HEAVY ION THEORY 2

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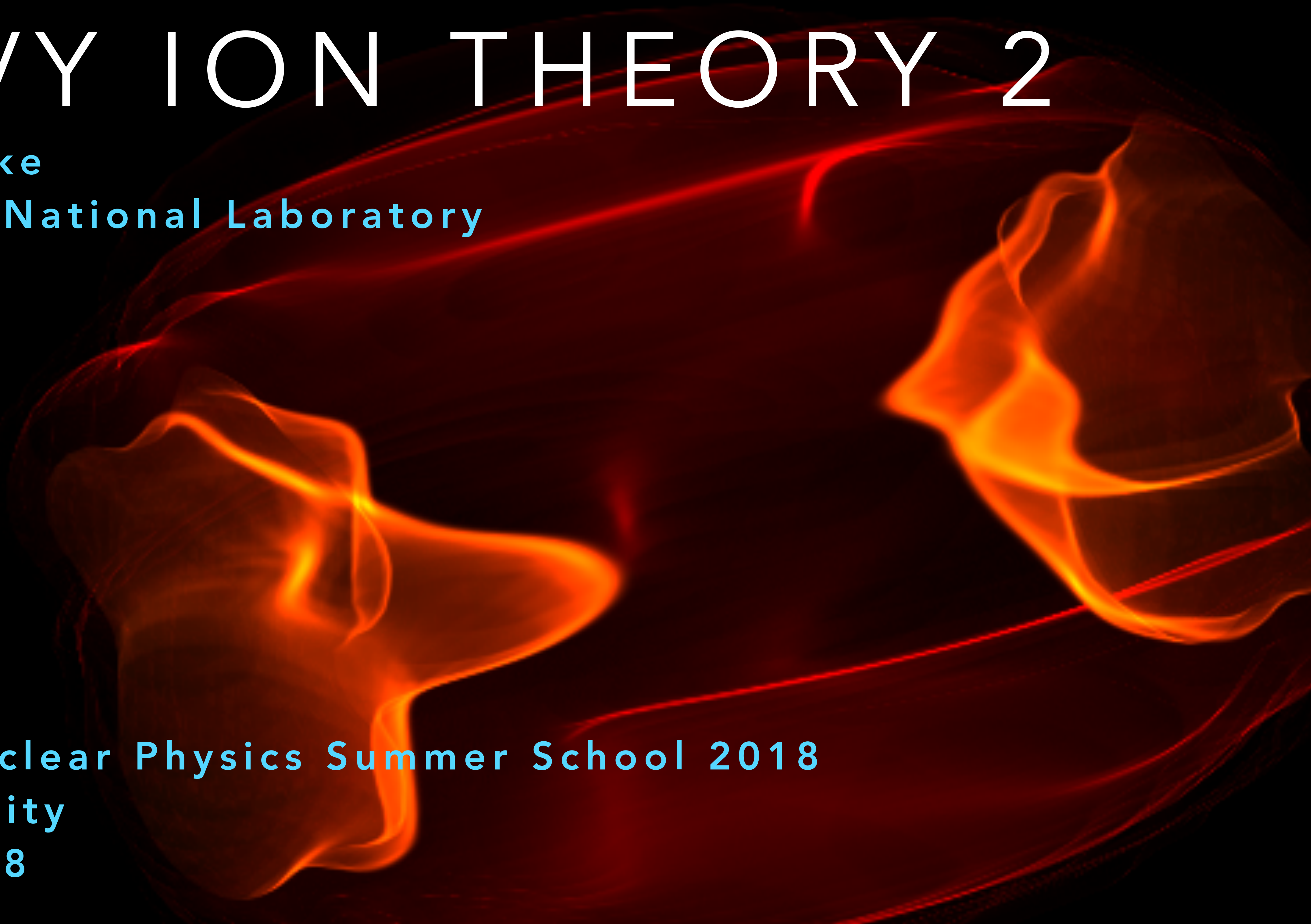

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COLOR GLASS CONDENSATE

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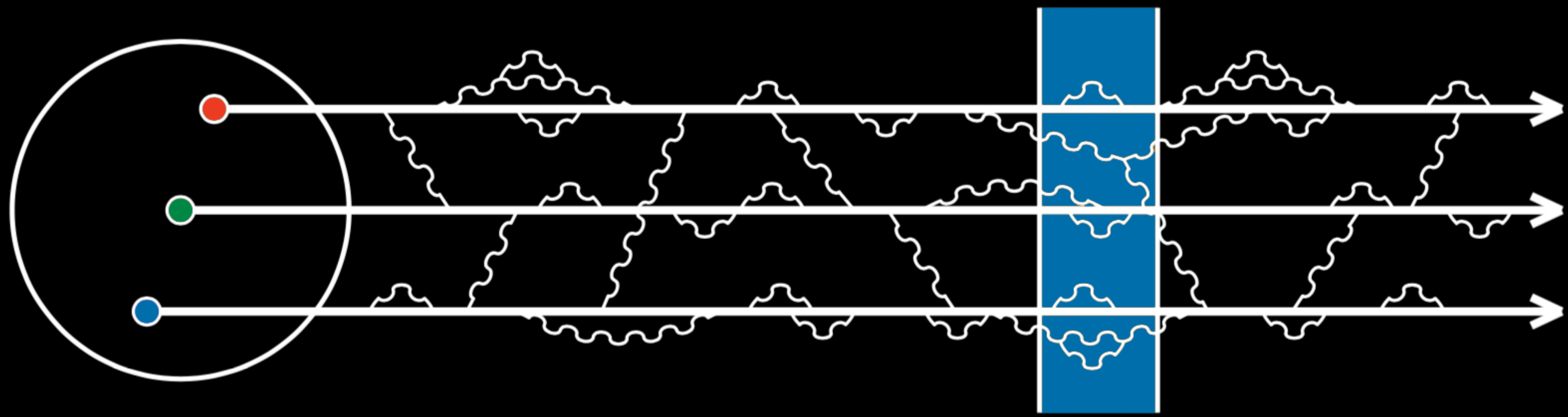


Figure from F. Gelis

Nucleon at rest:

- Complicated non-perturbative object
- Contains fluctuations at all scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of short lived fluctuations is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

COLOR GLASS CONDENSATE

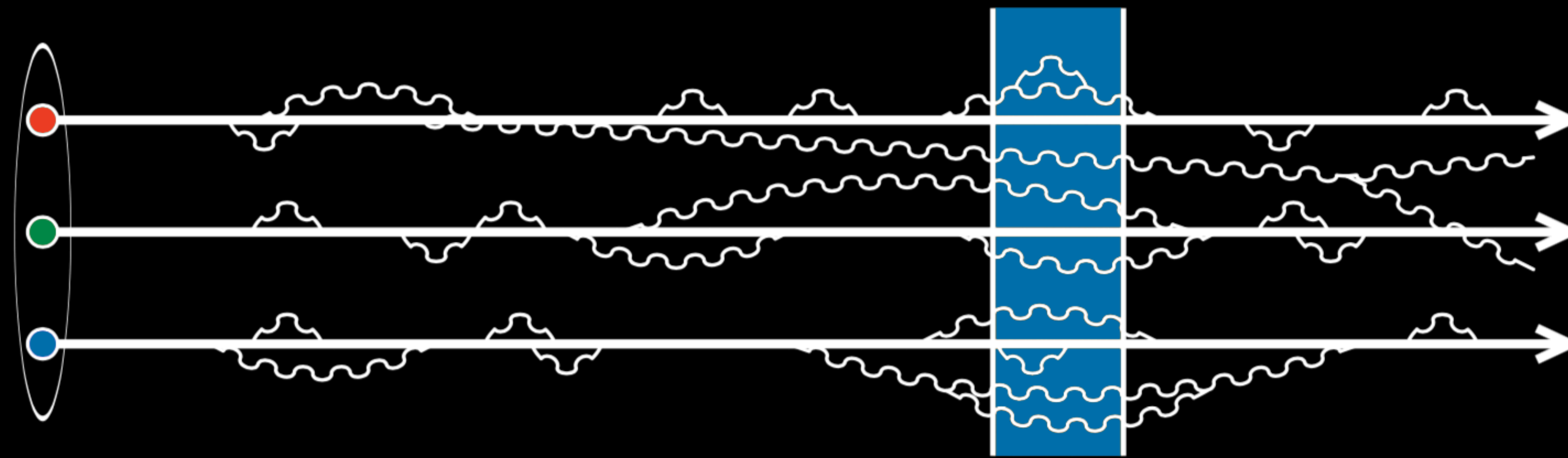


Figure from F. Gelis

Nucleon at high energy:

- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales longer than the characteristic time-scale of the probe
  → The constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. Nucleon appears denser at high energy (contains more gluons)
- Pre-existing fluctuations are totally frozen over the time-scale of the probe, and act as static sources of new partons

PARTON SATURATION

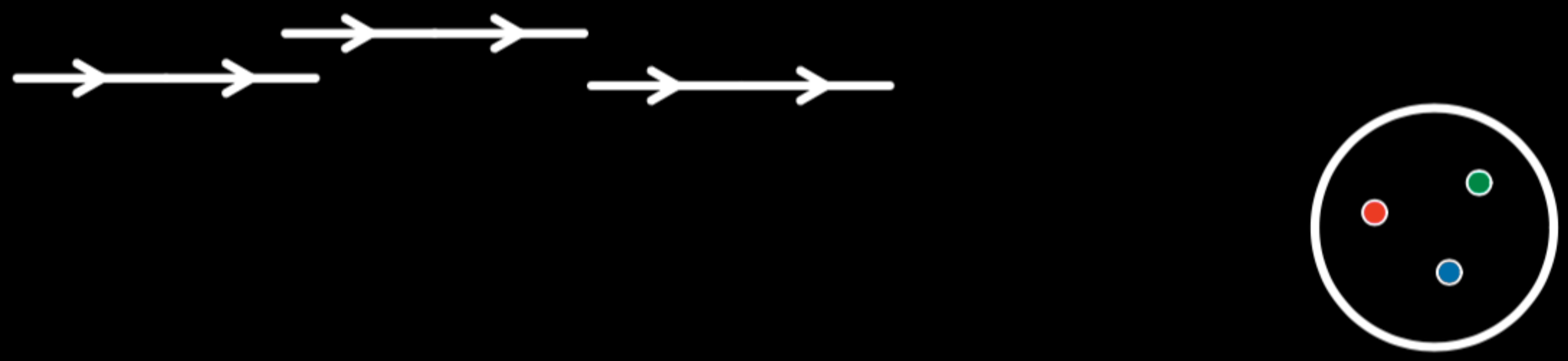


Figure from F. Gelis

At low energy, only valence quarks in the hadron wave function

PARTON SATURATION

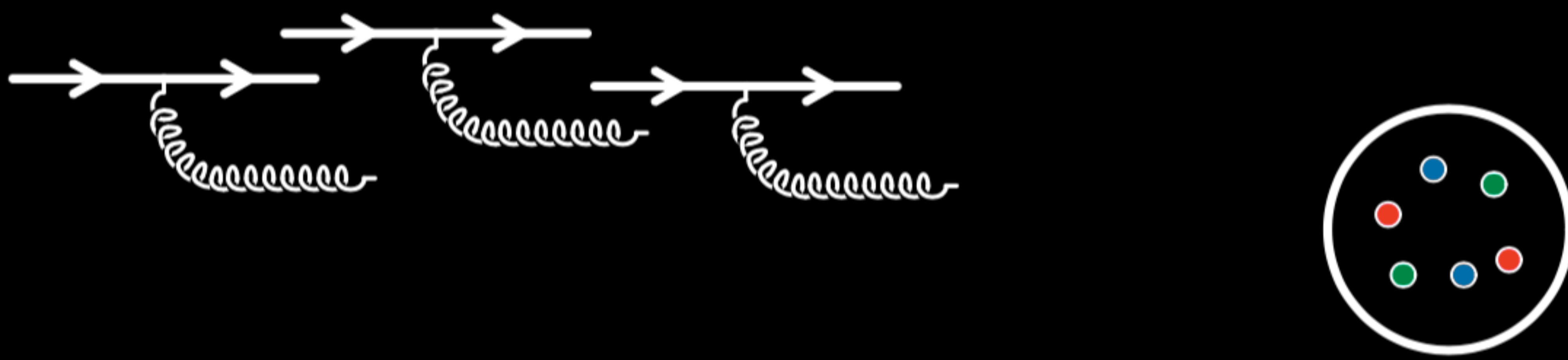


Figure from F. Gelis

- When energy increases, new partons are emitted
- The emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(1/x)$ with x the longitudinal momentum fraction of the gluon
- At small x (i.e. high energy), these logs need to be resummed

PARTON SATURATION

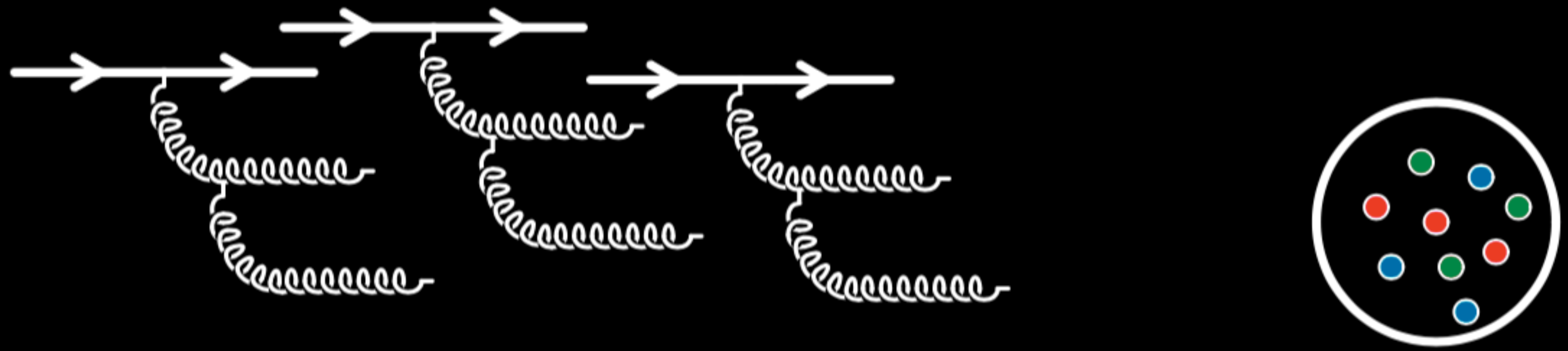


Figure from F. Gelis

- As long as the density of constituents remains small, the evolution is linear:
  The number of partons produced
  at a given step is proportional to
  the number of partons at the previous step (BFKL)

Balitsky-Fadin-Kuraev-Lipatov: L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 642;
V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B60 (1975) 50;
Sov. Phys. JETP 44 (1976) 443; 45 (1977) 199;
Ya. Ya. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822

PARTON SATURATION

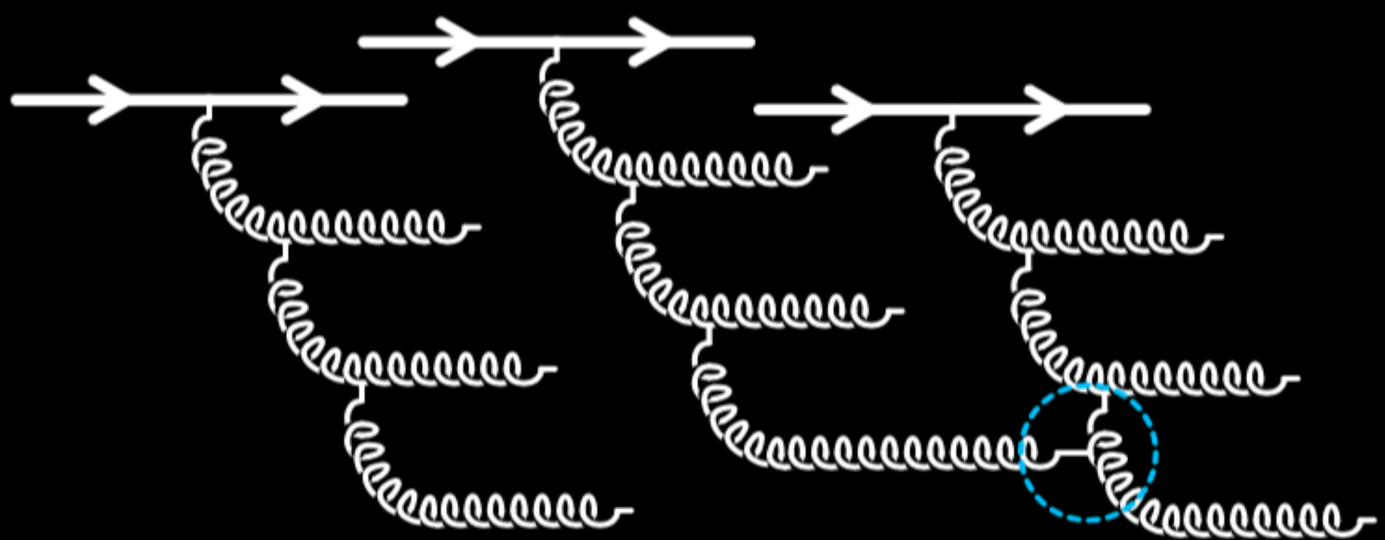


Figure from F. Gelis

- Eventually, the partons start overlapping in phase-space
  → parton recombination occurs

- Then the evolution becomes non-linear:
  The number of partons created at a given step depends non-linearly on the number of partons present previously

Balitsky (1996), Kovchegov (1996,2000)
Jalilian-Marian, Kovner, Leonidov, Weigert (1997,1999)
Iancu, Leonidov, McLerran (2001)

SATURATION CRITERION

L.V. Gribov, E.M. Levin and M.G. Ryskin, Physics Reports 100, Nos. 1 & 2 (1983) 1—150

- Number of gluons per area:

$$\rho \sim \frac{xG(x, Q^2)}{\pi R^2}$$

- Recombination cross section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination important when $\rho\sigma_{gg\to g} \gtrsim 1, \mathrm{i.e.}\ Q^2 \lesssim Q_s^2$

$$\text{with}\ \ Q_s^2 \sim \frac{\alpha_s xG(x, Q_s^2)}{\pi R^2} \sim A^{1/3}x^{-0.3}$$

- At saturation the phase-space density is:

$$\frac{dN_g}{d^2x_\perp d^2p_\perp} \sim \frac{\rho}{Q_s^2} \sim \frac{1}{\alpha_s}$$

DEGREES OF FREEDOM

McLerran, Venugopalan (1994) Iancu, Leonidov, McLerran (2001)

- Small x modes have a large occupation number
  → they can be described by a classical color field A^μ
- Large x modes, slowed down by time dilation, are described as static color sources **ρ**
- The classical field obeys Yang-Mills equations:

$$D_\nu F^{\nu\mu} = J^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{x}_\perp)$$

  The color sources **ρ** are random, and described by a statistical distribution $W_{x_0}[\rho]$, where x_0 is the separation between "small x" and "large x"
- An evolution equation (JIMWLK) controls the changes of $W_{x_0}[\rho]$ with x_0 (generalizes BFKL to the saturated regime)

SEMANTICS

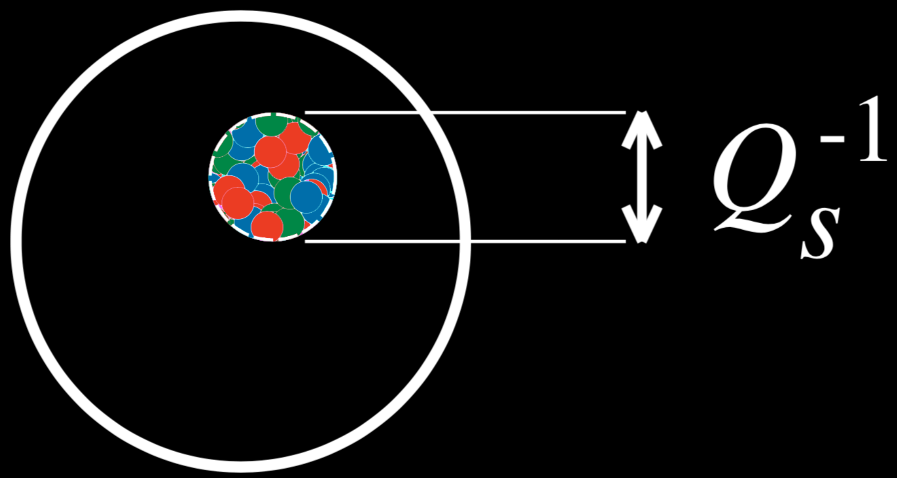
McLerran (mid 2000)

- **Color:** Gluons carry color charge
- **Glass:** The system has degrees of freedom whose time-scale is much larger than the typical time-scales for interaction processes. Moreover, these degrees of freedom are stochastic variables, like in "spin glasses" for instance
- **Condensate:** The soft degrees of freedom are as densely packed as they can (the density remains finite, of order α_s^{-1}, due to repulsive interactions between gluons)

CORRELATION LENGTH

- In a nucleon at low energy, the typical correlation length among color charges is of the order of the nucleon size, i.e. $\Lambda_{QCD}^{-1} \sim 1$ fm. Indeed, at low energy, color screening is due to confinement, controlled by the non-perturbative scale

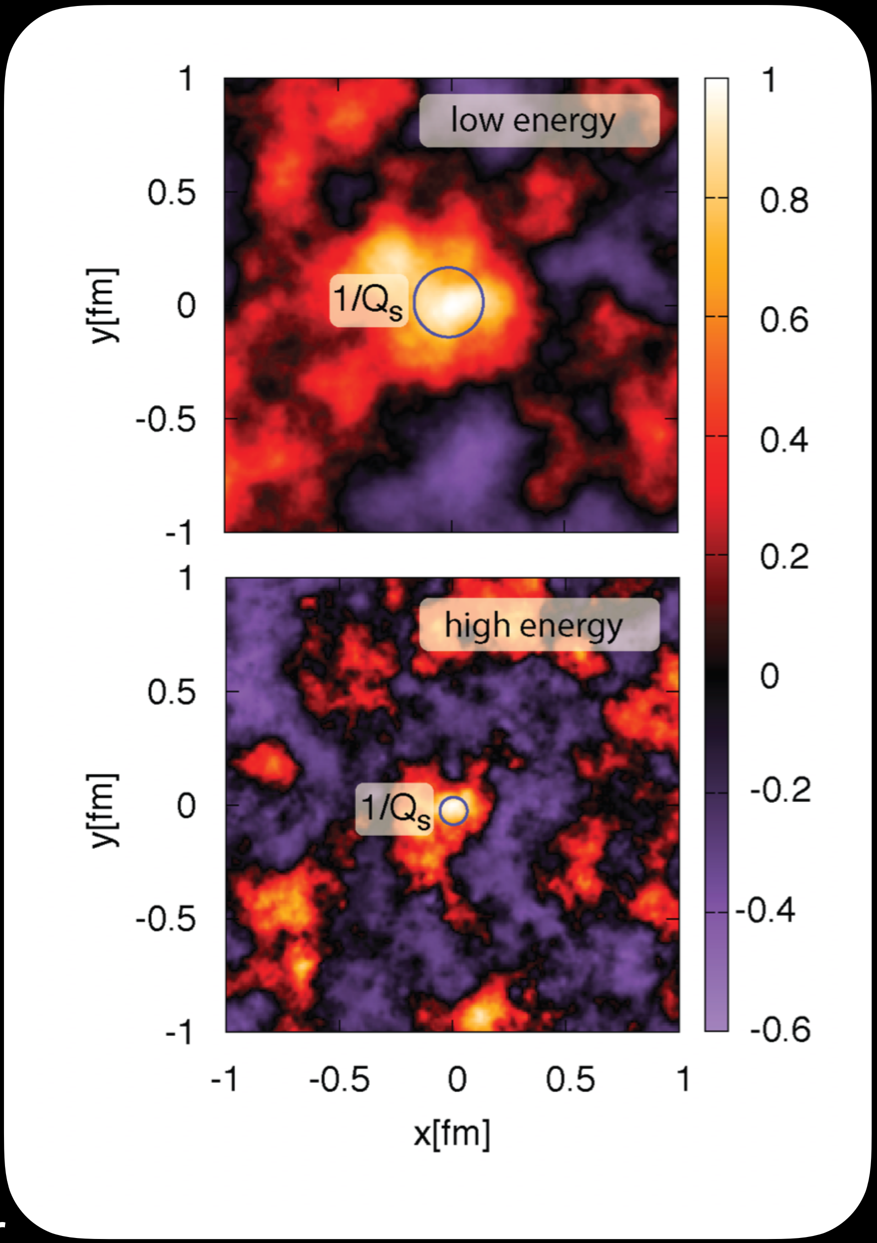
- At high energy (small *x*), partons are much more densely packed, and it can be shown that color neutralization occurs in fact over distances of the order of $Q_s^{-1} \ll \Lambda_{QCD}^{-1}$



- This implies that all hadrons and nuclei behave in the same way at high energy. In this sense, the small x regime described by the CGC is universal.

COLOR GLASS CONDENSATE

- **BFKL equation:** evolution with x of parton distributions, in the linear regime
- **McLerran-Venugopalan model:** a model in which the degrees of freedom are separated in fields (small-x partons) and color sources (large-x partons).
  This model assumes a fixed, gaussian, distribution of the color sources
- **CGC and non-linear evolution:** from renormalization group arguments, one derives the non-linear evolution equation for the distribution of color sources (JIMWLK).
  A mean field approximation of this is the Balitsky-Kovchegov equation



Wilson line correlator, evolved from low to high energy

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

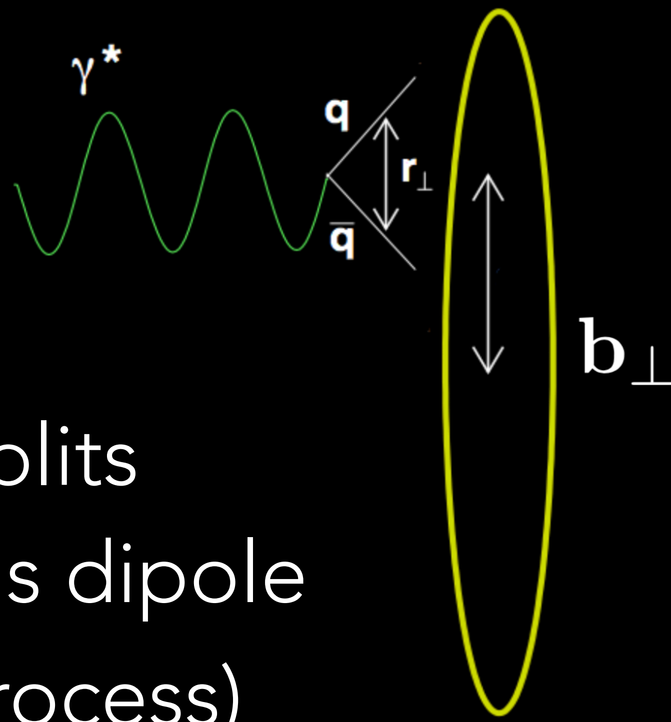
- I will not discuss in detail the derivation and form of the JIMWLK or BK evolution equations

- Instead, we will study a simple saturation model, that parametrizes the *x*-dependence and includes the spatial geometry of the target

- This model will be used to provide input to the IP-Glasma model for heavy ion collisions

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005



- Deeply inelastic scattering off a proton
- Factorized processes: electron emits γ*, γ* splits into quark-antiquark pair of spatial size $r_\perp$, this dipole interacts elastically with the target (eikonal process)
- The splitting is determined by the photon light-cone wave function computed in light-cone perturbation theory

  J.D. Bjorken, J.B. Kogut and D.E. Soper, Phys. Rev. D3, 1382 (1970)

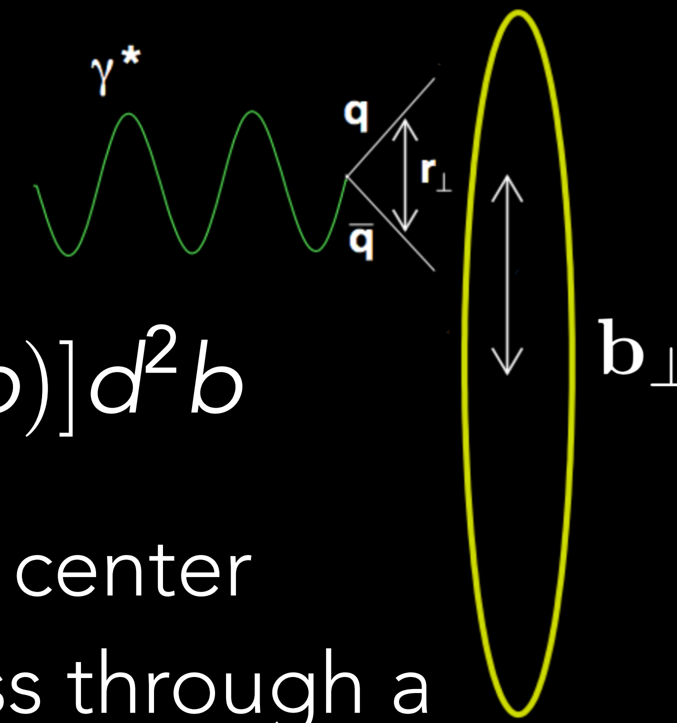
- The elastic scattering of the dipole with transverse momentum transfer $\Delta^2=-t$ described by the scattering amplitude $A_{\rm el}^{q\bar{q}}(x,r,\Delta)$
- As discussed before, the total cross section is given by

$$\sigma_{q\bar{q}}(x,r) = {\rm Im}\, iA_{\rm el}^{q\bar{q}}(x,r,0) = 2\int [1-{\rm Re}S(b)]d^2b$$

note that relates to the previously used f by $f=\dfrac{iAk}{4\pi}$

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005



- $\sigma_{q\bar{q}}(x,r) = \mathrm{Im}\, iA_{\mathrm{el}}^{q\bar{q}}(x,r,0) = 2\int [1 - \mathrm{Re}S(b)] d^2b$
- Here *S(b)* is the S-matrix at distance b from the center
- The total cross section for a small dipole to pass through a dilute gluon cloud is proportional to the dipole area, the strong coupling constant, and the number of gluons in the cloud

  L. Frankfurt, A. Radyushkin, and M. Strikman, Phys. Rev. D55, 98 (1997)

$$\sigma_{q\bar{q}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2)$$

where $xg(x,\mu^2)$ is the gluon density at some scale μ^2

- If the target is dense, the probability that the dipole does not scatter inelastically at impact parameter b is

$$P(b) = 1 - \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) \rho(b,z) dz$$

IPSAT MODEL

total prob. for no inel. interaction

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

$$
\begin{aligned}
P(-L < z \leq L) &= \lim_{n\to\infty} \Pi_{i=0}^{n-1} P(z_i < z \leq z_{i+1}) \\
&= \lim_{n\to\infty} \Pi_{i=0}^{n-1} (1 - \sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \lim_{n\to\infty} \Pi_{i=0}^{n-1} \exp(-\sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \exp(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \sigma_{q\bar{q}}\rho(b, z_i < z \leq z_{i+1})dz) \\
&= \exp\left(-\int_{-L}^{L} \sigma_{q\bar{q}}\rho(b, z)dz\right) \\
&= \exp(-\sigma_{q\bar{q}}T(b)) = P_{\mathrm{tot}}(b) \qquad \text{letting } L \to \infty
\end{aligned}
$$

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

- So the probability for the dipole not to interact inelastically passing through the entire target is:

$$|S(b)|^2 = P_{\rm tot}(b) = \exp\left(-\frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)$$

- Assuming the S-matrix element is predominantly real, we have

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2(1 - {\rm Re} S(b)) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$

- This is the Glauber-Mueller dipole cross section
  A.H. Mueller, Nucl. Phys. B335, 115 (1990)

- $T(b)$ and $xg(x,\mu^2)$ are determined from fits to HERA DIS data (b, x, and initial scale $(\mu_0)^2$ dependence) and DGLAP evolution in μ^2

IPSAT MODEL

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

- The impact parameter dependent function *T(b)* for a proton is assumed to be Gaussian:

$$T(b) = \frac{1}{2\pi B_G} \exp\left(\frac{-b^2}{2B_G}\right)$$

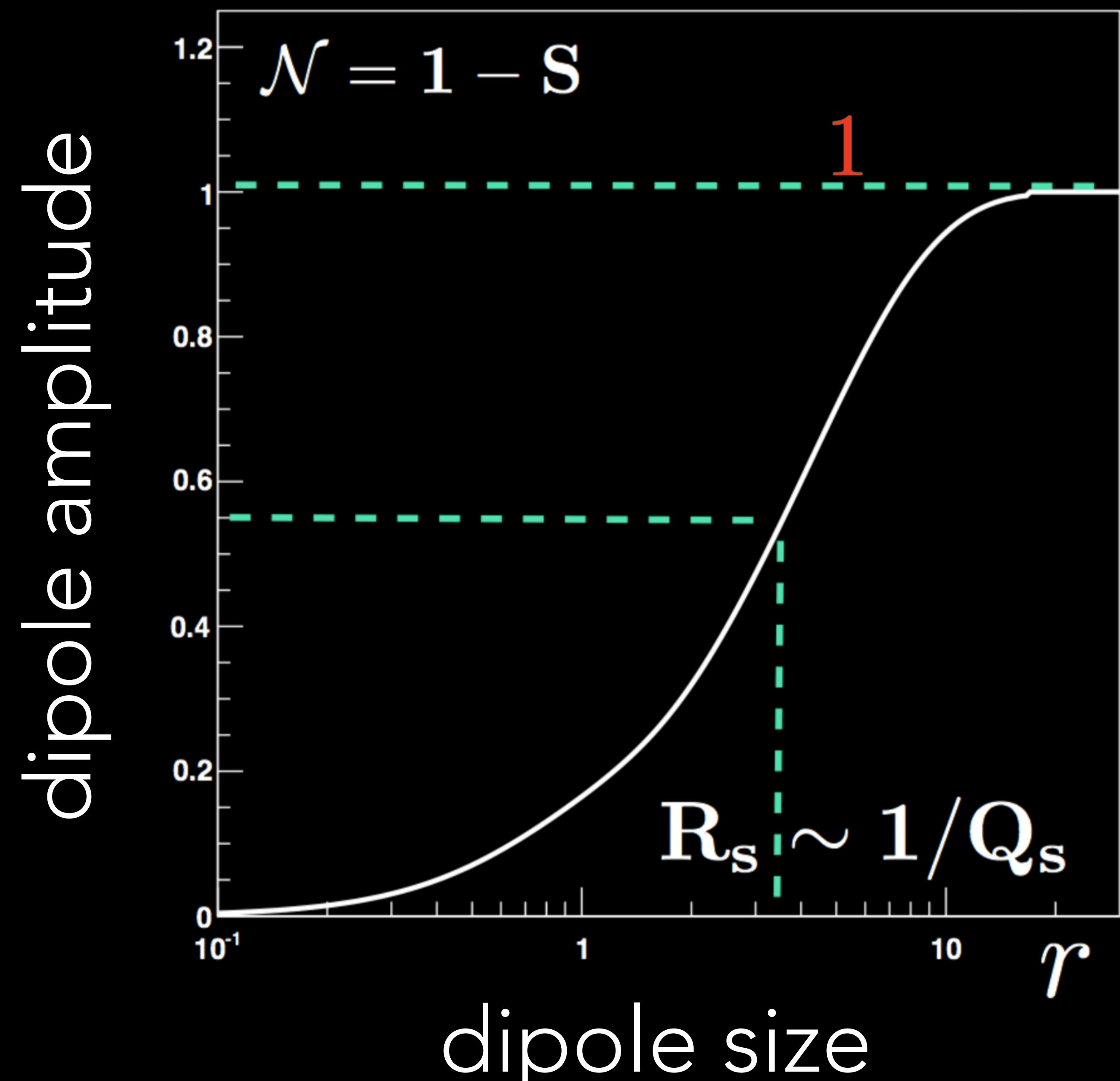
  B_G is assumed to be energy independent and fit yields ~4 GeV^{-2}

- It is related to the average squared gluonic radius $\langle b^2 \rangle = 2B_G$
  b is smaller than the charge radius: *b*=0.56 fm
  (c.f. R_p = 0.8751(61) fm)

- We will later discuss how additional sub-nucleonic fluctuations of this shape affect observables

EXTRACTING Q_S

$$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2(1 - \mathrm{Re}S(b)) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$



Dipole amplitude saturates at 1!
Q_s is defined as the inverse scale where saturation effects begin

$$N(R_s, x, b) = 1 - e^{-1/2}$$

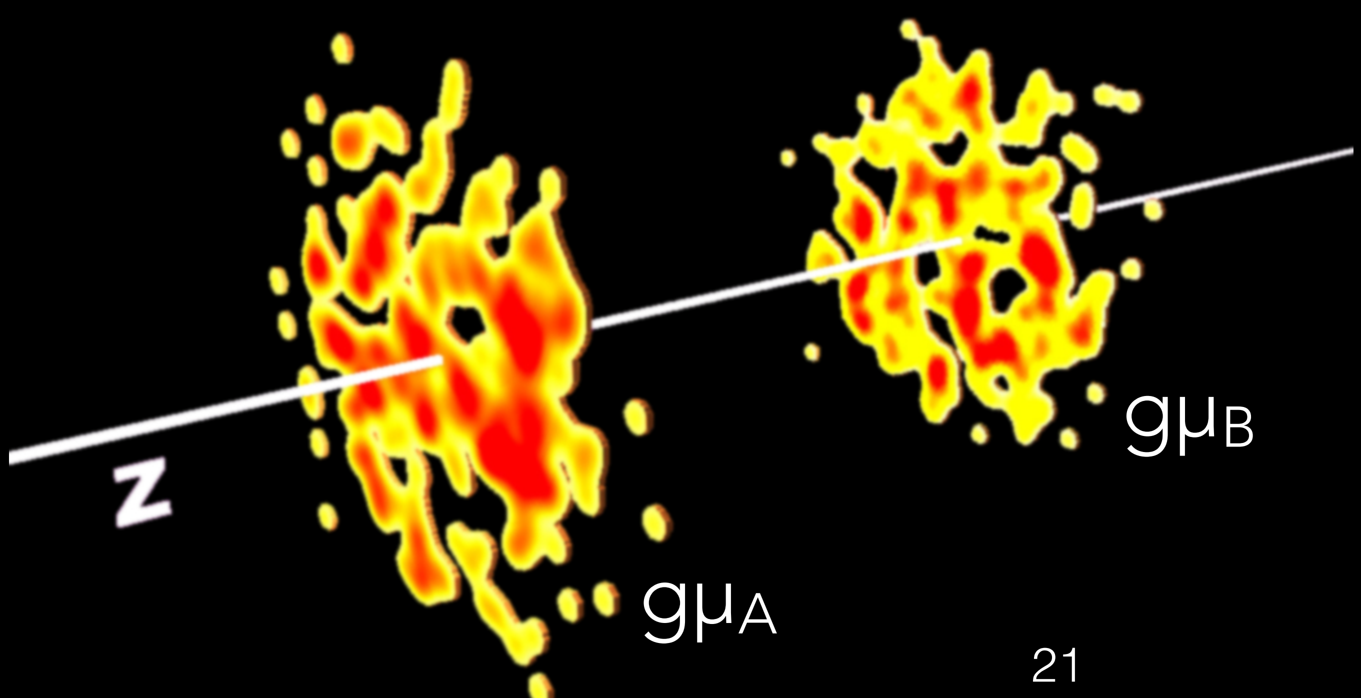
$$Q_s^2 = 2/R_s^2$$

IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Incoming nuclei described within color glass condensate: large x d.o.f. are color sources, small x classical gluon fields
- Incoming currents need to be constructed first:
  - Sample nucleons from nuclear density distributions like in MC-Glauber model
  - Add the *T(b)* at every transverse position
  - Extract Q_s from the IPSat dipole amplitude
  - Obtain the color charge density: $g^4\mu^2 \sim (Q_s)^2$

(the precise proportionality factor is of order 1, see **Lappi, arXiv:0711.3039**: $Q_s/(g^2\mu) = 0.75$)



IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Incoming currents need to be constructed first:
  - Sample color charges ρ^a from local Gaussian distributions with $\langle \rho^a_{A(B)}(\vec{x}_\perp) \rangle = 0$ and

$$\langle \rho^a_{A(B)}(\vec{x}_\perp) \rho^b_{A(B)}(\vec{y}_\perp) \rangle = g^2 \mu^2_{A(B)}(x, \vec{x}) \delta^{ab} \delta^2(\vec{x} - \vec{y})$$

THIS IS THE MV MODEL

  - The sampled color charges comprise the eikonal color current that sources the small-x classical gluon fields

$$J^\nu = \delta^{\nu\pm} \rho_{A(B)}(x^\mp, \vec{x})$$

  - Gluon fields are determined via the Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$

IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

- In covariant gauge the Yang-Mills equations take the form of a Poisson equation:

$$A^{\pm}_{A(B)} = -\frac{\rho_{A(B)}}{\nabla^2_\perp}$$

- The fields before the collision are pure gauge fields and can be transformed to lightcone gauge via the Wilson line

$$V_{A(B)}(\vec{x}) = P\exp\left(-ig\int dx^- \frac{\rho_{A(B)}(x^-,\vec{x})}{\nabla^2_\perp - m^2}\right)$$

infrared regulator $m \sim \Lambda_{QCD}$

- They read

$$A^i_{A(B)} = \theta(x^{-(+)})\frac{i}{g}V_{A(B)}(\vec{x})\partial_i V^\dagger_{A(B)}(\vec{x})$$

and $A^{-(+)}_{A(B)} = A^{+(-)}_{A(B)} = 0$

IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

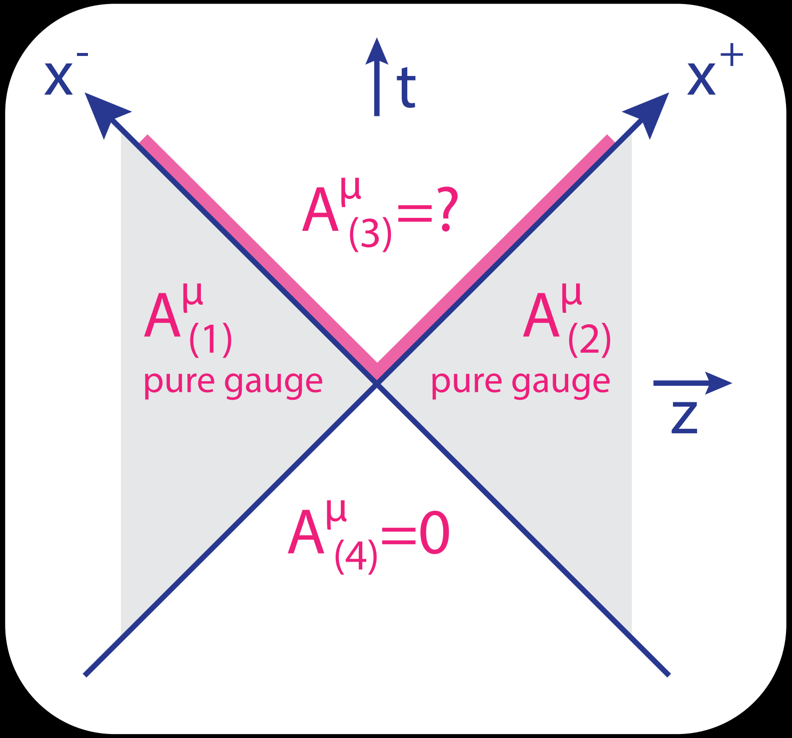
- The fields in the forward light cone at time $\tau = 0^+$ are determined by the requirement that the equations of motion do not contain any singular terms for $\tau \to 0$
  Kovner, McLerran, Weigert, Phys. Rev. D52, 6231 (1995)

- More precisely, choose gauge $x^+A^- + x^-A^+ = 0$, use the ansatz

$$A^i = \theta(x^-)\theta(-x^+)A^i_A + \theta(-x^-)\theta(x^+)A^i_B + \theta(x^-)\theta(x^+)\alpha^i_3$$

$$A^+ = \theta(x^-)\theta(x^+)x^+\alpha$$

$$A^- = -\theta(x^-)\theta(x^+)x^-\alpha$$



and demand that

$$[D_\mu, F^{\mu i}] = 0 \text{ and } [D_\mu, F^{\mu +}] = J^+$$

are not singular on the boundary as $\tau \to 0$

IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

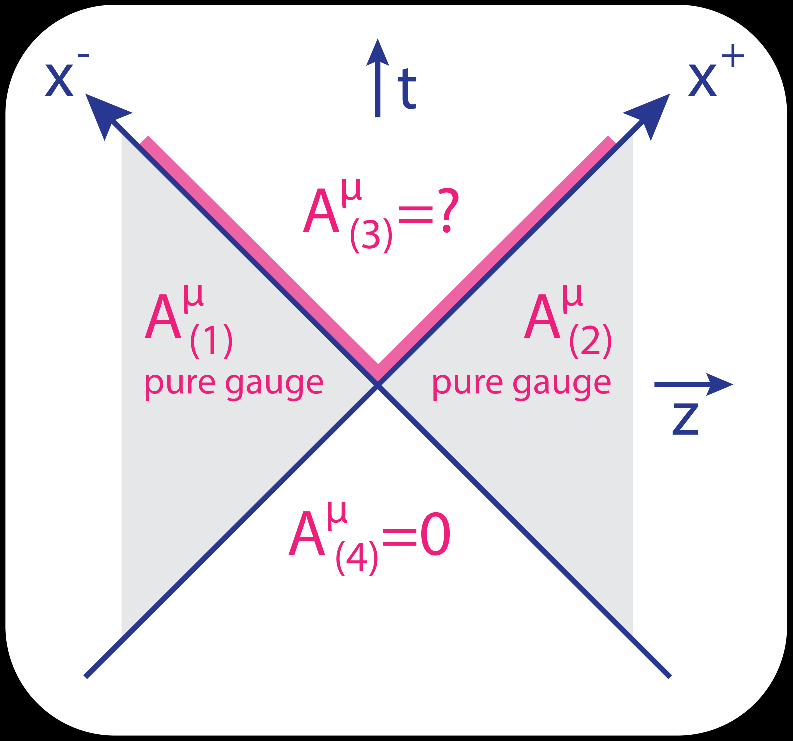
- We find in the forward lightcone:

$$A^i|_{\tau=0^+} = \alpha_3^i = A_A^i + A_B^i$$

$$A^\eta|_{\tau=0^+} = \alpha = \frac{ig}{2}[A_A^i, A_B^i]$$

$$\partial_\tau A^\eta|_{\tau=0} = \partial_\tau A^i|_{\tau=0} = 0$$

Kovner, McLerran, Weigert, Phys. Rev. D52, 6231 (1995)



- These are evolved in time with the source-free Yang Mills equations

- All this can be implemented on a spatial 2D lattice. Care has to be taken with parallel transporting when taking derivatives etc.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

IP-GLASMA MODEL: FIELDS

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

Action:

$$S = \int \mathcal{L}\sqrt{-\det(g_{\mu\nu})}d^4x = -\frac{1}{4}\int \tau F_{\mu\nu}F^{\mu\nu}d\tau dx dy d\eta$$

because $g_{\mu\nu} = (1, -1, -1, -\tau^2)$

$$S = \int \tau d\tau dx dy d\eta \left(-\frac{1}{2}F_{\tau\eta}F^{\tau\eta} - \frac{1}{2}F_{\tau i}F^{\tau i} - \frac{1}{4}F_{ij}F^{ij} - \frac{1}{2}F_{i\eta}F^{\eta}\right)$$

Gauge condition: $A^\tau = x^+A^- + x^-A^+ = 0$

$$E^i = \frac{\delta S}{\delta(\partial_\tau A_i)} = -\tau F^{\tau i} = -\tau g^{\tau\tau}g^{ij}F_{\tau j} = \tau\partial_\tau A_i$$

is the conjugate momentum to A^i

INITIAL FIELDS

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

$$E^i = \frac{\delta S}{\delta(\partial_\tau A_i)} = -\tau F^{\tau i} = -\tau g^{\tau\tau} g^{ij} F_{\tau j} = \tau \partial_\tau A_i$$

$$E^\eta = \frac{\delta S}{\delta(\partial_\tau A_\eta)} = -\tau F^{\tau\eta} = -\tau g^{\tau\tau} g^{\eta\eta} F_{\tau\eta} = \frac{1}{\tau}\partial_\tau A_\eta$$

So the initial conditions are

$$A^\eta|_{\tau=0^+} = \alpha = \frac{ig}{2}[A_A^i, A_B^i]$$

$$E^\eta|_{\tau=0} = \frac{1}{\tau}\partial_\tau(-\tau^2 A^\eta|_{\tau=0}) = -2A^\eta|_{\tau=0} - \tau\partial_\tau A^\eta|_{\tau=0}$$

$$A_\eta|_{\tau=0} = -\tau^2 A^\eta|_{\tau=0} = 0 \quad \text{and} \quad E^i|_{\tau=0} = 0$$

INITIAL FIELDS - SUMMARY

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

- The initial color electric and magnetic fields are

$$E^i = \tau\partial_\tau A_i = 0$$

$$B^{x/y} = -\frac{1}{2}\epsilon^{(x/y)jk}F_{jk} = 0$$

because $A_\eta = 0$ initially and gradients in η vanish $j,k \in \{x,y,\eta\}$

$$E^\eta = -ig[A^i_A, A^i_B]$$

$$B^\eta = F^{yx} = F_{yx} = \partial_y A_x - \partial_x A^y$$

Initially, the color electromagnetic fields only have longitudinal components!

IP-GLASMA MODEL

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

- Finally, let's compute the stress energy tensor, defined by

$$T^{\mu\nu} = -g^{\mu\alpha}g^{\nu\beta}g^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} + \frac{1}{4}g^{\mu\nu}g^{\alpha\gamma}g^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}$$

- The energy density is $T^{\tau\tau}$

$$T^{\tau\tau} = \frac{1}{2}(E^\eta)^2 + \frac{1}{2\tau^2}[(E^x)^2 + (E^y)^2] + \frac{1}{2}F_{xy}F_{xy} + \frac{1}{2\tau^2}(F_{x\eta}^2 + F_{y\eta}^2)$$

- $\frac{1}{2}(E^\eta)^2$: longitudinal electric field
- $\frac{1}{2\tau^2}[(E^x)^2 + (E^y)^2]$: transverse electric field
- $\frac{1}{2}F_{xy}F_{xy}$: longitudinal magnetic field
- $\frac{1}{2\tau^2}(F_{x\eta}^2 + F_{y\eta}^2)$: transverse magnetic field

EVOLUTION

B.Schenke, P.Tribedy, R.Venugopalan, PRL108, 252301 (2012), PRC86, 034908 (2012)

$$T^{\tau\tau} = \frac{1}{2}(E^\eta)^2 + \frac{1}{2\tau^2}[(E^x)^2 + (E^y)^2] + \frac{1}{2}F_{xy}F_{xy} + \frac{1}{2\tau^2}(F_{x\eta}^2 + F_{y\eta}^2)$$