Hadron Structure Theory III

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The plan:

- Lecture I:

  Transverse spin structure of the nucleon

- Lecture II

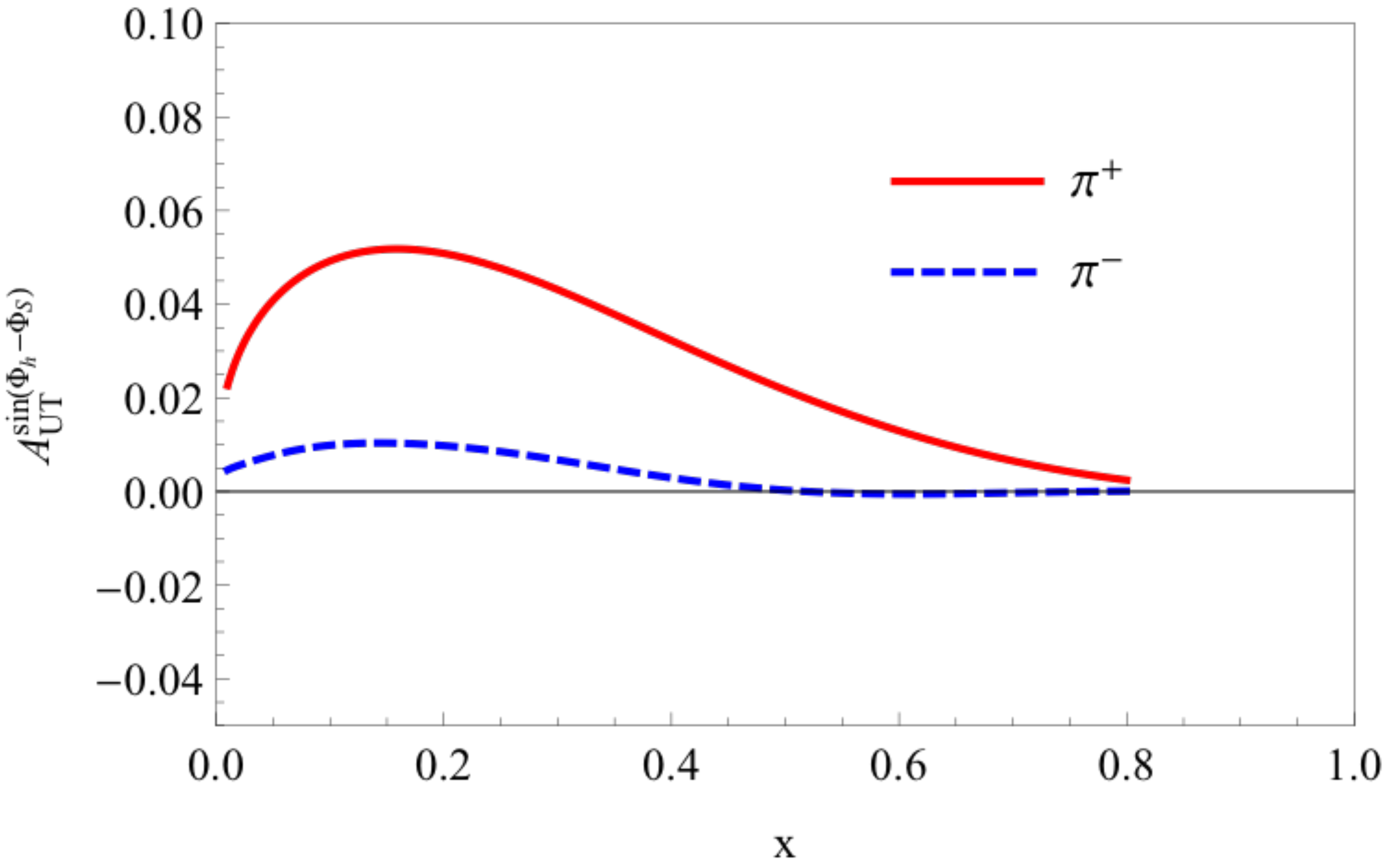
  Transverse Momentum Dependent distributions (TMDs)
  Semi Inclusive Deep Inelastic Scattering (SIDIS)

- Tutorial

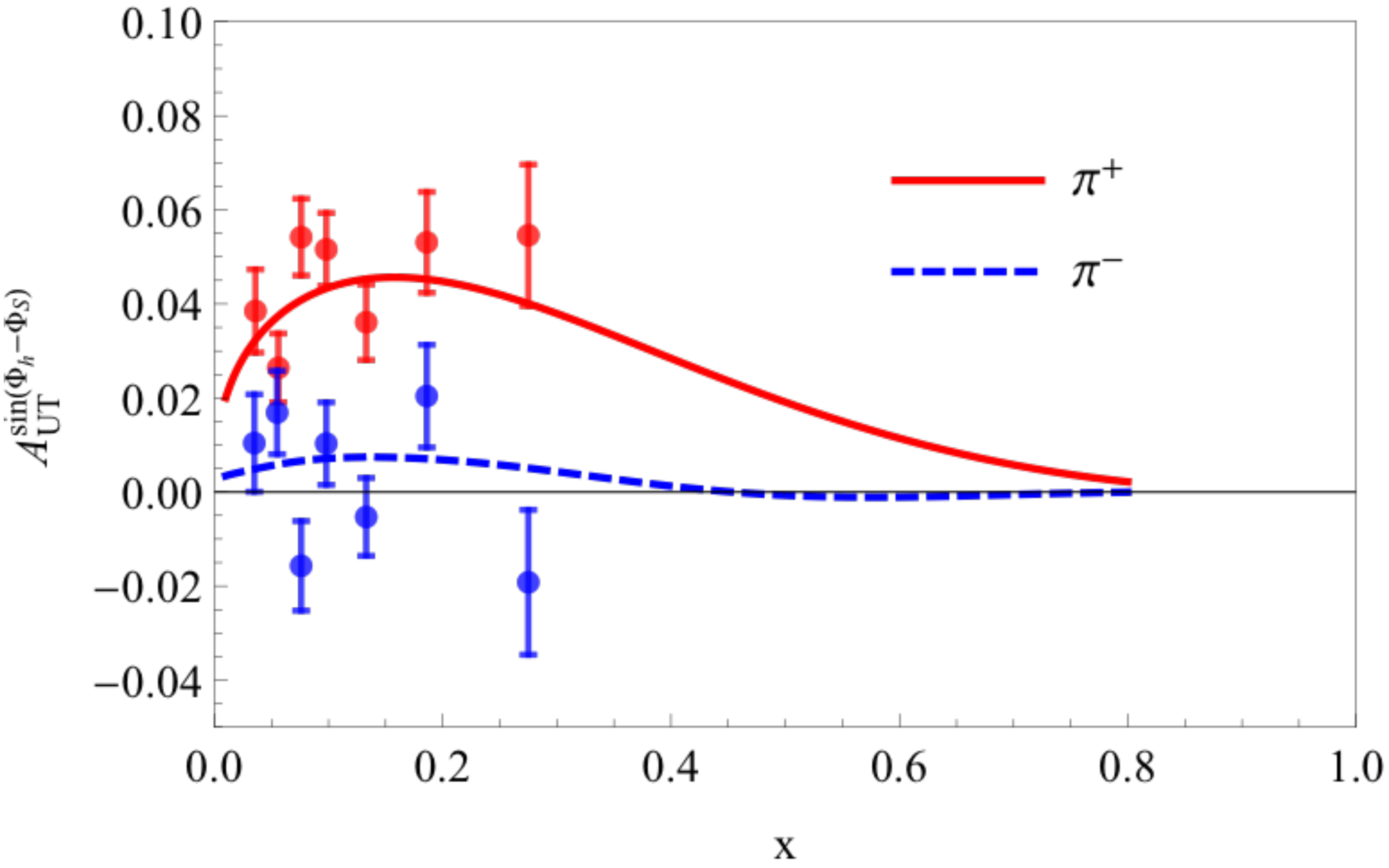
  Calculations of SIDIS structure functions using Mathematica

- Lecture III

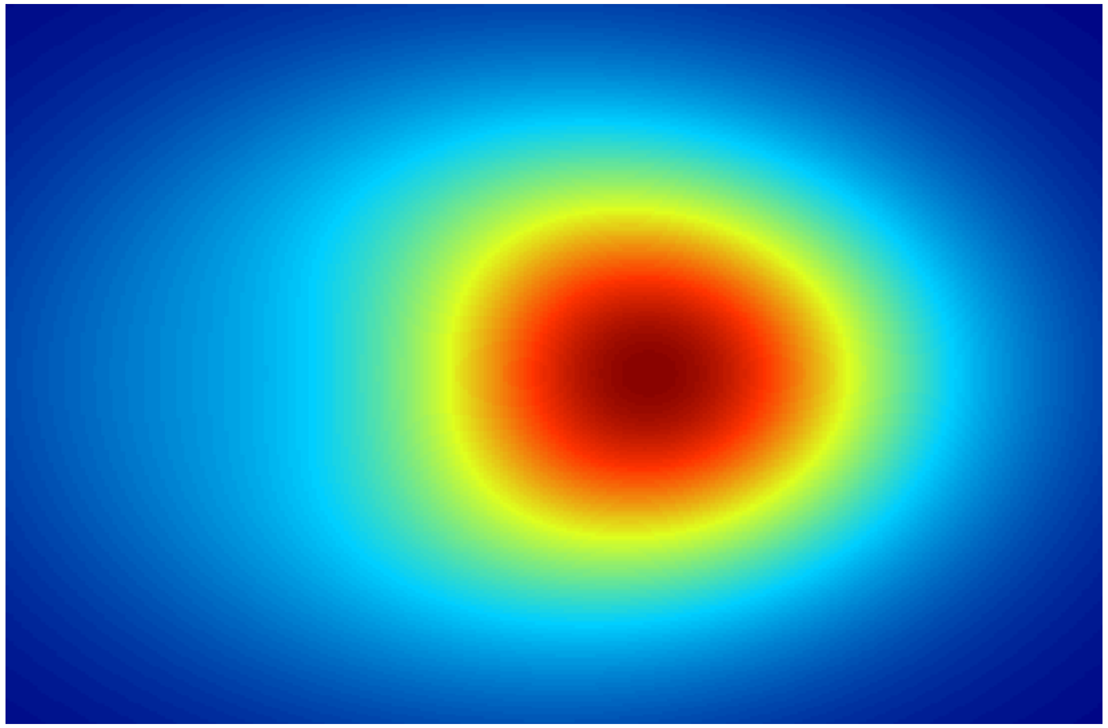
  Advanced topics. Evolution of TMDs


$A_{UT}^{\sin(\Phi_h-\Phi_S)}$
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
0.0
0.2
0.4
0.6
0.8
1.0
x
π^+
π^-

If you have any comment on the mathematica package, or the tutorial, send me a message **prokudin@jlab.org**



Nucleon landscape



The polarized proton in momentum space as "seen" by the virtual photon

Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

Factorization is a ***controllable approximation*** and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

Hadron structure is the ultimate goal of measurements and phenomenology

Transverse Momentum Dependent distributions

Individual TMDs can be projected out of the correlator

Unpolarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^{+}\;\Phi(x,k_\perp)\right] = f_1 - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M_N}\,f_{1T}^\perp$$

Longitudinally polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^{+}\gamma_5\;\Phi(x,k_\perp)\right] = S_L\,g_1 + \frac{k_\perp\cdot S_T}{M_N}\,g_{1T}^\perp$$

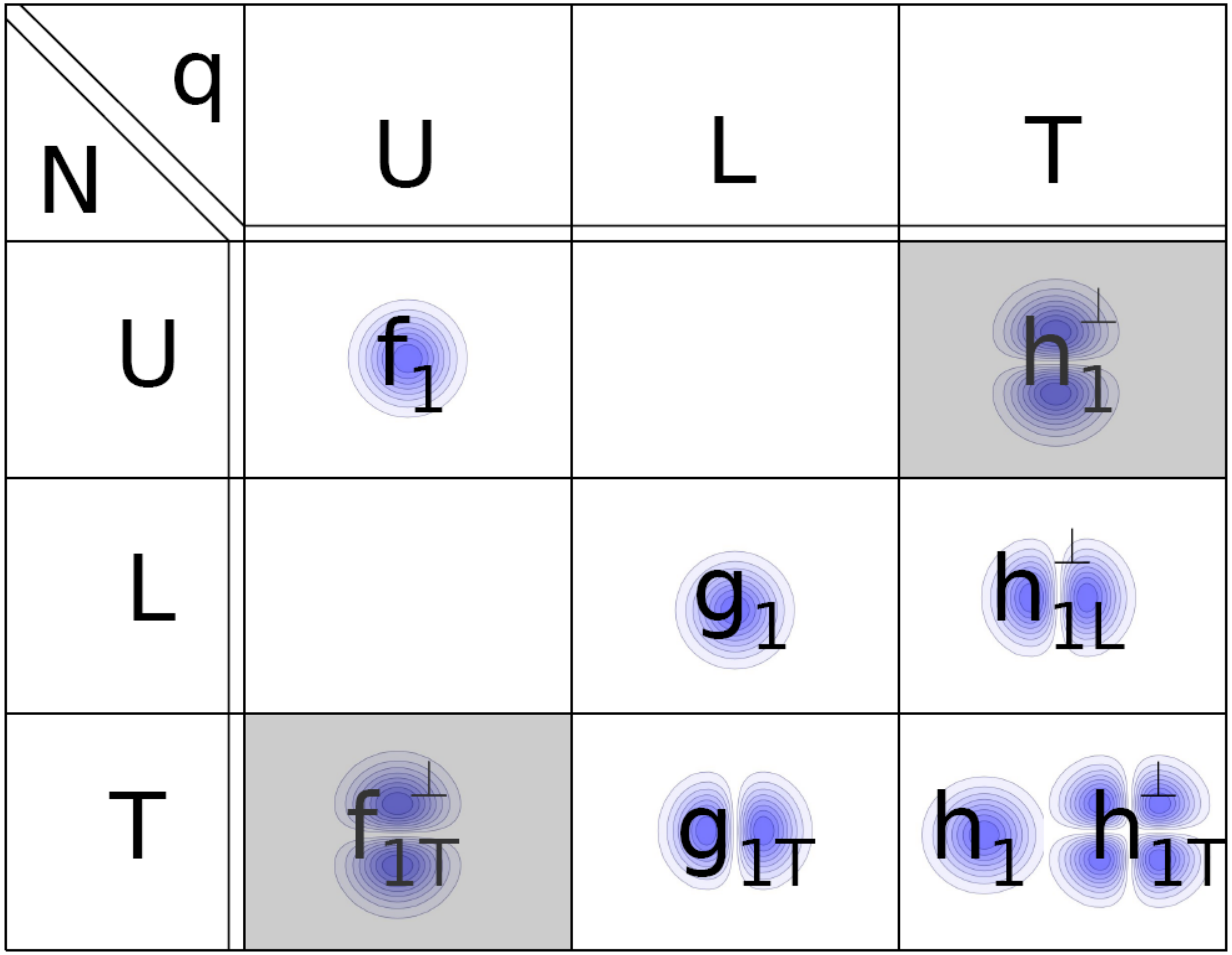
Transversely polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[i\sigma^{j+}\gamma^{+}\;\Phi(x,k_\perp)\right] = S_T^j\,h_1 + S_L\,\frac{k_\perp^j}{M_N}\,h_{1L}^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,h_{1T}^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,h_1^\perp$$

$$\kappa^{jk} \equiv (k_\perp^j k_\perp^k - \frac{1}{2}\,k_\perp^{\,2}\delta^{jk})$$

TMD distributions

Quark TMDs



N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

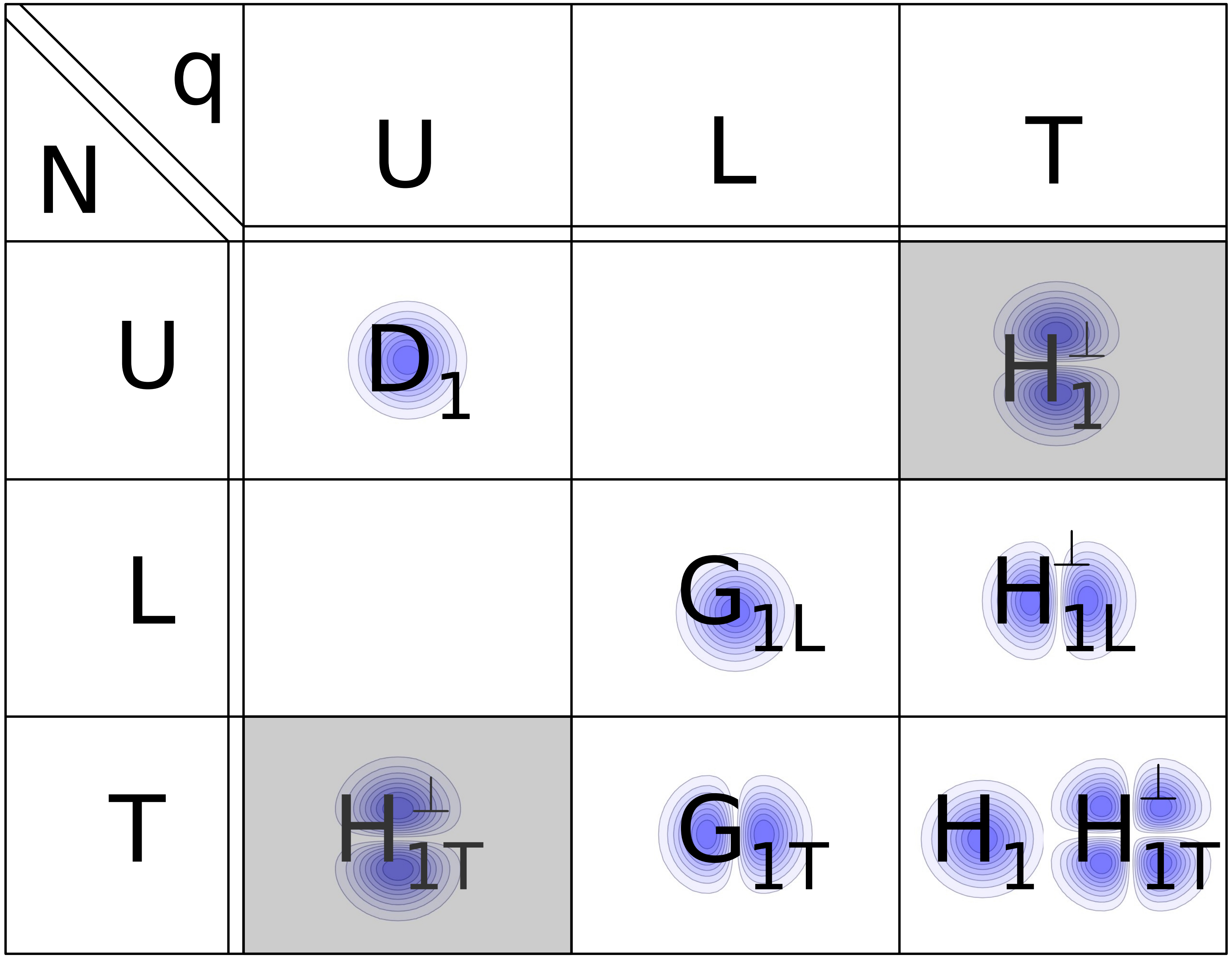
Each depends on Bjorken-x, transverse momentum, the scale

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD distributions

Quark TMD Fragmentation Functions



N \ q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$H_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$

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TMD distributions

More at higher twist!

$$\frac{1}{2}\mathrm{Tr}\Big[1\;\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[\quad e - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M_N}\,e_T^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L e_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,e_T\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[\frac{k_\perp^j}{M_N}f^\perp + \varepsilon^{jk}S_T^k f_T + S_L\frac{\varepsilon^{jk}k_\perp^k}{M_N}f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M_N^2}f_T^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_T^j\,g_T + S_L\,\frac{k_\perp^j}{M_N}g_L^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,g_T^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,g^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{jk}\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[\frac{S_T^j k_\perp^k - S_T^k k_\perp^j}{M_N}\,h_T^\perp - \varepsilon^{jk}\,h\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{+-}\,\gamma_5\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L\,h_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,h_T\Big].$$

TMD distributions

More at higher twist!

$$\frac{1}{2}\mathrm{Tr}\Big[1\;\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[\quad e - \frac{\varepsilon^{jk}k^j\text{[illegible]}}{\text{[illegible]}}\Big]$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L\text{[illegible]}\frac{\text{[illegible]}}{\text{[illegible]}N} - e_T\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\;\Phi(x,\boldsymbol{k}_\perp)\Big] = \text{[illegible]} + \varepsilon^{jk}S_T^k f_T + S_L\frac{\varepsilon^{jk}k_\perp^k}{M_N}f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M_N^2}f_T^\perp\Big],$$

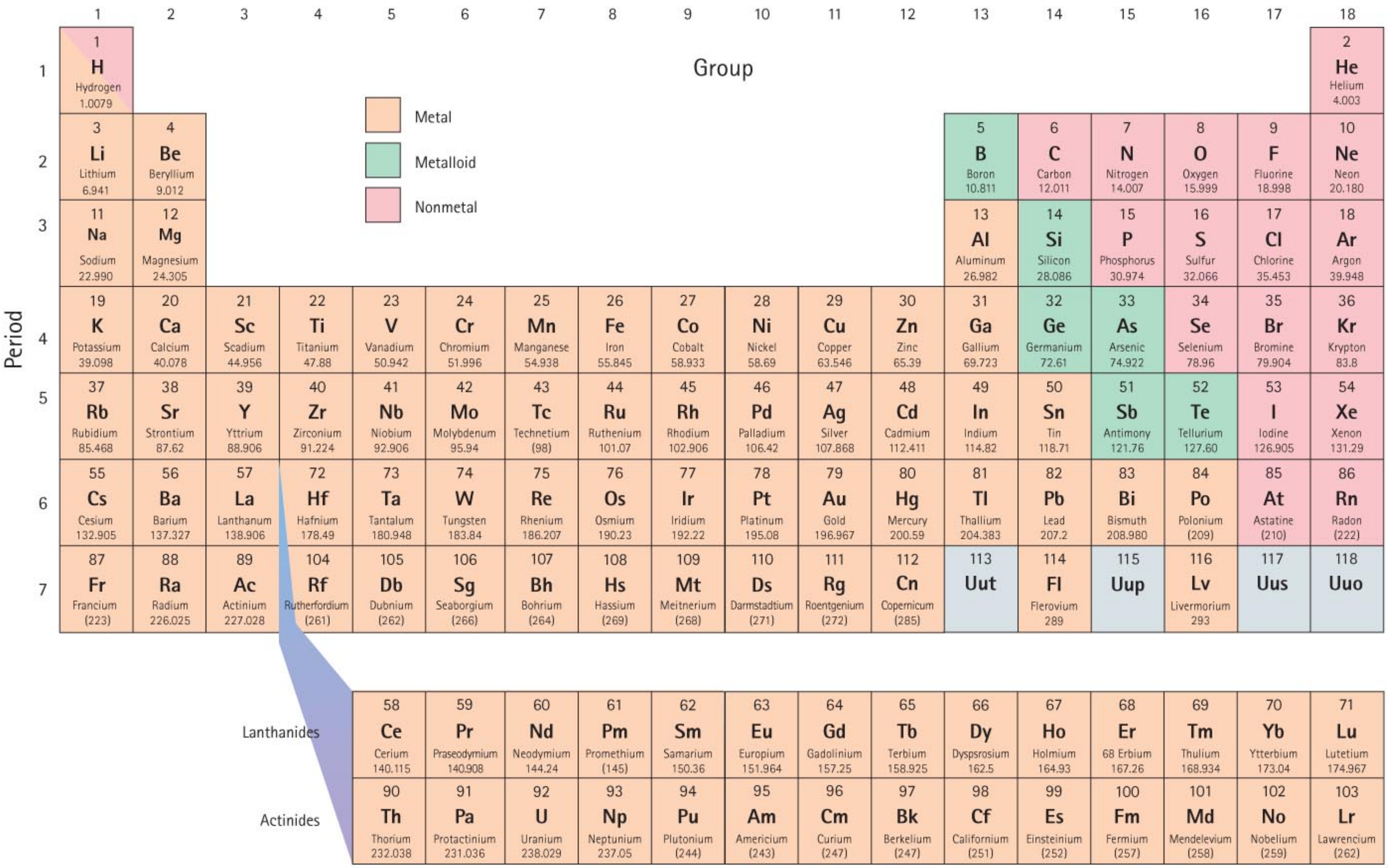
$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\gamma_5\Phi(x,\text{[illegible]}\Big] = \text{[illegible]}\Big[S_T^j\,g_T + S_L\,\frac{k_\perp^j}{M_N}g_L^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,g_T^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,g^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{jk}\text{[illegible]}\Big] = \frac{M_N}{P^+}\Big[\frac{S_T^jk_\perp^k - S_T^kk_\perp^j}{M_N}\,h_T^\perp - \varepsilon^{jk}\,h\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\text{[illegible]}(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L\,h_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,h_T\Big].$$

STAMP COLLECTING?

We are in a good company



Group

🟧 Metal

🟩 Metalloid

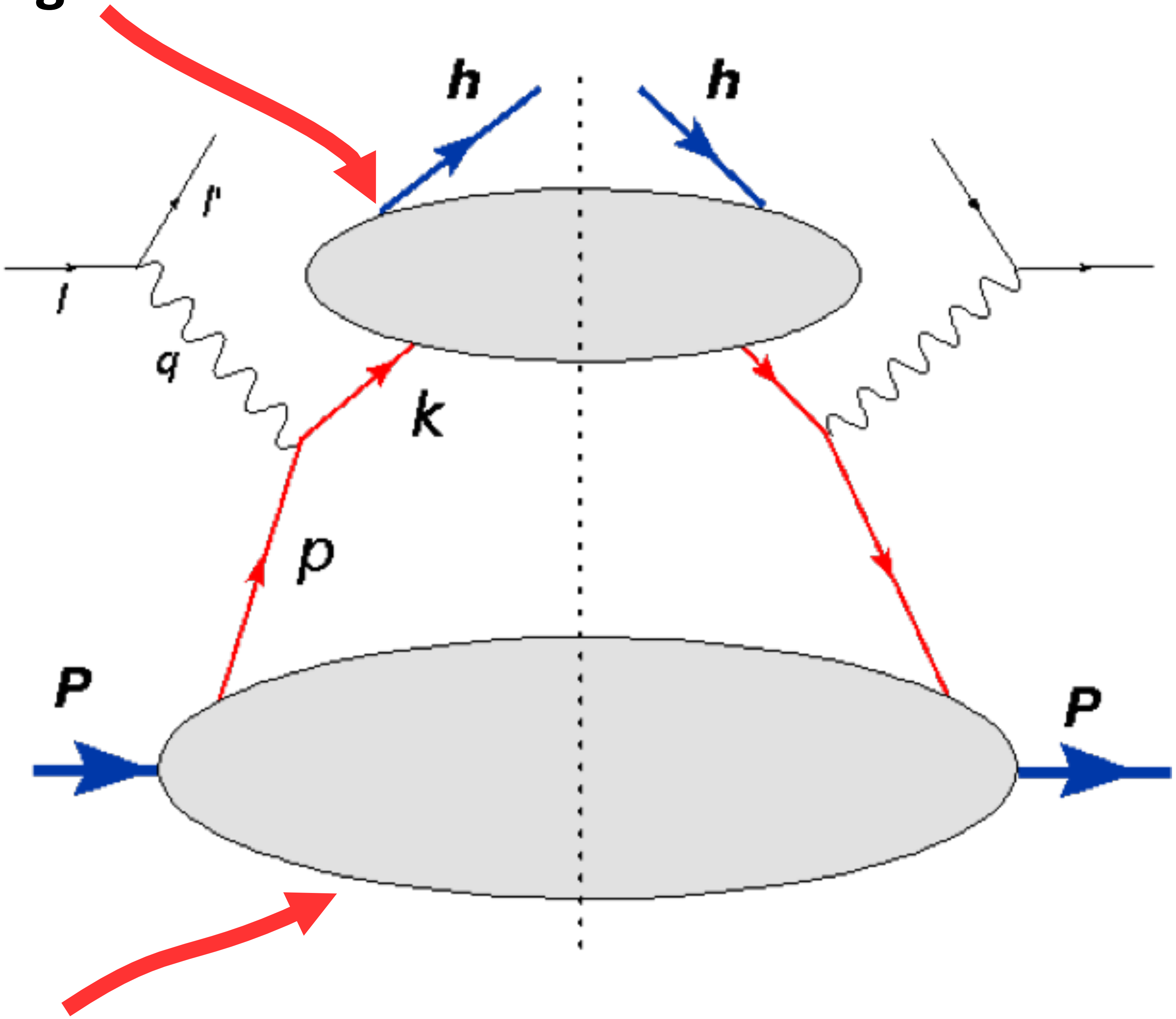
🟥 Nonmetal

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H Hydrogen 1.0079																	2 He Helium 4.003
2	3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
3	11 Na Sodium 22.990	12 Mg Magnesium 24.305											13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
4	19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scadium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.69	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.8
5	37 Rb Rubidium 85.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.94	43 Tc Technetium (98)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.82	50 Sn Tin 118.71	51 Sb Antimony 121.76	52 Te Tellurium 127.60	53 I Iodine 126.905	54 Xe Xenon 131.29
6	55 Cs Cesium 132.905	56 Ba Barium 137.327	57 La Lanthanum 138.906	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)
7	87 Fr Francium (223)	88 Ra Radium 226.025	89 Ac Actinium 227.028	104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (269)	109 Mt Meitnerium (268)	110 Ds Darmstadtium (271)	111 Rg Roentgenium (272)	112 Cn Copernicum (285)	113 Uut	114 Fl Flerovium 289	115 Uup	116 Lv Livermorium 293	117 Uus	118 Uuo
			Lanthanides		58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dyspsrosium 162.5	67 Ho Holmium 164.93	68 Er 68 Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967
			Actinides		90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.05	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)

Semi Inclusive Deep Inelastic Scattering (SIDIS)

$\ell P \to \ell' \pi X$ Factorization

Fragmentation



Distribution

$\sigma_{\mathbf{SIDIS}}$

$||$

$\mathbf{D_{q/h}}$

$\otimes$

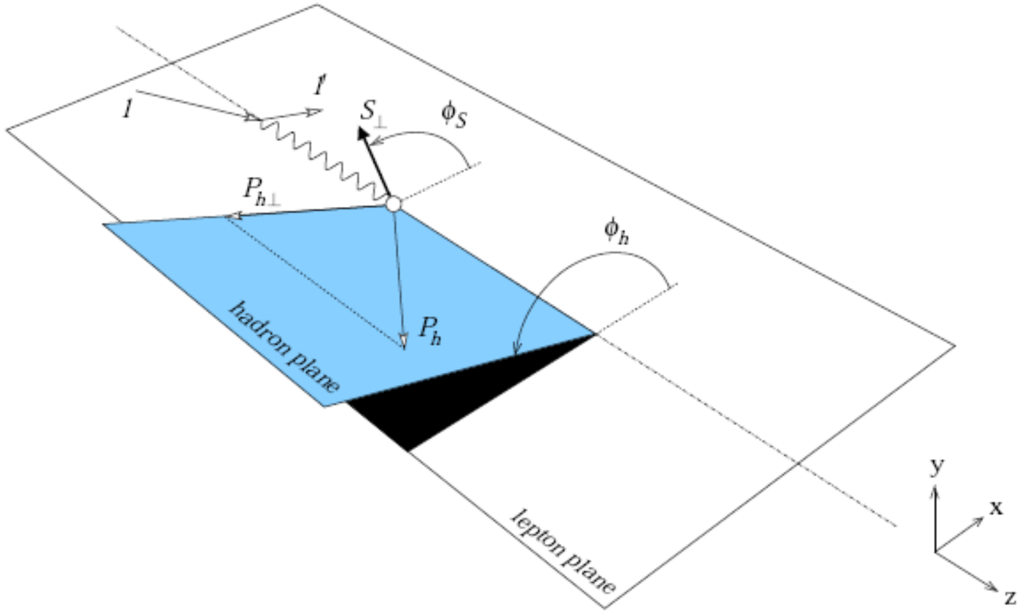
$\hat{\sigma}_{\mathbf{lq \to l'q'}}$

$\otimes$

$\mathbf{f_{q/P}}$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



One can rewrite the cross-section in terms of 18 structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

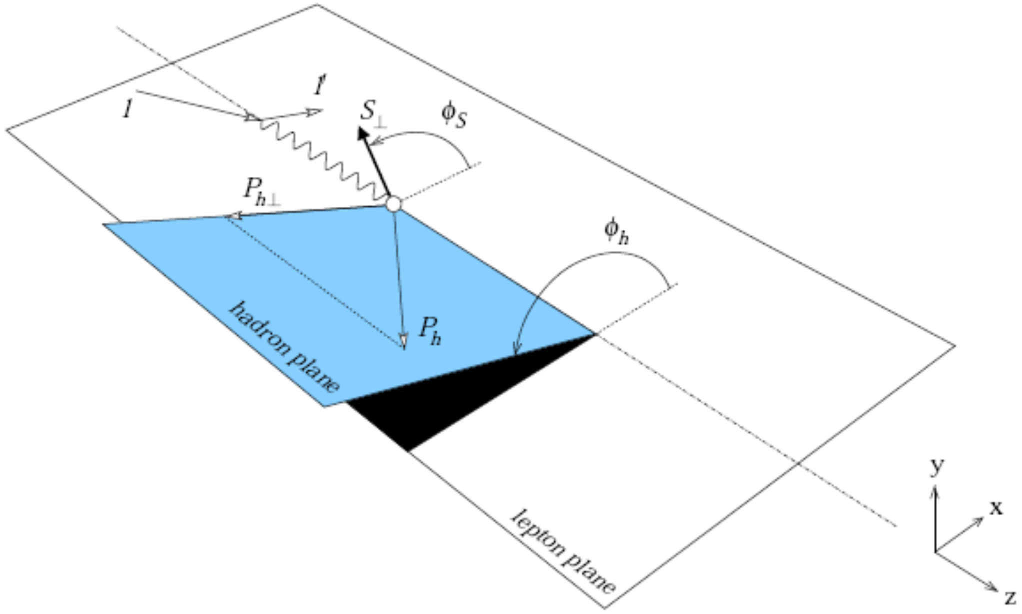
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xy\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\;+\ldots$$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



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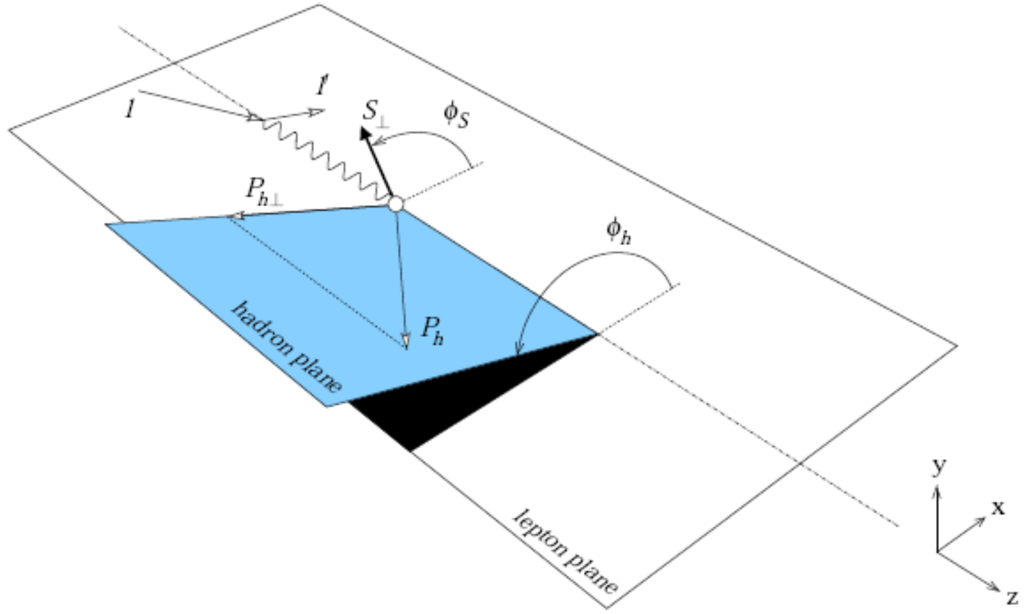
$$x = \frac{Q^2}{2\,P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad Q^2 = -q^2$$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



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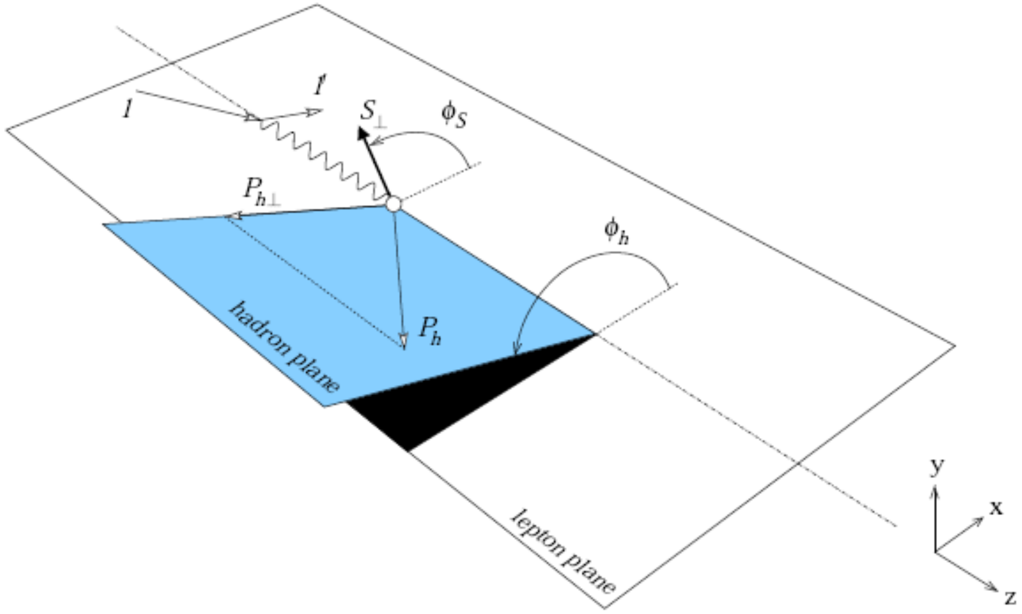
The TMD factorization is valid in the region

$$P_{hT}/z \ll Q$$

Interesting QCD regime, when recoil is happening from a low transverse momentum – important for studies of non perturbative physics.

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



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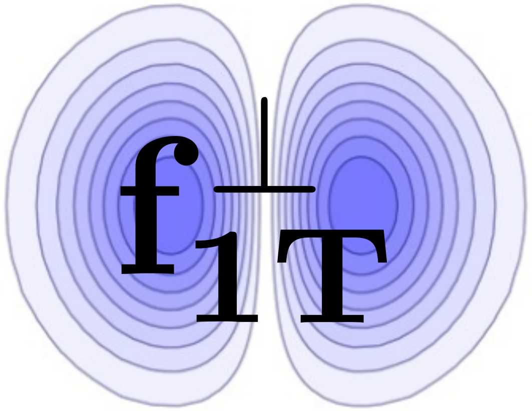
Boer, Mulders (1998)
Bacchetta et al (2007)

The TMD factorization is valid in the region

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{hT})\omega f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)$$

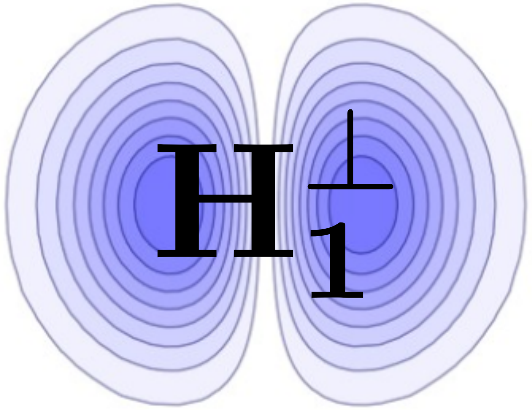
Final transverse momentum is related to transverse momenta of parent and fragmenting partons

What do we know about structure functions in SIDIS?



Sivers function

Non universal

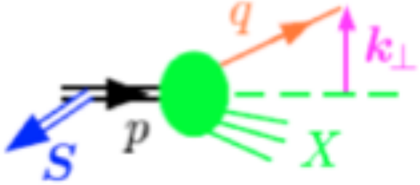


Collins function

Universal

Definitions

Sivers function: unpolarized quark distribution inside a transversely polarized nucleon



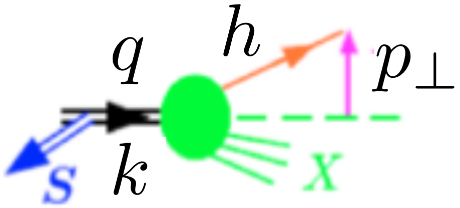
Sivers 1989

$$f_{q/h^\uparrow}(x,\vec{k}_\perp,\vec{S}) = f_{q/h}(x,k_\perp^2) - \frac{1}{M} f_{1T}^{\perp q}(x,k_\perp^2)\vec{S}\cdot(\hat{P}\times\vec{k}_\perp)$$

Spin independent

Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark



Collins 1992

$$D_{q/h}(z,\vec{p}_\perp,\vec{s}_q) = D_{q/h}(z,p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z,p_\perp^2)\vec{s}_q\cdot(\hat{k}\times\vec{p}_\perp)$$

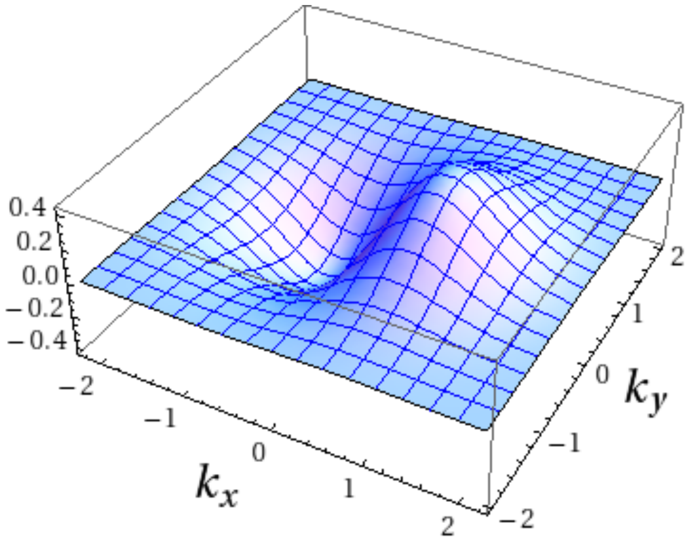
$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

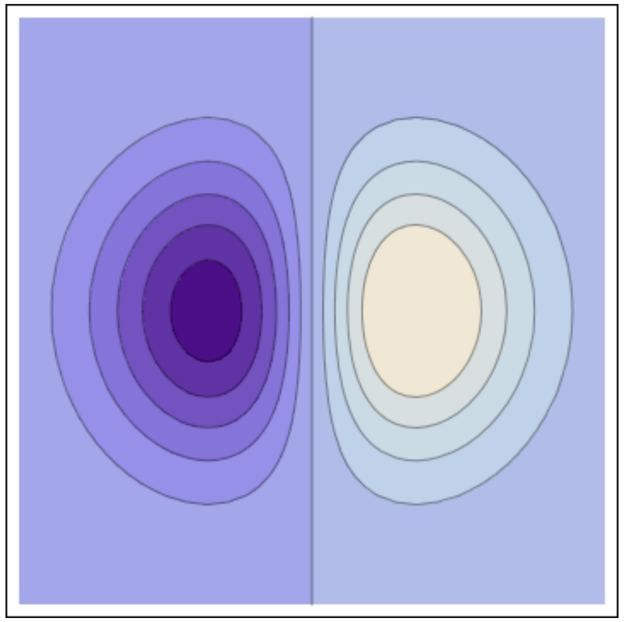
Suppose the spin is along Y direction: $S_T = (0, 1)$

Deformation in momentum space is:

This is the "dipole" deformation.

$$x \cdot f(x^2 + y^2)$$





$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0,1)$

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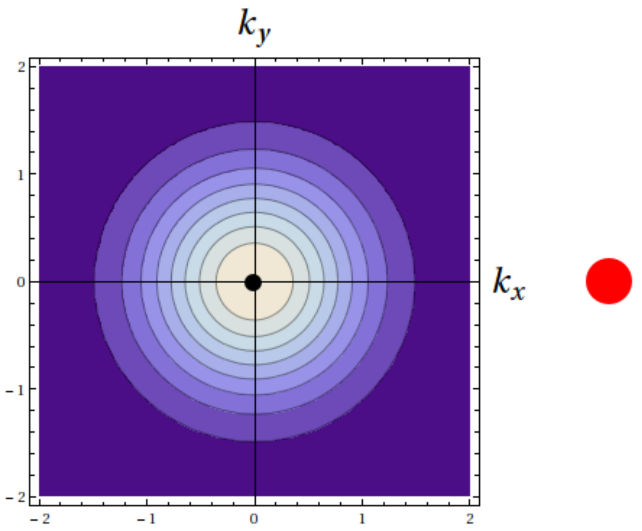
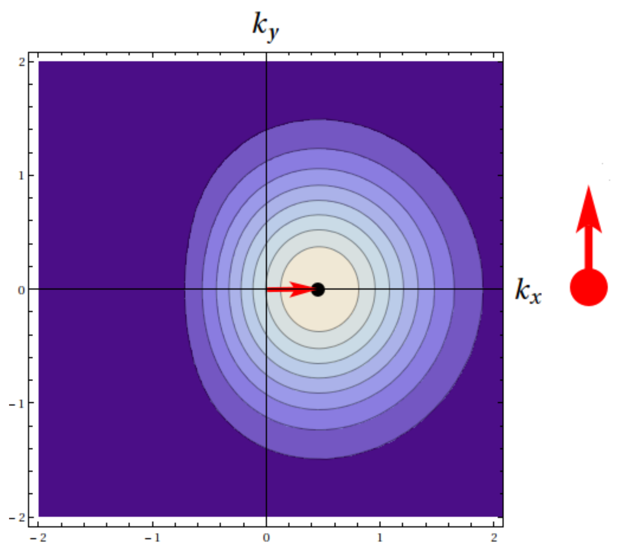
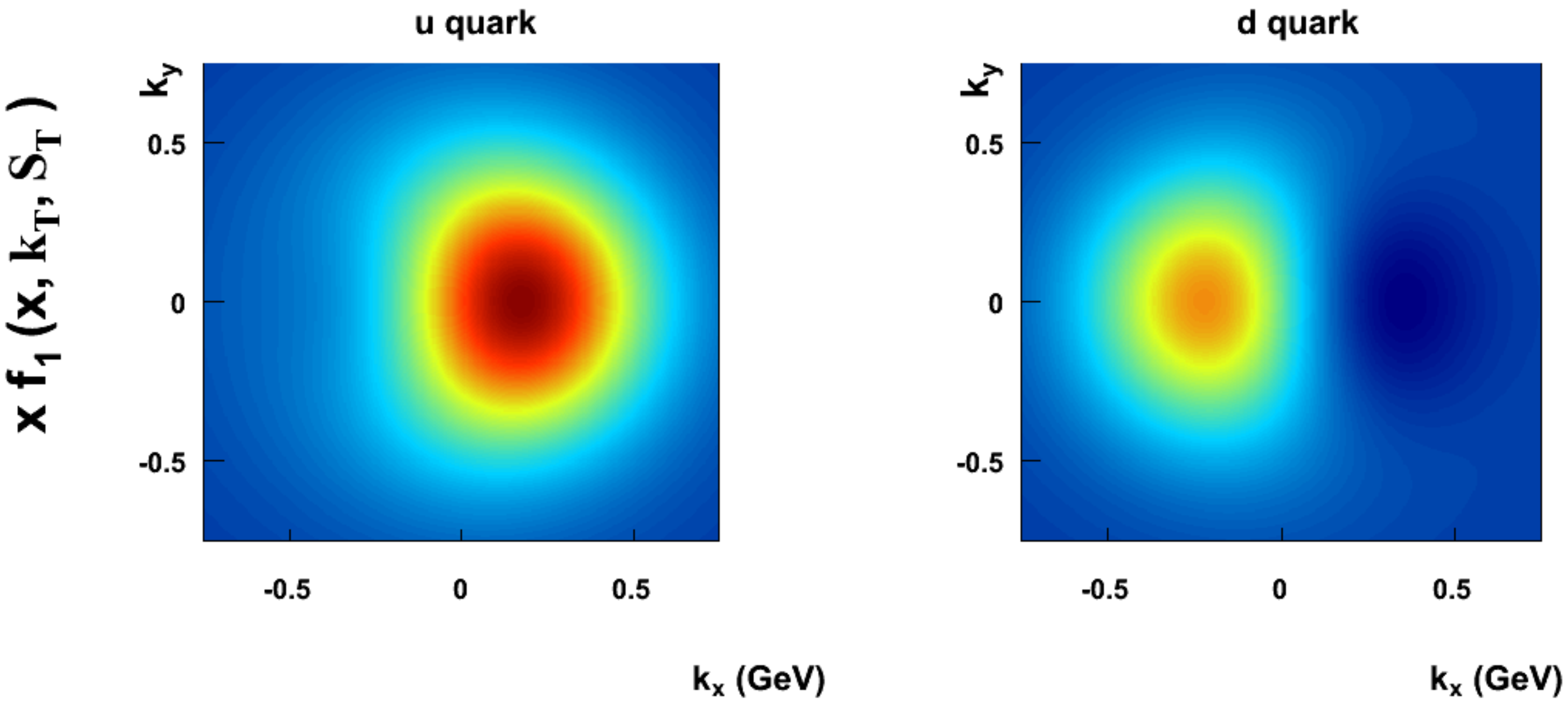
No correlation:



Correlation:

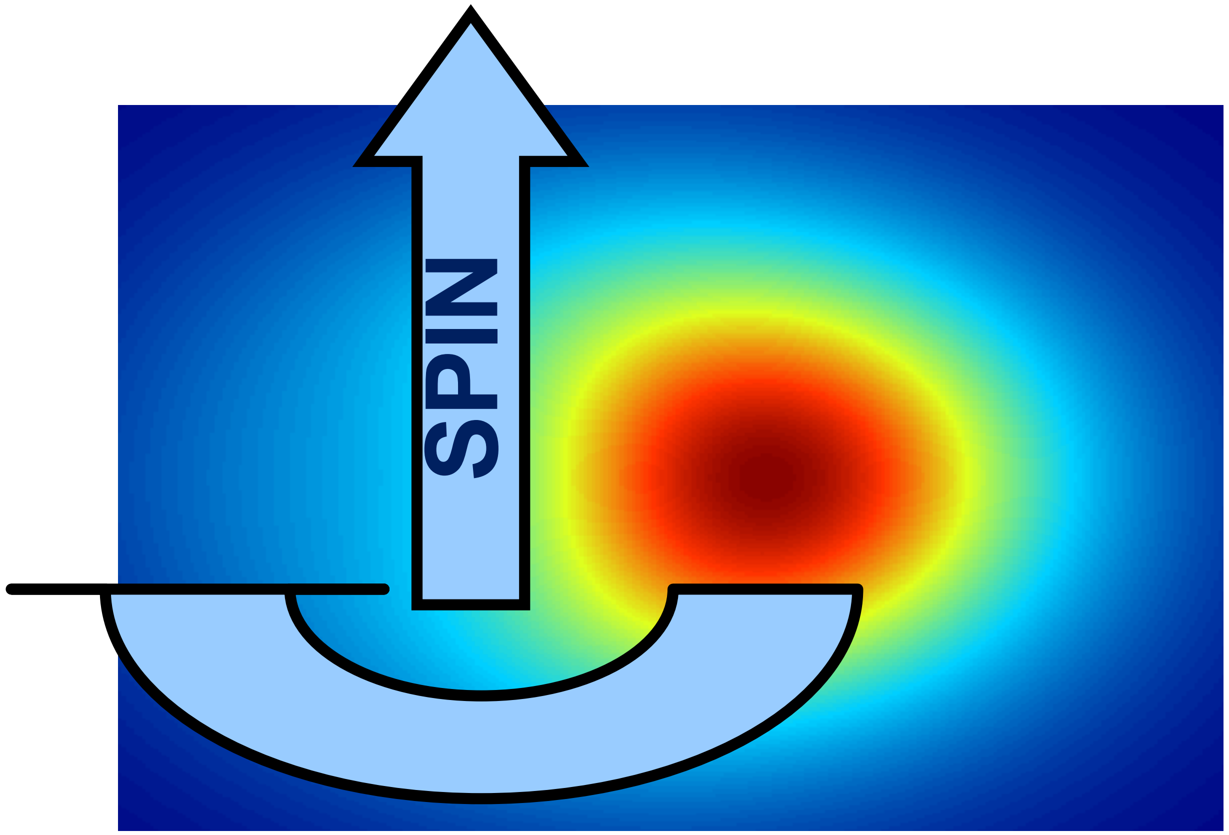


Tomographic scan of the nucleon



Anselmino et al 2009

Tomographic scan of the nucleon



Internal motion of quarks is correlated with the spin of the proton!

Definitions

Sivers function: $f_{1T}^{\perp q}$ describes strength of correlation $\vec{S} \cdot (\hat{P} \times \vec{k}_{\perp})$

Sivers 1989

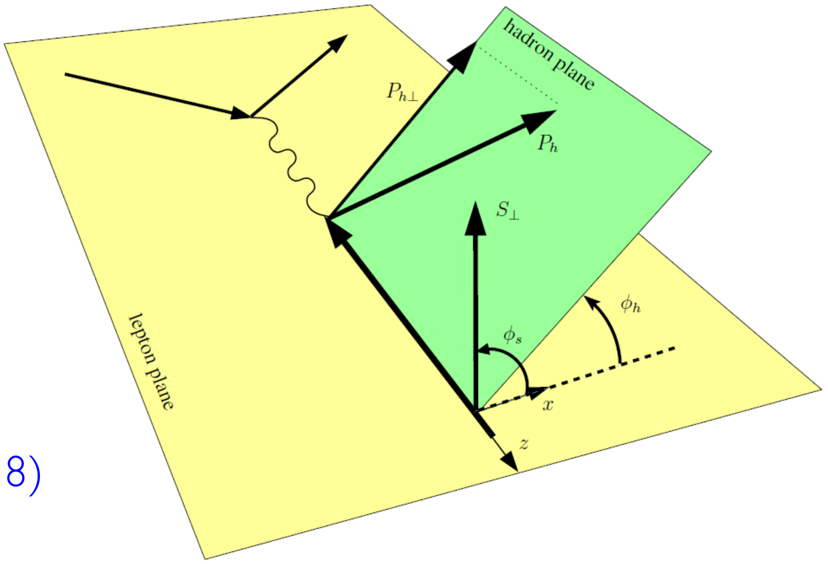
Collins function: $H_{1}^{\perp q}$ describes strength of correlation $\vec{s}_q \cdot (\hat{k} \times \vec{p}_{\perp})$

Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

$$\ell P \to \ell' \pi X$$

Sivers function and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process



Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^{\perp} + \sin(\phi_h - \phi_S) f_{1T}^{\perp} \otimes D_1 + \ldots$$

Sivers function

Large – N$_c$ result $f_{1T}^{\perp u} = - f_{1T}^{\perp d}$

Pobylitsa 2003

→ Confirmed by phenomenological extractions

→ Confirmed by experimental measurements

Relation to GPDs (E) and anomalous magnetic moment

Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

→ Predicted correct sign of Sivers asymmetry in SIDIS

→ Shown to be model-dependent

Meissner, Metz, Goeke 2007

→ Used in phenomenological extractions

Bacchetta, Radici 2011

Sivers function

Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

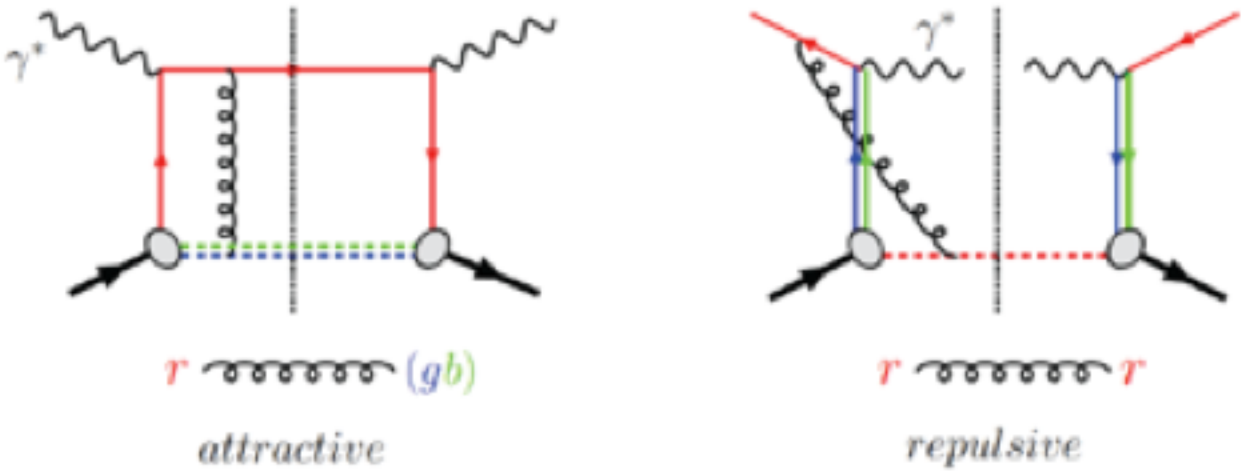
$$f_{1T}^{\perp(1)q}(x) = \int d^2k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^{\perp q}(x, k_\perp^2)$$

→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \qquad \sum_{a=q,g} \int_0^1 dx\, f_{1T}^{\perp(1)a}(x) = 0$$

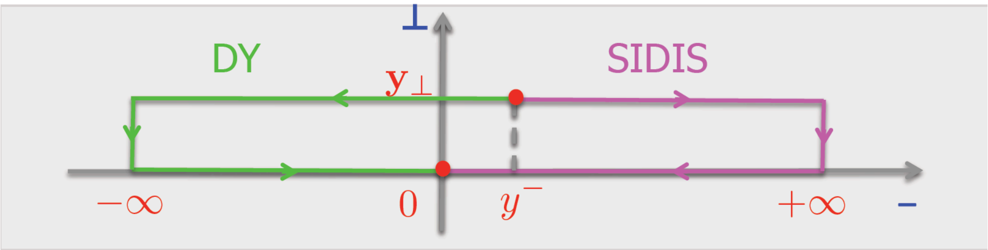
Sign change of Sivers function

Colored objects are surrounded by gluons, profound consequence of gauge invariance:
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky,Hwang,Schmidt;
Belitsky,Ji,Yuan;
Collins;
Boer,Mulders,Pijlman;
Kang, Qiu;
Kovchegov, Sievert;
etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$



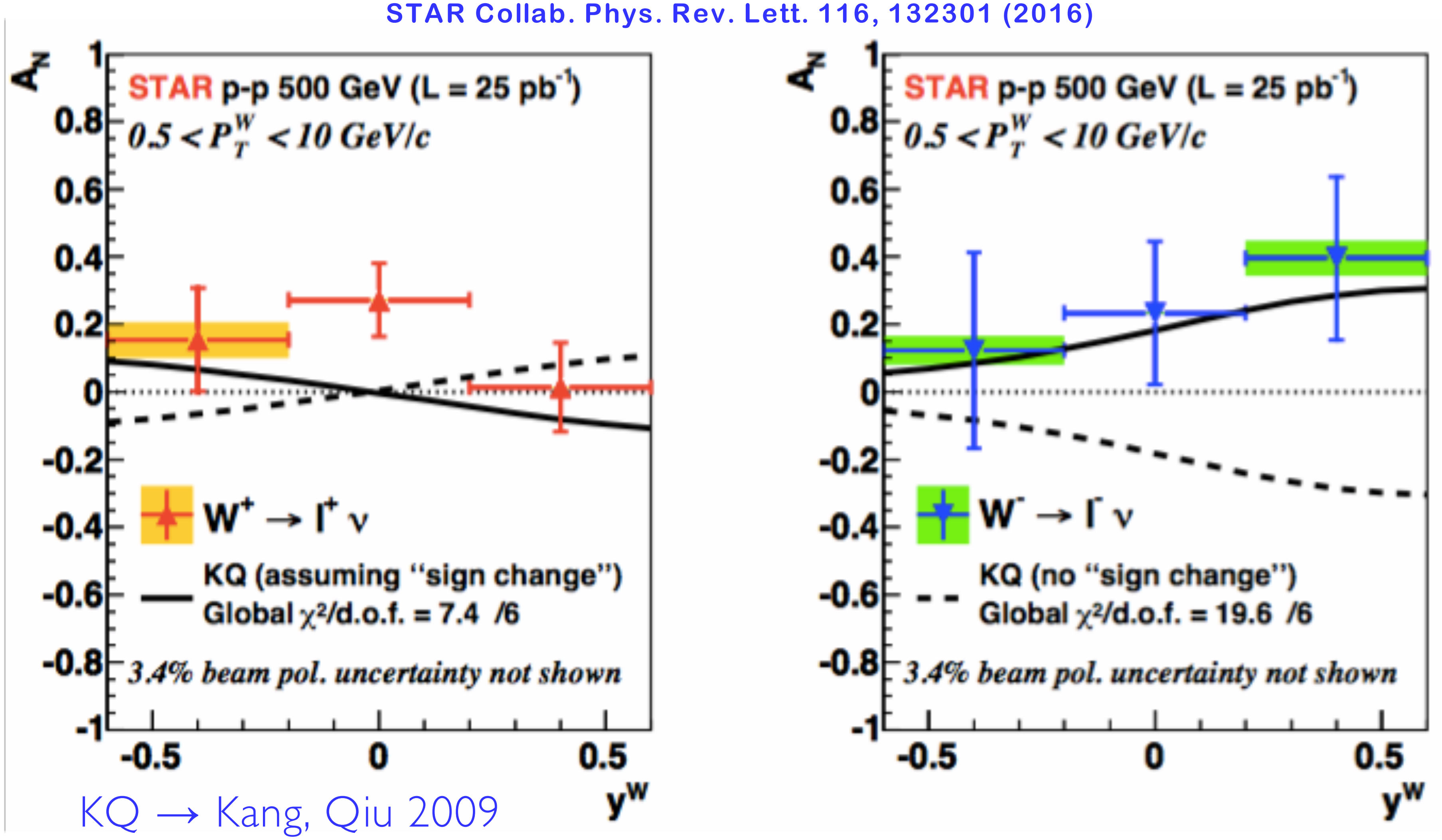
Crucial test of TMD factorization and collinear twist-3 factorization
Several labs worldwide aim at measurement of Sivers effect in Drell-Yan
BNL, CERN, GSI, IHEP, JINR, FERMILAB etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou etc
The verification of the sign change is an NSAC (DOE and NSF) milestone

Process dependence of Sivers function

STAR 2016

- First experimental hint on the sign change: A_N in W and Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



KQ → Kang, Qiu 2009

$$p^\uparrow p \to W^\pm X$$
$$p^\uparrow p \to Z^0 X$$

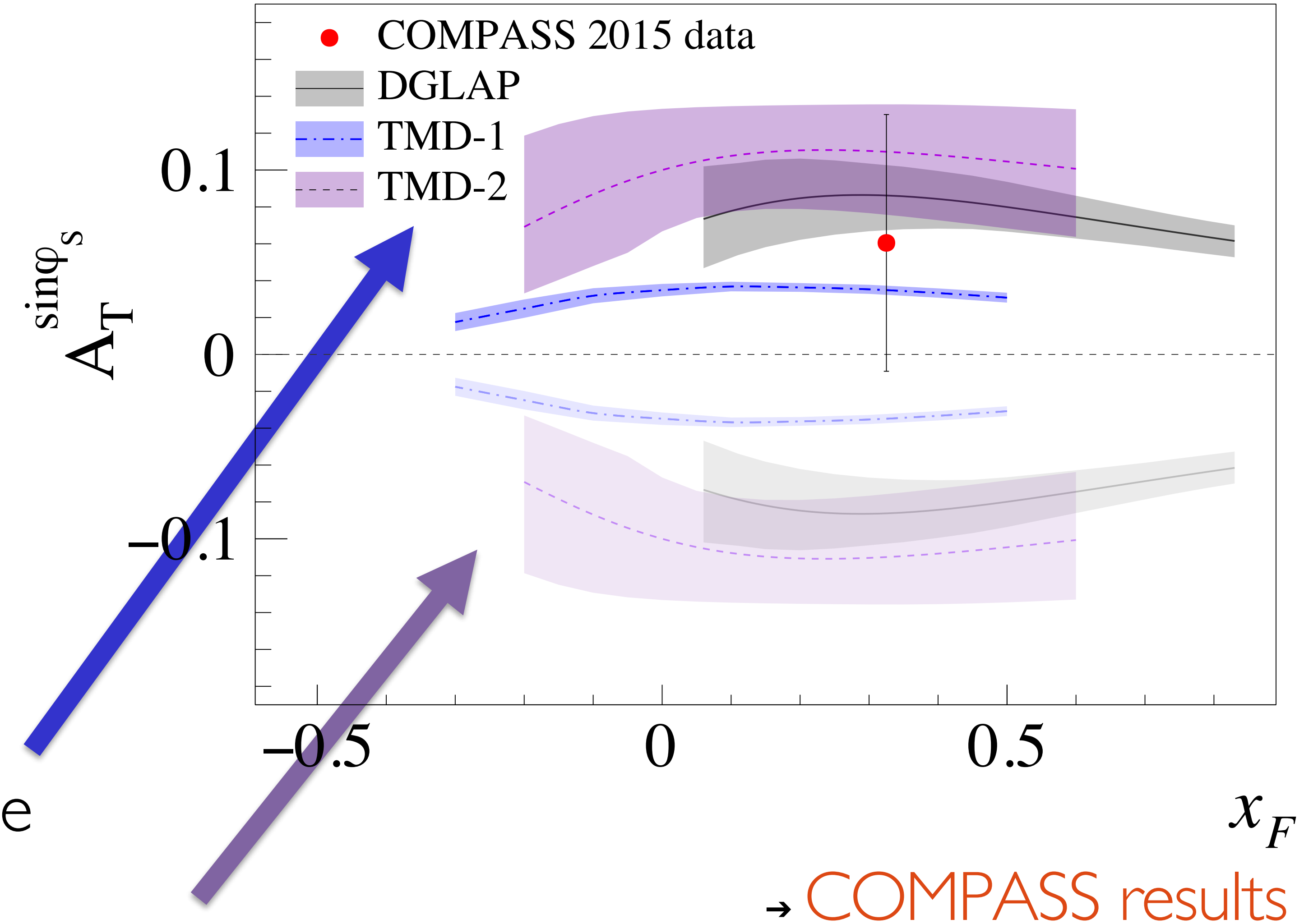
- Sign change $\chi^2/d.o.f \sim 1.2$
- No sign change $\chi^2/d.o.f \sim 3.2$

- Large uncertainties of predictions
- No antiquark Sivers functions

Process dependence of Sivers function

COMPASS 2017

→ First experimental hint on the sign change in Drell-Yan



→ Sign change

→ No sign change

→ COMPASS results hint on sign change

Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z)\rangle \sim H_1^{\perp(1)}(z) \qquad H_1^{\perp(1)}(z) = \int d^2p_\perp \frac{p_\perp^2}{2z^2M_h^2} H_1^\perp(z,p_\perp^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z)\rangle = 0$$

→ If only pions are considered $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

Metz 2002, Metz, Collins 2004, Yuan 2008
Gamberg, Mukherjee, Mulders 2011
Boer, Kang, Vogelsang, Yuan 2010

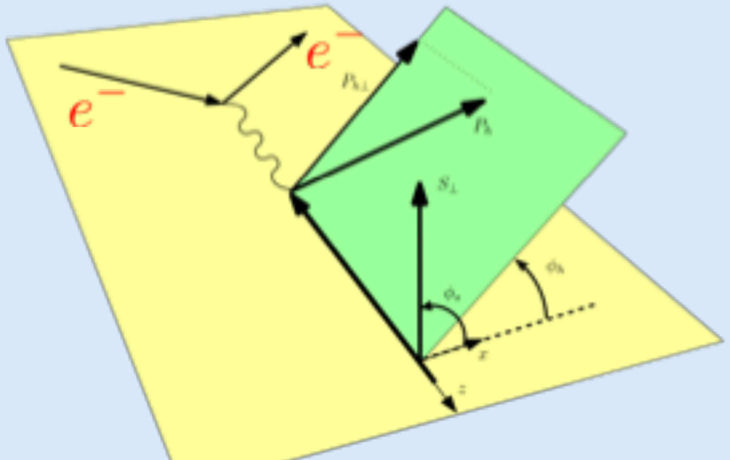
$$H_1^\perp(z)|_{SIDIS} = H_1^\perp(z)|_{e^+e^-} = H_1^\perp(z)|_{pp}$$

→ Very non trivial results

→ Agrees with phenomenology, allows global fits

Transversity and Collins FF

SIDIS and e+e-: combined global analysis

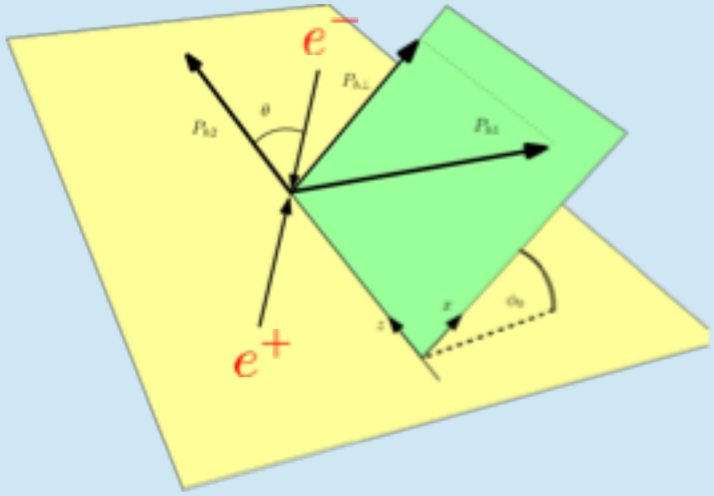


$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B,k_\perp) H_1^\perp(z_h,p_\perp)$$

transversity

Collins function

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0(x_B,y,Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



$$Z_{\text{collins}}^{h_1h_2} \sim H_1^\perp(z_1,p_{1\perp}) H_1^\perp(z_2,p_{2\perp})$$

Collins function

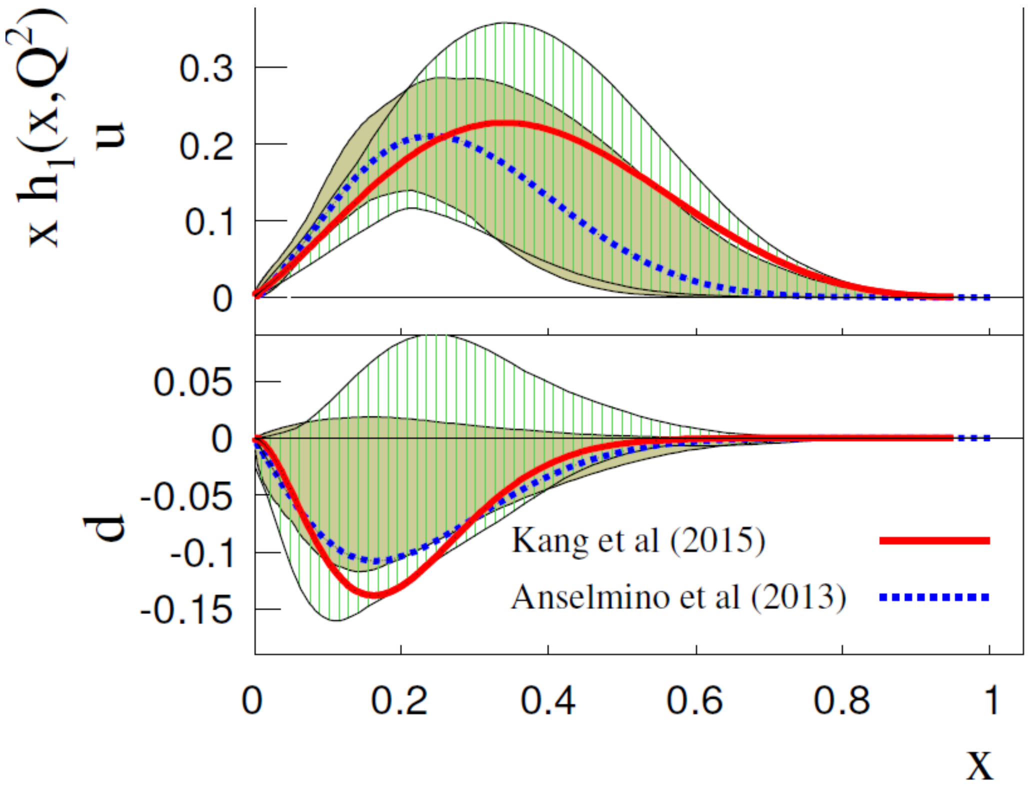
Collins function

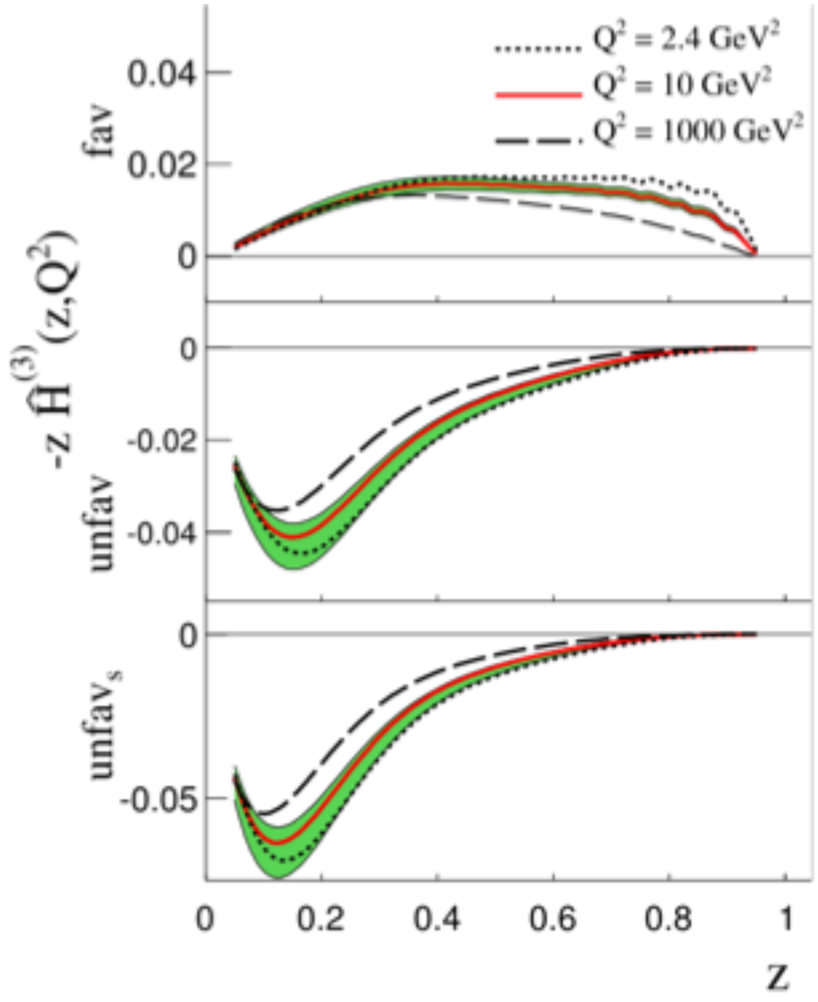
$$\frac{d\sigma^{e^+e^-\to h_1h_2+X}}{dz_{h1}dz_{h2}d^2P_{h\perp}d\cos\theta} = \frac{N_c\pi\alpha_{\text{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right) Z_{uu}^{h_1h_2} + \sin^2\theta\cos(2\phi_0) Z_{\text{collins}}^{h_1h_2}\right]$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2015
Anselmino et al 2015

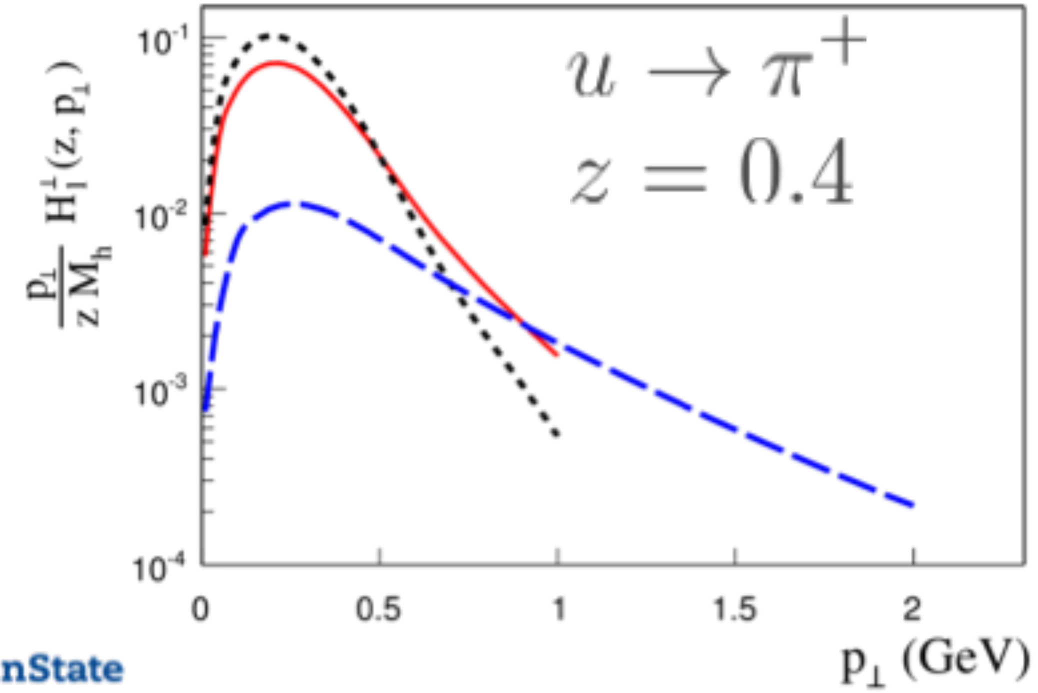
Fitted quark transversity and Collins function: x (z) -dependence





fav : $u \rightarrow \pi^+$
unfav : $d \rightarrow \pi^+$
unfav_s : $s \rightarrow \pi$

Collins function: pt-dependence



Compatible with LO extraction
Anselmino et al 2009, 2013, 2015

Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them

$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

$$\ell + P \to \ell' + h + X$$

$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

$$P_1 + P_2 \to \bar{\ell}\ell + X$$

$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$

$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

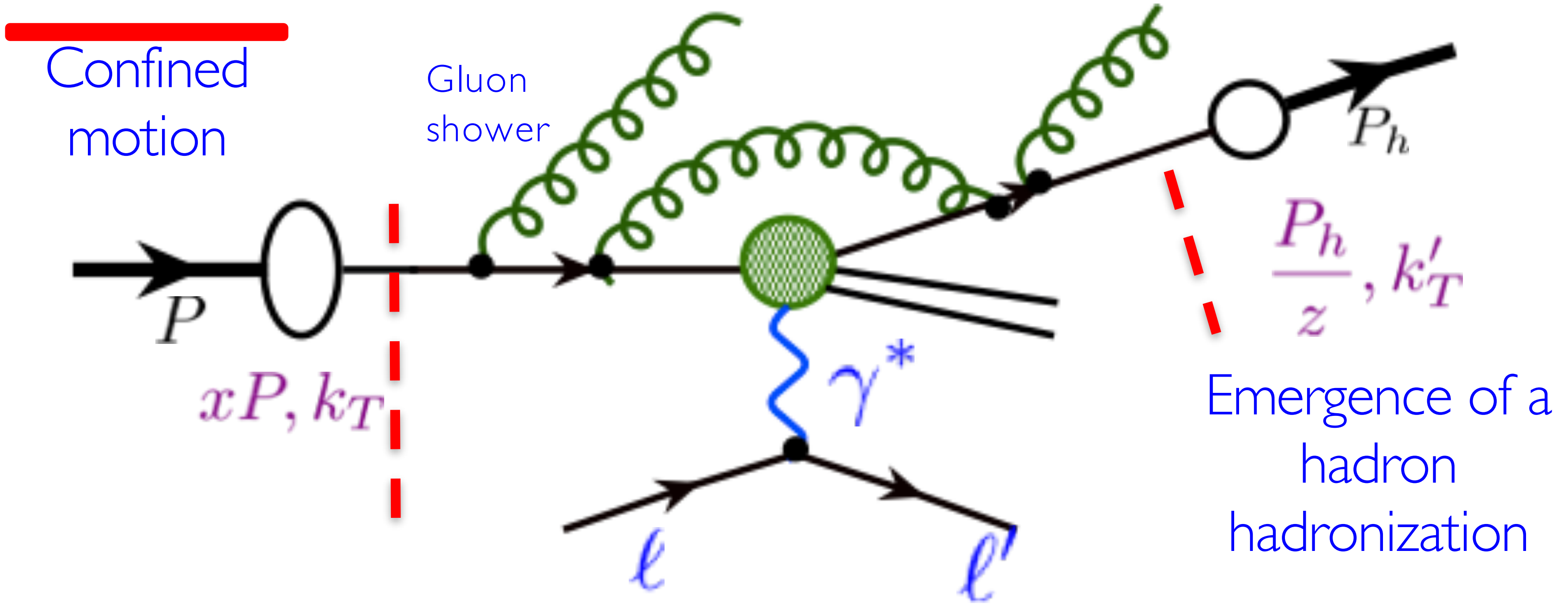
$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Combining measurements from all above is important

Why TMDs, factorization, and evolution

Why QCD evolution is interesting?

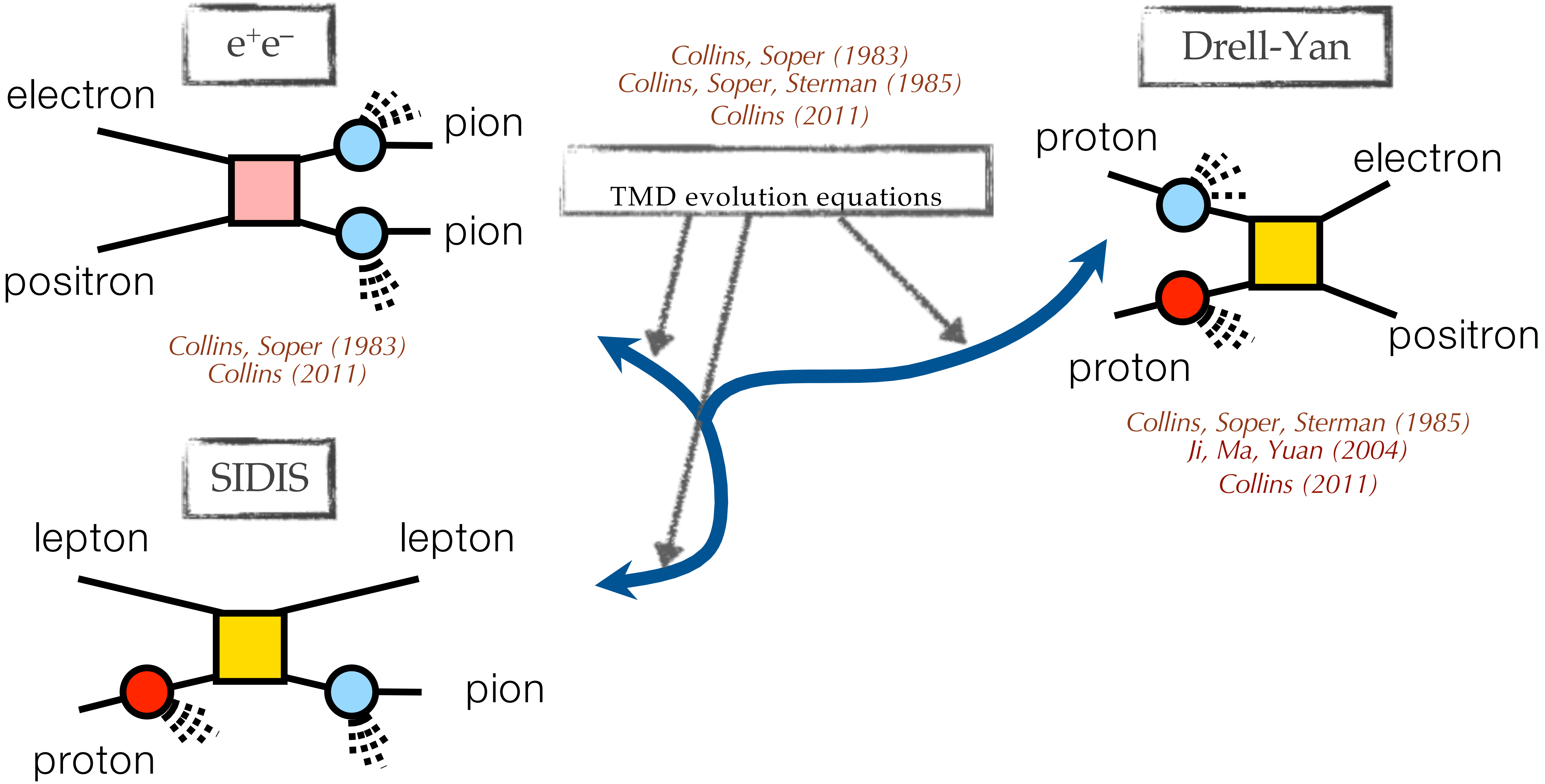
Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



Evolution allows to connect measurements at very different scales.

TMD evolution has also a universal non-perturbative part. The result of evolution cannot be uniquely predicted using evolution equations until the non-perturbative part is reliably extracted from the data.

TMD factorization



e^+e^-

Collins, Soper (1983)
Collins (2011)

Collins, Soper (1983)
Collins, Soper, Sterman (1985)
Collins (2011)

TMD evolution equations

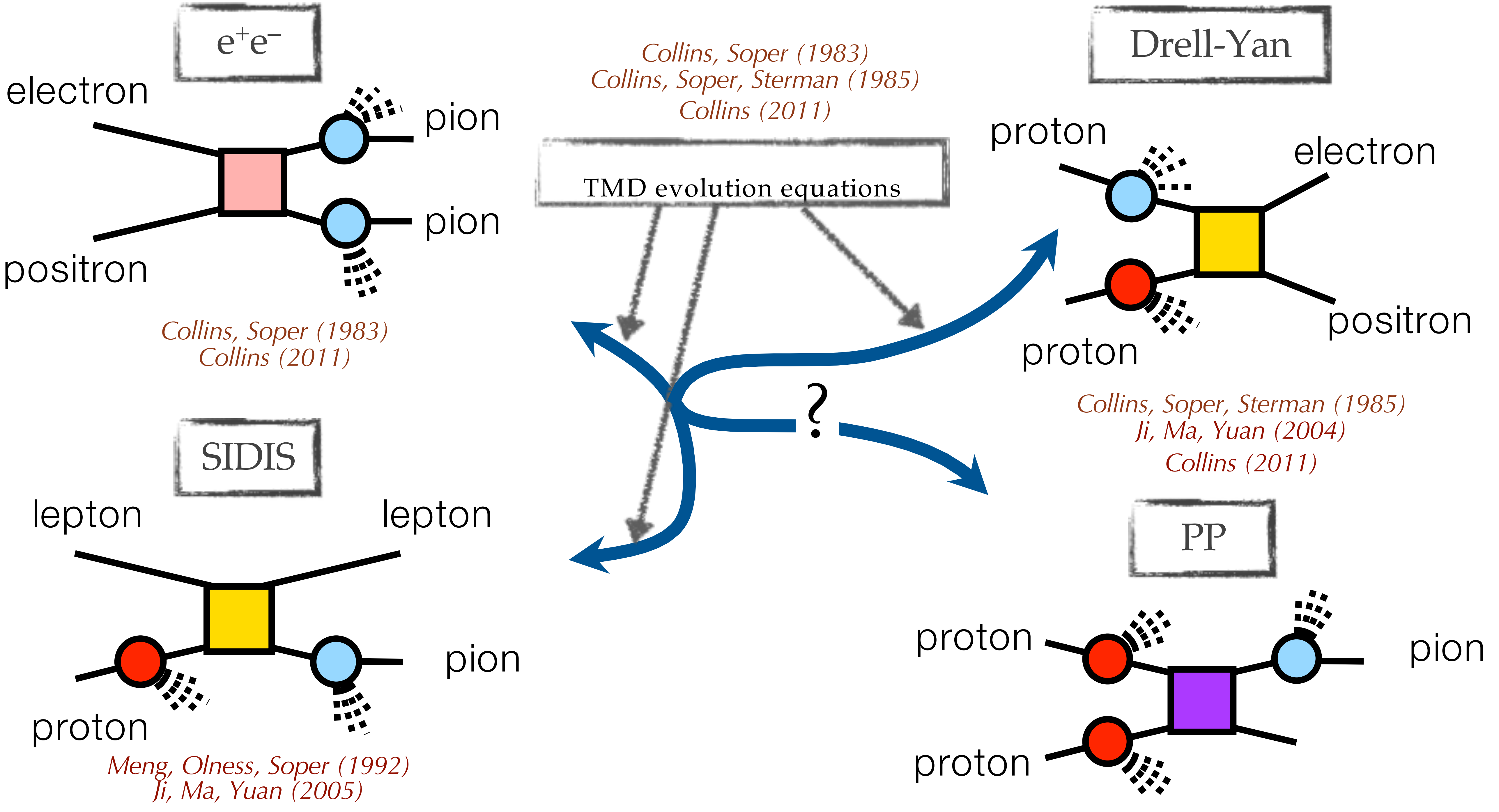
Drell-Yan

Collins, Soper, Sterman (1985)
Ji, Ma, Yuan (2004)
Collins (2011)

SIDIS

Meng, Olnes, Soper (1992)
Ji, Ma, Yuan (2005)
Collins (2011)

TMD factorization



e^+e^-

Collins, Soper (1983)
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TMD evolution equations

Drell-Yan

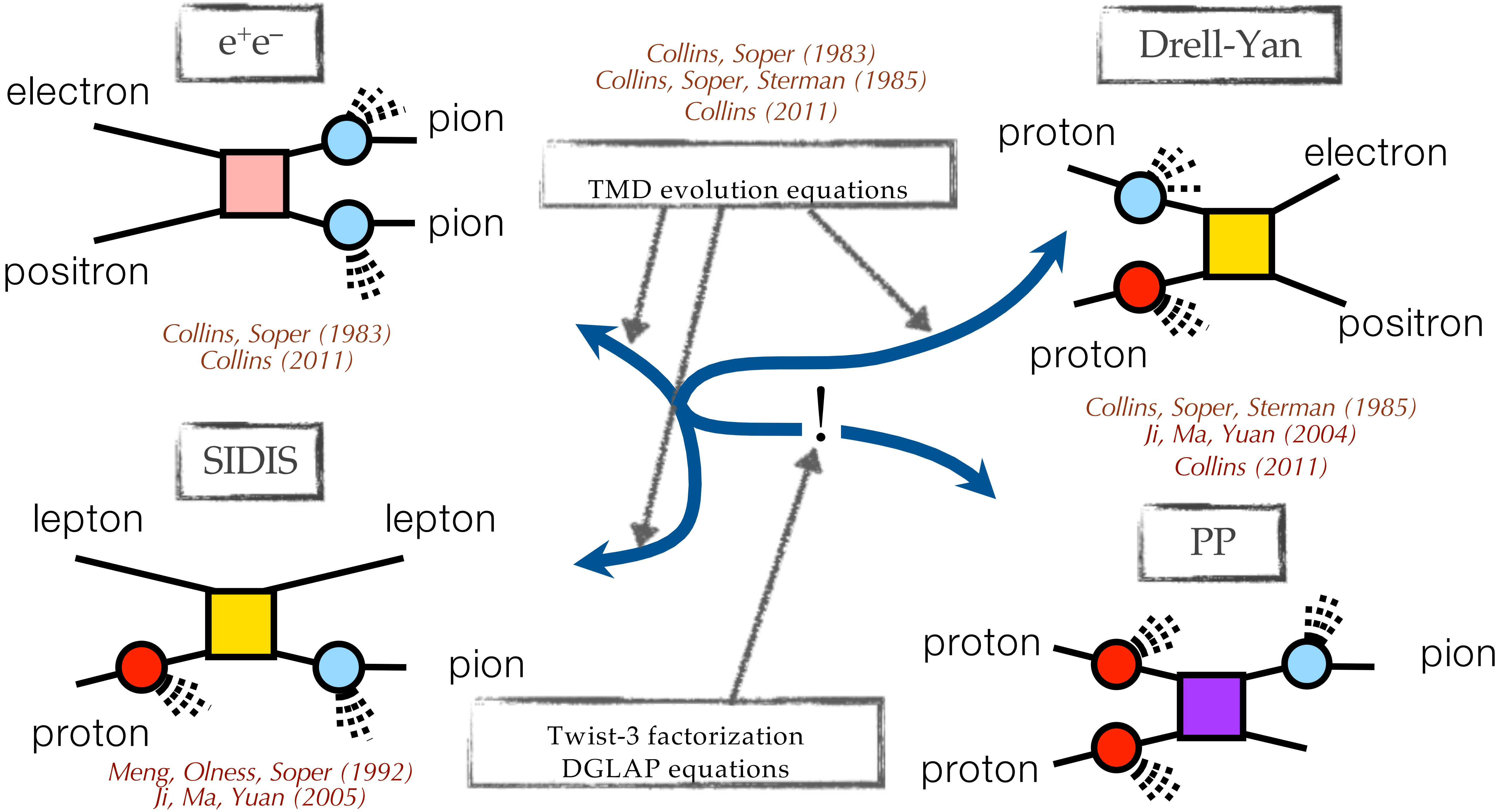
Collins, Soper, Sterman (1985)
Ji, Ma, Yuan (2004)
Collins (2011)

PP

Qiu, Sterman (1990)

Only one scale is measured in PP
TMD factorization is not applicable?

TMD factorization



Collins, Soper (1983)
Collins, Soper, Sterman (1985)
Collins (2011)

Collins, Soper (1983)
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Ji, Ma, Yuan (2004)
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Meng, Olness, Soper (1992)
Ji, Ma, Yuan (2005)
Collins (2011)

Qiu, Sterman (1990)

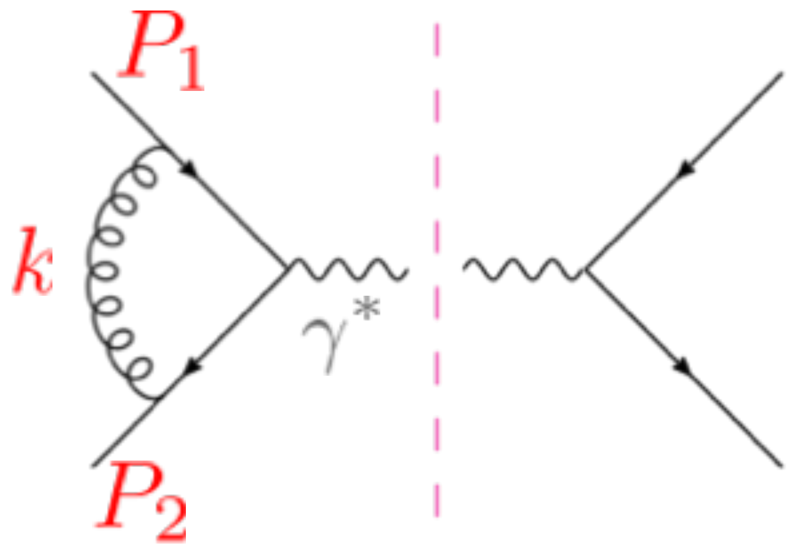
- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible: All four processes can be used.
- Data are from HERMES, COMPASS, JLab, BaBar, Belle, RHIC, LHC, Fermilab

Global fit is needed.
Work in progress

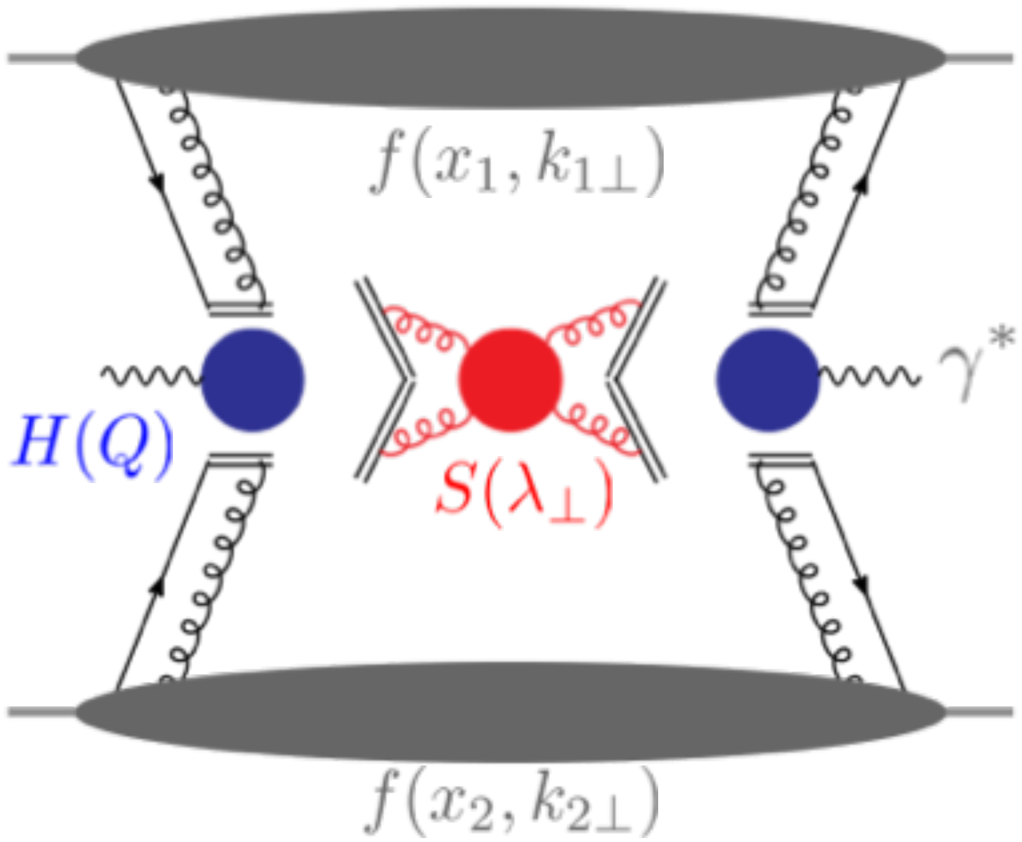
Why TMD Evolution?

TMD factorization in a nut-shell

Drell-Yan: $p+p\to[\gamma^*\to\ell^+\ell^-]+X$



Factorization of regions:
(1) k//P1, (2) k//P2, (3) k soft, (4) k hard



Factorized form and mimicking "parton model"

$$\frac{d\sigma}{dQ^2dyd^2q_\perp}\propto\int d^2k_{1\perp}\,d^2k_{2\perp}\,d^2\lambda_\perp H(Q)f(x_1,k_{1\perp})f(x_2,k_{2\perp})S(\lambda_\perp)\delta^2(k_{1\perp}+k_{2\perp}+\lambda_\perp-q_\perp)$$

$$=\int\frac{d^2b}{(2\pi)^2}e^{iq_\perp\cdot b}H(Q)f(x_1,b)f(x_2,b)S(b)$$

$$F(x,b)=f(x,b)\sqrt{S(b)}$$

$$\boxed{=\int\frac{d^2b}{(2\pi)^2}e^{iq_\perp\cdot b}H(Q)F(x_1,b)F(x_2,b)}$$

mimic "parton model"

slide courtesy of Z. Kang

TMDs evolve

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x,Q)$

- √ DGLAP evolution
- √ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- √ Kernel: purely perturbative

TMDs

$F(x,k_\perp;Q)$

- √ Collins-Soper/rapidity evolution equation
- √ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- √ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$\begin{tikzcd} F(x,Q_i) \arrow[d] \\ R^{\rm coll}(x,Q_i,Q_f) \arrow[d] \\ F(x,Q_f) \end{tikzcd}$$

$$\begin{tikzcd} F(x,k_\perp,Q_i) \arrow[d] \\ R^{\rm TMD}(x,k_\perp,Q_i,Q_f) \arrow[d] \\ F(x,k_\perp,Q_f) \end{tikzcd}$$

slide courtesy of Z. Kang

TMD evolution and non-perturbative component

Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q) = \frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17…

Eventually evolved TMDs in b-space

$$F(x,b;Q) \approx C\otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

longitudinal/collinear part

transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient – ln(Q) piece is spin-independent

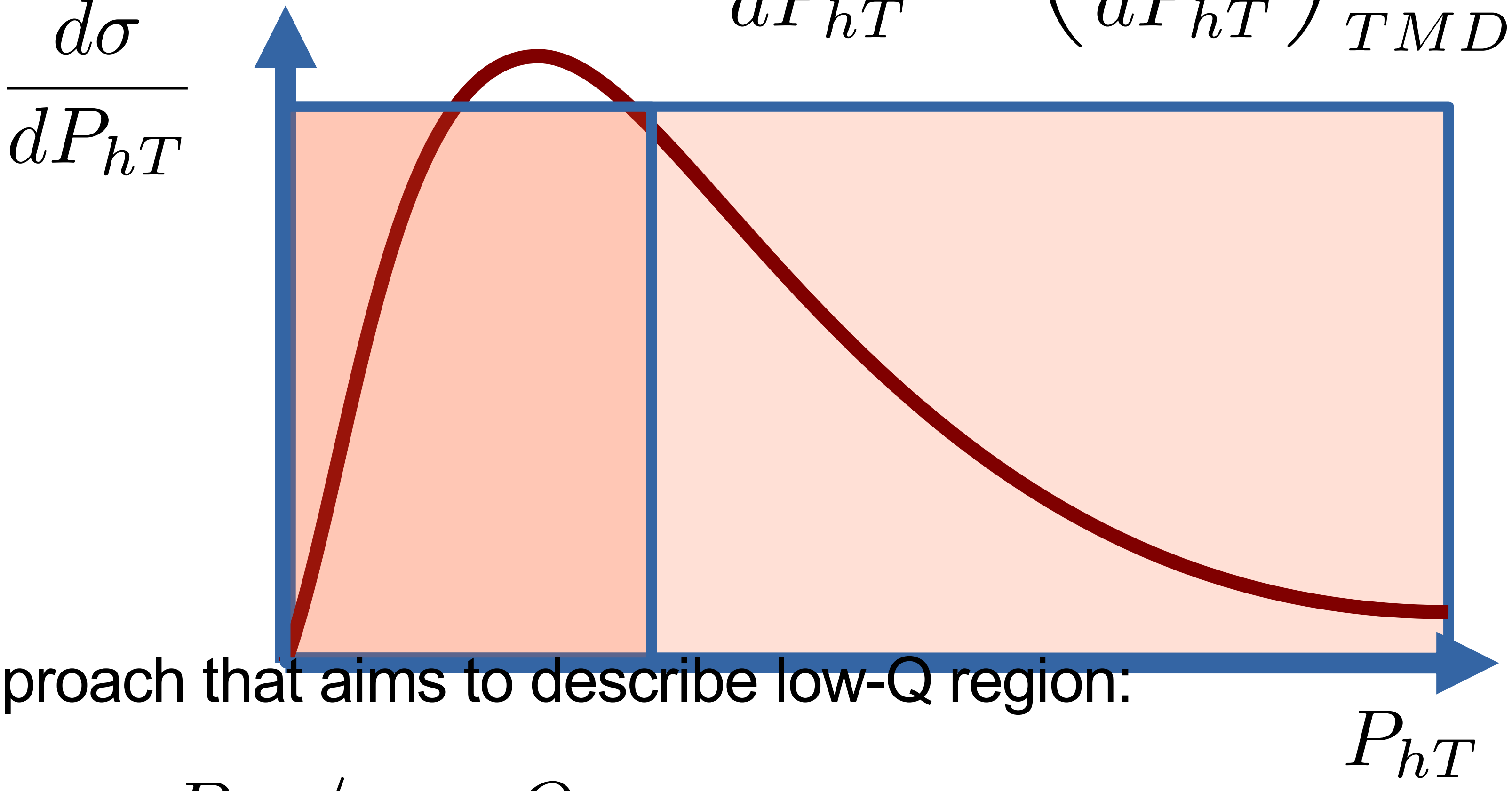
Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract key ingredients for the non-perturbative part

TMD

TMD factorization has a validity region $P_{hT}/z \ll Q$
(two scale problem)

In order to describe cross section in a wide region of transverse momentum one needs to add a Y term

$$\frac{d\sigma}{dP_{hT}} = \left(\frac{d\sigma}{dP_{hT}}\right)_{TMD} + Y$$

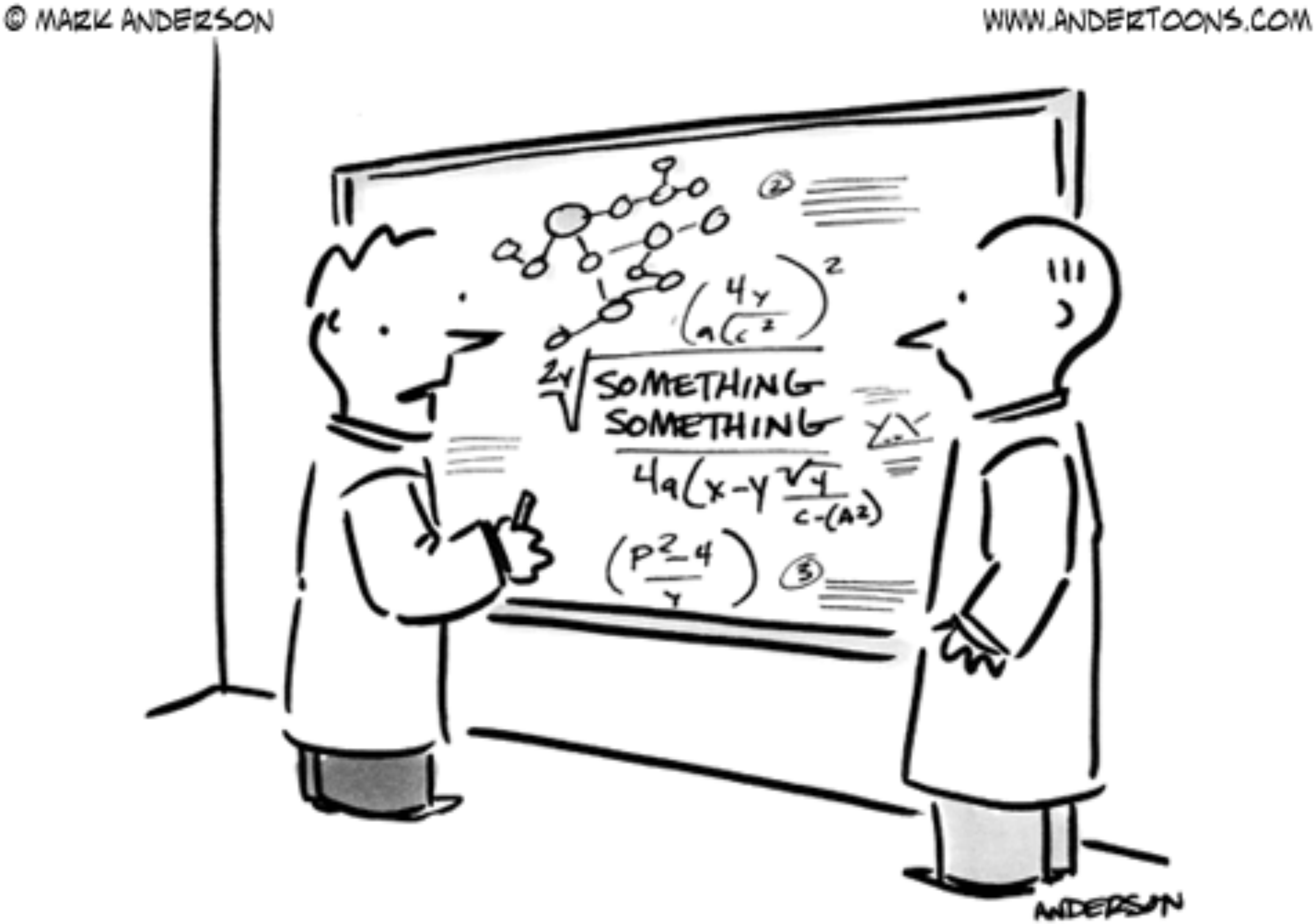


Improved approach that aims to describe low-Q region:

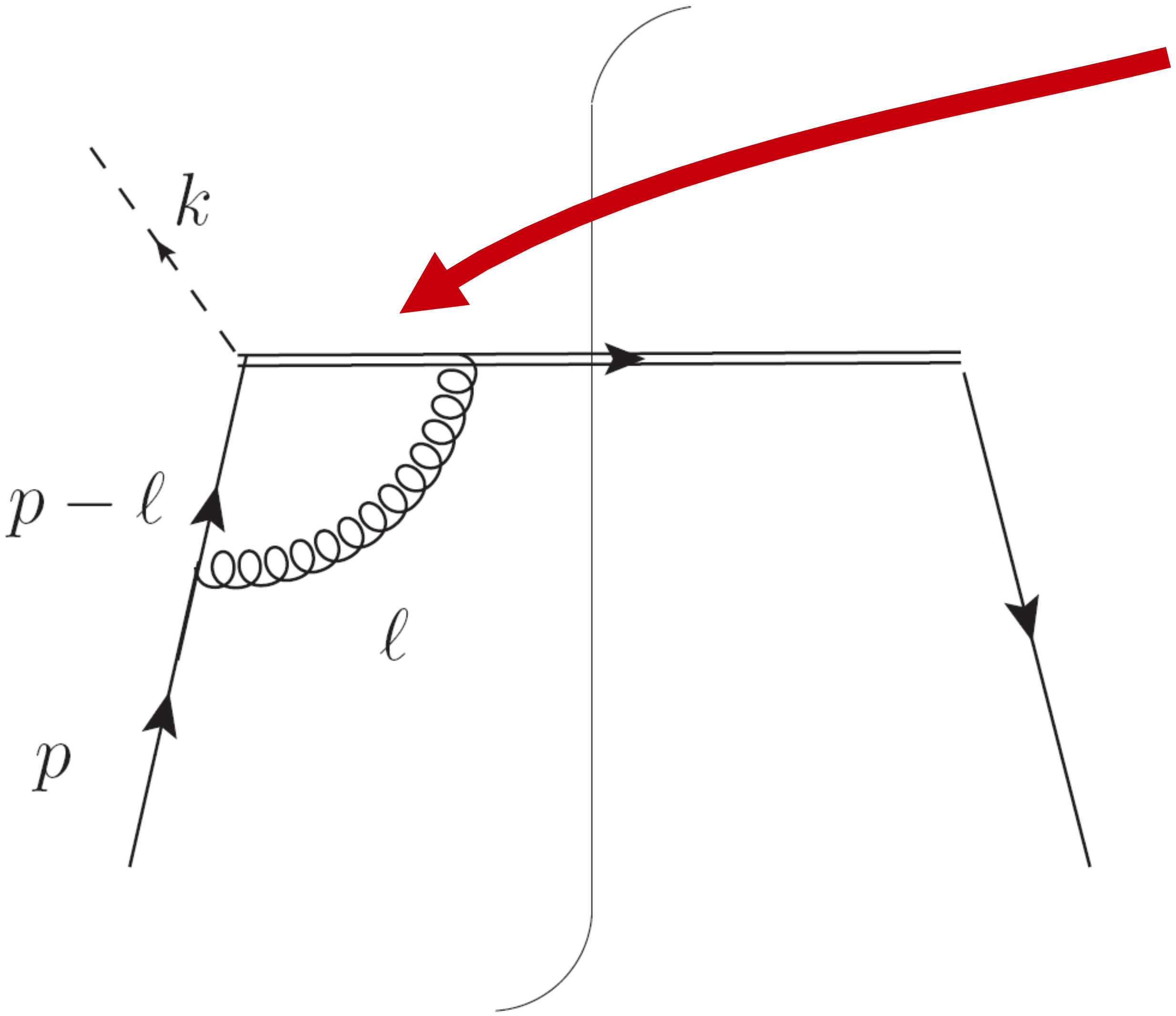
$$P_{hT}/z \ll Q$$

Collins, Gamberg, AP, Rogers, Sato, Wang arXiv:1605.00671

It seems too easy…



In fact it is not easy…

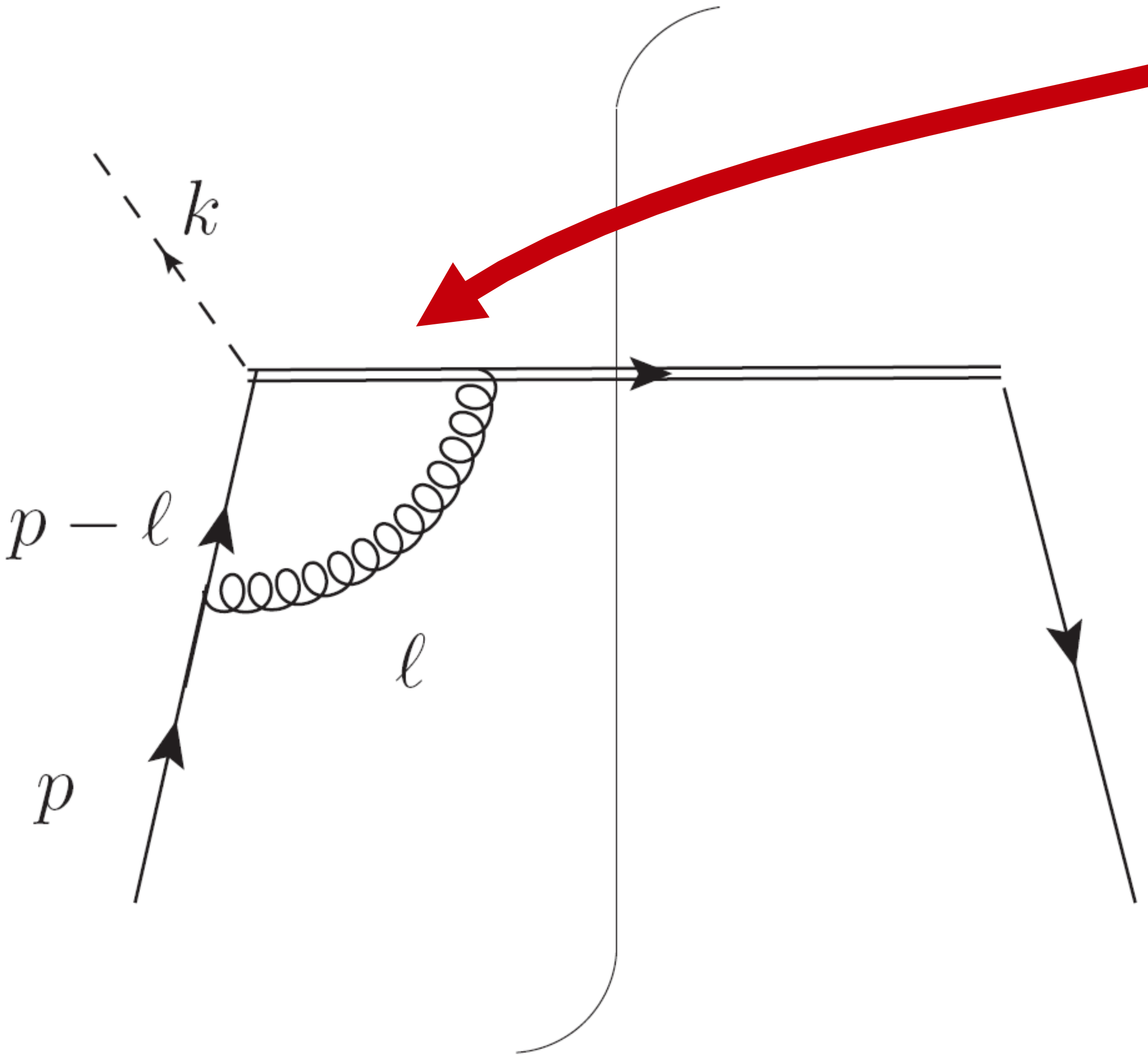


If we consider NLO corrections this diagram diverges

$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

In fact it is not easy...



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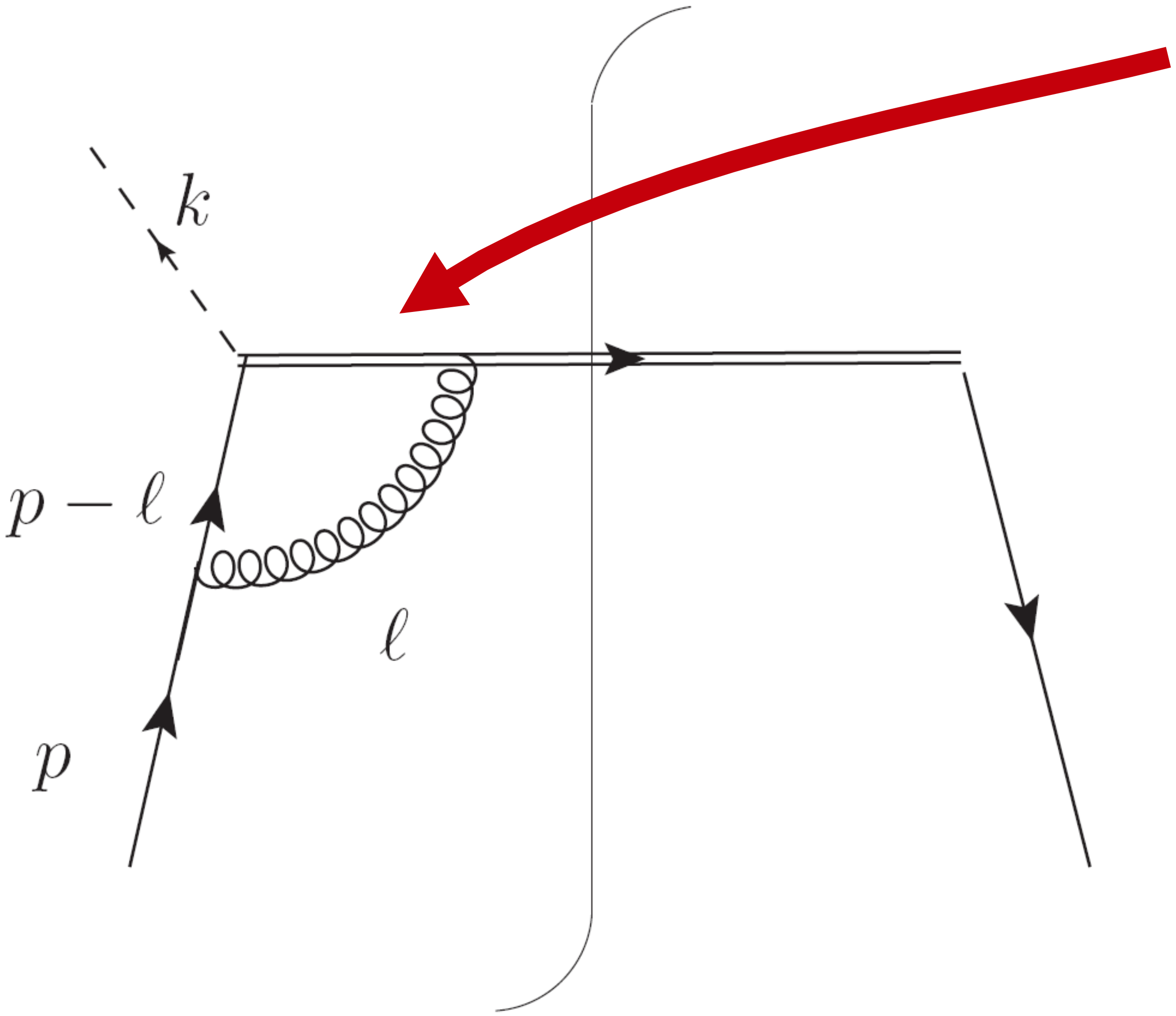


If we consider NLO corrections this diagram diverges

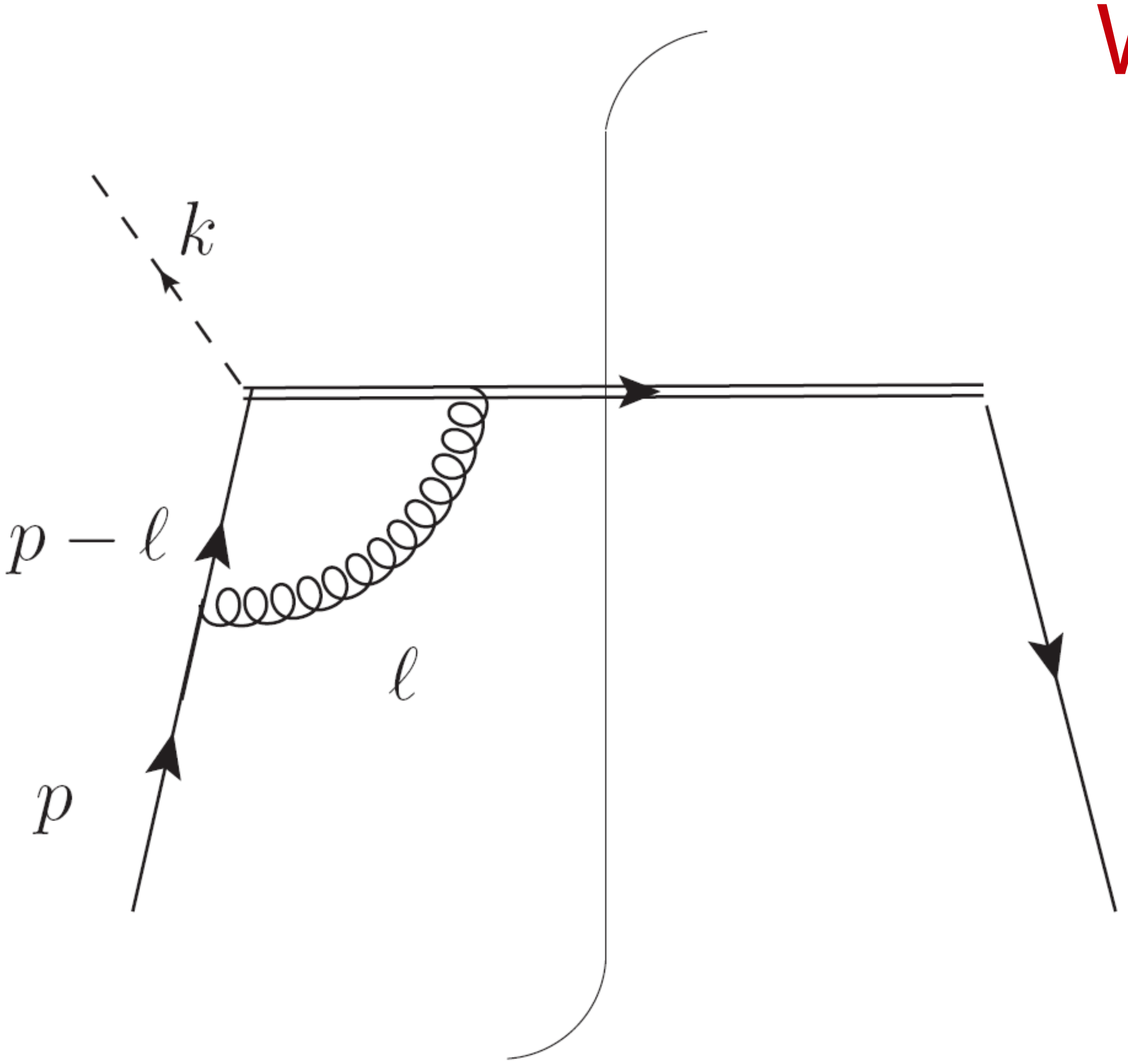
$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

Physics: The gluon becomes collinear to the Wilson line (struck quark) and its rapidity goes to $-\infty$

“Rapidity divergence”

In fact it is not easy…



We know how to deal with it:

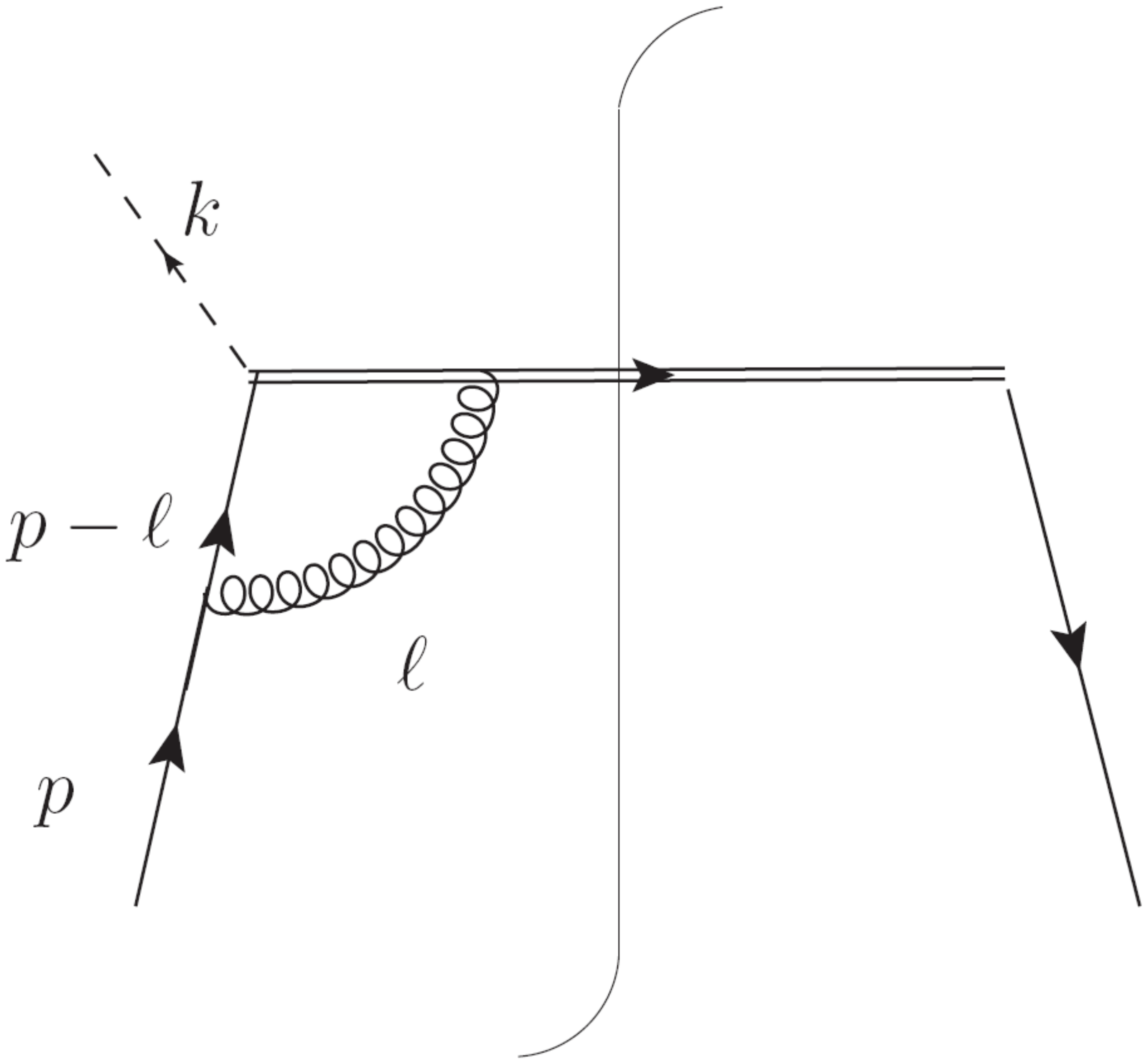
$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha) - T(1)}{(1-\alpha)}$$

"+ prescription"

$$T(\alpha = 1) - T(1) = 0$$

In fact it is not easy…



Not working for TMDs:

$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha, k_\perp) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha, k_\perp) - T(1, 0_\perp)}{(1-\alpha)}$$

“+ prescription”

$$T(\alpha = 1, k_\perp) - T(1, 0_\perp) \neq 0$$

John Collins, Acta Phys.Polon. B34 (2003) 3103

TMD related studies have been extremely active in the past few years, lots of progress have been made

We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider

Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA