Hadron Structure Theory II

Alexei Prokudin

PennState Berks

The plan:

- Lecture I:

  Structure of the nucleon

- Lecture II

  Transverse Momentum Dependent distributions (TMDs)
  Semi Inclusive Deep Inelastic Scattering (SIDIS)

- Tutorial

  Calculations of SIDIS structure functions using Mathematica

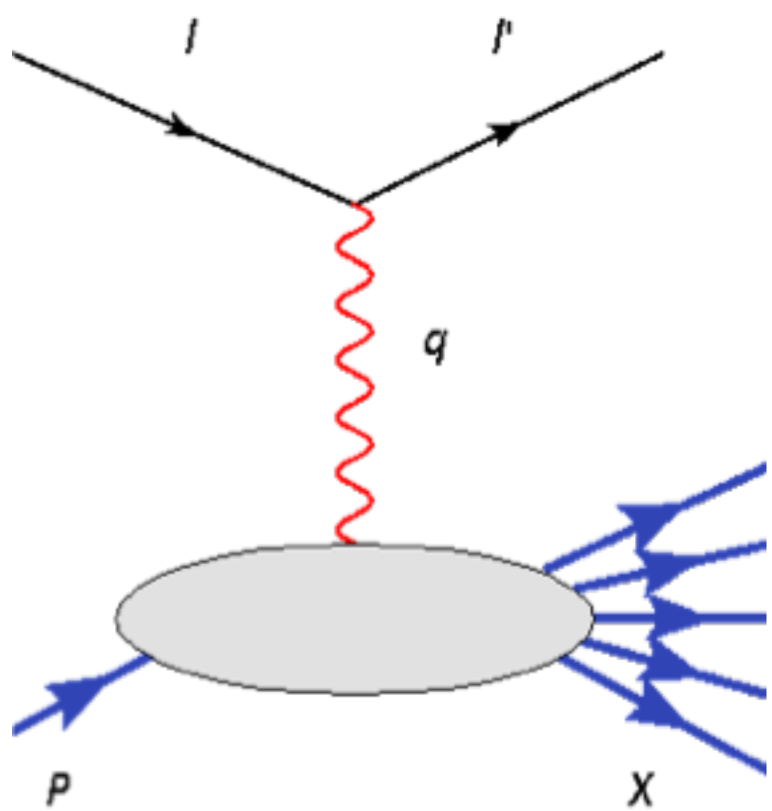
- Lecture III

  Advanced topics. Evolution of TMDs

How do we study the structure of the nucleon?

Deep Inelastic Scattering (DIS)

In order to access **distributions** we could use
deep inelastic scattering



The energy is big enough to
transform the proton in
a lot of final states

Bjorken limit is

$$\mathbf{Q^2 \to \infty}$$

$$\mathbf{P \cdot q \to \infty}$$

$$\mathbf{x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \to const}$$

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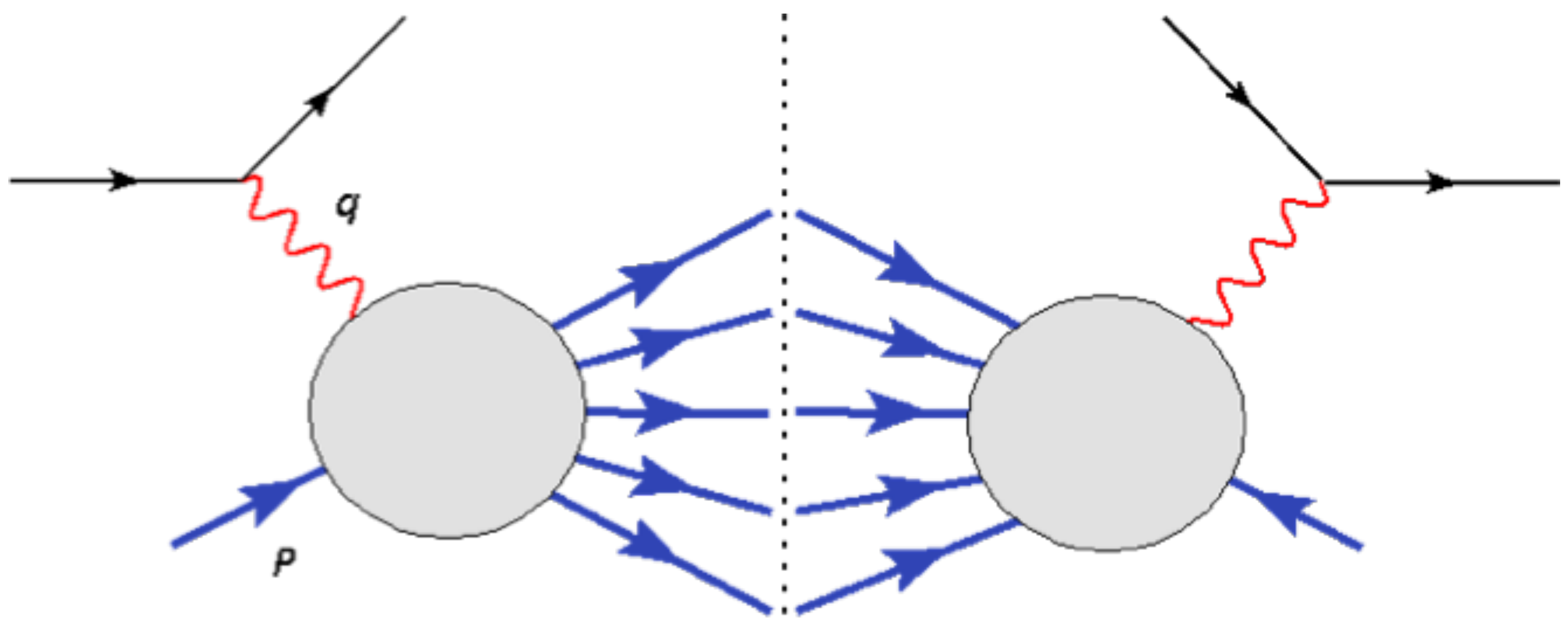
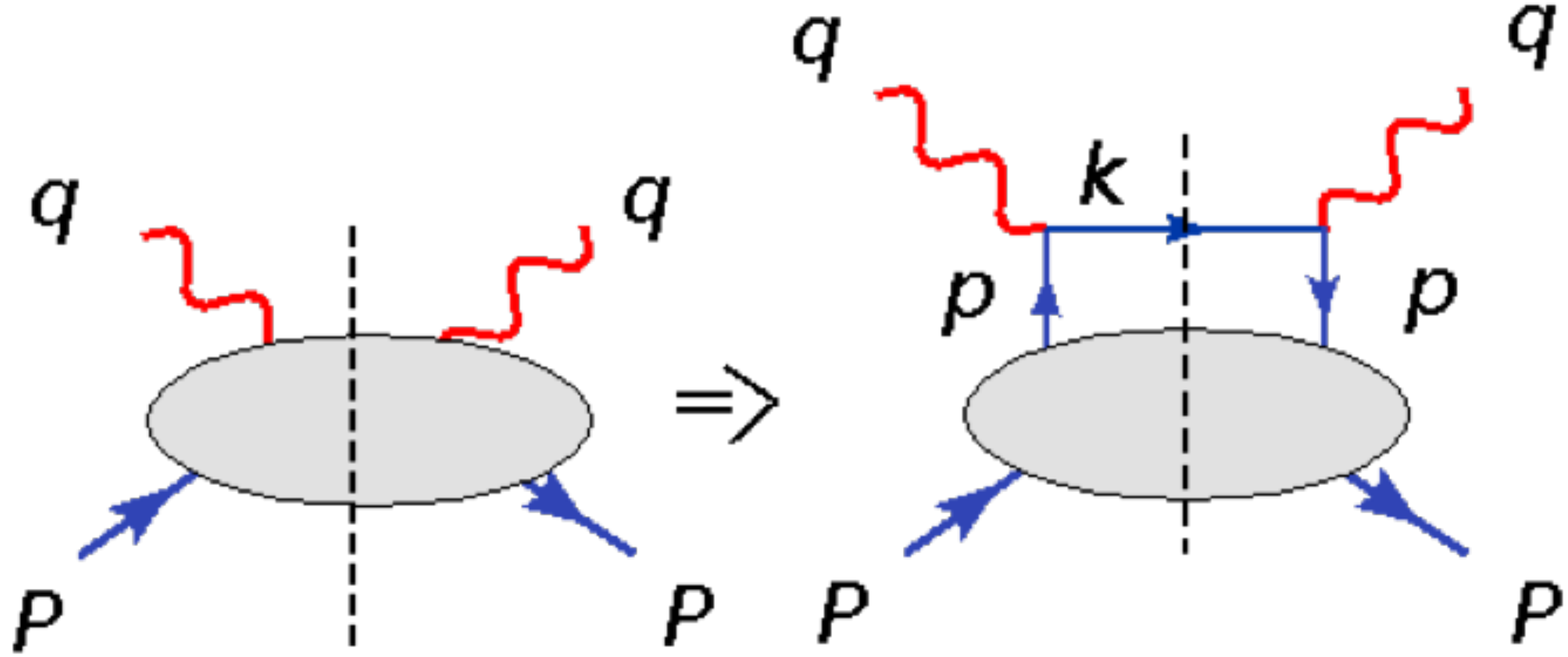
Distributions measured in deep inelastic scattering



$$\sum_{\mathbf{X}} |\mathbf{X}\rangle \langle \mathbf{X}|$$

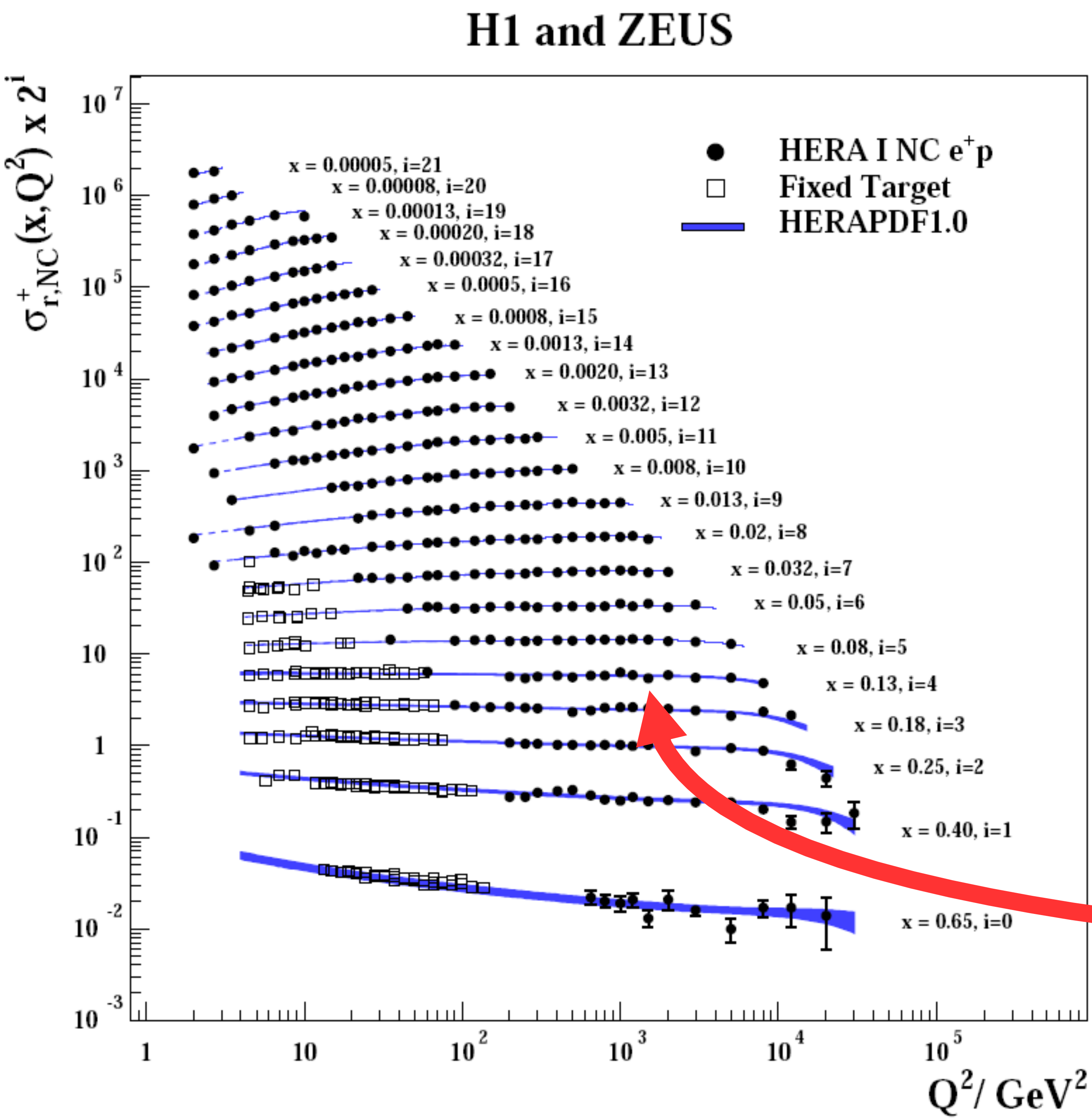
This sum makes it sensitive to parton structure!

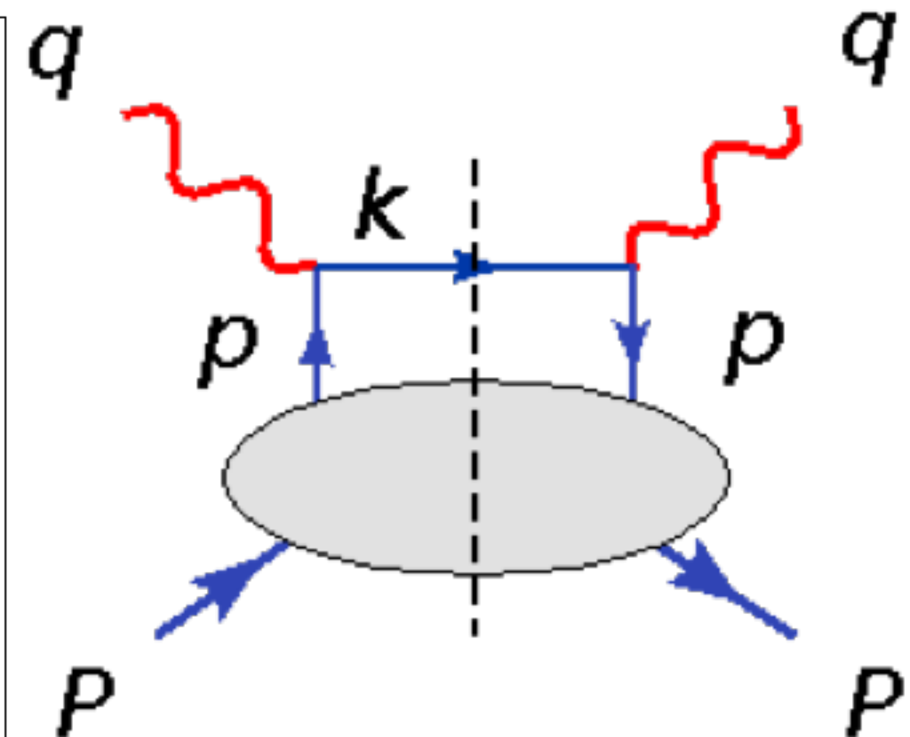
Parton model is a logical step, partons are pointlike and dilute, so the photon interacts with them incoherently



Distributions and parton model

Parton model is a logical step, partons are pointlike and dilute, so photon interacts with them incoherently

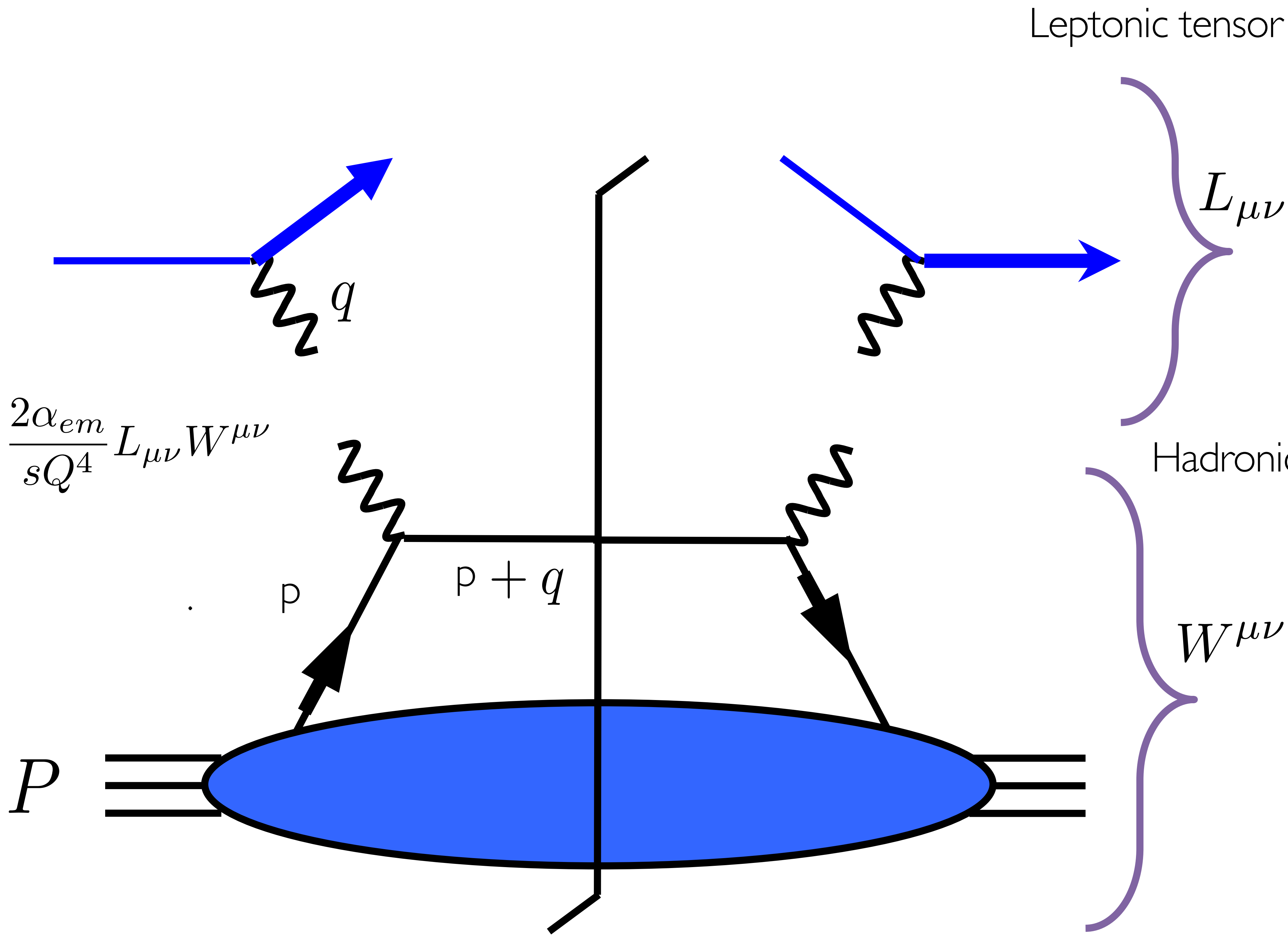




CONSTANT!

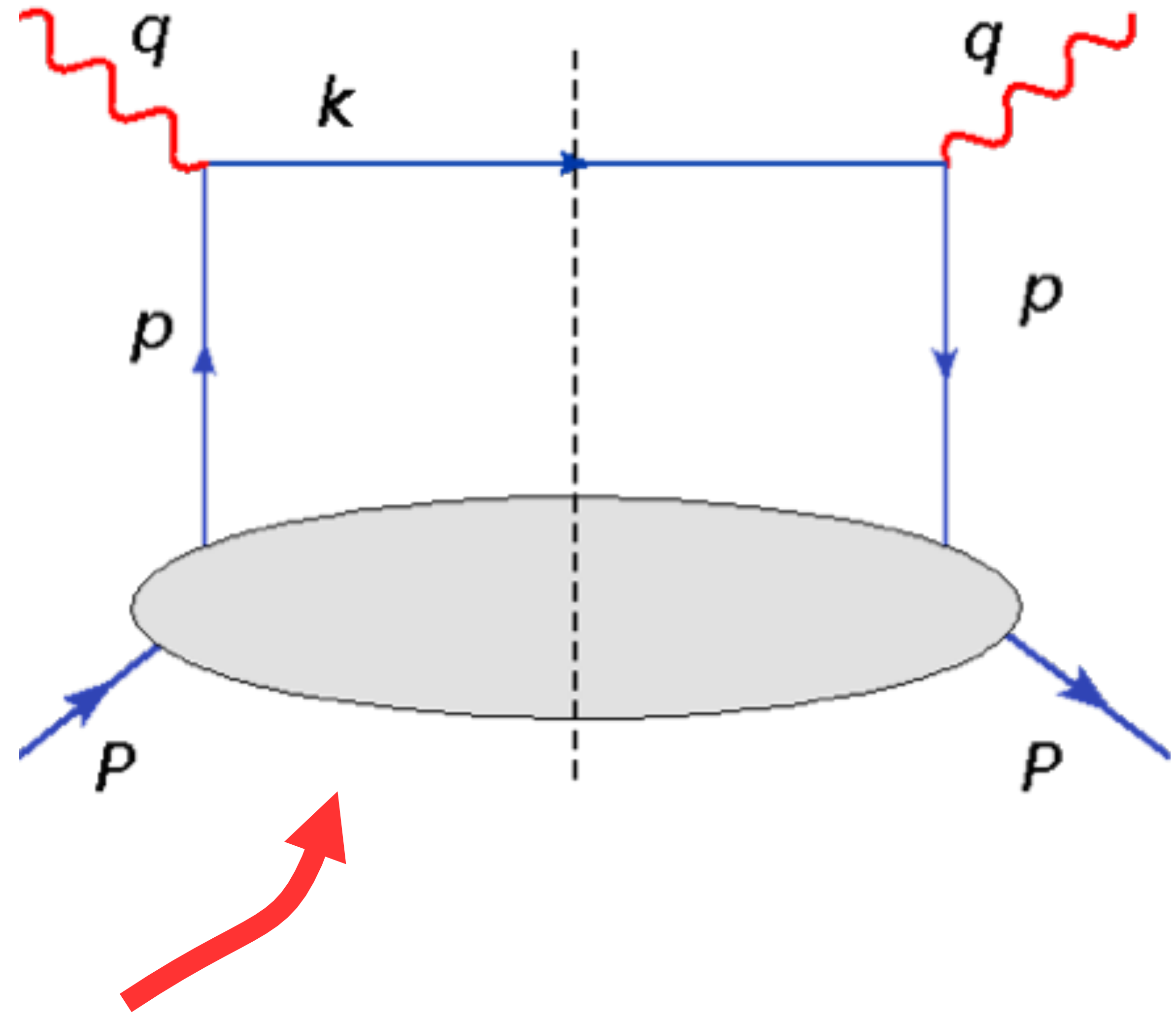
Factorization

$$E' \frac{d\sigma}{d^3\mathbf{l}'} = \frac{2\alpha_{em}}{sQ^4} L_{\mu\nu} W^{\mu\nu}$$



Distributions and parton model

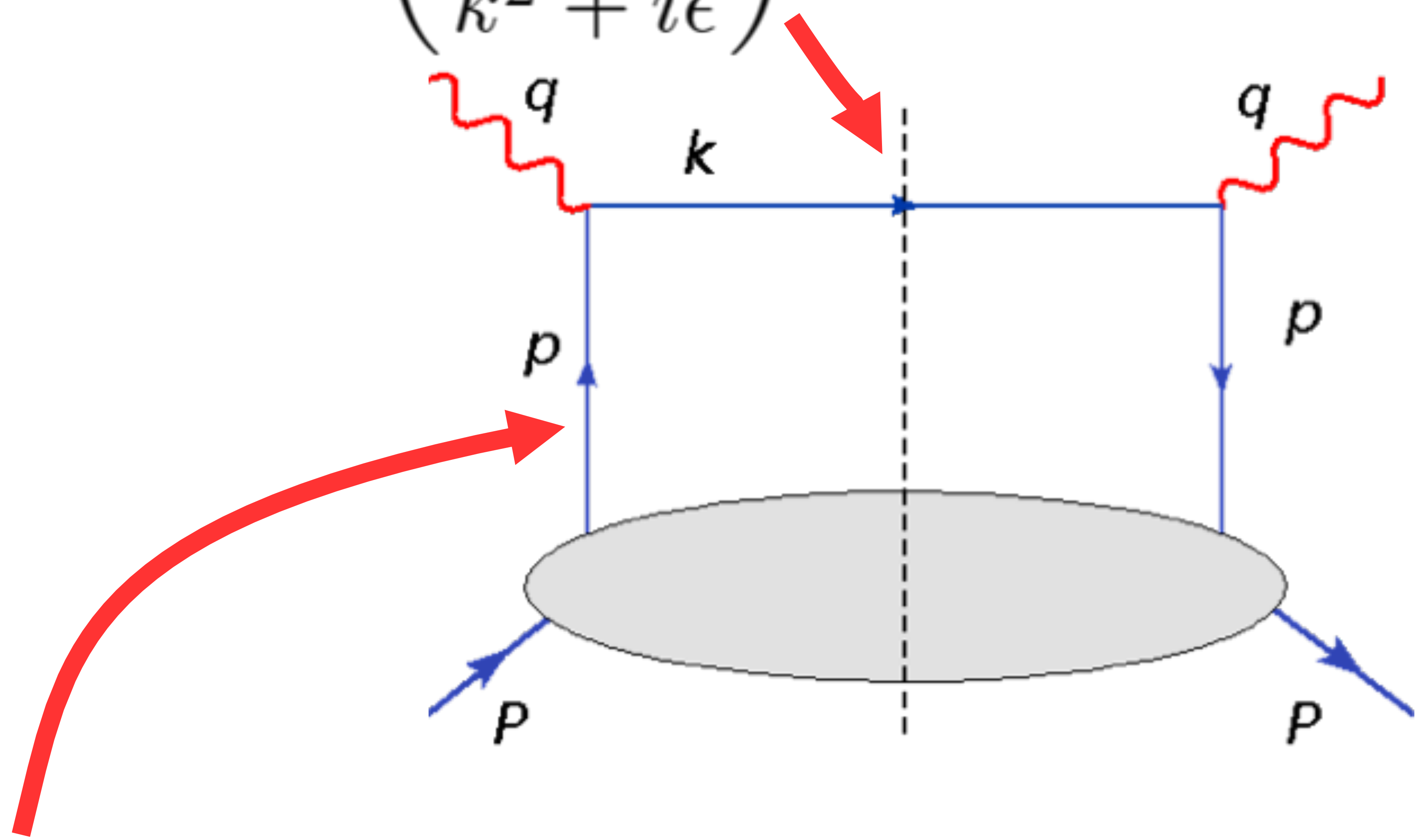
This diagram is called "handbag diagram"



$\mathbf{\Phi(p, P)}$ - parton distribution

Why quarks are on mass-shell?

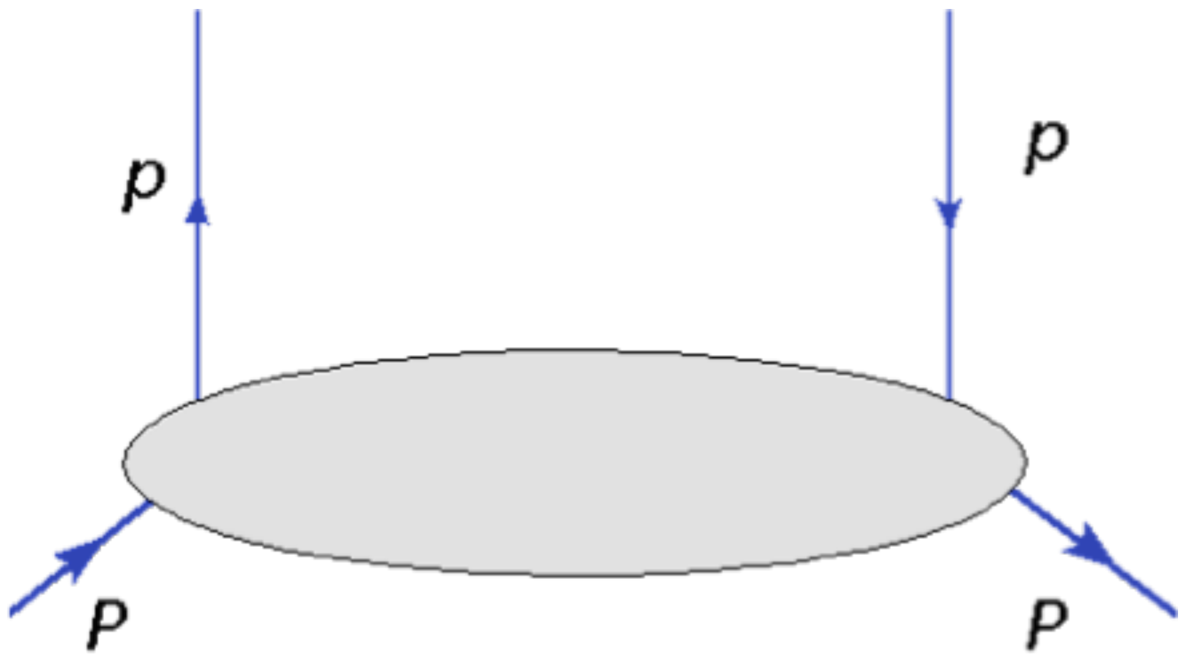
$$Im\left(\frac{1}{k^2+i\epsilon}\right) = \pi\delta(k^2) \quad \Rightarrow \; k^2 \approx 0$$



This one is virtual! However the main contribution comes from

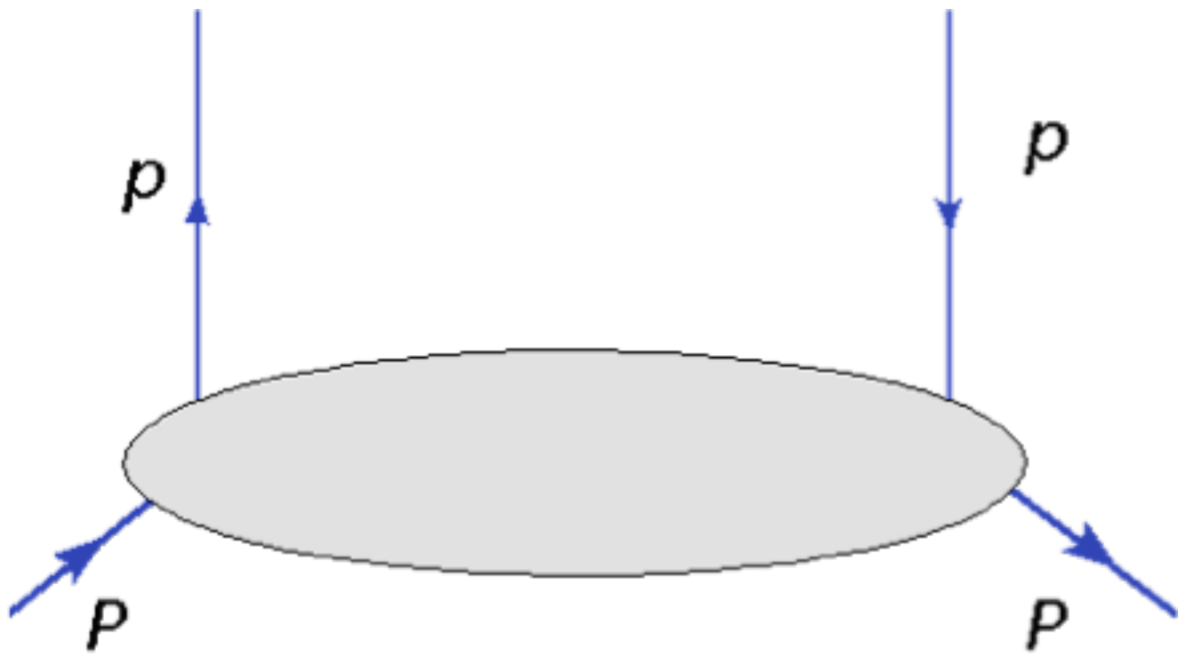
$$\int d^4p\left(\frac{1}{p^2+i\epsilon}\right)\left(\frac{1}{p^2-i\epsilon}\right) \;\Rightarrow\; p^2 \approx 0$$

Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P, S_P\rangle$$

Definition of parton distribution



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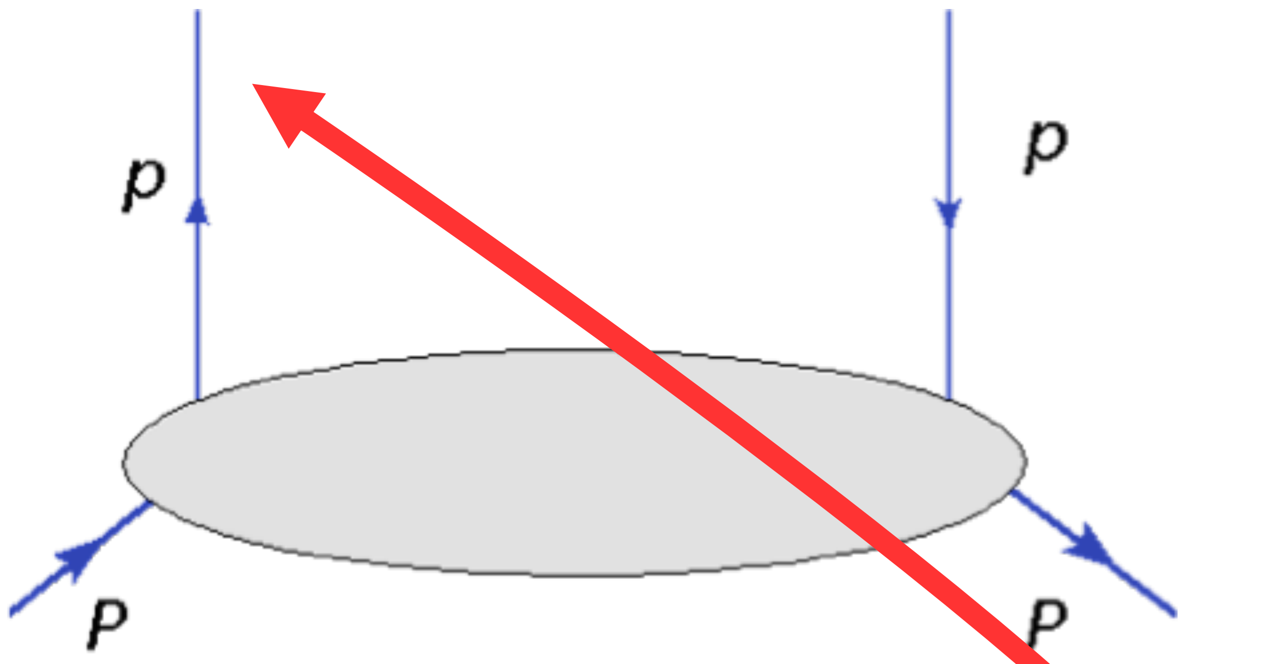
Fourier transform from coordinate to momentum space

Definition of parton distribution



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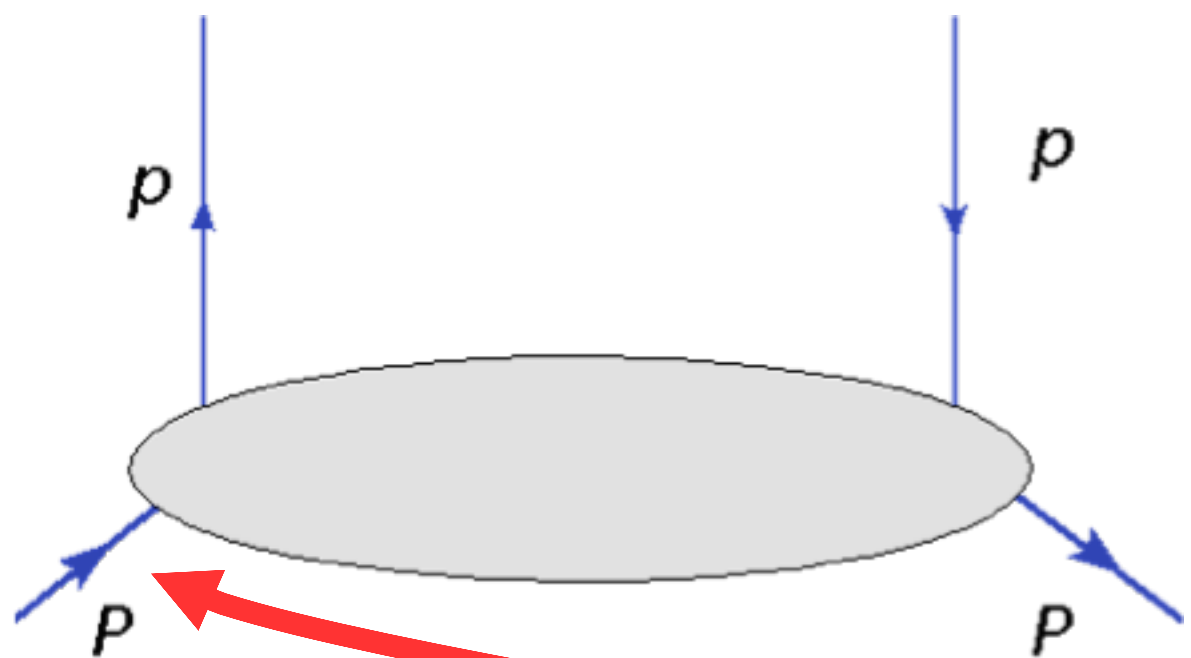
Quark field operator

Definition of parton distribution



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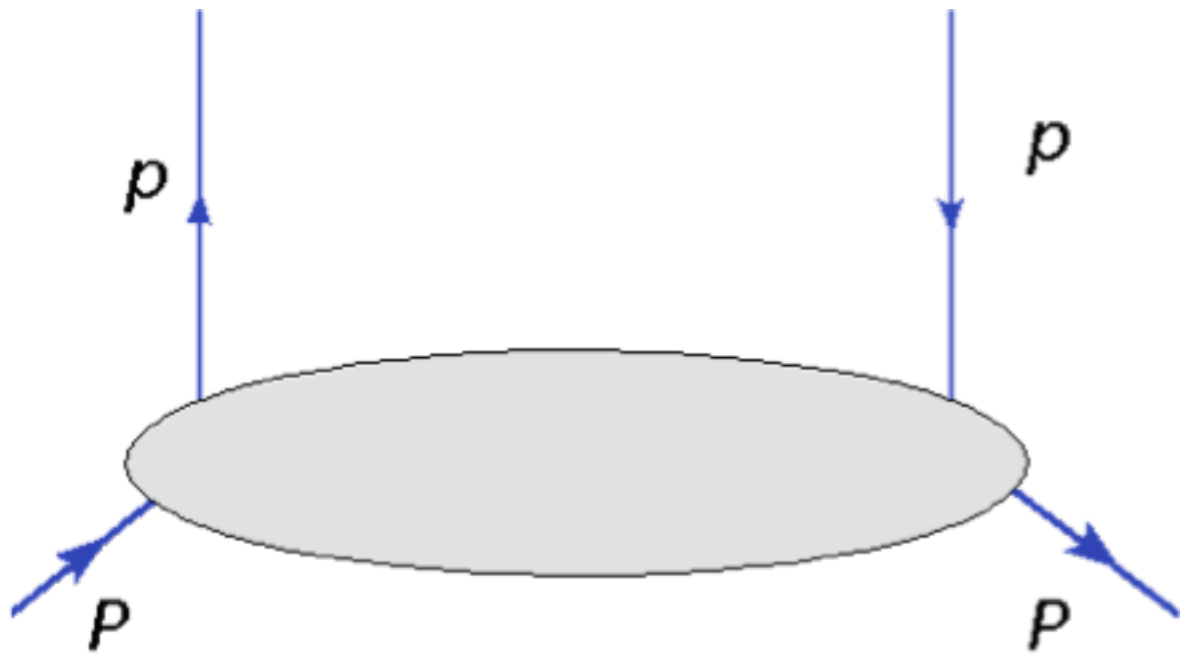
The proton state vector

Definition of parton distribution



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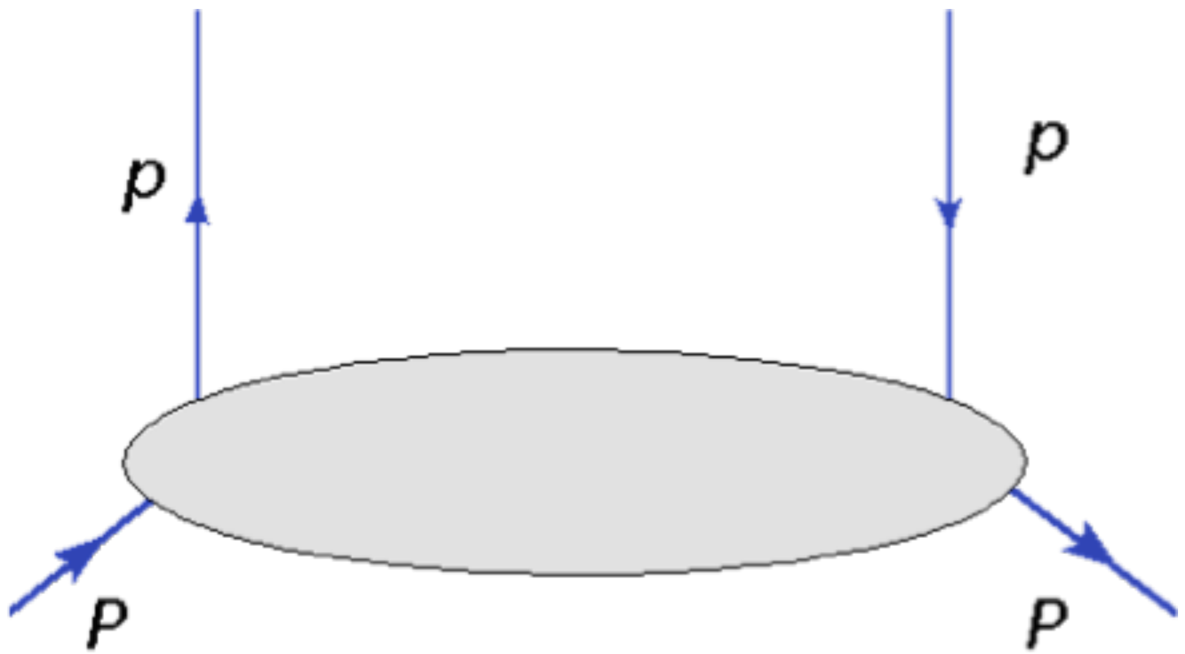
Position of the field in coordinate space

Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \underline{\langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P,S_P\rangle}$$

This matrix element is called "bilocal"

What do we know about quark momentum? Suppose that proton is moving along Z direction with a high momentum, then

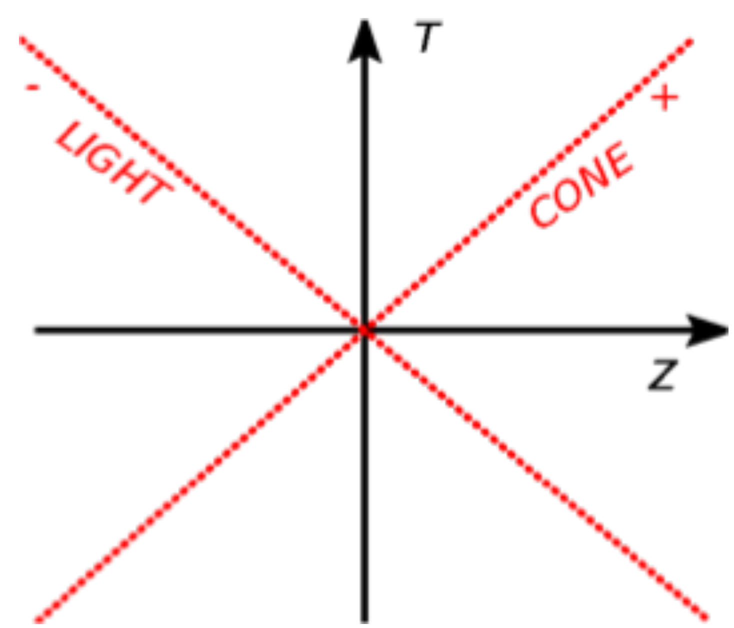
$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big" component $\sim Q$

$x = p^+/P^+$ is a new variable called lightcone momentum fraction

$$P^+ = \frac{1}{\sqrt{2}}\left(P^0 + P^z\right)$$

$$P^- = \frac{1}{\sqrt{2}}\left(P^0 - P^z\right)$$



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"Big" component $\sim Q$

"Small" component $\sim 1/Q$

Distributions and parton model

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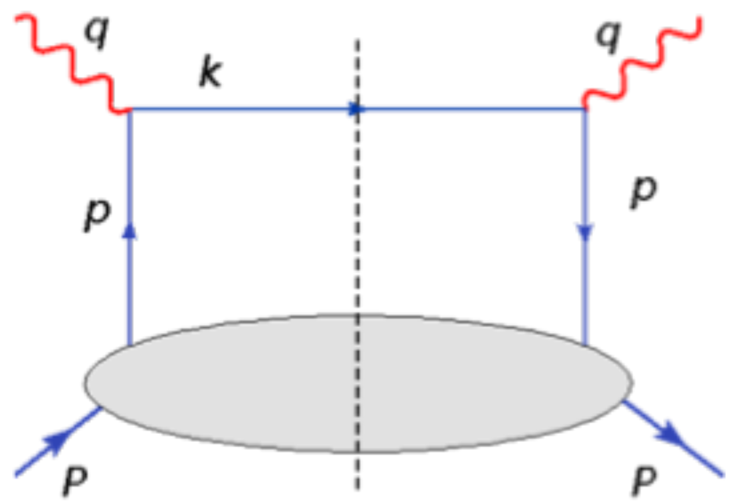
"Big" component $\sim Q$

"Small" component $\sim 1/Q$

"Transverse" component $\sim \Lambda_{QCD}$

What do we know about hadronic tensor?



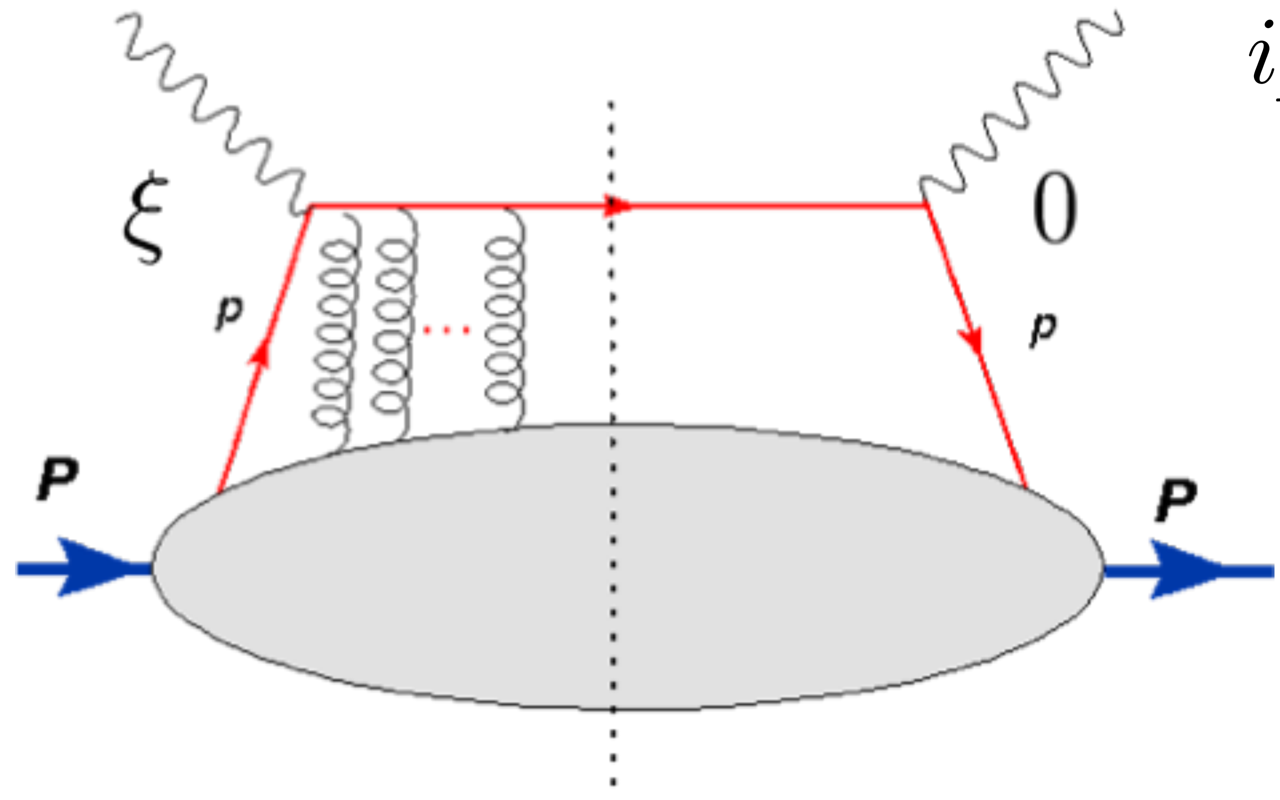
$$W^{\mu\nu} = \sum_q e_q^2 \int \frac{d^4p}{(2\pi)^4} Tr(\gamma^\mu (\not{p} + \not{q})\gamma^\nu \Phi(P,p))\delta((p+q)^2)$$

$$\delta((p+q)^2) \approx \delta(-Q^2 + 2xP\cdot q) = \frac{1}{2P\cdot q}\delta(x_{Bj} - x) \ ,$$

Quarks are "probed" at value of x_{Bj}

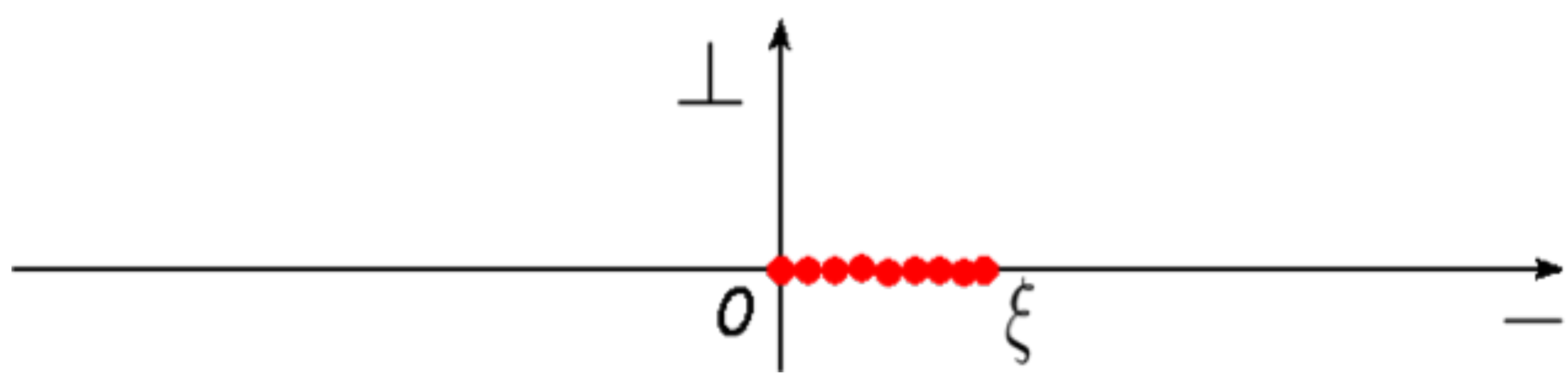
The quark and the remnant are colored thus they interact via gluon exchanges! If "–" and perpendicular component of parton momentum are neclected, than in configuration space only "–" component survives,

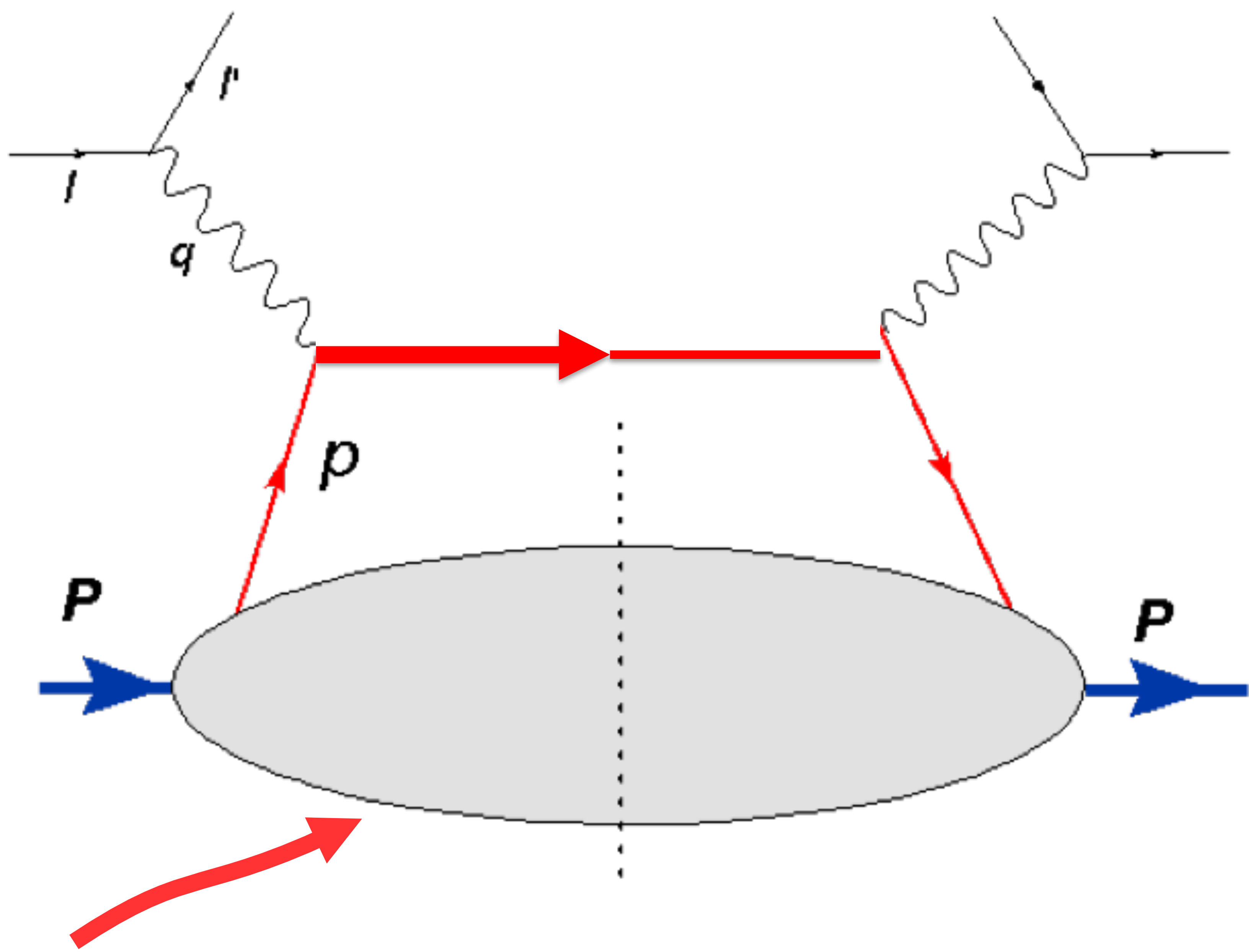
$$ip \cdot \xi = ixP^+\xi^-$$



This object is called Wilson line $\mathcal{W}(0,\xi)$

For DIS:





Distribution

$$\sigma_{\mathbf{DIS}}$$

$$||$$

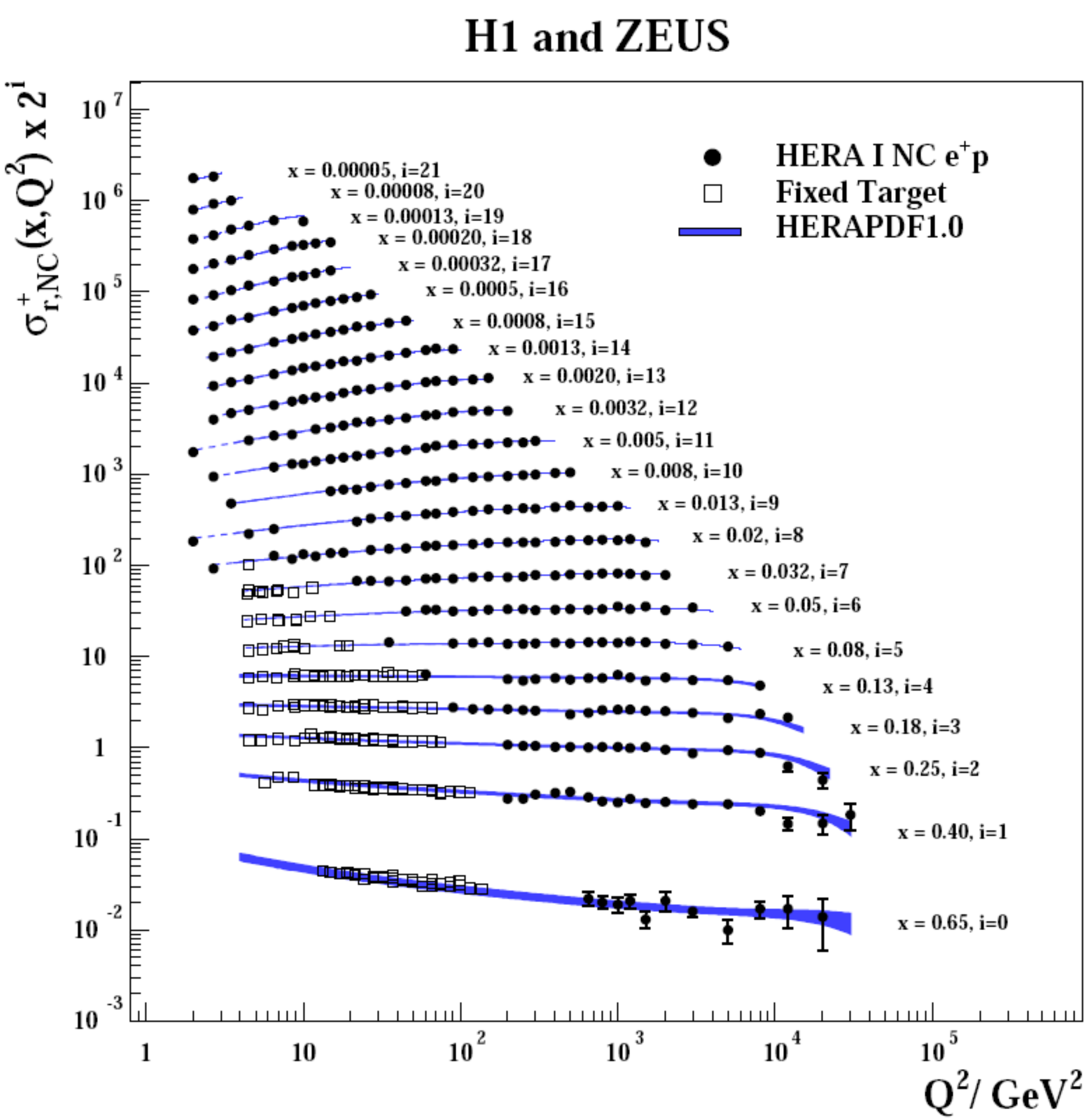
$$\hat{\sigma}_{\mathbf{lq \to l'q'}}$$

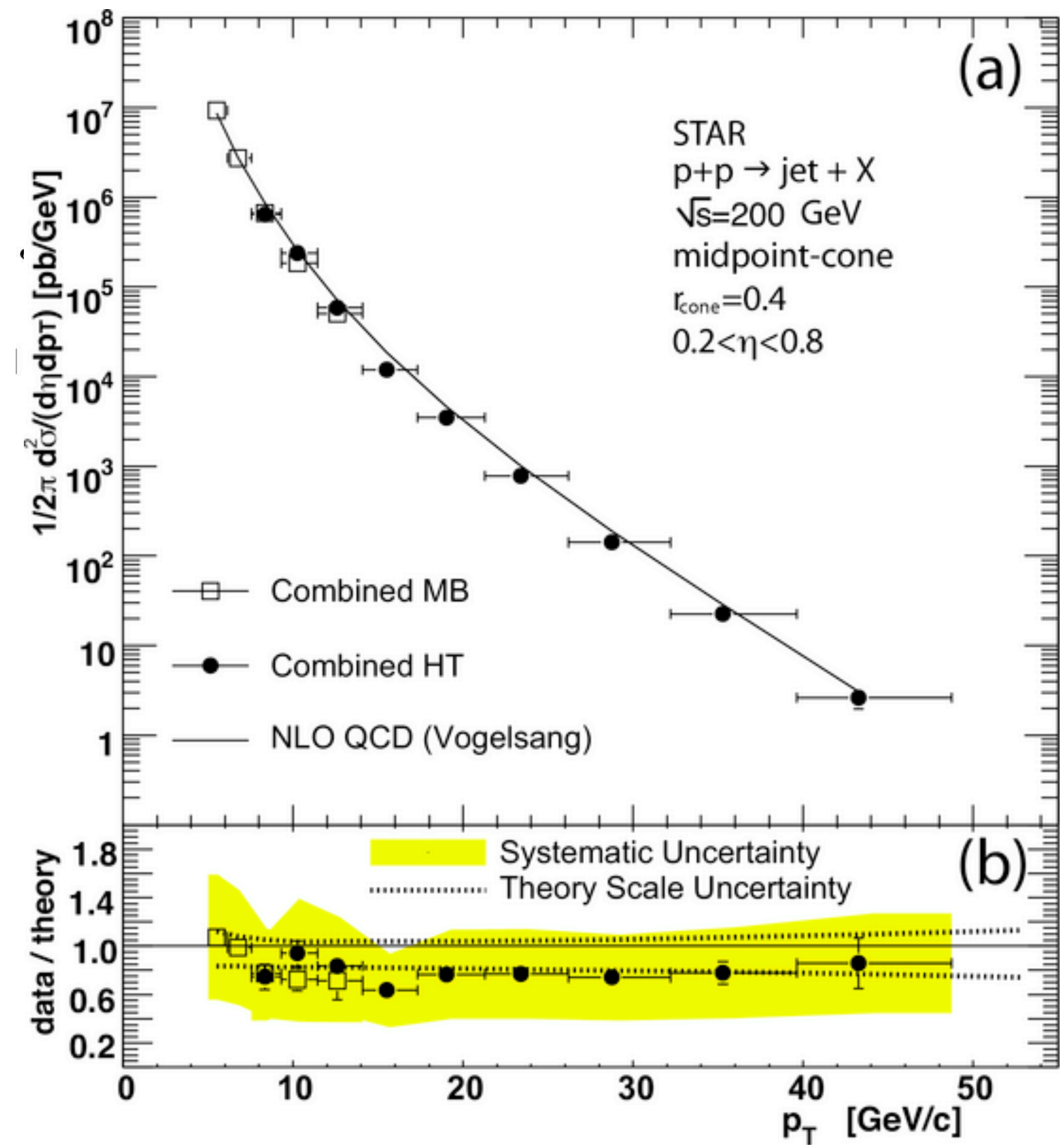
$$\otimes$$

$$\mathbf{f_{q/P}}$$

Success of QCD factorization

- Universality of PDFs: mapped in one process (say DIS), used in other processes





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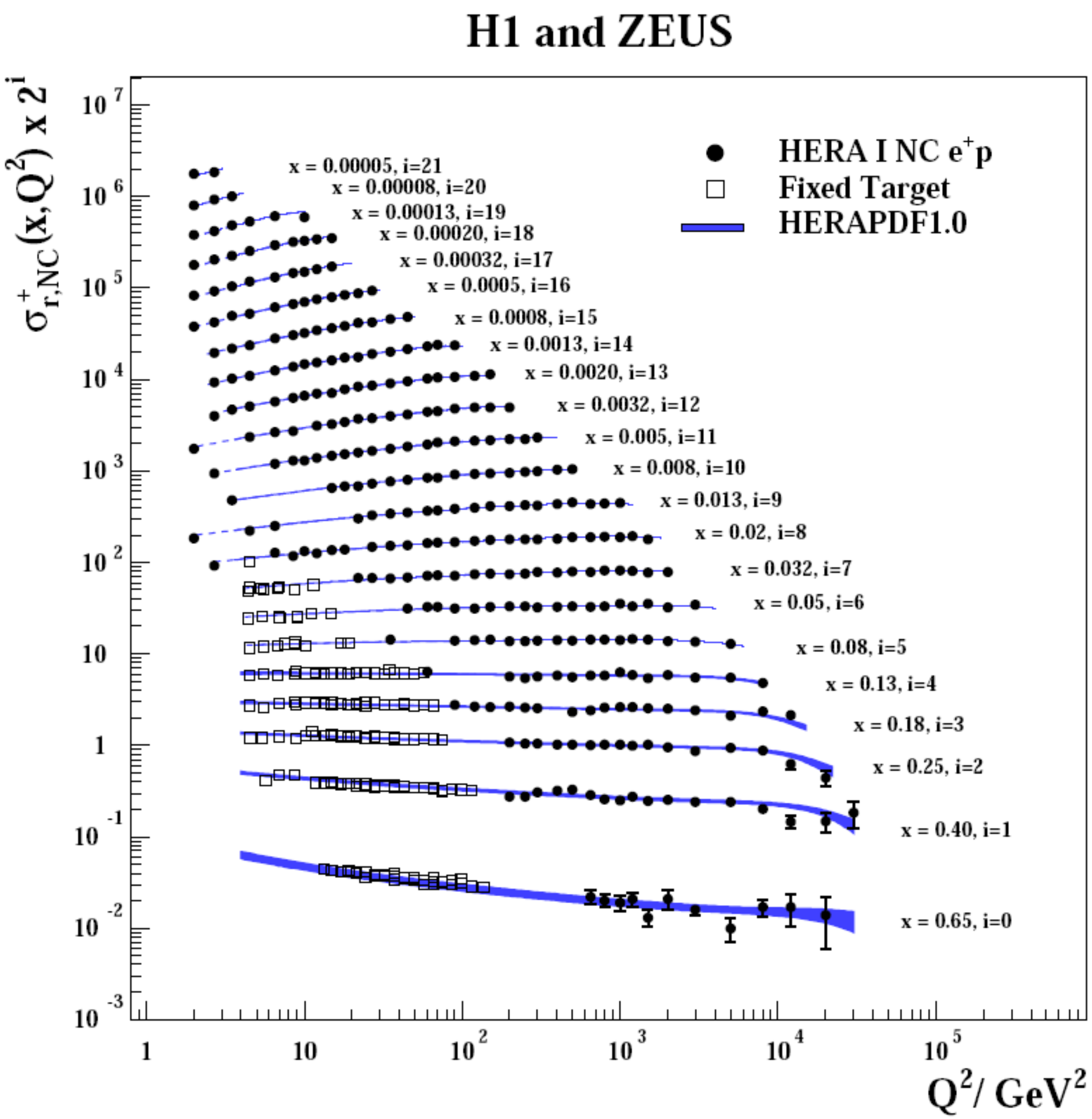

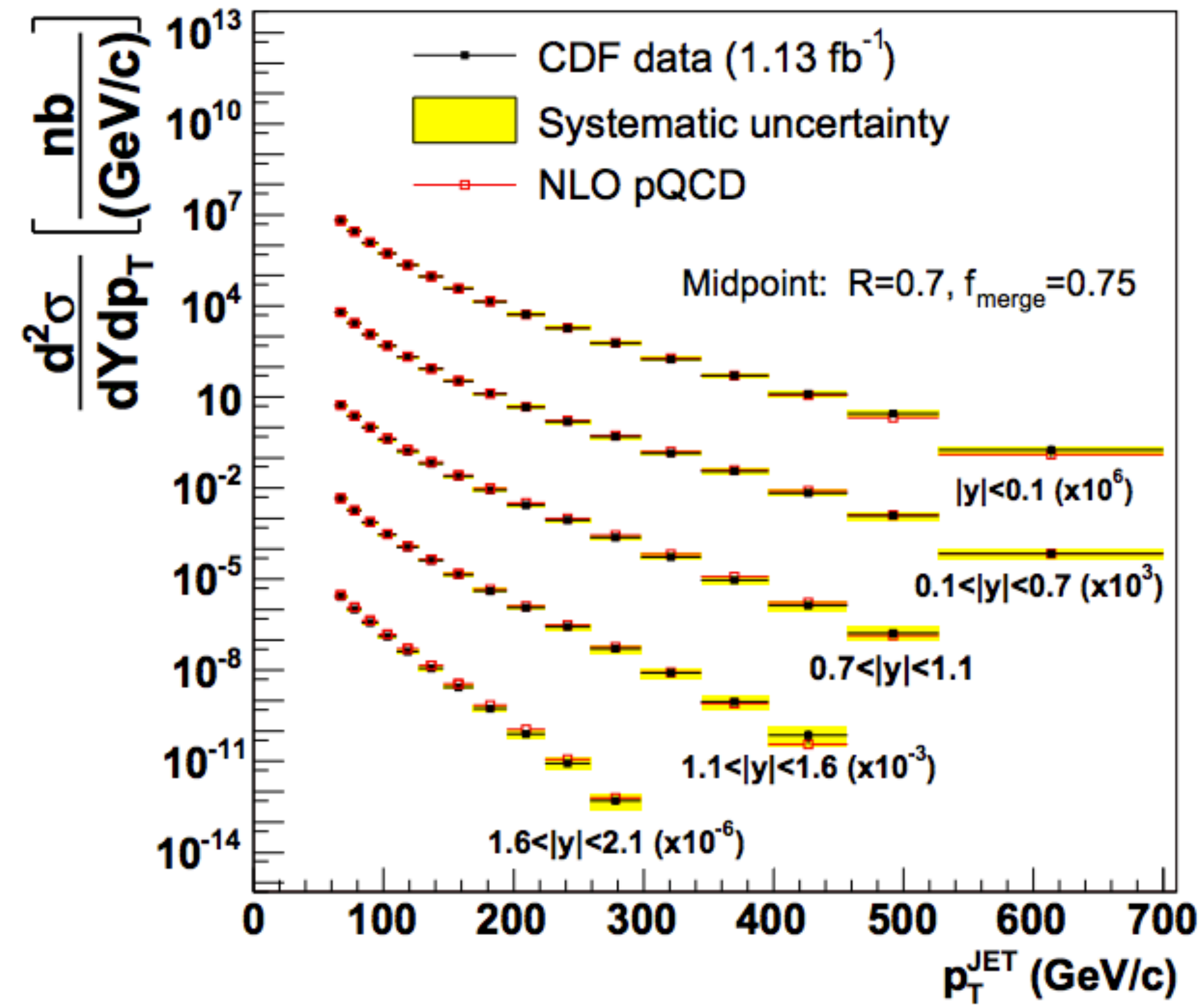

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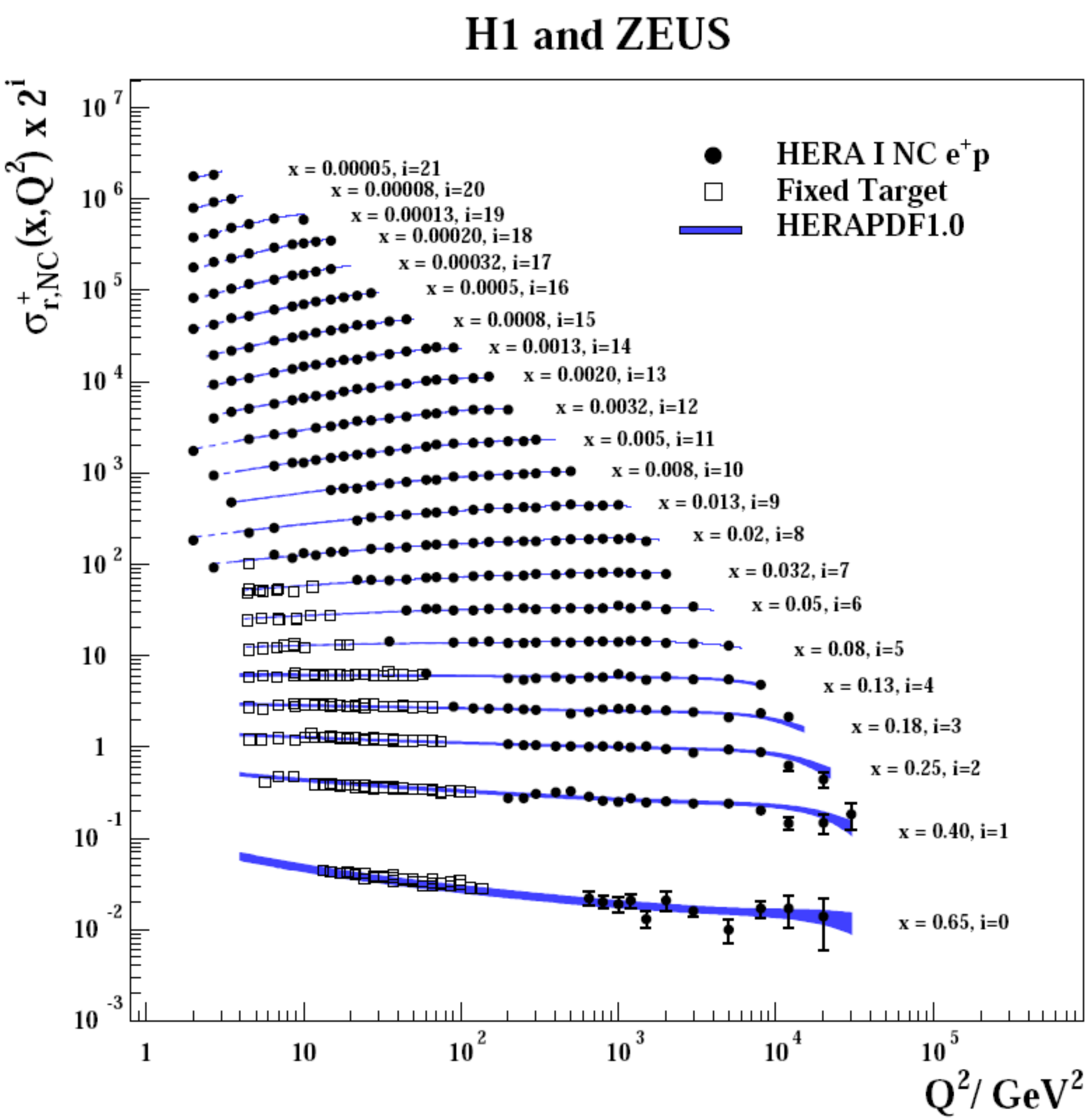

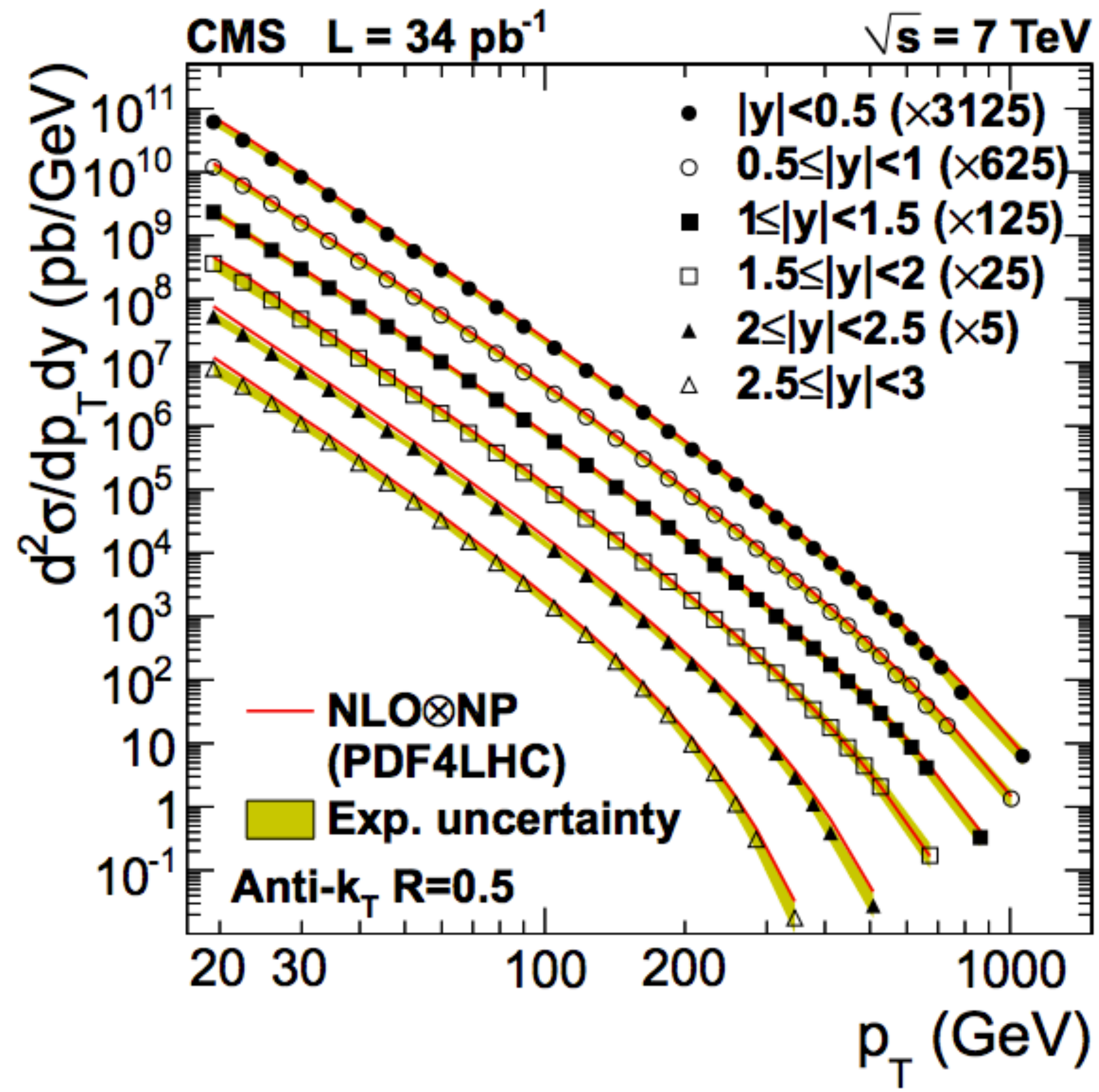

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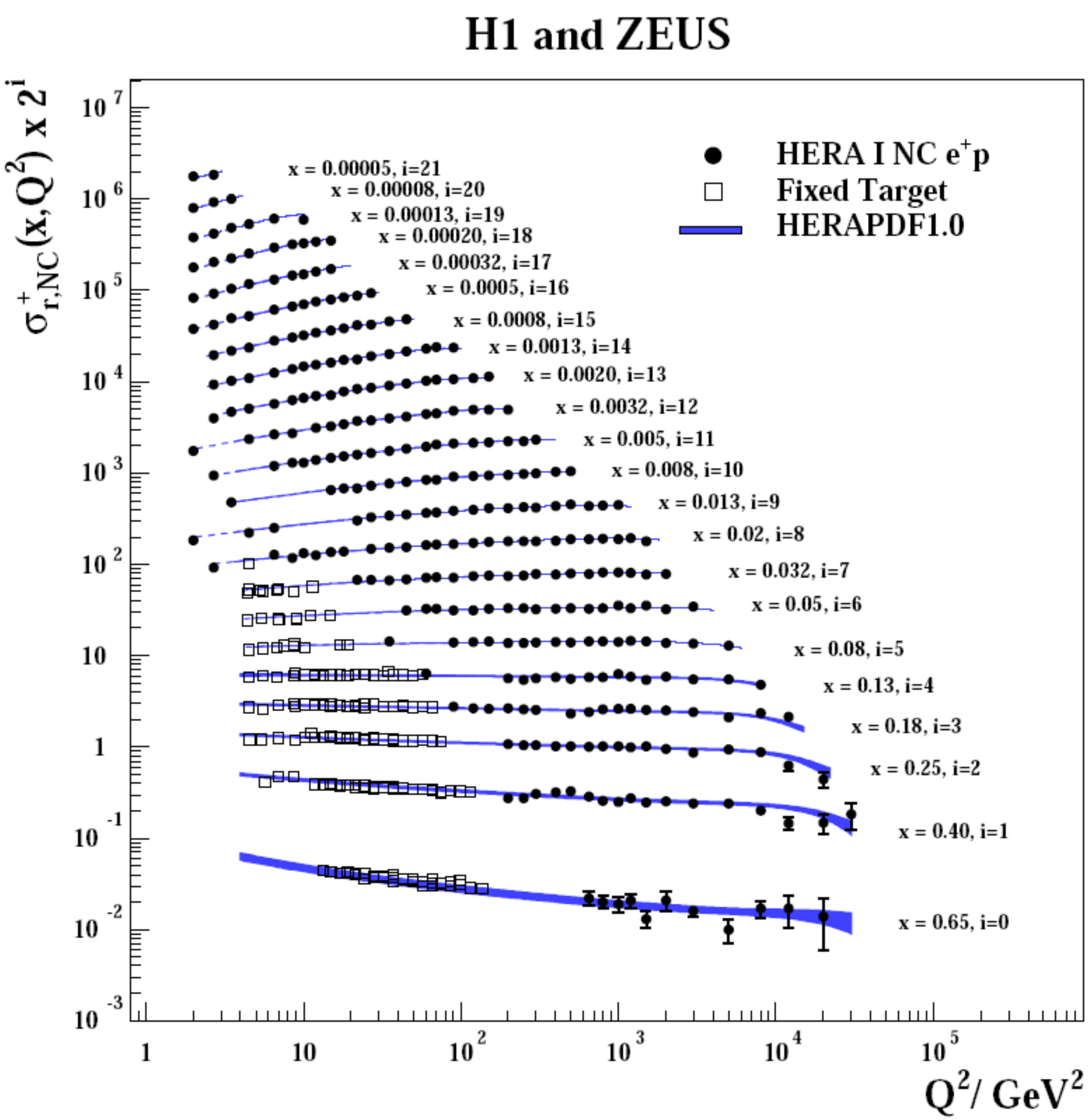
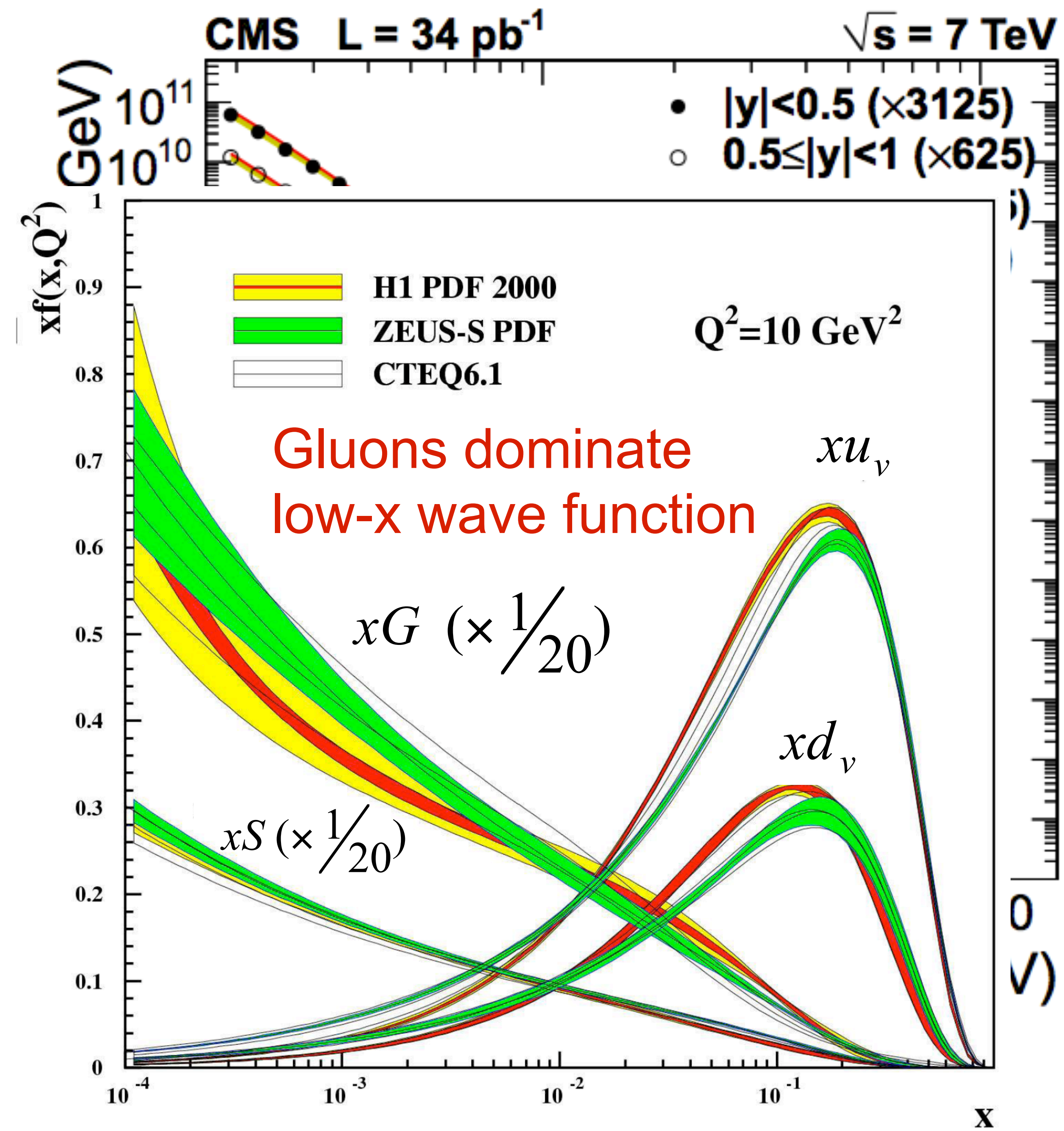




Transverse structure: Momentum vs Position

Variables are related by 2 dimensional Fourier transform

$$\bar{\tilde{\psi}}(k_\perp, z^-) = \int d^2 z_\perp e^{-iz_\perp k_\perp} \bar{\psi}(z_\perp, z^-)$$

At the level of squared amplitudes one has

$$\bar{\tilde{\psi}}(k_\perp)\tilde{\psi}(l_\perp) = \int d^2 z_\perp d^2 y_\perp e^{-i(z_\perp k_\perp - y_\perp l_\perp)} \bar{\psi}(z_\perp)\psi(y_\perp)$$

$$z_\perp k_\perp - y_\perp l_\perp = \frac{1}{2}(z_\perp - y_\perp)(k_\perp + l_\perp) + \frac{1}{2}(z_\perp + y_\perp)(k_\perp - l_\perp)$$

The 'average' transverse momentum is Fourier conjugate to position difference (TMD)

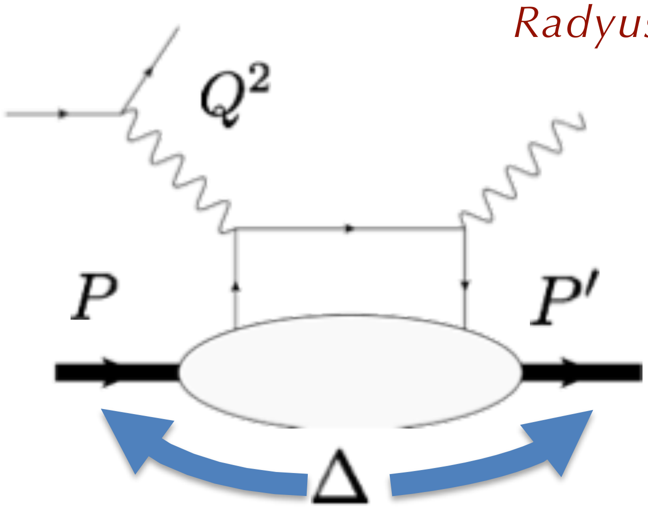
The momentum transfer is Fourier conjugate to 'average' position (GPD)

GPDs TMDs

DVCS

Ji (1997)

Radyushkin (1997)



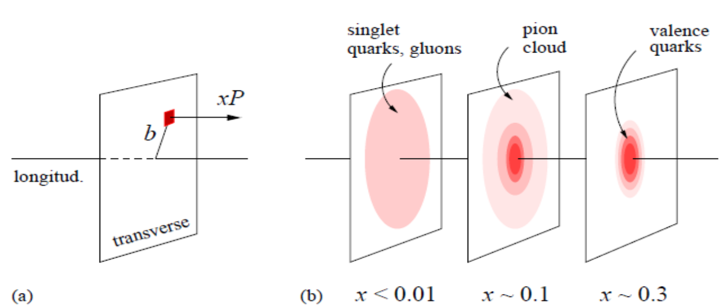
Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

Connection to 3D structure Burkardt (2000) Burkardt (2003)

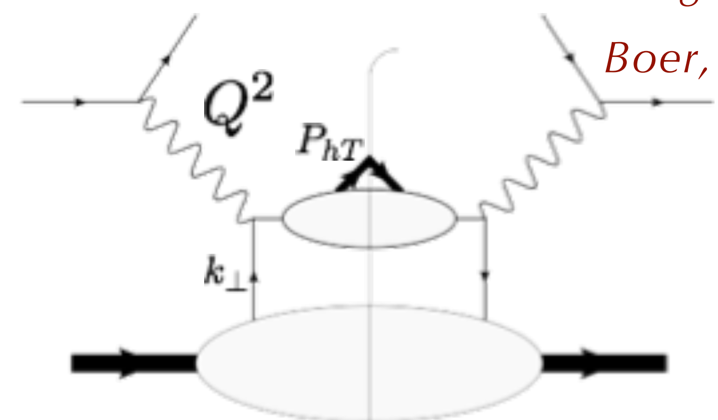
$$\rho(x,\vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{r}_\perp} H_q(x,\xi=0,t=-\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$ Weiss (2009)



SIDIS

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



Imaginary part, momentum transfer is zero

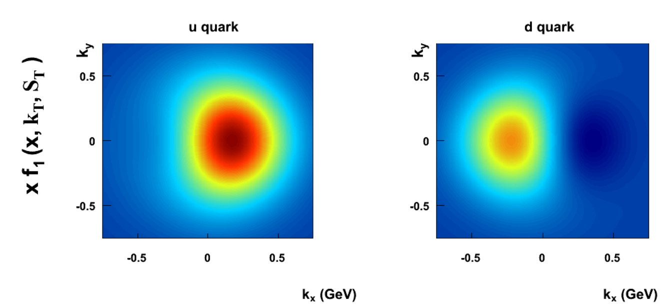
Q^2 ensures hard scale, pointlike interaction

P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure Ji, Ma, Yuan (2004) Collins (2011)

$$\tilde{f}(x,\vec{b}) = \int d^2k_\perp e^{-i\vec{b}\cdot\vec{k}_\perp} f(x,\vec{k}_\perp)$$

$\vec{b}$ is the transverse separation of parton fields in configuration space

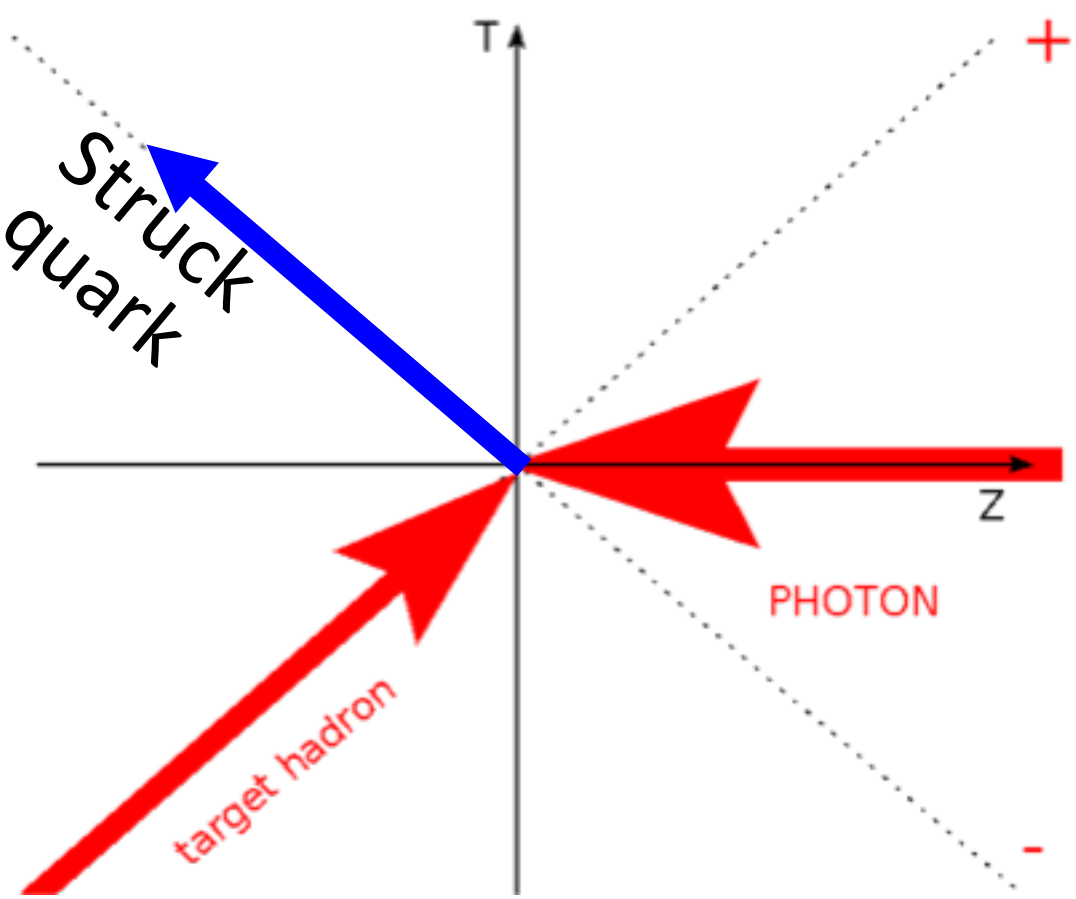


AP (2012)

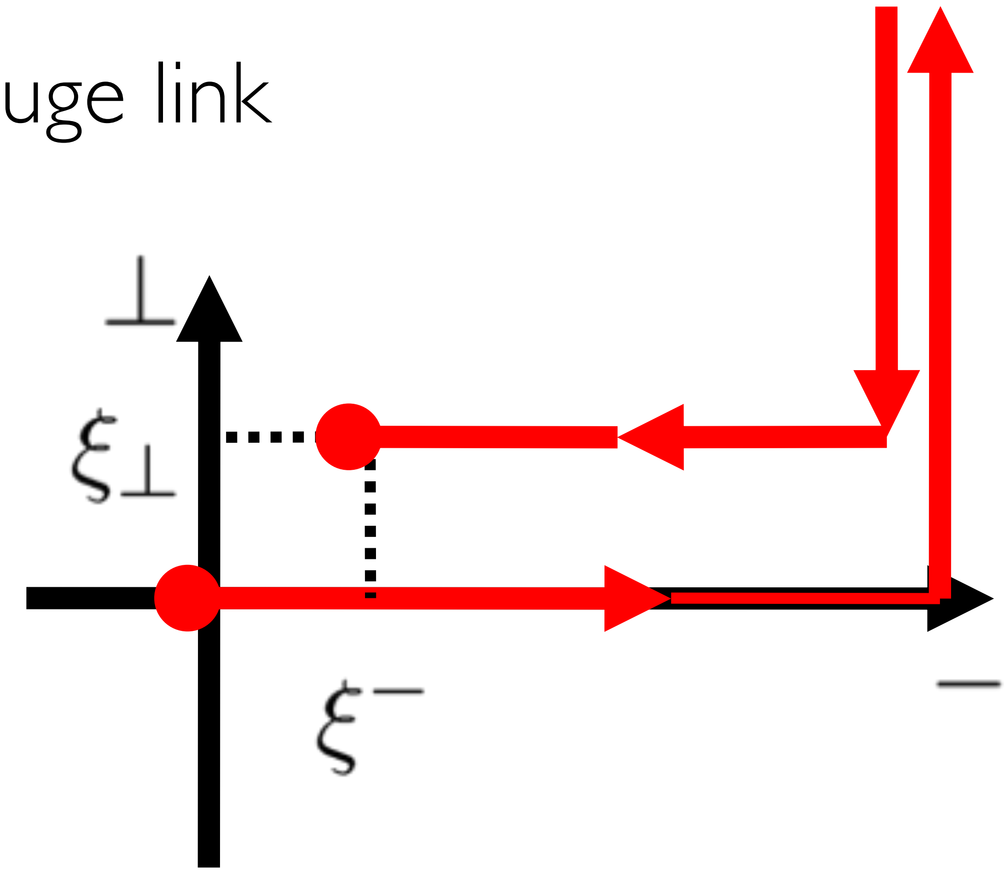
Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)}\frac{d^2\xi_\perp}{(2\pi)^2}\, e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp}\,\langle P,S_P|\bar\psi_j(0)\mathcal{U}(\mathbf{0},\xi)\psi_i(\xi)|P,S_P\rangle|_{\xi^+=0}$$

SIDIS in IMF:



Gauge link



$$\mathcal{U}(a,b;n) = e^{-ig\int_a^b d\lambda n\cdot A_\alpha(\lambda n)t_\alpha}$$

Ensures gauge invariance of the distribution, cannot be canceled by gauge choice

Transverse Momentum Dependent distributions

Individual TMDs can be projected out of the correlator

Unpolarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^{+}\;\Phi(x,k_\perp)\right] = f_1 - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M_N}\,f_{1T}^\perp$$

Longitudinally polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^{+}\gamma_5\;\Phi(x,k_\perp)\right] = S_L\,g_1 + \frac{k_\perp\cdot S_T}{M_N}\,g_{1T}^\perp$$

Transversely polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[i\sigma^{j+}\gamma^{+}\;\Phi(x,k_\perp)\right] = S_T^j\,h_1 + S_L\,\frac{k_\perp^j}{M_N}\,h_{1L}^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,h_{1T}^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,h_1^\perp$$

$$\kappa^{jk} \equiv \left(k_\perp^j k_\perp^k - \frac{1}{2}\,k_\perp^{\,2}\delta^{jk}\right)$$

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