Nuclear Astrophysics
Lecture 2: Nucleosynthesis

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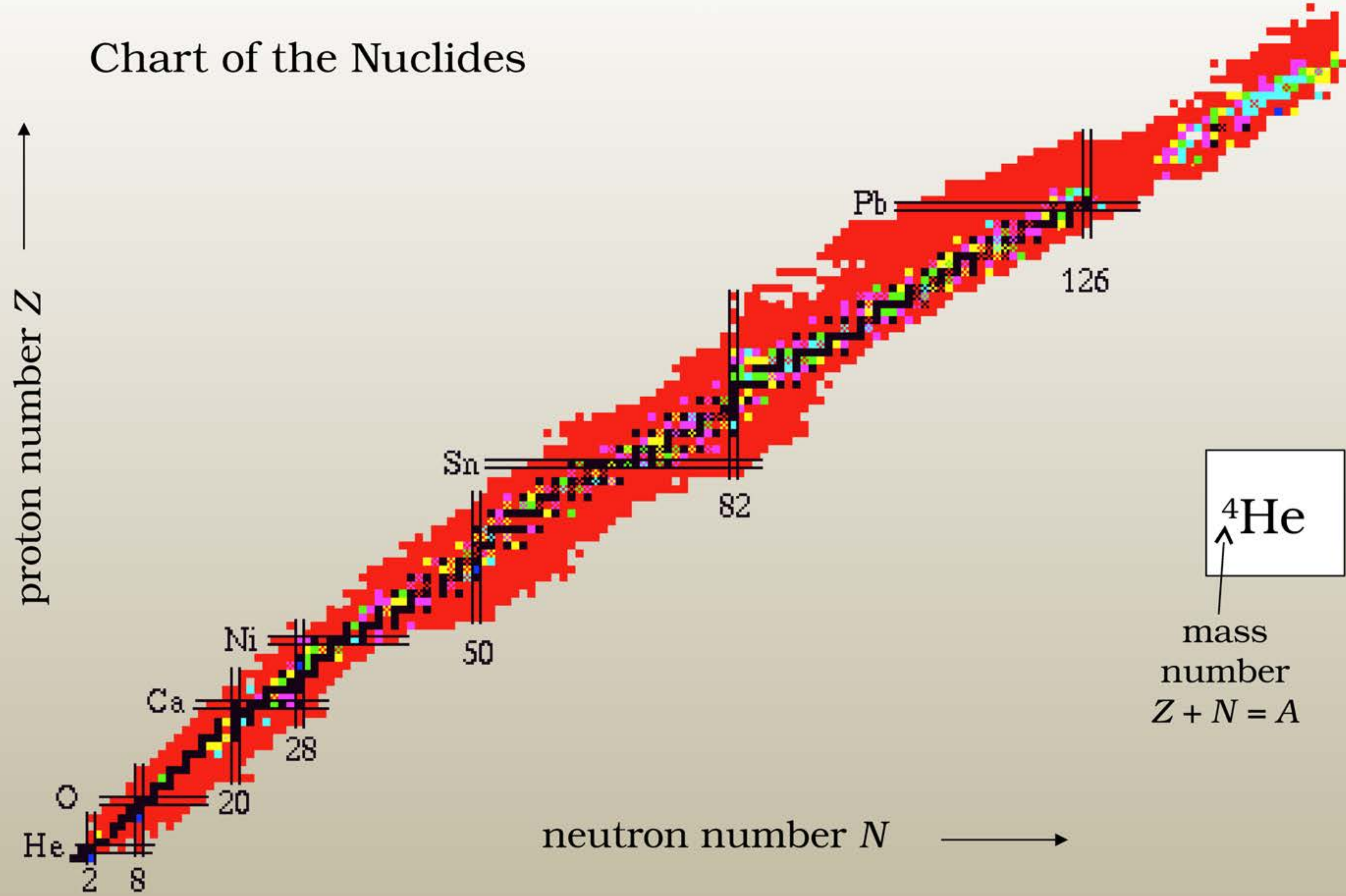
North Carolina State University

Objectives of Lecture 2

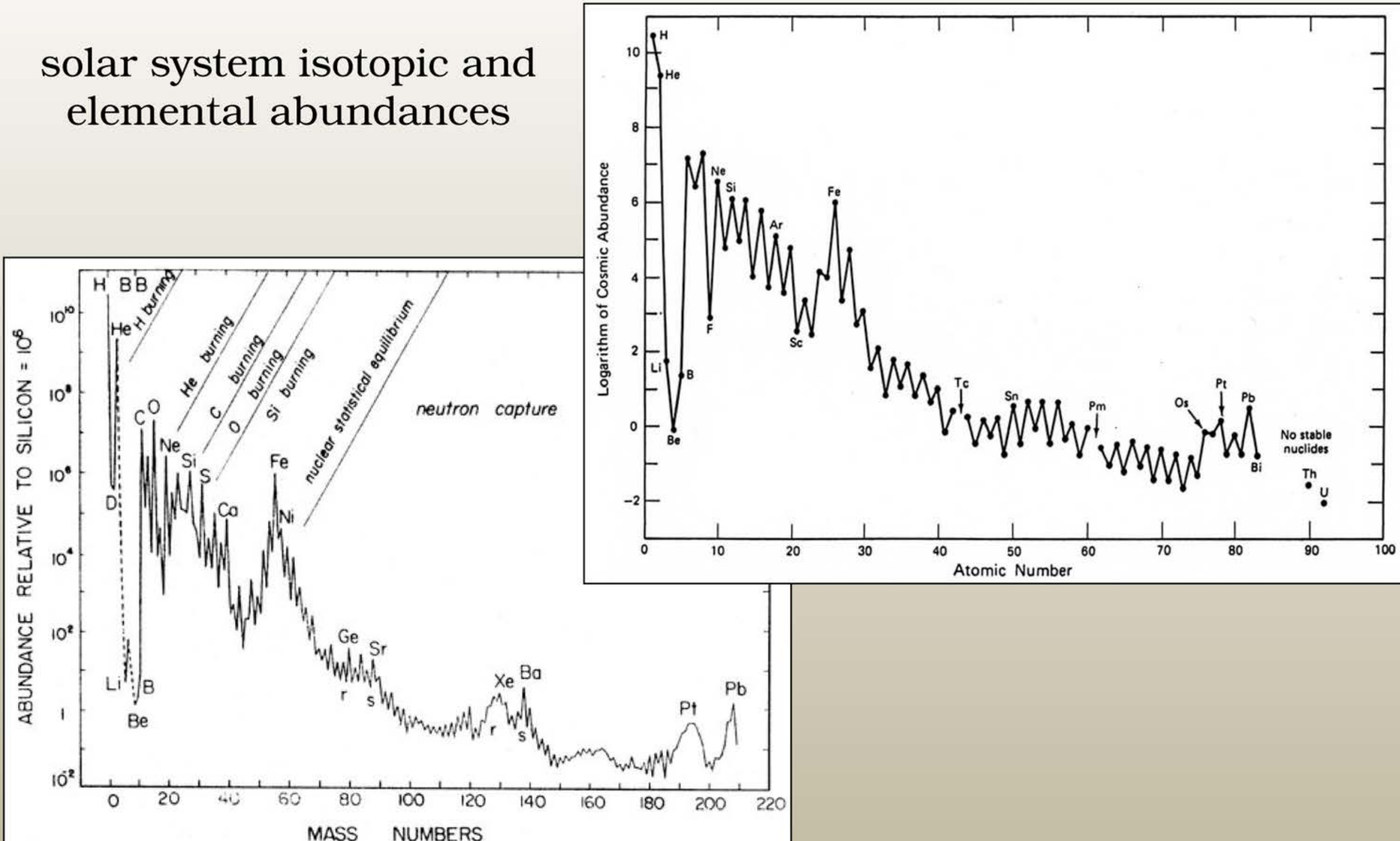
- Understand types of nucleosynthesis
- Understand where various types are made
- Be able to write down equations for a reaction network

the astrophysical formation of the elements



the astrophysical formation of the elements

solar system isotopic and elemental abundances



http://nedwww.ipac.caltech.edu/level5/Pagel/Figures/figure1_4.jpeg

some terminology

n_j number of species j per unit volume

W_j atomic weight (or molar mass) of species j

ρ_m mass density

$$\rho_m = \frac{\sum_j n_j W_j}{N_A}$$

ρ baryon mass density

$$\rho = \frac{\sum_j n_j A_j}{N_A}$$

X_j nucleon fraction, or mass fraction

$$X_j = \frac{n_j A_j}{\rho N_A}$$

Y_j mole fraction, or abundance

$$Y_j = \frac{X_j}{A_j} = \frac{n_j}{\rho N_A}$$

Note $\sum_j X_j = \sum_j \frac{n_j A_j}{\rho N_A} = \frac{1}{\rho} \frac{\sum_j n_j A_j}{N_A} = 1$

some terminology

y_j rescaled abundances

In meteoritics, abundances are normally scaled relative to silicon

(set number of silicon atoms to be 10^6):

$\log y_j = \log f_{Si} + \log Y_i$

where f_{Si} is the appropriate normalizing constant. For $Y_{Si} = 2.529\times10^{-5}$, $\log f_{Si} = 10.5970$.

Astronomers sometimes use a scale relative to hydrogen

(set number of hydrogen atoms to be 10^{12}):

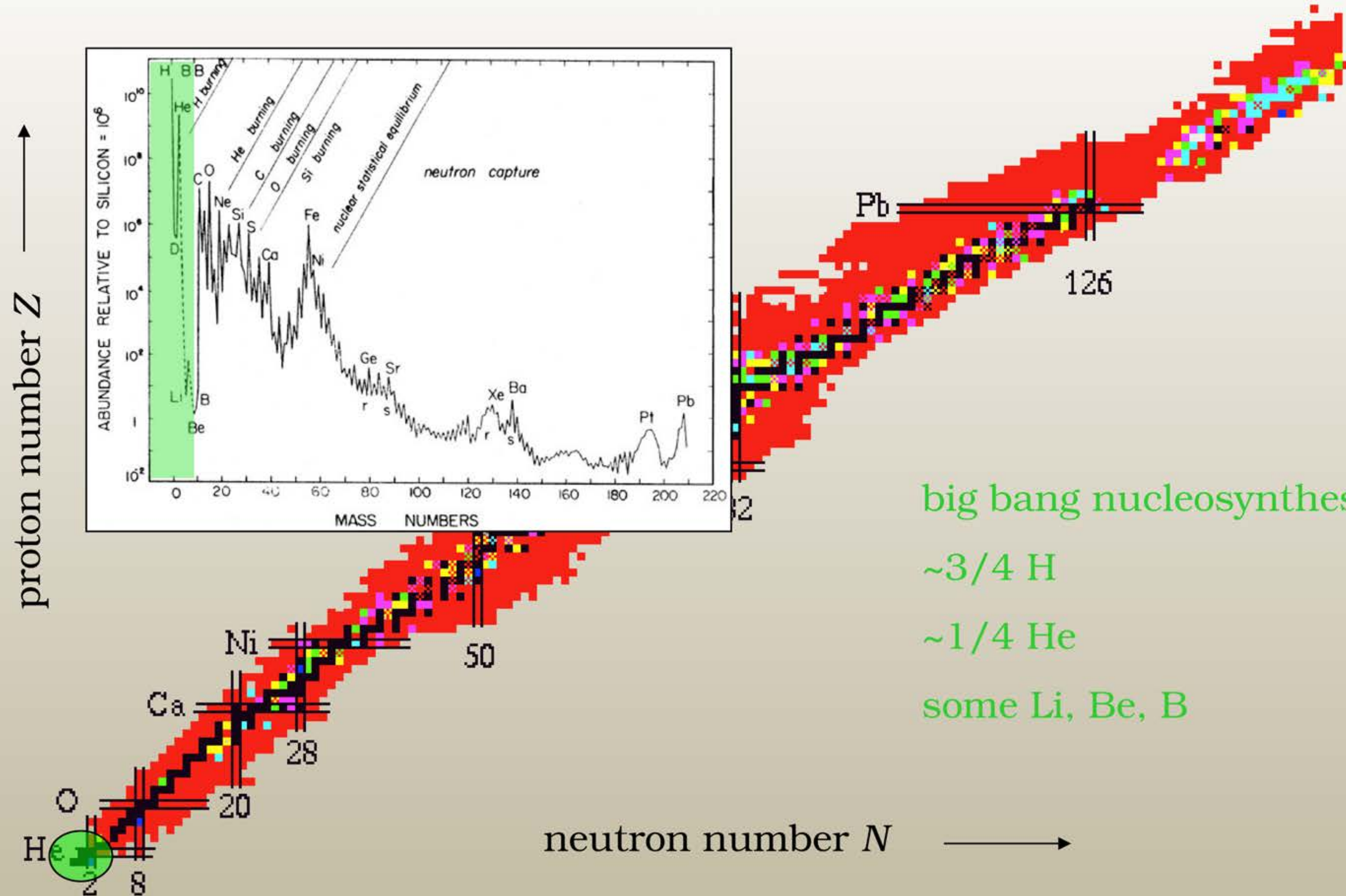
$\log y_j = \log f_H + \log Y_i$

More commonly, astronomers express abundances as ratios relative to solar:

$\log[n_i / n_j]_{\text{star}} - \log[n_i / n_j]_{\text{solar}} \equiv [i / j]$

For example, a ratio of sodium to iron which is half solar would be [Na/Fe] = -0.3.

the astrophysical formation of the elements



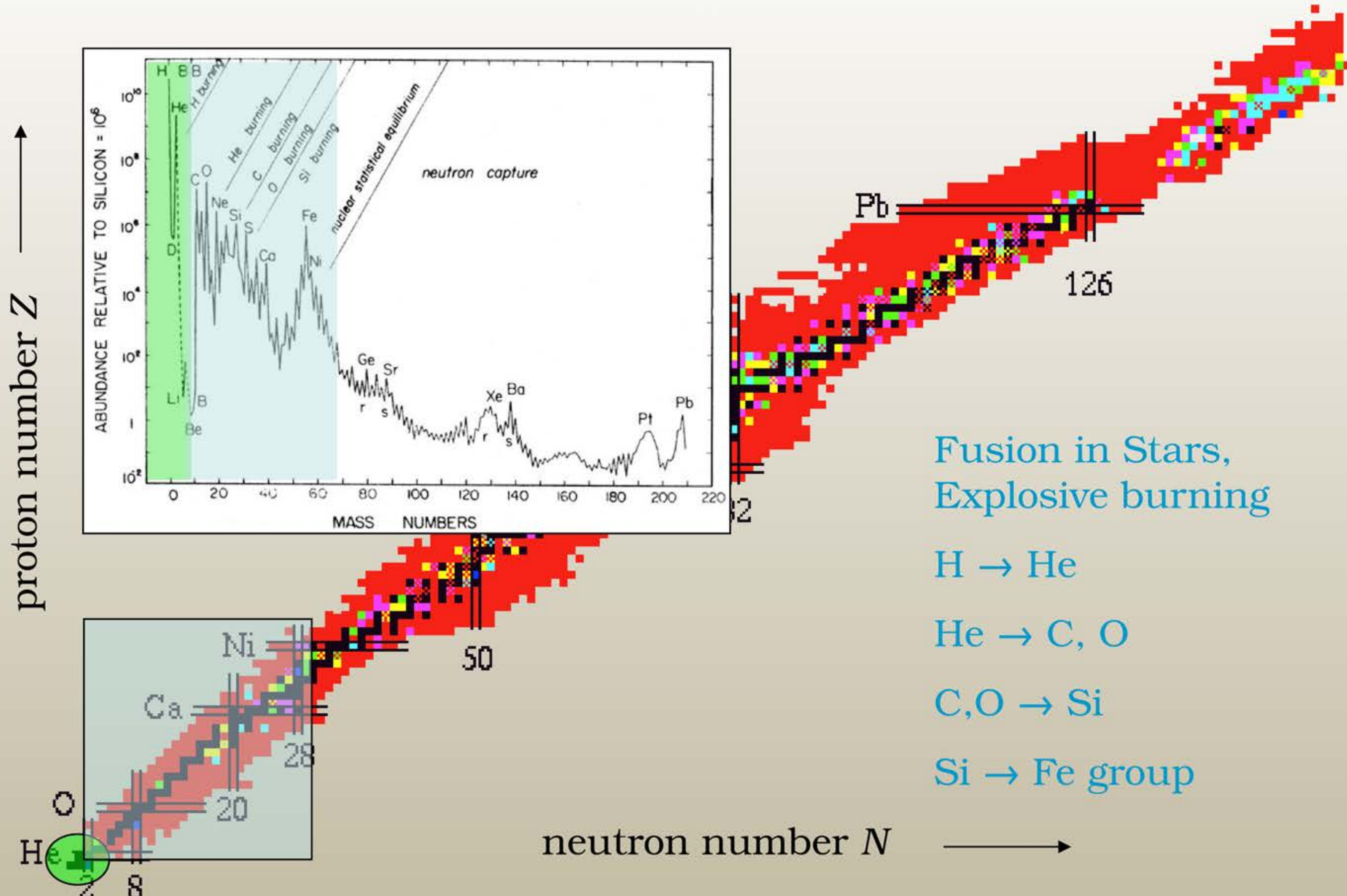
big bang nucleosynthesis

~3/4 H

~1/4 He

some Li, Be, B

the astrophysical formation of the elements



Fusion in Stars,
Explosive burning

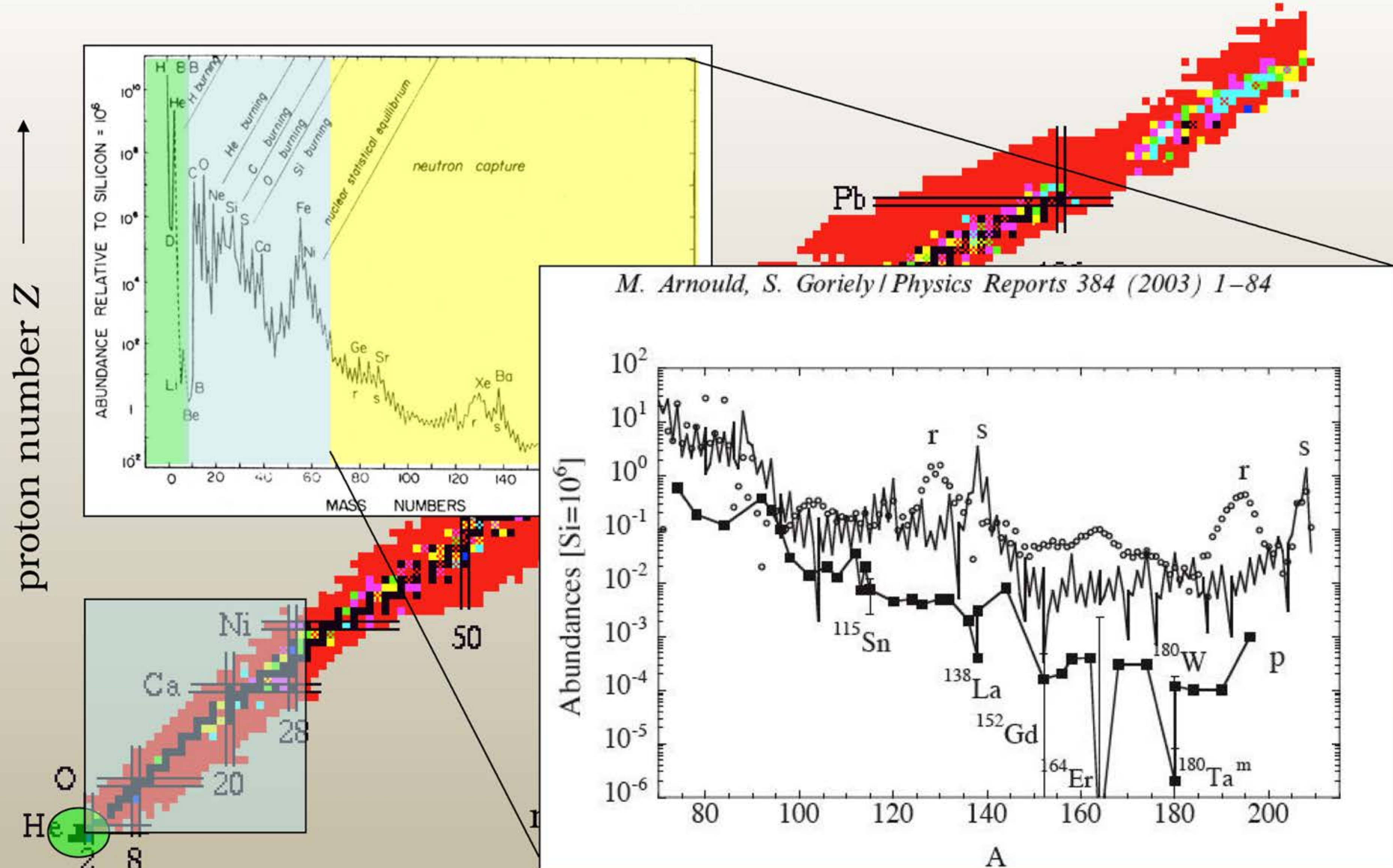
H → He

He → C, O

C,O → Si

Si → Fe group

the astrophysical formation of the elements



How to determine what elements an astrophysical environment produces

What information is needed?

How to determine what elements an astrophysical environment produces

My list:

- astrophysical conditions, i.e. temperature and density
- nuclear physics input (masses, reaction rates, decay rates)

How to determine what elements an astrophysical environment produces

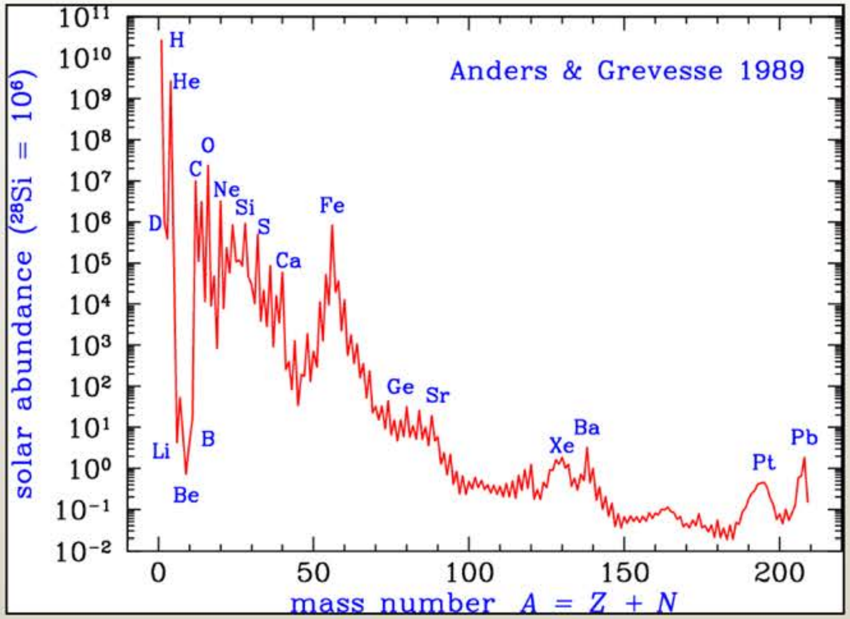
What data can you compare with?

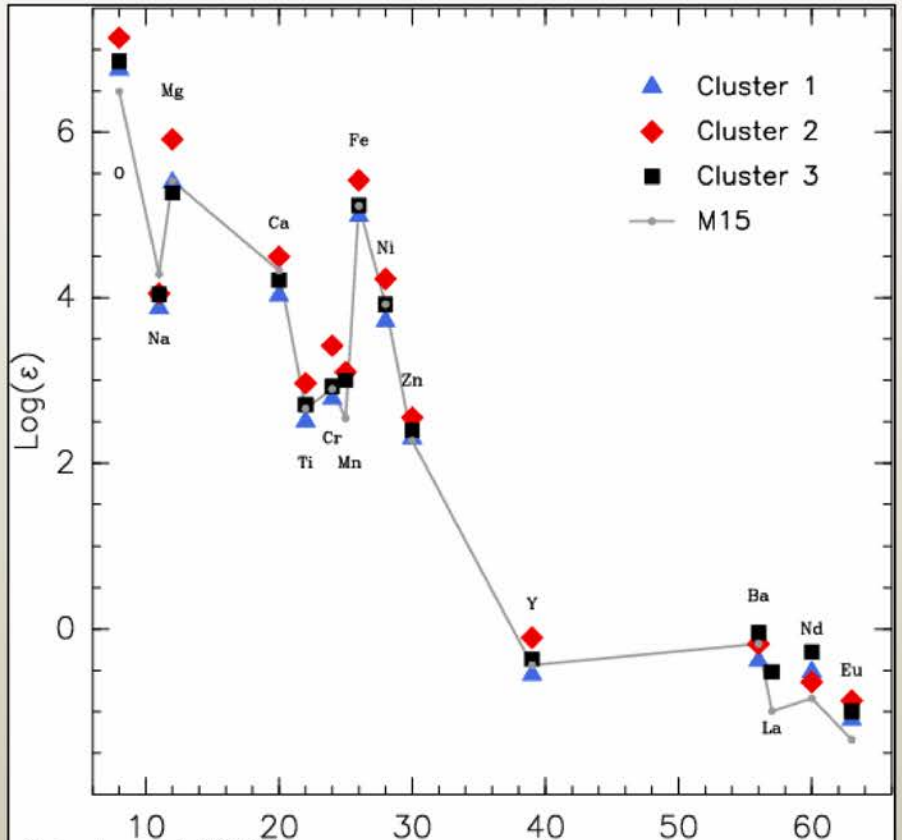
How to determine what elements an astrophysical environment produces

My list:

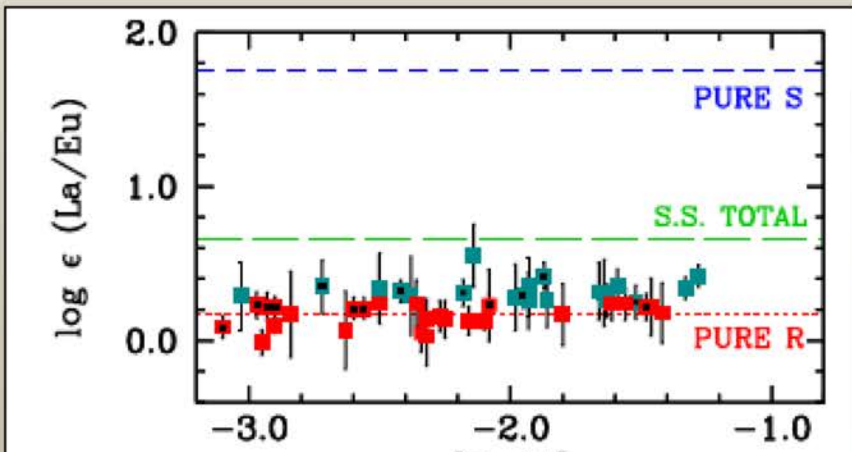
- solar system abundances (sun, earth)
- meterorites
- spectroscopic data from other stars

example abundance patterns





Letarte et al (2006)



Roederer et al (2009)

$$\log(\varepsilon_i) = \log_{10}(n_i / n_{\rm H}) + 12$$

some terminology

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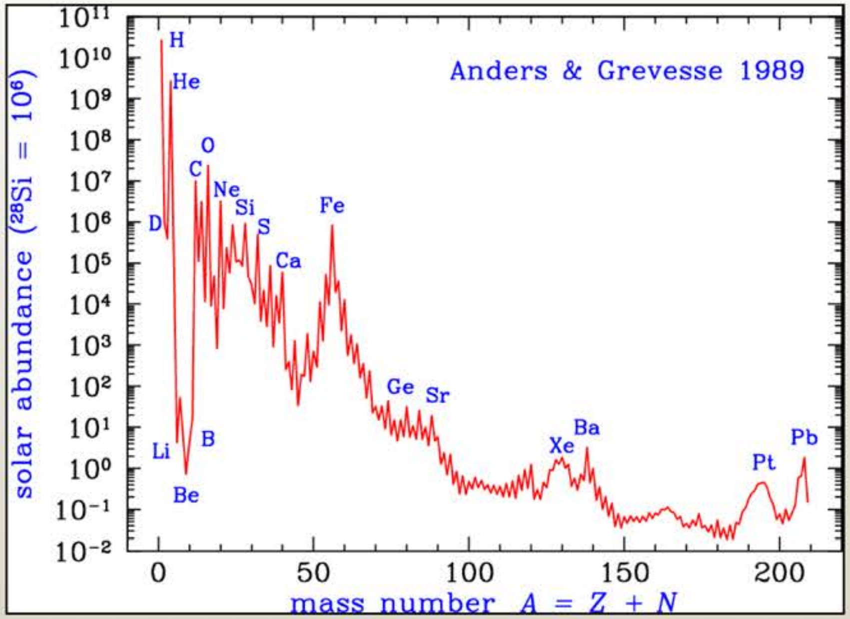
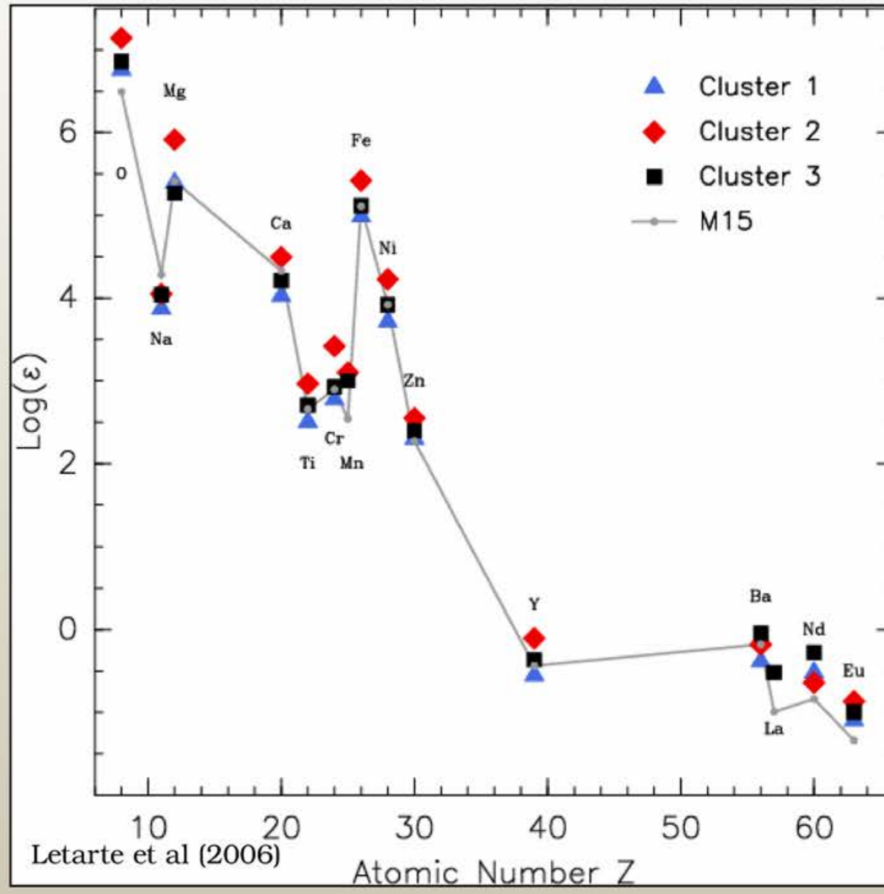
example abundance patterns





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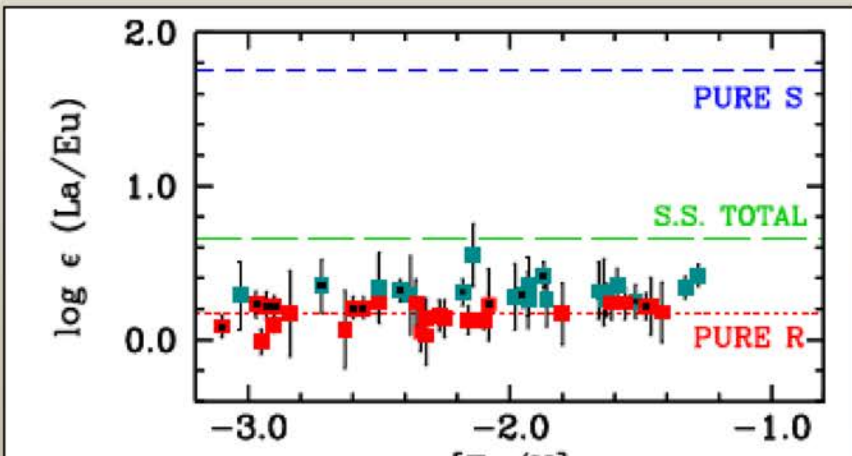
$$\log(\varepsilon_i) = \log_{10}\left(n_i / n_{\mathrm{H}}\right) + 12$$



Roederer et al (2009)

the astrophysical formation of the elements

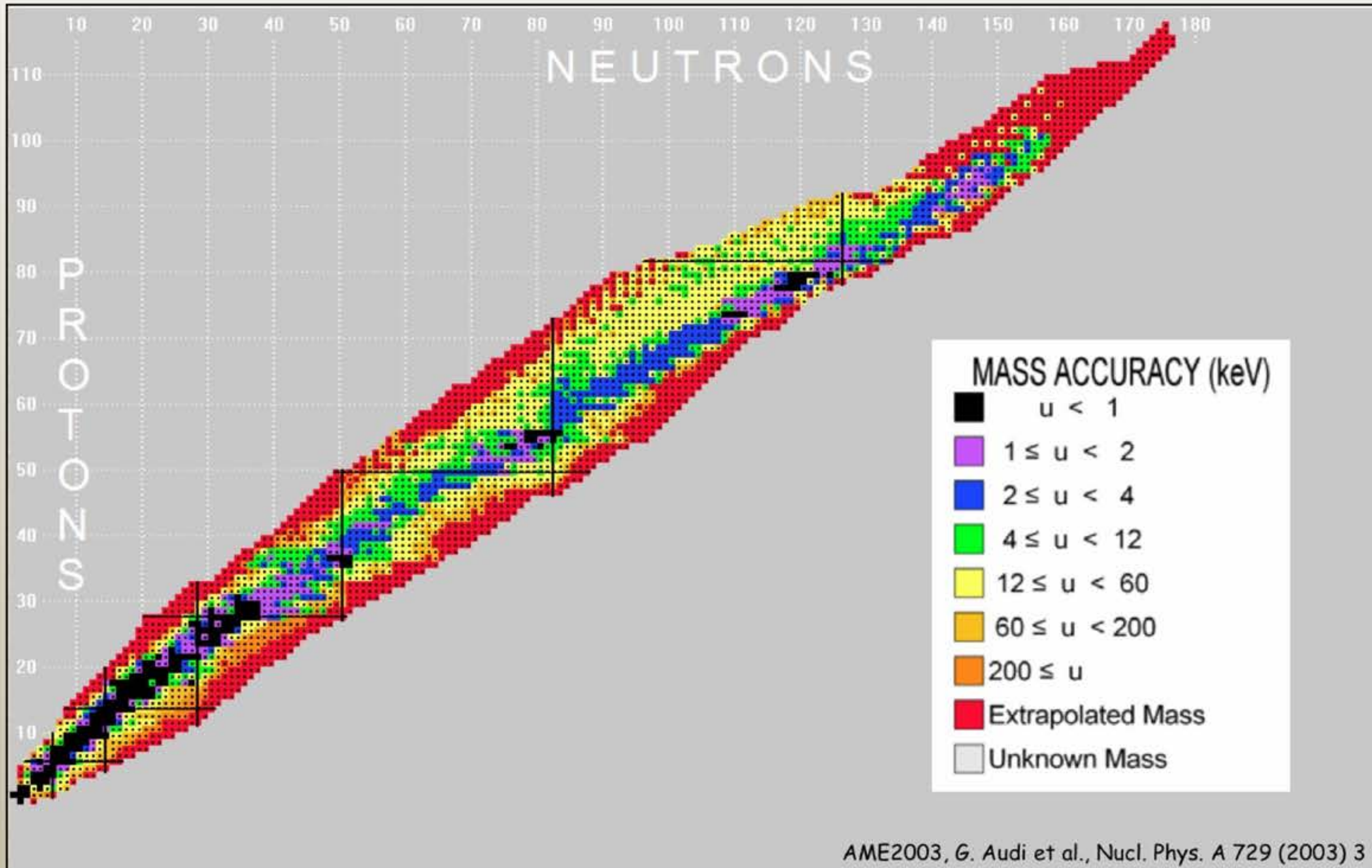
nuclear properties ↘

nuclear reaction rates → **nuclear network code** → evolution of nuclear abundances

astrophysical conditions ↗

building a nuclear network code: nuclear properties

atomic/nuclear masses



building a nuclear network code: nuclear properties

radioactive decay rates $N(t) = N_0 e^{-\lambda t}$

λ decay constant

τ mean lifetime, $\tau = 1/\lambda$

$T_{1/2}$ half - life, $T_{1/2} = \ln 2/\lambda$

How determined?

Fermi's 'Golden Rule'

$$rate = \frac{2\pi}{\hbar} \left| \langle f | H_{\text{int}} | i \rangle \right|^2 \rho(E)$$

f, i final and initial state wavefunctions

H_{int} weak interaction Hamiltonian

$\rho(E)$ density of states for the final particles

building a nuclear network code: reaction rates

cross section for the reaction $i + j \rightarrow k + l$

$$\sigma_{ij}(v) = \frac{\text{number of reactions per nucleus } i \text{ per second}}{\text{flux of incoming projectiles } j}$$

$$\sigma_{ij}(v) = \frac{r_{ij} / n_i}{n_j v_{ij}}$$

r_{ij} number of interactions $i(j,k)l$ per second

v_{ij} relative velocity of particles i, j

So the reaction rate per unit volume is just:

$$r_{ij} = n_i n_j v_{ij} \sigma_{ij}(v)$$

building a nuclear network code: reaction rates

In astrophysical environments the relative velocity v_{ij} is not constant, but instead there exists a distribution of relative velocities, which can be described by the probability function *P(v)*, where:

$$\int_0^\infty P(v)dv = 1$$

So the reaction rate can be generalized to:

$$r_{ij} = n_i n_j \int_0^\infty v P(v) \sigma_{ij}(v) dv$$

$$r_{ij} = n_i n_j \langle \sigma v \rangle_{ij}$$

building a nuclear network code: reaction rates

If the nuclei are nonrelativistic and nondegenerate, their velocities can be described by a Maxwell-Boltzmann distribution

$$P(v)dv = \left(\frac{m_{ij}}{2\pi kT}\right)^{3/2} e^{-m_{ij}v^2/2kT} 4\pi v^2 dv$$

where :

m_{ij} reduced mass, $m_{ij} = m_i m_j/(m_i + m_j)$

T temperature

k Boltzmann constant, $k = 8.6173\times10^{-5}$ eV/K

The velocity distribution can be written as an energy distribution, since $E = m_{ij}v^2/2$

$$P(v)dv = P(E)dE = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\sqrt{E}e^{-E/kT}dE$$

building a nuclear network code

Now consider the rate of change in the number density of species *j*:

$$\frac{dn_j}{dt} = n_k n_l \langle \sigma v \rangle_{kl,j} - n_j n_l \langle \sigma v \rangle_{jl,n} + n_i \lambda_{i,j} - n_j \lambda_{j,m} + \ldots$$

Note for reactions involving identical particles, a term of the form:

$$\frac{n_i^2}{2!} \langle \sigma v \rangle_{ii,j} \text{(two body)} \qquad \text{or} \qquad \frac{n_i^3}{3!} \langle \sigma v \rangle_{iii,j} \text{(three body)}$$

is needed.

The above can be written in terms of abundances as:

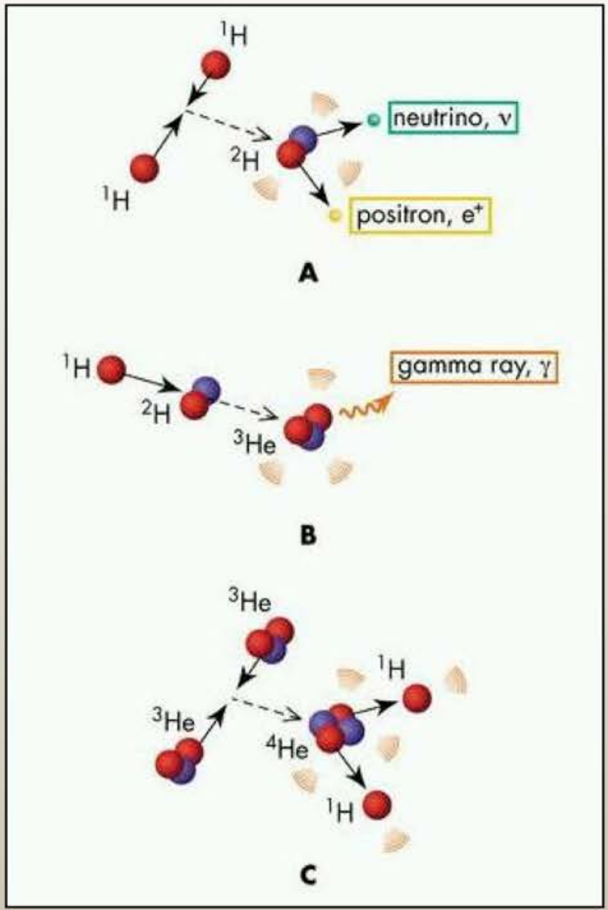
$$\frac{dY_j}{dt} = Y_k Y_l \boxed{\rho N_A \langle \sigma v \rangle_{kl,j}} - Y_j Y_l \rho N_A \langle \sigma v \rangle_{jl,n} + Y_i \lambda_{i,j} - Y_j \lambda_{j,m} + \ldots$$

this is often what we call the reaction rate in a network code

the basic proton-proton chain (PPI)

Conversion of ^{1}H to ^{4}He via:

$$^{1}H + ^{1}H \rightarrow ^{2}H + e^{+} + \nu_{e}$$
$$^{2}H + ^{1}H \rightarrow ^{3}He + \gamma$$
$$^{3}He + ^{3}He \rightarrow ^{4}He + ^{1}H + ^{1}H$$



PPI network equations

Recall the abundances evolve as

$$\frac{dY_j}{dt} = Y_k Y_l \rho N_A \langle \sigma v \rangle_{kl,j} - Y_j Y_l \rho N_A \langle \sigma v \rangle_{jl,n} + Y_i \lambda_{i,j} - Y_j \lambda_{j,m} + \ldots$$

So here we have a system of four differential equations:

$$\frac{dY_{\mathrm{H}}}{dt} = 2Y_{^3\mathrm{He}}{}^2 \rho N_A \langle \sigma v \rangle_{^3\mathrm{He}^3\mathrm{He},^4\mathrm{He}} - Y_{\mathrm{H}}{}^2 \rho N_A \langle \sigma v \rangle_{\mathrm{HH,D}} - Y_{\mathrm{D}} Y_{\mathrm{H}} \rho N_A \langle \sigma v \rangle_{\mathrm{DH},^3\mathrm{He}}$$

$$\frac{dY_{\mathrm{D}}}{dt} = \frac{1}{2} Y_{\mathrm{H}}{}^2 \rho N_A \langle \sigma v \rangle_{\mathrm{HH,D}} - Y_{\mathrm{D}} Y_{\mathrm{H}} \rho N_A \langle \sigma v \rangle_{\mathrm{DH},^3\mathrm{He}}$$

$$\frac{dY_{^3\mathrm{He}}}{dt} = Y_{\mathrm{D}} Y_{\mathrm{H}} \rho N_A \langle \sigma v \rangle_{\mathrm{DH},^3\mathrm{He}} - Y_{^3\mathrm{He}}{}^2 \rho N_A \langle \sigma v \rangle_{^3\mathrm{He}^3\mathrm{He},^4\mathrm{He}}$$

$$\frac{dY_{^4\mathrm{He}}}{dt} = \frac{1}{2} Y_{^3\mathrm{He}}{}^2 \rho N_A \langle \sigma v \rangle_{^3\mathrm{He}^3\mathrm{He},^4\mathrm{He}}$$

PPI network equations

But note first D, then ^{3}He will come into steady-state:

$$\frac{dY_{\mathrm{D}}}{dt} = \frac{1}{2} Y_{\mathrm{H}}{}^{2} \rho N_A \langle \sigma v \rangle_{\mathrm{HH,D}} - Y_{\mathrm{D}} Y_{\mathrm{H}} \rho N_A \langle \sigma v \rangle_{\mathrm{DH},{}^3\mathrm{He}} = 0, \text{ so}$$

$$\frac{Y_{\mathrm{D}}}{Y_{\mathrm{H}}} = \frac{\langle \sigma v \rangle_{\mathrm{HH,D}}}{2 \langle \sigma v \rangle_{\mathrm{DH},{}^3\mathrm{He}}} \sim 10^{-17}$$

$$\frac{dY_{{}^3\mathrm{He}}}{dt} = Y_{\mathrm{D}} Y_{\mathrm{H}} \rho N_A \langle \sigma v \rangle_{\mathrm{DH},{}^3\mathrm{He}} - Y_{{}^3\mathrm{He}}{}^{2} \rho N_A \langle \sigma v \rangle_{{}^3\mathrm{He}{}^3\mathrm{He},{}^4\mathrm{He}}$$

$$\frac{dY_{{}^3\mathrm{He}}}{dt} = \frac{1}{2} Y_{\mathrm{H}}{}^{2} \rho N_A \langle \sigma v \rangle_{\mathrm{HH,D}} - Y_{{}^3\mathrm{He}}{}^{2} \rho N_A \langle \sigma v \rangle_{{}^3\mathrm{He}{}^3\mathrm{He},{}^4\mathrm{He}} = 0$$

$$\frac{Y_{{}^3\mathrm{He}}}{Y_{\mathrm{H}}} = \sqrt{\frac{\langle \sigma v \rangle_{\mathrm{HH,D}}}{2 \langle \sigma v \rangle_{{}^3\mathrm{He}{}^3\mathrm{He},{}^4\mathrm{He}}}} \sim 10^{-5}$$

Higher T than the sun; nuclear statistical equilibrium

When strong and electromagnetic interactions come into equilibrium at high temperatures, the nuclear abundances are no longer sensitive to individual reaction rates and only depend on the temperature T, density ρ, and the neutron-richness of the composition.

Nuclear statistical equilibrium (NSE) abundances are given by:

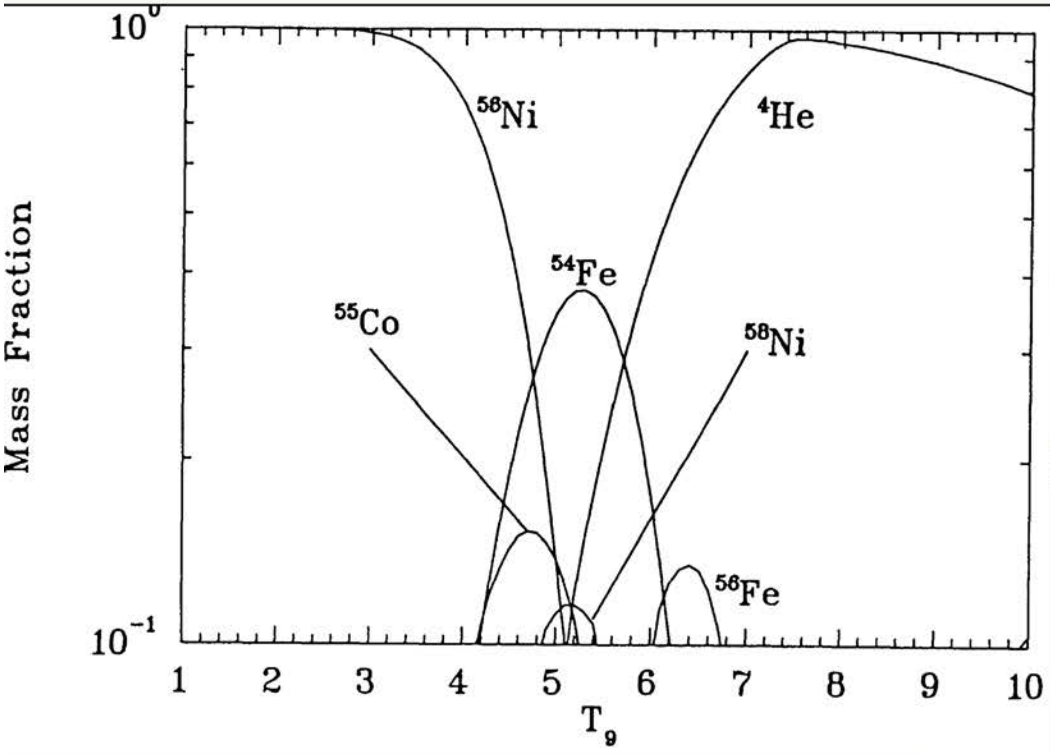
$$Y_i = (\rho N_A)^{A_i - 1} \frac{G_i}{2^{A_i}} A_i^{3/2} \left(\frac{2\pi\hbar^2}{m_u kT} \right)^{\frac{3}{2}(A_i - 1)} \exp\left[\frac{B_i}{kT} \right] Y_p^{Z_i} Y_n^{N_i}$$

where G_i is the nuclear partition function and B_i is the binding energy.

We also require mass and charge conservation :

$$\sum_i Y_i A_i = 1 \qquad \sum_i Y_i Z_i = \underset{\substack{\uparrow \\ \text{electron fraction}}}{Y_e}$$

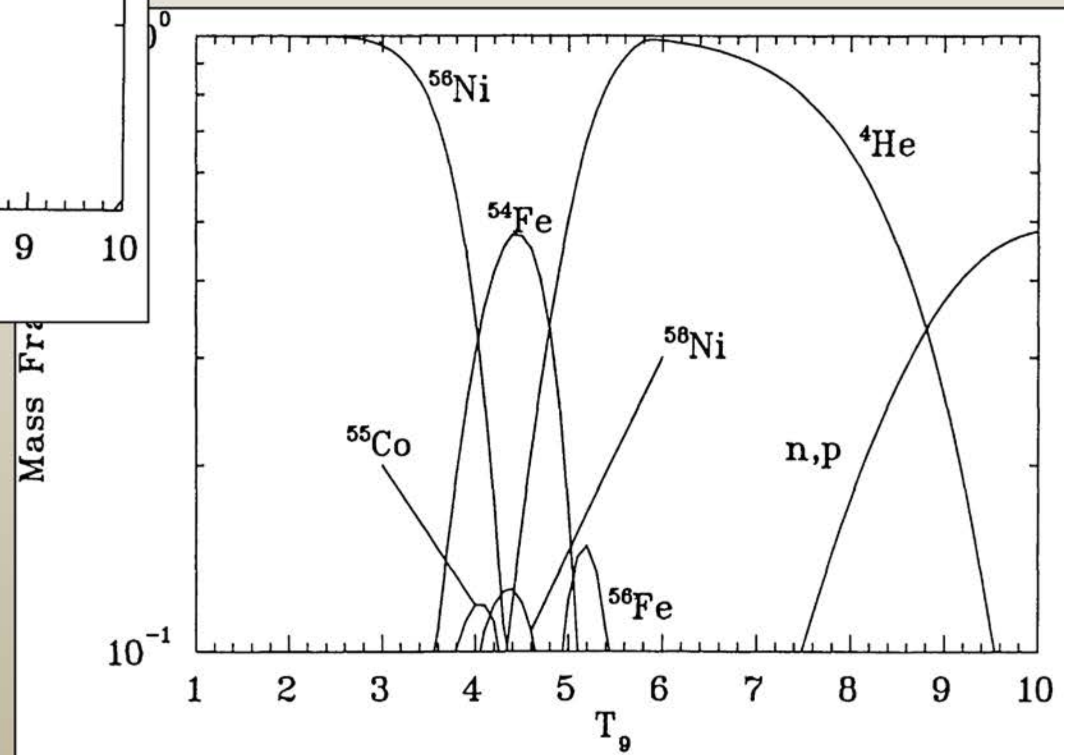
sample NSE compositions



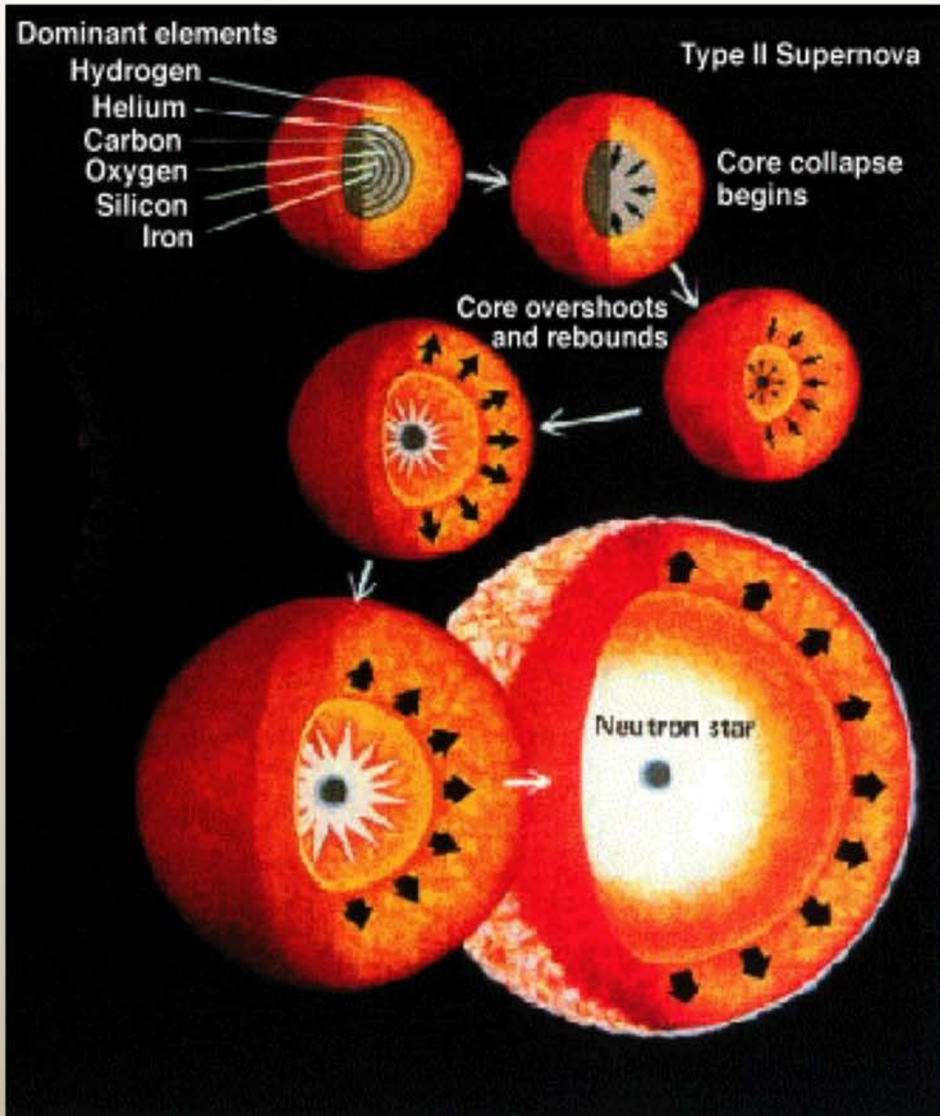
entropy s/k = 10, Y_e = 0.5

from Meyer (1994)

entropy s/k = 100, Y_e = 0.5



explosive burning in core-collapse supernovae



Outgoing shock wave heats inner Si and O layers – NSE is achieved and rapidly freezes out as shock passes

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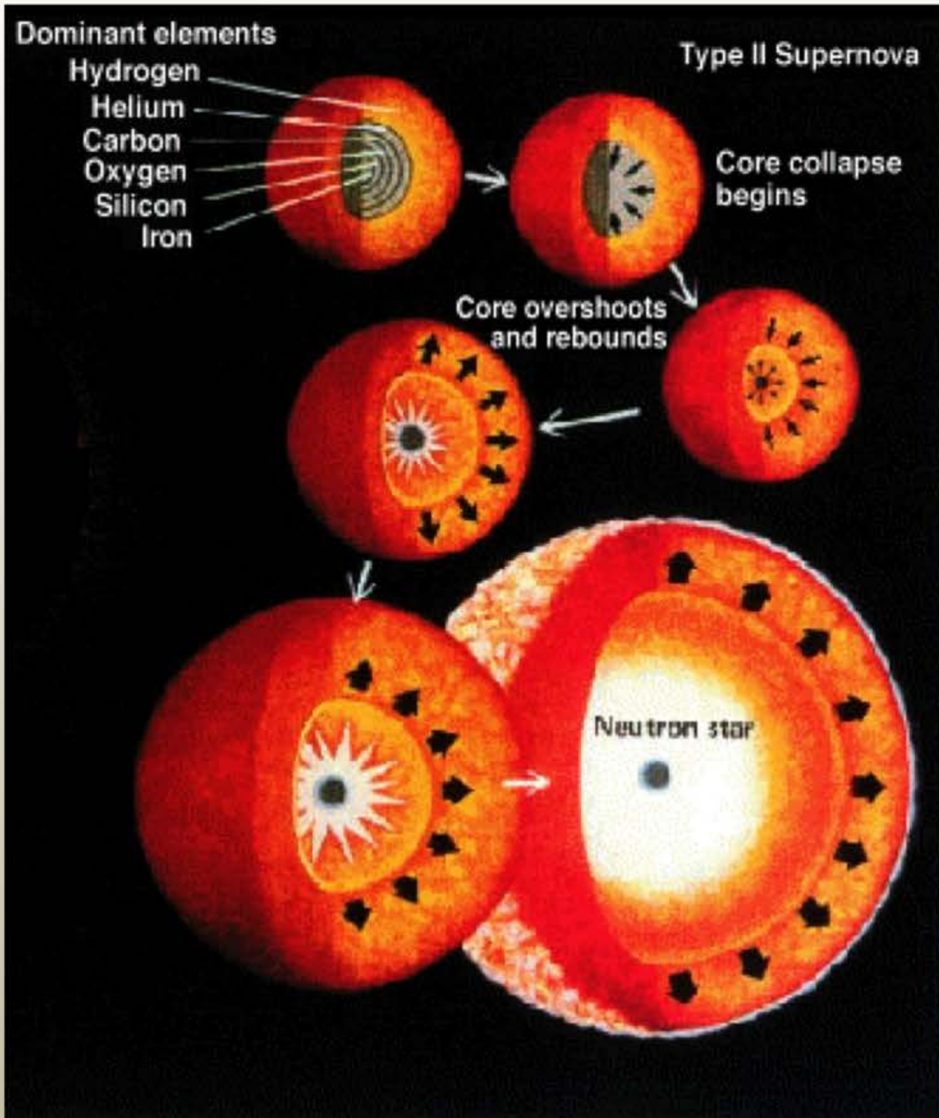

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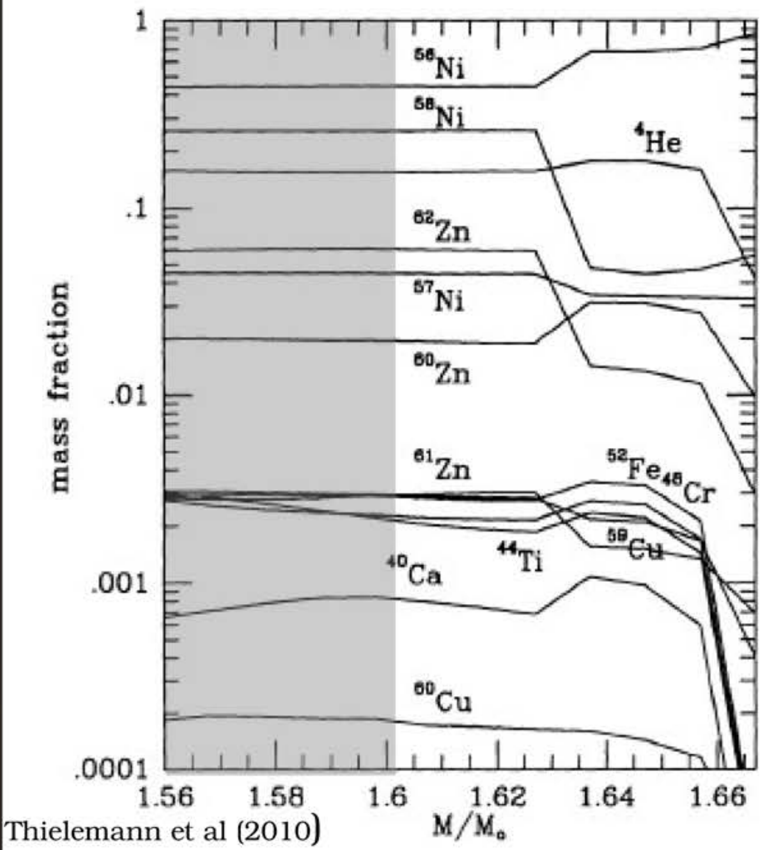
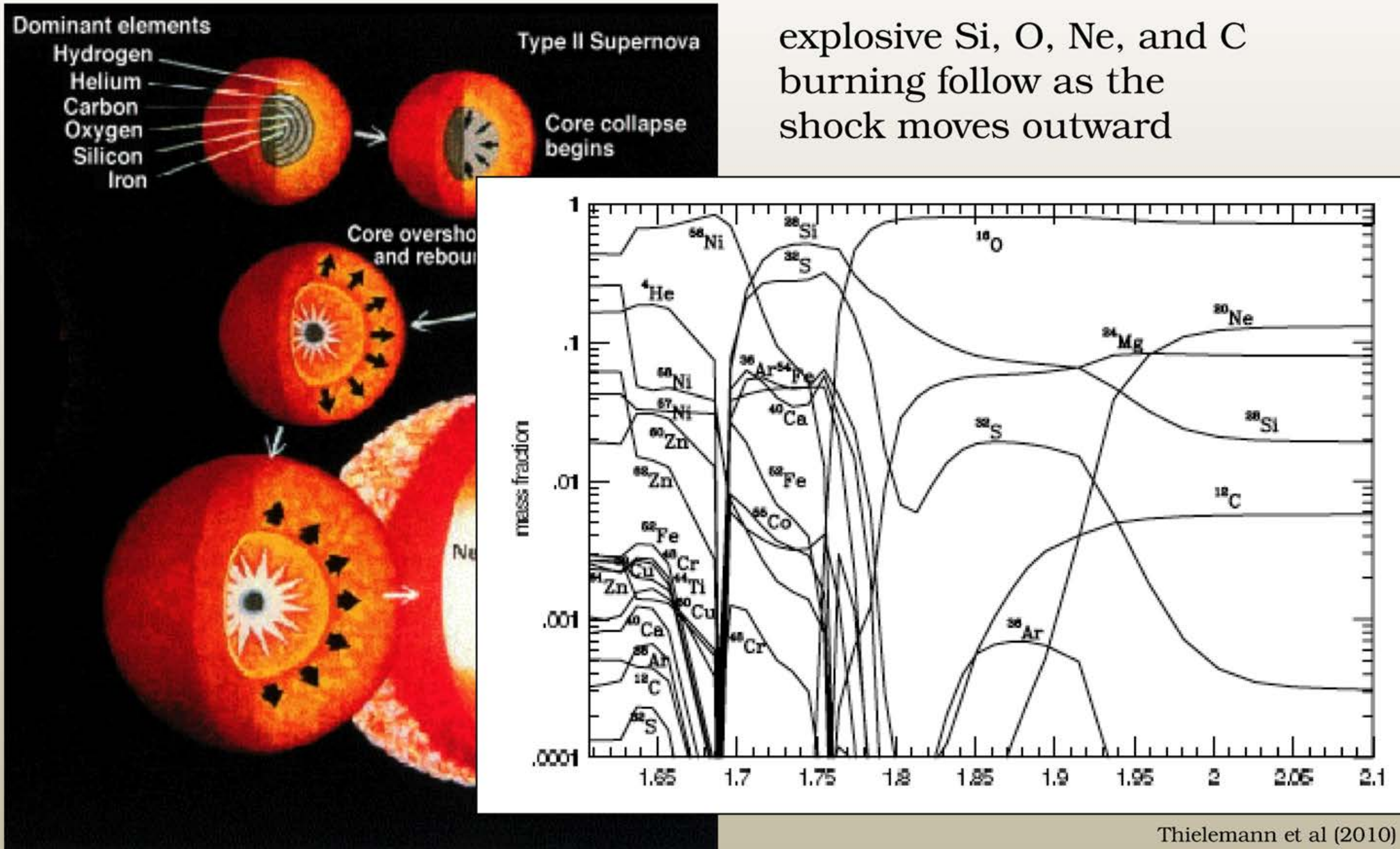


Thielemann et al (2010)

explosive burning in core-collapse supernovae

explosive Si, O, Ne, and C burning follow as the shock moves outward



Thielemann et al (2010)

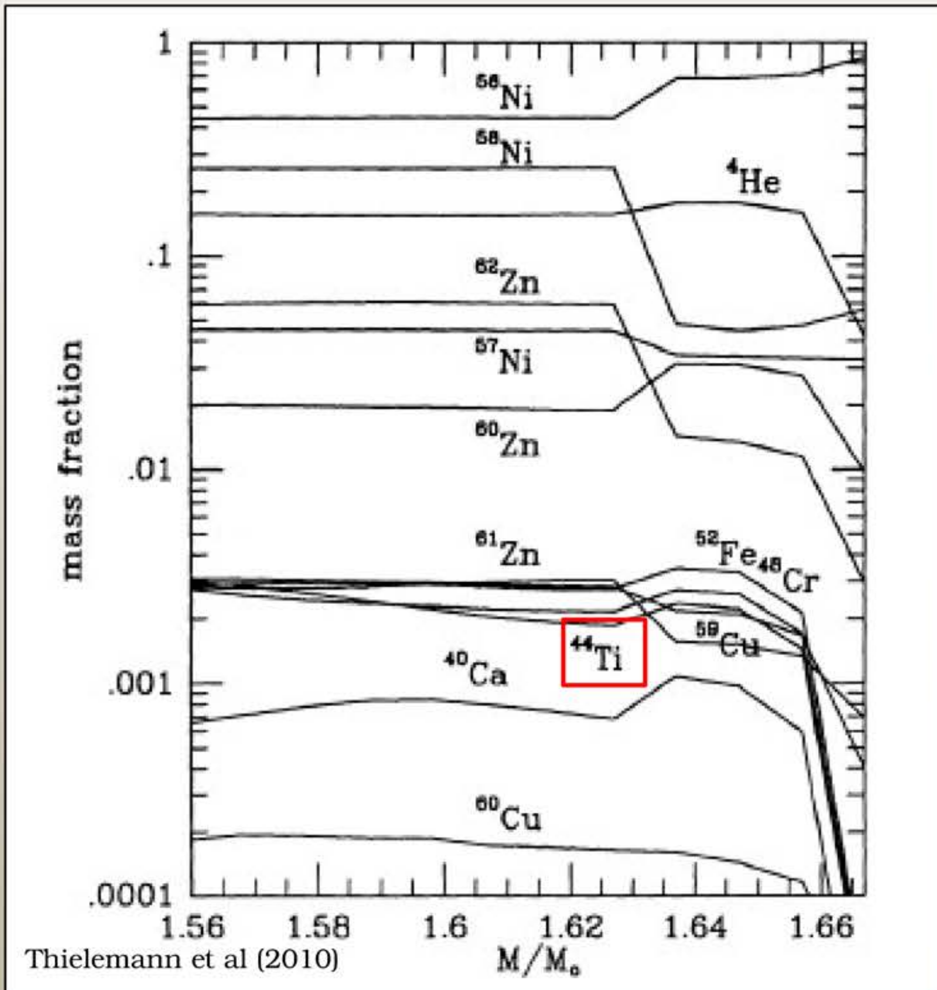
SNe nucleosynthesis – long-lived radionuclides

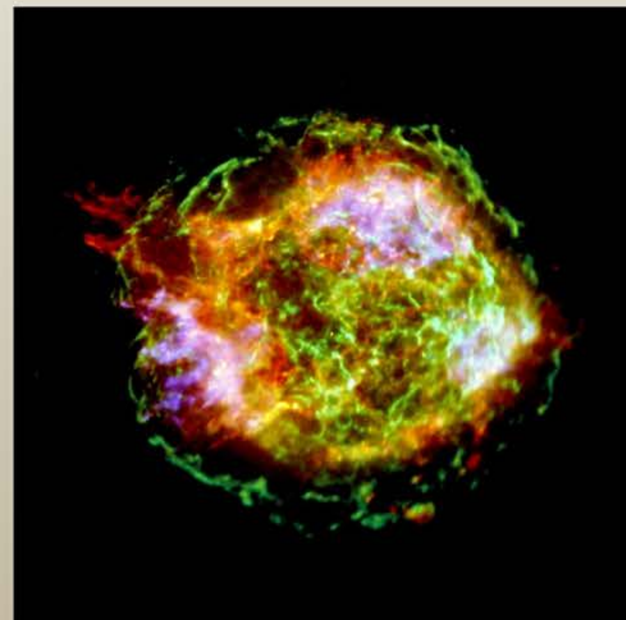
^{44}Ti

created in alpha-rich freezeout from NSE, close to the mass cut

$T_{1/2}$ ~ 60 years

x-rays from the decay chain observed in SNe remnants Cas-A and RX J0852.0-4622



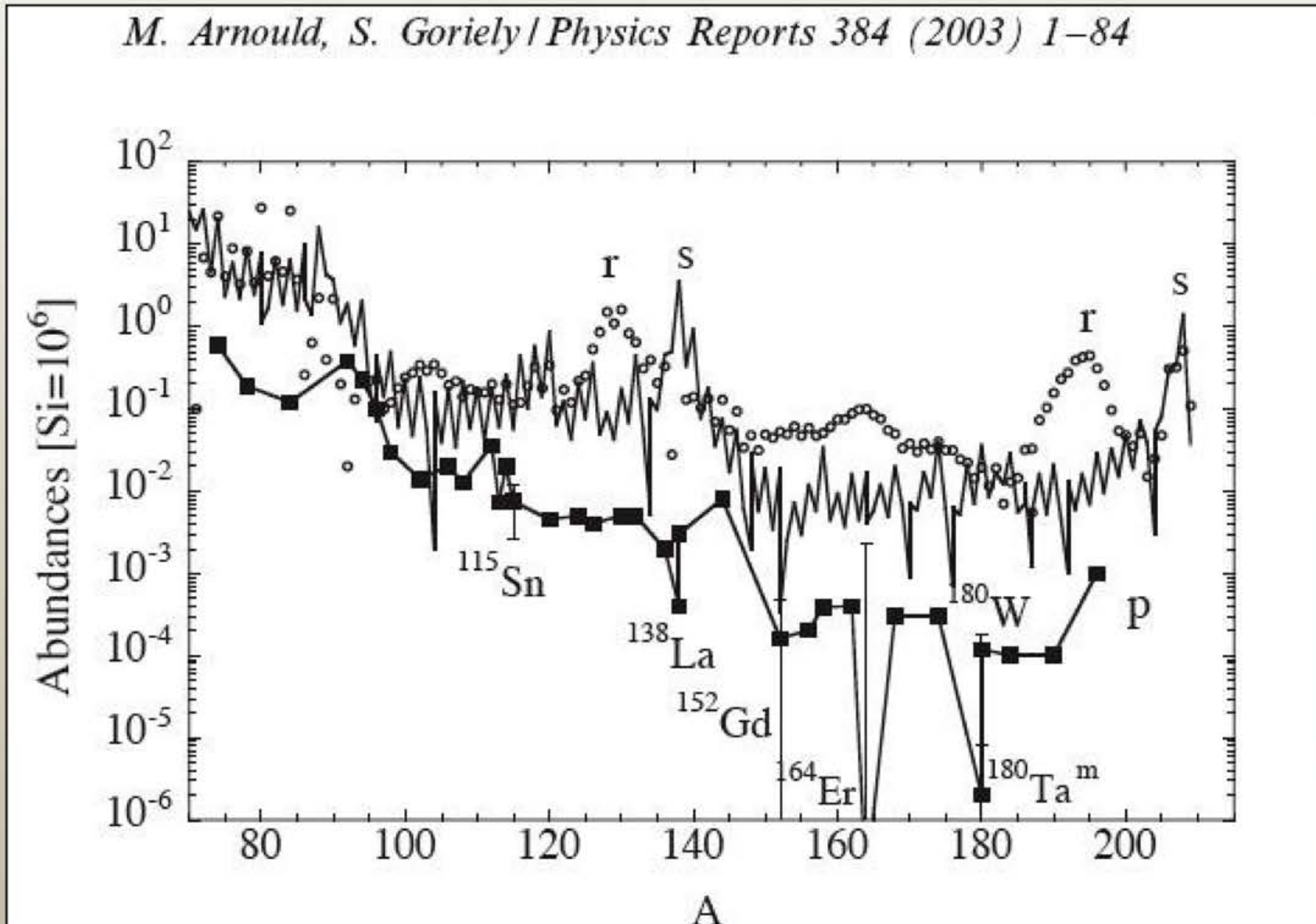


What about the heavier elements?

- p-process
- s-process
- r-process

SNe nucleosynthesis – proton-rich heavy elements

heavy *p*-process nuclei are made by (γ,n) photodissociations of pre-existing *r*- and *s*- process nuclei



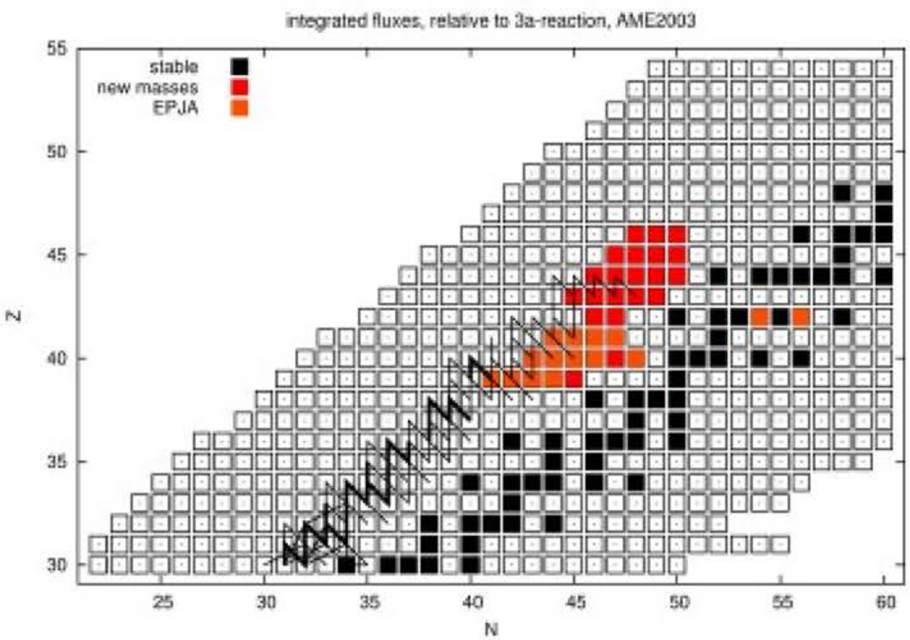
SNe nucleosynthesis – proton-rich heavy elements

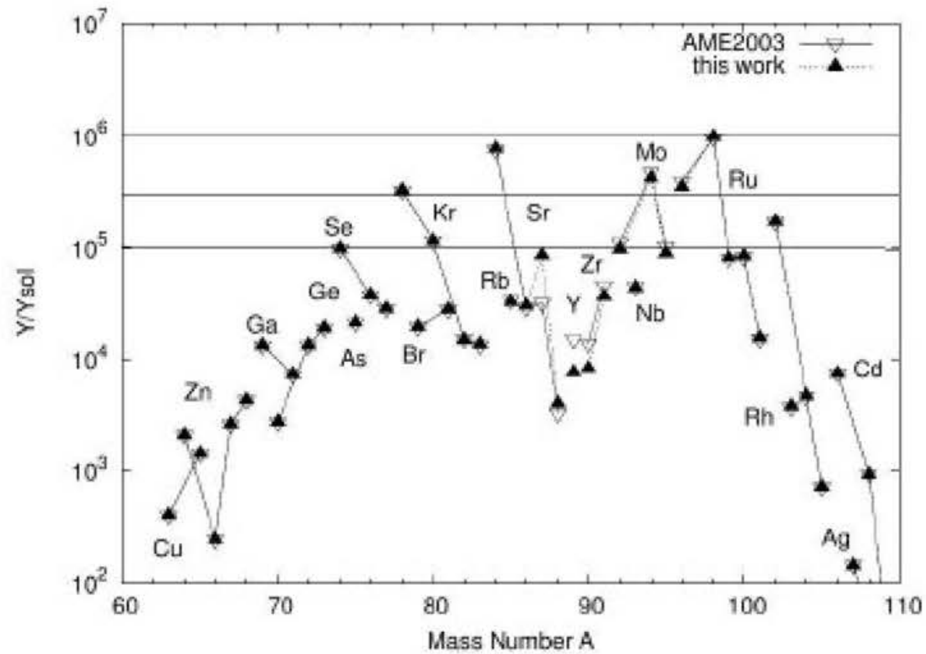
νp-process
(Frohlich et al 2006, Pruet et al 2006, Wanajo 2006)

Thought to occur in proton-rich ejecta from the inner regions of the SNe

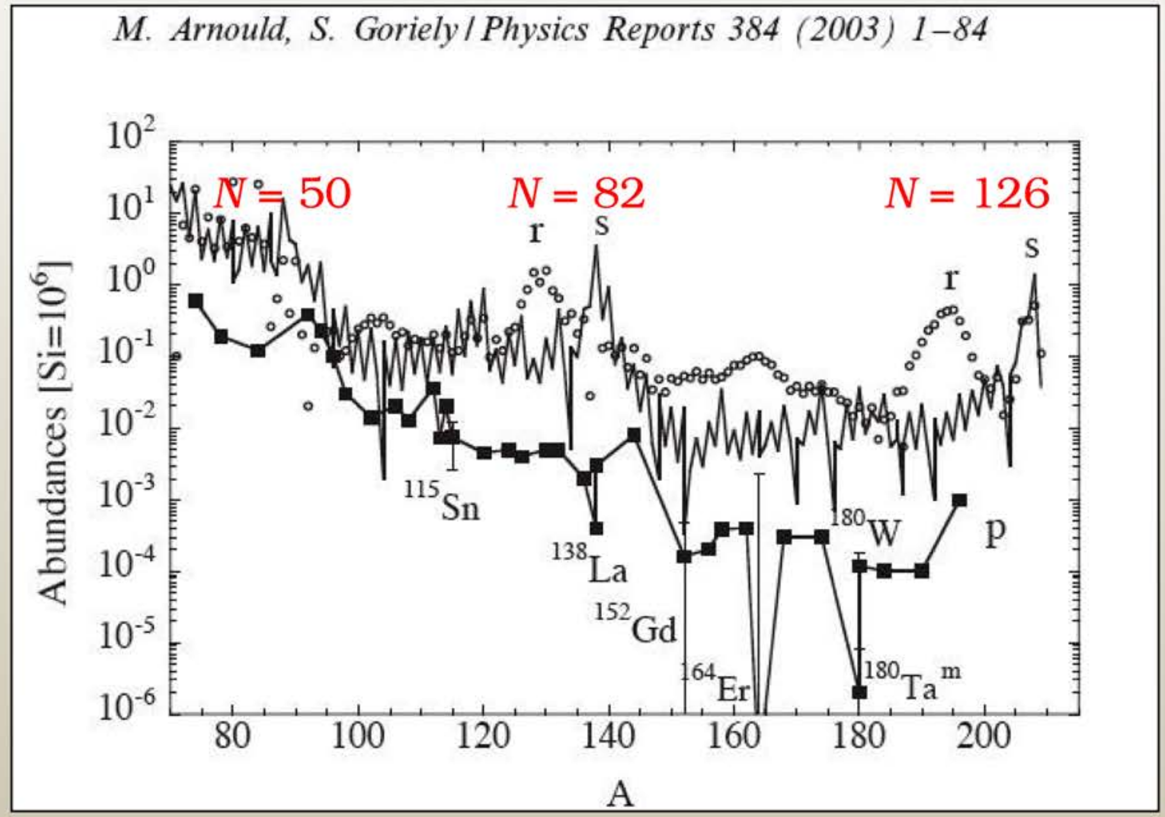
$$p+\bar{\nu}_e \rightarrow n+e^+$$

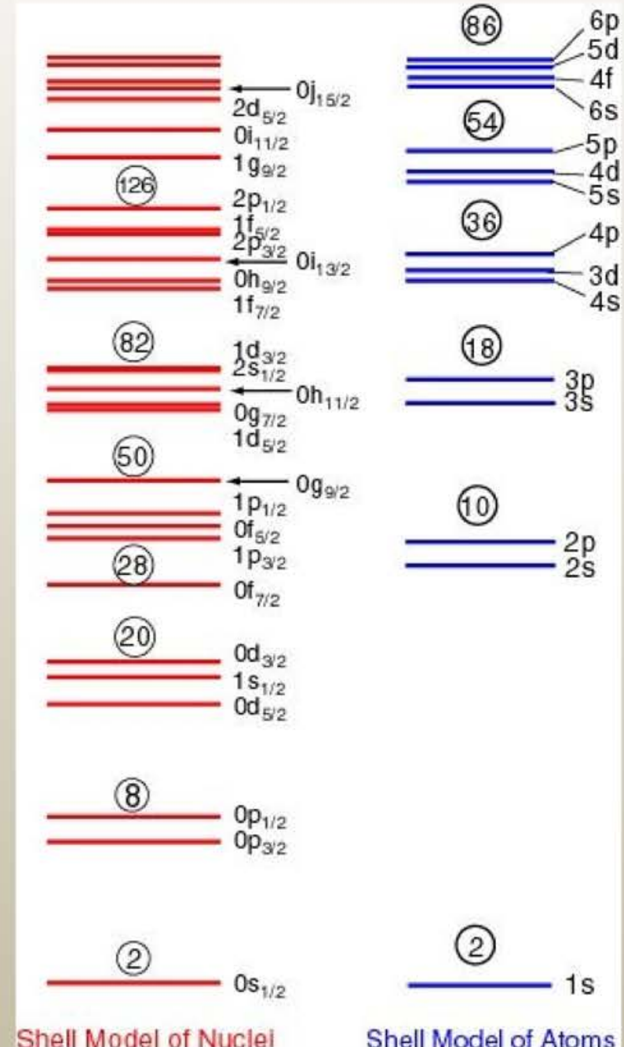
Thielemann et al (2010)





neutron capture nucleosynthesis





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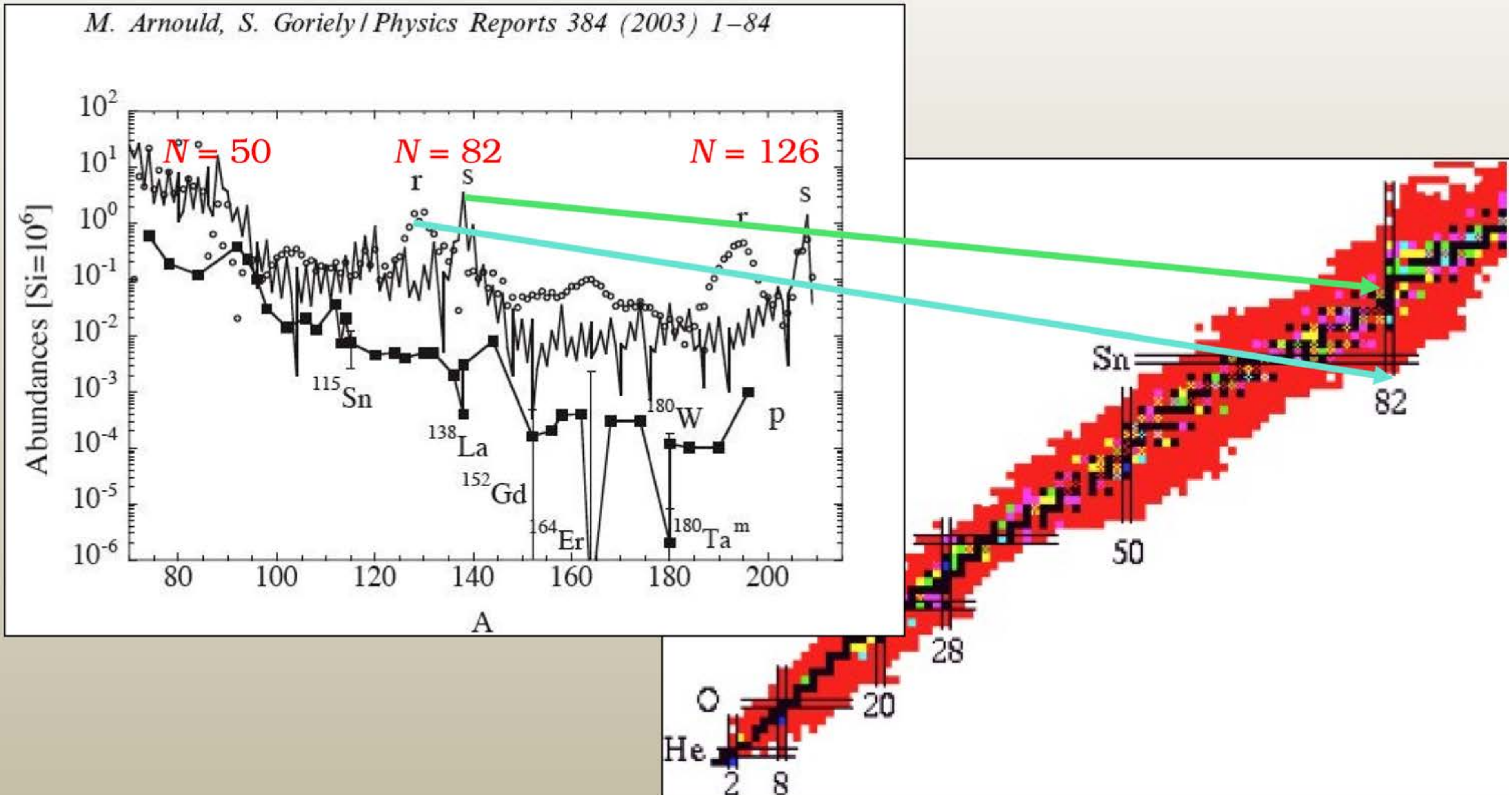
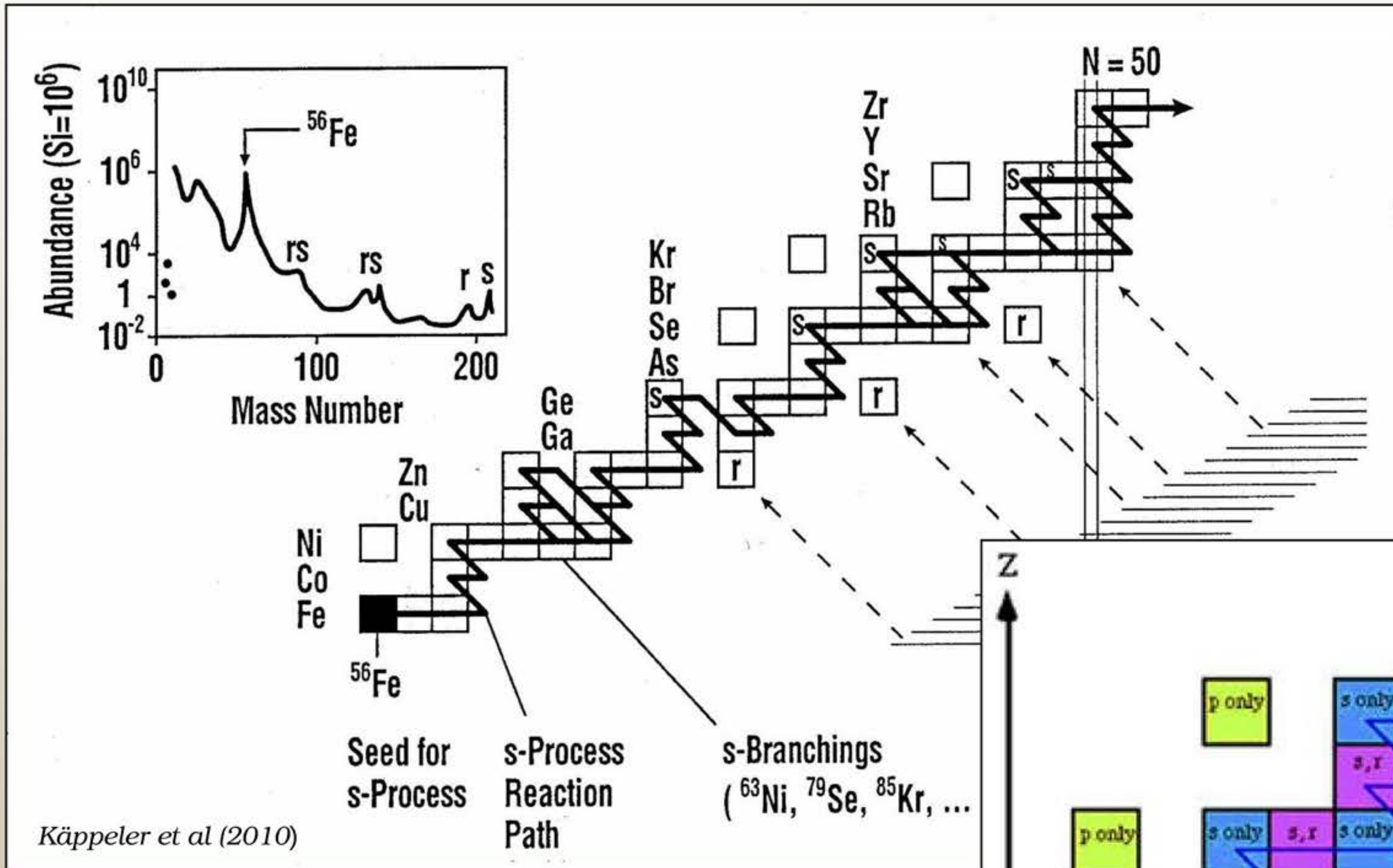
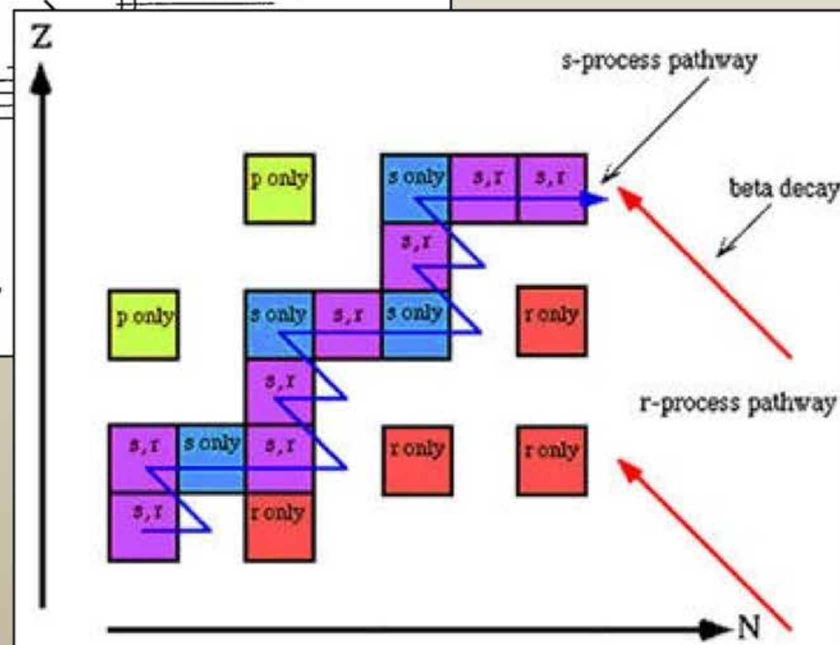
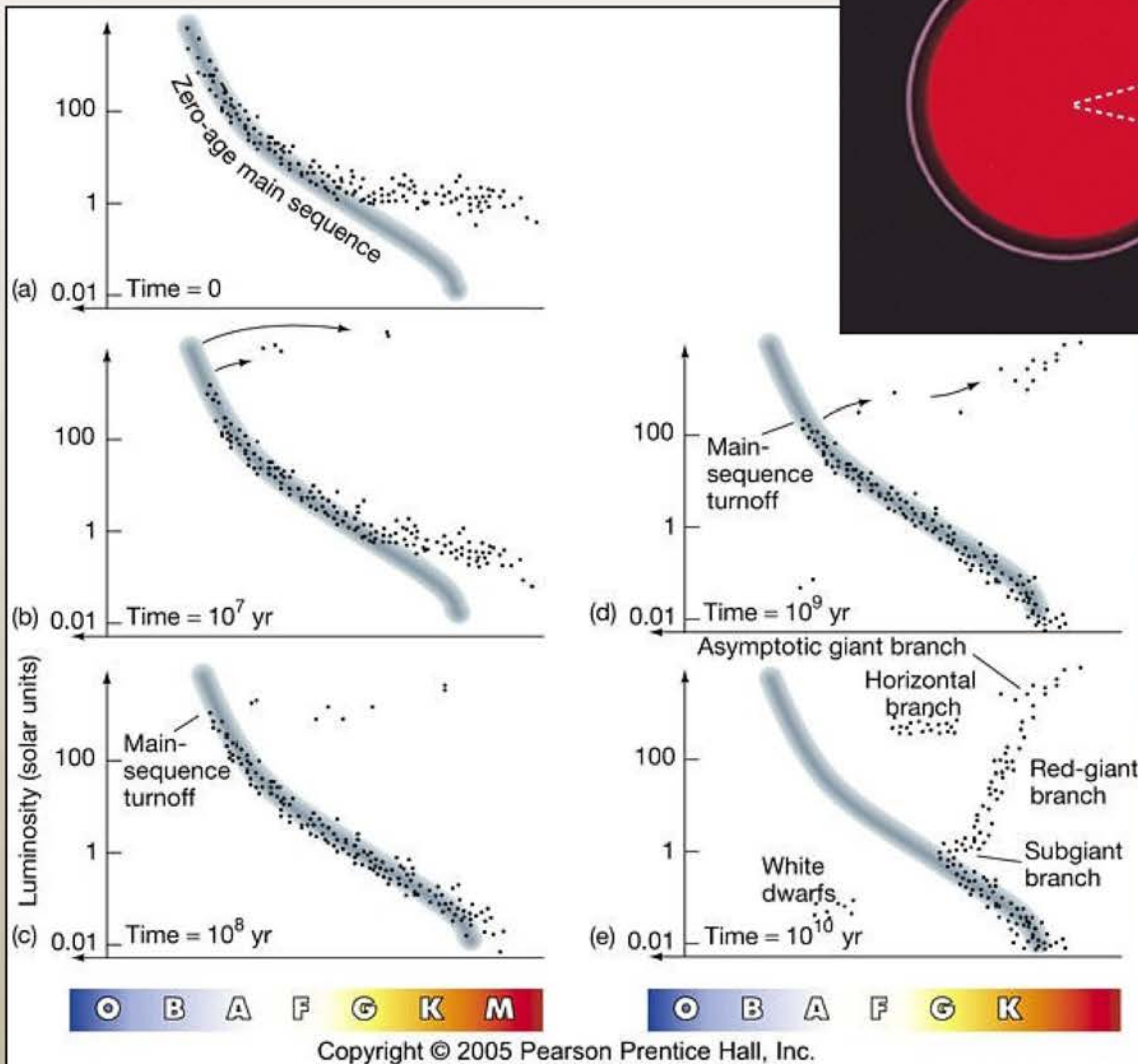


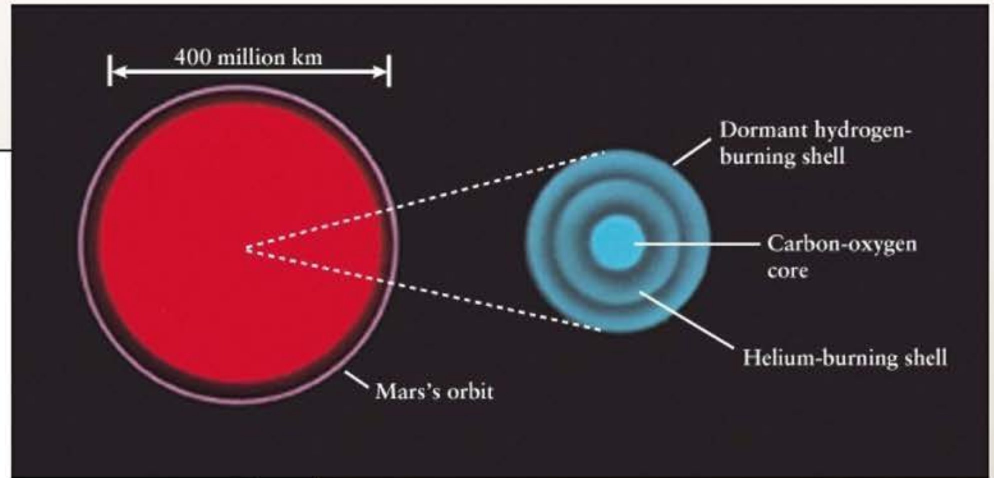
separating *s*- and *r*-process abundance patterns





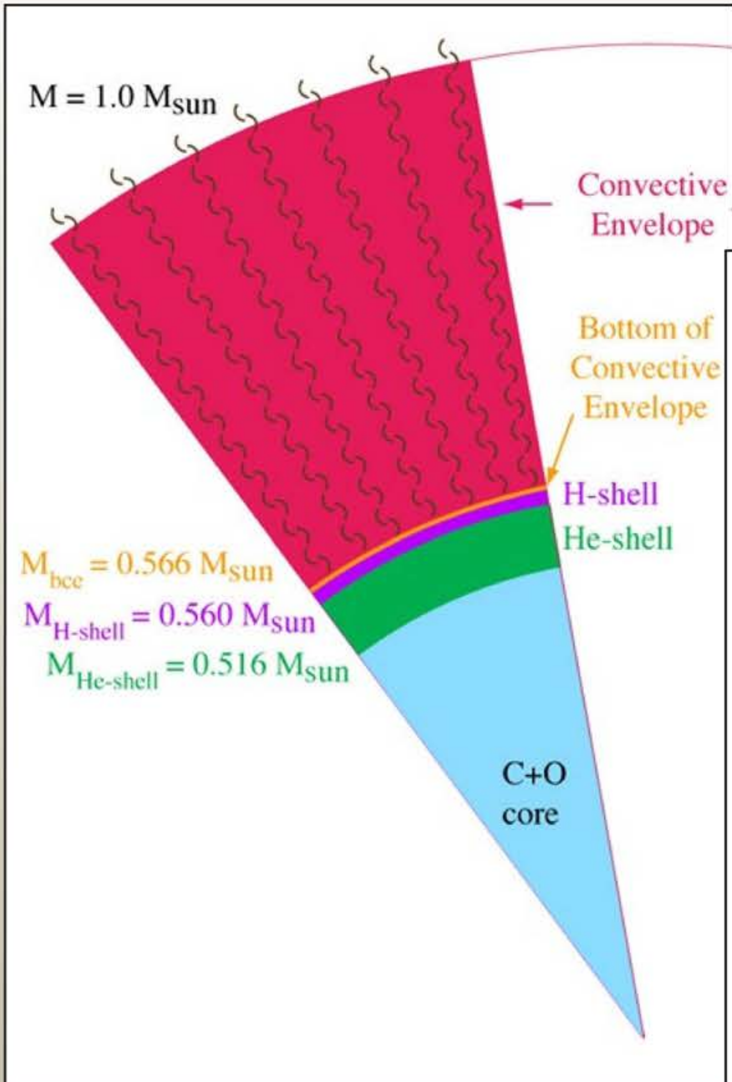
the astrophysical site of the *s*-process



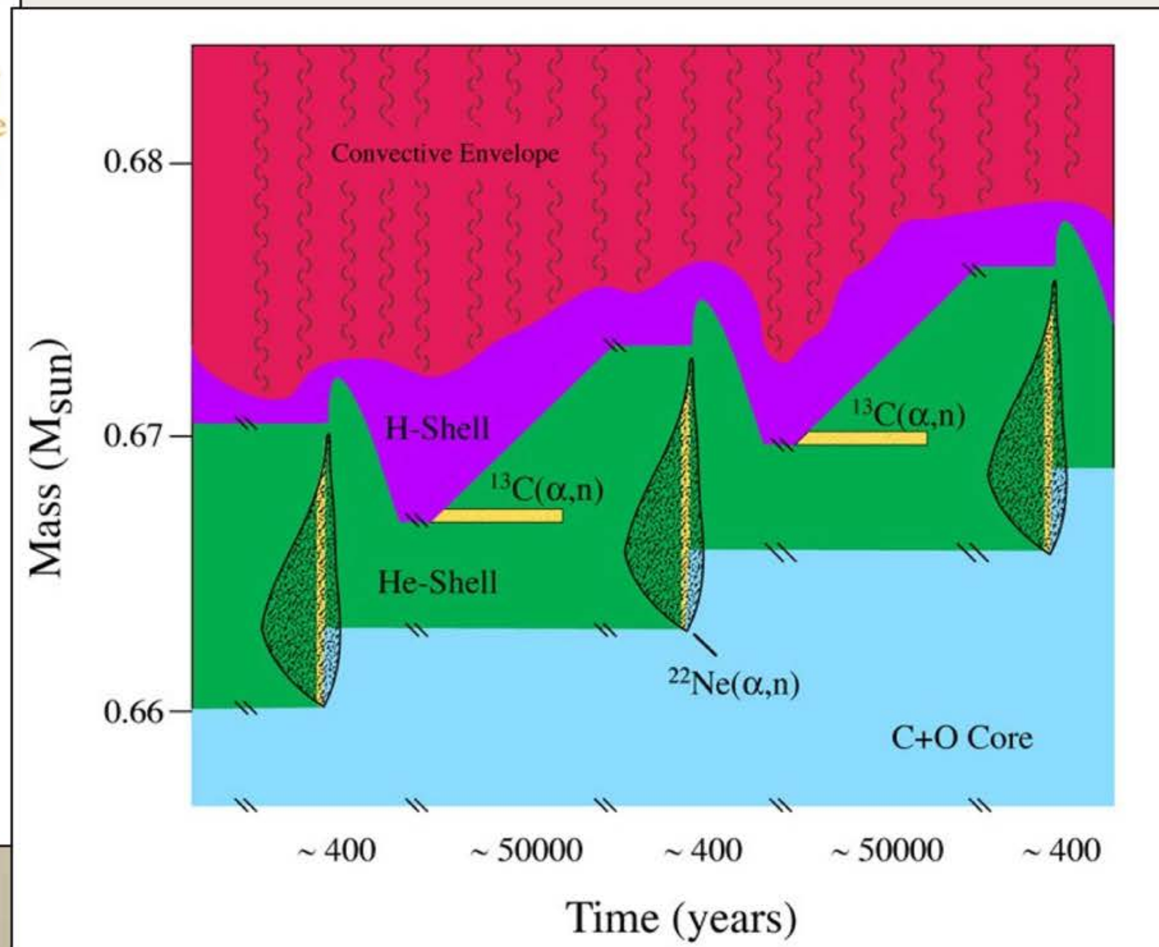


main component –
low mass AGB stars

the astrophysical site of the *s*-process



$$^{12}C(p,\gamma)^{13}N(\beta^{+}\nu)^{13}C \longrightarrow {}^{13}C(\alpha,n)$$



Next lecture: r-process