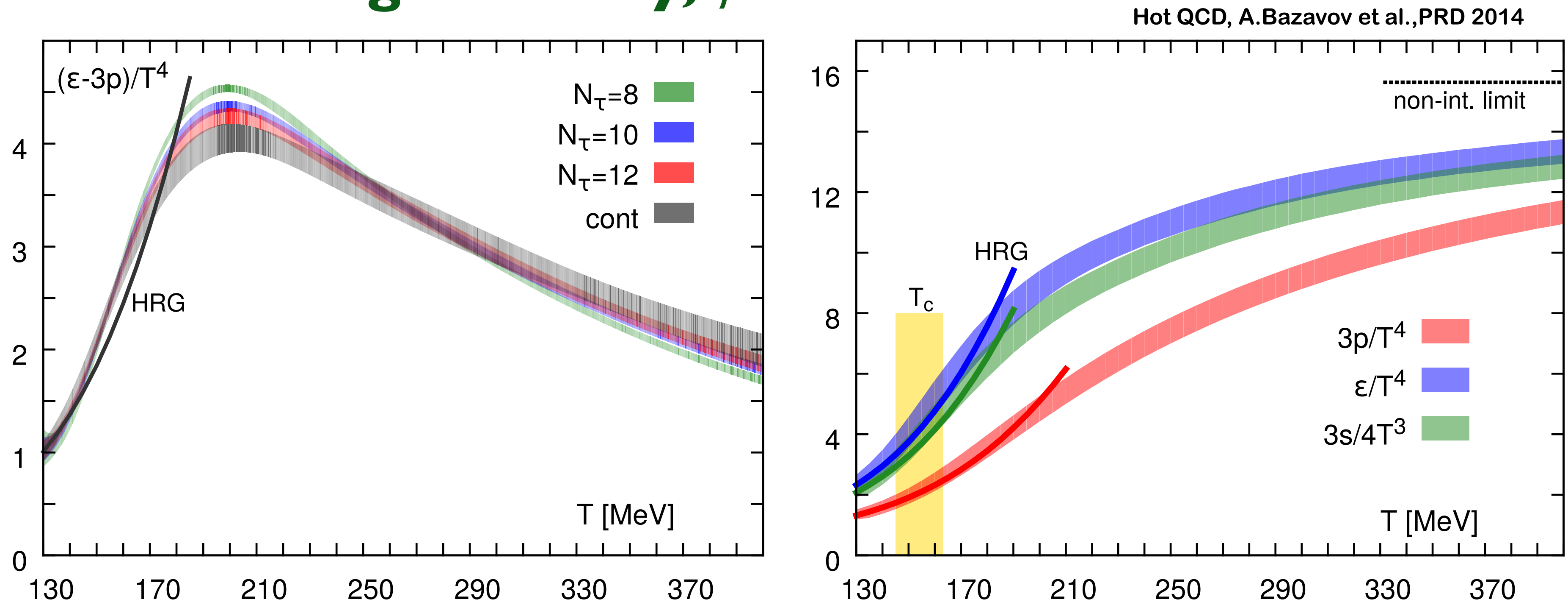
$$Z = \sum_i \langle i|e^{-\beta H}|i\rangle = \sum_{i_1, i_2, \cdots, i_n} \langle i_1|e^{-\delta\beta H}|i_2\rangle\langle i_2|e^{-\delta\beta H}|i_3\rangle\langle i_3|\cdots|i_n\rangle\langle i_n|e^{-\delta\beta H}|i_1\rangle$$

$$e^{-\delta\beta H} \approx 1 - \delta\beta H$$

$$\begin{aligned} |\eta\rangle &= \exp\left(\eta a^\dagger - \eta^* a\right)|0\rangle, \\ a|\eta\rangle &= \eta|\eta\rangle, \\ H(a^\dagger, a) &\to H(\eta^*, \eta), \\ \sum_i |i\rangle\langle i| &\to \frac{1}{\pi}\int d\eta_r d\eta_i |\eta\rangle\langle\eta| \end{aligned}$$

10x10x10x10 lattice ➜ 520,000 dimensional path integral
Needs VERY efficient Monte Carlo

Eq. of State:
Lattice Gauge Theory, μ=0

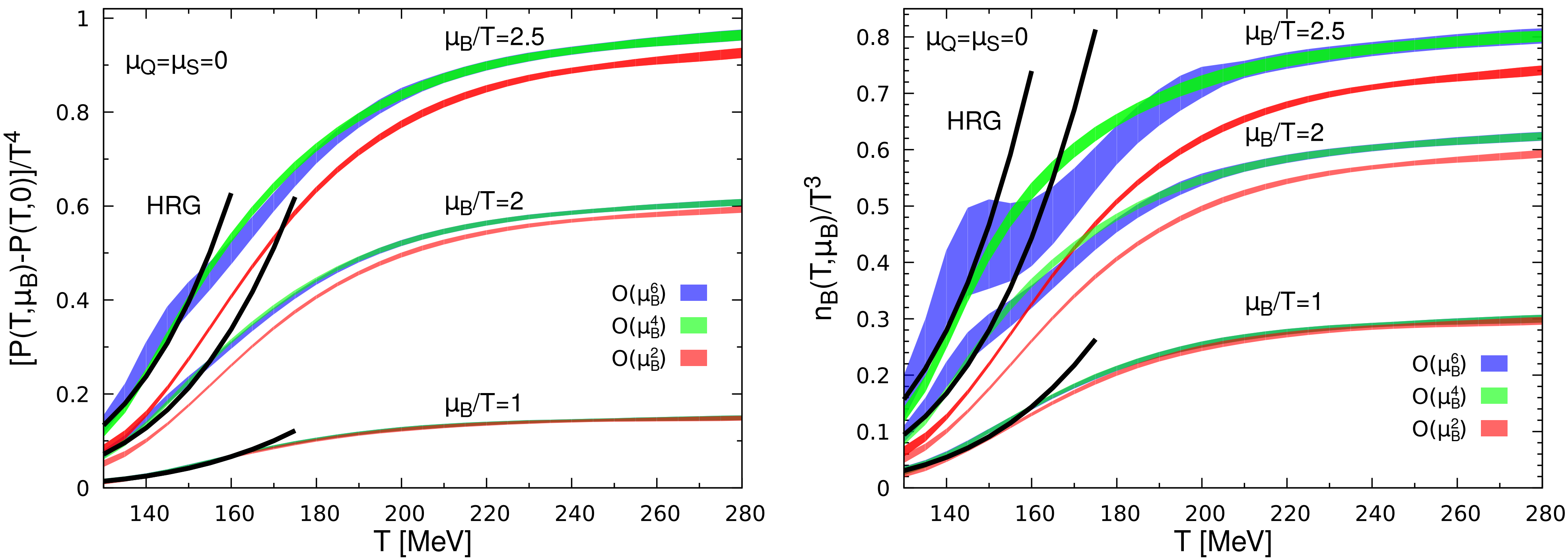


Sharp rise in ε/T⁴ near T~160 MeV

Rise from hadron resonances coming into play
Levels off in QGP

I. Eq. of State: Lattice Gauge Theory, $\mu \neq 0$

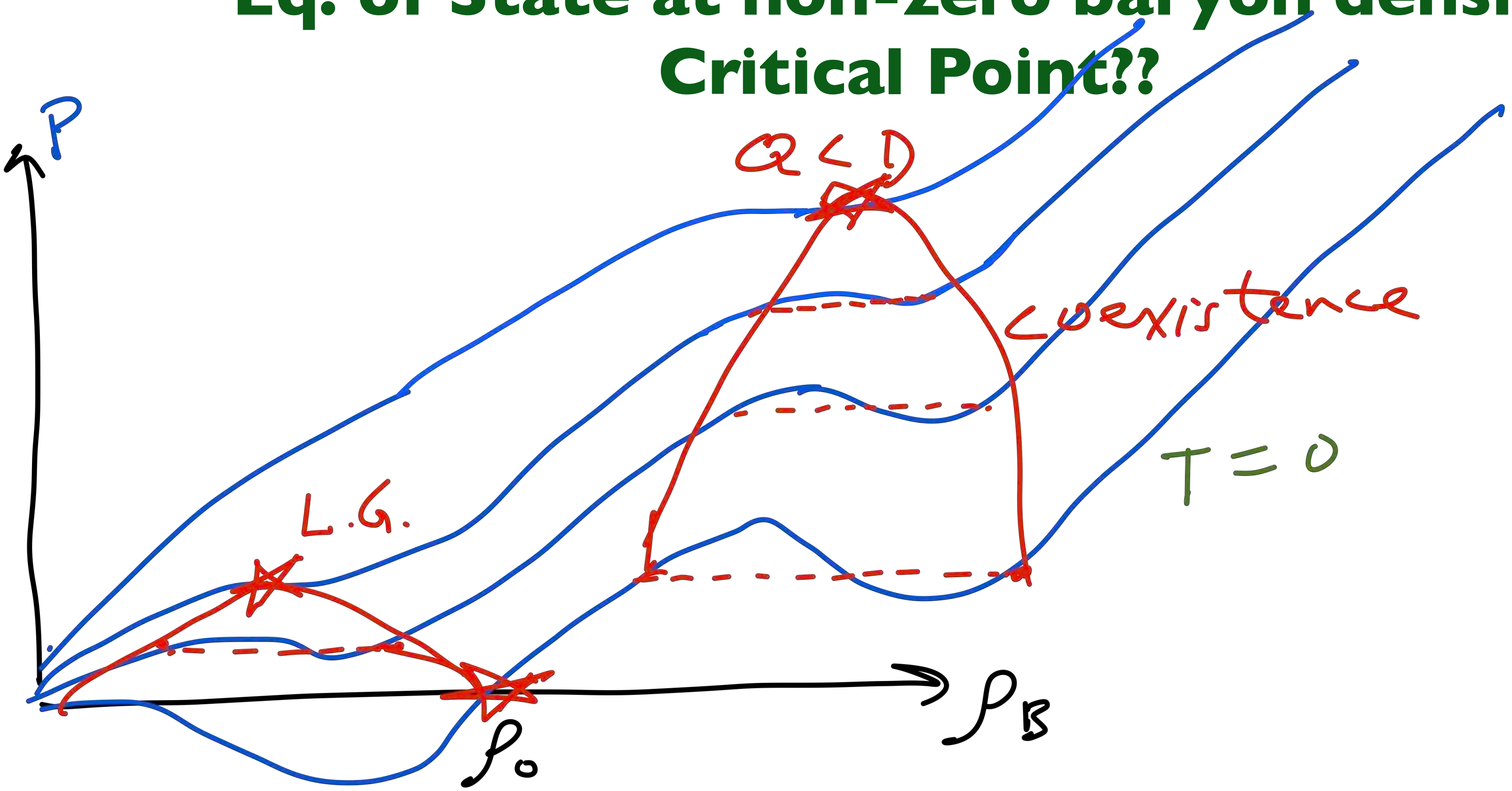
Hot QCD, A.Bazavov et al.,PRD 2017



Taylor expansion in μ_B.

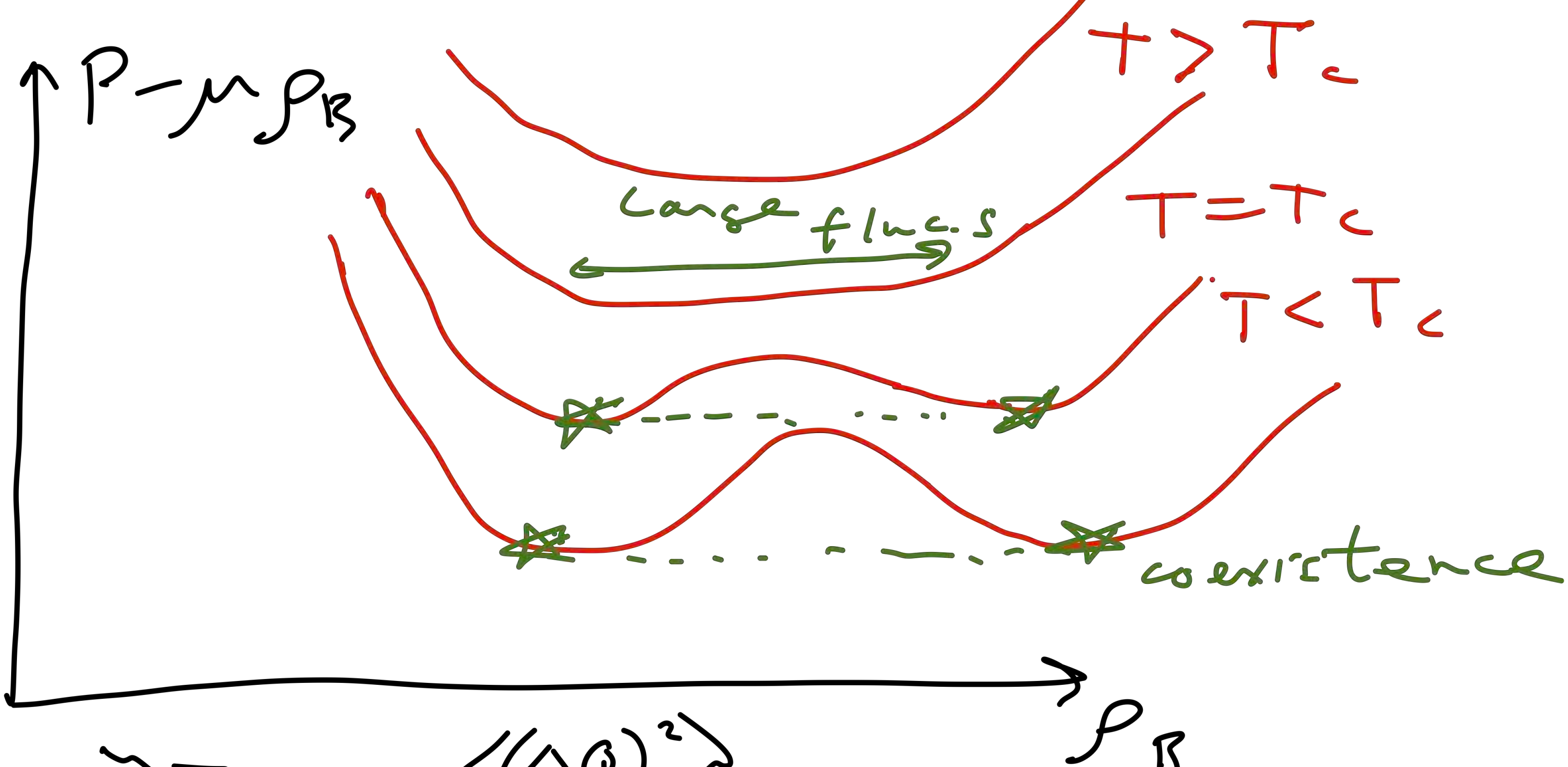
Funky behavior for T<150 MeV, Phase transition????

Eq. of State at non-zero baryon density
Critical Point??



QCD transition
might be 1st order

Critical Behavior



$$T \gtrsim T_c \to \frac{\langle (\Delta Q)^2 \rangle}{V} \to \infty$$

$$T < T_c \to \frac{\langle (\Delta Q)^2 \rangle}{V^2} = \infty$$

Fluctuations (things to keep in mind)

- **Over total volume charge does not fluctuate**
- **For $T \sim T_c$ fluctuations are slow to develop**
- **For $T << T_c$ phase separation can be fast (unstable)**
- **Dynamics are important**
- **Must study correlations,**

$$\langle \delta\rho(\boldsymbol{r},t)\delta\rho(\boldsymbol{r}',t)\rangle$$

Charge Susceptibility

For *μ*=0, $\langle \rho_a \rangle = 0$ **"*a*" refers to up, down, strange or baryon, strangeness elec. charge**

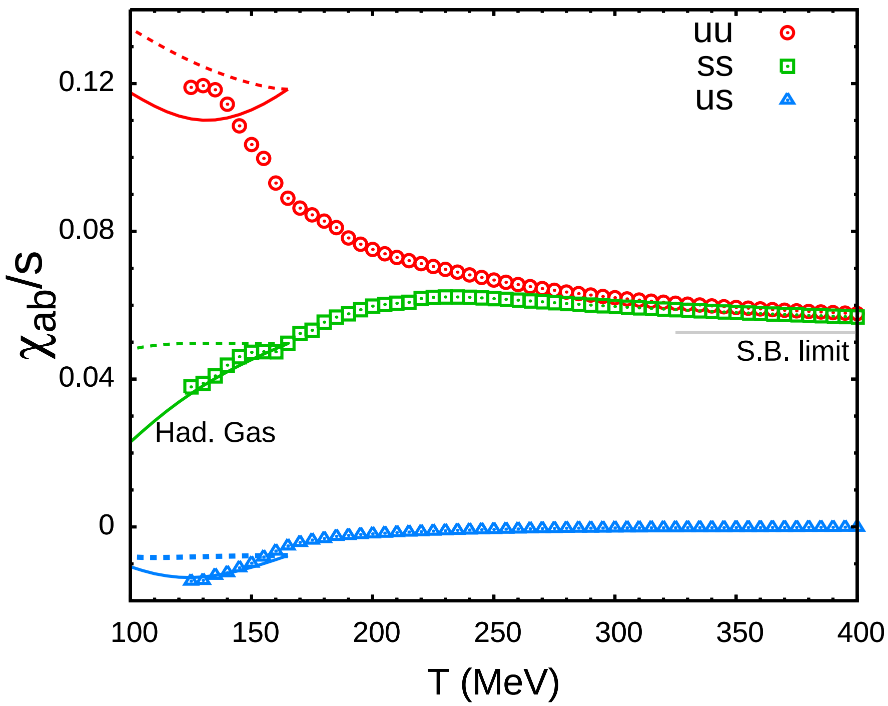
$$\langle (\delta\rho_a)(\delta\rho_b) \rangle \neq 0$$

$$= \sum_a (n_a + n_{\bar{a}})\delta_{ab}, \quad \text{quark gas}$$

$$= \sum_h n_h q_{ha} q_{hb}, \quad \text{hadron gas}$$

$\langle (\delta\rho)^3 \rangle$ **compared to** $\langle (\delta\rho)^2 \rangle$ **depends on existence of hadrons**

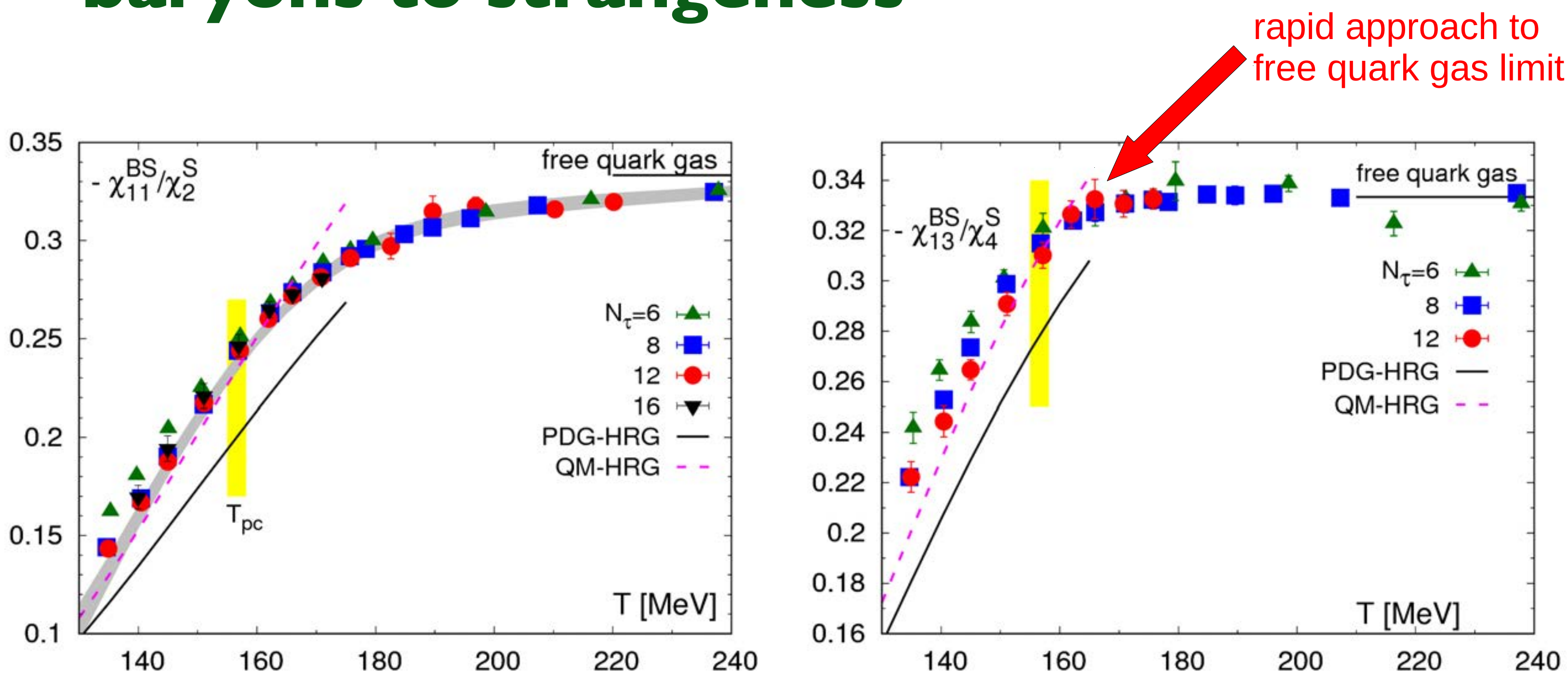
Charge Susceptibility from lattice



Off-diagonal elements disappear for T>190

Approach quark-gas limit at higher T

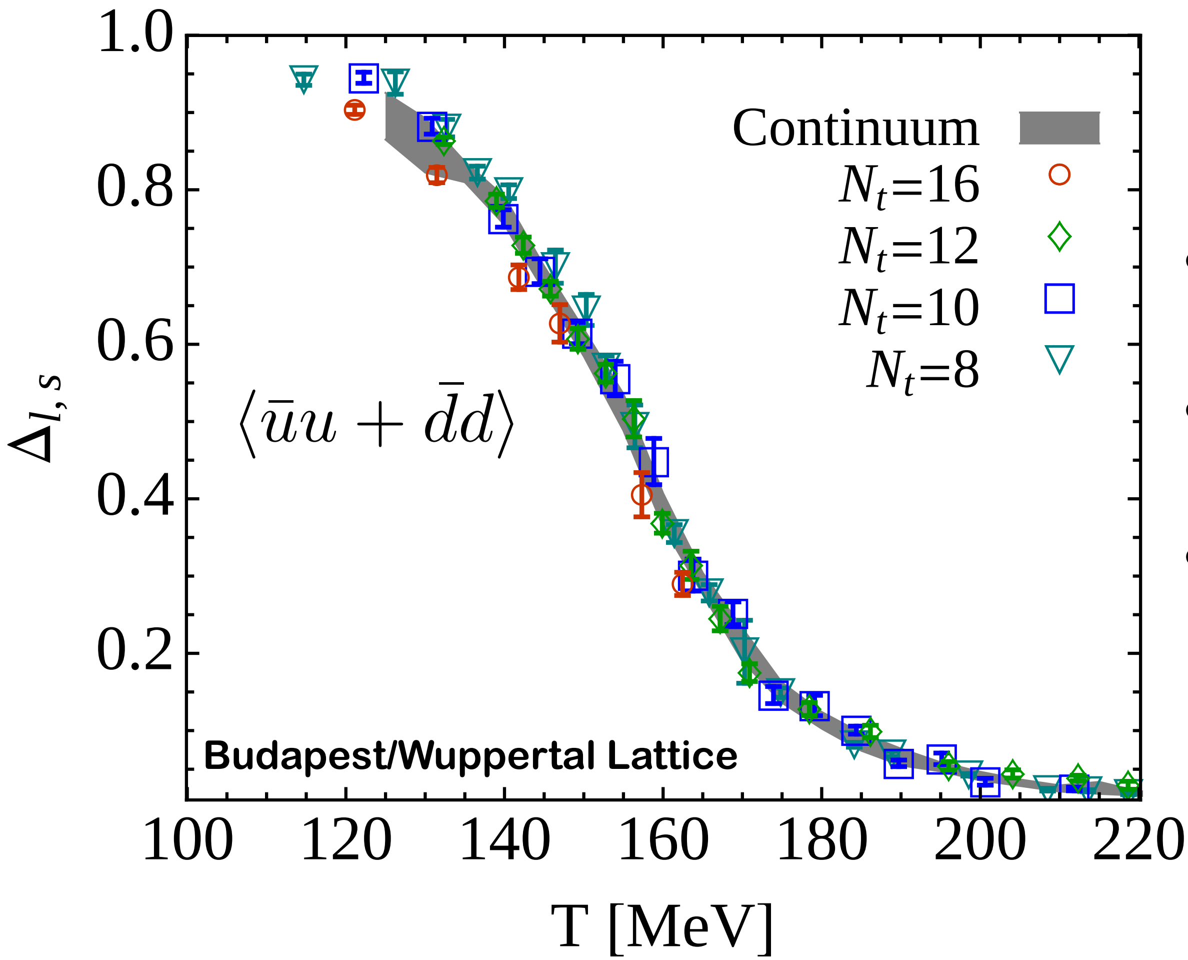
Susceptibilities: correlating baryons to strangeness



Bazavov et al., arXiV:1404.6511

Baryons dissolve for T>170 MeV

Quark-antiquark Condensate $\langle \bar{u}u + \bar{d}d \rangle$



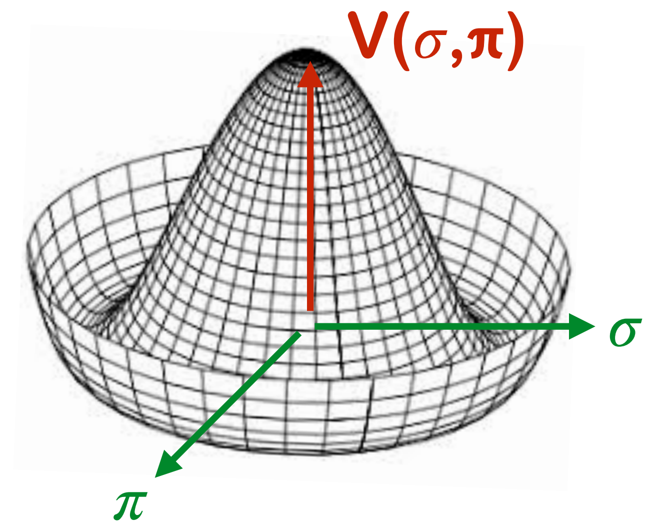
- Gives quarks "constituent" mass
- melts ~ T=160 MeV same T as QGP transition
- would be 2nd order if u,d quarks massless

Linear-sigma model

Chiral Symmetry in QCD

$$\Psi \rightarrow \exp(i\gamma_5 \vec{\tau} \cdot \vec{\theta})\Psi$$

Invariant for massless quarks



Hadronic degrees of freedom

$$\mathcal{L} = \frac{1}{2}\left\{\vec{\pi} \cdot \partial^2 \vec{\pi} + \sigma \partial^2 \sigma\right\} + \mathcal{V}(\sigma, \vec{\pi}),$$

$$\mathcal{V}(\sigma, \vec{\pi}) = -\frac{1}{2} m_0^2 \left(\vec{\pi}^2 + \sigma^2\right) + \frac{\lambda}{4}\left(\vec{\pi}^2 + \sigma^2\right)^2 - g\sigma,$$

$$\langle \sigma \rangle_{T=0} = f_\pi = 93 \text{ MeV}$$

Pion is Goldstone boson

Add coupling to baryons

$$\mathcal{L}_{\text{baryons}} = g_{\pi N N}\left[\bar{\Psi}\sigma\Psi + \bar{\Psi}(\vec{\pi} \cdot \vec{\tau})\gamma_5\Psi\right]$$

$$g_A M_B = g_{\pi N N} f_\pi$$

Goldberger-Treiman relation

Linear-sigma model
(chiral symmetry breaking)

Many variants: non-linear sigma model, NJL,
couple to quarks, gluons .....

Open issues

1. As σ ➜ zero,

   Do hadron masses go to ~zero
   or do they pair up? e.g. ρ and a_1

2. Is there any window with both restoration and where hadronic degrees of freedom are relevant? Lattice: maybe not

Transport Coefficients...

Diffusivity/Conductivity, Viscosity, Opacity, ...
Can be written in terms of thermal averages:

Linear response theory & Kubo relations

Example: Conductivity

$$\langle\Psi|J(x=0,t=0)|\Psi\rangle = \sigma E,$$

$$|\Psi(t=0)\rangle \approx |\Psi_0\rangle - \frac{i}{\hbar}\int_{-\infty}^{0} dt\ (-E(t)x\rho(x,t)dx|\Psi_0\rangle$$

$$\sigma = \frac{i}{\hbar}\int_{-\infty}^{0} dtdx\ x\langle[J(0,0),\rho(x,t)]\rangle$$

$$= \frac{-i}{\hbar}\int_{-\infty}^{0} dtdx\ xt\langle[J(0,0),\partial_t\rho(x,t)]\rangle$$

$$= \frac{i}{\hbar}\int_{-\infty}^{0} dtdx\ t\langle[J(0,0),x\partial_x J(x,t)]\rangle$$

$$= \frac{-i}{\hbar}\int_{-\infty}^{0} dtdx\ t\langle[J(0,0),J(x,t)]\rangle$$

$$= \frac{-i}{\hbar}\int_{-\infty}^{\infty} dtdx\ t\langle J(0,0)J(x,t)\rangle$$

Kubo relation (commutator)

Deriving "anti-commutator" Kubo relation

Connection to classical physics

$$G(t) \equiv \langle A(0)A(t)\rangle$$

$$= \mathrm{Tr}\; e^{-\beta H} A e^{iHt/\hbar} A e^{-iHt/\hbar}$$

Cyclic property of trace

$$G(i\hbar\beta/2 + z) = \mathrm{Tr}\; e^{-\beta H} A e^{iHz/\hbar - \beta H/2} A e^{-iHz/\hbar + \beta H/2}$$

$$= \mathrm{Tr}\; e^{-\beta H} A e^{-iHz/\hbar - \beta H/2} A e^{iHz/\hbar + \beta H/2}$$

$$= G(i\hbar\beta/2 - z)$$

“anti-commutator" Kubo relation

$$\sigma = \frac{-i}{\hbar}\int_{-\infty}^{0} dt\ t(G(t) - G(-t))$$

$$= \frac{-i}{\hbar}\int_{-\infty}^{\infty} dt\ tG(t)$$

$$0 = \oint_P dz\ (z - i\hbar\beta/2)\ G(z)$$

$$= I_1 + I_2 + I_3$$

$$I_2 = 0$$

$$I_1 = I_4$$

cyclic prop. of trace

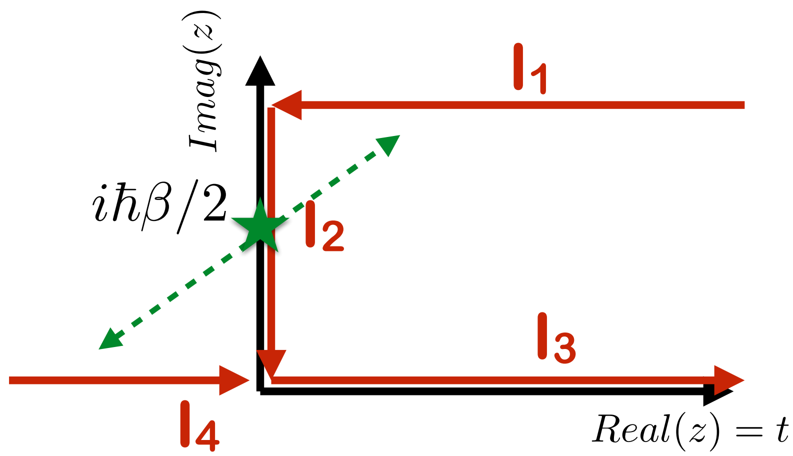
$$0 = I_4 + I_3$$

$$= \int_{-\infty}^{\infty} dt\ (t - i\hbar\beta/2)G(t)$$

$$\sigma = \frac{1}{2T}\int_{-\infty}^{\infty} dt\ G(t)$$

$$\sigma = \frac{1}{2T}\int_{0}^{\infty} dt\ (G(t) + G(-t))$$

$$\sigma = \frac{1}{2T}\int_{-\infty}^{0} dt\ dx\ \langle\{J(0,0), J(x,t)\}\rangle$$



$$I_1 + I_2 + I_3 = 0, \quad \text{analyticity}$$

$$I_2 = 0, \quad \text{symmetry}$$

$$I_1 = I_4, \quad \text{symmetry}$$

$$I_4 + I_3 = 0$$

anti-commutator
classical or quantum

Transport Coefficients

$$\sigma = \frac{-i}{\hbar}\int_{-\infty}^{\infty} dtdx\ t\langle J(0,0)J(x,t)\rangle,$$
$$= \frac{1}{2T}\int_{0}^{\infty} dtdx\ \langle\{J(0,0),J(x,t)\}\rangle$$

Kubo: correlations in real time

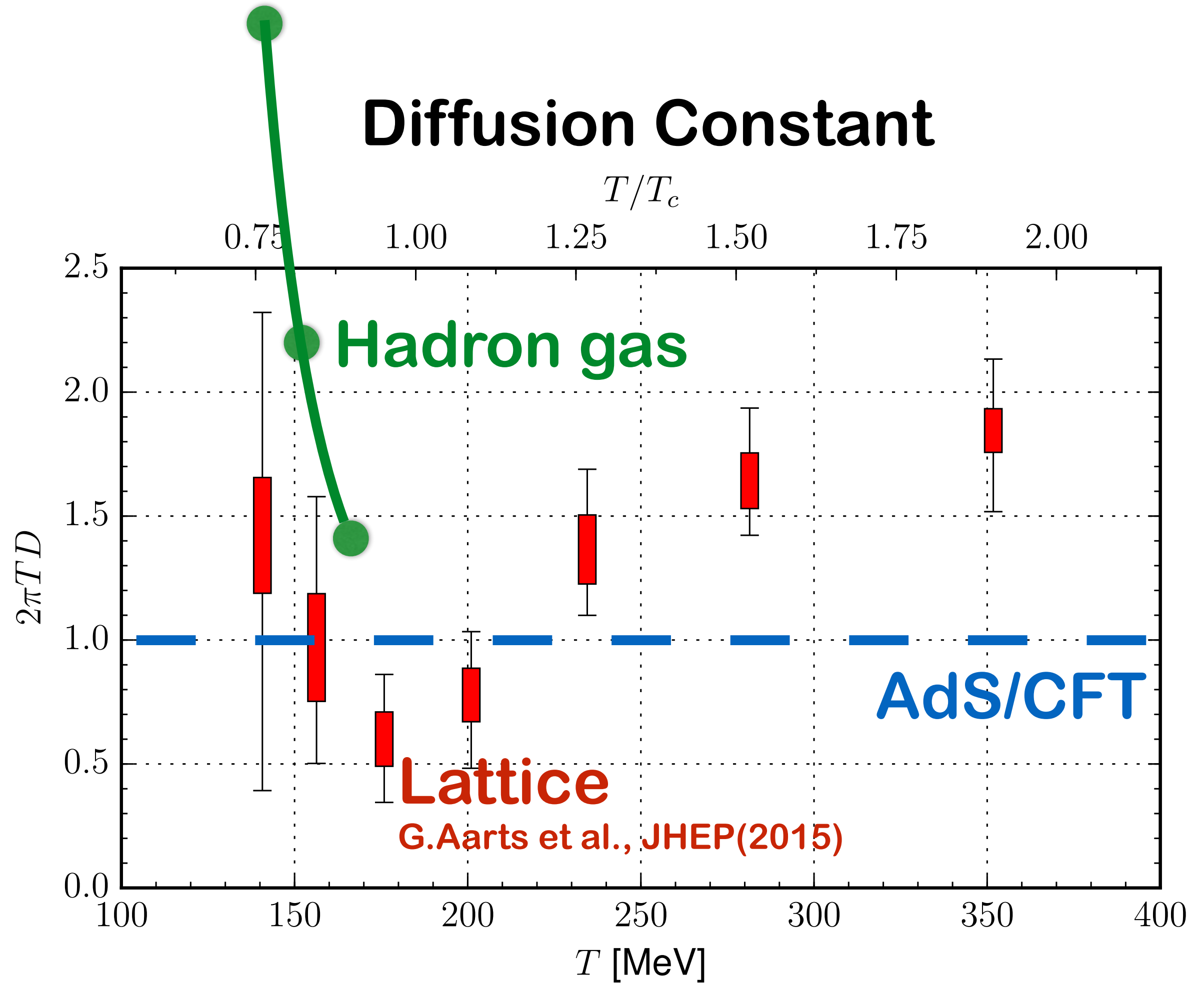
$$Z = \sum_i \langle i|e^{-\beta H}|i\rangle = \sum_{i_1,i_2,\cdots,i_n} \langle i_1|e^{-\delta\beta H}|i_2\rangle\langle i_2|e^{-\delta\beta H}|i_3\rangle\langle i_3|\cdots|i_n\rangle\langle i_n|e^{-\delta\beta H}|i_1\rangle$$
$$e^{-\delta\beta H} \approx 1 - \delta\beta H$$

Lattice: calculates in imaginary time

Analytic continuation involves difficult-to-constrain errors
Lattice results for conductivity, diffusivity, viscosity…
are suspect

Diffusivity/Conductivity

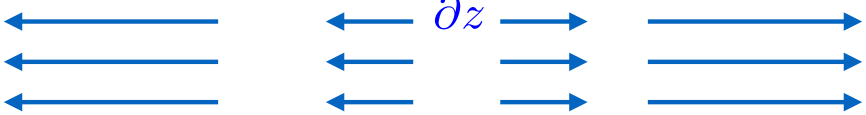
$$J = -\sigma \frac{dV}{dx}$$
$$= -e\sigma \frac{d\mu}{dx}$$
$$= -e\sigma \frac{d\rho}{dx} \frac{1}{d\rho} d\mu$$
$$= -\frac{e\sigma}{\chi} \frac{d\rho}{dx}$$
$$J/e = -D \frac{d\rho}{dx}$$
$$D = \frac{\sigma}{\chi}$$



Viscosity

$$\pi_{ij} = -\eta\left[\partial_i v_j + \partial_j v_i - (2\nabla\cdot\boldsymbol{v})\delta_{ij}/3\right] - \zeta\nabla\cdot\boldsymbol{v}$$

$$\frac{\partial v_z}{\partial z} > 0$$



$$T_{zz} - T_{xx} = -2\eta\partial_z v_z$$

Less work, less cooling

Kubo relation

Same steps as conductivity:

$$J \to T_{ij},$$
$$E \to \partial_i v_j,$$
$$H_{\rm int} = T_{0i} r_j \partial_j v_i,$$
$$\eta = \frac{1}{2T}\int_{-\infty}^{\infty} dt d^3r\, \langle T_{xy}(0,0)T_{xy}(\boldsymbol{r},t)\rangle$$

For gas,

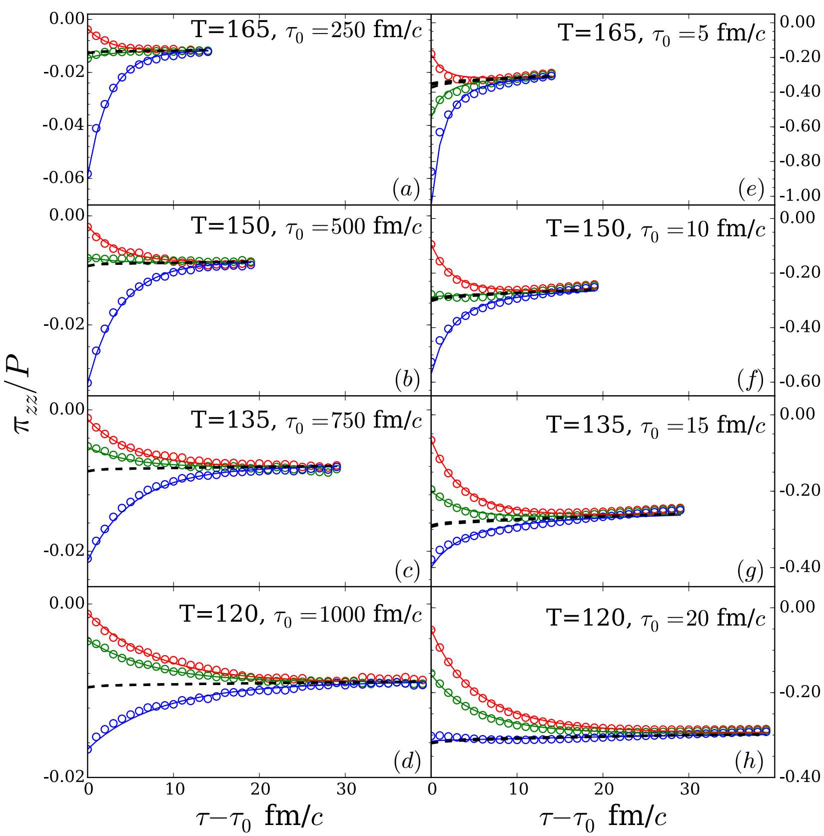
$$\eta = \sum_h \frac{(2S_h+1)}{2T}\int \frac{d^3p}{(2\pi\hbar)^3} e^{-E/T}\frac{p_x^2 p_y^2}{E^2}\tau_{\rm relax.},$$

$$\tau_{\rm relax.} \approx 2\tau_{\rm coll.}$$

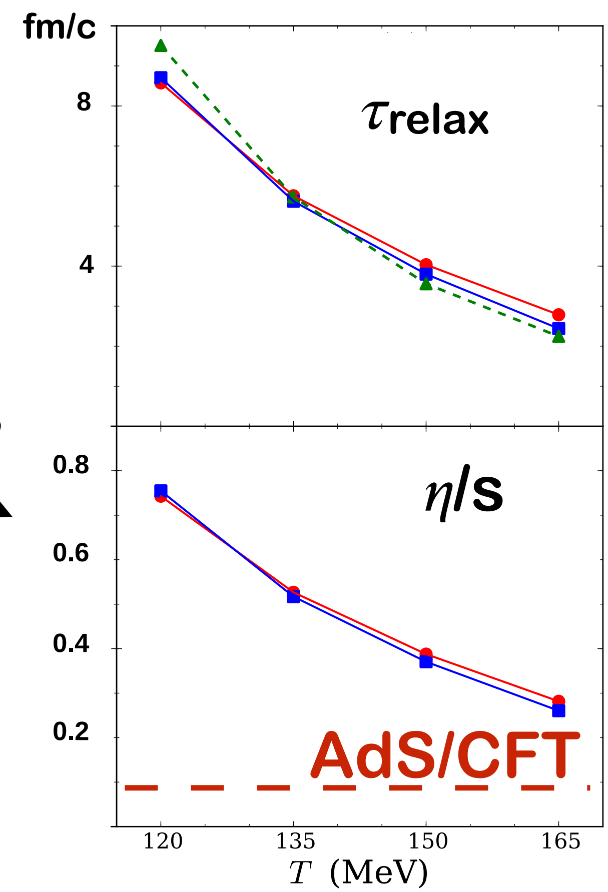
Viscosity: Kubo relation

Kubo: $$\eta = \sum_h \frac{(2S_h+1)}{2T} \int \frac{d^3p}{(2\pi\hbar)^3} e^{-E/T} \frac{p_x^2 p_y^2}{E^2} \tau_{\rm relax.},$$

$$\tau_{\rm relax.} \approx 2\tau_{\rm coll.}$$



Simulation of hadron gas



Electromagnetic Emissivity and Opacity

Photons make it through QGP fireball (90% of time)
"Penetrating probe"
Both direct photons and dilepton production

$$E\frac{dN}{d^4x\ d^3p} = \sum_h \epsilon_{hi}\epsilon_{hk} \int d^3r\ dt\ e^{iEt - i\boldsymbol{p}\cdot\boldsymbol{r}} \langle J_i(0,0) J_k(\boldsymbol{r},t)\rangle$$

In practice, emissivity calculated as sum of MANY microscopic processes:

hadronic decays, Bremmstrahlung

Jet Opacity

- Similar to photons
  — non-Abelian nature
- Anne Sickles will cover this

Open questions for HOT matter

- Equation of state for large ρ_B
  — Can you calculate it?
  — Is there a 1st-order phase transition?

- Transport coefficients
  — correlators in real time

- Role of chiral symmetry restoration in hadronic gas?