QCD and String Theory

(A modest title for the NNPSS)

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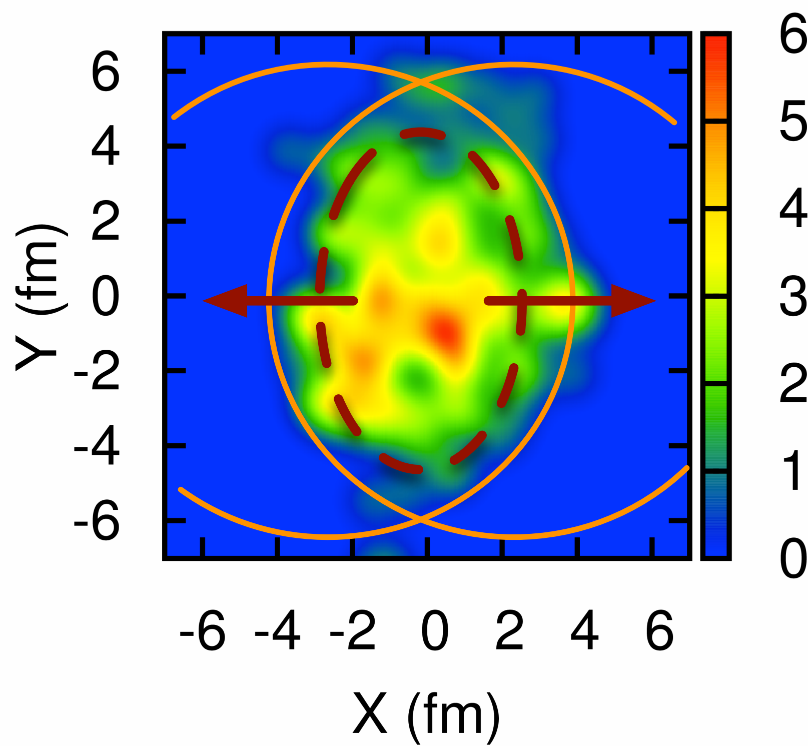
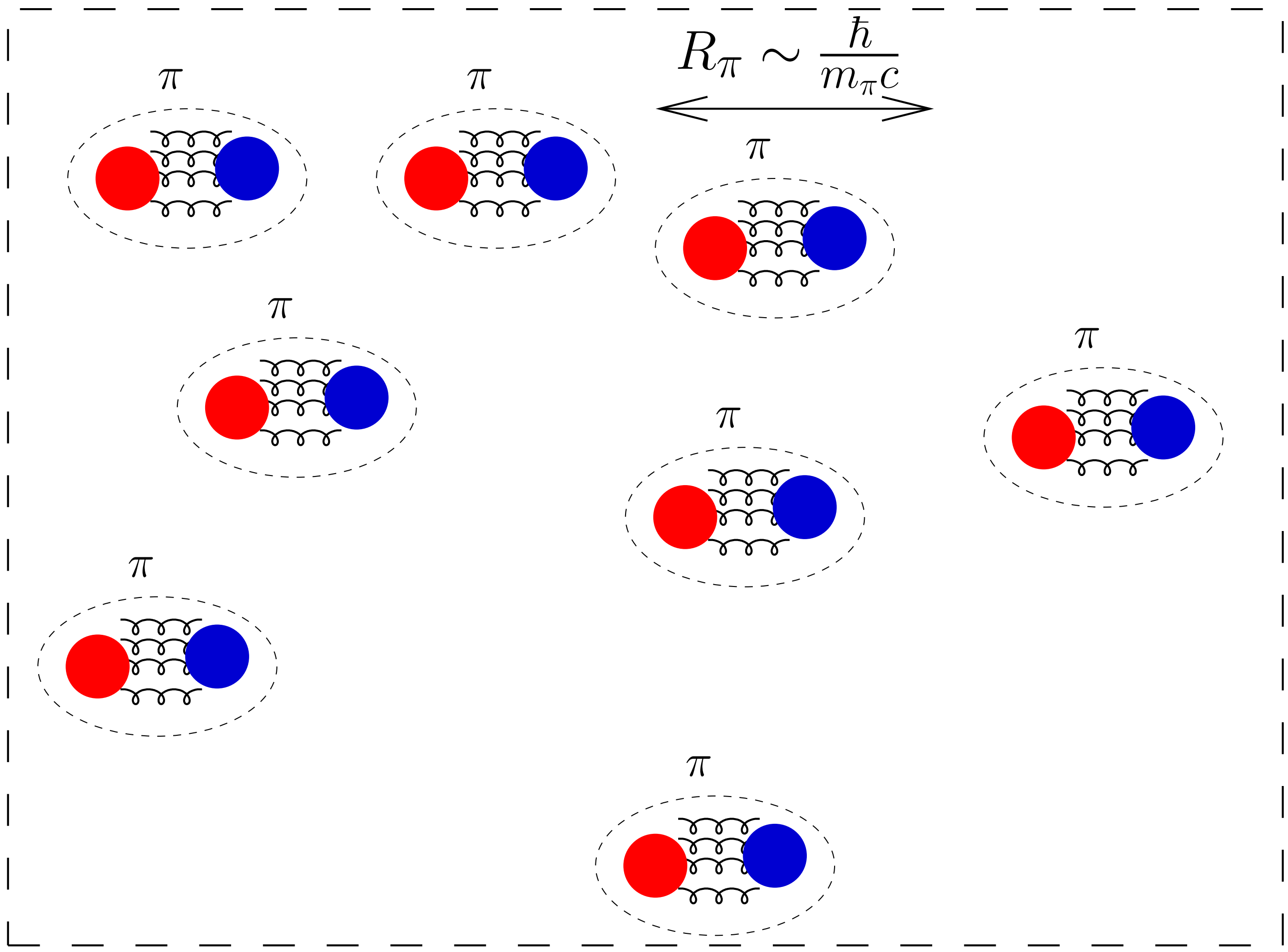
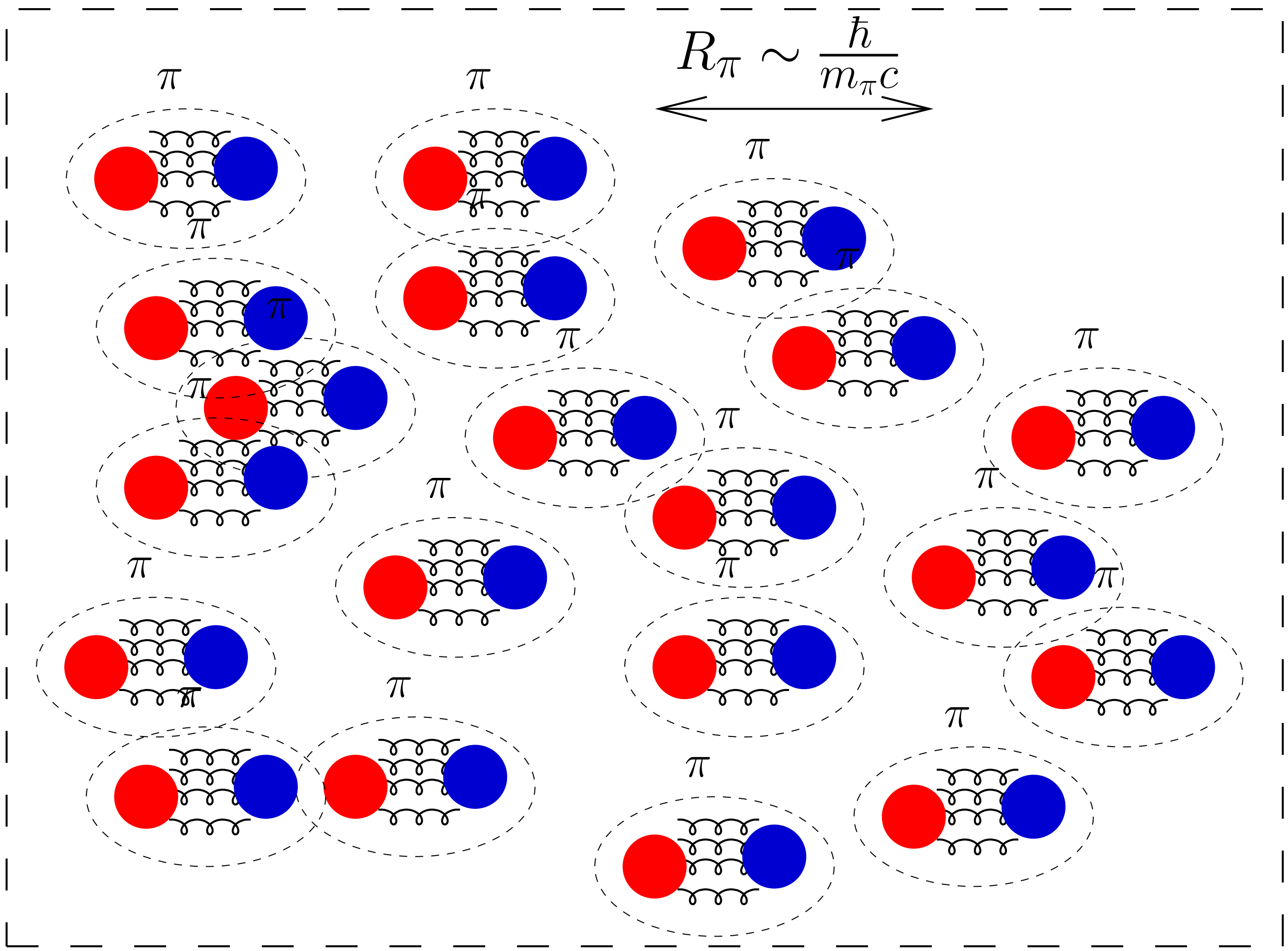


Nuclear physics at low temperatures:



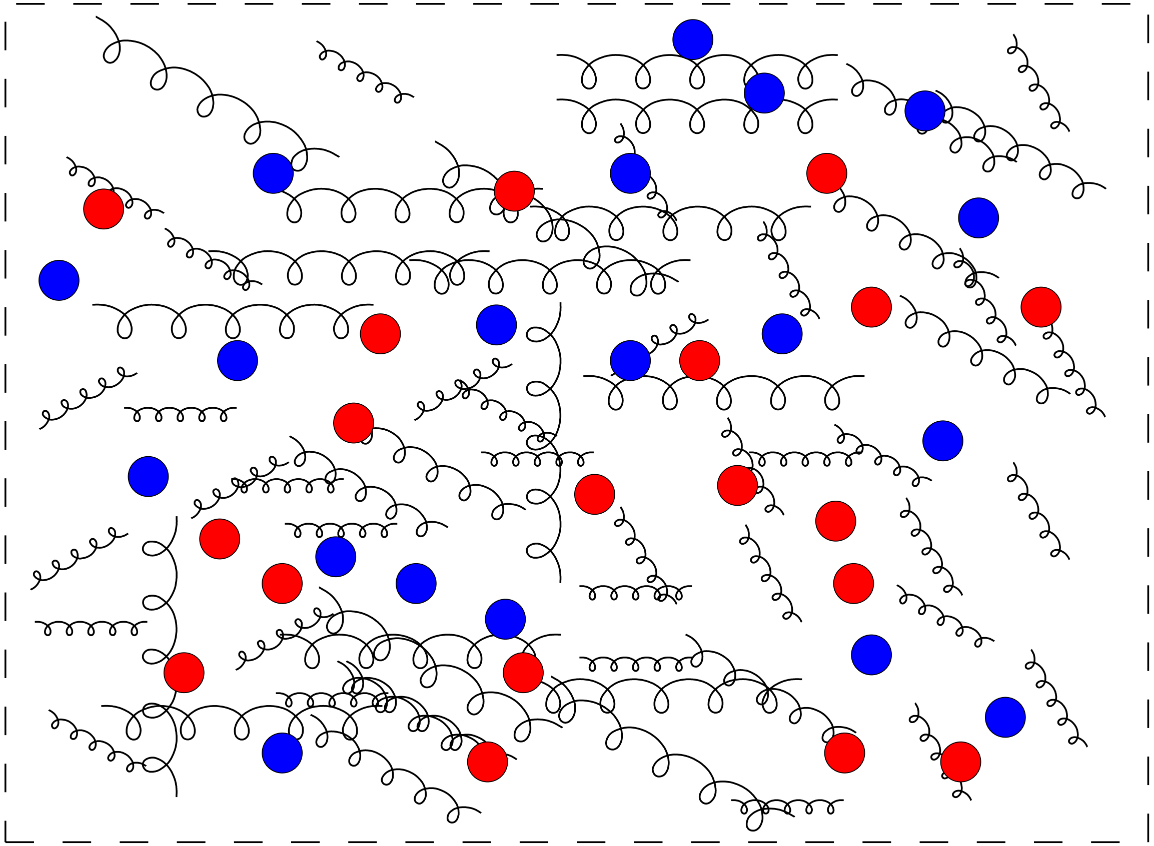
“Low” temperature nuclear physics of a dilute pion gas.

Nuclear physics at modest temperatures:



At modest temperatures the pion density increases like $n_\pi \propto \left(\frac{T}{\hbar c}\right)^3$

Nuclear physics at high temperatures:



Non-abelian plasma is special:

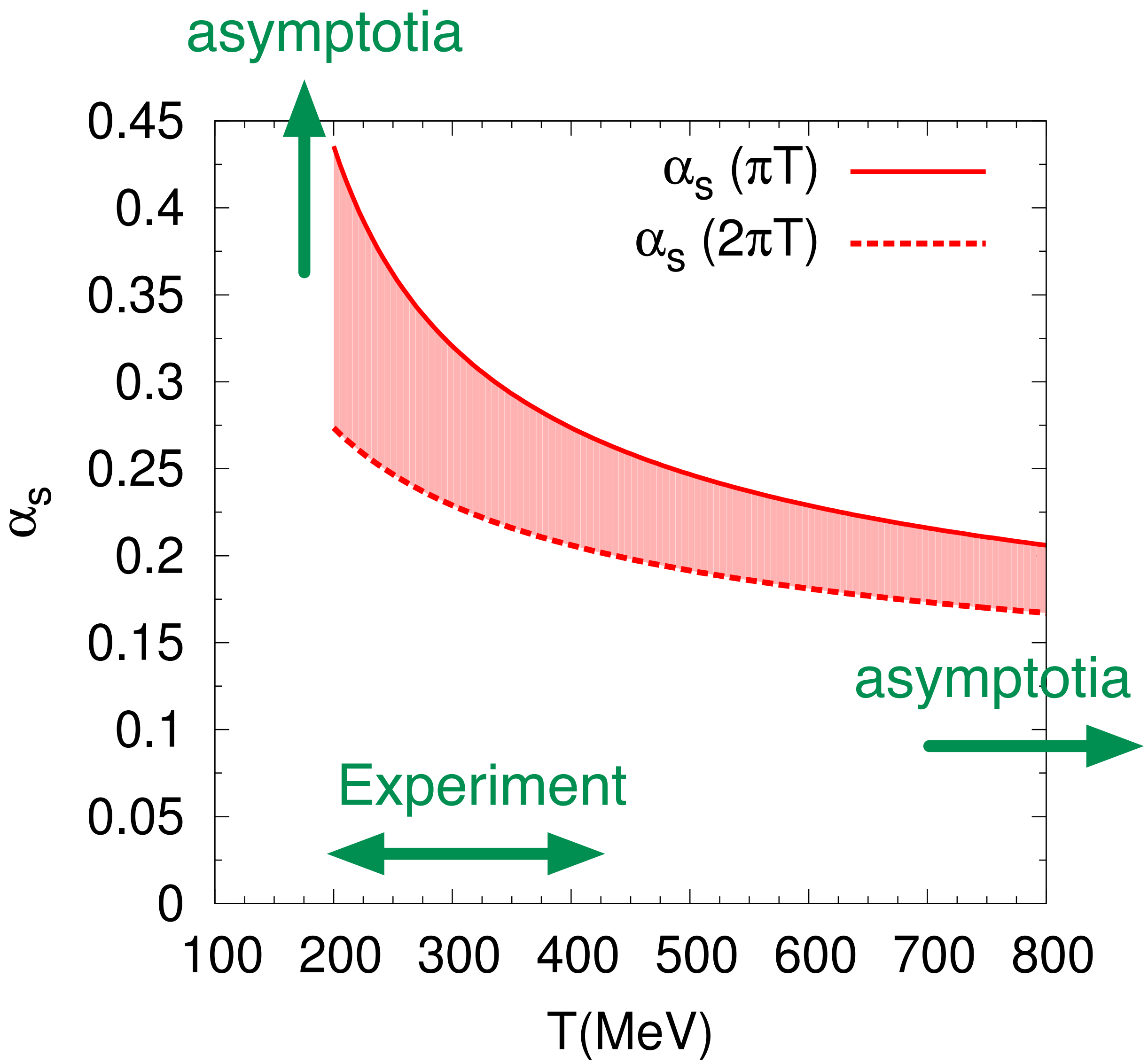
1. Ultra-relativistic

2. Non-linear

Expect a transition at for temperatures $T \sim m_\pi c^2 \simeq 140\,\mathrm{MeV}$

How nonlinear? The strong coupling constant at finite temperature:

$$\alpha_s = \frac{g^2}{4\pi}$$



The real QGP is neither completely weakly nor strongly coupled making life hard!

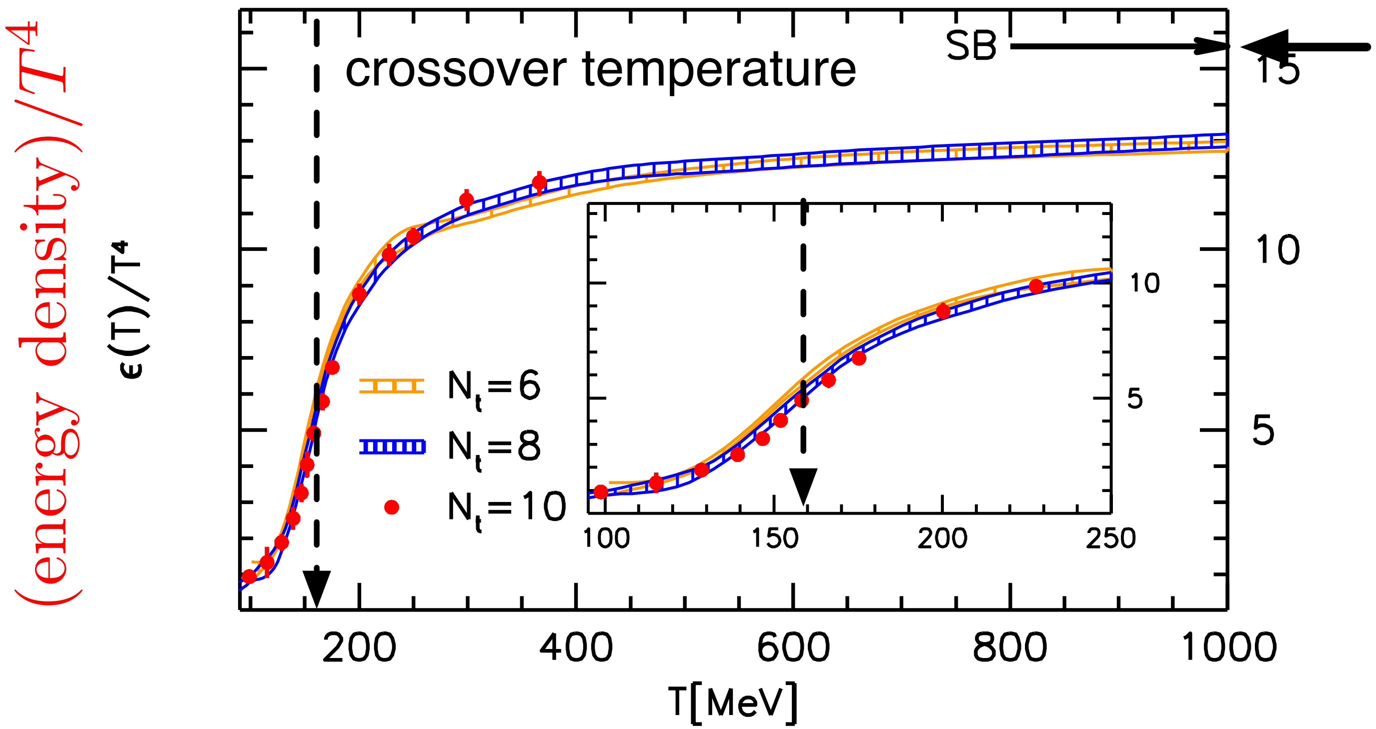
Lattice QCD and the QCD equation of state:

Compute the equation of state by sampling fields with the statistical weight:

$$Z \sim \int [DA] e^{-S_{QCD}[A]}$$

The largest single computational project in human history!

The QCD Equation of State (Budapest-Wuppertal Collaboration)



Ideal gas limit

1. The “critical” energy density and temperature are

$$e_c \simeq 1\,\mathrm{GeV/fm^3} \qquad\qquad T_c \simeq 160\,\mathrm{MeV}$$

2. The EOS state should be computable at high temperatures

$$p(T) = T^4\left(1 + g^2 + g^3 + g^4 + g^5 + g^6\log(1/g) + \ldots\right)$$

The equation of state is close to ideal gas – but important 20% deviations exist.

Discussion influenced by, Braaten and Nieto, hep-ph/9508406, and 9501375.
Also influenced by discussions with Mike Strickland

High Temperature Plasma

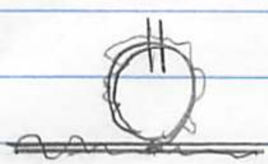
$\nu_g = 2 \cdot 8 \qquad N_f \cdot 2 \cdot 2 \cdot N_c$

$$P_{QGP} = \frac{\pi^2}{90} T^4 \left(\nu_g + \frac{7}{8} \nu_q \right)$$

- Ideal Gas of massless quarks and gluons

$V_p^\mu = (1, \hat{p})$ ← light like vectors

$O(g^2)$ Corrections:



a medium induced mass is given to all particle

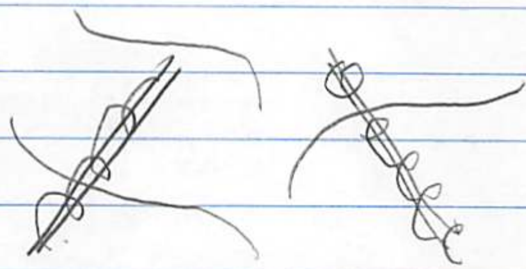
$m_\infty^2 \sim g^2 T^2$ ← dimensions

Then the energy density is corrected

$$\mathcal{E} \sim \int \frac{d^3p}{(2\pi)^3} E_p \, n(\omega) \qquad n(\omega) = \frac{1}{e^{E_p/T} - 1}$$

$$E_p = \sqrt{p^2 + m_\infty^2} \simeq p + \frac{m_\infty^2}{2p}$$ ← this gives the first correction

$\mathcal{O}(g^3)$ Corrections:



The hard particles create a bath of softer excitations with $p \sim gT$

Propagators $\sim \dfrac{1}{p^2 + m_D^2}$ (for electric modes)
(but still $1/p^2$ for magnetic modes)

$$\mathcal{E}_{\substack{glue\\soft}} \sim \int \frac{d^3p}{(2\pi)^3} \, E_p \, n(E_p)$$

$$\sim (gT)^3 \, E_p \, \frac{T}{E_p}$$

we used

$$n(E_p) = \frac{1}{e^{E_p/T} - 1} \simeq \frac{T}{E_p} \sim \frac{T}{gT} \sim \frac{1}{g} \leftarrow \text{large } \#$$

Since the number is large these modes are approximately classical in nature

Higher Orders

$O(g^4)$ first order where coupling runs. Describes $g^2(\mu) \to g^2(2\pi T)$

$O(g^5)$ Determines the renormalization of the Debye sector $g^2(m_D)$

$O(g^6)$ The magnetic modes become important. $p \sim g^2 T$

$$n \sim \frac{T}{p} \sim \frac{1}{g^2} \qquad n \sim \langle A^2 \rangle$$

Thus for these modes: $A \sim \frac{1}{g}$

Thus the interactions become intrinsically non-perturbative (and non-abelian)

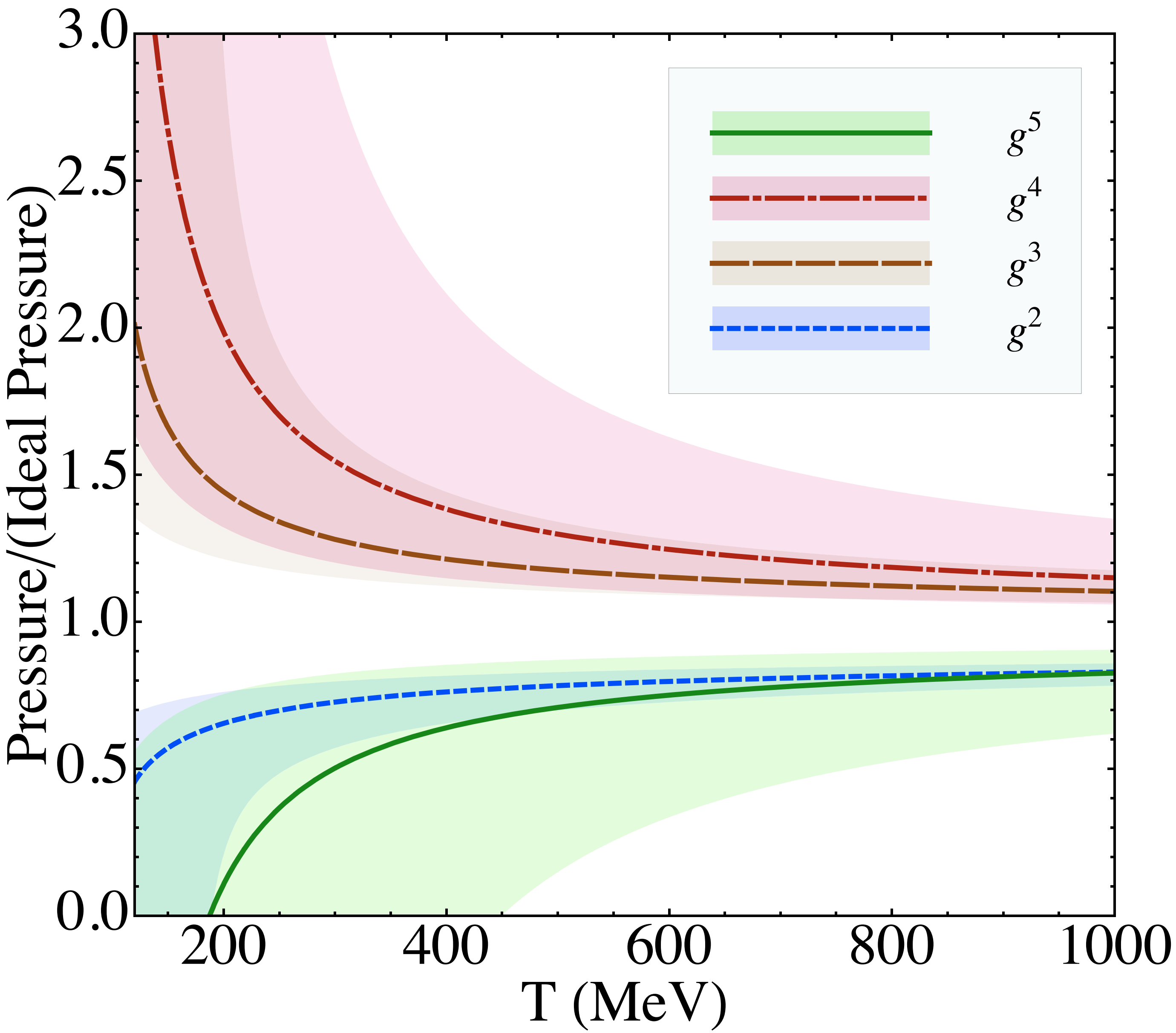
$$i\partial - \underbrace{gA}_{\text{Order unity}}$$

The appropriate Lattice theory to describe these modes is Magnetic QCD $\equiv$ MQCD

Lattice simulations show that a mass (gap) develops in this theory and chromo magnetic fields are damped out. It is for these reasons that the long wavelength Effective theory of the QGP is hydro, rather than some non-abelian generalization of magneto-hydro.

Failure of the strict perturbative expansion

from Strickland, Andersen, and Su



A disaster!

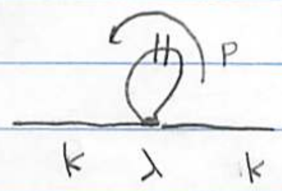
Reorganization of the Perturbative Expansion (HTL PT)

- want to perturb around a state of massive quasi-particles

Take a scalar theory

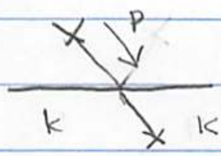
$$\mathcal{L}_0 = -\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4$$

The mass



$$m_{th}^2 \sim \lambda \int \frac{d^3p}{(2\pi)^3} \frac{n_p}{2p}$$

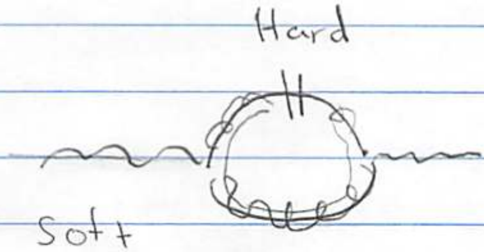
or



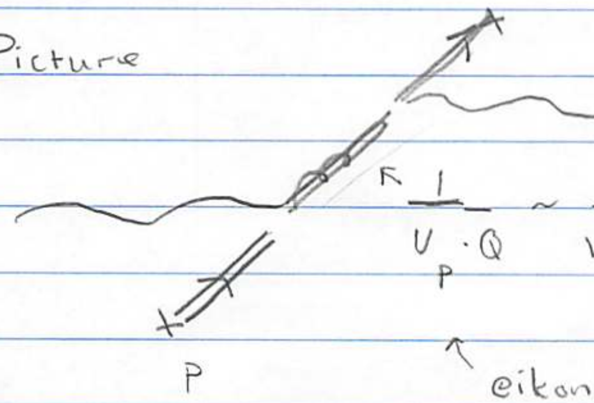
Add and subtract

$$\mathcal{L} = \underbrace{\mathcal{L}_0 - \frac{1}{2} m_{th}^2 \phi^2}_{\substack{\text{do not expand} \\ \text{resummed propgator}}} + \underbrace{\frac{1}{2} m_{th}^2 \phi^2}_{\substack{\text{treat as} \\ \text{perturbation}}}$$

QCD is much more complicated



Picture



$\frac{1}{V_p \cdot Q} \sim \frac{1}{v \cdot \partial}$

eikonal, or straight line.

The "mass" term in QCD is

integrate ove $\hat{p}$ direction

$$\mathcal{L}_{HTL} = -\frac{1}{2} m_D^2 \, G_{\mu\alpha} \int \frac{d\Omega_{v_p}}{4\pi} \frac{v_p^\alpha v_p^\beta}{(v_p \cdot D)^2} G^{\mu}{}_{\beta}$$

covariant derivative

Hard Thermal Loop Perturbation Theory

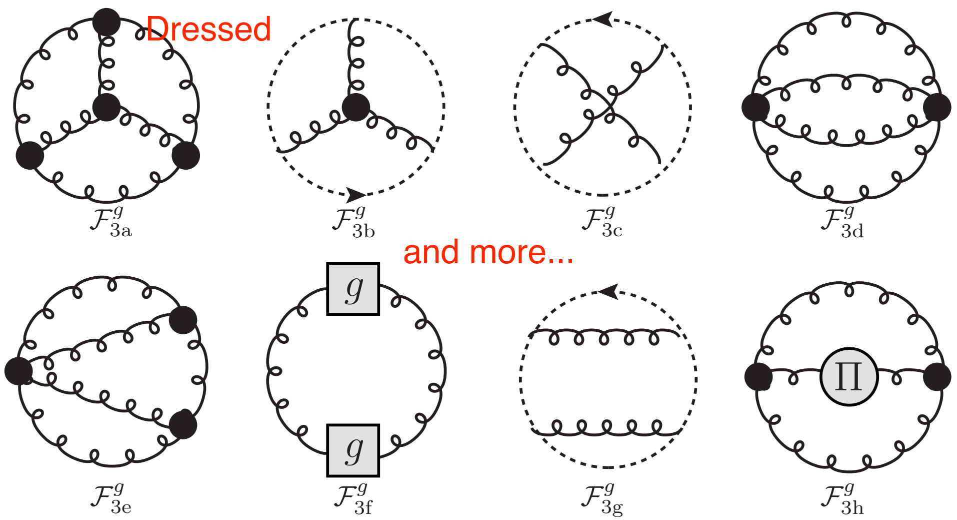
from Braaten, Andersen, Leganger, Strickland, Su

$$\mathcal{L} = \underbrace{\mathcal{L}_{QCD} + \mathcal{L}_{HTL}}_{\text{treat without expansion}} - \underbrace{\mathcal{L}_{HTL}}_{\text{treat as perturbation}}$$

where

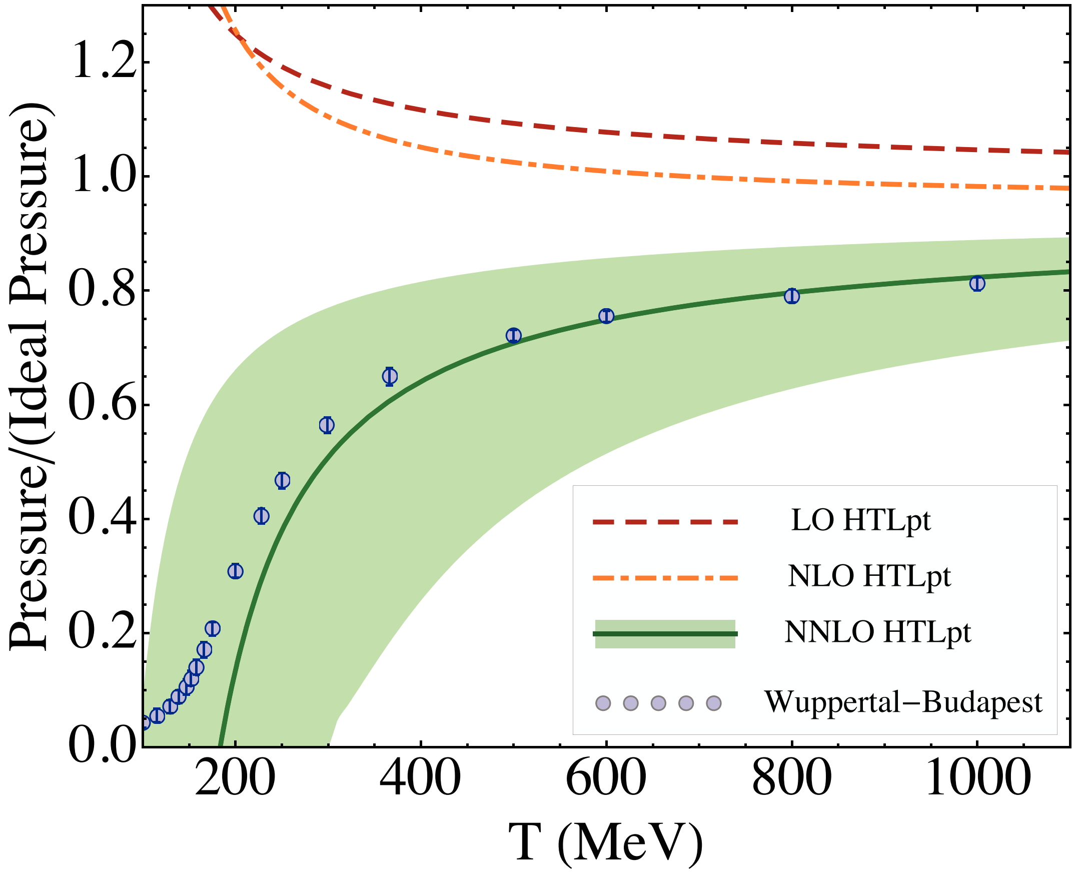
$$\underbrace{\mathcal{L}_{HTL}}_{\text{provides a mass term}} = -\frac{1}{2} m_D^2 G_{\mu\alpha} \underbrace{\int \frac{d\Omega_v}{4\pi} \frac{v_{\boldsymbol{p}}^{\alpha} v_{\boldsymbol{p}}^{\beta}}{(v_{\boldsymbol{p}} \cdot D)^2}}_{\text{Hard thermal loop self energy+vertices}} G^{\mu}{}_{\beta}$$

then work very hard . . .



Result

from Braaten, Andersen, Strickland, Su



Gives a much better agreement with lattice results! But . . .

EQCD resummation scheme gives similar results

Comments on Mike's Plot

- There is some (rather small) uncertainty in how the mass is determined

- There are largeish scale uncertainties

  - The NNLO correction is largish (the largest) It is unclear to me how serious this is. It may simply be a consequence of new physics (running coupling) appearing at NNLO

Why Study N=4 SYM? And Gauge Gravity Duals

- We have a very elaborate theory of weakly coupled quasi-particles. Good to have a foil for these calculations (There are some similarities and some differences)
- Extremely Interesting in its own right (and very physical)

- At least with coarse glasses, the theory effortlessly and naturally produces what we're seeing

  - Hydro everyWhere
  - Rapid thermalization, etc

Entropy of Super Yang Mills Theory at Large N_c

1. The coupling constant in Super Yang Mills Theory

$$\underbrace{\lambda = g^2 N_c = 4\pi\alpha_s N_c}_{\text{“theorist's” coupling}}$$

2. The entropy is function of coupling

$$\frac{s}{s_0} = \frac{\text{entropy}}{\text{ideal gas entropy}} = \underbrace{f(\lambda)}_{\text{function of coupling}}$$

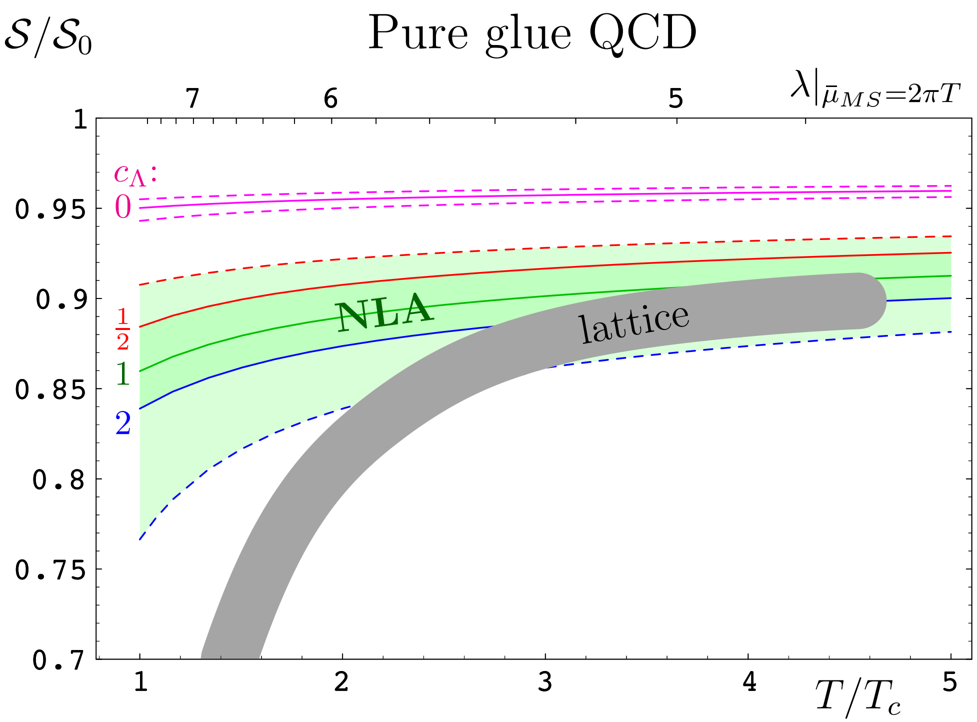
(a) Weak coupling (quasi-particles), $\lambda \ll 1$

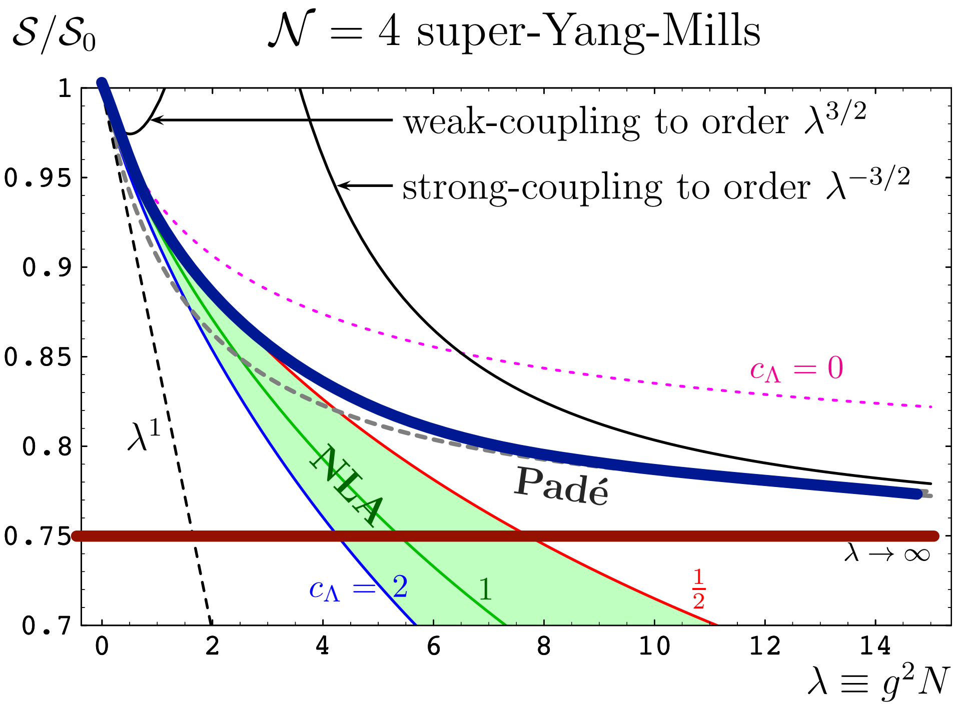
$$\frac{s}{s_0} = 1 \underbrace{- \frac{3}{2\pi^2}\lambda}_{\sim g^2} + \underbrace{\frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2}}_{\sim g^3} + \dots$$

Blaizot, Iancu, Kraemmer, Rebhan

- Strong coupling result, $\lambda \gg 1$:

$$\frac{s}{s_0} = \underbrace{\frac{3}{4}}_{\text{constant as } \lambda \to \infty!} + \frac{45}{32}\zeta(3)\lambda^{-3/2}$$





Green = quasi-particle resumption

Red = $\lambda \to \infty$

At strong coupling, all of those interactions add up to $\sim 25\%$ correction

References:
1. My favorite d'Hoker and Freedman. hep-th/0201253

2. The disscussion follows, Teaney and Schaefer, Section 4.

Overview of AdS/CFT correspondence

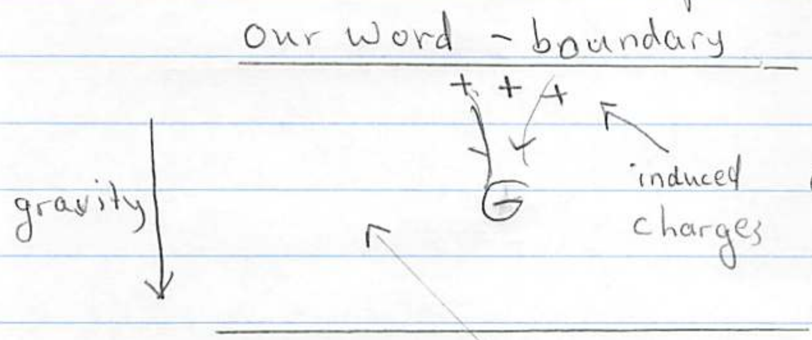
The AdS/CFT correspondence is an equivalence between $N=4$ SYM theory at large λ and large N_c and type IIB supergravity on $AdS_5 \times S_5$ (A five dimensional space)

$N=4$ SYM	Gravity
3+1 dimensional	4+1
Operators	Corresponding sources
$T^{\mu\nu}$ $\longleftrightarrow$	g_{ab}
J^{μ} $\longleftrightarrow$	A_a
$\vdots$	$\vdots$

radial direction

$\nu, \mu = 0, 1, 2, 3$ $\qquad\qquad$ $a, b = 0, 1, 2, 3, 4$

Picture of AdS Space



fifth dimension

$f(r)$

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)\,dt^2 + dx^2\right) + \frac{L^2}{f(r)r^2}\,dr^2$$

Black brane

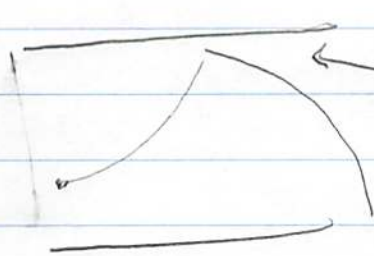
3+1 dimensional surface

We are to solve for the fields in this five dimension. This determines the charges at $r = \infty$. The AdS space acts like a parallel plate capacitor with

The AdS space acts like a parallel plate capacitor, with a reflecting upper wall (the boundary) and an absorbing lower wall (the black brane)

You can see this by studying geodesics in this 5d-space

$$m\frac{d^2x^a}{d\tau^2} + \Gamma^a_{bc}\,\dot{x}^a\dot{x}^b = 0$$

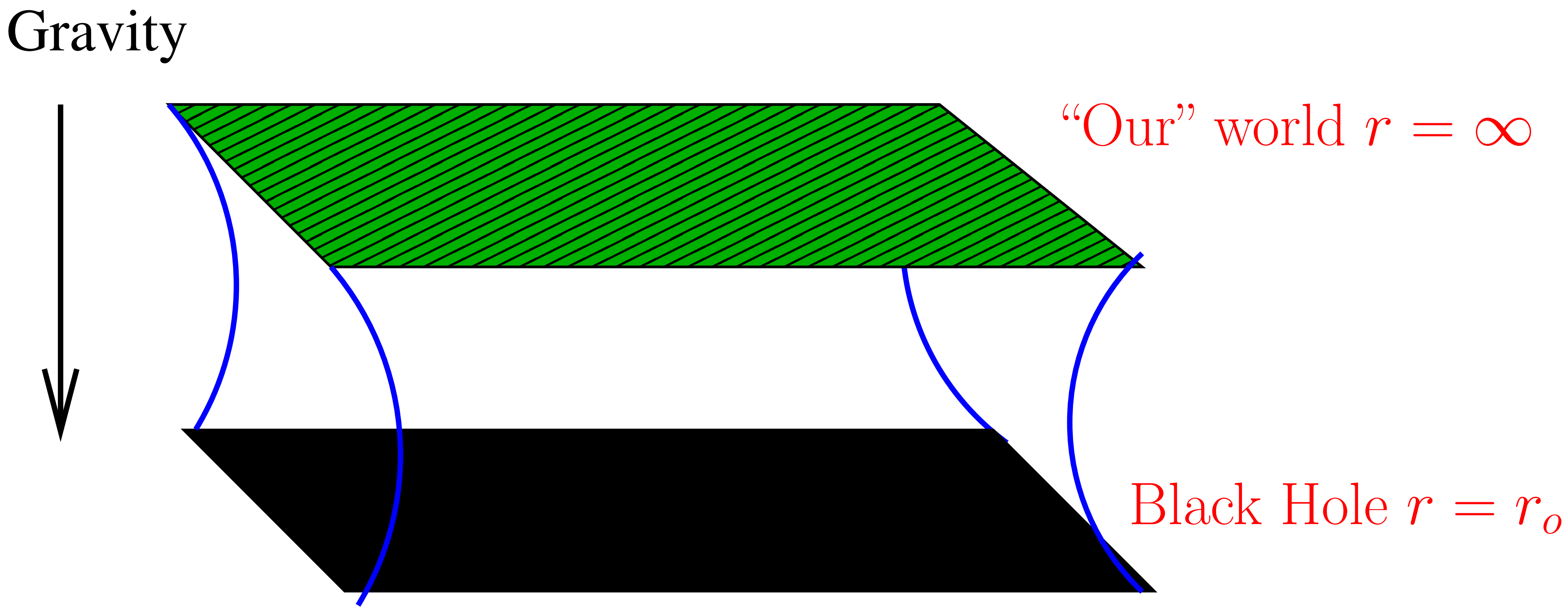


refl

particle reflecting off the boundary and falling into the black brane

The AdS Black Hole

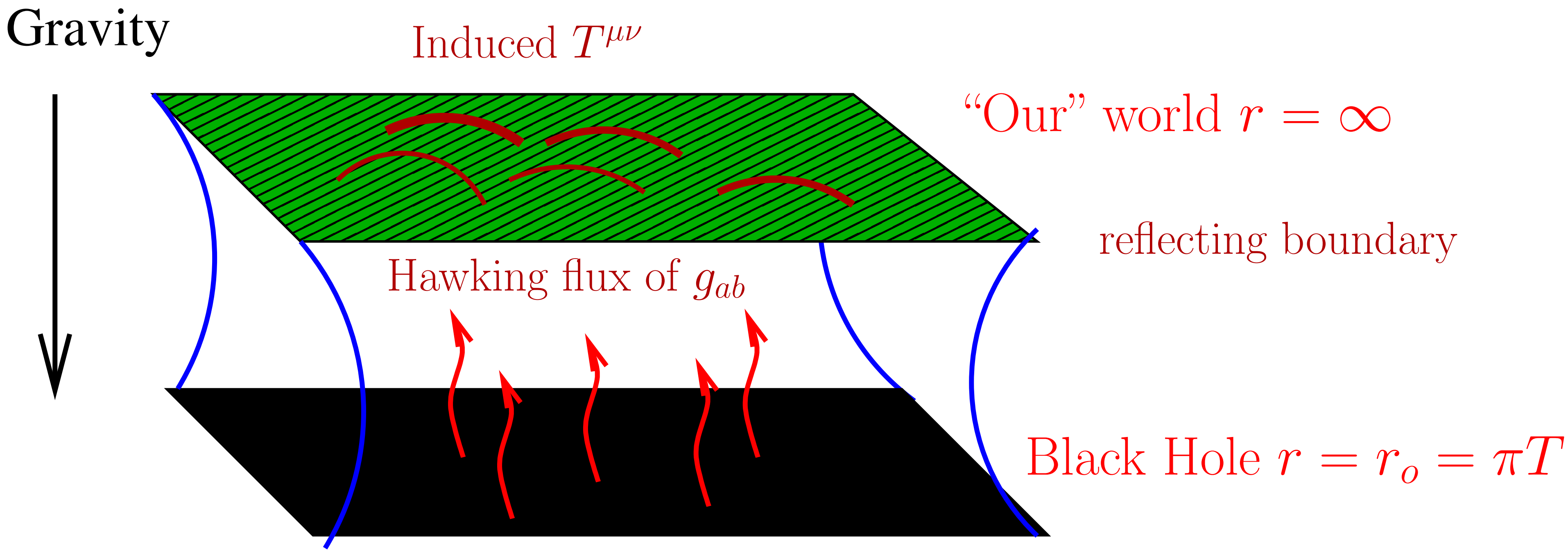
$$ds^2 = \underbrace{\frac{r^2}{L^2}\left(-f(r)dt^2 + d\mathbf{x}^2\right) + \frac{L^2}{f(r)r^2}dr^2}_{\text{AdS geometry}} \qquad \text{where} \qquad f(r) = 1 - \left(\frac{r_o}{r}\right)^4$$



I'm not seeing the QGP here!

The AdS Cavity with Hawking Radiation

$$ds^2 = \underbrace{\frac{r^2}{L^2}\left(-f(r)dt^2 + d\mathbf{x}^2\right) + \frac{L^2}{f(r)r^2}dr^2}_{\text{AdS geometry}} \qquad \text{where} \qquad f(r) = 1 - \left(\frac{r_o}{r}\right)^4$$



The fluctuations are small in large N_c, justifying the classical gravity approximation

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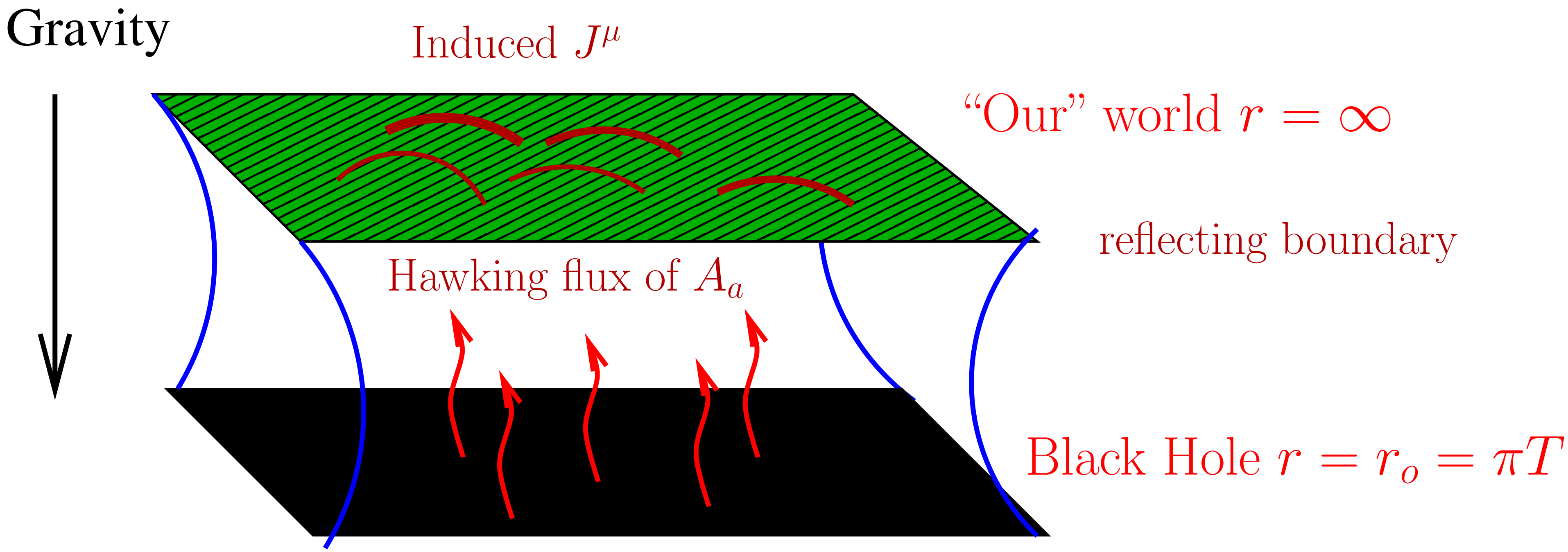

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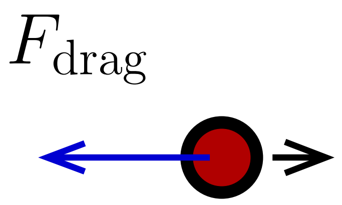
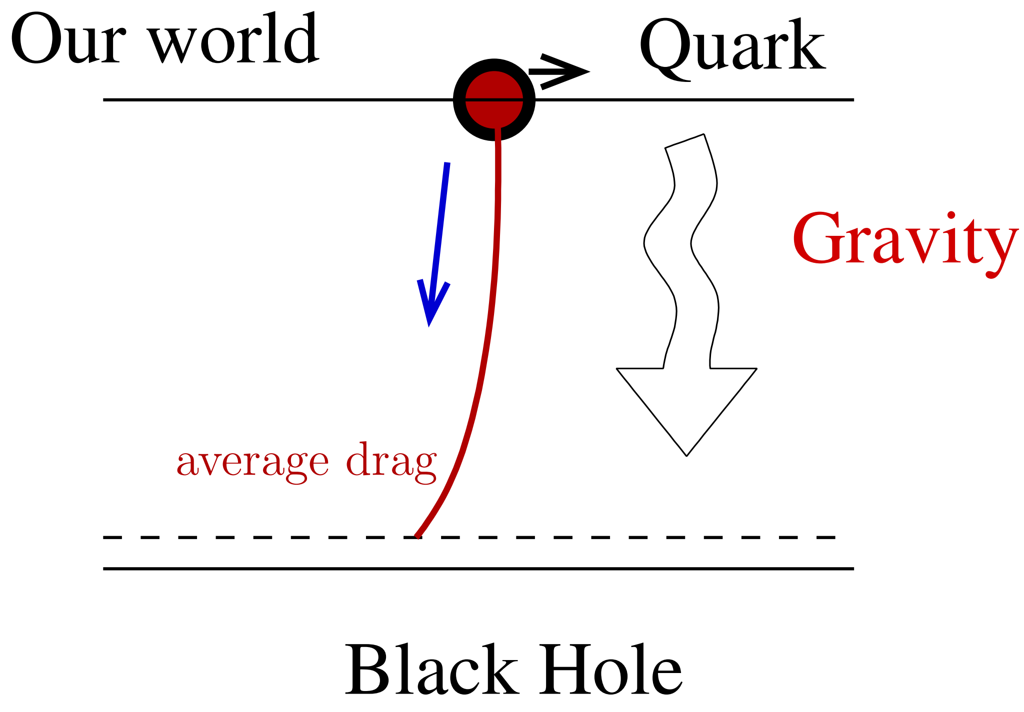
Gauge-Gravity Duality

Quark Drag



$$M\frac{dv}{dt} = -\eta v$$

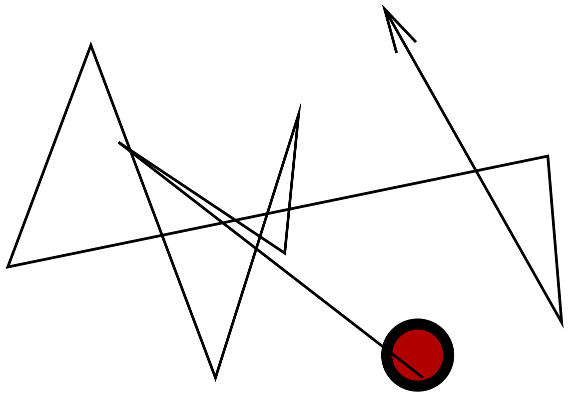
String Pulling on Quark



Its physics not math!

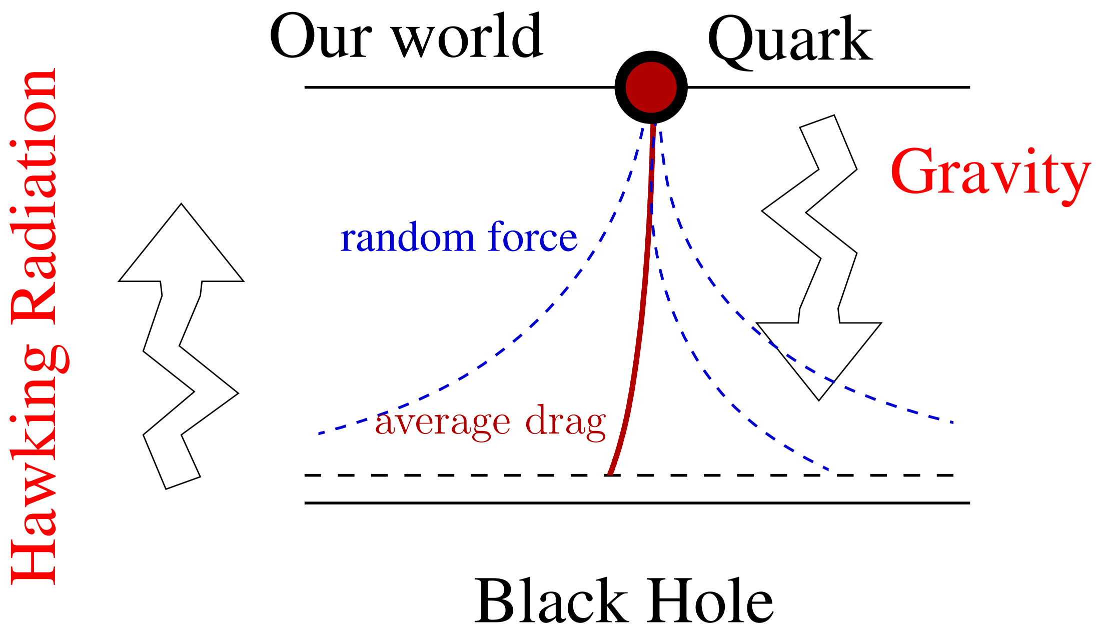
Gauge-Gravity Duality

Brownian Quark



$$M\frac{dv}{dt} = -\eta v + \xi(t)$$

Stochastic String Pulling on Quark



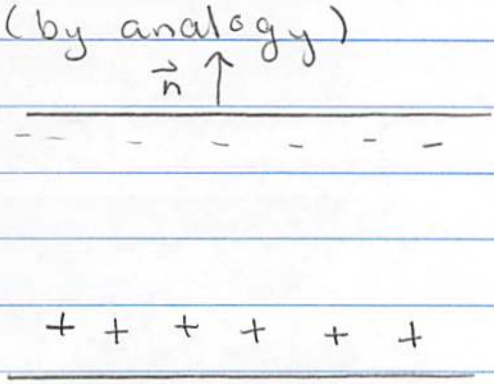
5D equilibrium is a competition between dissipative gravity and hawking radiation:

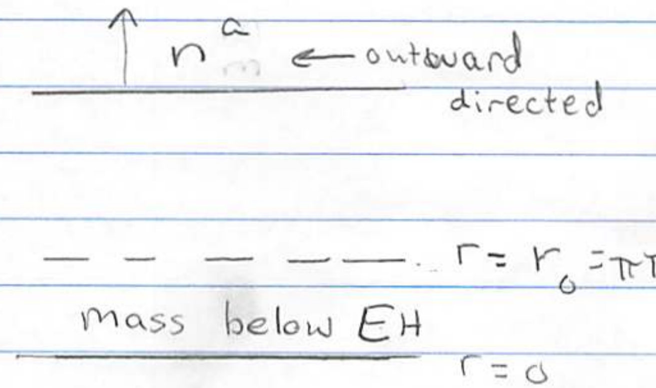
classical probability $\propto e^{-\beta H[x,\pi_x]}$

Again, its physics not math!

Calculation of The Pressure of SYM

(by analogy)





	Parallel Plate	Gravity
Potential	ϕ	g_{ab}
Field	$\vec{E}$	$\Gamma^c_{ab} \sim \partial g$
EOM	$\nabla \cdot E = \rho$	$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \kappa_5^2 T_{ab}$

$$R \sim \partial \Gamma$$

After solving the equations find metric.
Then we need to find the surface charge/stress

$$\sigma = \cancelto{0}{\vec{n} \cdot \vec{E}_{out}} - n \cdot \vec{E}_{in} \qquad \tau^\mu{}_\nu = -\frac{1}{\kappa_5^2} \underbrace{\left[K^\mu{}_\nu - K \delta^\mu{}_\nu \right]}_{\text{Jump in extrinsic curvature}}$$

$$= -n \cdot E_{in}$$

$$\kappa_5^2 = \underbrace{8\pi G}_{\text{newton constant}} = \frac{4\pi^2}{N_c^2} \qquad K_{\mu\nu} = n_a \Gamma^a_{\mu\nu}$$

Find

$$\frac{\varepsilon}{3} = p = \frac{N_c^2}{8\pi^2} (\pi T)^4 = (\text{ideal gas}) \, \frac{3}{4}$$

Greatly influenced by, Arnold, Moore, Yaffe, hep-ph/0209353. For a review with progress to "NLO" see Ghiglieri and Teaney arXiv:1502.03730.

Transport in QCD

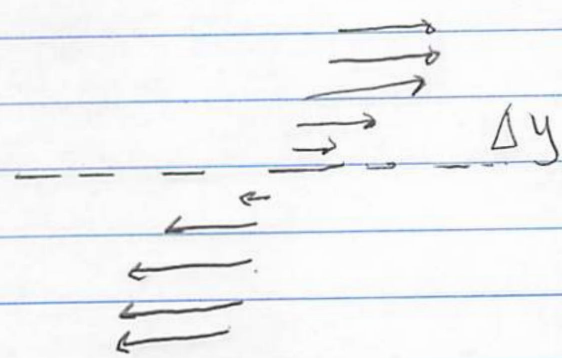
Want to compute how long it takes to transport energy and momentum, i.e. the shear-visosity

$y=0$

$$\frac{F_x}{A^y} = -\eta \frac{\partial u_x}{\partial y} = T^{vis}_{xy}$$

Now the momentum <u>diffuses</u> from the lower to upper stream, So after some time have the following picture

$y=0$



The momentum transferred is $\quad T^{\mu\nu} = (e+p)u^\mu u^\nu + p\, g^{\mu\nu}$

$$\frac{\Delta p^x}{A} = \int_{y>0}^{\Delta y} T^{0x}\, dy \qquad\qquad T^{0\nu} = (e+p)\, u^x$$

$$= \int_{y>0}^{\Delta y} (e+p)\, \Delta u^x(y) = -\int_0^{\Delta y} (e+p) \frac{\partial u^x}{\partial y}\, y$$

$$\Delta p^x = -(e+p)\,\frac{\partial u^x}{\partial y}\,\frac{\Delta y^2}{2}$$

Now the process is diffusive:

$$(\Delta y)^2 = 2 D_\eta \, \Delta t$$

So

$$\frac{F^x}{A} = \frac{\Delta p^x}{A \Delta t} = -(e+p)\, D_\eta \frac{\partial u^x}{\partial y} = -\eta \frac{\partial u^x}{\partial y}$$

Showing

$$\boxed{D_\eta = \frac{\eta}{e+p}}$$ ← the momentum diffusion coefficient

Then we estimate from kinetic theory

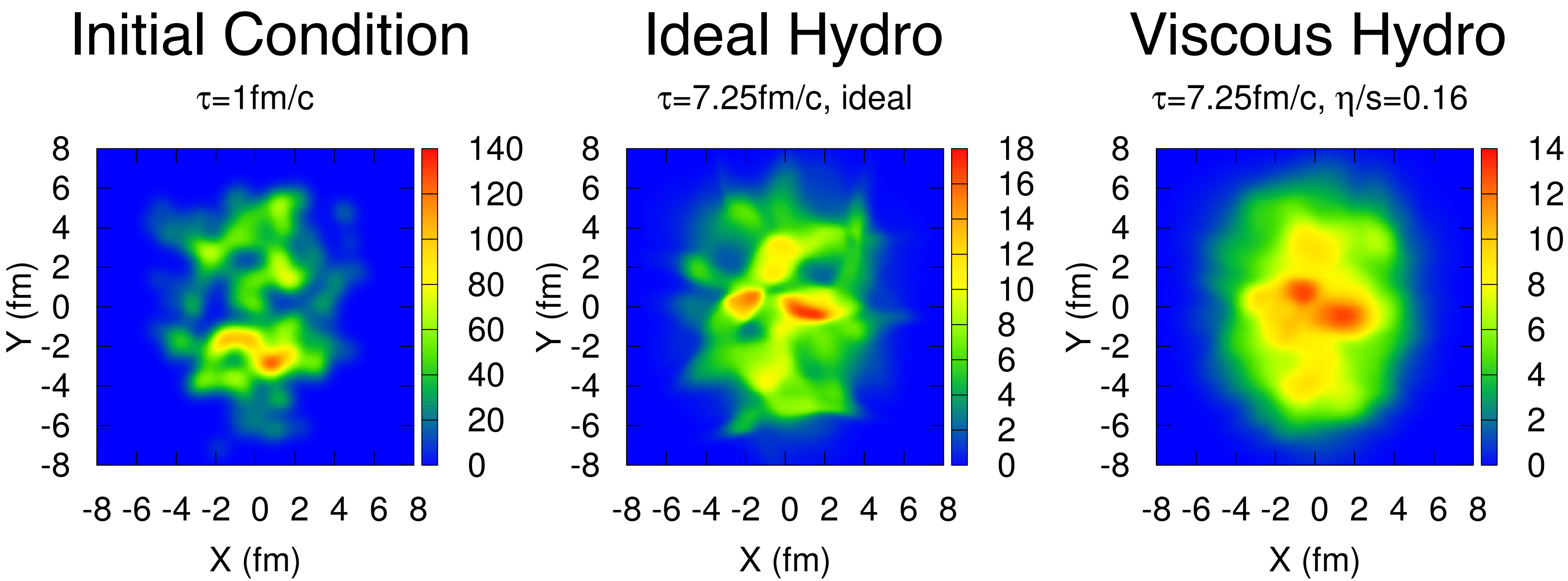
$$D \sim v_{th}^2 \, \tau_R$$ ← typical momentum relaxation time

see Reif, Kittel, ...

$$l_{mfp} = v_{th} \tau_R \qquad D = v_{th}\, l_{mfp}$$

$$[D] = \frac{(\text{distance})^2}{\text{time}}$$

Plots of energy density in a heavy ion collision:



Viscosity diffuses out fluctuations!

Estimate of Shear Viscosity - Weakly Coupled

- Use Kinetic Theory

rough model of this is $C[f] = \frac{\delta f}{\tau_R}$

$$\left(\partial_t + v_p \frac{\partial}{\partial x} \right) f = - C[f]$$

Assumes that the debroglie wavelength is short compared to distance / time between collisions

Three processes

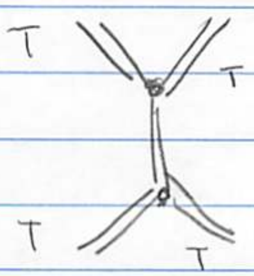
① Collisions $C_{coll}[f]$

② Momentum Diffusion $C_{Diff}[\delta f]$

③ Collinear Bremmsstrahlung $C_{brem}[f]$

The total relaxation rate is $C[f]$ = sum of these

① **Collisions (Hard - Randomizing Collisions)**



$$t_{coll} \sim \frac{1}{n\sigma} \qquad n \sim T^3$$

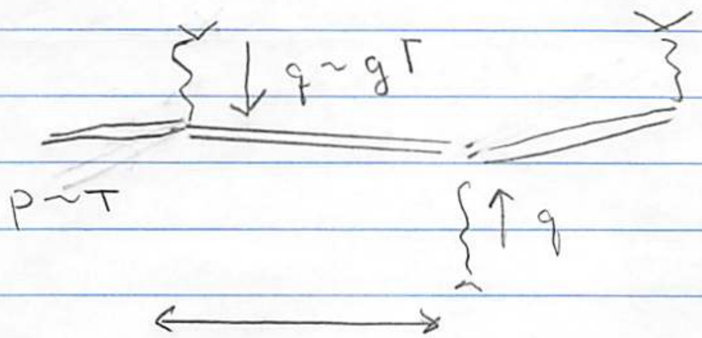
$$\sim \frac{1}{g^4 T} \qquad \sigma \sim \frac{g^4}{T^2}$$

Thus the momentum relaxation time for this process is of order

$$\tau_R \sim \frac{1}{g^4(T)\,T} \qquad \frac{\eta}{s} \sim \tau_R T \sim \frac{1}{g^4(T)}$$

i.e. it is large when g is small

(2) Diffusion of Momentum



$$t_{sc} \sim \frac{1}{g^2 T}$$

It is a random process. The mean squared momentum increases with time

$$(\Delta p)^2 \equiv \hat{q}_{soft}\, t$$

momentum diffusion coefficient

$$\Delta p^2 \sim N_{coll}\, q^2 \sim \frac{t}{t_{sc}} (gT)^2 \sim t \frac{1}{1/g^2T} (gT)^2$$

$$\Delta p^2 \sim \underbrace{g^4(m_D)\, T^3}_{\text{estimate of momentum diffusion coefficient } \hat{q}_{soft}}\; t$$

Normally define

$$(\Delta p)^2 = \hat{q}_{soft}\, t$$

mean

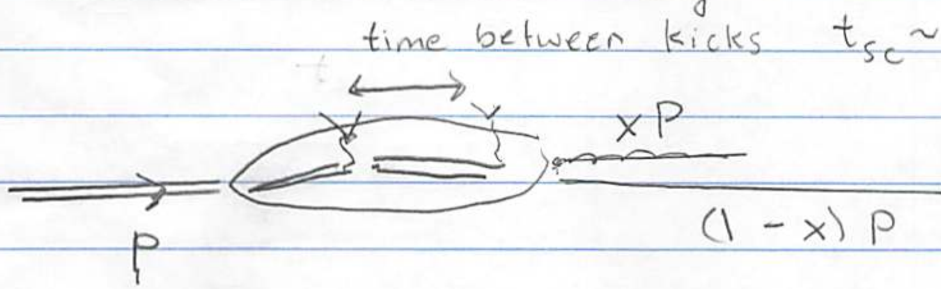
Then when $(\Delta p)^2 \sim T^2$ the momentum is fully relaxed. This happens when $t = \tau_R$ is

$$\tau_R \sim \frac{T^2}{\hat{q}_{soft}} \sim \frac{1}{g^4(m_D)\, T}$$

agin

$$\frac{\eta}{s} \sim \frac{1}{g^4(m_D)} \sim \frac{T^3}{\hat{q}_{soft}}$$

③ <u>Collinear Bremstrahlung</u>



- The charged particle is acclerating due to diffusion. Relativistic particles radiate. Every kick has a probability of g^2 of radiating, i.e, it takes $\frac{1}{g^2}$ soft kicks before you radiate

- Thus the time scale for a radiation is

$$\frac{t_{rad}}{t_{sc}} \sim \frac{1}{g^2} \qquad t_{rad} \sim \frac{1}{g^4 T}$$

A radiation completely changes the momentum

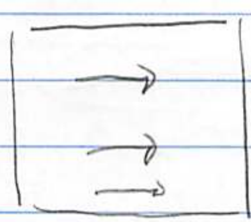
$$\tau_R \sim t_{rad} \sim \frac{1}{g^4 T}$$

Thus

$$\frac{\eta}{s} \sim \frac{1}{g^4}$$

Linear Response & Shear Viscosity

How to compute the conductivity



Turn on a time dependent electric field $E(t) = E_0 e^{-i\omega t}$

$$J^x = \sigma E_0^x = \sigma F^x{}_t$$

Then since $E^x(t) = -\partial_t A^x(t)$

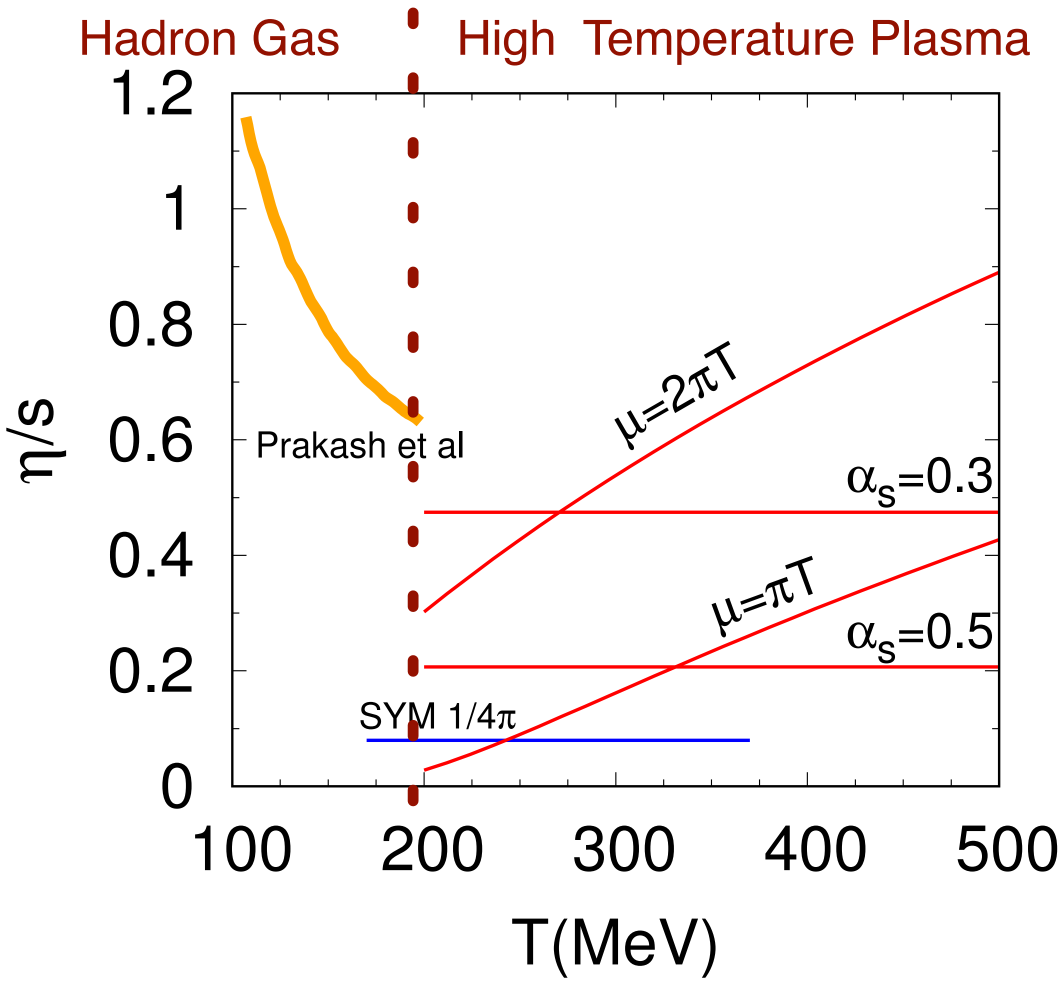
$$J^x(\omega) = \underbrace{+i\omega\sigma}_{\text{green function}} A^x$$

For the shear viscosity we have something similar

Real calculation of η/s at LO

Arnold, Moore, Yaffe (red curves)

$$\frac{\eta}{s} = \frac{1}{\alpha_s^2}\Big(\underbrace{C_0 + C_1 \log(m_D/T)}_{\text{AMY-Leading order}} + \underbrace{C_2\,(m_D/T)}_{\text{“NLO”, i.e. order g}} + \ldots\Big)$$

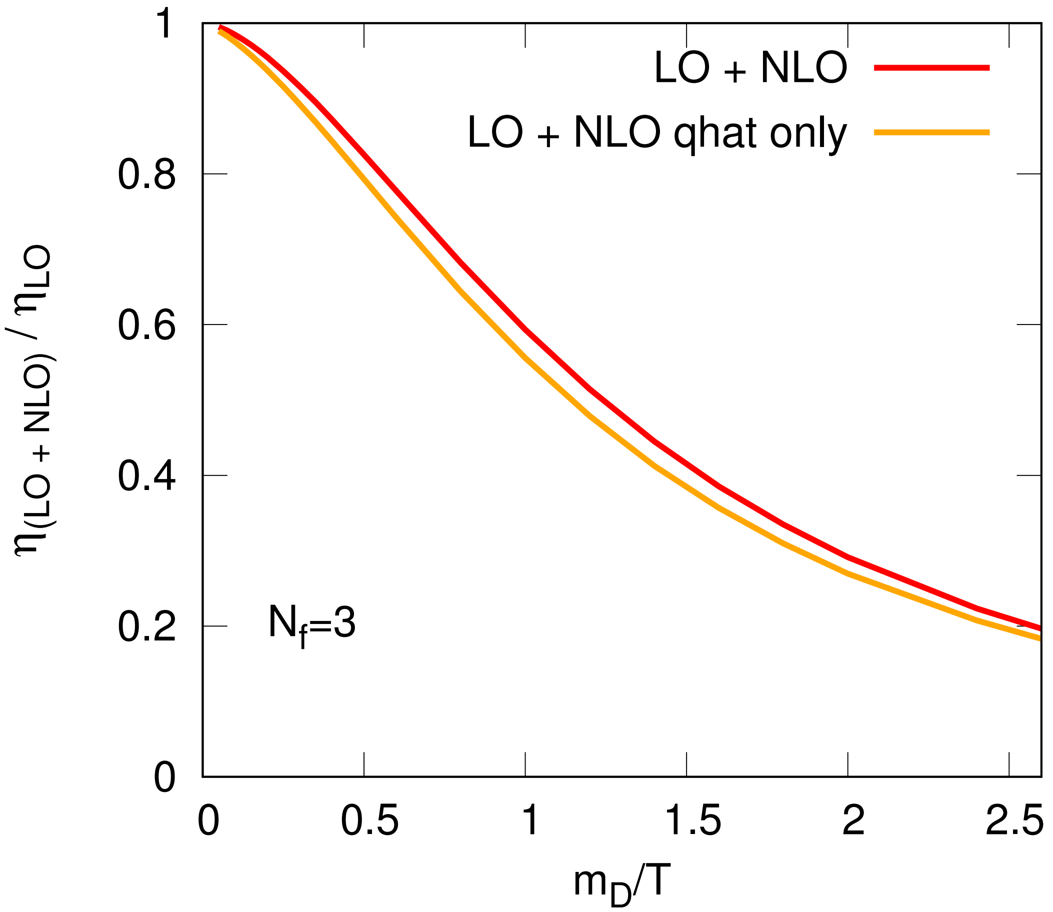


At leading order you can get whatever you want for η/s!

Calculation of η/s at NLO

Moore, Ghiglieri, Teaney

$$\frac{\eta}{s} = \frac{1}{\alpha_s^2}\Big(\underbrace{C_0 + C_1 \log(m_D/T)}_{\text{AMY-Leading order}} + \underbrace{C_2\,(m_D/T)}_{\text{“NLO”, i.e. order g}} + \ldots\Big)$$



For any reasonable value of the coupling the first correction is huge!

Correction is dominated by $\hat{q}_{\rm soft}$

A radiation completely changes the momentum

$$\tau_R \sim t_{rad} \sim \frac{1}{g^4 T}$$

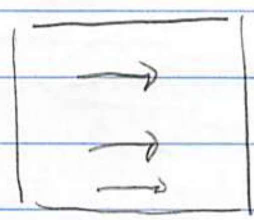
Thus

$$\frac{\eta}{s} \sim \frac{1}{g^4}$$

Start of strong coupling calculation of shear viscosity. The orginal paper, hep-th/0205051, is very clear.

Linear Response & Shear Viscosity

How to compute the conductivity



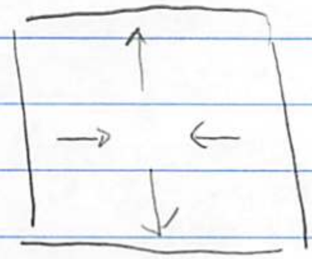
Turn on a time $E(t) = E_0 e^{-i\omega t}$ dependent electric field

$$J^x = \sigma E_0^x = \sigma F^x{}_t$$

Then since $E^x(t) = -\partial_t A^x(t)$

$$J^x(\omega) = \underbrace{+i\omega\sigma}_{\text{green function}} A^x$$

For the shear viscosity we have something similar



Γ^y_{xt}

$$h_{xy}(t) = h^0_{xy}\, e^{-i\omega t}$$

Create grav forces (per area) by turning on a gravitational field. The induced forces (stress) compensate the gravitational ones

$$T^y_x = \eta\, \Gamma^y_{xt} = -\eta\, \partial_t h_{xy}$$

This follows from the hydro eqns

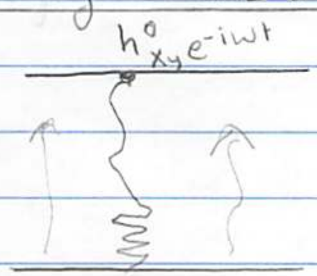
$$T_x{}^y = -\eta \nabla_x u^y + \nabla^y u_x - \frac{2}{3}\delta_x{}^y \nabla\cdot u$$

covariant deriv

Find

$$T^{xy} = +i\omega\, \eta\, h^0_{xy}$$

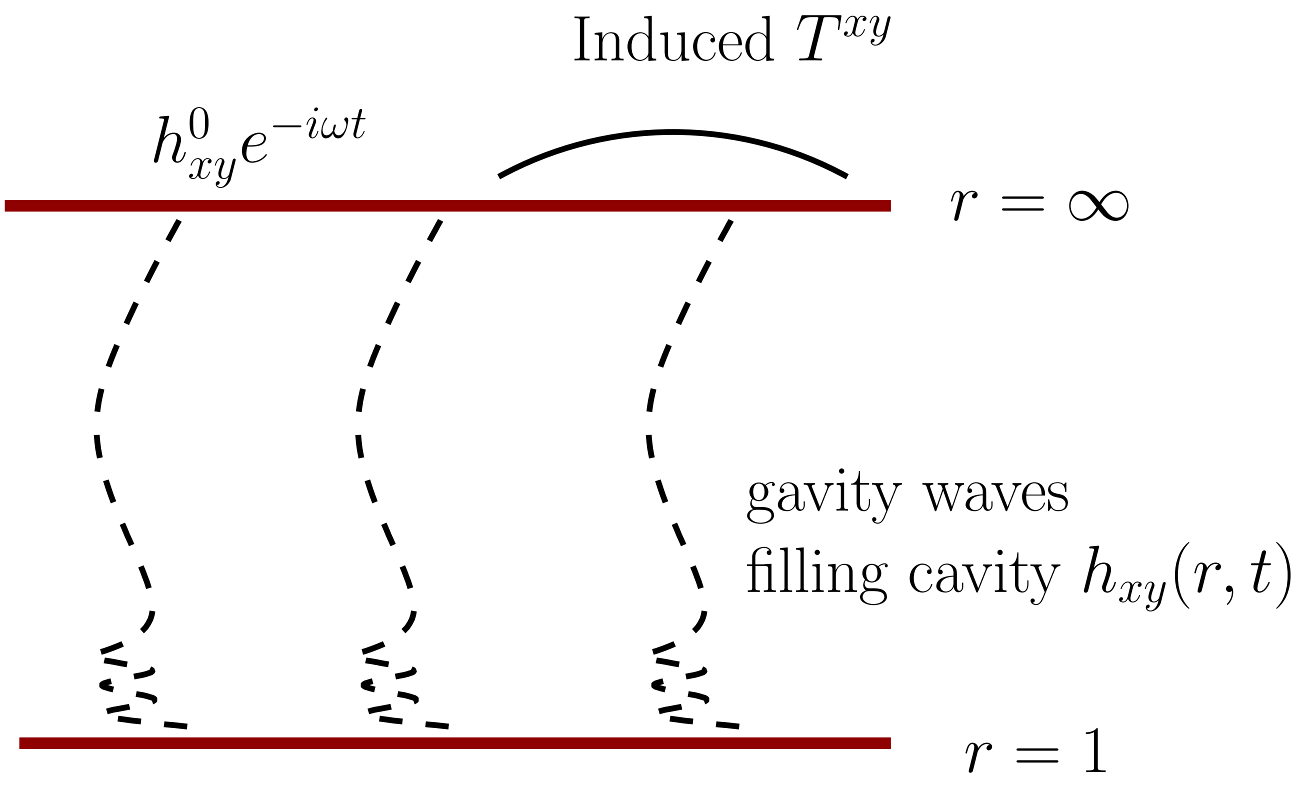
Doing Linear Response in AdS/CFT



bndry $r=\infty$

Black Hole $r=1$

η/s from AdS/CFT



where, $u \equiv 1/r^2$

$$h_{xy} = h^o_{xy}(\omega)\,(1-u)^{-i\omega/4\pi T}\left[1 - \frac{i\omega}{4\pi T}\log(1+u) + O(\omega^2)\right].$$

Solve and evaluate the induced stress from the discontinuity in the extrinsic curvature

① Turn on an external field $h^0_{xy} e^{-i\omega t}$ in the boundary

② Solve for the steady state solution in the interior

$$h_{xy}(t,r) = h_{xy}(r)\, e^{-i\omega t}$$

Technically this means that we solve the wave equation with the constraints that

$$h_{xy}(r,t) \xrightarrow[r \to \infty]{} h^0_{xy}$$

And require that the waves are ingoing at the horizon

③ Once we have the steady state solution we may evaluate the boundary stress.

Find

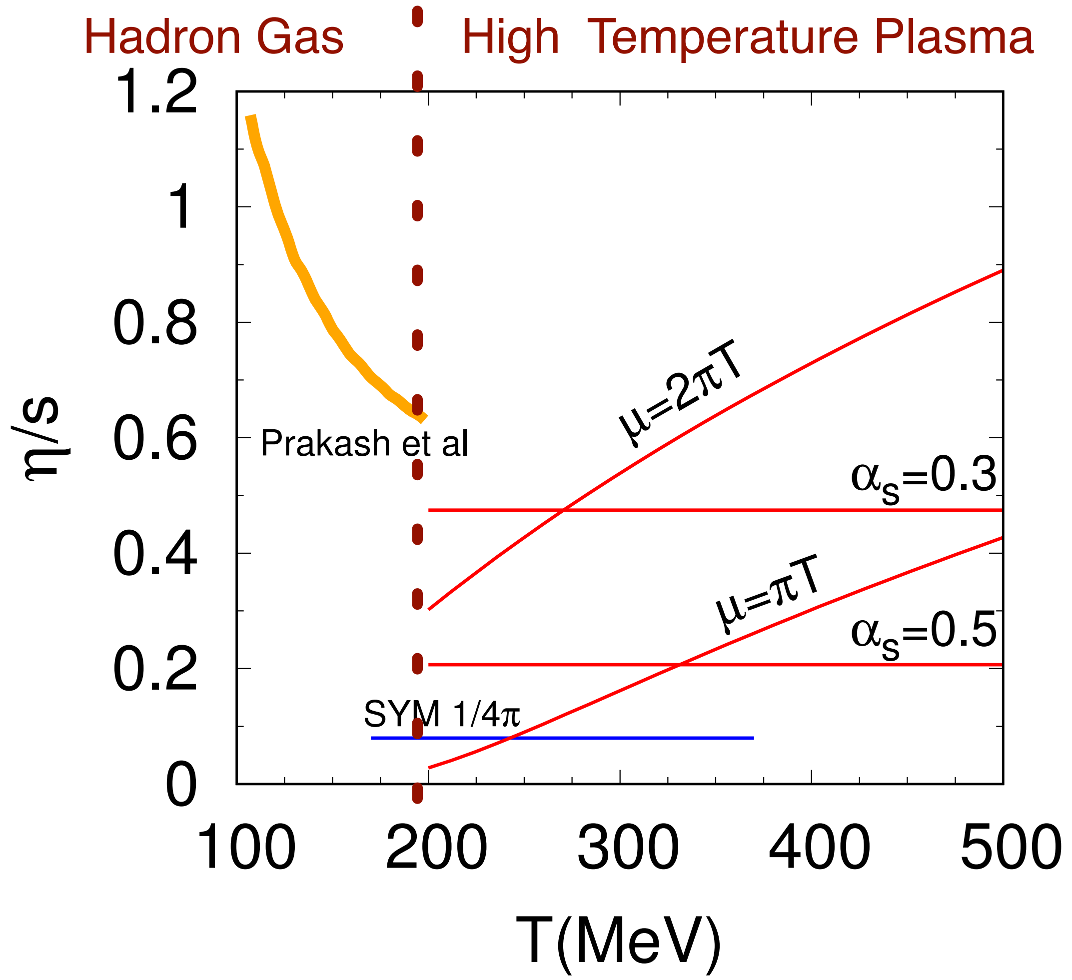
$$T_{xy} = +i\omega \eta\, h^0_{xy}$$

where $\frac{\eta}{s} = \frac{1}{4\pi}$

Real calculation of η/s

Arnold, Moore, Yaffe (red curves)

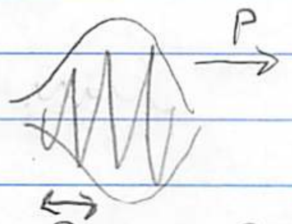
$$\frac{\eta}{s} = \frac{1}{\alpha_s^2}\Big(\underbrace{C_0 + C_1 \log(m_D/T)}_{\text{AMY-Leading order}} + \underbrace{C_2\, g}_{\text{“NLO”}} + \ldots\Big)$$



Summary

Weak Coupling → Dressed Quasi-particles

$p \sim T$ $\qquad\qquad$ $\frac{\lambda_{debrog}}{c} \sim \frac{\hbar}{T}$



$\frac{\lambda_{debrog}}{c} \sim \frac{\hbar}{T} \ll \tau_R$

Then $\quad \tau_R \sim \frac{\hbar}{g^4 T}$ $\quad$ was the time scale for order 1 change in momentum

Thus

$$\frac{\eta}{s} \sim \tau_R T \sim \frac{\hbar}{g^4} \gg \hbar$$

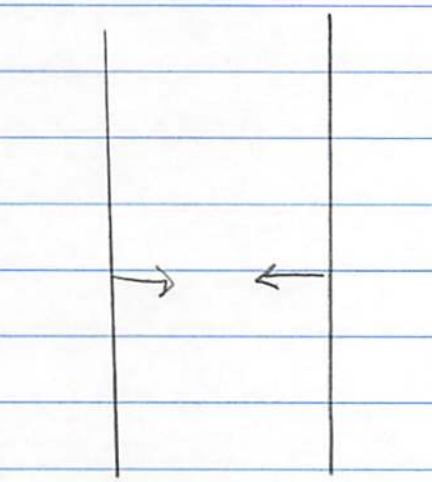
- Strong Coupling

$\frac{\eta}{s} = \frac{\hbar}{4\pi}$ ← any notion of independent quarks & gluons is impossible

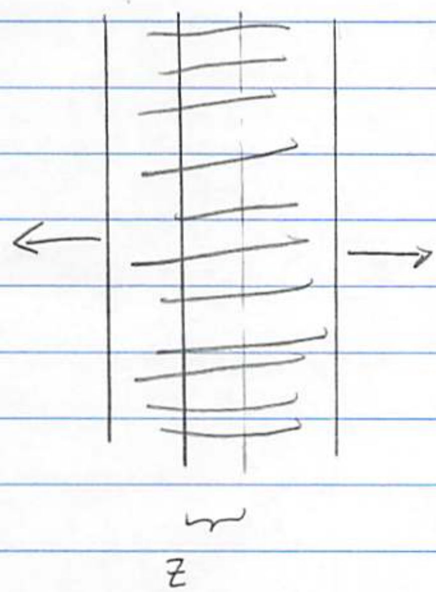
Since debroglie wavelength is the same order as the relaxation time

Thermalization of Strongly Coupled Plasmas

The Setting



Two nuclei pass through each other



Picture at time t

stuff at position z is moving with velocity

$$V^z = \frac{z}{t}$$

At early times the expansion is fast. Indeed the expansion rate is

$$\text{expansion rate} = \frac{1}{V}\frac{dV}{dt} = \partial_z V^z = \frac{1}{t}.$$

We should compare the collisional

rate to the expansion rate. For Hydro, to be valid requre

$$\underbrace{\frac{1}{\tau_R}}_{\text{collisional}} \gg \underbrace{\frac{1}{\tau}}_{\text{expansion rate}}$$

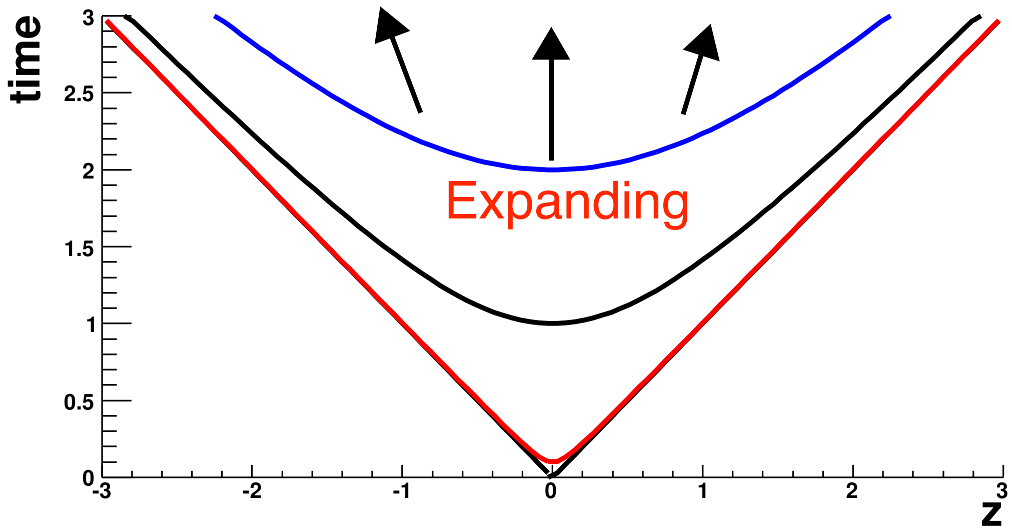
i.e.

$$\alpha_s^2 T \gg \frac{1}{\tau}$$

or

$$\frac{1}{\alpha_s^2} \ll \tau T$$

Bjorken Expansion at weak coupling:



- Condition for hydro to apply:

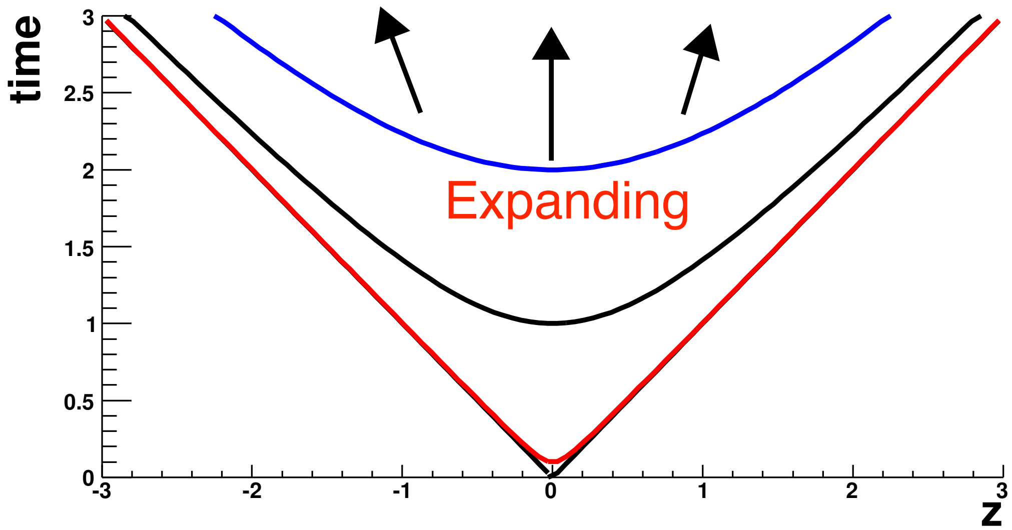
$$\underbrace{\text{Collision rate}}_{\sim\alpha_s^2 T_o} \gg \underbrace{\text{Expansion Rate}}_{\sim 1/\tau_o}$$

Find:

1. For a fixed coupling α_s, need $T\tau$ larger enough to have hydro

$$\frac{1}{\alpha_s^2} \ll \tau T$$

Bjorken Expansion at weak coupling:



- Condition for hydro to apply:

$$\underbrace{\text{Collision rate}}_{\sim\alpha_s^2 T_o} \gg \underbrace{\text{Expansion Rate}}_{\sim 1/\tau_o}$$

Find:

1. For a fixed coupling α_s, need $T\tau$ larger enough to have hydro

$$\underbrace{\frac{1}{\alpha_s^2} \ll \tau T}_{\text{What about when } \alpha_s \to \infty \text{ ???}}$$

Strong coupling answer: (M. Heller et al, PRL)

- Find at strong coupling must have

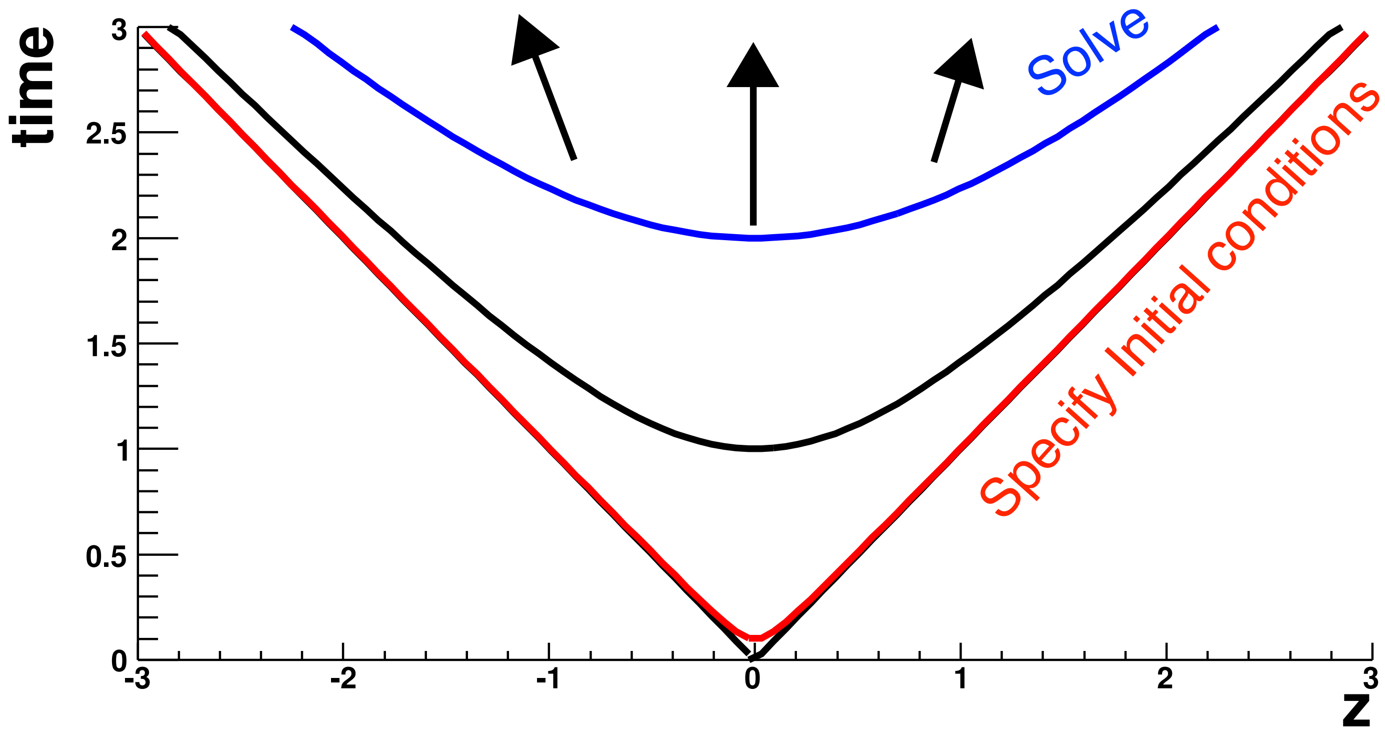
$$0.65 \lesssim \tau_o T_o$$

  before we can use (viscous) hydro

  - I will review work of Michal Heller, R. Janik, R. Pechanski
  - See also recent work by Keegan, Kurkela, Romatschke, van der Schee, Zhu

The setup

- Specify intial conditions and solve

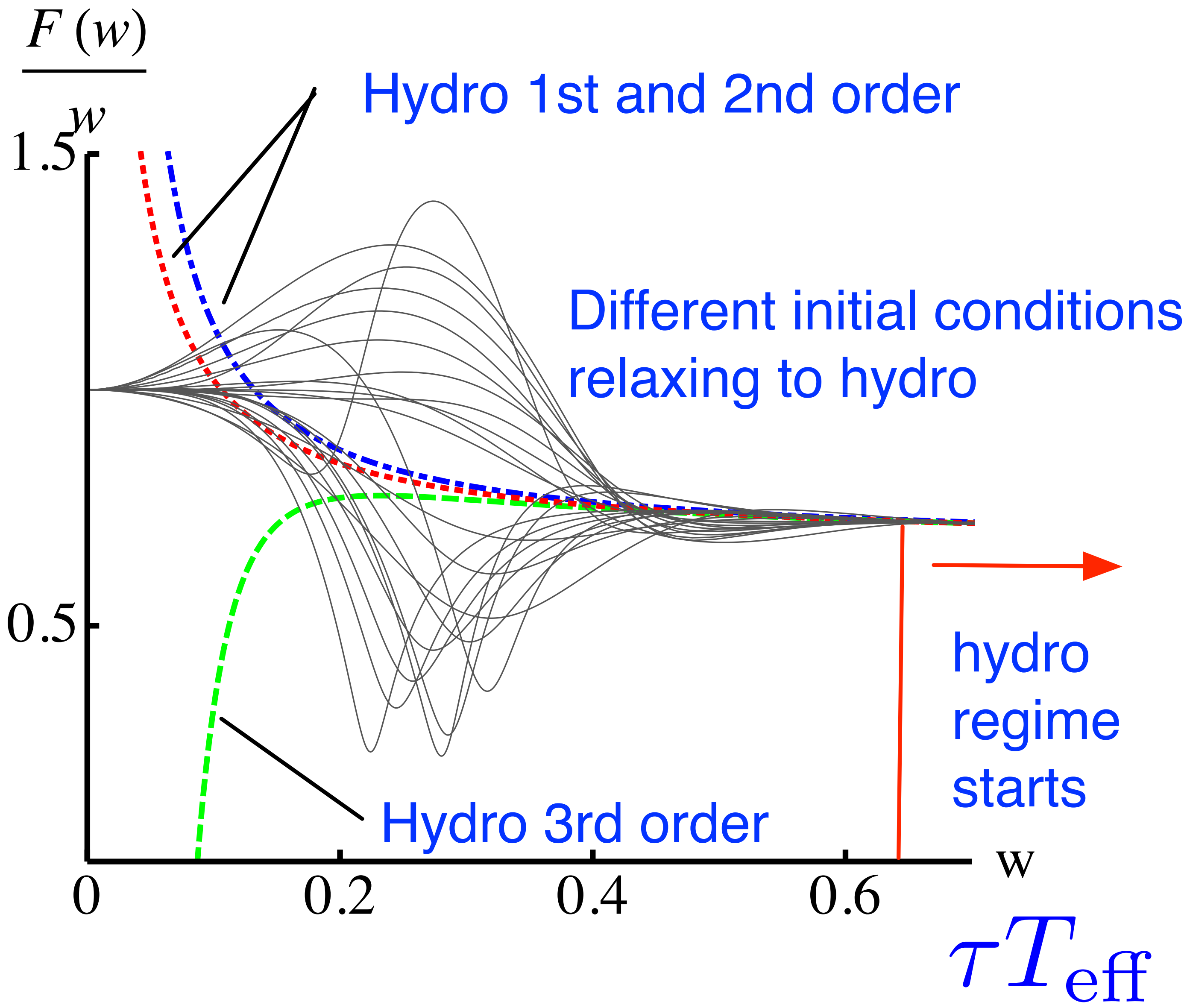


- Immense number of initial conditions with the same initial energy density
  - * Specify initial conditions in the fifth dimension
- Specify an *effective* temperature $T_{\rm eff}(\tau)$ from the energy density at all times

$$\text{energy density}(\tau) \equiv \text{(constant)}\,(T_{\rm eff}(\tau))^4$$

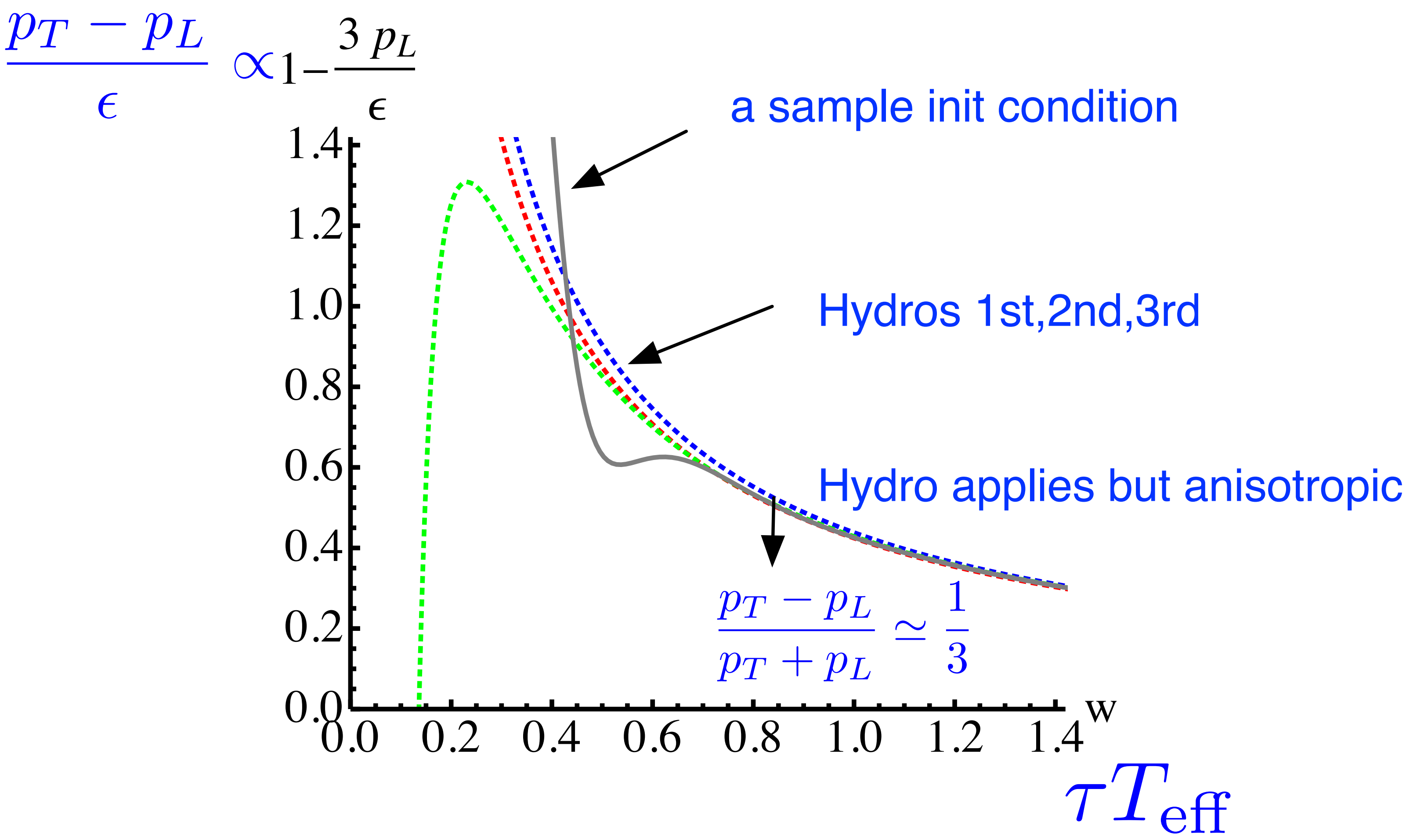
Result:

$$\frac{1}{T_{\text{eff}}}\,\frac{\mathrm{d}(\tau T_{\text{eff}})}{d\tau}$$



Remarkably fast convergence to the universal hydro regime

But, viscous (anisotropic) corrections are important for everything



Viscous (anisotropic) corrections are important for *all* observables.

Hydrodynamics is seemingly much more robust than what one expects!