Parton Distributions in an Instanton Vacuum

Starting with the π^0 PDA

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Where I'm headed

1. Background
2. The Specific Problem
3. The Plan
4. Results (Preliminary)
5. Future Work

Background

- Parton Distributions (PDs) characterize non-perturbative contribution to many experimental hadronic observables

- They can be written as light-cone wavefunctions.

$$\begin{pmatrix} F_q(\xi) \\ \tilde{F}_q(\xi) \end{pmatrix} = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-it\xi(n_\mu P^\mu)} \langle P' | \bar{\psi}_q(tn_\mu) \frac{\not{n}}{2} \begin{pmatrix} 1 \\ \gamma^5 \end{pmatrix} W(tn^\mu, 0) \psi_q(0) | P \rangle$$

- In 2013, quasi-PDs were found [1]. They are the spacelike-separated, Euclidean version of GPDs (as expressed by light-cone wavefunctions). They match normal PDs to leading order in $\Lambda_{QCD}{}^2/Q^2$.

Background

- For example, the simplest of parton distributions is the pion parton distribution amplitude†. The expression for the pion qPDA is:

$$\tilde{\phi}_\pi(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi} e^{-i(x-\bar{x})zP_z/2} \langle \pi(p)| \psi^\dagger(z_-)\gamma^z\gamma^5 [z_-, z_+] \psi(z_+) |0\rangle$$

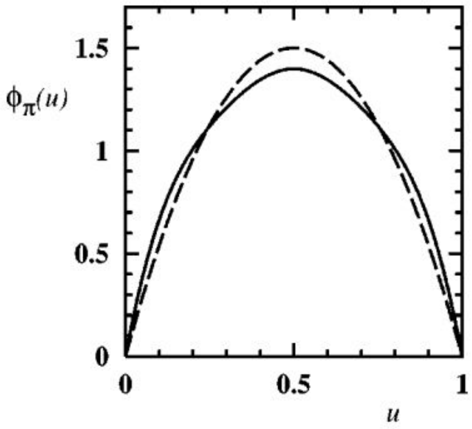
- Evaluating quasi-distributions directly is hard - requires non-perturbative calculational techniques.

- A guide: Ward-Takahashi Identities. They are exact, non-perturbative identities. They give rise to certain PD sum rules, which endow PDs with physical interpretation.

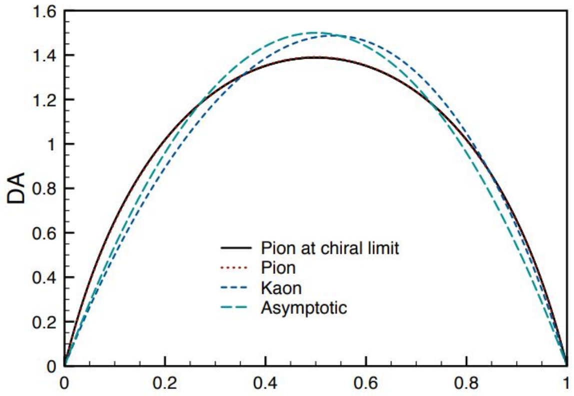
The Specific Problem

- The pion PDA has never been calculated in a scheme that **both** follows from QCD semiclassics and upholds the requisite Ward identities.

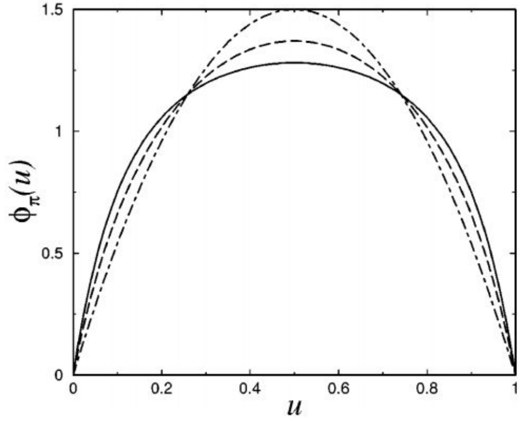
- Despite this, sensible pion PDAs have been obtained. [3 4 5]



V. Y. Petrov et al., (1999)



S. Nam, (2017)



A. Rostworowski, (2001)

The Specific Problem

- Improving their schemes may not change affairs much - or they may.

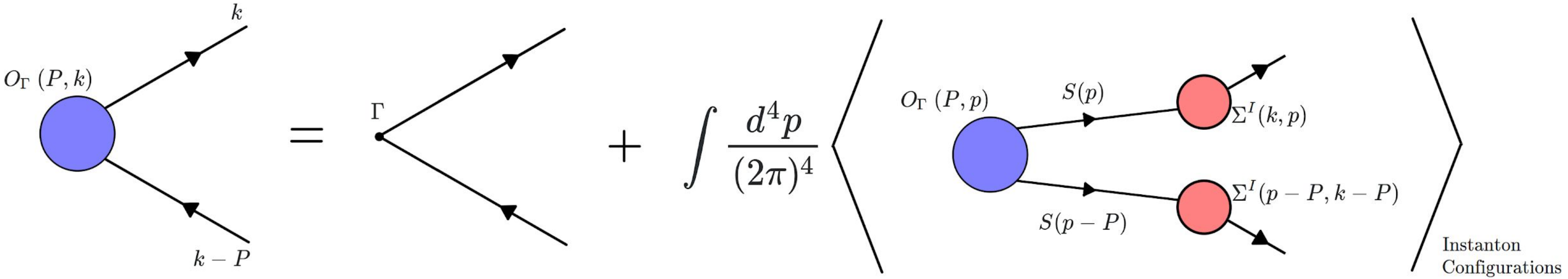
- There has been renewed interest in the exact shape of the pion PDA, especially near the physical endpoints $x \rightarrow 0^+, 1^-$. [7]

The Plan

- Approximate QCD vacuum with dilute, non-interacting gas of instantons & anti-instantons.[6] Expansion parameter becomes "packing-coefficient":

$$\alpha = \sqrt{\frac{n\rho^4}{2N_c}}$$

- Calculate **resummed** propagator and vertex factors in this approximate QCD.



The Plan

- Feynman rules then become a piece of cake, but with bloody analytical expressions.

$$M(p) = \frac{\alpha}{2} \frac{|p\phi'(p)|^2}{||q\phi'^2(q)||^2} + \mathcal{O}(\alpha^2)$$

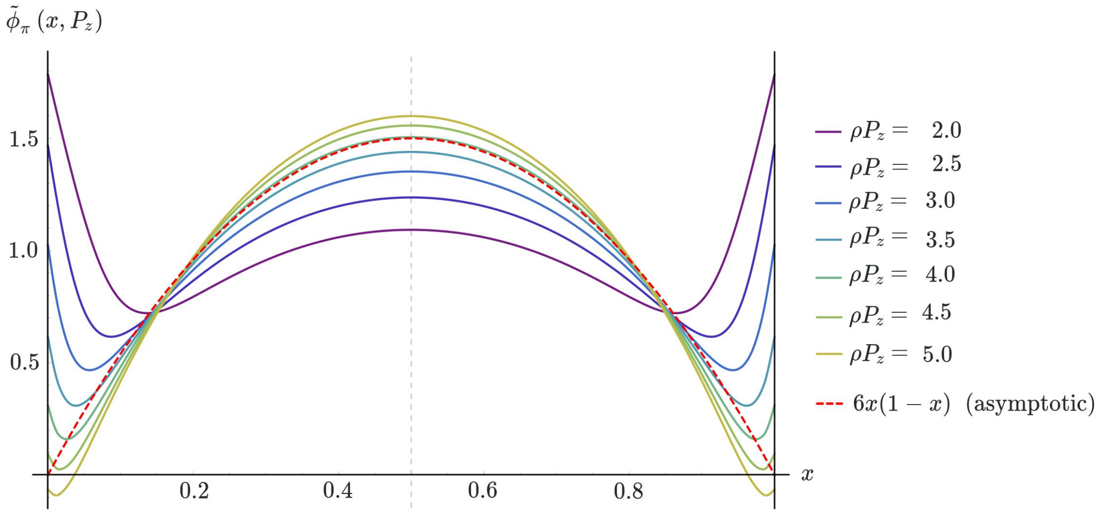
$$\phi'(p) = \pi\rho^2 \frac{d}{dz}(I_0(z)K_0(z) - I_1(z)K_1(z))|_{z=p\rho/2}$$

- Check Ward Identity

- Calculate pion qPDA

Results (Preliminary)

- We have confirmed that at LO and NLO in α, these resummed vertices satisfy the requisite Ward-identity.
- Our pion PDA does not currently match previous literature, but it appears to match recent lattice data. 2



Future Work

- Recheck calculations: Our (my) calculations are probably wrong.
- Calculate pion PDF.
- Extend to other light mesons for further analysis.
- Corroborate with numerical tests, whose details are classified at the moment. This will hopefully establish error bounds on our model!

Conclusion

- PDs are important for nuclear physics, theoretically and experimentally.
- We're coming up with a(nother) calculation scheme to calculate PDs: modeling the QCD vacuum as a dilute set of instantons and anti-instantons, and expanding systematically in the packing-coefficient α. We're testing it on the simplest distribution - the (neutral) pion distribution amplitude.
- Our preliminary results make some sense, and some non-sense. More work to come...
- Hopefully we can calculate polarized nucleon GPDs soon!

Fin.

References

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