Lecture 2

Low Energy Nuclear Theory

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LONI
Louisiana Optical Network Initiative

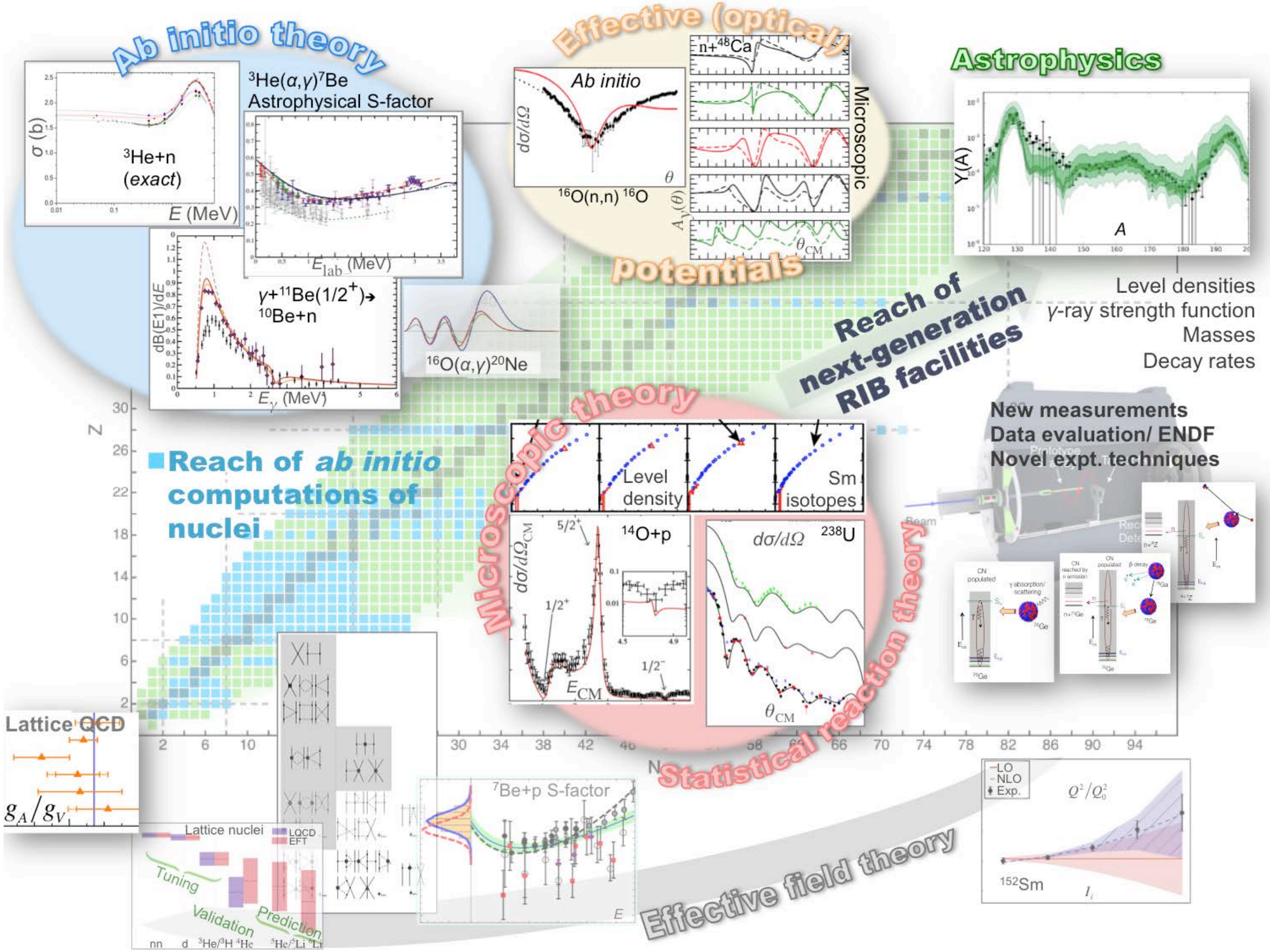
LIGO

Lecture 2

Modeling nuclei: structure …and reactions

I will focus on how to build on first principles (rooted in QCD)

… EFT approaches are also powerful (halo-EFT, EFT for deformed nuclei, etc.)



From INT-17-1a program "Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart"

Interaction Renormalization

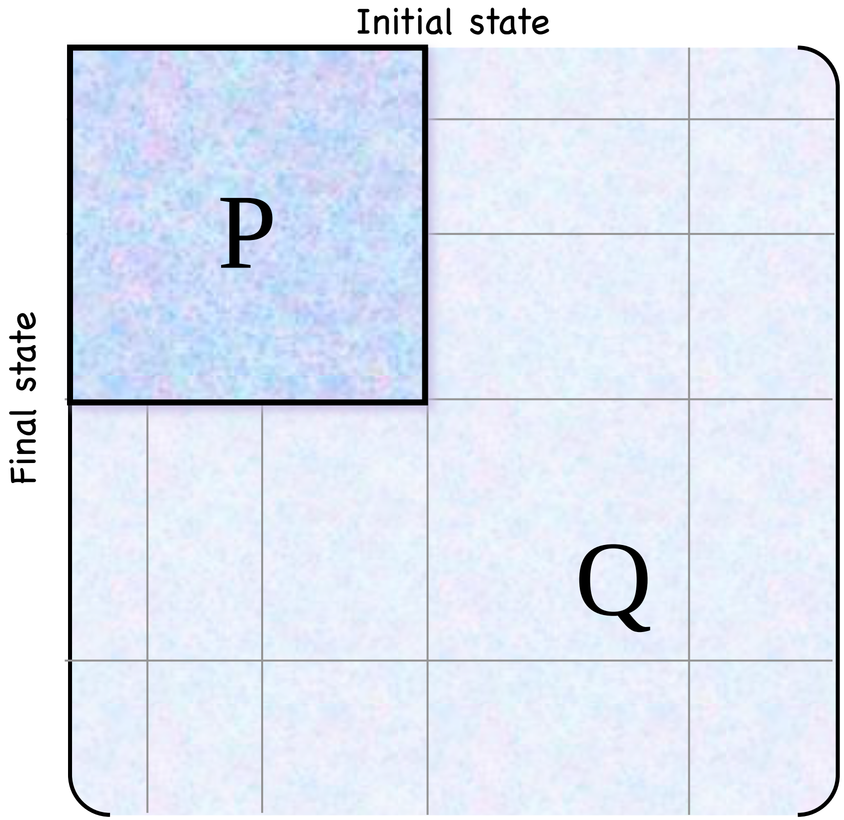
Effective interactions...

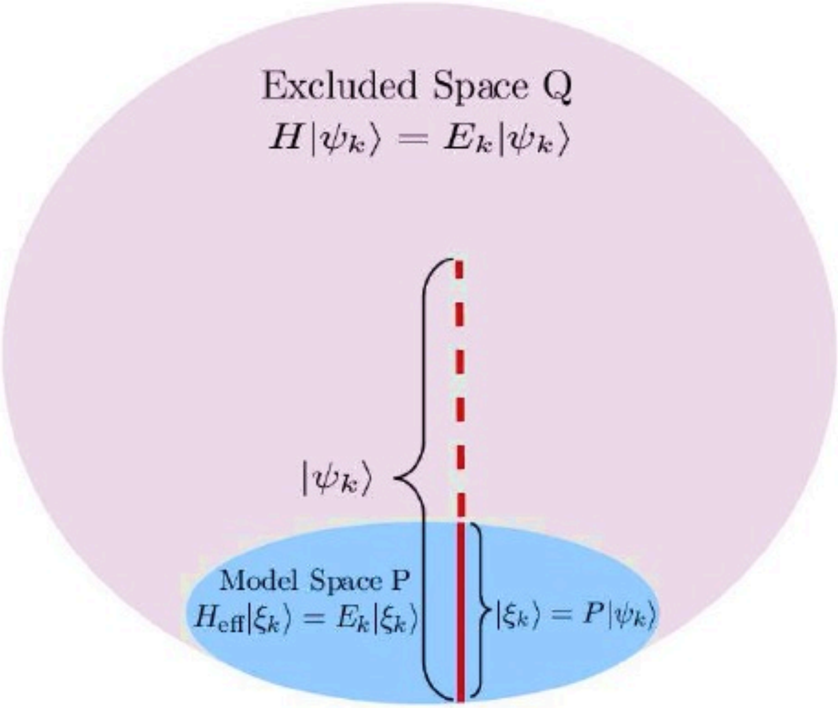
Effective interactions: renormalization

P+Q … Infinite space → P … Model space

$UHU^\dagger$

2 major techniques: Projection & Infinitesimal Rotations





Lee-Suzuki technique:

Project to P space, while retaining the eigenvalues

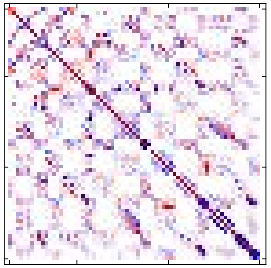
Similarity Renormalization Group (SRG)

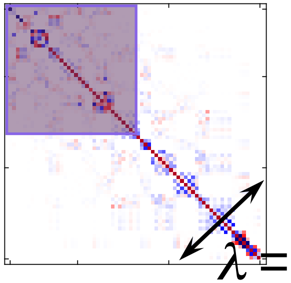
"Bare" $H_{s=0}$

$$H_s = U(s) H_{s=0} U^{\dagger}(s)$$

Renormalized…
H_s

Unitary transformation





$\lambda = \frac{1}{\sqrt{s}}$

Large model space

Many-body approach

Small subspace

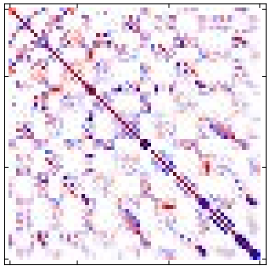
"Simple"… NN, 3N

Interaction

Complex … many-body interactions (3b, 4b, 5b, etc.)

Similarity Renormalization Group (SRG)

"Bare" $H_{s=0}$

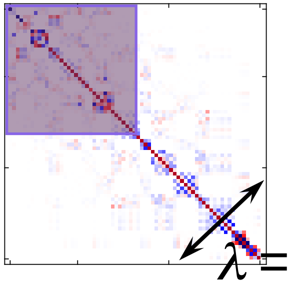


$$\boxed{H_s = U(s) H_{s=0} U^\dagger(s)}$$

Unitary transformation

Renormalized…

H_s



$$\lambda = \frac{1}{\sqrt{s}}$$

flow equation: $$\boxed{\frac{dH_s}{ds} = \left[\left[C, H_s\right], H_s\right]}$$

reference operators

(C could be Trel, symmetry operator, etc.)

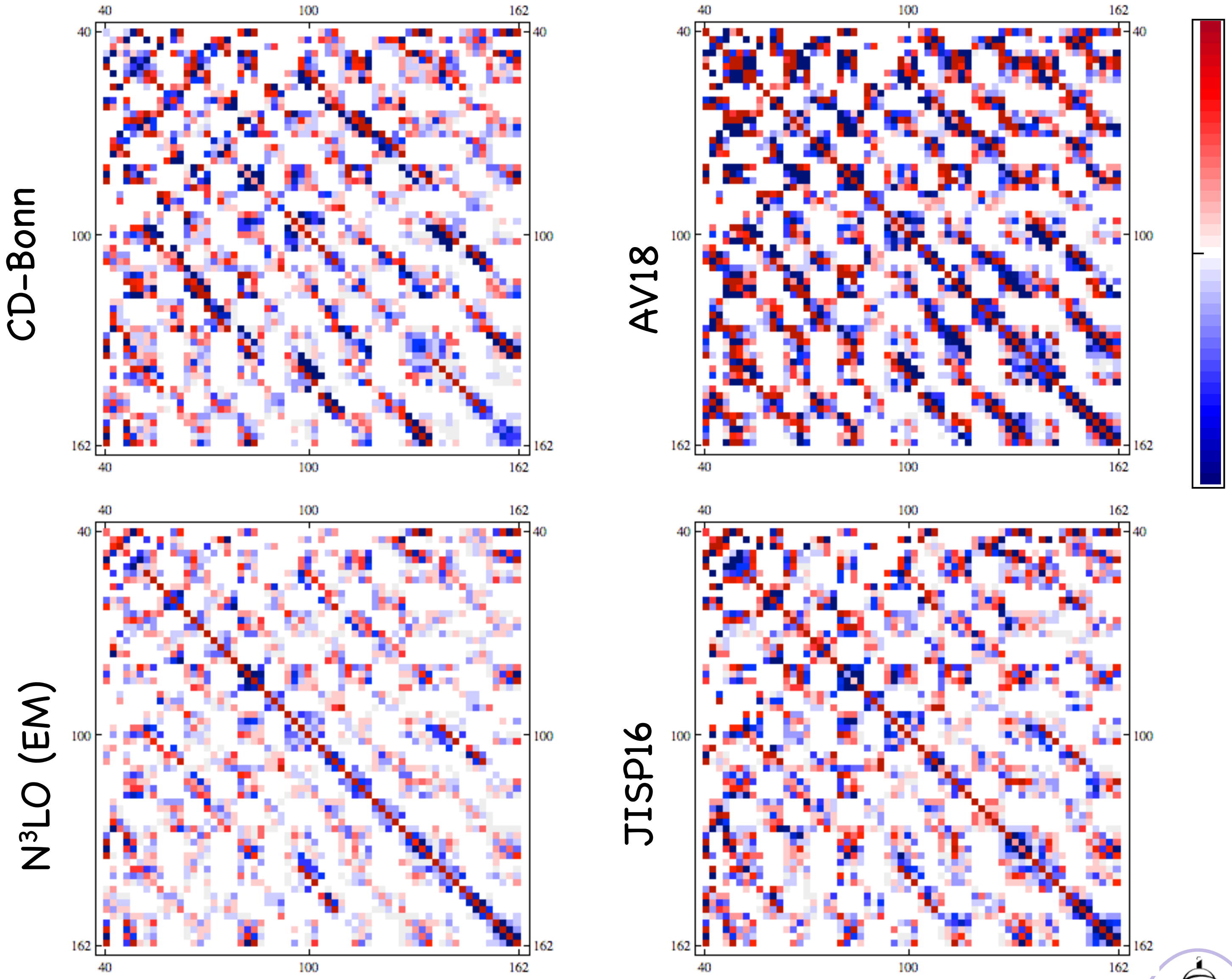
With 1b C: $\xrightarrow[2b]{H_0}$ $$\boxed{\frac{dH_0}{ds} = \left[\left[C, H_0\right], H_0\right]}$$ $\xrightarrow[3b]{H_1}$ $$\boxed{\frac{dH_1}{ds} = \underbrace{\left[\left[C, H_1\right], H_1\right]}_{\text{up to 5b}}}$$ … and so on

two-body

three-body

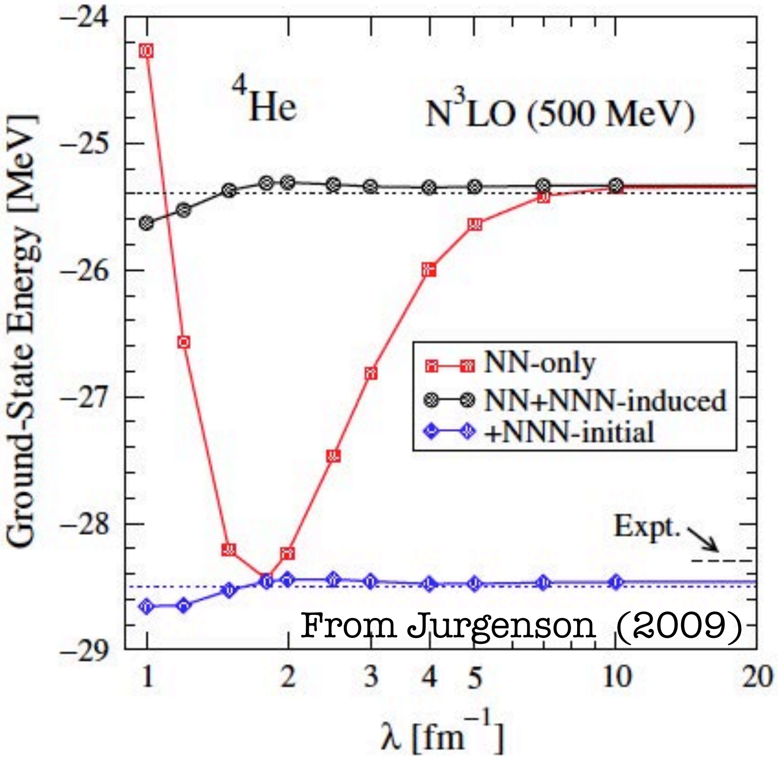
SRG - Simple Illustration

- Bare NN+ Relative Kinetic Energy
- Decouples model space



Similarity Renormalization Group (SRG) for Nuclear Physics

- He-4
  - SRG-evolved chiral potentials
    - 3-body important
    - 4-body negligible in He-4 (for binding energy)



From Jurgenson (2009)

Similarity Renormalization Group (SRG) for Nuclear Physics

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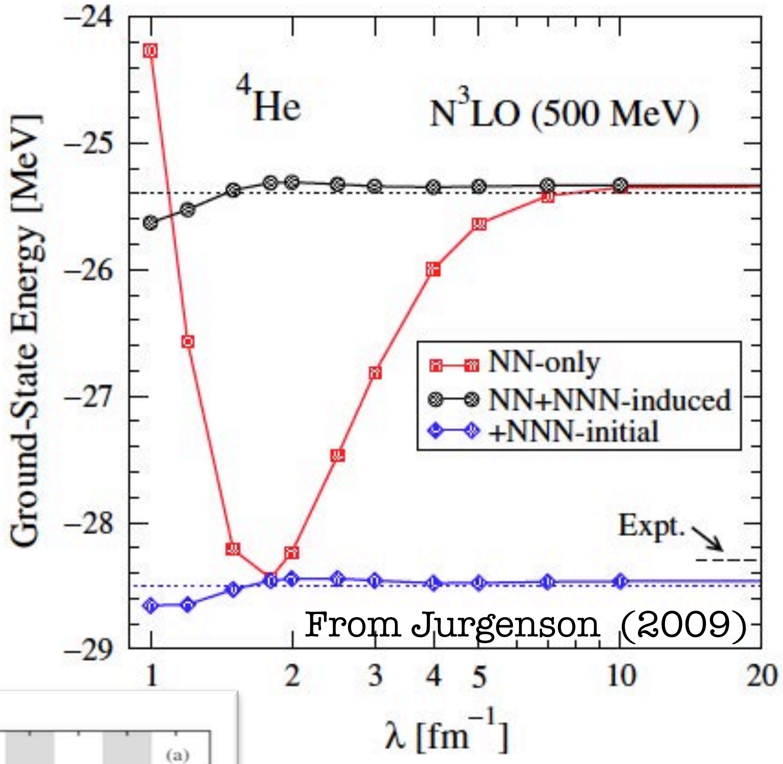
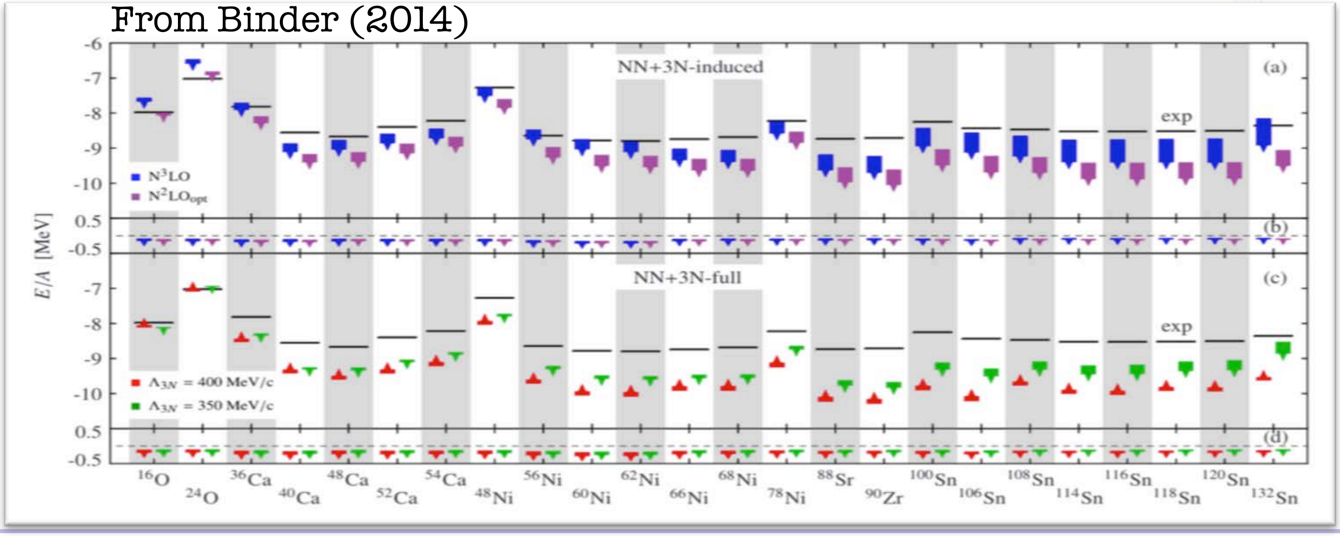
- SRG-induced interactions become important in heavy nuclei!



From Jurgenson (2009)

From Binder (2014)



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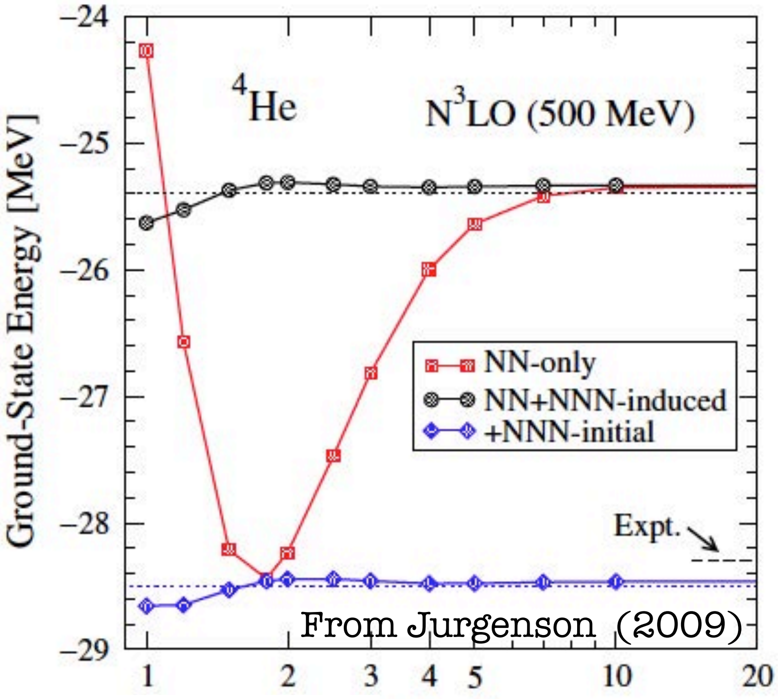
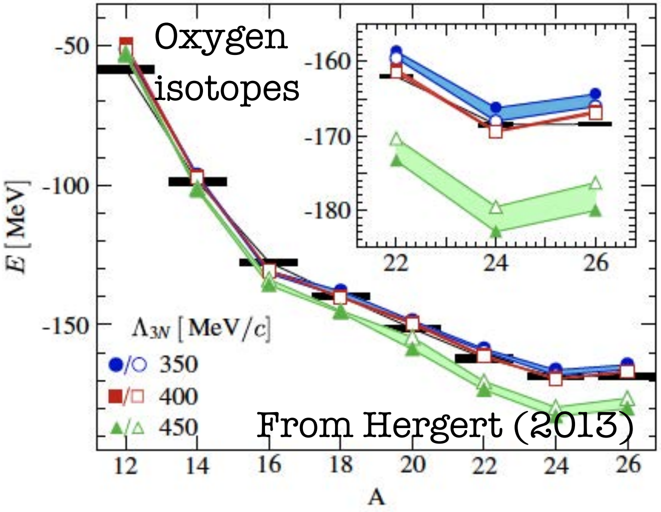
- SRG-induced interactions become important in heavy nuclei!

- SRG can evolve the entire nuclear Hamiltonian: In-medium SRG (IM-SRG)

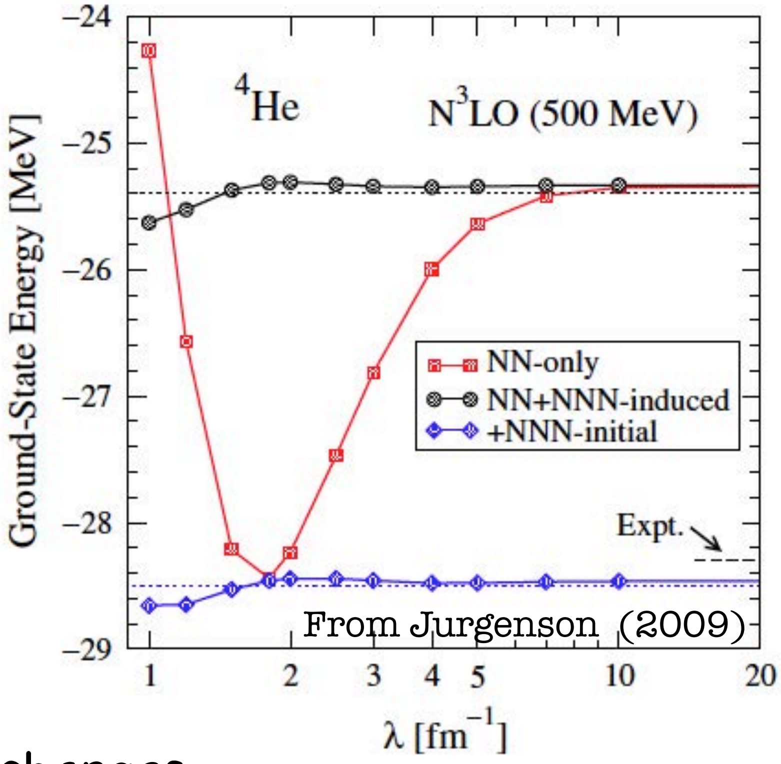


From Jurgenson (2009)



From Hergert (2013)

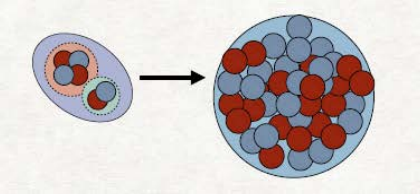
Similarity Renormalization Group (SRG) for Nuclear Physics



From Jurgenson (2009)

- He-4
  - SRG-evolved chiral potentials
    - 3-body important
    - 4-body negligible in He-4 (for binding energy)
- SRG-induced interactions become important in heavy nuclei!
- Important: interaction renormalization changes nuclear wave functions $|\Psi_s\rangle$; to calculate observables, the operators need to be renormalized too
  - E.g., for rms radii, need to use $\langle \Psi_s | U(s) \left(\sum_i \hat{r}_i^2 \right) U^+(s) | \Psi_s \rangle$
  - Nontrivial (handling many-body operators)

Effective interaction for reactions

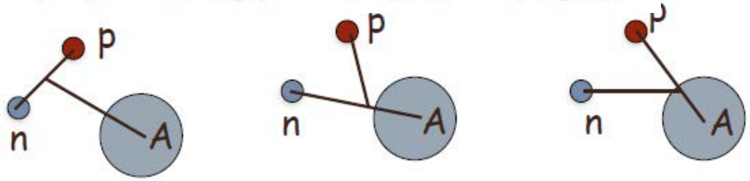


Many-body
problem

Exact solutions…

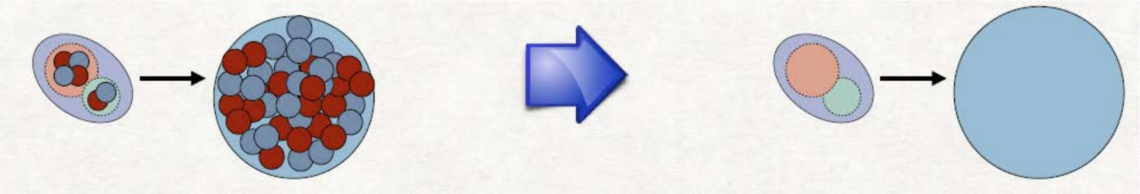
Faddeev equations (A=1):

$$|\Psi\rangle = |\psi_{np}\rangle + |\psi_{nA}\rangle + |\psi_{pA}\rangle$$



Exact solutions exist to about 5 nucleons.

Can we use this technique for larger A?

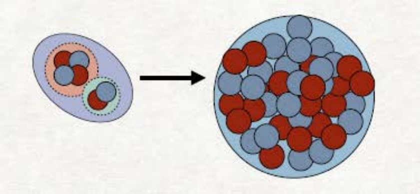


Many-body
problem

Few-body
problem

Reducing the many-body
problem to a few-body problem
induces effective interaction
(optical potentials)

Effective interaction for reactions



Many-body problem

Exact solutions…

Faddeev equations (A=1):

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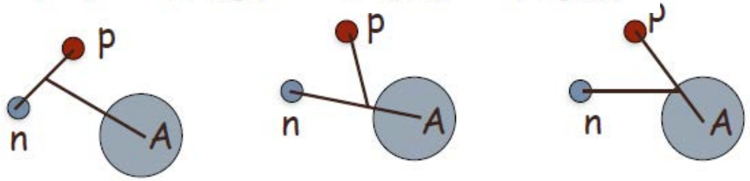

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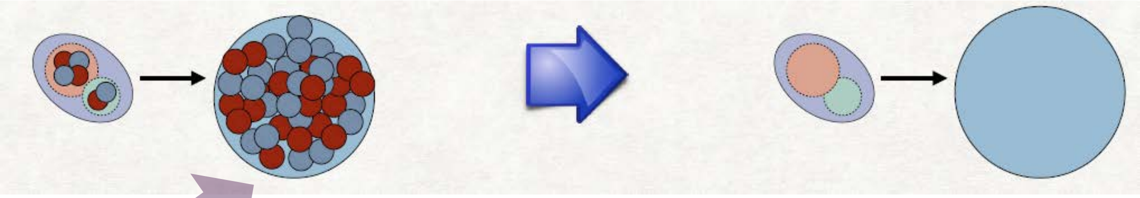


Many-body problem

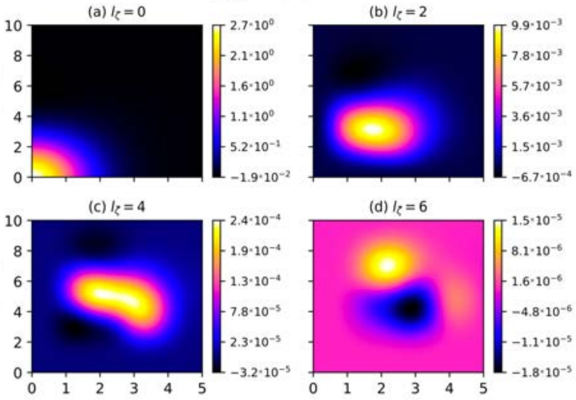
Few-body problem

Use ab initio techniques to solve for the structure of target, and to derive effective interactions

Reducing the many-body problem to a few-body problem induces effective interaction (optical potentials)

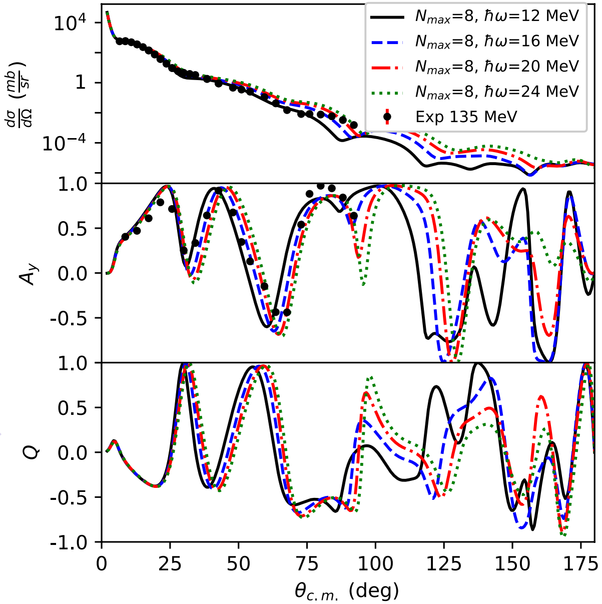
Scattering observables from first principles

^{6}Li, ab initio densities

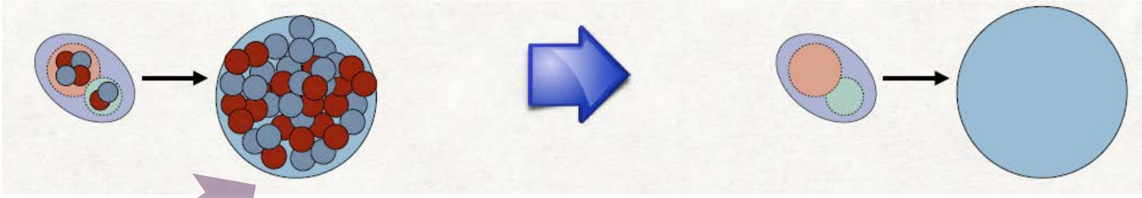


Proton scattering
off target ^{16}O
@ 135MeV
(NNLOopt)

First-principle derived
effective interaction



From Burrows (2018, 2019)



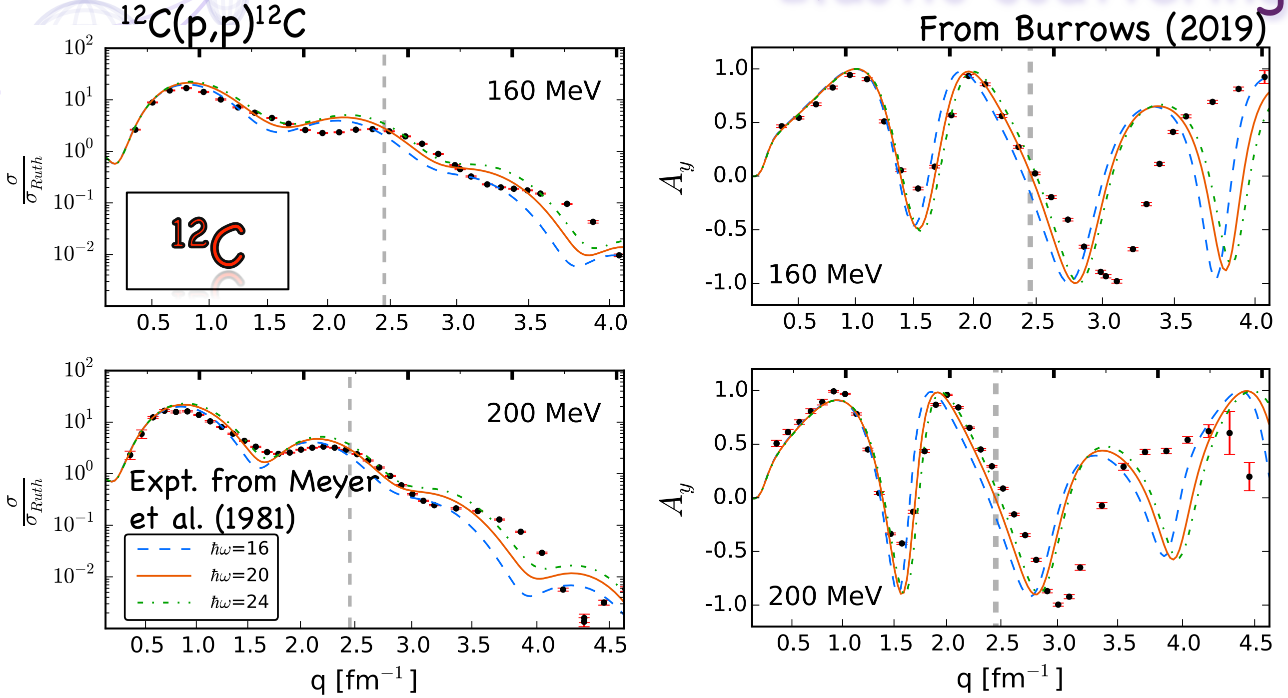
Many-body
problem

Few-body
problem

Use ab initio
techniques to solve
for the structure of
target, and to derive
effective interactions

Reducing the many-body
problem to a few-body problem
induces effective interaction
(optical potentials)

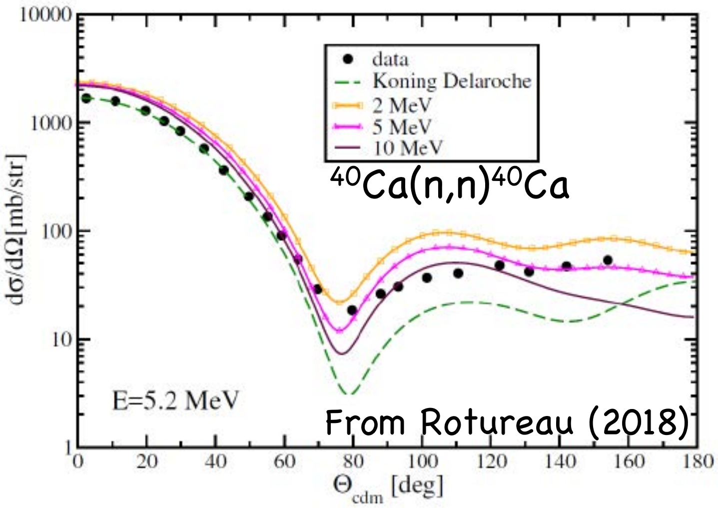
Elastic scattering



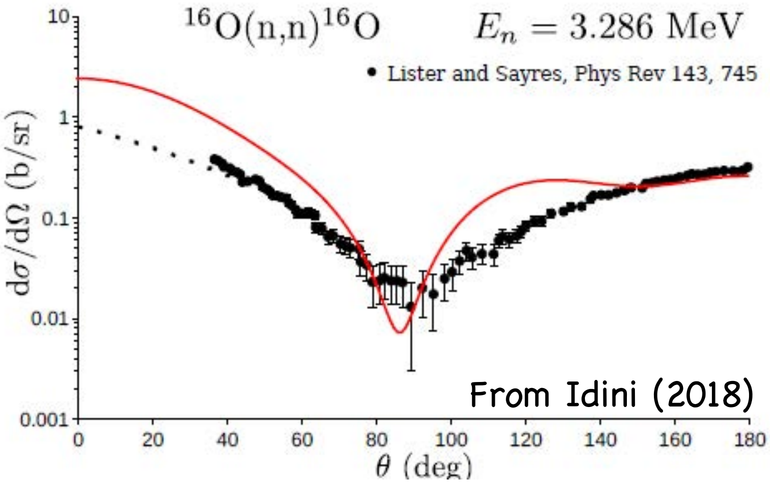
From Burrows (2019)

Higher energies
(multiple scattering approach)

Low energy (~10 MeV)
(Feshbach projection)

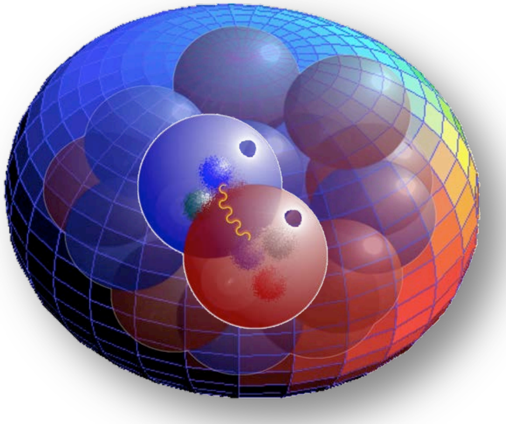


From Rotureau (2018)



From Idini (2018)

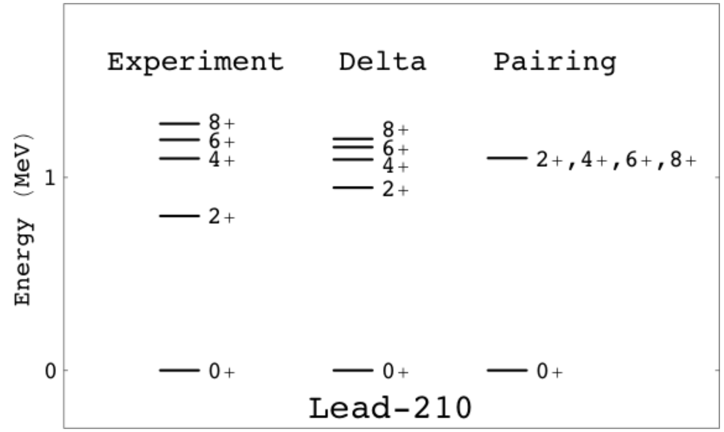
Nuclei



Many-body approaches

Inside the Nucleus … the Insights

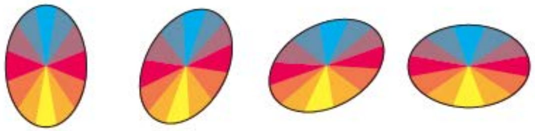
Irrotational flow



Nuclear ``superfluidity”:

- Pairing gap: higher first 2^+.
- Two-particle (2n or 2p) transfer enhancement.
- Low moment of inertia

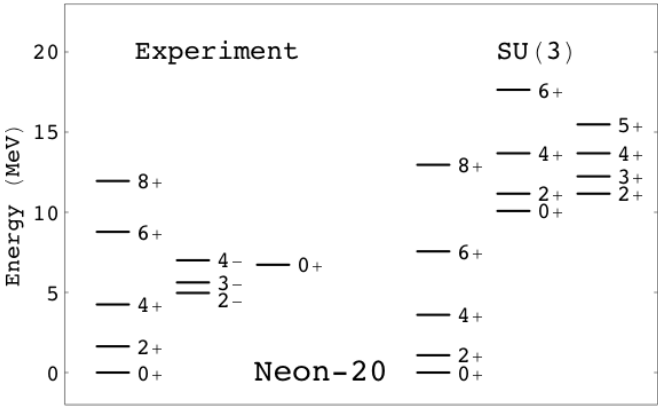
Irrotational-flow rotation



From Rowe (2013)

Rotational modes

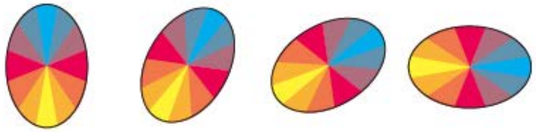
SU(3) model (Elliott model): shell model of deformation/rotations



Nuclear “rigid rotor”:

- E_J~J(J+1) => E_{4+}/E_{2+}=3.33: “3.33-rule”
- Rotations of deformation
- High moment of inertia

Rigid flow rotation

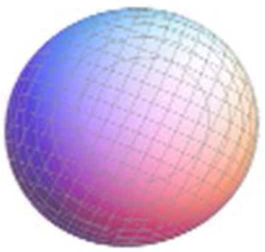


From Rowe (2013)

Inside the Nucleus … the Insights

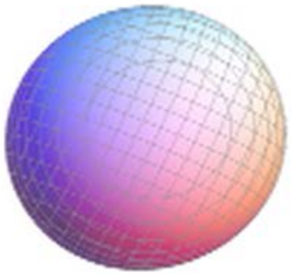
Vibrational modes

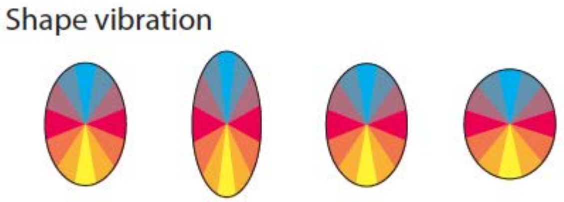
Intrinsic frame:



Giant resonance
- monopole -
(breathing mode)

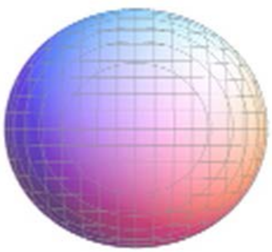
Lab frame:





From Rowe (2013)

<u>Nuclear compressibility</u>
rather stiff: ~$80A^{-1/3}$ MeV



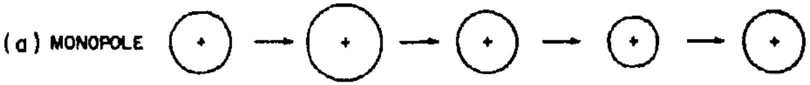
- quadrupole -

Low-energy vibrations not likely

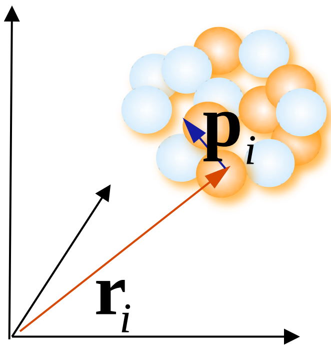
Surface radius for λ–multipole vibrations (<u>lab frame</u>):

$$R(\theta,\varphi;t) = R_0\left\{1+\sum_{\lambda\mu}\alpha_{\lambda\mu}(t)Y_{\lambda\mu}(\theta,\varphi)\right\}$$

$\lambda=0$



The shell model



- A nucleons of mass m_N: $\mathbf{r}_1$, $\mathbf{p}_1$; $\mathbf{r}_2$, $\mathbf{p}_2$;…; $\mathbf{r}_A$, $\mathbf{p}_A$;

$$\boxed{\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)}$$

- Many-body Hamiltonian = kinetic energy + potential energy):

$[\mathbf{p} = -i\hbar\nabla]$

$$\boxed{H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i,j=1(i<j)}^{A} V_{NN}\left(\mathbf{r}_i - \mathbf{r}_j\right) + \sum_{i<j<k} (V_{NNN})_{ijk} + \ldots}$$

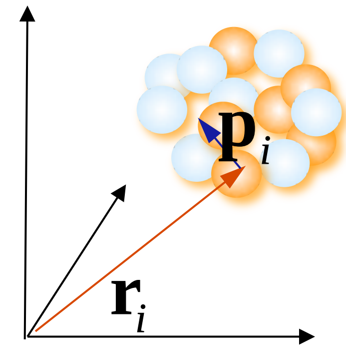
- …actually, relative kinetic energy:

$$\boxed{\frac{1}{A} \sum_{i,j=1(i<j)}^{A} \frac{\left(\mathbf{p}_i - \mathbf{p}_j\right)^2}{2m_N}}$$

- Solve Schrödinger equation

$$\boxed{H\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)}$$

The shell model



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- Solve Schrödinger equation

$$\boxed{H\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)}$$

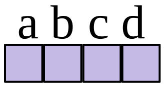
- Identify dominant average potential (mean field): dictates choice for s.p. states (basis states)

$$\boxed{\sum_{i,j=1(i<j)}^{A} V_{NN}\left(\mathbf{r}_i - \mathbf{r}_j\right) = \sum_{i}^{A} V\left(\mathbf{r}_i\right) + \sum_{i,j=1(i<j)}^{A} V_{res}\left(\mathbf{r}_i - \mathbf{r}_j\right)}$$

The shell model

Single-particle states

$$\varphi_a, \varphi_b, \varphi_c, \varphi_d$$



Single-particle energies

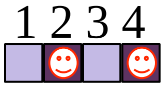
$$e_a, e_b, e_c, e_d$$

- Many-particle state $\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)\ldots\varphi_d(\mathbf{r}_A)$
- Anti-symmetric many-particle **basis states**

(Slater determinant):

$$\Phi_{ab\ldots d}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\frac{1}{\sqrt{A!}}\begin{array}{|cccc|l}
\varphi_a(\mathbf{r}_1) & \varphi_a(\mathbf{r}_2) & \cdots & \varphi_a(\mathbf{r}_A) & \text{State a}\\
\varphi_b(\mathbf{r}_1) & \varphi_b(\mathbf{r}_2) & \cdots & \varphi_b(\mathbf{r}_A) & \text{State b}\\
\vdots & \vdots & \ddots & \vdots & \\
\varphi_d(\mathbf{r}_1) & \varphi_d(\mathbf{r}_2) & \cdots & \varphi_d(\mathbf{r}_A) & \text{State d}\\
\hline
\text{Particle 1} & \text{Particle 2} & & \text{Particle A} &
\end{array}$$

- Example for *A=2* particles:



State:

$$\Phi_{24}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_2(\mathbf{r}_1)\phi_4(\mathbf{r}_2)-\phi_2(\mathbf{r}_2)\phi_4(\mathbf{r}_1)\right]$$

$$E=e_2+e_4$$

The shell model

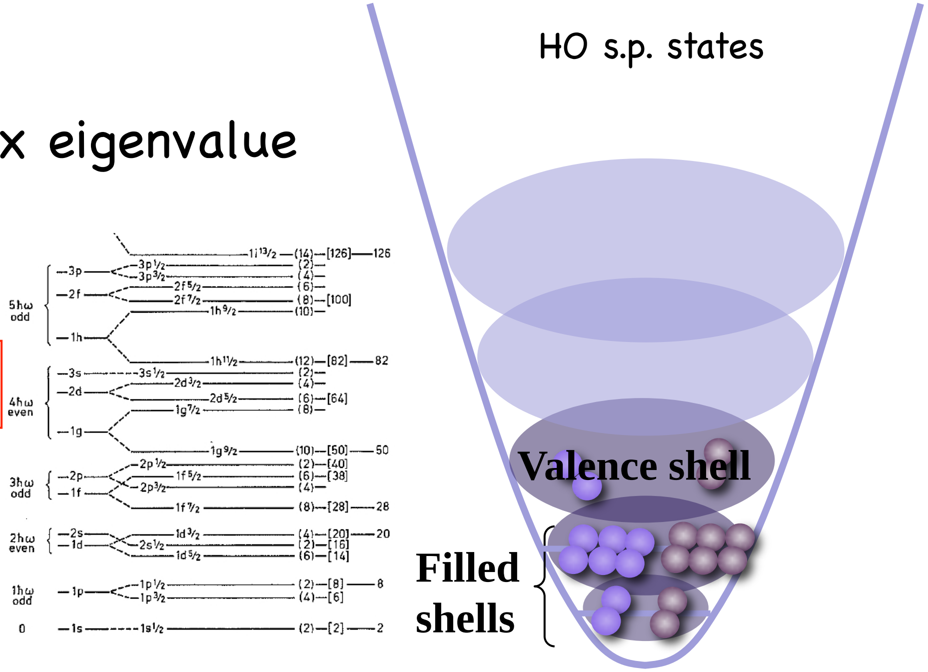
- Choice for s.p. states (basis states):
  often Harmonic Oscillator (HO)

- Solve Schrödinger equation: matrix eigenvalue problem

$$H\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=E\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

$$\Psi_\alpha(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\sum_{k=1}^{D}C_k^\alpha\Phi_k(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

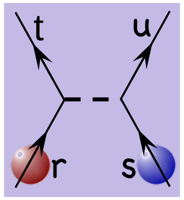
$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1D} \\ H_{21} & H_{22} & \cdots & H_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ H_{D1} & H_{D2} & \cdots & H_{DD} \end{pmatrix}\begin{pmatrix} C_1^\alpha \\ C_2^\alpha \\ \vdots \\ C_D^\alpha \end{pmatrix}=E_\alpha\begin{pmatrix} C_1^\alpha \\ C_2^\alpha \\ \vdots \\ C_D^\alpha \end{pmatrix}$$



✧ *… in Hilbert space (infinite!)*

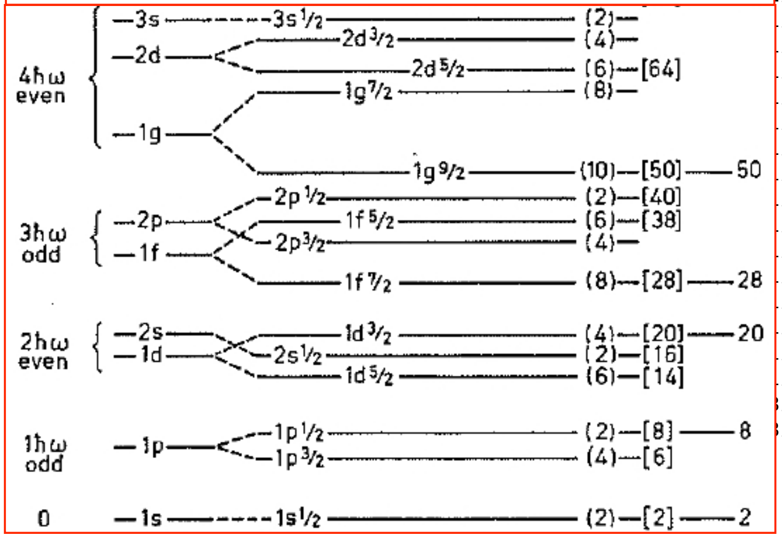
✧ *What is the best choice for basis?*

Two-Body Matrix elements (TBME)
fp shell



jr	js	jt	ju	J	T	Hsp4	GXPF1
p1/2	p1/2	p1/2	p1/2	1	0	-1.077001	-1.2431
p1/2	p1/2	p1/2	p1/2	0	1	-0.209086	-0.4469
p1/2	p1/2	p1/2	p3/2	1	0	0	-0.849
p1/2	p1/2	p3/2	p3/2	1	0	0	0.7675
p1/2	p1/2	p3/2	p3/2	0	1	-0.444876	-1.4928
p1/2	p1/2	p3/2	f5/2	1	0	0	0.8137
p1/2	p1/2	f5/2	f5/2	1	0	0	-0.3161
p1/2	p1/2	f5/2	f5/2	0	1	-0.54486	-0.8093
p1/2	p1/2	f5/2	f7/2	1	0	0	-0.1928
p1/2	p1/2	f7/2	f7/2	1	0	0	0.0271
p1/2	p1/2	f7/2	f7/2	0	1	-0.816667	-0.38
p1/2	p3/2	p1/2	p3/2	1	0	-1.077001	-2.5068
p1/2	p3/2	p1/2	p3/2	2	0	-1.077001	-2.3122
p1/2	p3/2	p1/2	p3/2	1	1	0.105489	-0.1594
p1/2	p3/2	p1/2	p3/2	2	1	0.105489	-0.2938
p1/2	p3/2	p1/2	f5/2	2	0	0	-0.69
p1/2	p3/2	p1/2	f5/2	2	1	0	0.249
p1/2	p3/2	p3/2	p3/2	1	0	0	-1.8059
p1/2	p3/2	p3/2	p3/2	2	1	0	0.634
p1/2	p3/2	p3/2	f5/2	1	0	0	0.993
p1/2	f5/2	p3/2	p3/2	2	1	0	-0.1923
p1/2	f5/2	p3/2	f5/2	2	0	0	-0.354
p1/2	f5/2	p3/2	f5/2	3	0	0	1.0151
p1/2	f5/2	p3/2	f5/2	2	1	0	-0.4043
p1/2	f5/2	p3/2	f5/2	3	1	0	-0.06
p1/2	f5/2	p3/2	f7/2	2	0	0	1.0933
p1/2	f5/2	p3/2	f7/2	3	0	0	0.7227
p1/2	f5/2	p3/2	f7/2	2	1	0	-0.803
p1/2	f5/2	p3/2	f7/2	3	1	0	-0.1814
p1/2	f5/2	f5/2	f5/2	3	0	0	-0.6276
p1/2	f5/2	f5/2	f5/2	2	1	0	-0.3208
p1/2	f5/2	f5/2	f7/2	2	0	0	-0.5447
p1/2	f5/2	f5/2	f7/2	3	0	0	-0.6262
p1/2	f5/2	f5/2	f7/2	2	1	0	0.1537
p1/2	f5/2	f5/2	f7/2	3	1	0	-0.1105
p1/2	f5/2	f7/2	f7/2	3	0	0	-0.1082
p1/2	f5/2	f7/2	f7/2	2	1	0	-0.1295
p1/2	f7/2	p1/2	f7/2	3	0	-1.638477	-1.6968
p1/2	f7/2	p1/2	f7/2	4	0	-1.638477	-1.0602
p1/2	f7/2	p1/2	f7/2	3	1	0.08819	0.4873
1/2	f7/2	p1/2	f7/2	4	1	0.08819	-0.1347
/2	f7/2	p3/2	p3/2	3	0	0	-0.6411
/2	f7/2	p3/2	f5/2	3	0	0	0.0354
/2	f7/2	p3/2	f5/2	4	0	0	-1.3607
/2	f7/2	p3/2	f5/2	3	1	0	0.3891
/2	f7/2	p3/2	f5/2	4	1	0	0.6111
/2	f7/2	p3/2	f7/2	3	0	0	-1.685
/2	f7/2	p3/2	f7/2	4	0	0	-0.1706
/2	f7/2	p3/2	f7/2	3	1	0	0.1048
/2	f7/2	p3/2	f7/2	4	1	0	0.3351
/2	f7/2	f5/2	f5/2	3	0	0	0.2621
/2	f7/2	f5/2	f5/2	4	1	0	0.2248
/2	f7/2	f5/2	f7/2	3	0	0	-0.4252
/2	f7/2	f5/2	f7/2	4	0	0	-0.3789
/2	f7/2	f5/2	f7/2	3	1	0	0.3224
/2	f7/2	f5/2	f7/2	4	1	0	0.1907
/2	f7/2	f7/2	f7/2	3	0	0	-0.8883
/2	f7/2	f7/2	f7/2	4	1	0	0.2096
/2	p3/2	p3/2	p3/2	1	0	-1.077001	-0.6308
/2	p3/2	p3/2	p3/2	3	0	-1.077001	-2.289
p3/2	p3/2	p3/2	p3/2	0	1	-0.523662	-1.1165
p3/2	p3/2	p3/2	p3/2	2	1	0.105489	-0.0887
p3/2	p3/2	p3/2	f5/2	1	0	0	0.2373
p3/2	p3/2	p3/2	f5/2	3	0	0	0.2276
p3/2	p3/2	p3/2	f5/2	2	1	0	-0.4631
p3/2	p3/2	p3/2	f7/2	3	0	0	-0.4309
p3/2	p3/2	p3/2	f7/2	2	1	0	-0.3738
p3/2	p3/2	f5/2	f5/2	1	0	0	0.0483
p3/2	p3/2	f5/2	f5/2	3	0	0	-0.0546
p3/2	p3/2	f5/2	f5/2	0	1	-0.770548	-1.2457
p3/2	p3/2	f5/2	f5/2	2	1	0	0.0719
p3/2	p3/2	f5/2	f7/2	1	0	0	-0.8914
p3/2	p3/2	f5/2	f7/2	3	0	0	-0.6264
p3/2	p3/2	f5/2	f7/2	2	1	0	-0.0717
p3/2	p3/2	f7/2	f7/2	1	0	0	-0.4313
p3/2	p3/2	f7/2	f7/2	3	0	0	-0.3415
p3/2	p3/2	f7/2	f7/2	0	1	-1.154941	-0.7174
p3/2	p3/2	f7/2	f7/2	2	1	0	-0.2021
p3/2	f5/2	p3/2	f5/2	1	0	-1.077001	-2.7262
p3/2	f5/2	p3/2	f5/2	2	0	-1.077001	-1.511
p3/2	f5/2	p3/2	f5/2	3	0	-1.077001	-0.5859
p3/2	f5/2	p3/2	f5/2	4	0	-1.077001	-1.0882
p3/2	f5/2	p3/2	f5/2	1	1	0.105489	0.3284
p3/2	f5/2	p3/2	f5/2	2	1	0.105489	0.3608
p3/2	f5/2	p3/2	f5/2	3	1	0.105489	0.346
p3/2	f5/2	p3/2	f5/2	4	1	0.105489	-0.2584
p3/2	f5/2	p3/2	f7/2	2	0	0	1.2708
p3/2	f5/2	p3/2	f7/2	3	0	0	0.579
p3/2	f5/2	p3/2	f7/2	4	0	0	0.7103
p3/2	f5/2	p3/2	f7/2	2	1	0	-0.5436
p3/2	f5/2	p3/2	f7/2	3	1	0	-0.1836
p3/2	f5/2	p3/2	f7/2	4	1	0	-0.4546
p3/2	f5/2	f5/2	f5/2	1	0	0	0.477
p3/2	f5/2	f5/2	f5/2	3	0	0	0.32
p3/2	f5/2	f5/2	f5/2	2	1	0	-0.056
p3/2	f5/2	f5/2	f5/2	4	1	0	-0.3615
p3/2	f5/2	f5/2	f7/2	1	0	0	1.2721
p3/2	f5/2	f5/2	f7/2	2	0	0	-0.598
p3/2	f5/2	f5/2	f7/2	3	0	0	0.7716
p3/2	f5/2	f5/2	f7/2	4	0	0	-0.6408

HO: Single-particle basis

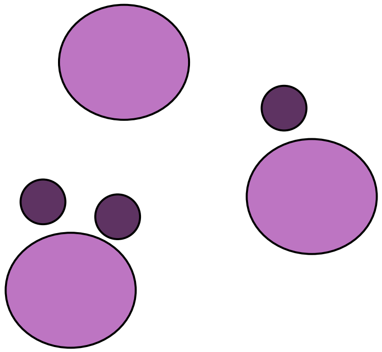


… and more two-body matrix elements

p3/2	f5/2	f5/2	f7/2	1	1	0	0.0521
p3/2	f5/2	f5/2	f7/2	2	1	0	0.4247
p3/2	f5/2	f5/2	f7/2	3	1	0	-0.0268
p3/2	f5/2	f5/2	f7/2	4	1	0	0.2699
p3/2	f5/2	f7/2	f7/2	1	0	0	-0.0907
p3/2	f5/2	f7/2	f7/2	3	0	0	0.0752
p3/2	f5/2	f7/2	f7/2	2	1	0	-0.1725
p3/2	f5/2	f7/2	f7/2	4	1	0	-0.2224
p3/2	f7/2	p3/2	f7/2	2	0	-1.638477	-0.5391
p3/2	f7/2	p3/2	f7/2	3	0	-1.638477	-1.0055
p3/2	f7/2	p3/2	f7/2	4	0	-1.638477	-0.3695
p3/2	f7/2	p3/2	f7/2	5	0	-1.638477	-2.967
p3/2	f7/2	p3/2	f7/2	2	1	0.08819	-0.6081
p3/2	f7/2	p3/2	f7/2	3	1	0.08819	0.1561
p3/2	f7/2	p3/2	f7/2	4	1	0.08819	-0.1398
p3/2	f7/2	p3/2	f7/2	5	1	0.08819	0.5918
p3/2	f7/2	f5/2	f5/2	3	0	0	0.166
p3/2	f7/2	f5/2	f5/2	5	0	0	0.0334
p3/2	f7/2	f5/2	f5/2	2	1	0	0.088
p3/2	f7/2	f5/2	f5/2	4	1	0	-0.2146
p3/2	f7/2	f5/2	f7/2	2	0	0	0.6381
p3/2	f7/2	f5/2	f7/2	3	0	0	-0.254
p3/2	f7/2	f5/2	f7/2	4	0	0	-0.1951
p3/2	f7/2	f5/2	f7/2	5	0	0	-0.6743
p3/2	f7/2	f5/2	f7/2	2	1	0	-0.0959
p3/2	f7/2	f5/2	f7/2	3	1	0	0.523
p3/2	f7/2	f5/2	f7/2	4	1	0	0.2486
p3/2	f7/2	f5/2	f7/2	5	1	0	0.481
p3/2	f7/2	f7/2	f7/2	3	0	0	-0.8807
p3/2	f7/2	f7/2	f7/2	5	0	0	-0.4265
p3/2	f7/2	f7/2	f7/2	2	1	0	-0.516
p3/2	f7/2	f7/2	f7/2	4	1	0	-0.2969
f5/2	f5/2	f5/2	f5/2	1	0	-1.077001	-0.8551
f5/2	f5/2	f5/2	f5/2	3	0	-1.077001	-0.5599
f5/2	f5/2	f5/2	f5/2	5	0	-1.077001	-2.2816
f5/2	f5/2	f5/2	f5/2	0	1	-0.838236	-1.2081
f5/2	f5/2	f5/2	f5/2	2	1	0.105489	-0.4621
f5/2	f5/2	f5/2	f5/2	4	1	0.105489	-0.1624
f5/2	f5/2	f5/2	f7/2	1	0	0	0.2735
f5/2	f5/2	f5/2	f7/2	3	0	0	-0.6378
f5/2	f5/2	f5/2	f7/2	5	0	0	-1.1302
f5/2	f5/2	f5/2	f7/2	2	1	0	0.5022
f5/2	f5/2	f5/2	f7/2	4	1	0	0.2709
f5/2	f5/2	f7/2	f7/2	1	0	0	0.6511
f5/2	f5/2	f7/2	f7/2	3	0	0	0.4358
f5/2	f5/2	f7/2	f7/2	5	0	0	0.1239
f5/2	f5/2	f7/2	f7/2	0	1	-1.414508	-1.3832
f5/2	f5/2	f7/2	f7/2	2	1	0	-0.2038
f5/2	f5/2	f7/2	f7/2	4	1	0	-0.0331
f5/2	f7/2	f5/2	f7/2	1	0	-1.638477	-4.5802
f5/2	f7/2	f5/2	f7/2	2	0	-1.638477	-3.252
f5/2	f7/2	f5/2	f7/2	3	0	-1.638477	-1.4019
f5/2	f7/2	f5/2	f7/2	4	0	-1.638477	-2.2583
f5/2	f7/2	f5/2	f7/2	5	0	-1.638477	-0.6084
f5/2	f7/2	f5/2	f7/2	6	0	-1.638477	-3.0351
f5/2	f7/2	f5/2	f7/2	1	1	0.08819	-0.0889
f5/2	f7/2	f5/2	f7/2	2	1	0.08819	-0.175
f5/2	f7/2	f5/2	f7/2	3	1	0.08819	0.6302
f5/2	f7/2	f5/2	f7/2	4	1	0.08819	0.4763
f5/2	f7/2	f5/2	f7/2	5	1	0.08819	0.7433
f5/2	f7/2	f5/2	f7/2	6	1	0.08819	-0.9916
f5/2	f7/2	f7/2	f7/2	1	0	0	-1.8998
f5/2	f7/2	f7/2	f7/2	3	0	0	-1.0917
f5/2	f7/2	f7/2	f7/2	5	0	0	-1.2853
f5/2	f7/2	f7/2	f7/2	2	1	0	-0.2167
f5/2	f7/2	f7/2	f7/2	4	1	0	0.4999
f5/2	f7/2	f7/2	f7/2	6	1	0	0.5643
f7/2	f7/2	f7/2	f7/2	1	0	-2.078472	-1.2838
f7/2	f7/2	f7/2	f7/2	3	0	-2.078472	-0.8418
f7/2	f7/2	f7/2	f7/2	5	0	-2.078472	-0.7839
f7/2	f7/2	f7/2	f7/2	7	0	-2.078472	-2.6661
f7/2	f7/2	f7/2	f7/2	0	1	-1.845204	-2.4385
f7/2	f7/2	f7/2	f7/2	2	1	0.062016	-0.9352
f7/2	f7/2	f7/2	f7/2	4	1	0.062016	-0.1296
f7/2	f7/2	f7/2	f7/2	6	1	0.062016	0.2783

Ca Isotopes
Empirical interactions: from available data

Question



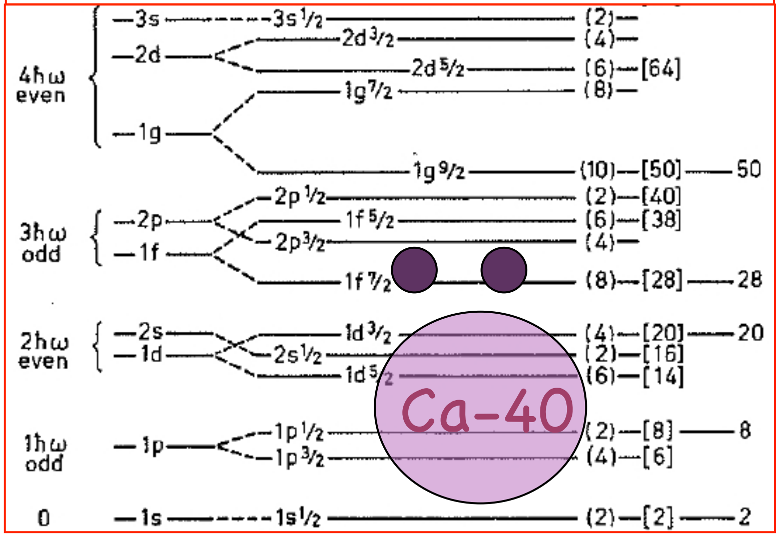
Binding Energies: 342.05 MeV (Ca-40)
350.41 MeV (Ca-41, $7/2^-$) + 1 neutron in f7/2
361.90 MeV (Ca-42, 0^+) + 2 neutrons in f7/2

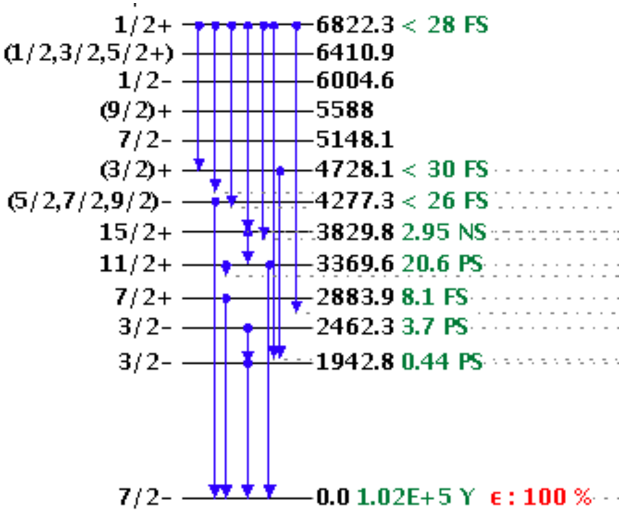
Ec = ? (energy due to core)

$e_{f7/2}$=? (energy of single nucleon)

$V^{01}_{f7/2f7/2f7/2f7/2}$= ? (energy of two nucleons, J=0, T=1)

HO: Single-particle energies (SPE)





Ca-41

Ca Isotopes
Empirical interactions: from available data

Binding Energies: 342.05 MeV (Ca-40)
350.41 MeV (Ca-41, $7/2^-$) + 1 neutron in f7/2
361.90 MeV (Ca-42, 0^+) + 2 neutrons in f7/2

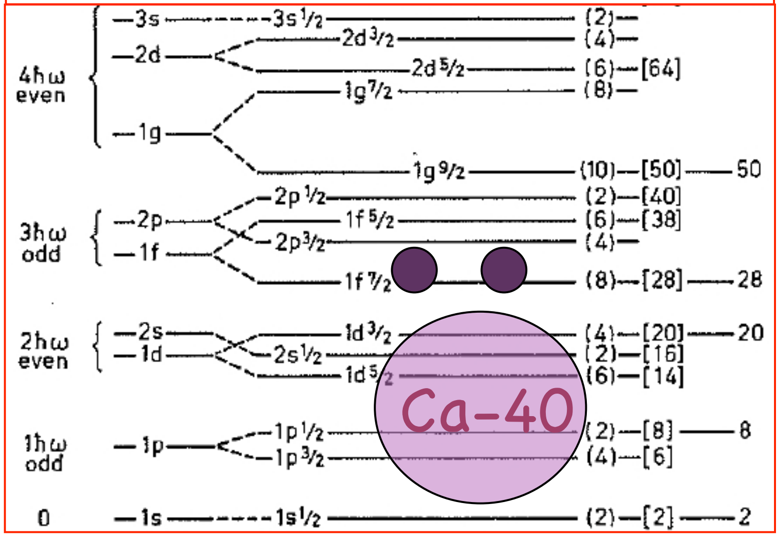
↕ -8.36 MeV
↕ -19.84MeV

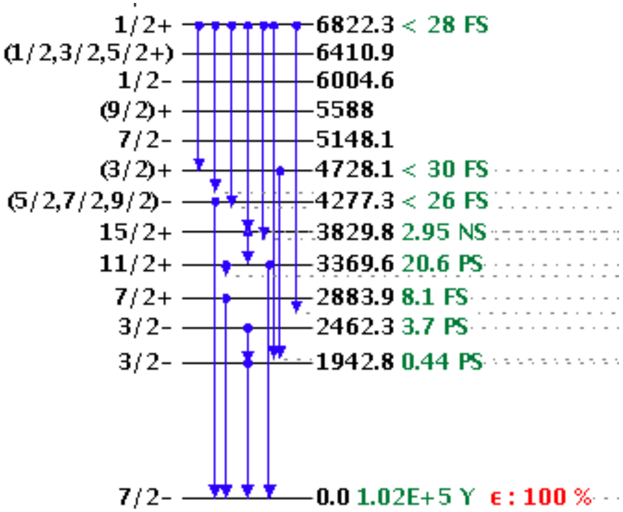
Ec = -342.05 MeV (energy due to core)

$e_{f7/2}$=-350.41-(-342.05)=-8.36 MeV

$V^{01}_{f7/2f7/2f7/2f7/2}$=-361.90-(-342.05)-2*($e_{f7/2}$)=-3.12 MeV ($J$=0, T=1)

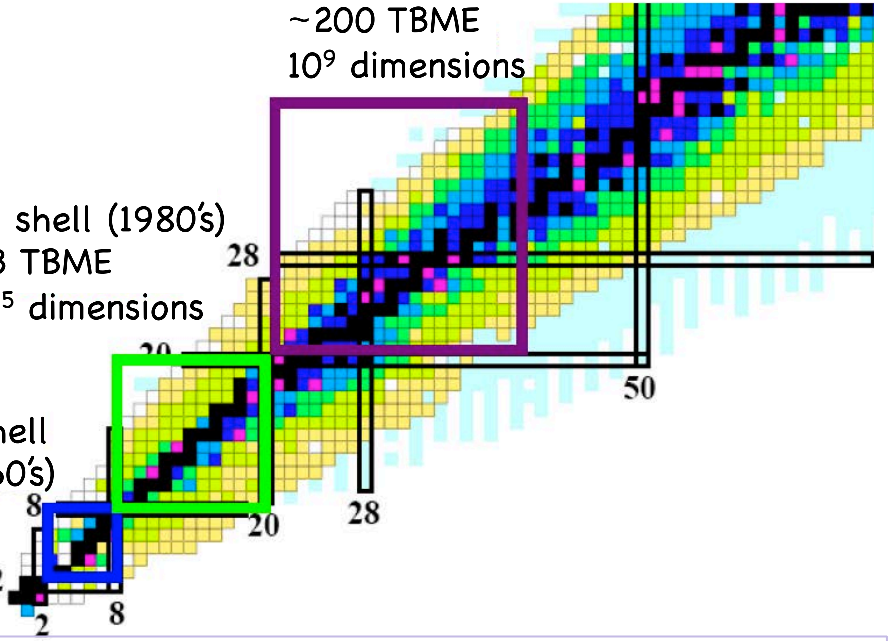
HO: Single-particle energies (SPE)





Ca-41

Valence shell model

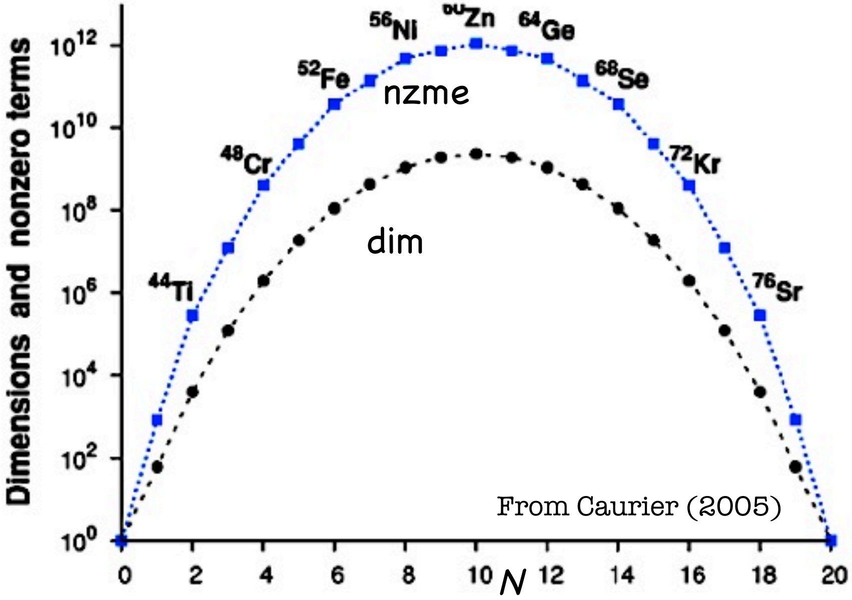


pf shell (1990's)
~200 TBME
10^9 dimensions

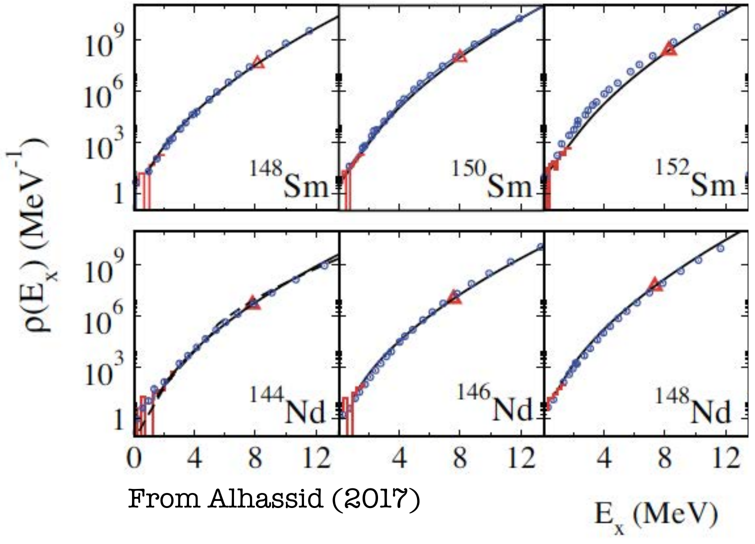
sd shell (1980's)
63 TBME
10^5 dimensions

p shell
(1960's)

At present: large-scale SM calculations, $pfg_{9/2}$ shell



From Caurier (2005)



From Alhassid (2017)

Current limits of model space sizes:

- Diagonalization: ~10^9
- Monte Carlo: ~10^{15}

Publicly available codes:

- Oxbash (MSU): arXiv:nucl-th/9406020
- Antoine (Strasbourg): www.iphc.cnrs.fr/nutheo/code_ant

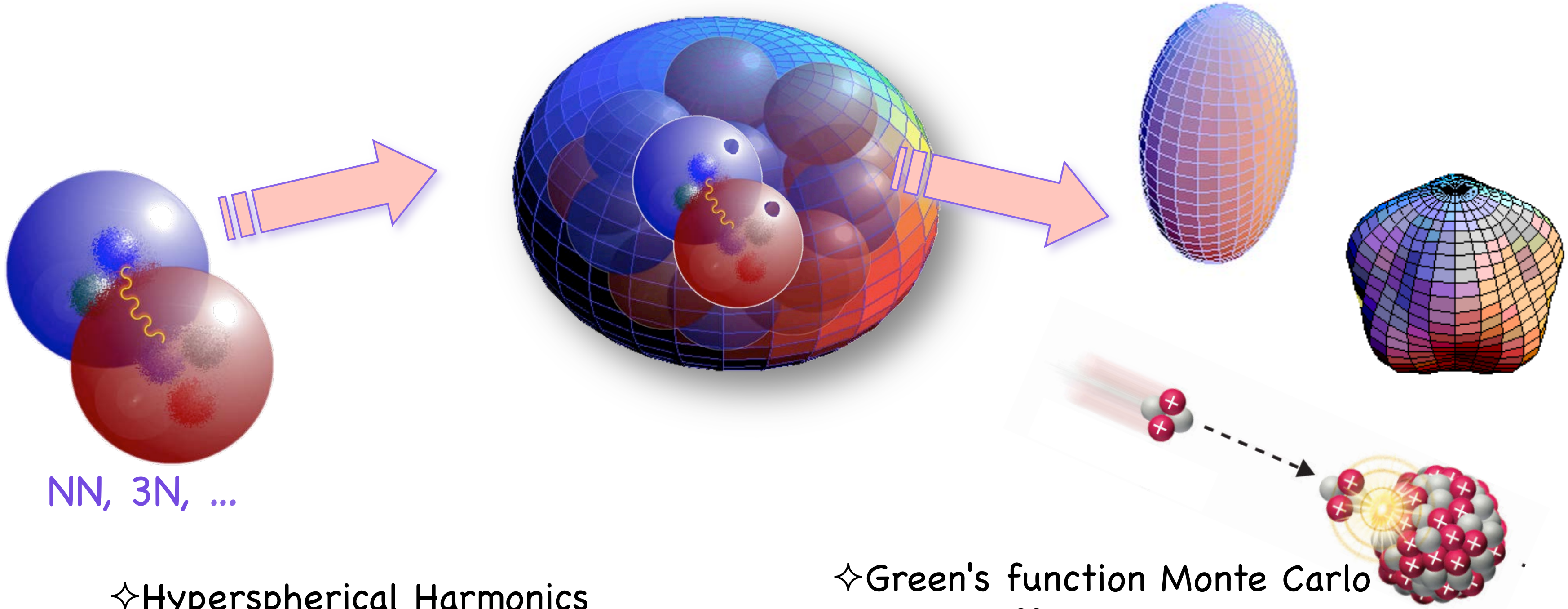
Lecture 2

Ab initio models

Nuclear force

Many-body Approach

Nuclear properties:
structure & reactions



NN, 3N, ...

✧Hyperspherical Harmonics
✧No-core Shell Model
✧NCSM/Resonating Group Method
✧Symmetry-adapted NCSM
✧Importance Truncation NCSM
✧Monte Carlo NCSM

✧Green's function Monte Carlo
✧Lattice Effective Field Theory
✧Coupled-cluster method
✧In-Medium SRG
✧Gorkov-Green's function
✧Many-body perturbation theory

I will give a few examples...

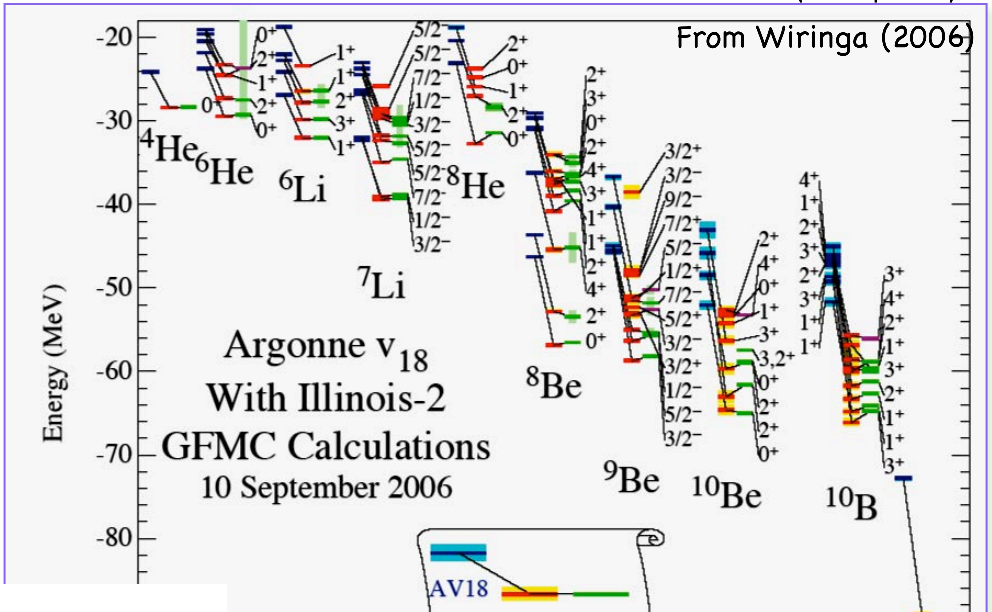
Ab initio Variational and Green's Function Monte Carlo

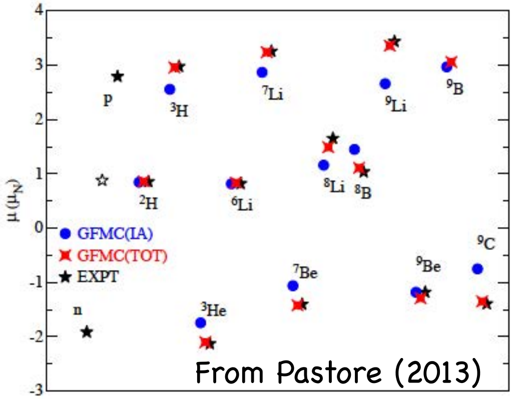
- Variational Monte Carlo Ψ_T:
  - contains variational parameters adjusted via energy minimization, $E_T = \dfrac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle}$
  - excellent approximation
- GFMC propagates the VMC Ψ_T to imaginary time

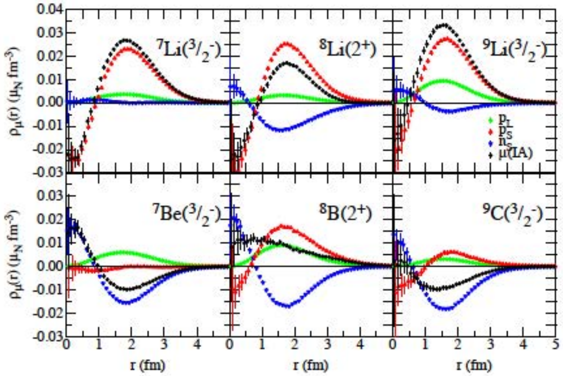
$$|\Psi(\tau)\rangle = e^{-(H-E_0)\tau}\Psi_T \xrightarrow[\tau\to\infty]{} |\Psi_0\rangle$$

(filters out excited-state contamination to leave lowest state of given J^π ; T)

Virtually exact method
Limited to local interactions
Light nuclei


From Wiringa (2006)


From Pastore (2013)



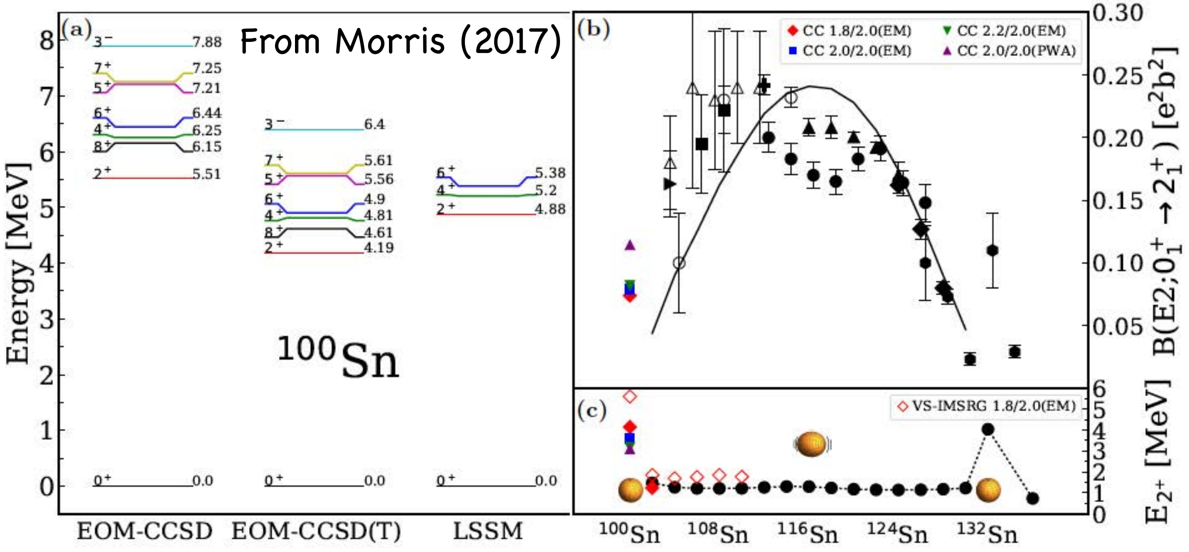
Ab initio Coupled-cluster Theory

- Ansatz: $|\Psi\rangle = e^{T}|\Phi\rangle \quad E = \langle\Phi|e^{-T}He^{T}|\Phi\rangle$
- Expansion in number of particle-hole pairs:

Scales gently with increasing *A*
-> first calculations of ^{100}Sn!
Limited near closed-shell nuclei
Missing correlations

$T=1+T_1+T_2+\ldots$

Part of np-nh excitations included



From Morris (2017)

Ab Initio No-Core Shell Model

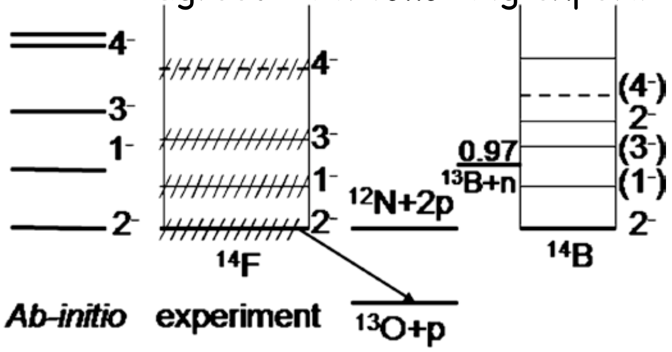
- Harmonic-oscillator single-particle basis
- Construct many-body basis states (Slater determinants)
- Express Hamiltonian in this basis (huge matrix)
- Find low-lying states (eigenfunctions)

Convergence to exact solutions with increasing model space

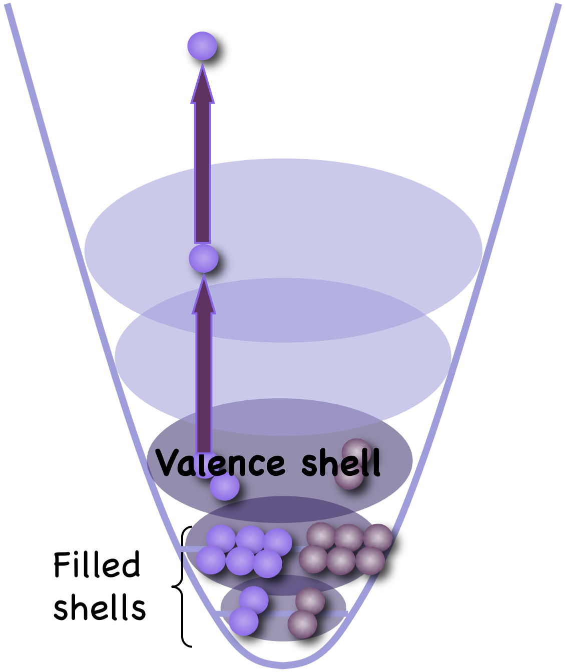
Limited to light nuclei

No restrictions on interaction/nucleus

First prediction of ^{14}F:
agrees with following experiment



From Maris (2010)

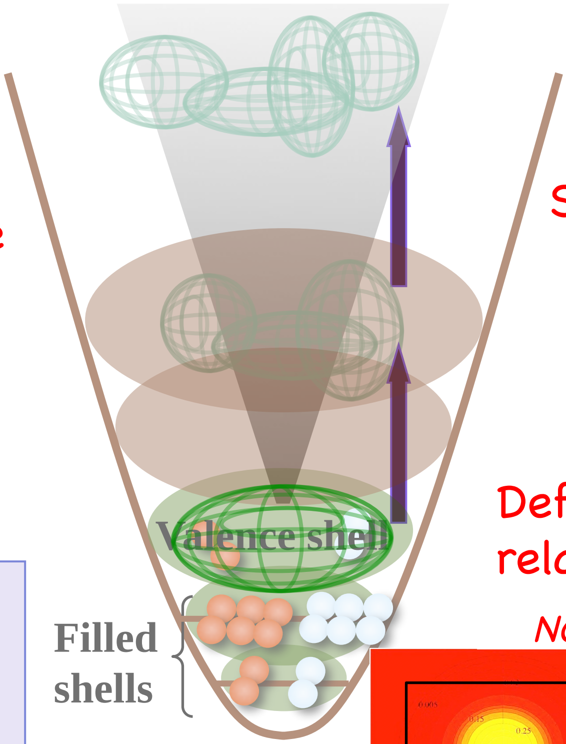


Solve
Schrödinger equation
in infinite space

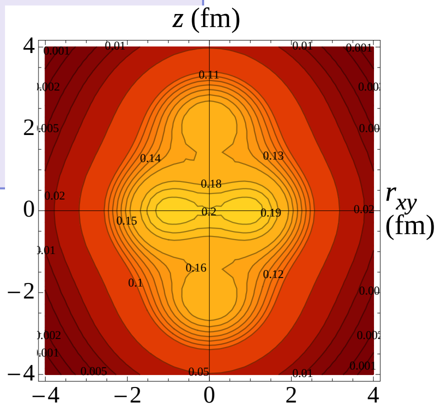
$$H\Psi(1,2,\ldots,A) = E\Psi(1,2,\ldots,A)$$

Ab Initio Symmetry-adapted (SA) NCSM

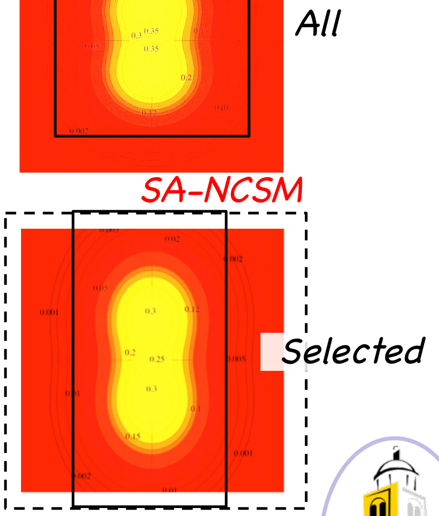
- Harmonic-oscillator single-particle basis
- Construct many-body basis states – all possible *shapes*
- Take physically relevant shapes
- Express Hamiltonian in this basis (huge matrix)
- Find low-lying states (eigenfunctions)



Up to Ca region (A<50, so far)
Accounts for collective correlations & clustering
Convergence to exact solutions with increasing model space
CPU-bound calculations
Selected model spaces may be too restrictive

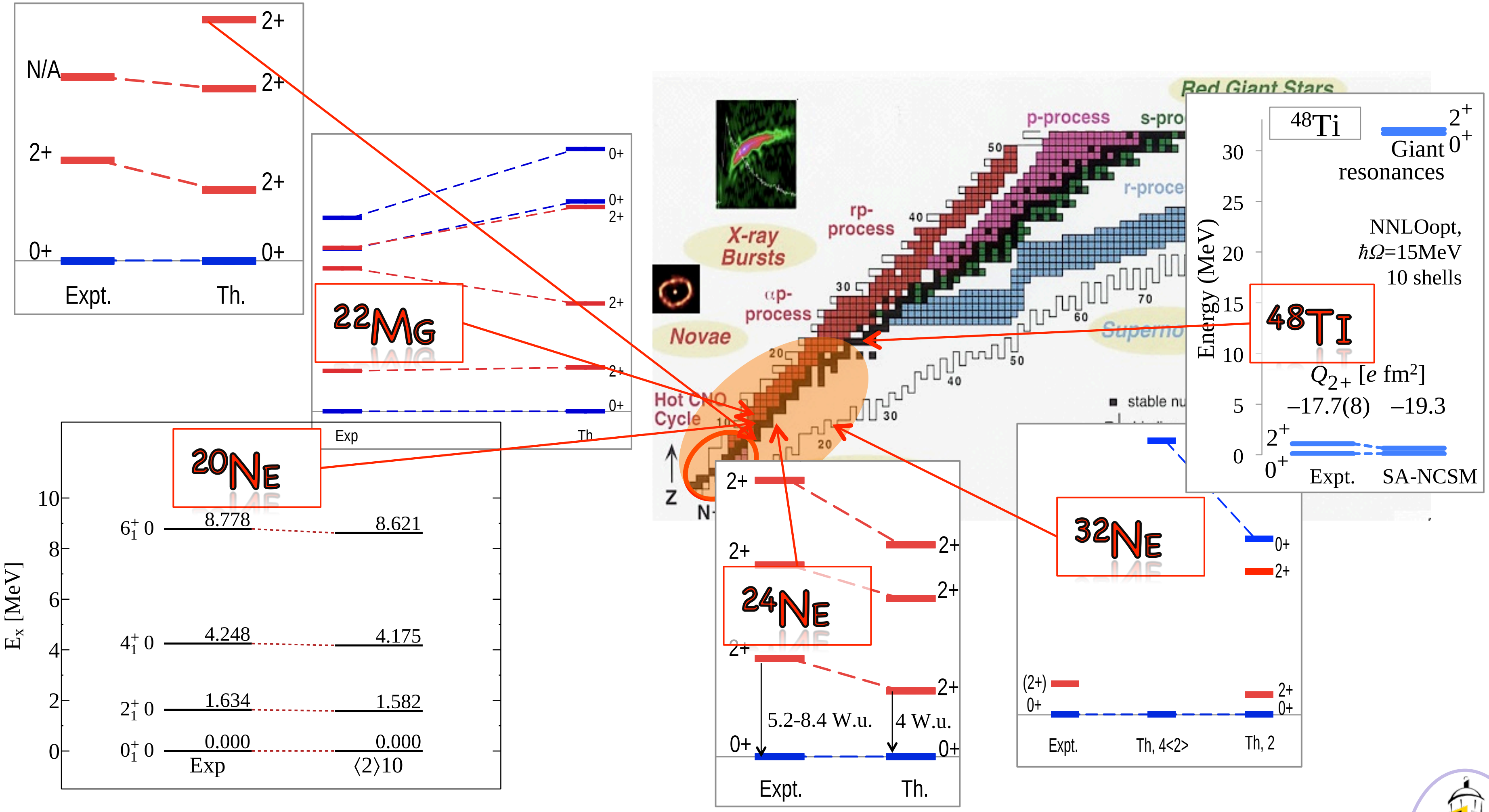


Ab initio modeling of ^{20}Ne



Code available at:
https://sourceforge.net/projects/lsu3shell/

Ab Initio Symmetry-adapted (SA) NCSM



Ab initio Lattice EFT

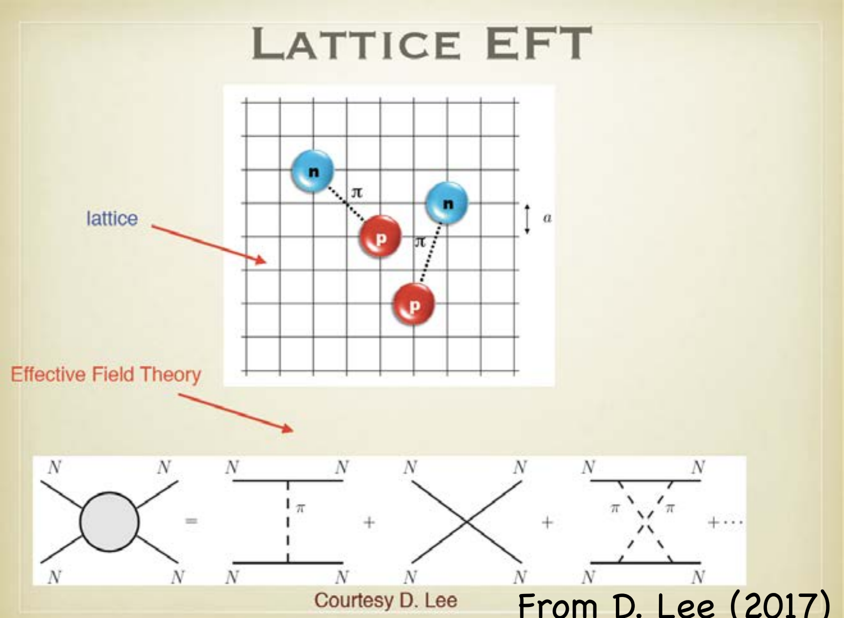
- Nucleon and pion fields on lattice
- Use Auxiliary Field Method:
  - Replace contact interaction by interaction of each nucleon with a background field (particles decouple and interact only with the auxiliary field

$$\exp\left[-\frac{C}{2}(N^{\dagger}N)^{2}\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} ds\ \exp\left[-\frac{1}{2}s^{2}+\sqrt{-C}s\ (N^{\dagger}N)\right]$$

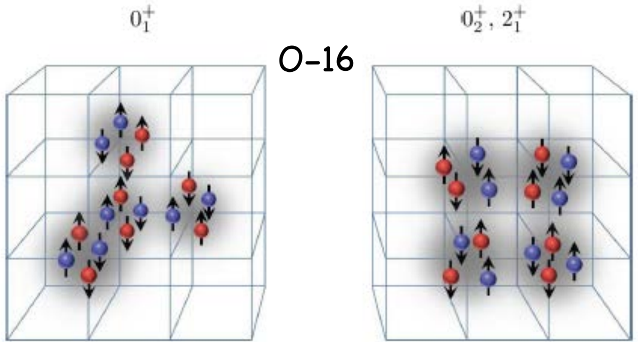
2-body

1-body

Scales gently with increasing *A*
Descriptions of clustering
Lattice spacing may be too large



From D. Lee (2017)



O-16

From Epelbaum (2014)

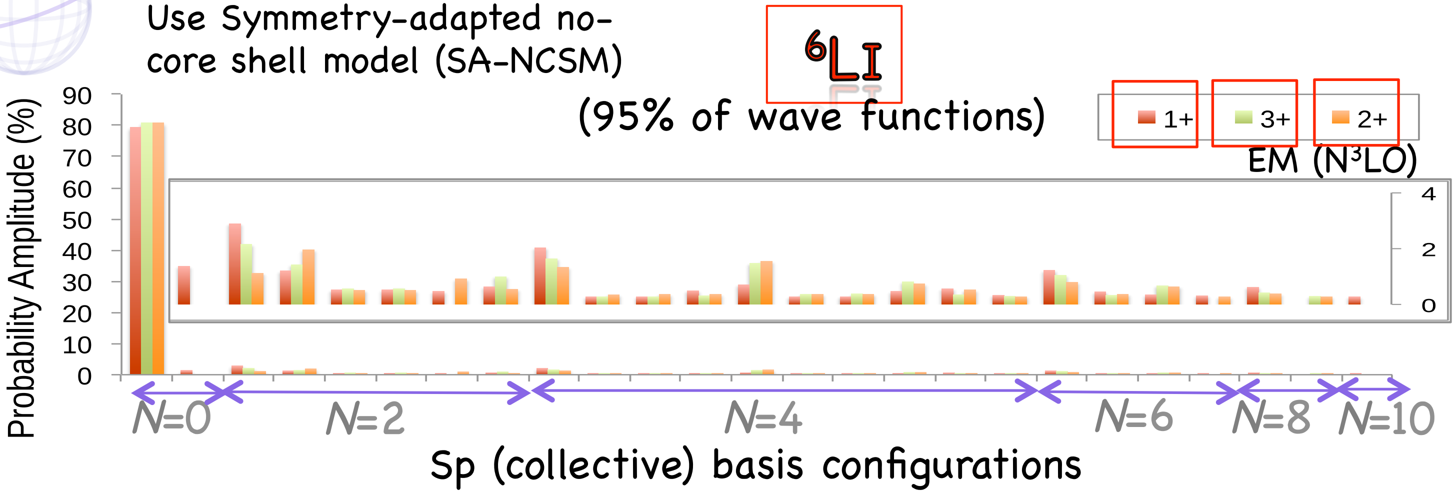
Lecture 2

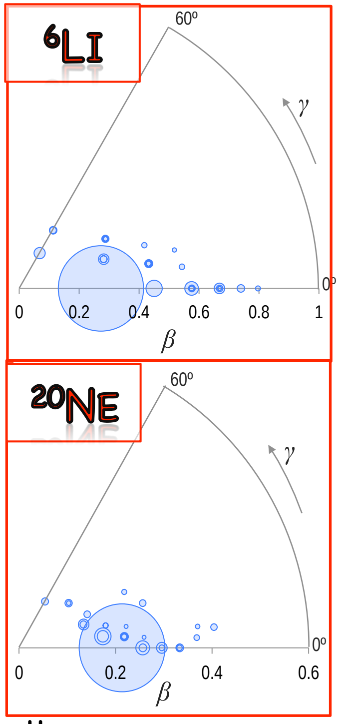
Symmetries (Exact &Approximate)

Emergent symmetries within nuclei

Preference of Nature

Use Symmetry-adapted no-core shell model (SA-NCSM)



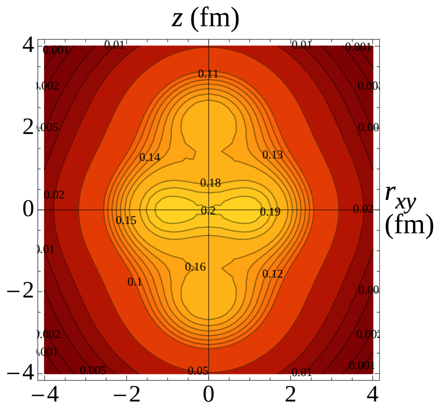


^{6}Li, 14 shells

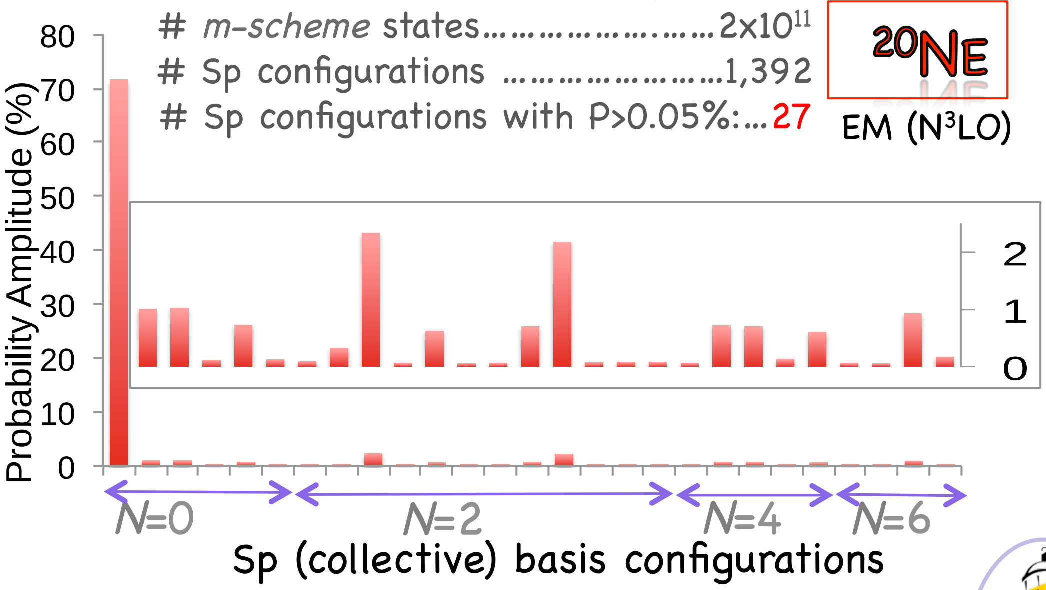
\# J=1,2,3 states……………………………2x10^7

\# Sp configurations……………….……….528

\# Sp configurations with P>0.2%……25



^{20}Ne, 11 shells



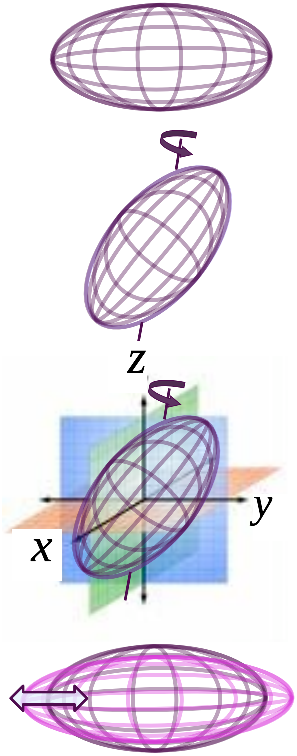
\# *m-scheme* states……………….……2x10^{11}

\# Sp configurations ……………………1,392

\# Sp configurations with P>0.05%:…27

What physics can we learn from Sp basis?

Sp (collective) basis configuration:



one equilibrium deformation ("shape")

rotations

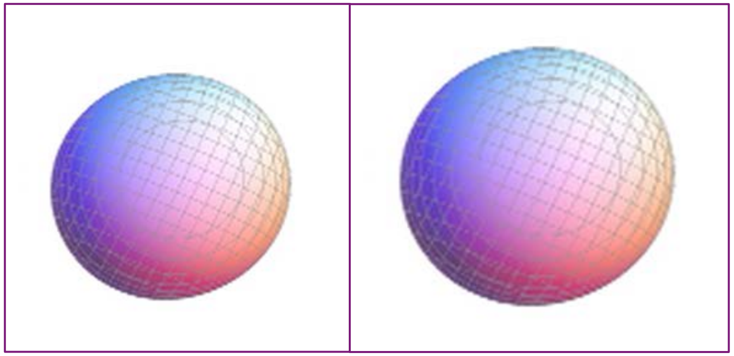
space orientation

Vibrations

(of the giant resonance monopole (r^2)/ quadrupole (Q) type)

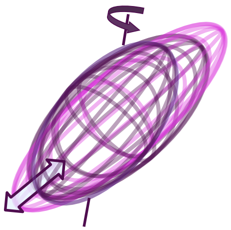
All states preserve the equilibrium shape…

Symmetry?



Symplectic Sp(3,R) Symmetry!

Formal definition



All linear canonical transformations of the single-particle phase-space observables

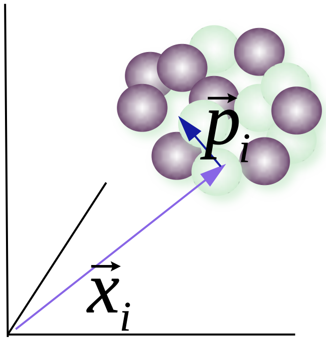
$$x_{i\alpha} \rightarrow \sum_{\beta=x,y,z} a_{\alpha\beta} x_{i\beta} + b_{\alpha\beta} p_{i\beta}$$

$$p_{i\alpha} \rightarrow \sum_{\beta=x,y,z} c_{\alpha\beta} x_{i\beta} + d_{\alpha\beta} p_{i\beta}$$

that preserve the canonical commutation relation

$$\left[x_{i\alpha}, p_{j\beta} \right] = i\hbar \delta_{ij} \delta_{\alpha\beta}$$

Nucleus with *A* nucleons



Generators:

$$Q_{ij} = \sum_n x_{ni} x_{nj},$$

$$S_{ij} = \sum_n (x_{ni} p_{nj} + p_{ni} x_{nj}),$$

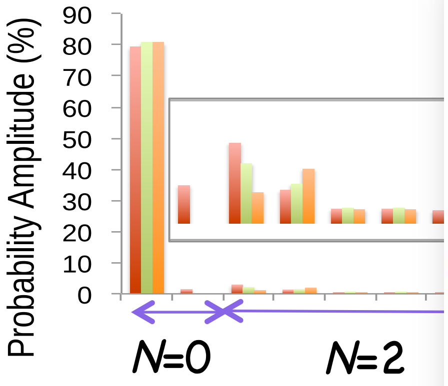
$$L_{ij} = \sum_n (x_{ni} p_{nj} - x_{nj} p_{ni}),$$

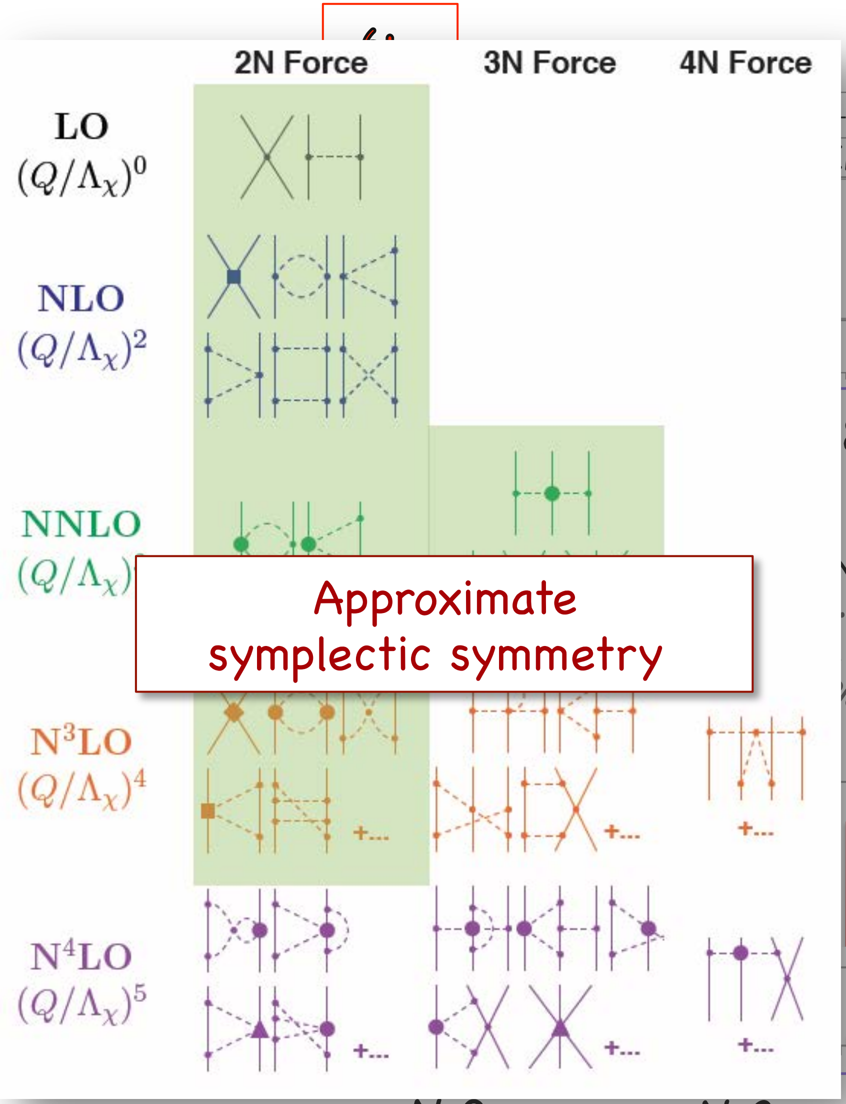
$$K_{ij} = \sum_n p_{ni} p_{nj},$$

SU(3)
in a HO shell
(Elliott, 1958)

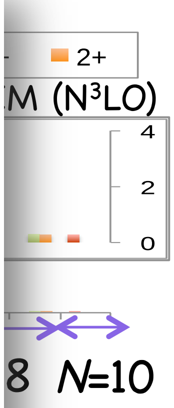
Rowe, Rosensteel, Draayer, Hecht, Suzuki, Escher, Bahri, ….

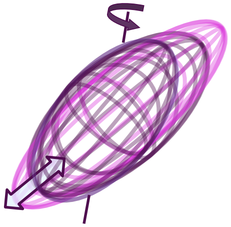
Approximate Symmetry in Nuclei





Approximate
symplectic symmetry

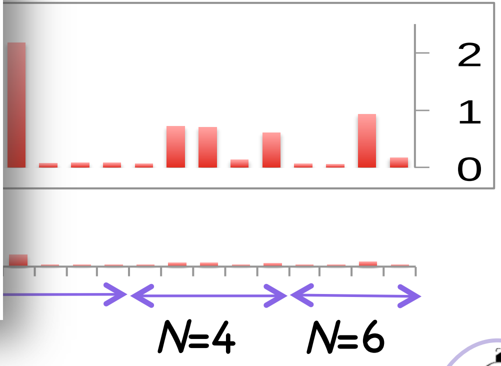




^{6}Li, Nmax=12
J=1,2,3 states……………2
Sp(3,R) irreps………………
Sp(3,R) with P>0.2%………

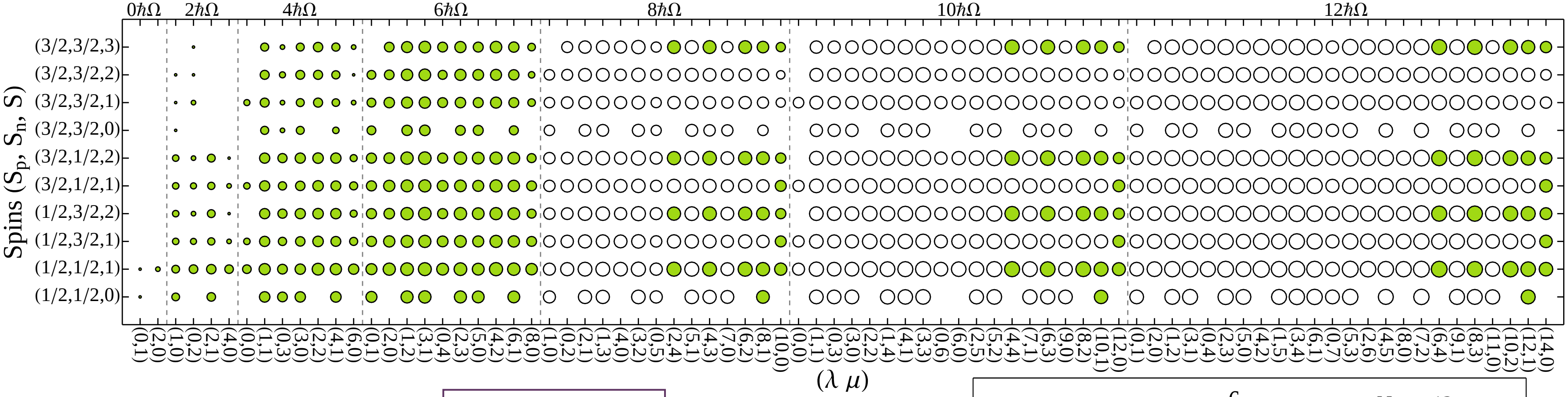
Nmax=8
…2x10^{11}
…1,392
%: ….27

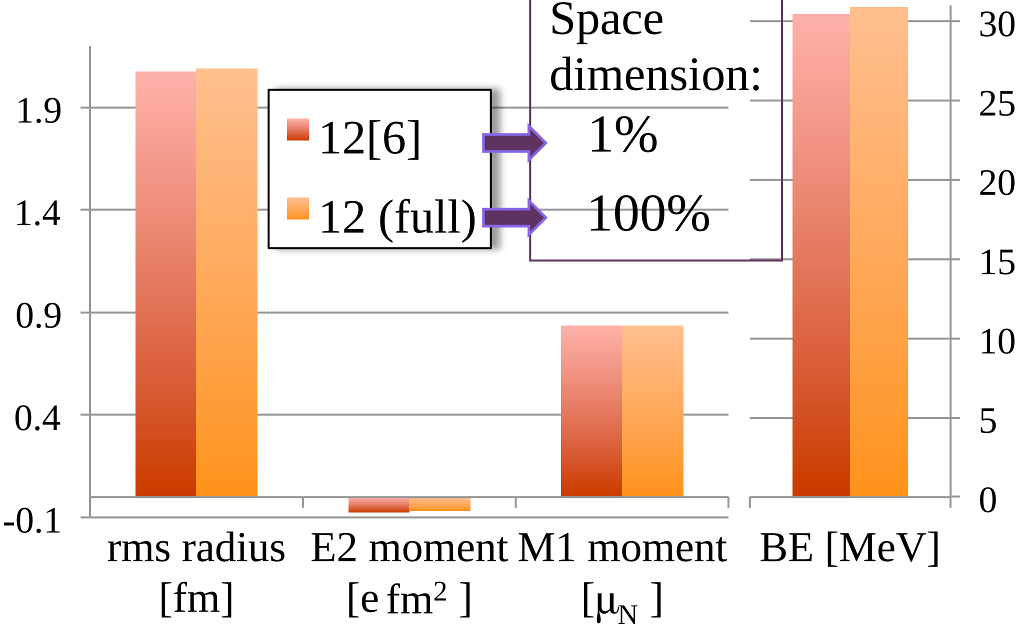
^{20}NE
EM (N^{3}LO)



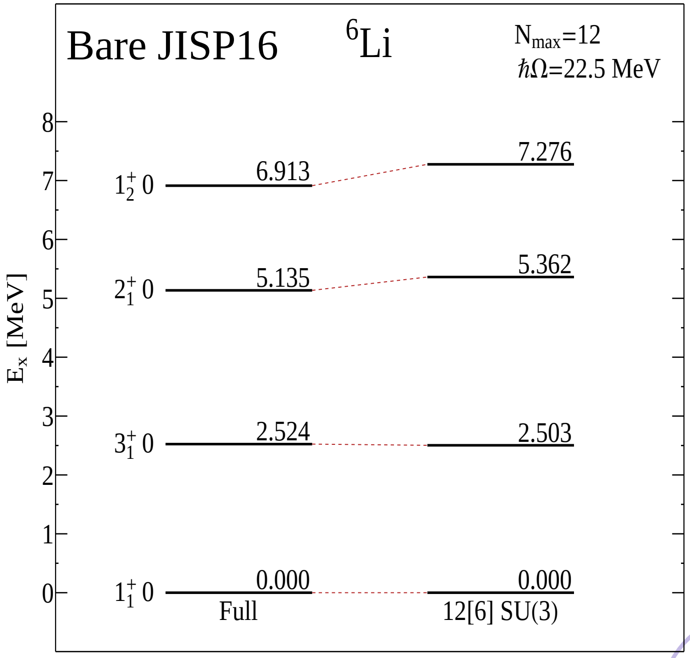
Efficacy of SA-NCSM: Li-6

Winnowing the model space: $N_{\max}=12[6]$ (full up to $6\hbar\Omega$; selected configurations in 8-12 $\hbar\Omega$)

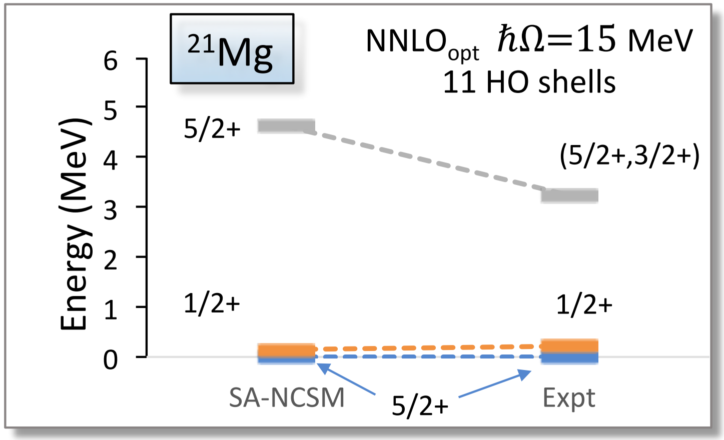


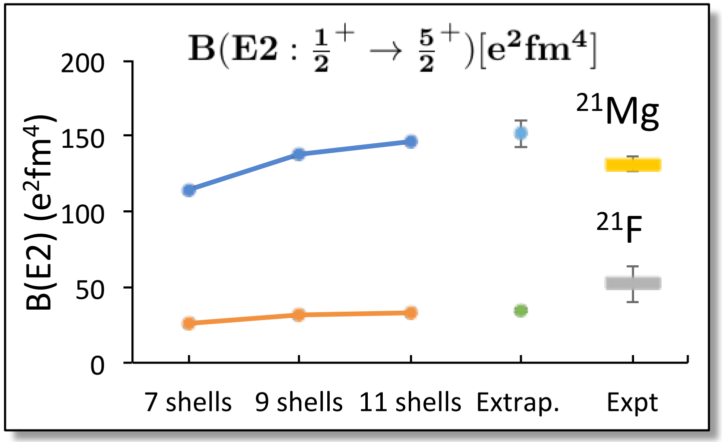


Space dimension:
1%
100%



Collectivity in intermediate-mass nuclei





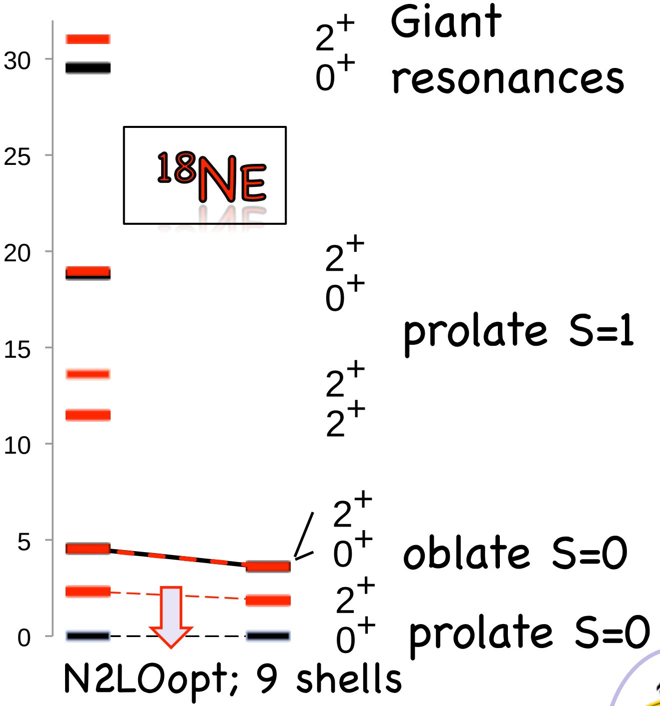
Ne & Mg isotopes

^{18}Ne, B(E2: $2^+ \rightarrow 0^+$)

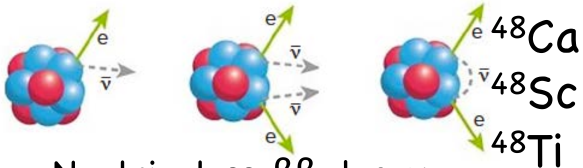
Experiment……… 17.7(18) W.u.

9 shells ………… 1.13 W.u.

33 shells ………… 13.0(7) W.u.
(no effective charges)



Structure of Ca-48 and Ti-48



Neutrinoless ββ decay

^{48}CA

8 shells, N2LOopt

0^+

SA-NCSM (selected): ……………966,152

Complete model space: ……3,162,511,819

2^+

SA-NCSM (selected): ……………3,055,554

Complete model space: …14,522,234,982

^{48}TI

8 shells, N2LOopt

0^+

SA-NCSM (selected): ………………602,493

Complete model space: ……24,694,678,414

2^+

SA-NCSM (selected): ……………1,178,834

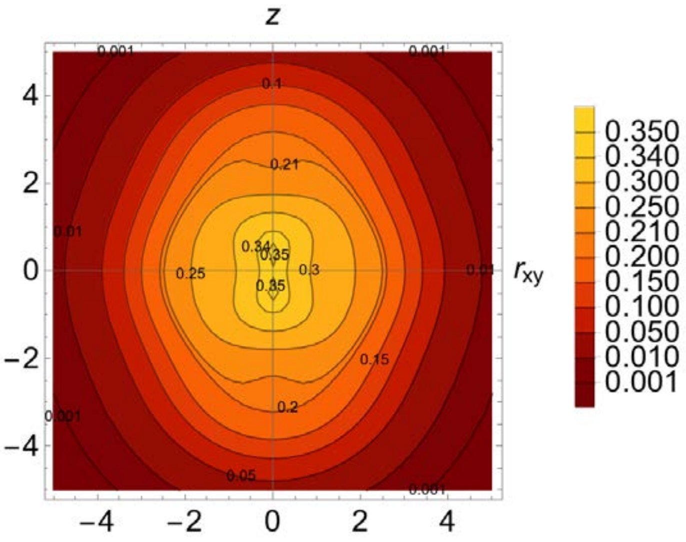
Complete model space: …113,920,316,658

^{48}Ti, Q(2^+) [e fm^2]

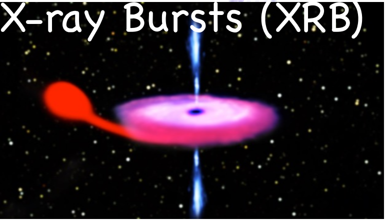
Experiment……… -17.7

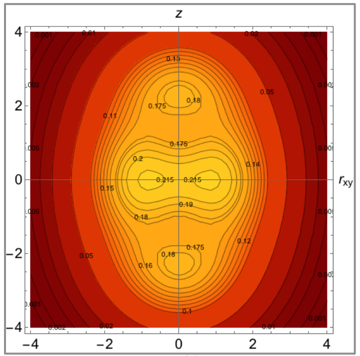
8 shells …………… -19.3

(no effective charges)

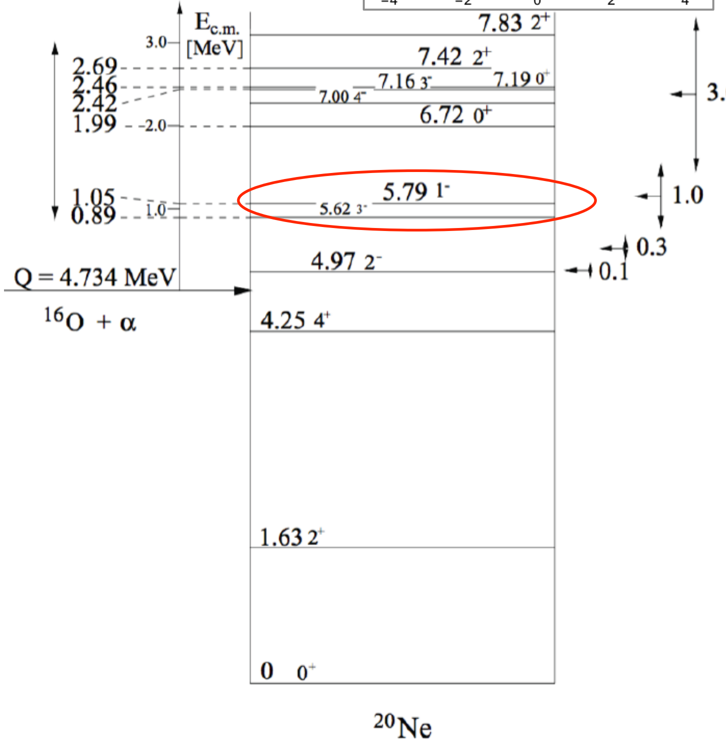


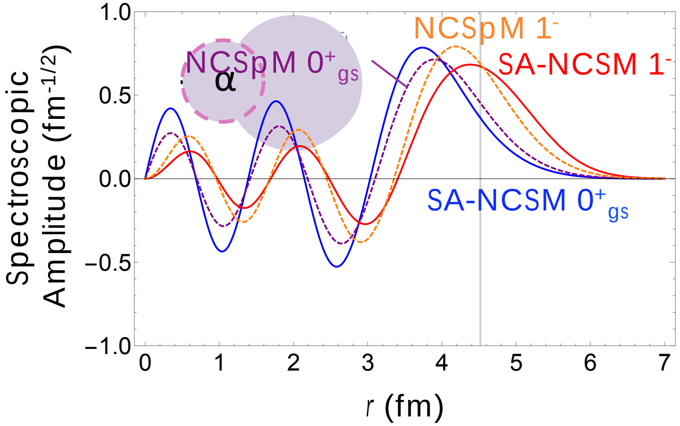
XRB nucleosynthesis abundances from *ab initio* wave functions

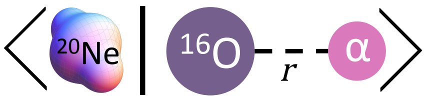




$^{16}O(\alpha,\gamma)^{20}Ne$



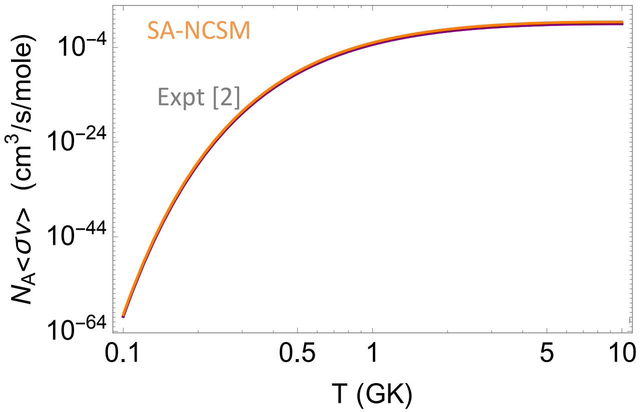




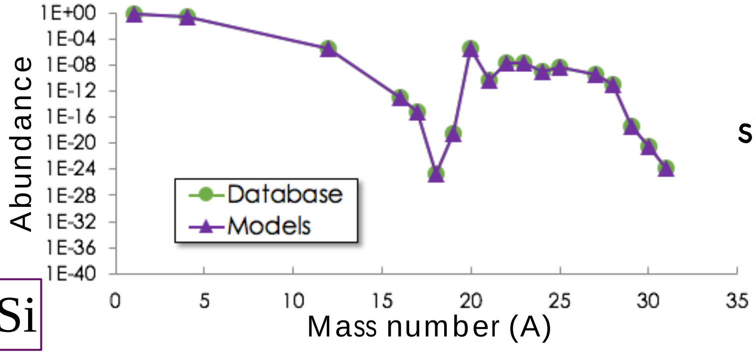
Wave functions from *ab initio* SA-NCSM with N2LOopt

Γ_α width

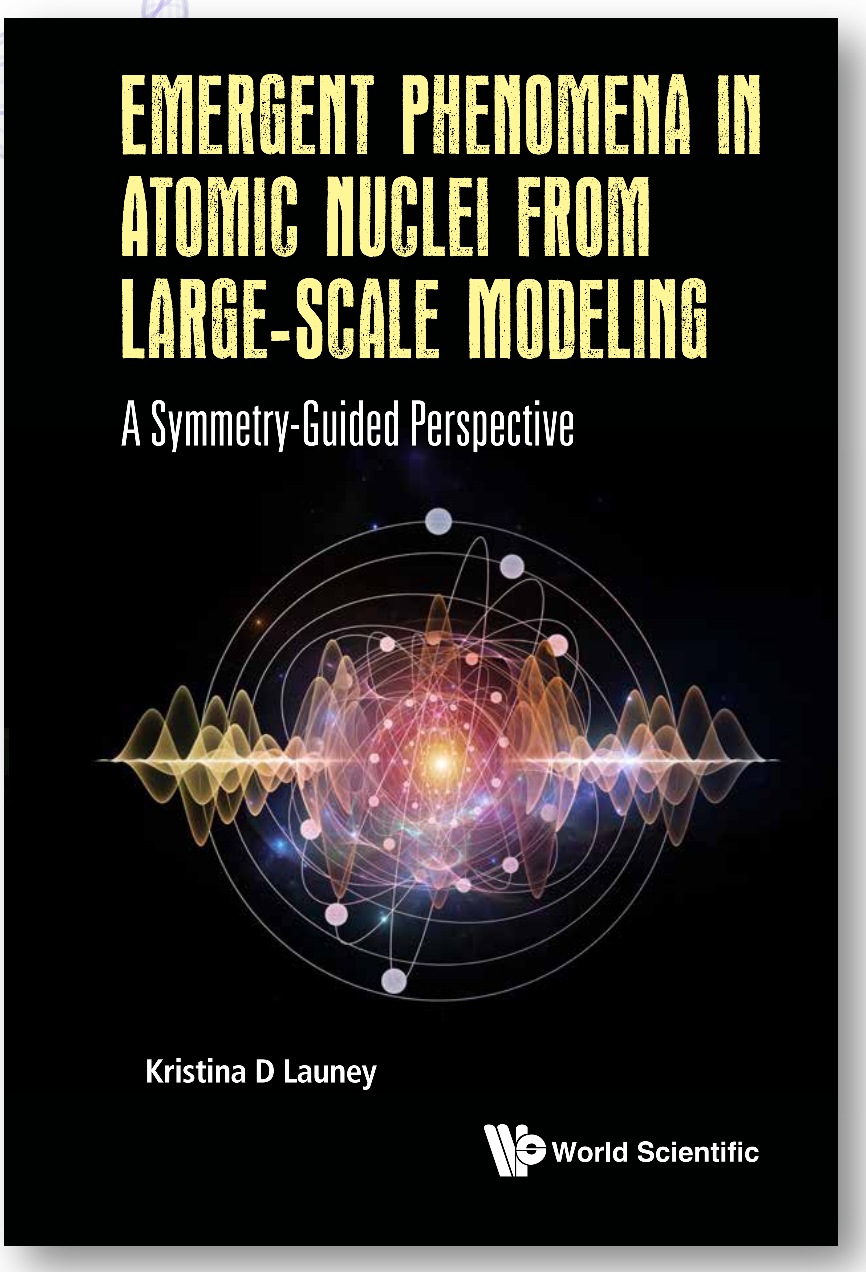
Reaction rates



Nucleosynthesis simulations (Xnet): XRB abundance pattern



$^{23}Al(p,\gamma)^{24}Si$



EMERGENT PHENOMENA IN ATOMIC NUCLEI FROM LARGE-SCALE MODELING

A Symmetry-Guided Perspective

Kristina D Launey

World Scientific

Nuclear Collectivity – Experimental perspective
(John L Wood)

Configuration-interaction models
(Calvin W Johnson)

Symplectic rotor model
(David J Rowe)

Electron Scattering in the Symplectic Shell Model
(Jutta E Escher)

Lattice QCD
(Thomas Luu and Andrea Shindler)

Ab Initio Lattice Effective Field Theory
(Dean Lee)

Correlated Gaussian Approach and Clustering
(Yasuyuki Suzuki and Wataru Horiuchi)

Symmetry-Adapted No-Core Shell Model
(Jerry P Draayer, Tomas Dytrych and KD Launey)

Auxiliary-Field Quantum Monte Carlo Methods
(Yoram Alhassid)

Lie Density Functional Theory
(George Rosensteel)

Exactly Solvable Pairing
(Feng Pan, Xin Guan & Jerry P Draayer)