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BROOKHAVEN NATIONAL LABORATORY

HEAVY ION THEORY

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Brookhaven National Laboratory

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Yale University
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LITERATURE

Glauber model:

Michael L. Miller, Klaus Reygers, Stephen J. Sanders, Peter Steinberg
"Glauber modeling in high energy nuclear collisions"
Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243

Color Glass Condensate:

Edmond Iancu, Raju Venugopalan
"The Color glass condensate and high-energy scattering in QCD"
Hwa, R.C. (ed.) et al.: Quark gluon plasma 249-3363

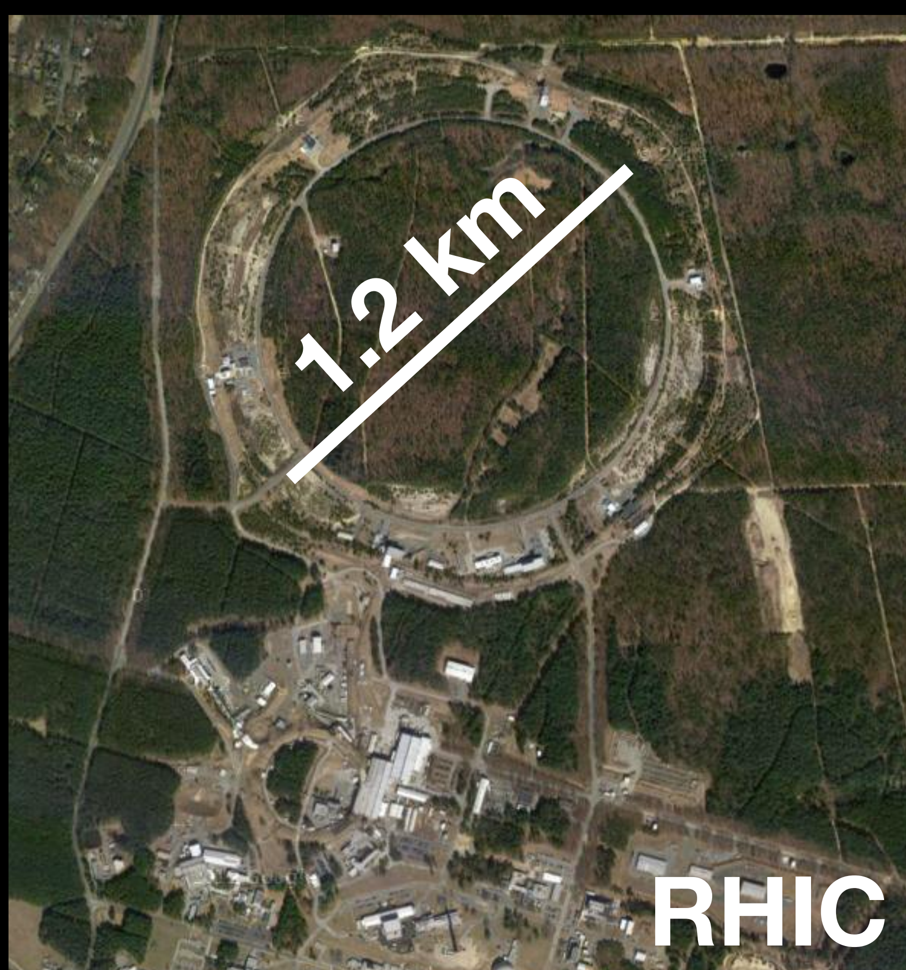
Relativistic Hydrodynamics:

Paul Romatschke
"New Developments in Relativistic Viscous Hydrodynamics"
Int.J.Mod.Phys. E19 (2010) 1-53

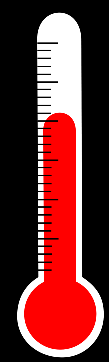
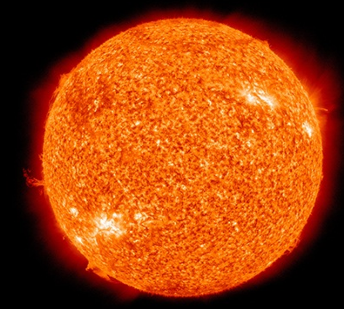
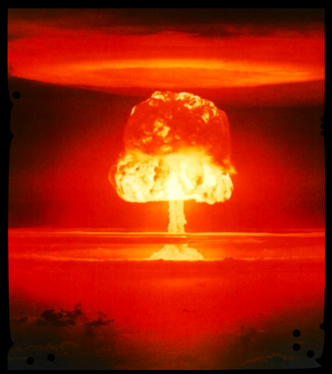
Charles Gale, Sangyong Jeon, Bjoern Schenke
"Hydrodynamic Modeling of Heavy-Ion Collisions"
Int.J.Mod.Phys. A28 (2013) 1340011

HEAVY ION COLLISIONS

>99.9999% of the speed of light



THE STUFF CREATED

Hot:  **>10^{12} K** ≫  

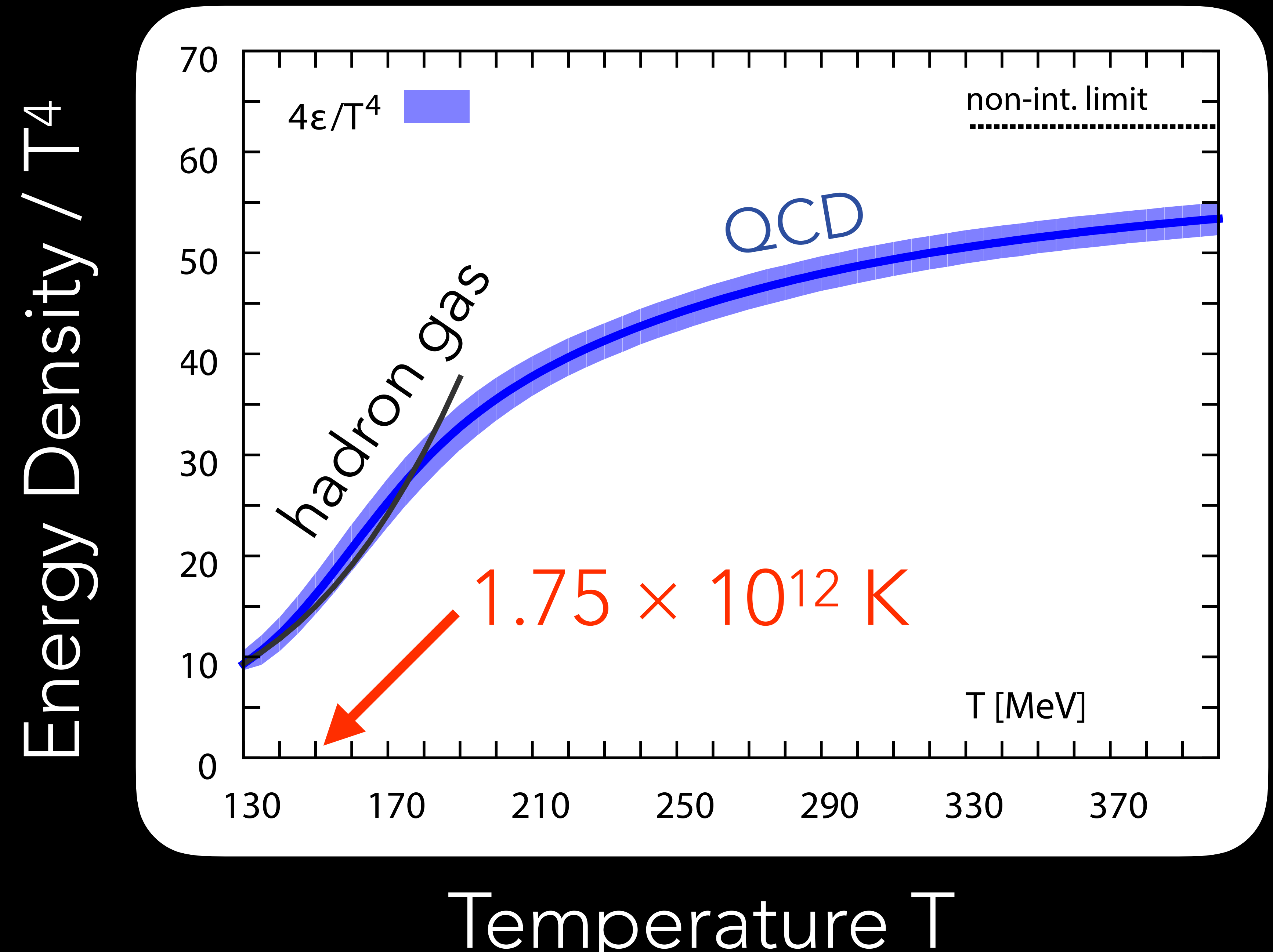
100,000 times hotter than the sun or a hydrogen bomb (10^7 to 10^8 K)

Small:  **10^{-14} m** ≪ 

100,000,000,000 times smaller than a typical water droplet

QUARK GLUON PLASMA

Quantum Chromodynamics (QCD) predicts crossover to system of liberated quarks and gluons at temperatures of ~ 2×10^{12} K

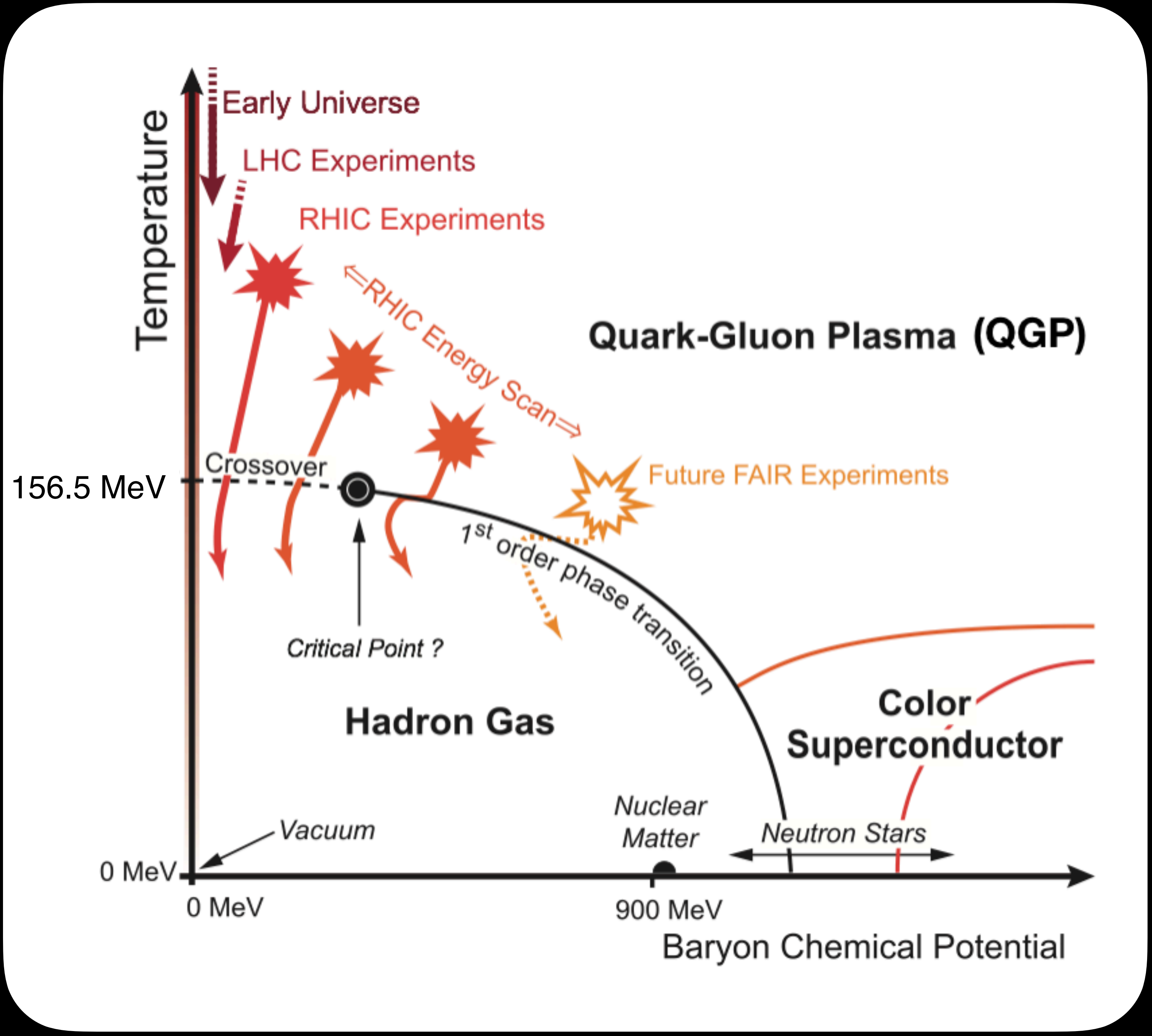


HotQCD Collaboration, Phys.Rev. D90 (2014) 094503

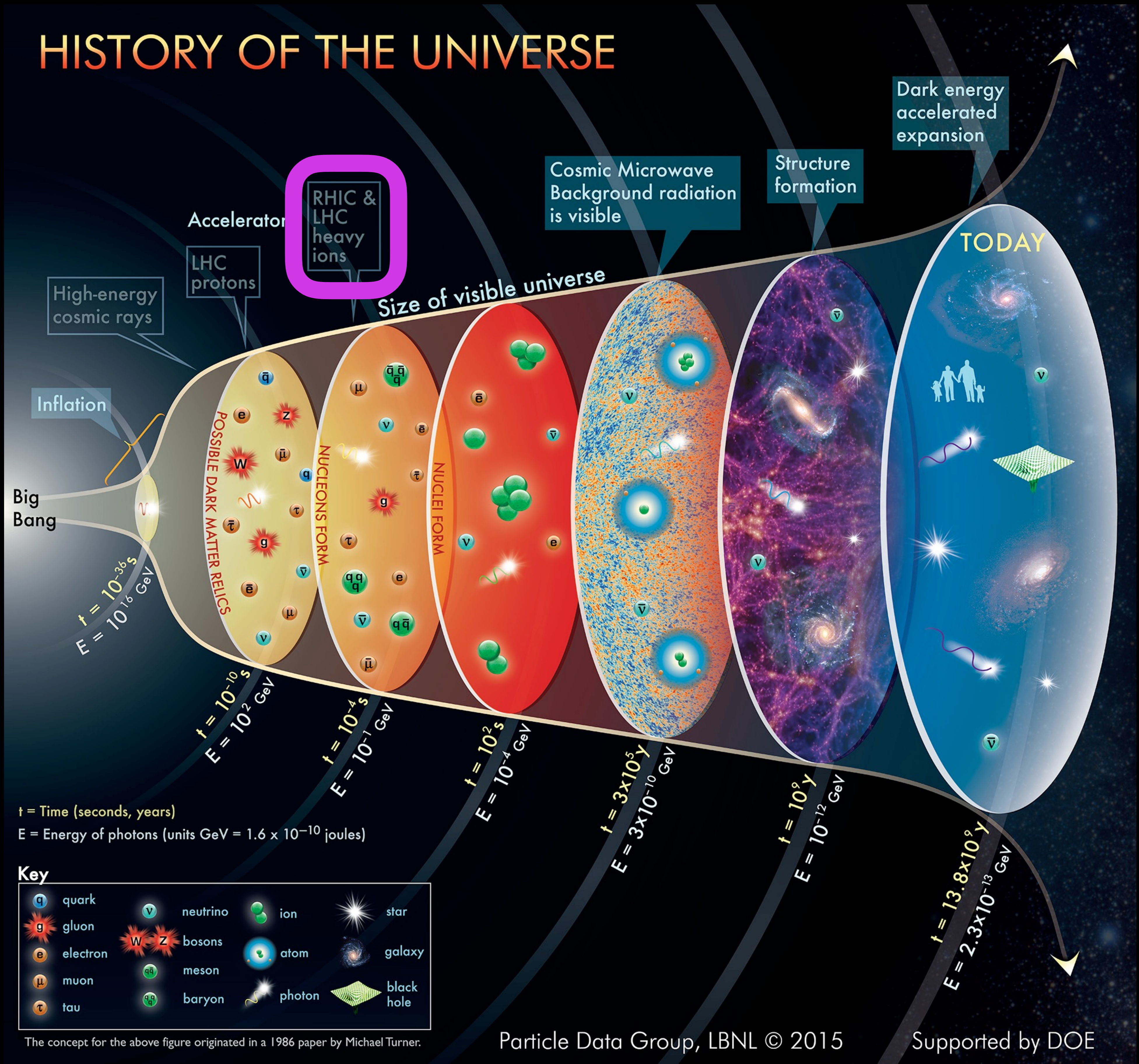
also see: Sz. Borsányi et al, JHEP 1208 (2012) 053

PHASE DIAGRAM

We aim to understand the phase structure of nuclear matter

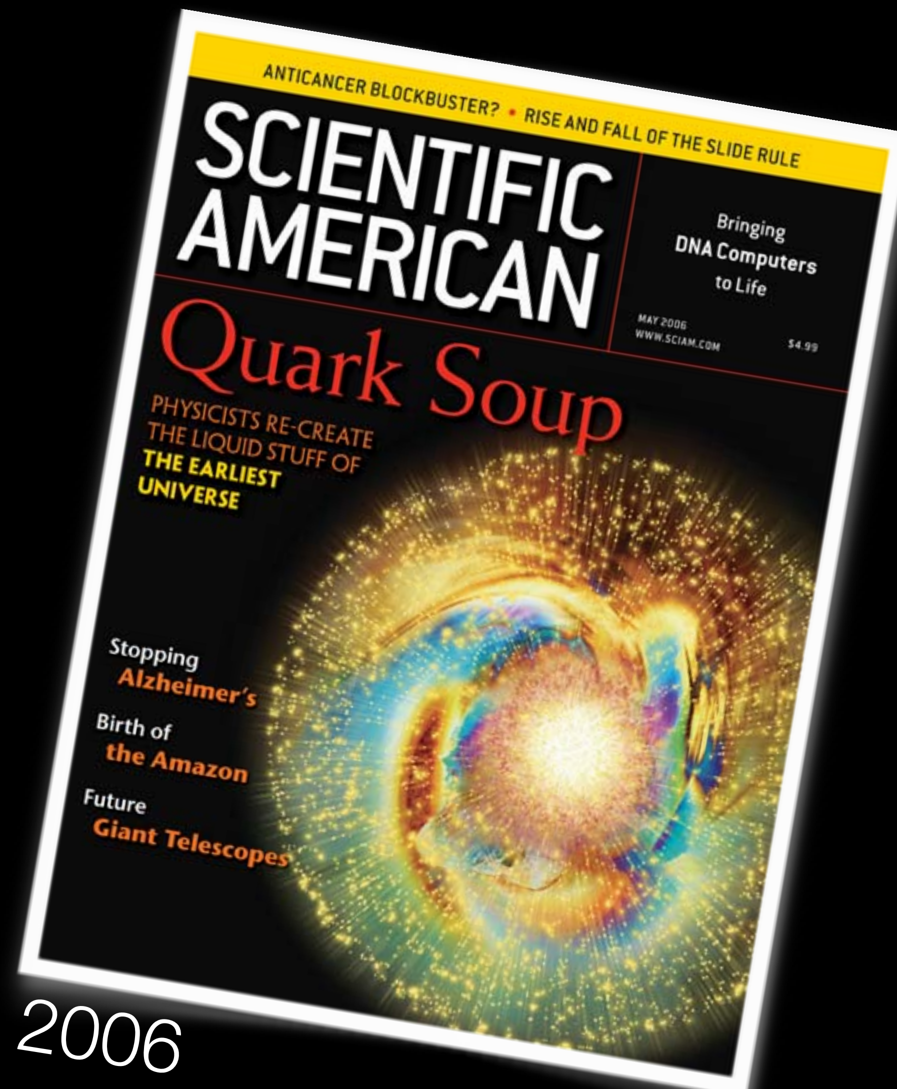


AFTER 10 MICROSECONDS...



THE "PERFECT FLUID"

- **Discovery at RHIC:**
  The Quark Gluon Plasma behaves like an almost perfect fluid



2006

- confirmed by results from LHC in 2010

- We conclude this from comparison of measured azimuthal anisotropies in particle spectra to theoretical calculations, in particular hydrodynamics
  Data only explained if the produced matter flows

OUTLINE

- **Initial State:**
  Glauber model of particle production
  Effective theories at high energy (color glass condensate)

- **Hydrodynamics:**
  Ideal and viscous relativistic hydrodynamics
  Equation of state, transport parameters

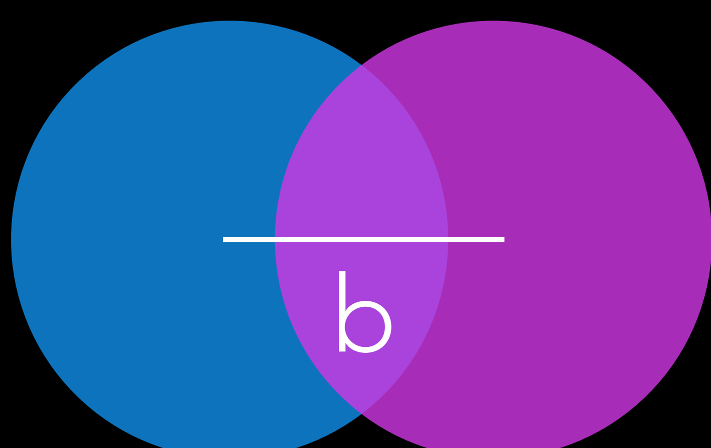
- **Some results and comparison to experimental data**

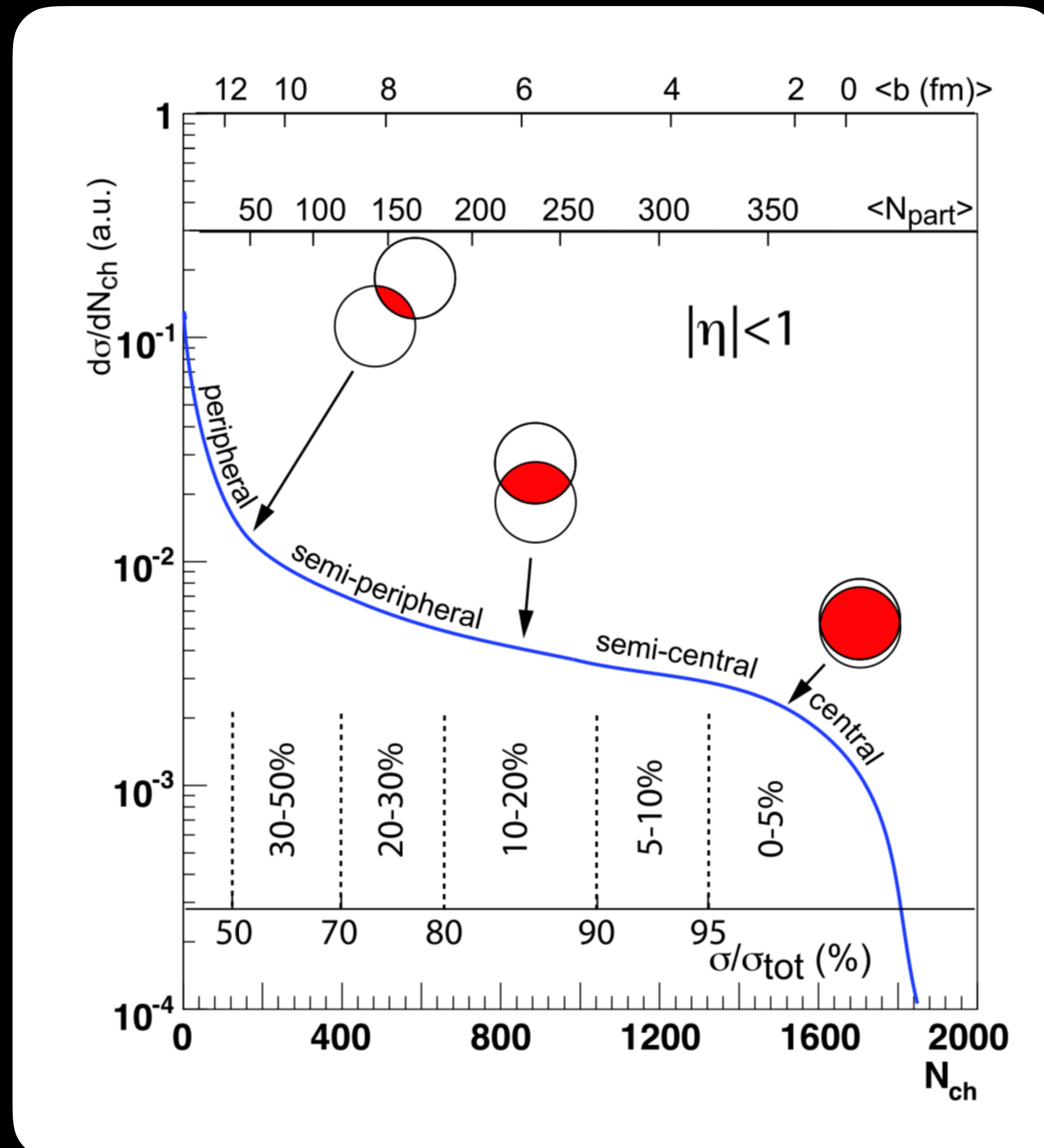
- **Small systems (p+p, p+A, d+A, ^{3}He+A):**
  Initial vs. final state effects
  Is there a coherent framework from low to high multiplicity?

DEFINITIONS

- **Centrality**
  Percentage bins counting from the highest multiplicities

- **Impact parameter**

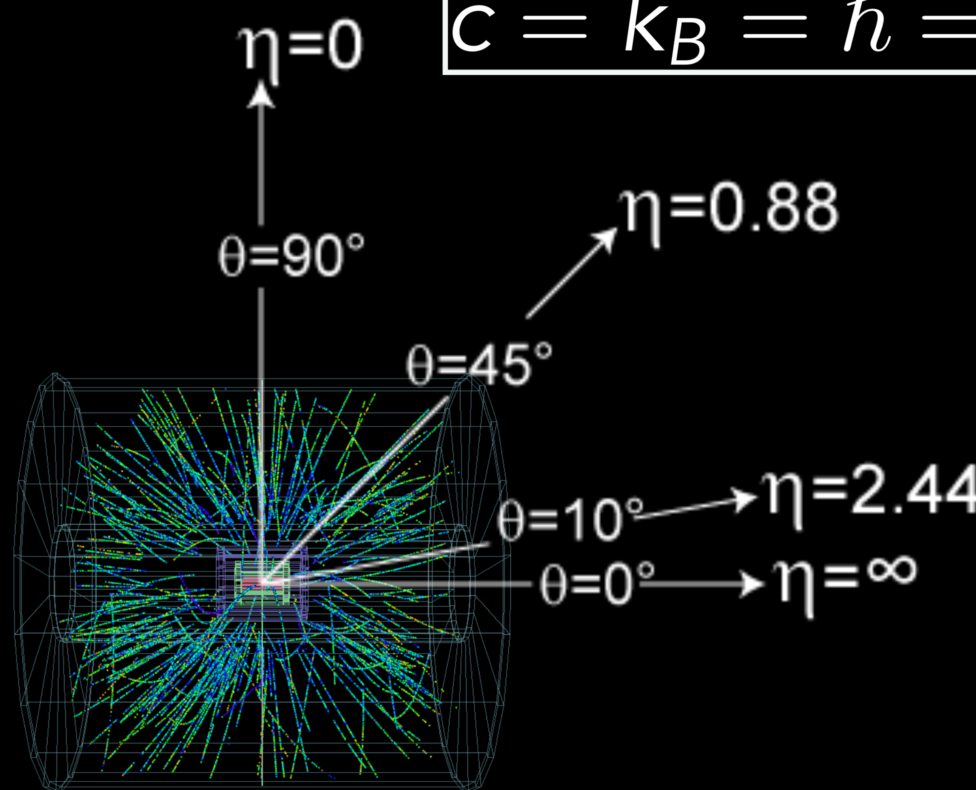




Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243

DEFINITIONS

$c = k_B = \hbar = 1$



- **Rapidity *y*, Pseudo-rapidity *η***

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}$$

$$\eta = \frac{1}{2}\ln\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z} = \tanh^{-1}[\cos(\theta)] = -\ln[\tan(\theta/2)]$$

$\vec{v} = \vec{p}/E$

$$\begin{aligned}E &= m\cosh y\\ &= m\cosh\left[\frac{1}{2}\ln\left(\frac{1+v_z}{1-v_z}\right)\right]\\ &= \frac{m}{\sqrt{2}}\left\{\cosh\left[\ln\left(\frac{1+v_z}{1-v_z}\right)\right]+1\right\}^{\frac{1}{2}}\\ &= \frac{m}{\sqrt{2}}\left[\frac{1}{2}\left(\frac{1+v_z}{1-v_z}+\frac{1-v_z}{1+v_z}\right)+1\right]^{\frac{1}{2}}\\ &= m\sqrt{\frac{1}{1-v_z^2}} = m\gamma\end{aligned}$$

$$\begin{aligned}p_z &= m\sinh y\\ &= m\sinh\left[\frac{1}{2}\ln\left(\frac{1+v_z}{1-v_z}\right)\right]\\ &= \frac{m}{\sqrt{2}}\mathrm{sgn}(v_z)\left\{\cosh\left[\ln\left(\frac{1+v_z}{1-v_z}\right)\right]-1\right\}^{\frac{1}{2}}\\ &= \frac{m}{\sqrt{2}}\mathrm{sgn}(v_z)\left[\frac{1}{2}\left(\frac{1+v_z}{1-v_z}+\frac{1-v_z}{1+v_z}\right)-1\right]^{\frac{1}{2}}\\ &= m\,\mathrm{sgn}(v_z)\sqrt{\frac{v_z^2}{1-v_z^2}} = m\,\mathrm{sgn}(v_z)|v_z|\gamma = m v_z\gamma\end{aligned}$$

DEFINITIONS

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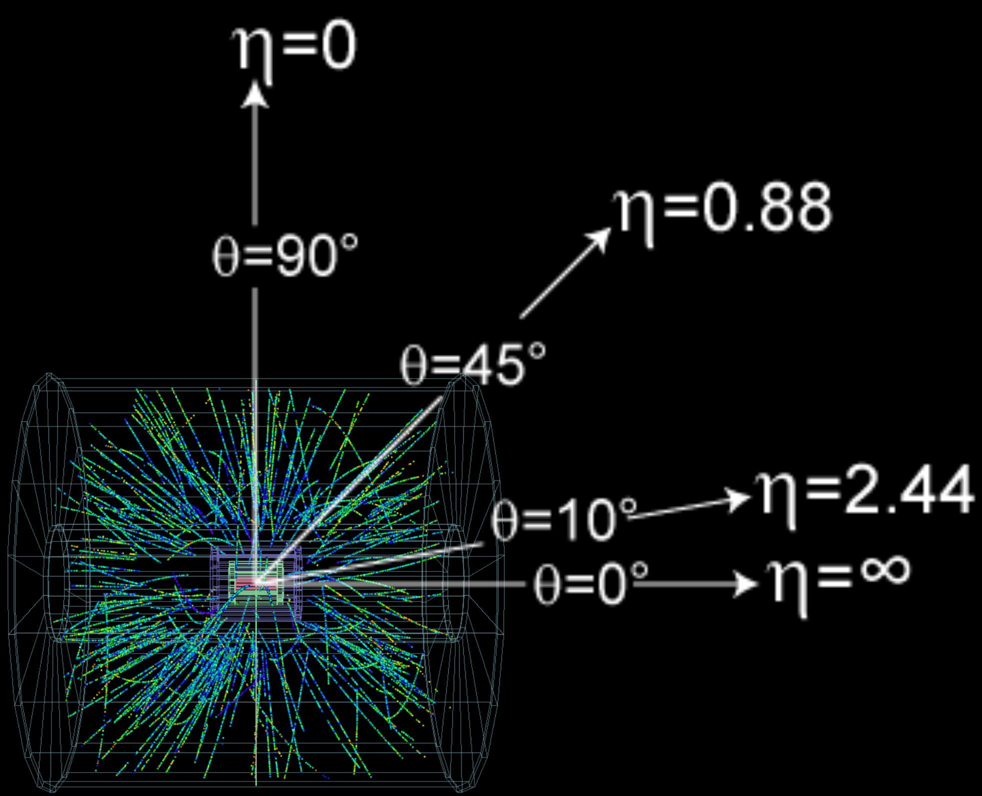

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&= m\sqrt{\frac{1}{1-v_z^2}} = m\gamma
\end{aligned}$$

$$\vec{v} = \vec{p}/E$$

Björn Schenke, BNL

DEFINITIONS

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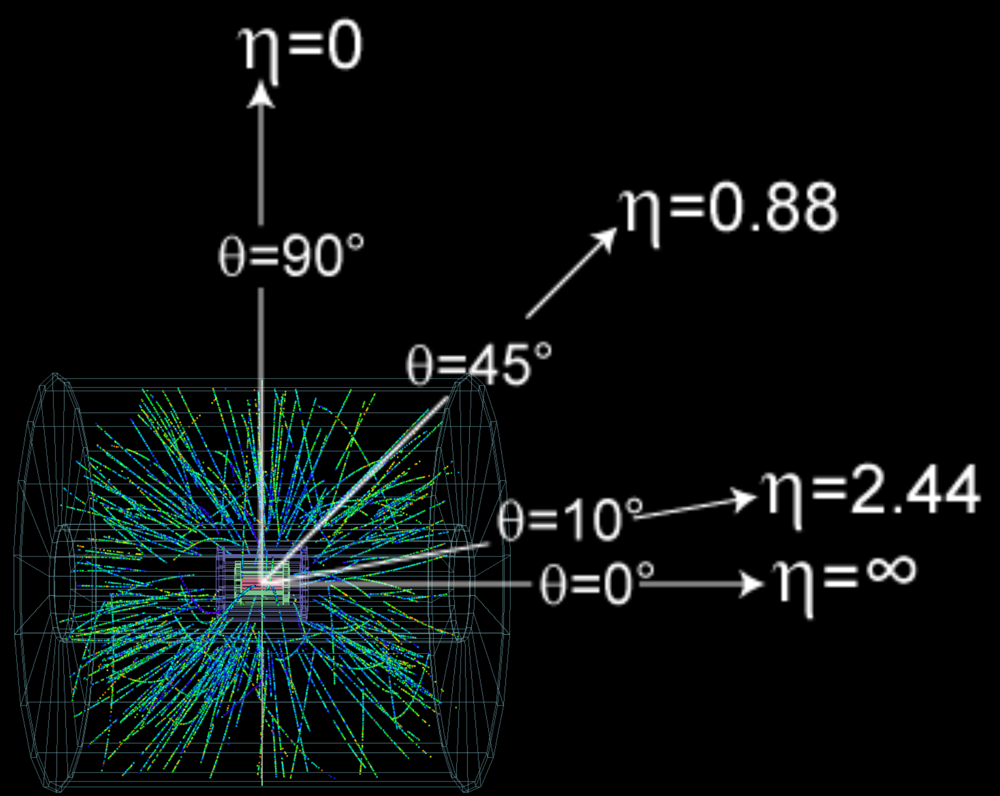


Rapidity is additive under boosts in z-direction

$$u'_z = \gamma u_z + \gamma v_z u_0$$

$$\gamma = \cosh y_1 \qquad \gamma v_z = \sinh y_1 \qquad u_0 = \cosh y_2 \qquad u_z = \sinh y_2$$

$$\Rightarrow u'_z = \cosh y_1 \sinh y_2 + \sinh y_1 \cosh y_2 = \sinh(y_1 + y_2)$$

$$u = p/m = \gamma(1, \vec{u})$$

DEFINITIONS

- **Beam rapidity**

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \ln\frac{E+p_z}{\sqrt{E^2-p_z^2}} = \ln\frac{E+p_z}{m}$$

$$\approx \ln\frac{2E}{m} = \ln\frac{\sqrt{s_{NN}}}{m_N}$$

CENTER OF MASS ENERGY [GEV]	BEAM RAPIDITY
19.6	3.04
200	5.36
2760	7.99
5020	8.58
7000	8.92

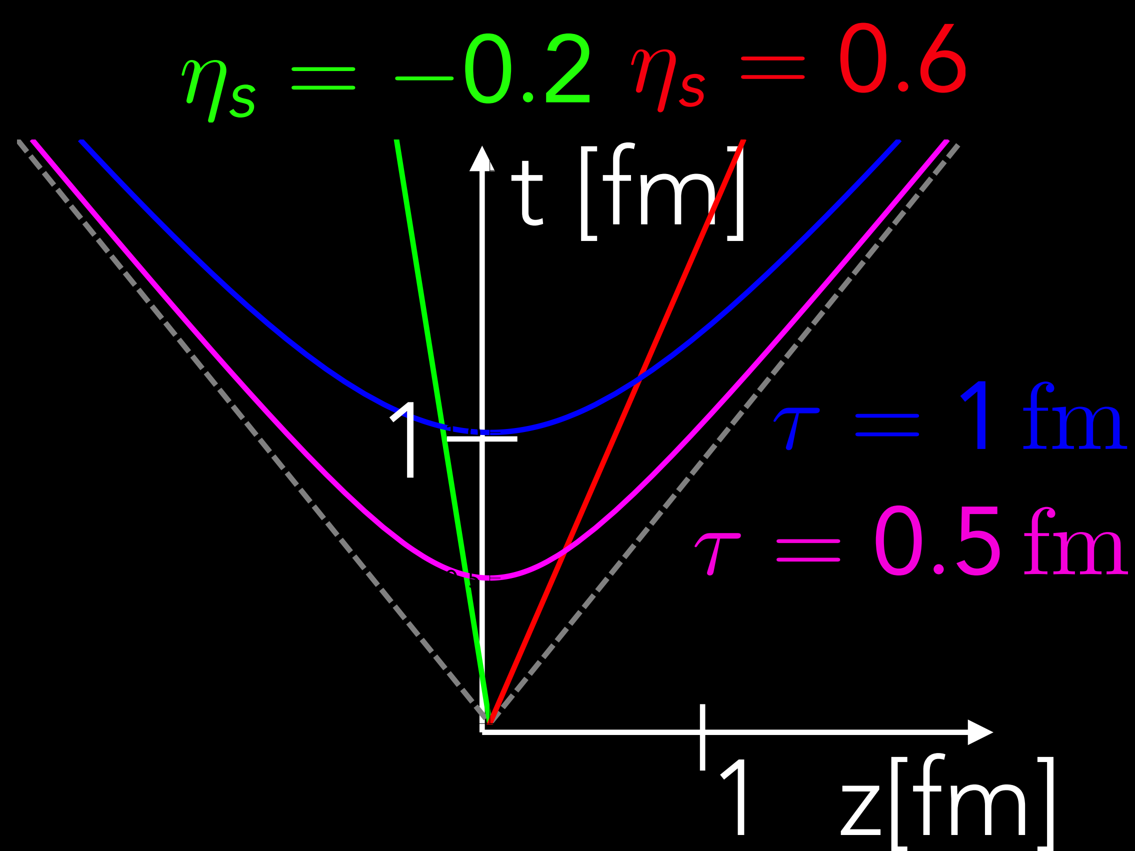
m_N=nucleon mass $\sqrt{s_{NN}}$ =center of mass energy per nucleon

DEFINITIONS

- **Space-time rapidity and proper time**

$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z} \qquad \tau = \sqrt{t^2 - z^2}$$

Inversely: $t = \tau\cosh\eta_s$ $\qquad z = \tau\sinh\eta_s$



BJORKEN EXPANSION

J. D. Bjorken, Phys. Rev. D 27, 140 (1983)

- **Matter streams freely from *t=z=0***

$$z = v_z t \qquad \Rightarrow v_z = z/t$$

Then space-time rapidity = momentum rapidity:

$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z} = \frac{1}{2}\ln\frac{1+\frac{z}{t}}{1-\frac{z}{t}} = \frac{1}{2}\ln\frac{1+v_z}{1-v_z} = y$$

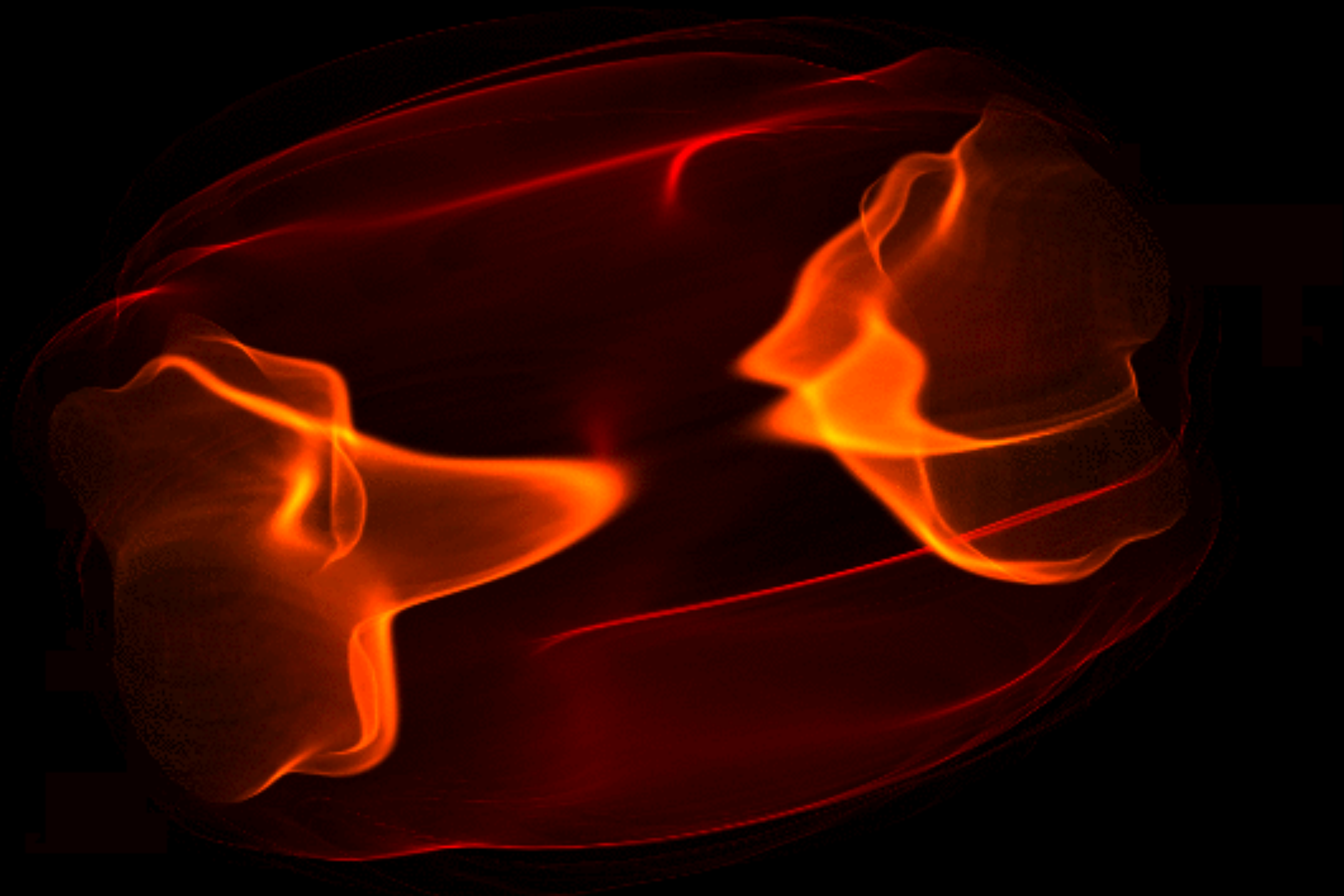
A central rapidity plateau in the data thus translates to a boost-invariant (η_s independent) system

τ is the proper time in the (longitudinally) moving rest-frame of the matter:

$$\tau = \sqrt{t^2 - z^2} = t\sqrt{1 - z^2/t^2} = t/\gamma \quad \Rightarrow t = \gamma\tau$$ time dilation

TIME DILATION

$$\tau = \sqrt{t^2 - z^2} = t\sqrt{1 - z^2/t^2} = t/\gamma \quad \Rightarrow t = \gamma\tau$$ time dilation



constant energy density contours

LIGHTCONE COORDINATES

$$v^+ = \frac{v^0 + v^3}{\sqrt{2}} \quad v^- = \frac{v^0 - v^3}{\sqrt{2}} \quad \mathbf{v}_T = (v^1, v^2)$$

$$x^+ = \frac{t + z}{\sqrt{2}} \qquad x^- = \frac{t - z}{\sqrt{2}}$$

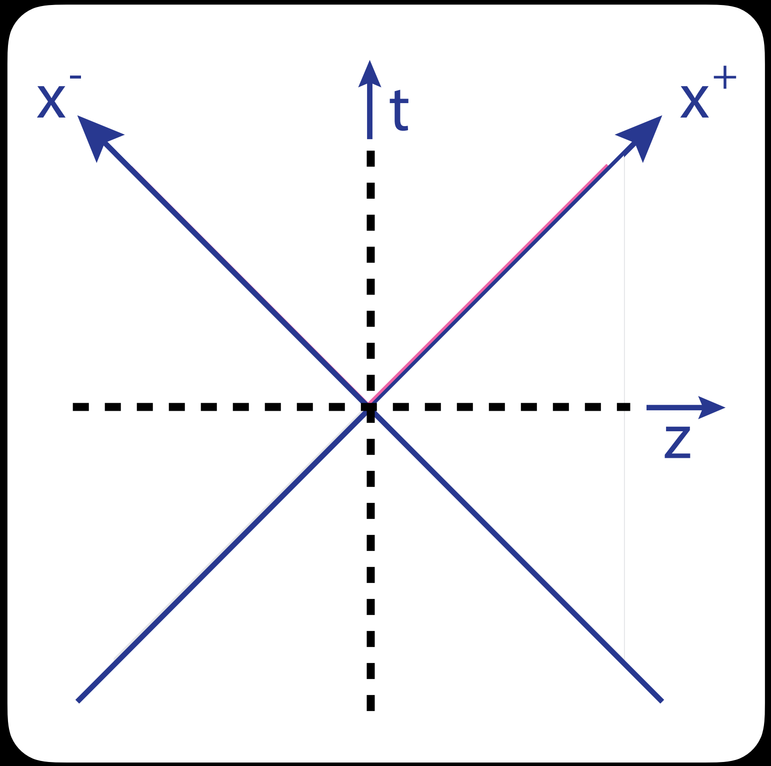
$$\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-} \quad \text{and} \quad \eta_s = \frac{1}{2}\ln\frac{x^+}{x^-}$$

$$ds^2 = dt^2 - \delta_{ij}dx^i dx^j \quad \text{for} \quad i, j = 1, \ldots, d$$

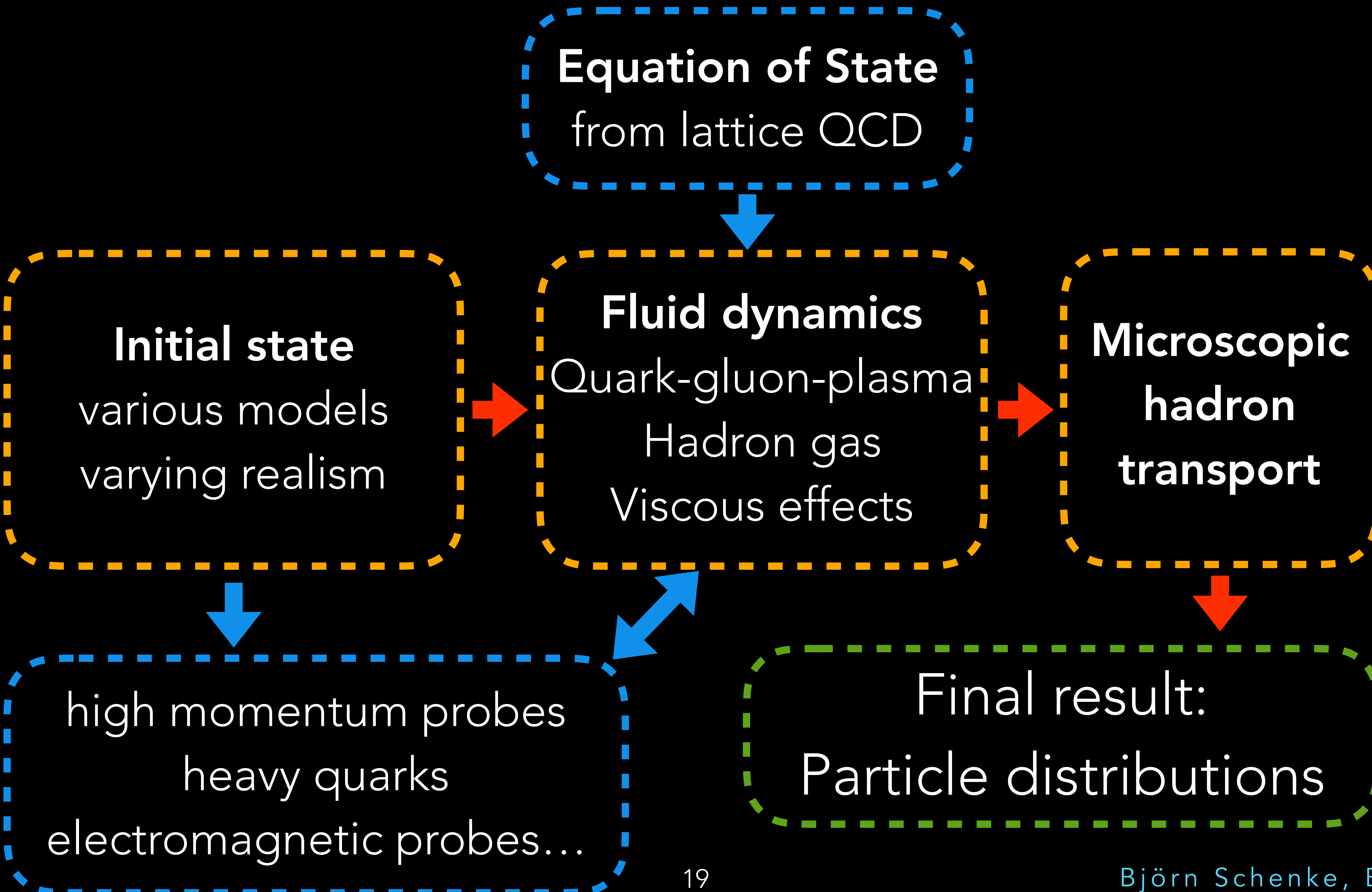
becomes

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j$$

for $i, j = 1, \ldots, d-1$



INGREDIENTS TO DESCRIBE HEAVY ION COLLISIONS



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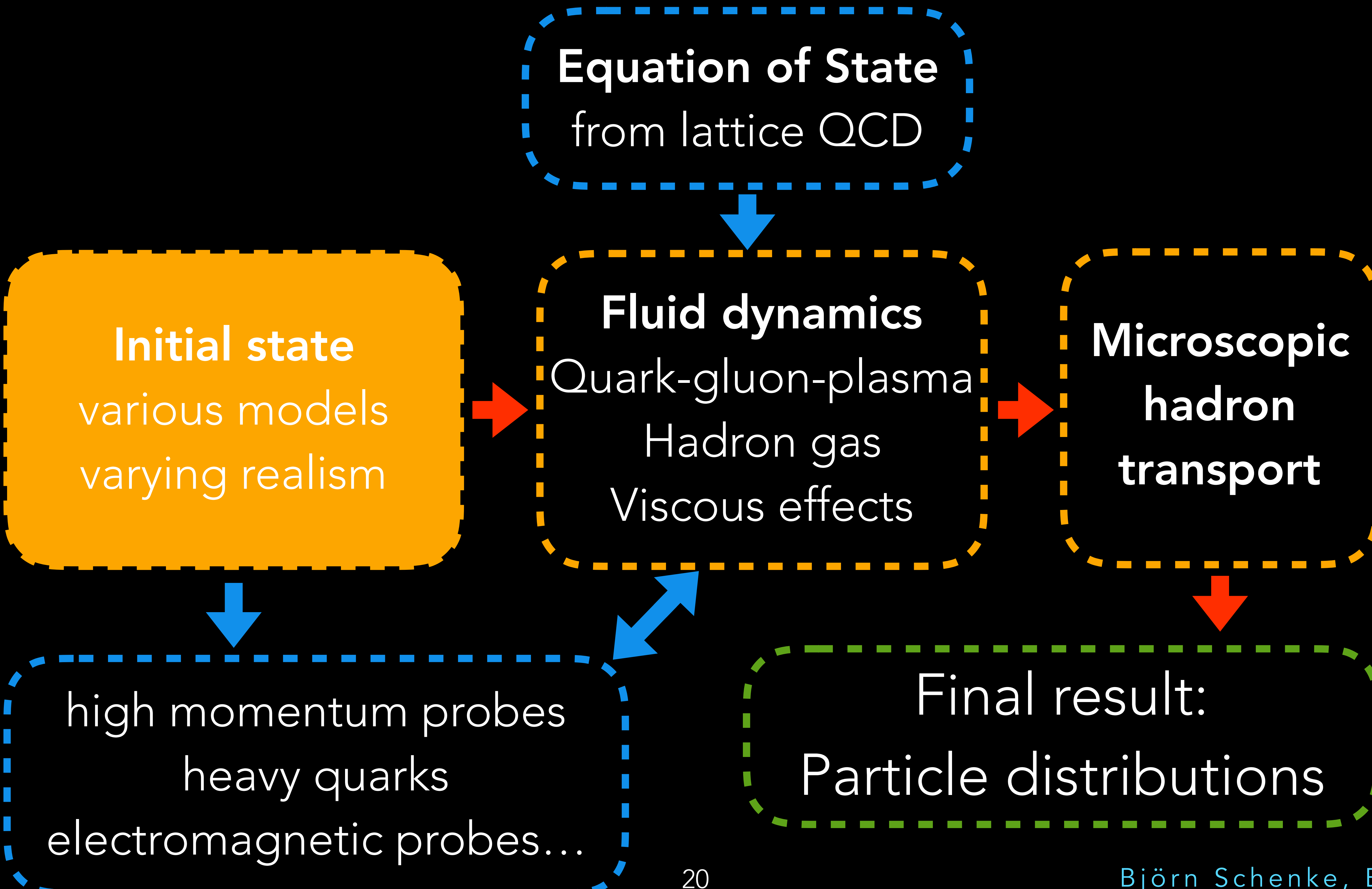

INITIAL CONDITIONS

- **Average** (smooth) **initial conditions**, e.g. from the Glauber model
- **MC-Glauber:** geometric model determining wounded nucleons based on the inelastic cross section (different implementations)
- **MC-KLN:** Color-Glass-Condensate (CGC) based model using k_T-factorization Same fluctuations in the wounded nucleon positions as MC-Glauber
- **MCrcBK:** Similar to MC-KLN but with improved energy/rapidity dependence following from solutions to the running coupling Balitsky Kovchegov equation
- **IP-Glasma:** CGC based model using classical Yang-Mills evolution of early-time gluon fields, including additional fluctuations in the particle production
- Also hadronic cascades UrQMD or NEXUS and partonic cascades (e.g. BAMPS) can provide initial conditions; AMPT as well

INITIAL PARAMETERS

- Historically, averaged initial conditions were first used
- Initial time from early thermalization argument (+fine tuning)
- Total energy density usually (fine-)tuned to fit multiplicities
- (Optical) Glauber model can provide cross section for nucleus-nucleus collisions in the high energy limit by treating it as multiple nucleon-nucleon collisions
- The nucleon position are random and given by the nuclear density profile $\rho_A(\vec{s})$

GLAUBER MODEL

Björn Schenke, BNL

GLAUBER MODEL

High energy: Assume independent straight line trajectories for all nucleons and small angle scattering

More precisely: Eikonal approximation:

Scattering of high energy (E) particle off a potential V with a finite range a, where V is strongly suppressed for r > a.

E ≫ |V| and k ≫ 1/a, meaning λ ≪ a.

Maximal angular momentum $l_{max} \approx ka \gg 1$

Partial wave analysis: Scattering amplitude expanded as:

$$f(\theta) = \frac{1}{2ik}\sum_{l}(2l+1)[e^{2i\delta_l} - 1]P_l(\cos\theta)$$

phase shift (δ_l) — Legendre polynomials (P_l)

GLAUBER MODEL

In the limit of large *l* and small angle θ, the scattering amplitude

$$f(\theta) = \frac{1}{2ik}\sum_l (2l+1)[e^{2i\delta_l} - 1]P_l(\cos\theta)$$

can be simplified by using

$$P_l(\cos\theta) \to J_0(l\theta) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{il\theta\cos(\phi)}$$

Also, for large *l*, angular momentum is essentially classical and we can replace the summation by an integral over *l*.
We further define the impact parameter *b* by kb ≈ l

$$\sum_{l=0}^{\infty} = \int_0^{\infty} dl = k\int_0^{\infty} db$$

GLAUBER MODEL

Next, rewrite the exponent in

$$P_l(\cos\theta) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{ikb\theta\cos(\phi)}$$

using the momentum transfer

$$\vec{q} = \vec{k} - \vec{k}' = (0, 0, k) - k(\sin\theta\cos\phi, \sin\theta\cos\phi, \cos\theta)$$

$$= k\left(\sin\theta\cos\phi, \sin\theta\sin\phi, \quad 2\sin^2\frac{\theta}{2}\right)$$

small θ ⇒ neglect

Then $\vec{q}$ lies in impact parameter $\vec{b}$ plane, and:

$$\vec{q}\cdot\vec{b} = kb\sin\theta\cos\phi \approx kb\theta\cos(\phi)$$

with $\vec{b} = (b, 0)$

$$\Rightarrow P_l(\cos\theta) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{i\vec{q}\cdot\vec{b}}$$

GLAUBER MODEL

Follows the scattering amplitude in the eikonal approximation:

$$f(\vec{k}, \vec{k}') = \frac{ik}{2\pi} \int \left(1 - e^{i\chi(\vec{b})}\right) e^{i\vec{q}\cdot\vec{b}} d^2 b$$

with $\chi(\vec{b}) = 2\delta_l$ determined by the potential V

Total cross section from optical theorem:

$$\sigma_{\mathrm{tot}} = \frac{4\pi}{k} \mathrm{Im} f(0) = 2 \int \left[1 - \mathrm{Re}\left(e^{i\chi(\vec{b})}\right)\right] d^2 b$$ (q=0)

Elastic cross section is

$$\sigma_{\mathrm{el}} = \int d\Omega |f(\vec{k}, \vec{k}')|^2 = \int \frac{d^2 k}{k^2} |f(\vec{k}, \vec{k}')|^2$$

$$= \int \frac{d^2 q}{4\pi^2} \int d^2 b \int d^2 b' e^{i\vec{q}\cdot\vec{b}} \left[1 - e^{i\chi(\vec{b})}\right] e^{-i\vec{q}\cdot\vec{b}'} \left[1 - e^{i\chi(\vec{b}')}\right]^* = \int \left|1 - e^{i\chi(\vec{b})}\right|^2 d^2 b$$

GLAUBER MODEL

Finally, the inelastic cross section is

$$\sigma_{\rm inel} = \sigma_{\rm tot} - \sigma_{\rm el} = \int 1 - \left| e^{i\chi(\vec{b})} \right|^2 d^2b$$

At high energies this is the main contribution

We introduce the probability for an inelastic nucleon-nucleon collision at impact parameter b:

$$P(\vec{b}) = 1 - \left| e^{i\chi(\vec{b})} \right|^2 = t(\vec{b})\sigma_{\rm inel}$$

Now, for a nucleon(nucleus)-nucleus collision, the phase shift functions χ are assumed to add. That leads to AA cross sections. We will derive the cross sections in AA on geometrical grounds.

GLAUBER MODEL

The nucleon-nucleus thickness function is

$$T_A(\vec{b}) = \int dz_A \int d^2 s_A \rho_A(\vec{s}_A, z_A) t(\vec{s}_A - \vec{b})$$

With $t(\vec{s}_A - \vec{b}) \approx \delta^{(2)}(\vec{s}_A - \vec{b})$

$$T_A(\vec{s}) = \int \rho_A(\vec{s}, z_A) dz_A$$

Nuclear density is e.g. given by a Woods-Saxon distribution:

$$\rho_A(\vec{s}, z_A) = \frac{\rho_0}{1 + \exp[(\sqrt{\vec{s}^2 + z_A^2} - R)/d]}$$

More complex descriptions including correlations among nucleons

GLAUBER MODEL

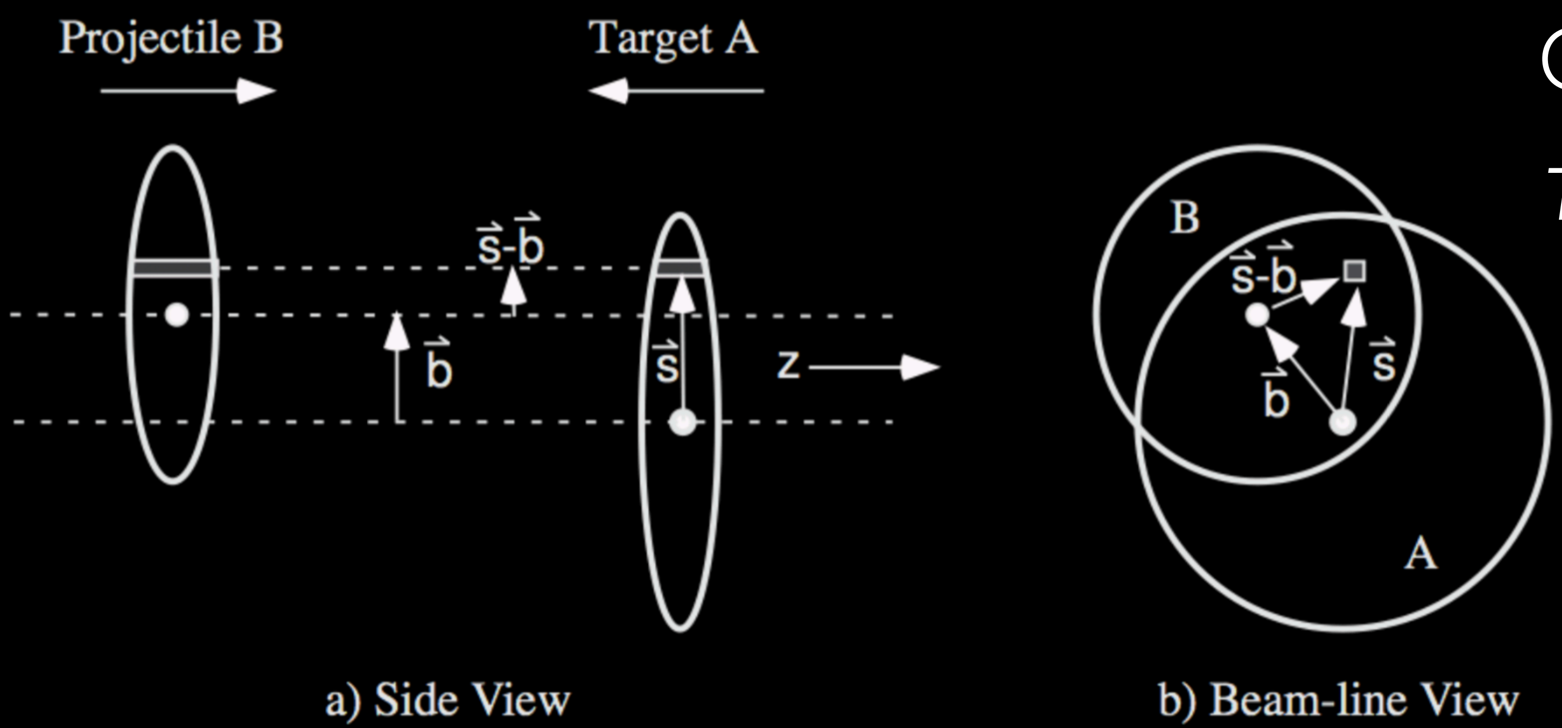

Figure from arXiv:nucl-ex/0701025

Overlap function:

$$T_{AB}(\vec{b}) = \int T_A(\vec{s})T_B(\vec{s}-\vec{b})d^2s$$

Probability for an interaction: $T_{AB}\,\sigma_{\rm inel}^{NN}$

Probability for n interactions (binomial):

$$P(n,\vec{b}) = \binom{AB}{n}(T_{AB}(\vec{b})\sigma_{\rm inel}^{NN})^n(1-T_{AB}(\vec{b})\sigma_{\rm inel}^{NN})^{AB-n}$$

GLAUBER MODEL

Total probability of an interaction between A and B:

$$P_{\rm inel}^{A+B}(\vec{b}) = \sum_{n=1}^{AB} P(n,\vec{b}) = 1 - P(0,\vec{b}) = 1 - [1 - T_{AB}(\vec{b})\sigma_{\rm inel}^{NN}]^{AB}$$

Density of wounded nucleons:

$$n_{WN}(x,y,b) = T_A(x+b/2,y)\left[1-\left(1-\frac{\sigma_{\rm inel}^{NN}}{B}T_B(x-b/2,y)\right)^B\right] + (A \leftrightarrow B)$$

integrate over x,y and multiply by A (resp. B) to get N_{part}

Density of binary collisions:

$$n_{BC}(x,y,b) = \sigma_{\rm inel}^{NN} T_A(x+b/2,y)T_B(x-b/2,y)$$

Now, either energy density or entropy density chosen to scale with n_{WN} or n_{BC} or a combination of both. Not clear from first principles.

MONTE-CARLO GLAUBER MODEL

Continuous nucleon density distributions → specific spatial coordinates for all nucleons:

Sample density distributions

Sample random impact parameter from $\frac{d\sigma}{db} = 2\pi b$

Nucleus-nucleus collision: a sequence of independent nucleon-nucleon collisions with $\sigma_{\rm inel}^{NN}$

(nucleons travel on straight-line trajectories, inelastic nucleon-nucleon cross-section is assumed to be independent of the number of collisions a nucleon underwent before)

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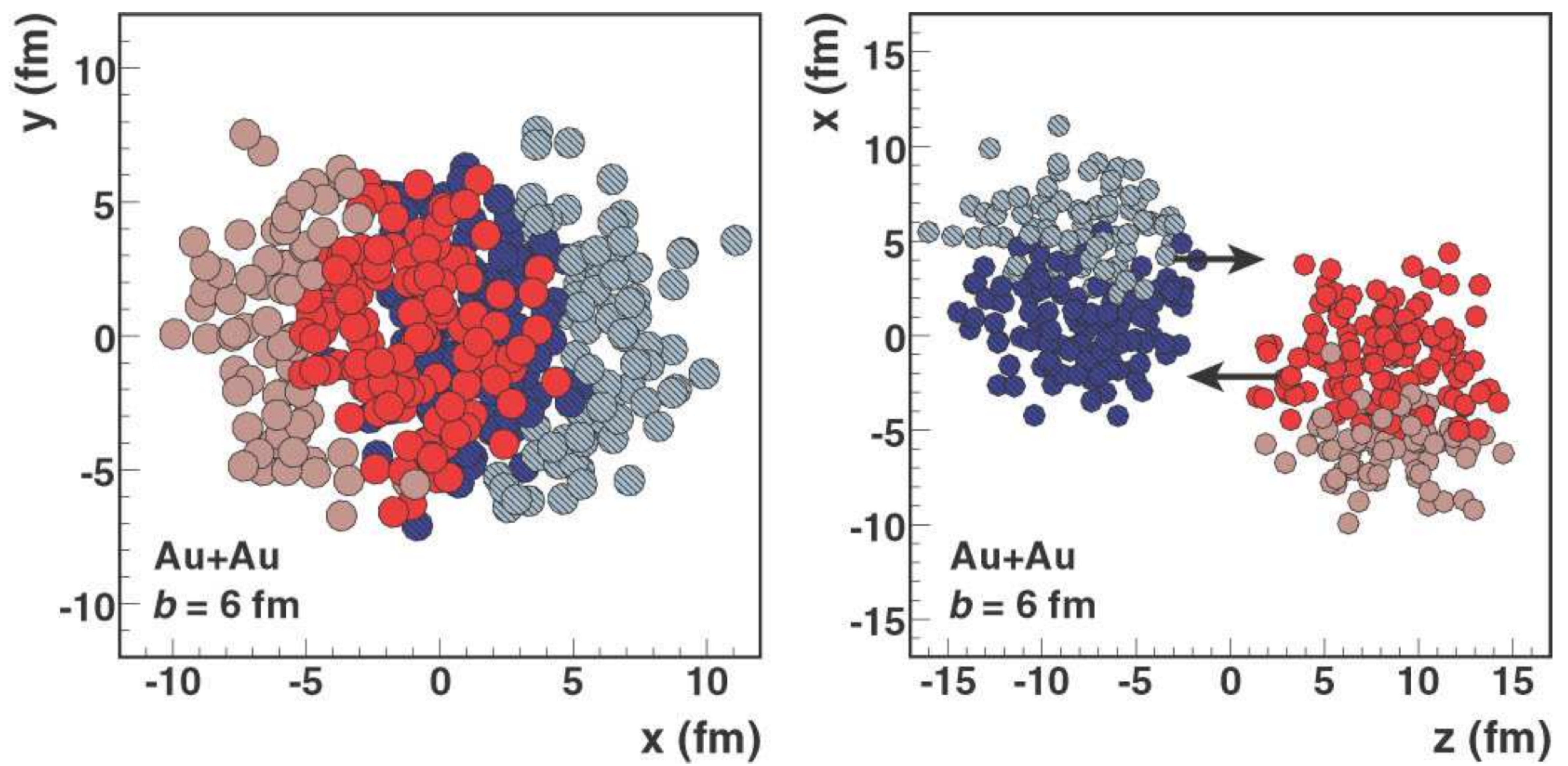
MONTE-CARLO GLAUBER MODEL

Participants in dark colors, spectators in light colors



Miller, Reygers, Sanders, Steinberg
Ann. Rev. Nucl. Part. Sci. 57, 205 (2007)