Lattice QCD at non-zero temperature and density

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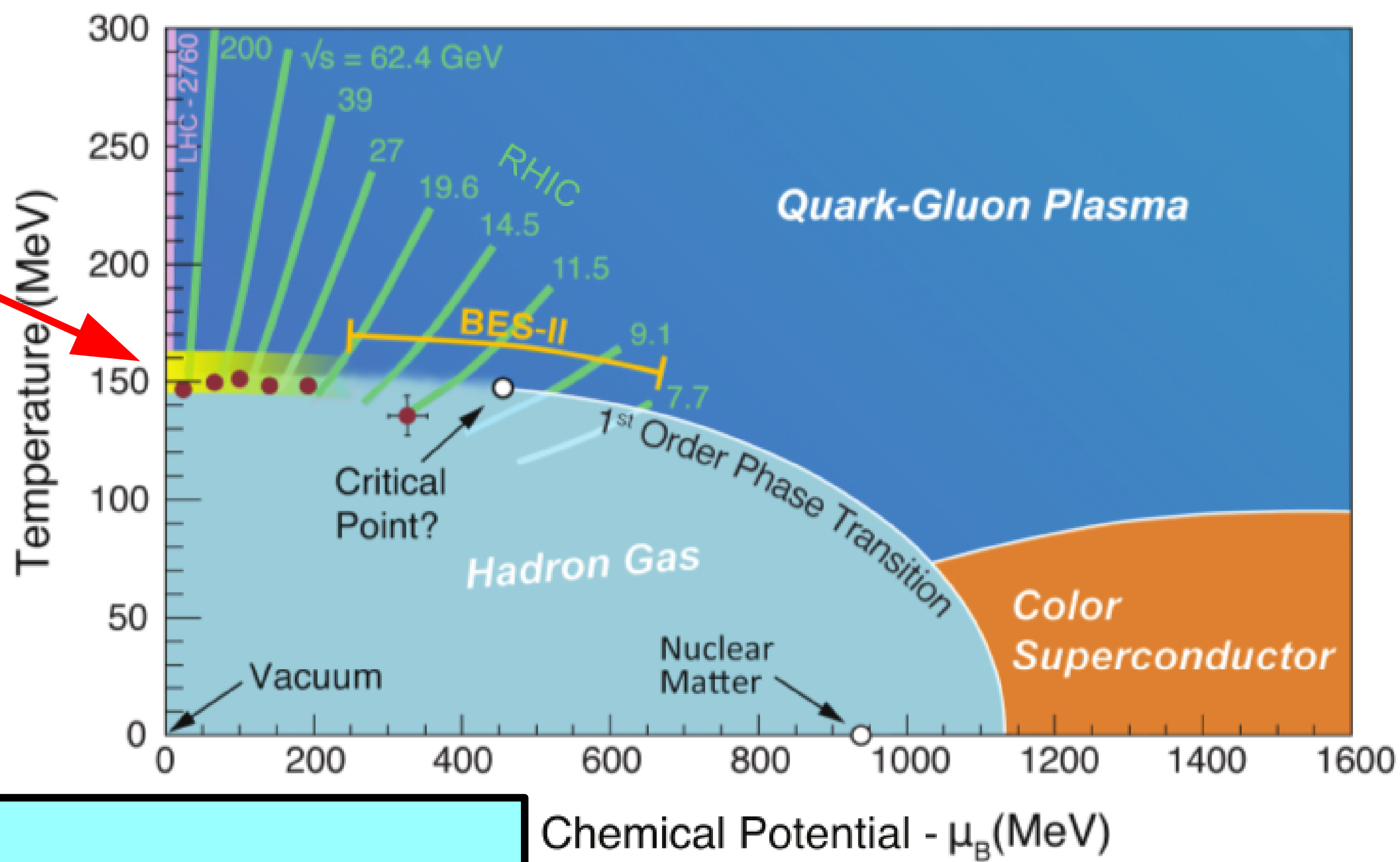
- QCD in a nutshell, non-perturbative physics, lattice-regularized QCD, Monte Carlo simulations
- the phase diagram on strongly interacting matter, chiral symmetry restoration, the equation of state
- finite density QCD, cumulants of conserved charge fluctuations, thermal masses ~~& transport properties~~

Phases of strong-interaction matter

chiral phase transition (crossover)



phase structure:

order parameter: $$\frac{\langle\bar{\psi}\psi\rangle_l}{T^3} = \frac{1}{VT^3}\frac{1}{4}\frac{\partial \ln Z}{\partial m_l/T}$$

chiral susceptibility: $$\frac{\chi_l}{T^2} = \frac{\partial\langle\bar{\psi}\psi\rangle_l/T^3}{\partial m_l/T}$$

phase transitions are related to the spontaneous breaking/restoration of global symmetries

Symmetries of QCD

$$\mathcal{L}_E = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_{j,a} \left(\sum_{\nu=0}^{3} \gamma_\nu \left(\partial_\nu - i \frac{g}{2} \mathcal{A}^a_\nu \lambda^a \right) + m_j \right)^{a,b} \psi_{j,b}$$

– symmetries of QCD: $U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$

– chiral decomposition: $\psi \equiv (\psi_1, ...\psi_{n_f}) = \psi_L + \psi_R$

$$P_\epsilon = \frac{1}{2}(1 + \epsilon\gamma_5) \ , \ \epsilon = \pm 1 \qquad P_\epsilon^2 = P_\epsilon \ , \ P_+ P_- = 0$$

$$\psi_L = P_+ \psi \ , \ \psi_R = P_- \psi \qquad \bar{\psi}_L = \bar{\psi} P_- \ , \ \bar{\psi}_R = \bar{\psi} P_+$$

$$\mathcal{L}_F \sim \bar{\psi}_L \not{D}_\mu \psi_L + \bar{\psi}_R \not{D}_\mu \psi_R - m_q (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$U_V(1)$ **:** baryon number $\psi^\Theta = \mathrm{e}^{i\Theta} \psi \ , \ \bar{\psi}^\Theta = \bar{\psi} \mathrm{e}^{-i\Theta}$

$U_A(1)$ **:** axial symmetry $\psi^\Theta = \mathrm{e}^{i\Theta\gamma_5} \psi \ , \ \bar{\psi}^\Theta = \bar{\psi} \mathrm{e}^{i\Theta\gamma_5}$

$SU_{L/R}(n_f)$ **:** flavor symmetry $G_\epsilon \equiv P_{-\epsilon} \cdot 1 + P_\epsilon U_\epsilon \ , \ U_\epsilon \in U(n_f)$

$$G \equiv G_+(U_+) G_-(U_-)$$

$$\psi' = G\psi \ , \ \bar{\psi}' = \bar{\psi} G^\dagger$$

Chiral phase transition

Which symmetry is restored?

$$U_L(n_f) \times U_R(n_f) \;\Leftrightarrow\; U_V(1) \times U_A(1) \times \underbrace{SU_L(n_f) \times SU_R(n_f)}_{n_f=2\ (u,d):\ O(4)}$$

exact: baryon number conservation

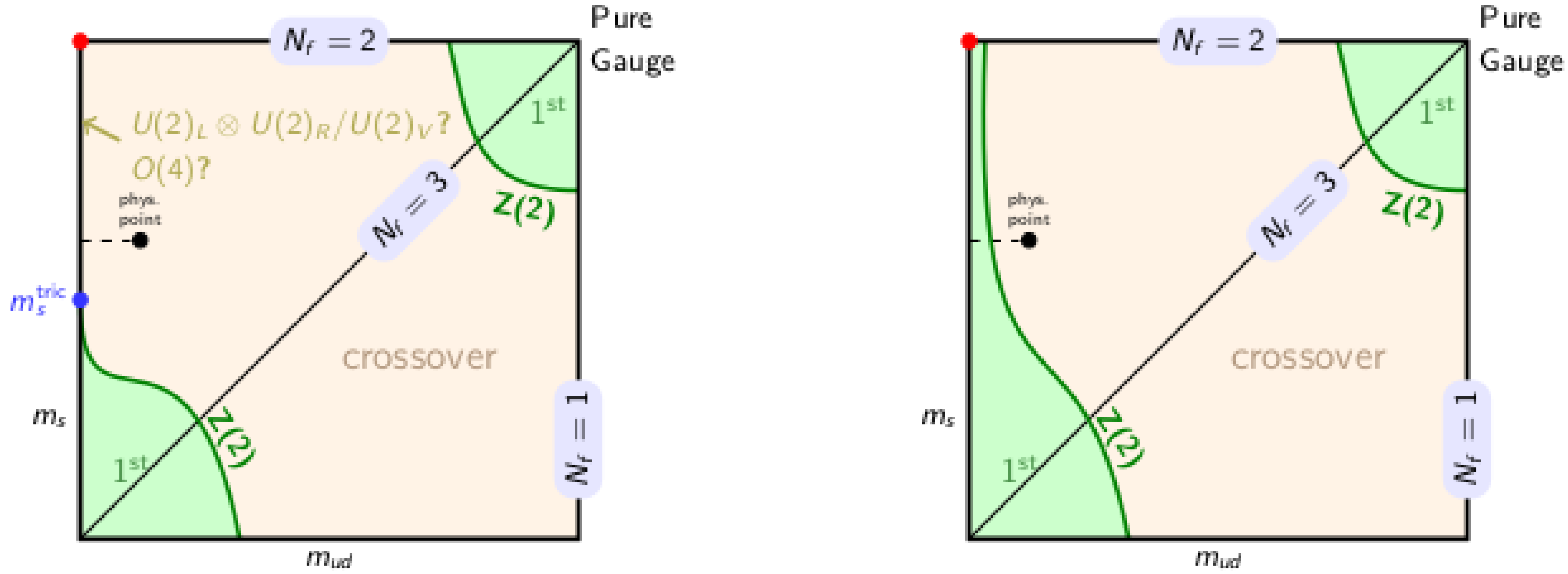
axial anomaly

$n_f = 2\ (u,d) : O(4)$

$n_f = 2:$

standard scenario: $U_A(1)$ remains broken, chiral limit controlled by $O(4)$

alternative scenario: $U_A(1)$ "effectively" restored, first order transition possible



R. Pisarski, F. Wilczek, PRD29 (1984) 338

Chiral symmetry breaking and restoration

staggered (or Kogut-Susskind) fermions do have a global U(1)xU(1) symmetry (remnant of the chiral SU(nf)xSU(nf))

$U(1) \times U(1)$: independent phase transformations on even and odd sites of the lattice

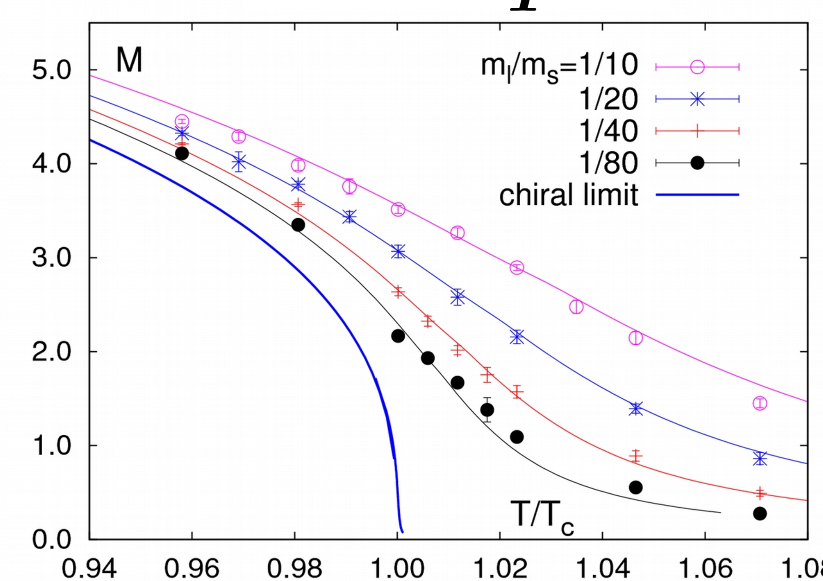
$$\psi'_e = \mathrm{e}^{i\theta_1}\psi_e \quad , \quad \bar{\psi}'_e = \mathrm{e}^{-i\theta_2}\bar{\psi}_e$$

$$\psi'_o = \mathrm{e}^{i\theta_2}\psi_o \quad , \quad \bar{\psi}'_o = \mathrm{e}^{-i\theta_1}\bar{\psi}_o$$

one parameter, continuous global symmetry

$$M \equiv \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4}$$

→ its spontaneous breaking generates one Goldstone pion $m_\pi \sim m_l^2$



Universality and the Chiral Phase Transition

- close to the chiral limit thermodynamics in the vicinity of the QCD transition is controlled by a universal O(4) scaling function

singular

regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,) = -f_s(t/h^{1/\beta\delta}) - f_r(V,T)$$

critical point:
$t \equiv 0 \quad , \quad h \equiv 0$

$$t \sim \frac{T - T_c}{T_c} \quad , \quad h \sim \frac{m_l}{T}$$

	O(4)
α	-0.213
β	0.380
δ	4.824

$$(2-\alpha)/\beta\delta = 1 + 1/\delta$$

$$M_b \equiv \frac{m_s \langle \bar\psi\psi\rangle_l}{T^4} = \frac{1}{VT^3}\frac{m_s}{T}\frac{1}{2}\frac{\partial \ln Z}{\partial m_l/T} = h^{1/\delta} f_G(z)$$

$$\frac{\chi_l}{T^2} = \frac{\partial \langle \bar\psi\psi\rangle_l / T^3}{\partial m_l/T} \sim h^{1/\delta - 1} f_\chi(t/h^{1/\beta\delta}) \quad , \quad z = t/h^{1/\beta\delta}$$

$$\frac{\mathrm{d} f_\chi(z)}{\mathrm{d} z} = 0 \Leftrightarrow z_{max}$$

– defines pseudo-critical $T_c(m_l)$

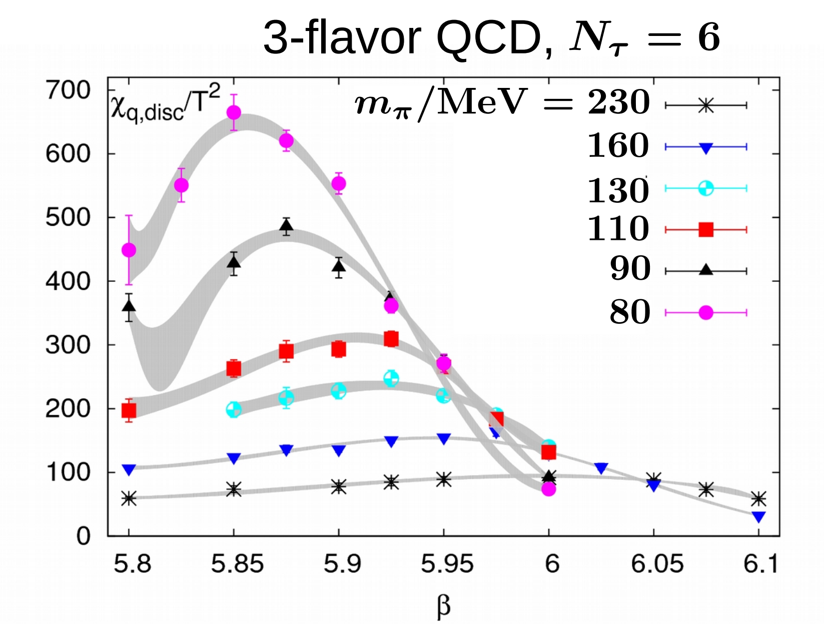
– scaling: $\chi_l(m_l)/T^2 \sim m_l^{1/\delta - 1}$

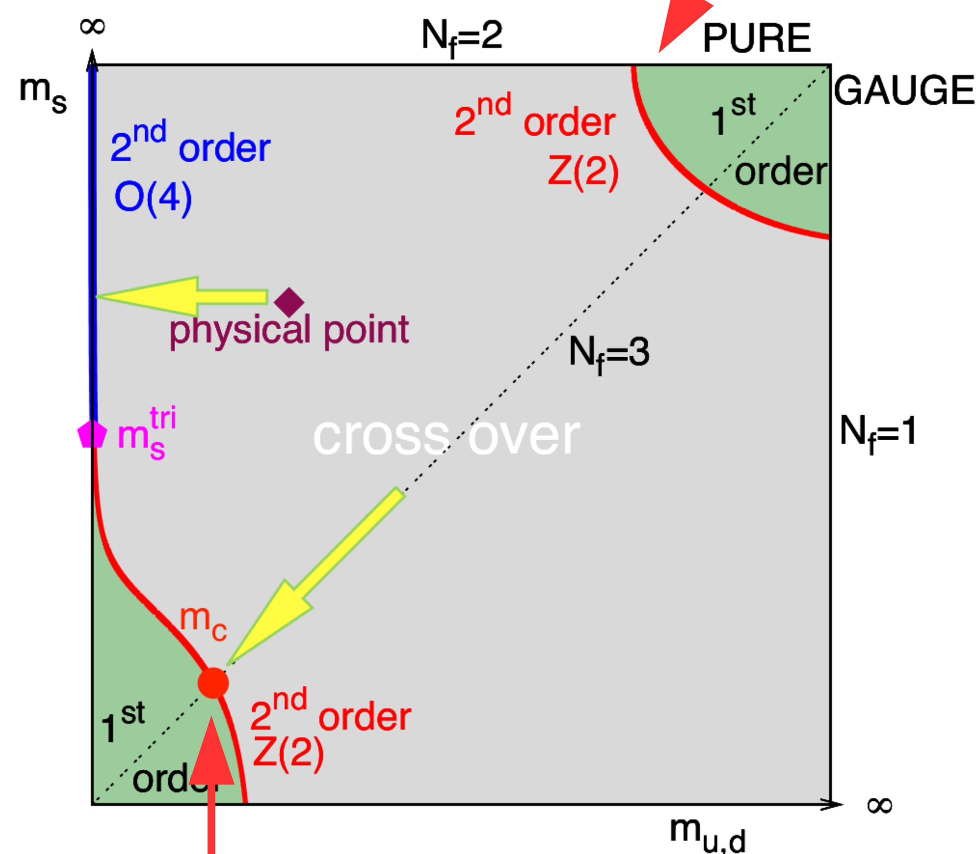
Chiral phase transition

Chiral susceptibility

$m_\pi^{crit.} \sim \ 3 \text{ GeV}$

$$\chi_l(t,h) \sim \frac{\partial M}{\partial h} = h^{1/\delta-1} f_\chi(t/h^{1/\beta\delta}) + \text{regular}$$





Exploring the chiral limit in (2+1)- and 3-flavor QCD

A. Bazavov et al, arXiv:1701.03548

$m_\pi^{crit.} \leq \ 50 \text{ MeV}$

$$(\chi_l/T^2)^{max} \sim m_l^{1/\delta-1}$$

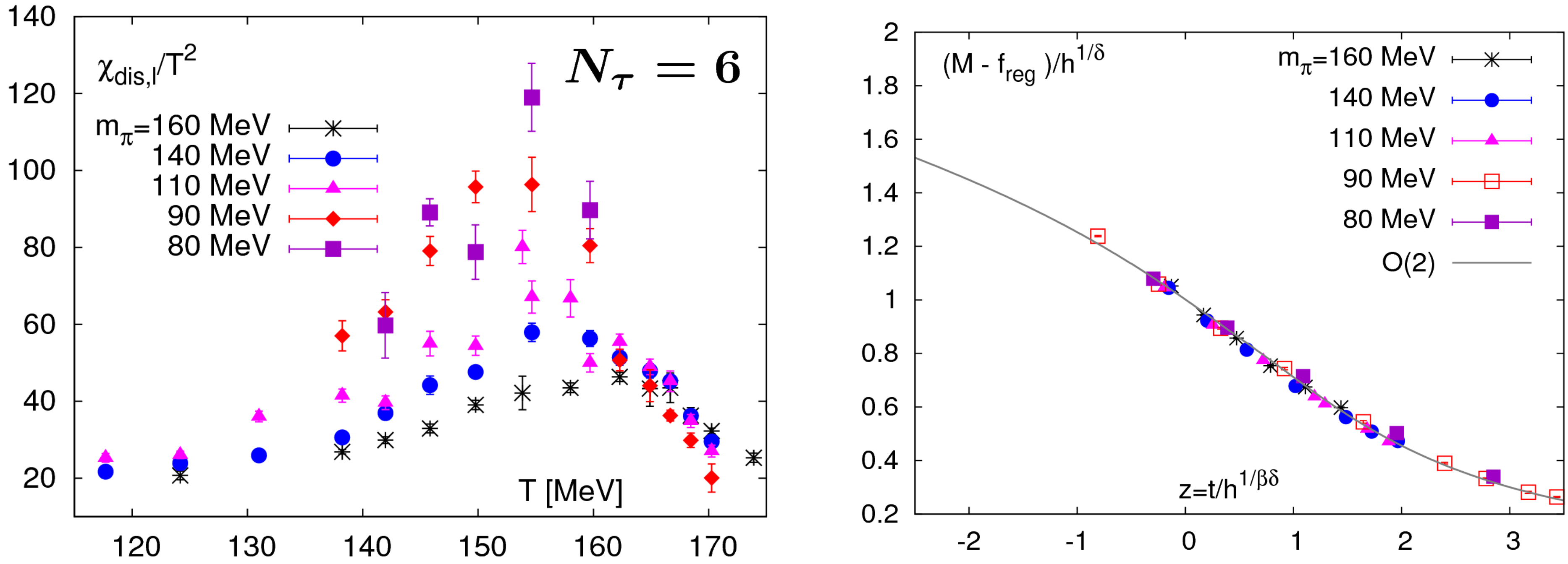
or $$(\chi_l/T^2)^{max} \sim (m_l - m_c)^{1/\delta-1}$$

3-flavor QCD:

Action	N_t	m_π^c	Year
standard staggered	4	$\sim$ **290** MeV	2001
p4 staggered	4	$\sim$ **67** MeV	2004
standard staggered	6	$\sim$ **150** MeV	2007
HISQ staggered	6	$\lesssim$ **50** MeV	2017 1)
stout staggered	4-6	could be zero	2014
Wilson-clover	6-8	$\sim$ **300** MeV	2014
Wilson-clover	4-10	$\sim$ **100** MeV	2016
Wilson-clover	4-10, cont. extrap.	$\lesssim$ **170** MeV	2017 2)

1) A. Bazavov et al., arXiv:1701.03548
2) X.-J. Jin et al., arXiv:1706.01178

2 and (2+1)-flavor QCD: **O(4) scaling?**



$$\chi_{\rm dis}/T^2 \sim m_\pi^{2(1/\delta-1)}$$

$$M = m_s \langle \bar\psi \psi \rangle / T^4$$

magnetic equation of state: $M = h^{1/\delta} f_G(z)$

– scaling analysis in (2+1)-flavor QCD with HISQ fermions

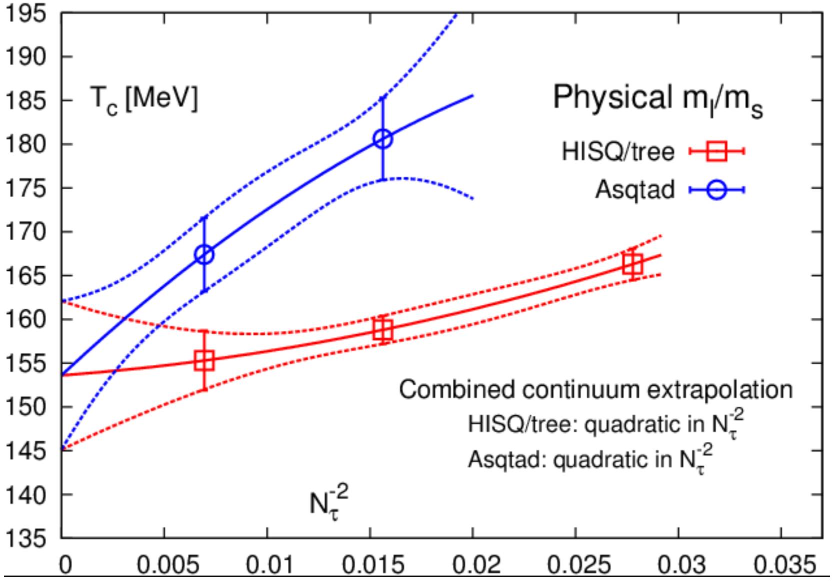
$\Rightarrow$ $m_\pi^{crit} < 80\mathrm{MeV}$

not yet sensitive to O(4) scaling in the chiral limit vs. Z(2) critical behavior at $m_c > 0$

staggered fermions: O(2) instead of O(4) for non-zero cut-off

The QCD crossover transition
– extracting the pseudo-critical temperature –

Crossover transition temperature



$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

- well defined pseudo-critical temperature
- quark mass dependence of susceptibilities consistent with O(4) scaling

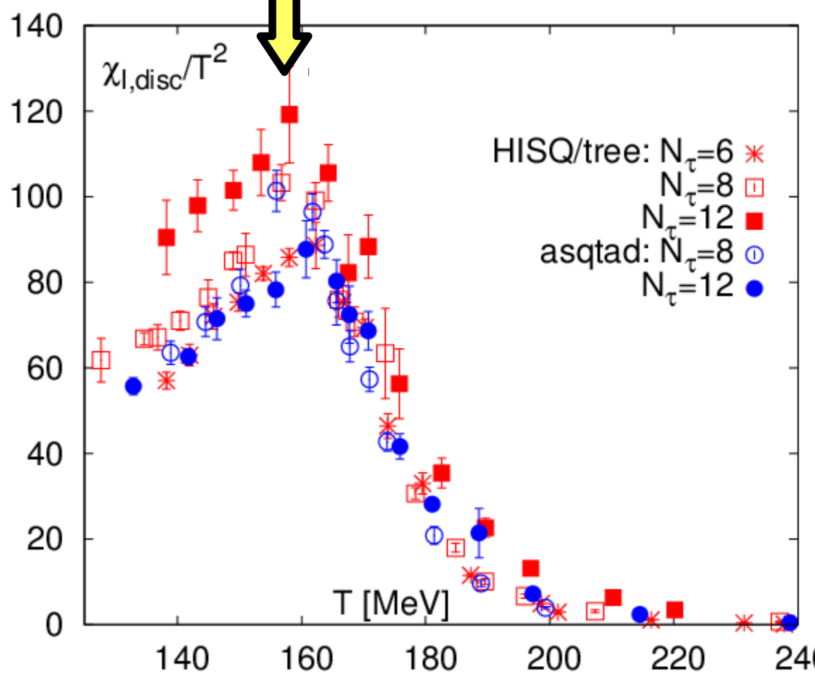
A. Bazavov et al. (hotQCD),
Phys. Rev. D85, 054503 (2012), arXiv:1111.1710

lattice: $N_\sigma^3 \cdot N_\tau$
temperature: $T = 1/N_\tau a$

Critical temperature from location of peak in the fluctuation of the chiral condensate (order parameter):

Chiral susceptibility

$$\chi_l = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} = \chi_{l,disc} + \chi_{l,con}$$



consistent with Y. Aoki et al, JHEP 0906 (2009) 088

Symmetries and in-medium properties of hadrons

Which symmetries are restored at Tc?

- thermal hadron correlation functions

Greens functions G of quark-antiquark pair in different quantum number channels H, controlled by operators J

$$J_H(x) = \bar{q}(x)\Gamma_H q(x) \qquad \Gamma_H = 1,\ \gamma_5,\ \gamma_\mu,\ \gamma_\mu\gamma_5$$

scalar, pseudo-scalar, vector, axial-vector

$$q(\bar{q}) = u(\bar{u}),\ d(\bar{d}), ... \Rightarrow$$

$\bar{q}q = \bar{u}u$ flavor singlet

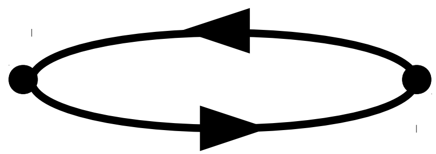
$\bar{q}q = \bar{u}d$ flavor non-singlet

$$G_H(\tau,\vec{x}) = \langle J_H(\tau,\vec{x})\ J_H^\dagger(0,\vec{0})\rangle \ \sim\ \mathrm{e}^{-m_H\tau}$$

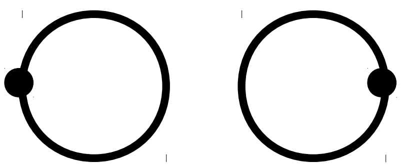
at T=0

Thermal modification of the hadron spectrum

quark propagator: $\bar{q}(x)q(0) = M_q^{-1}(x,0)$



$$
\begin{aligned}
G_\pi(x) &= \langle \mathrm{Tr}\, \gamma_5 M_l^{-1}(x,0)\gamma_5 M_l^{-1}(0,x)\, \rangle \\
G_\eta(x) &= G_\pi(x) - \langle \mathrm{Tr}\left[\gamma_5 M_l^{-1}(x,x)\right] \mathrm{Tr}\left[\gamma_5 M_l^{-1}(0,0)\right]\rangle \\
G_\delta(x) &= -\langle \mathrm{Tr}\, M_l^{-1}(x,0) M_l^{-1}(0,x)\, \rangle \\
G_\sigma(x) &= G_\delta(x) + \langle \mathrm{Tr} M_l^{-1}(x,x) \mathrm{Tr} M_l^{-1}(0,0)\rangle \\
&\quad - \langle \mathrm{Tr} M_l^{-1}(x,x)\rangle\, \langle \mathrm{tr} M_l^{-1}(0,0)\rangle
\end{aligned}
$$



hadronic susceptibilities

$$
\chi_\pi = \sum_x G_\pi(x) \equiv \chi_{5,\mathrm{con}} \quad , \quad \chi_\delta = \sum_x G_\delta(x) = \chi_{\mathrm{con}}
$$

$$
\chi_\eta = \sum_x G_\eta(x) \equiv \chi_{5,\mathrm{con}} - \chi_{5,\mathrm{disc}}
$$

$$
\chi_\sigma = \sum_x G_\sigma(x) = \chi_{\mathrm{con}} + \chi_{\mathrm{disc}}
$$

Thermal modification of the hadron spectrum

$T < T_c$ **:** broken chiral symmetry is reflected in the hadron spectrum

$a_1: \bar{q}\gamma_5\gamma_\mu q \longleftrightarrow \rho: \bar{q}\gamma_\mu q$

$SU(2)_L \times SU(2)_R$

$\chi_{5,con}$ $\pi: \bar{q}\gamma_5 \frac{\tau}{2} q \longleftrightarrow \sigma: \bar{q}q$ (f_0) $\chi_{con} + \chi_{disc}$

flavor non-singlet $U(1)_A \updownarrow$ $U(1)_A \updownarrow$ flavor singlet

χ_{con} $\delta: \bar{q}\frac{\tau}{2} q$ (a_0) $\longleftrightarrow \eta: \bar{q}\gamma_5 q$ $\chi_{5,con} - \chi_{5,disc}$

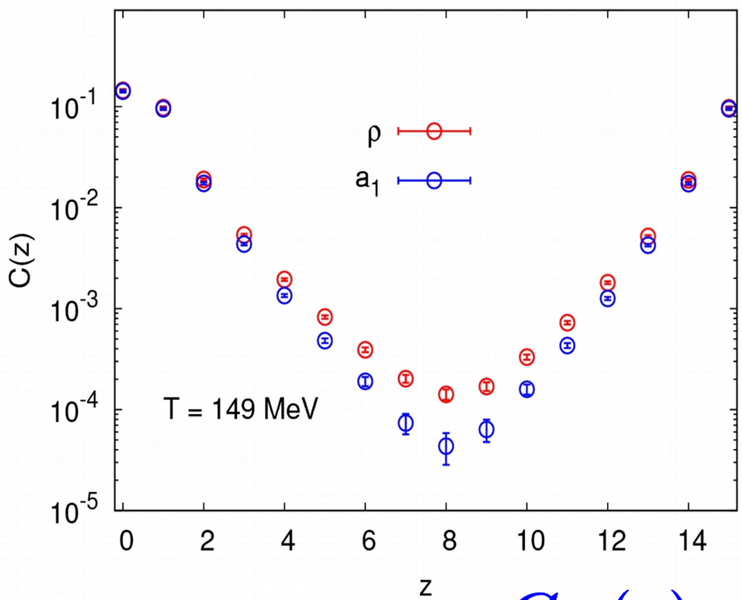
$SU(2)_L \times SU(2)_R$

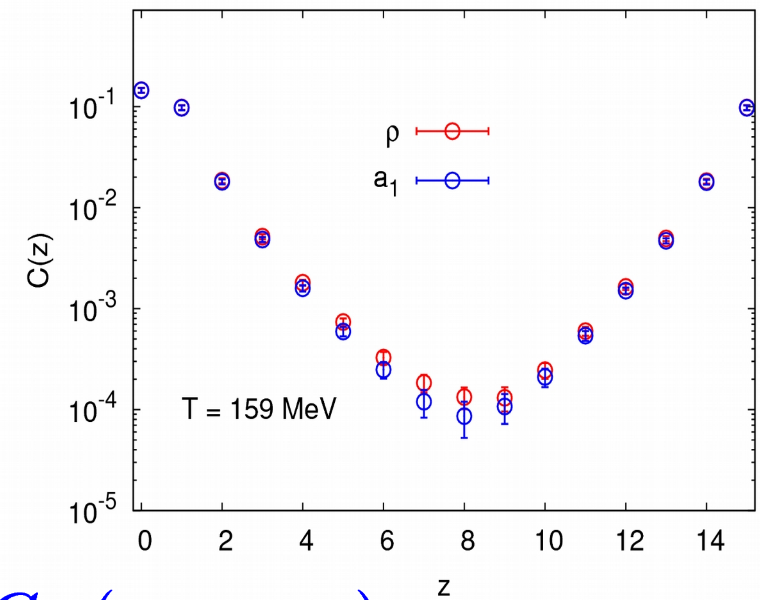
$T \geq T_c$ **:** restoration of symmetries is reflected in the (thermal) hadron spectrum

$SU(2)_L \times SU(2)_R$: $(\pi,\ \sigma)$, (a_1, ρ) degenerate

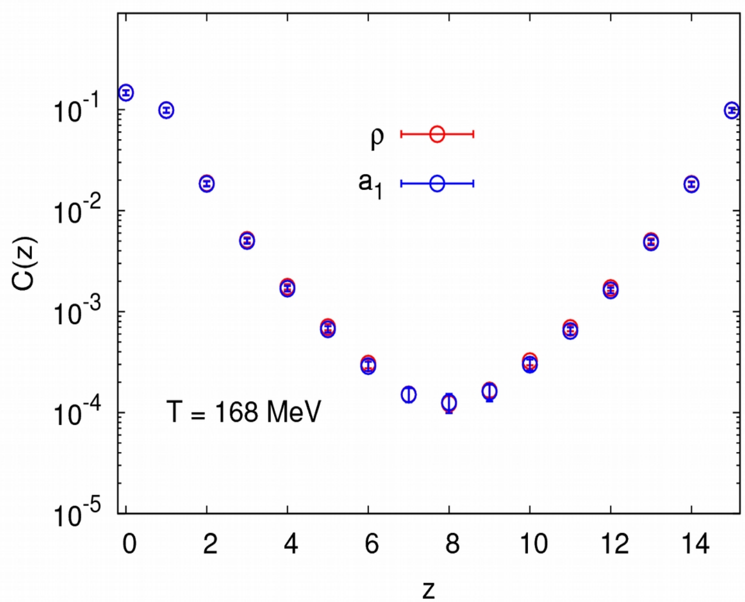
$U(1)_A$: $(\pi,\ \delta)$ degenerate

Symmetry restoration and correlation functions





$$C_H(z) = \sum_{x,y,\tau} G_H(x,y,z,\tau)$$



Thermodynamics with domain wall fermions, hotQCD, arXiv:1205.3535

chiral flavor symmetry is restored at

$T \gtrsim 160\ \mathrm{MeV}$ $\quad m_\pi \simeq 200\ \mathrm{MeV}$

no cont. extrap.

Restoration of the axial symmetry

$T < T_c$ **:** broken chiral symmetry is reflected in the hadron spectrum

$$
\begin{tikzcd}
\chi_{5,con} & \pi:\ \bar{q}\gamma_5\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma:\ \bar{q}q\ (f_0) \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{con}+\chi_{disc} \\
\chi_{con} & \delta:\ \bar{q}\frac{\tau}{2}q\ (a_0) \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta:\ \bar{q}\gamma_5 q & \chi_{5,con}-\chi_{5,disc}
\end{tikzcd}
$$

$T \geq T_c$ **:** $SU(2)_L \times SU(2)_R$ restored

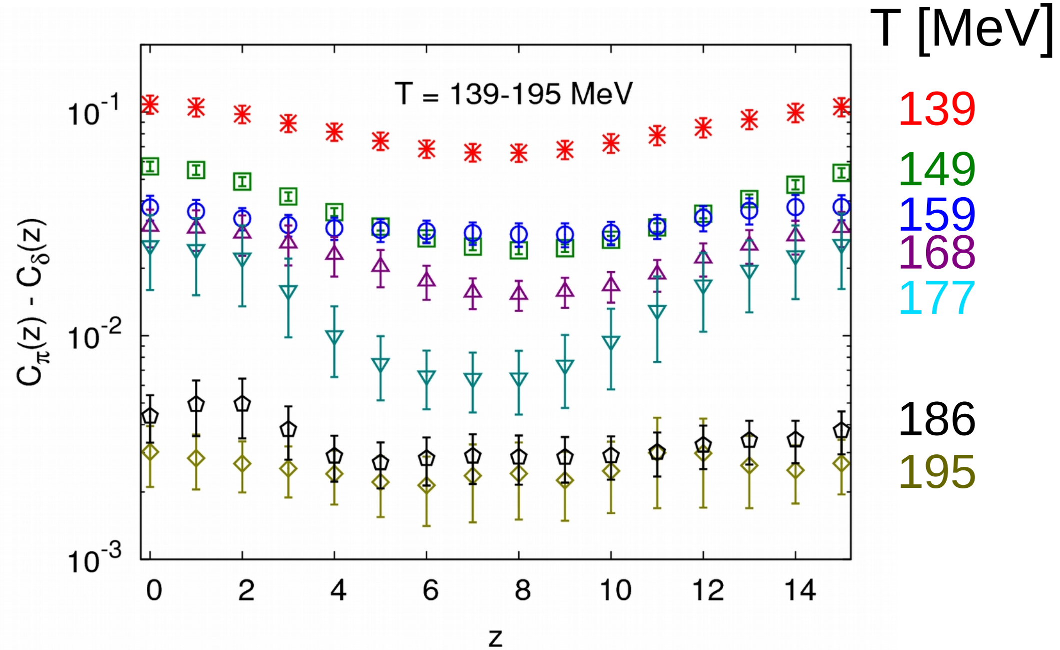
$$\Rightarrow \quad \chi_{5,con} = \chi_{con} + \chi_{disc}$$

$U(1)_A$ restored $\Rightarrow \quad \chi_{5,con} = \chi_{con}$

$$\Leftrightarrow \ \chi_{disc} = 0 \ \Leftrightarrow \ \chi_\pi(x) - \chi_\delta(x) = 0?$$

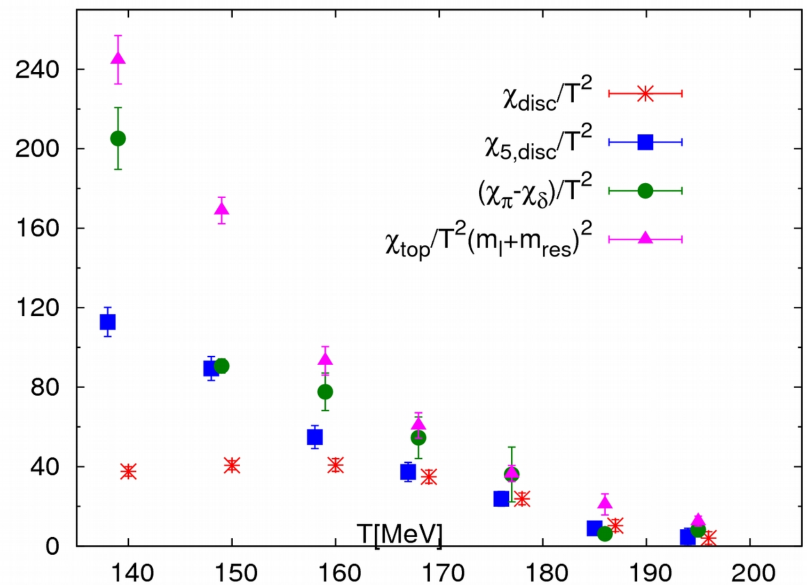
$U(1)_A$ remains broken

the difference of the scalar (δ) and pseudo-scalar(π) drops by an order of magnitude but stays non-zero



above Tc (but still for m>0):

$$\frac{\chi_\pi - \chi_\delta}{T^2} = \frac{\chi_{disc}}{T^2} = \frac{\chi_{5,disc}}{T^2} > 0$$



thermodynamics with domain wall fermions

hotQCD, arXiv:1205.3535

nonetheless, chiral limit remains controversial

S. Aoki et al., PR D86 (2012) 114512

Lattice QCD at non-zero baryon number density

$$\mu > 0$$

THE PROBLEM in QCD Thermodynamics

partition function again:

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{\bar{\psi}\mathcal{M}(\mathcal{A},m_q,\mu)\psi}\ \mathrm{e}^{-S_G}$$

$$= \int \mathcal{D}\mathcal{A}\ \mathrm{det}M(\mathcal{A},m_q,\mu)\ \mathrm{e}^{-S_G}$$

$$\mu > 0$$

The fermion determinant – is no longer positive definite

standard simulation techniques fail

$$\mathrm{detM}(\mathcal{A},\mathrm{m_q},\mu) = \mathrm{e}^{i\theta(\mu)}\,|\mathrm{det}M(\mathcal{A},m_q,\mu)|$$

Lattice QCD at non-zero baryon number density
– the infamous sign problem –

partition function:
$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\; \det M(\mathcal{A}, m_q, \mu)\; \mathrm{e}^{-S_G}$$

staggered fermion matrix:
$$M(\mu) = m_q \delta_{i,j} + \frac{1}{2}\eta_i \Big(\sum_{k=1}^{3} (U_{i,k}\delta_{i,j-\hat{k}} - U^\dagger_{i-\hat{k},k}\delta_{i,j+\hat{k}}$$
$$+ \mathrm{e}^{\mu}\, U_{i,0}\delta_{i,j-\hat{0}} - \mathrm{e}^{-\mu}\, U^\dagger_{i-\hat{0},0}\delta_{i,j+\hat{0}} \Big)$$
$$= m_q \cdot 1 + \sum_{i=1}^{3} D_i + D_0(\mu)$$
$$= \begin{array}{cc} \begin{array}{cc} e & o \end{array} & \\ \begin{pmatrix} m_q & D_{eo} \\ D_{oe} & m_q \end{pmatrix} & \begin{array}{c} e \\ o \end{array} \end{array}$$

Lattice QCD at non-zero baryon number density
– the infamous sign problem –

schematic:

$$= \begin{pmatrix} m_q & & \\ & \ddots \; e^{\mu} U_{i+\hat{0},0} & \\ -e^{-\mu} U^{\dagger}_{i-\hat{0},0} & \ddots & \\ & & m_q \end{pmatrix} \qquad = \begin{array}{cc} & \begin{array}{cc} e & o \end{array} \\ & \left(\begin{array}{c|c} m_q & D_{eo} \\ \hline D_{oe} & m_q \end{array}\right) \end{array} \begin{array}{c} e \\ o \end{array}$$

$\mu = 0:$

D is anti-hermitian
→ eigenvalues are purely imaginary

D has even-odd structure
→ eigenvalues come in pairs: $\pm\lambda$

$$\Longrightarrow \quad \det M \geq 0$$

$\mu^2 > 0:$

D is no-longer anti-hermitian
→ eigenvalues are no longer purely imaginary

$$\Longrightarrow \quad \det M = e^{i\theta} \, |\det M|$$

SIGN problem!!

$\mu^2 < 0:$

D is anti-hermitian
→ eigenvalues are purely imaginary

$$\Longrightarrow \quad \det M \geq 0$$

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

the pressure in hadron resonance gas (**HRG**) models:

$$\boxed{\frac{p}{T^4} = \sum_{m \in meson} \ln Z_m^b(T, V, \mu) + \sum_{m \in baryon} \ln Z_m^f(T, V, \mu)}$$

$$\sim \mathrm{e}^{-m_H/T} \mathrm{e}^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_S = \mu_Q = 0$

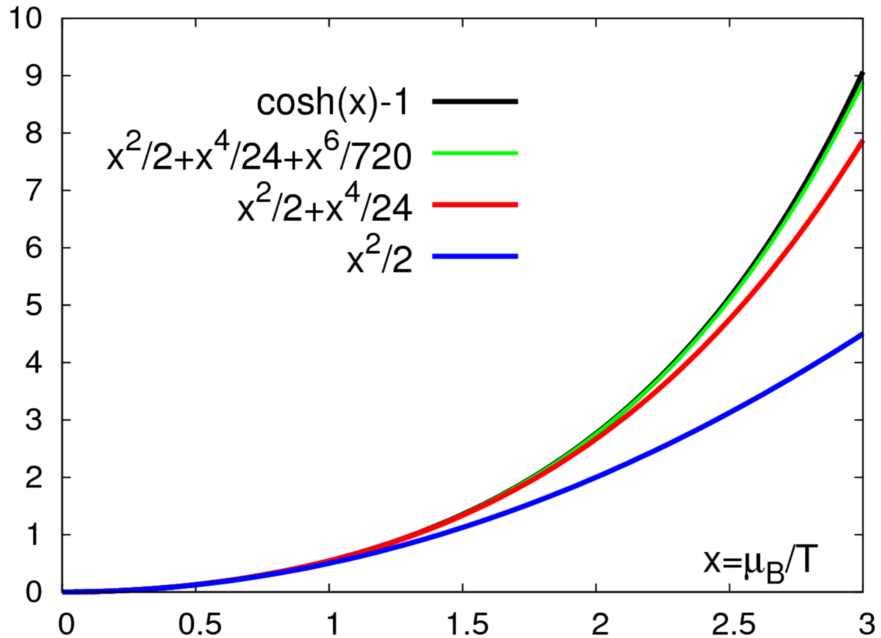
$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

HRG vs. QCD:

$\mathcal{O}((\mu_B/T)^4)$: difference is less than 3% at $\mu_B/T = 2$

$\mathcal{O}((\mu_B/T)^6)$: difference is less than 2% at $\mu_B/T = 3$



Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

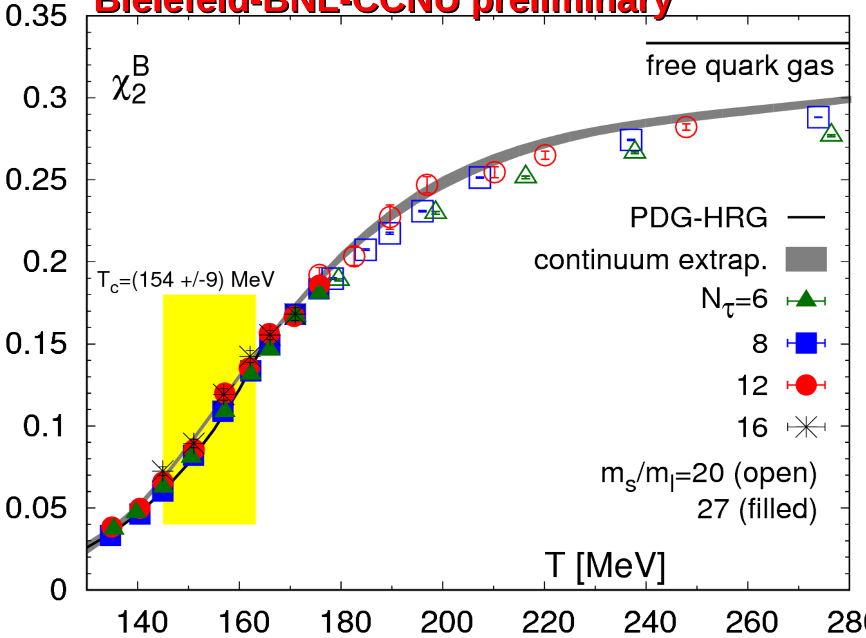
variance of net-baryon number distribution

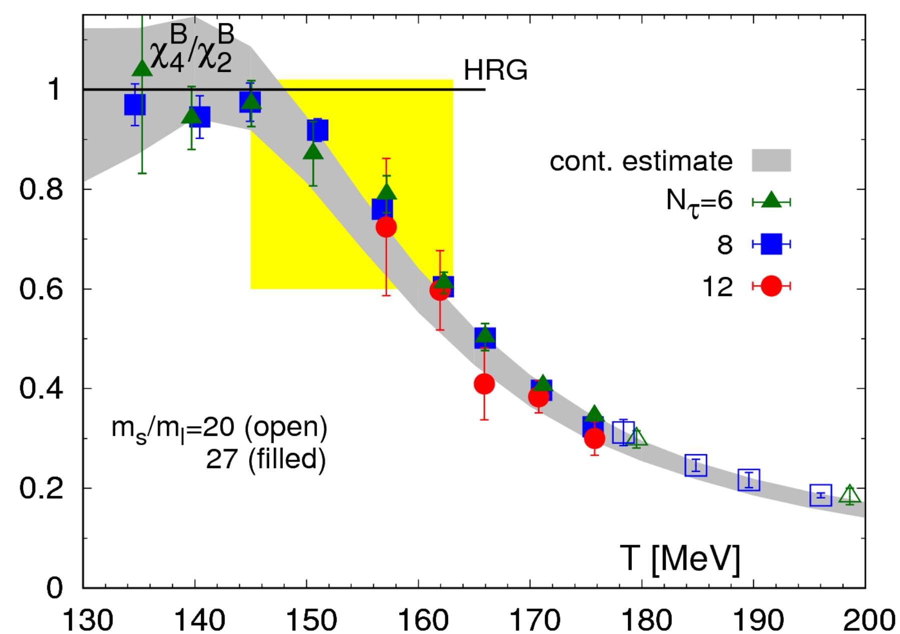
kurtosis*variance

$\kappa_B \sigma_B^2$

fits: A. Bazavov et al. (Bielefeld-BNL-CCNU) arXiv:1701.04325

data are updated: Bielefeld-BNL-CCNU preliminary



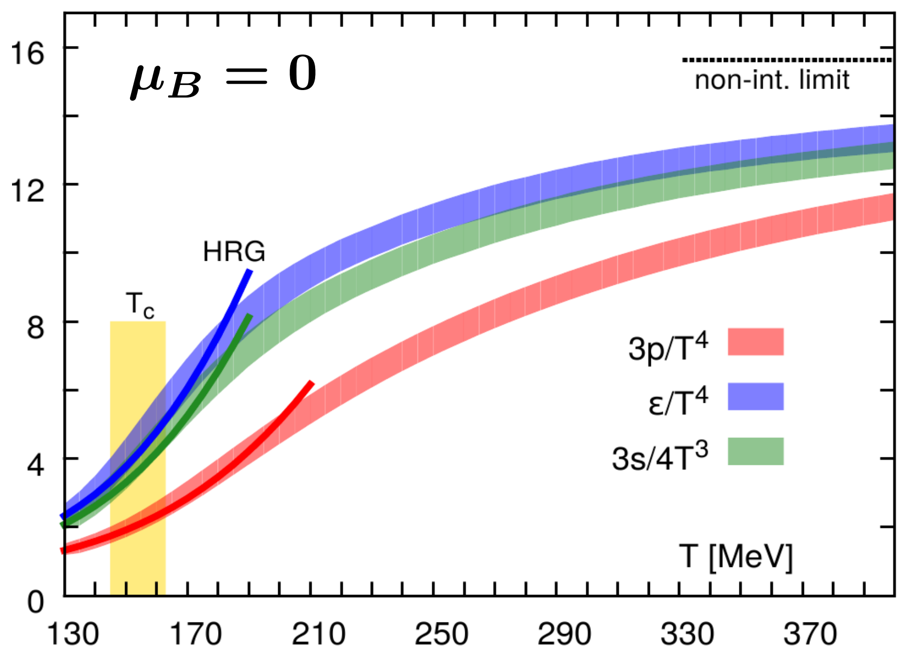


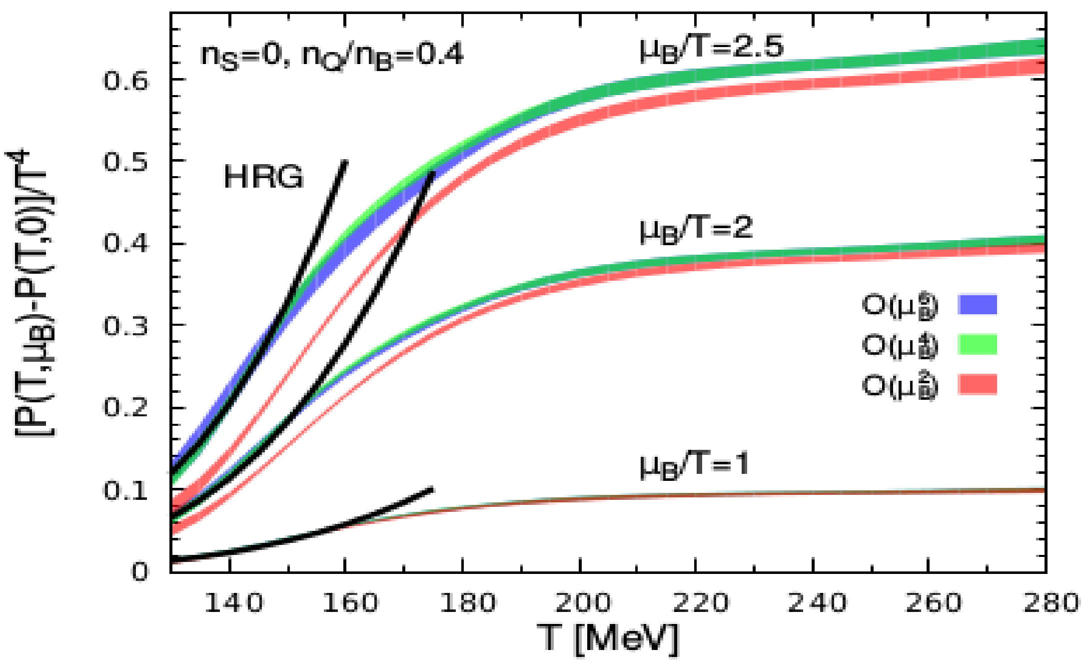
– leading and next-to-leading order corrections agree well with HRG for T<150 MeV

– already in the crossover region deviations from HRG can reach ~40% for T~165 MeV

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

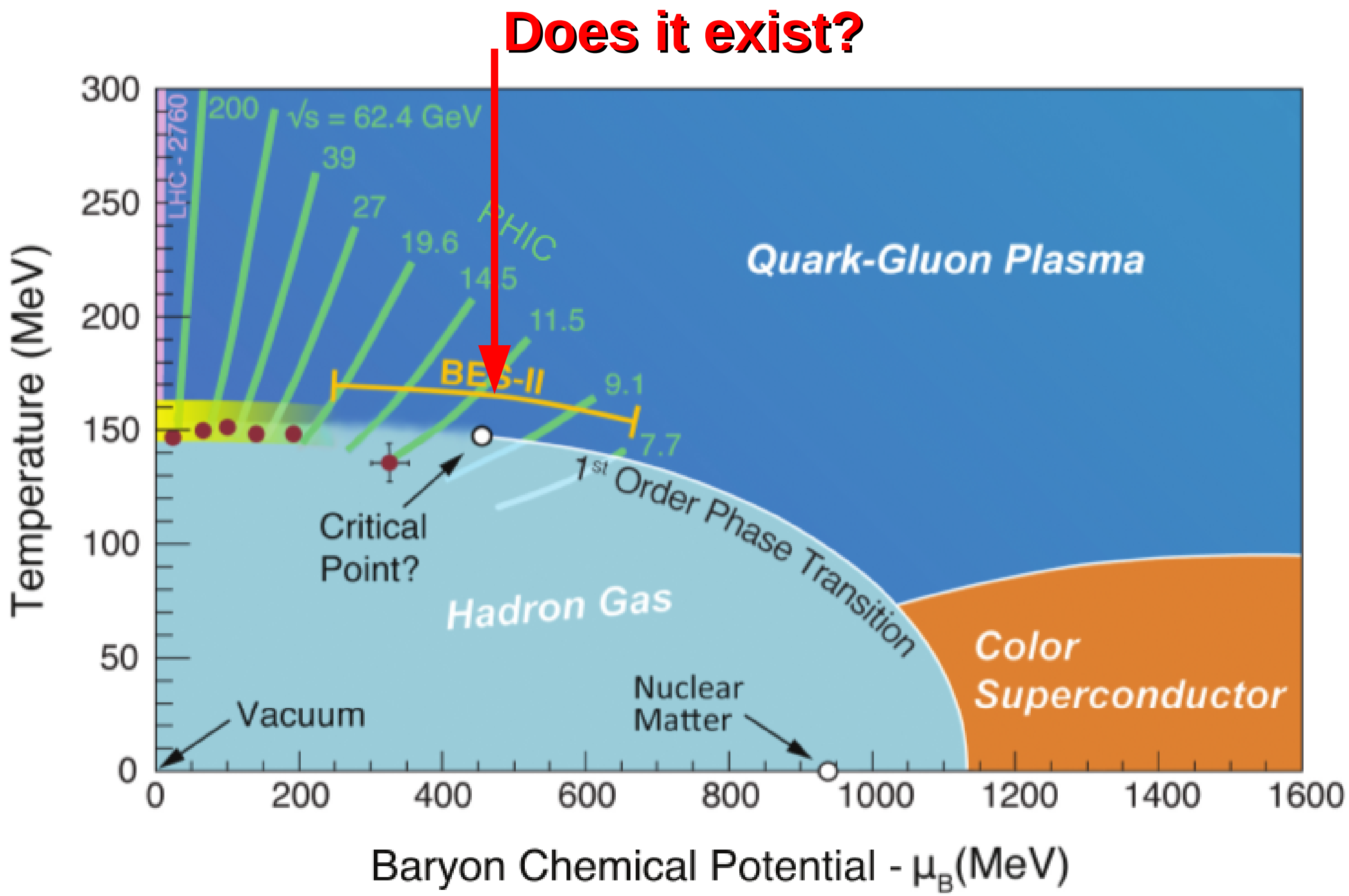
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right.$$
$$\left. + \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6\right)$$





The EoS is well controlled for $\mu_B/T \le 2$
or equivalently $\sqrt{s_{NN}} \ge 20$ GeV

Searching for a critical point at $\mu_B > 0$



– signatures for a critical point: large fluctuations in e.g. the net baryon-number

break-down of Taylor series expansion → **radius of convergence**

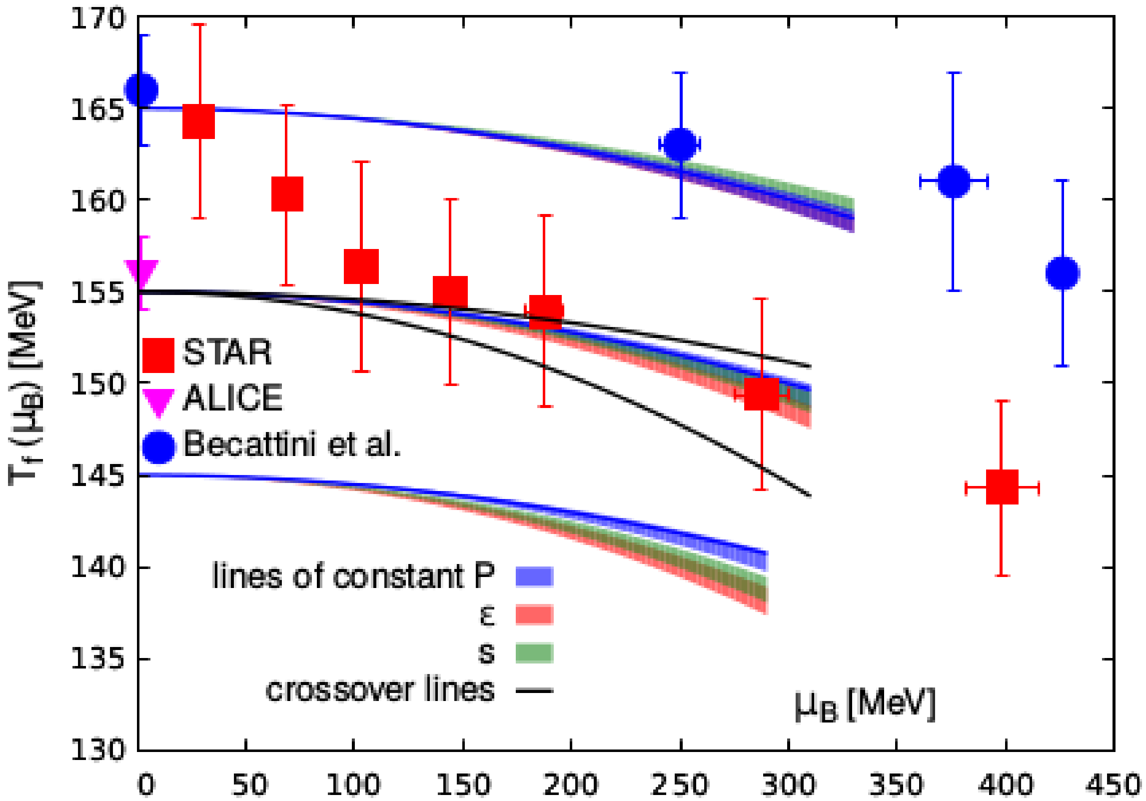
Chiral transition, hadronization and freeze-out

– pseudo-critical temperature $T_c = 154(9)\mathrm{MeV}$

– hadronization temperatures $T_h = 164(3)\ \mathrm{MeV}$

– freeze-out temperatures: $T_{fo} = 156(3)\ \mathrm{MeV}$

$T_{fo} = [164(5) - 168(4)]\ \mathrm{MeV}$



Where does hadronization set in?

physics is quite different at lower and upper end of the current error bar on Tc

probed with net-charge correlations&fluctuations

crossover transition lines:

G. Endrodi et al., arXiv:1102.1356, O. Kaczmarek et al., arXiv:1011.31.30

C. Bonati et al., arXiv:1507.03571, P. Cea et al., arXiv:1403.0821

HRG vs. QCD
net baryon-number fluctuations

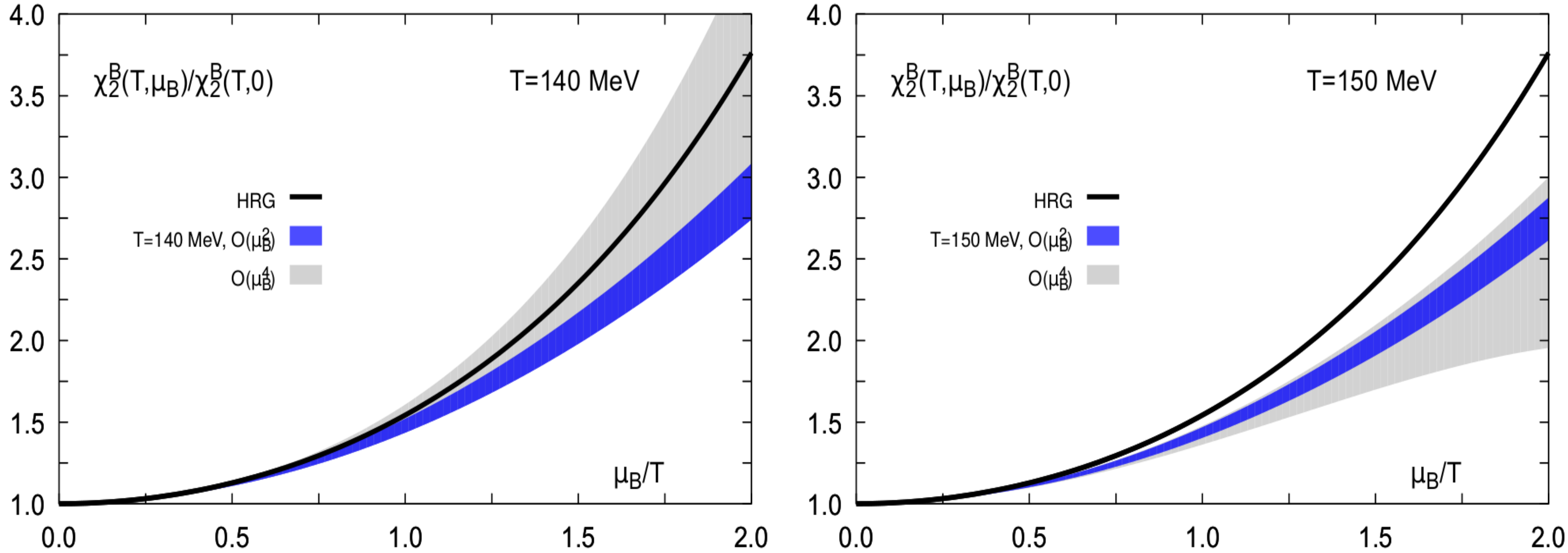
$$\boxed{\mu_B/T > 0}$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B \left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B \left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**



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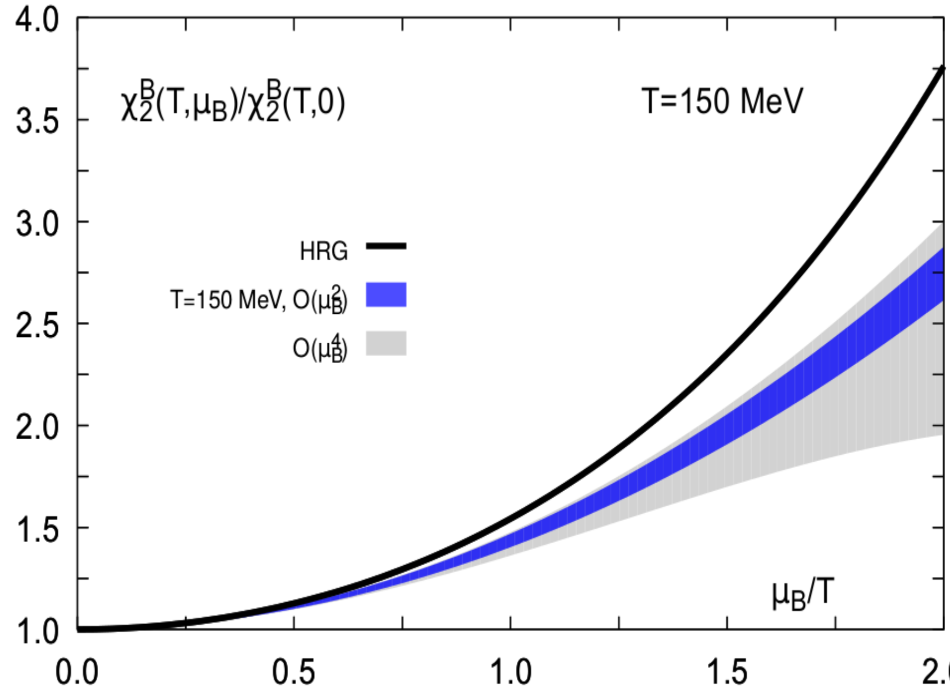
– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**

no evidence for enhanced net baryon-number fluctuations for

$$T \geq 135\mathrm{MeV}\ ,\ \mu_B \leq 2T$$

no evidence for getting closer to a "critical region"



Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T,\mu_B)$ to be monotonically growing with μ_B/T

$\Updownarrow$

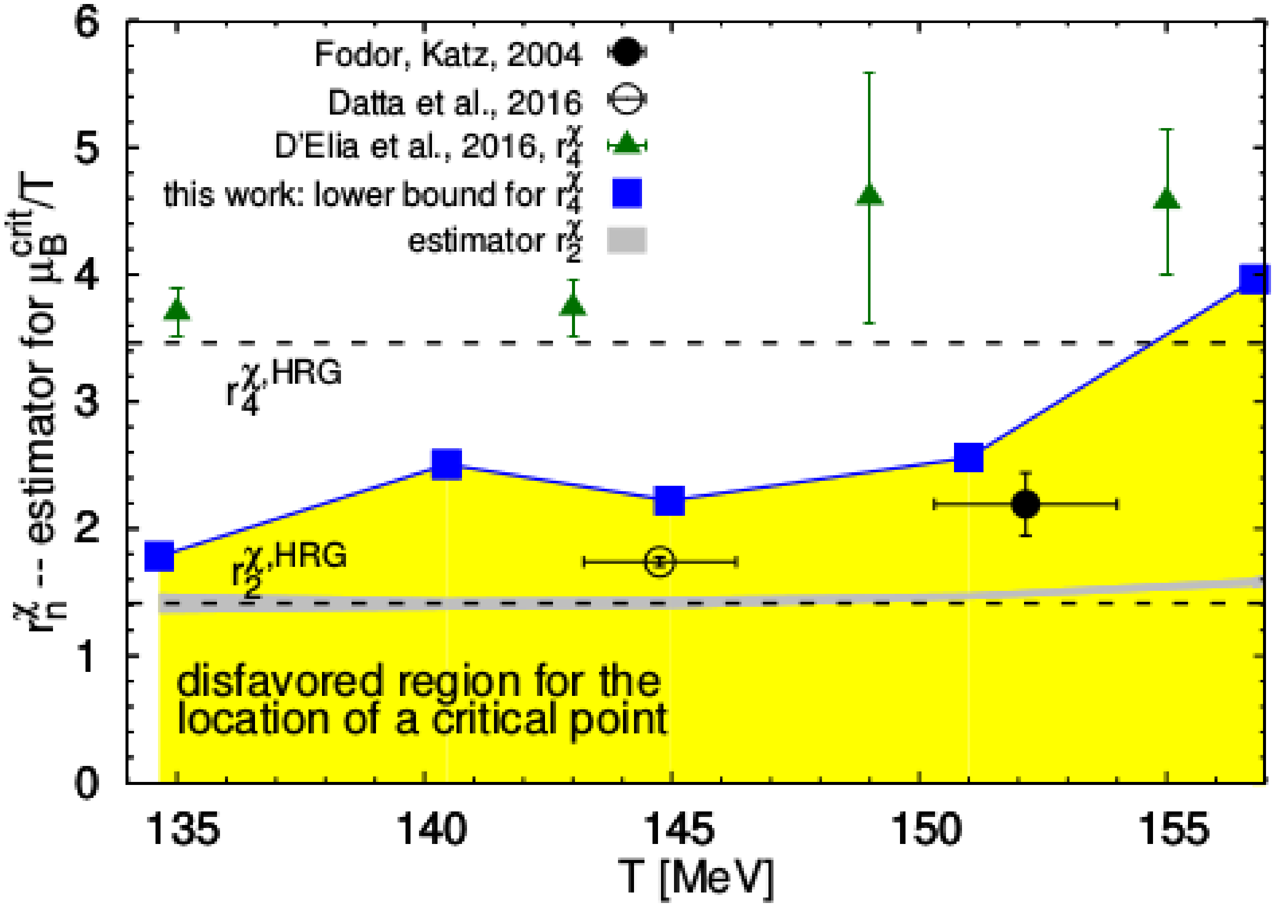
at T_{CP} : $\kappa_B \sigma_B^2 = \dfrac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

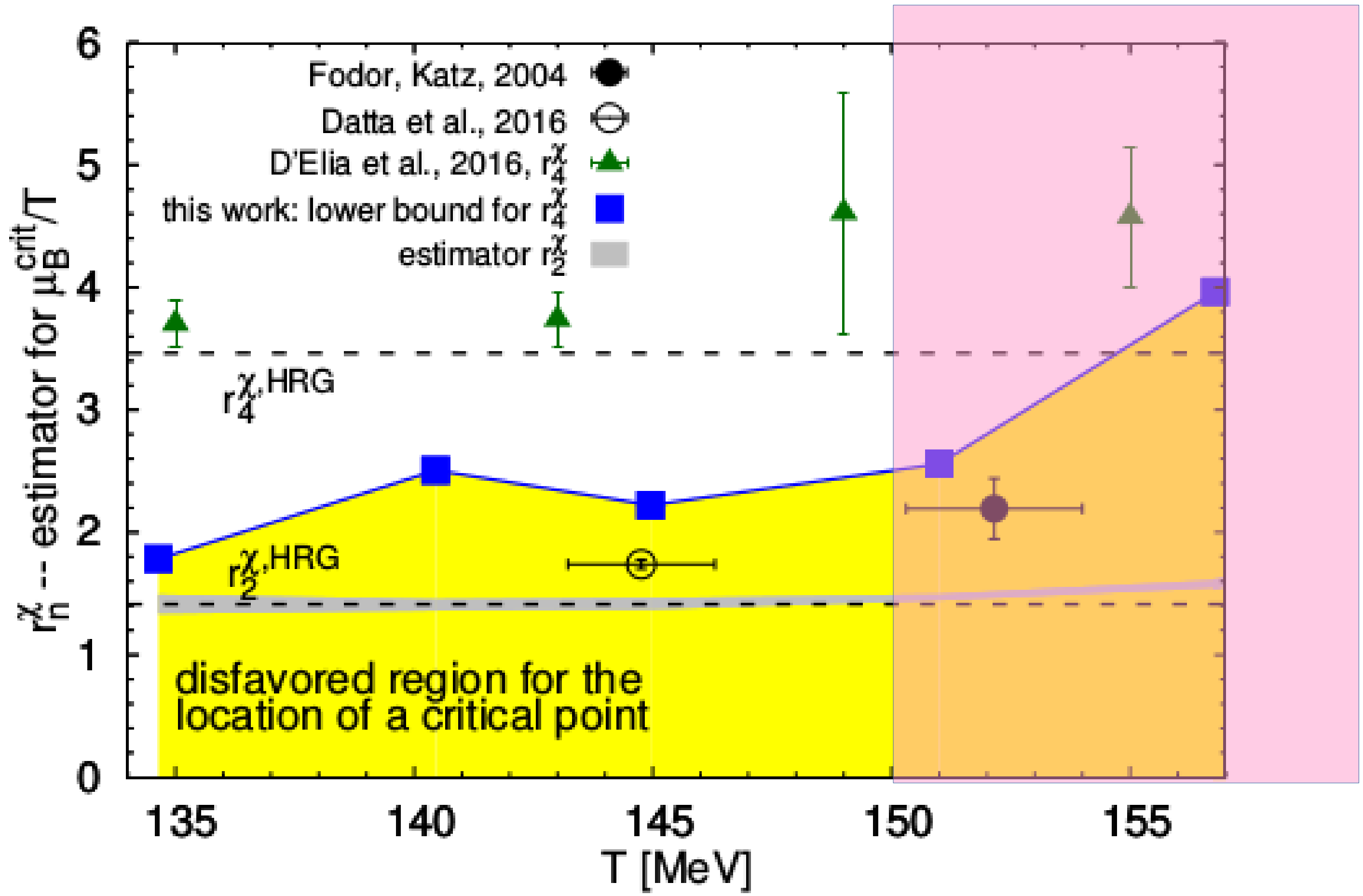
– Taylor expansion can not be used close to the critical point

estimates/constraints on critical point location



01/01/17:
based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

estimates/constraints on critical point location



strongly disvavored as

$$\chi_6^B < 0$$

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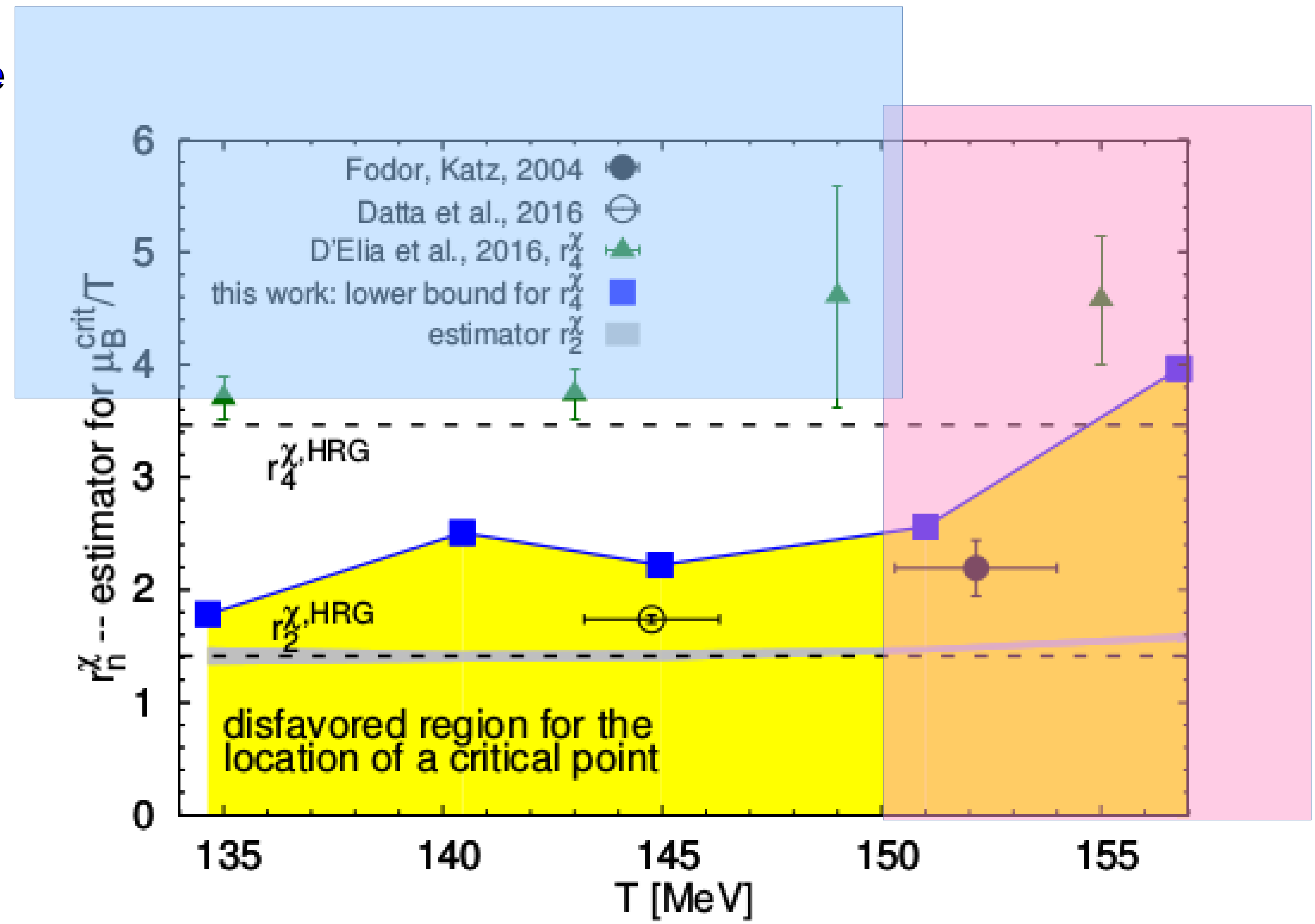
A. Bazavov et al., arXiv:1701.04325

estimates/constraints on critical point location

not accessible in BES@RHIC collider mode



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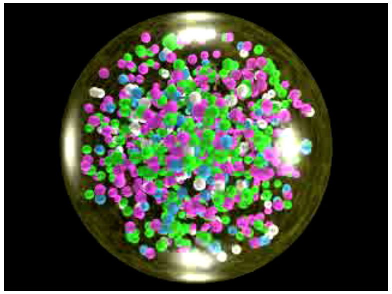
A. Bazavov et al., arXiv:1701.04325

Explore the **structure of matter** close to the QCD transition temperature using **fluctuations of conserved charges**

baryon number, strangeness, electric charge

High T: ideal gas

ideal quark (fermi) gas, m=0



fractional charges

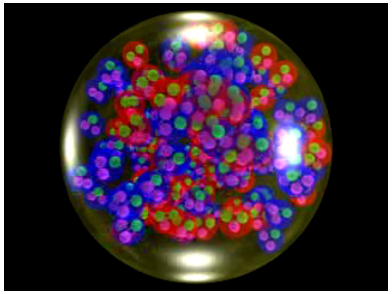
baryon number: B= +/- 1/3

electric charge: Q= +/- 1/3, +/- 2/3

strangeness: S= 0, +/- 1

Low T: HRG

hadron resonance gas



integer charges

baryon number: B= +/-1

electric charge: Q= 0 =+/- 1, +/- 2

strangeness: S= 0, +/- 1, +/- 2, +/- 3

Correlations and Fluctuations of conserved charges

– construct QCD observables that would project onto specific quantum numbers, if QCD = HRG

– obtain fluctuations of quantum numbers and correlations between them from the grand canonical potential (~pressure)

$$\boxed{\frac{P}{T^4} = \ln Z(T, V, \mu_B, \mu_Q, \mu_S, \ldots)}$$

charge fluctuations

$$\chi_n^X = \left.\frac{\partial^n \ln Z(T, V, ..\mu_X..)}{\partial \mu_X^n}\right|_{\mu=0}$$

$$n = 2: \ \chi_2^X = \langle X^2 \rangle - \langle X \rangle^2$$

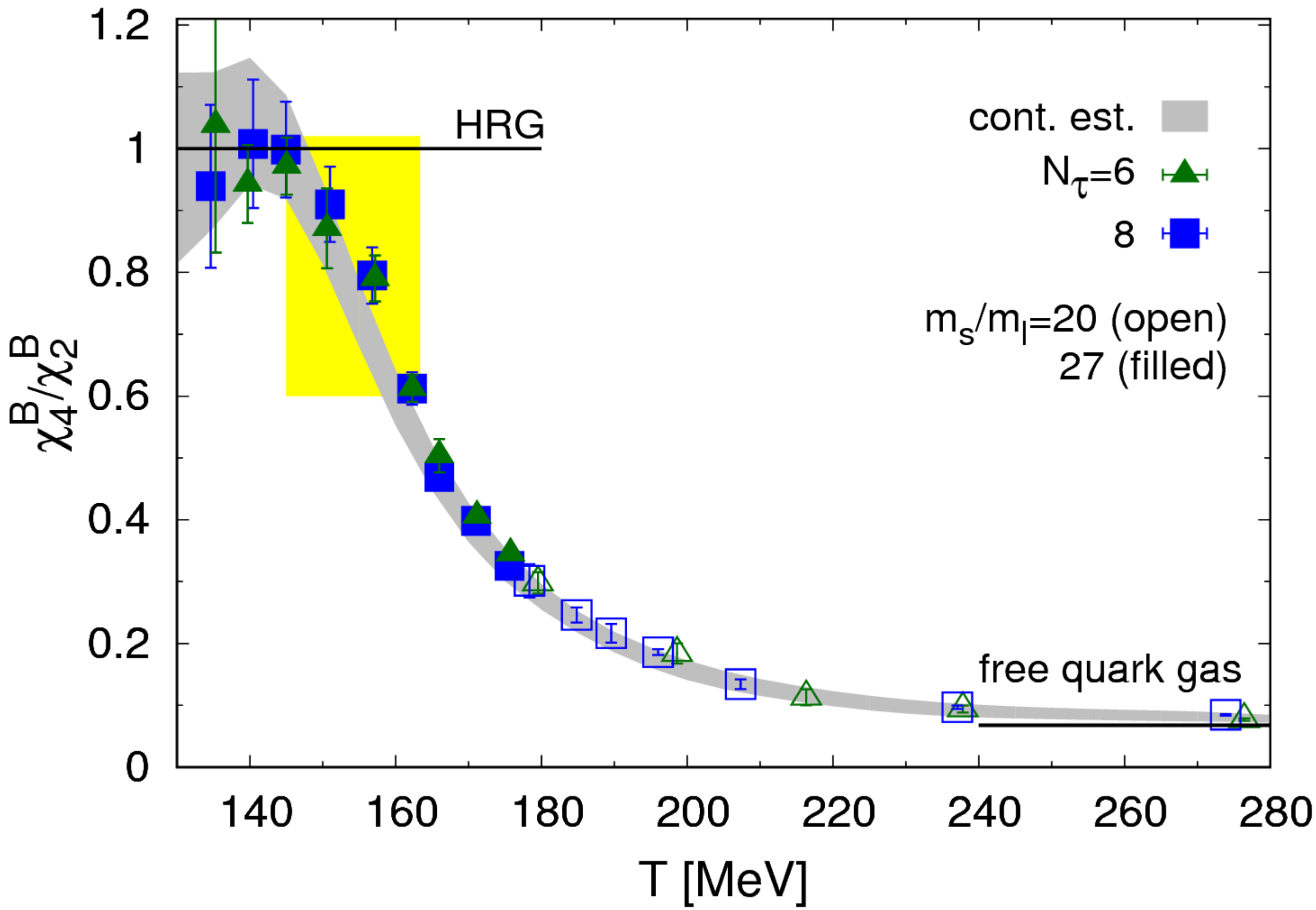
charge correlations:

$$\chi_{XY}^{nm} = \left.\frac{\partial^{n+m} \ln Z(T, V, ..\mu_X, \mu_Y..)}{\partial \mu_X^n \partial \mu_Y^m}\right|_{\mu=0}$$

$$n = m = 1: \ \chi_{11}^{XY} = \langle XY \rangle - \langle X \rangle \langle Y \rangle$$

Net baryon-number fluctuations

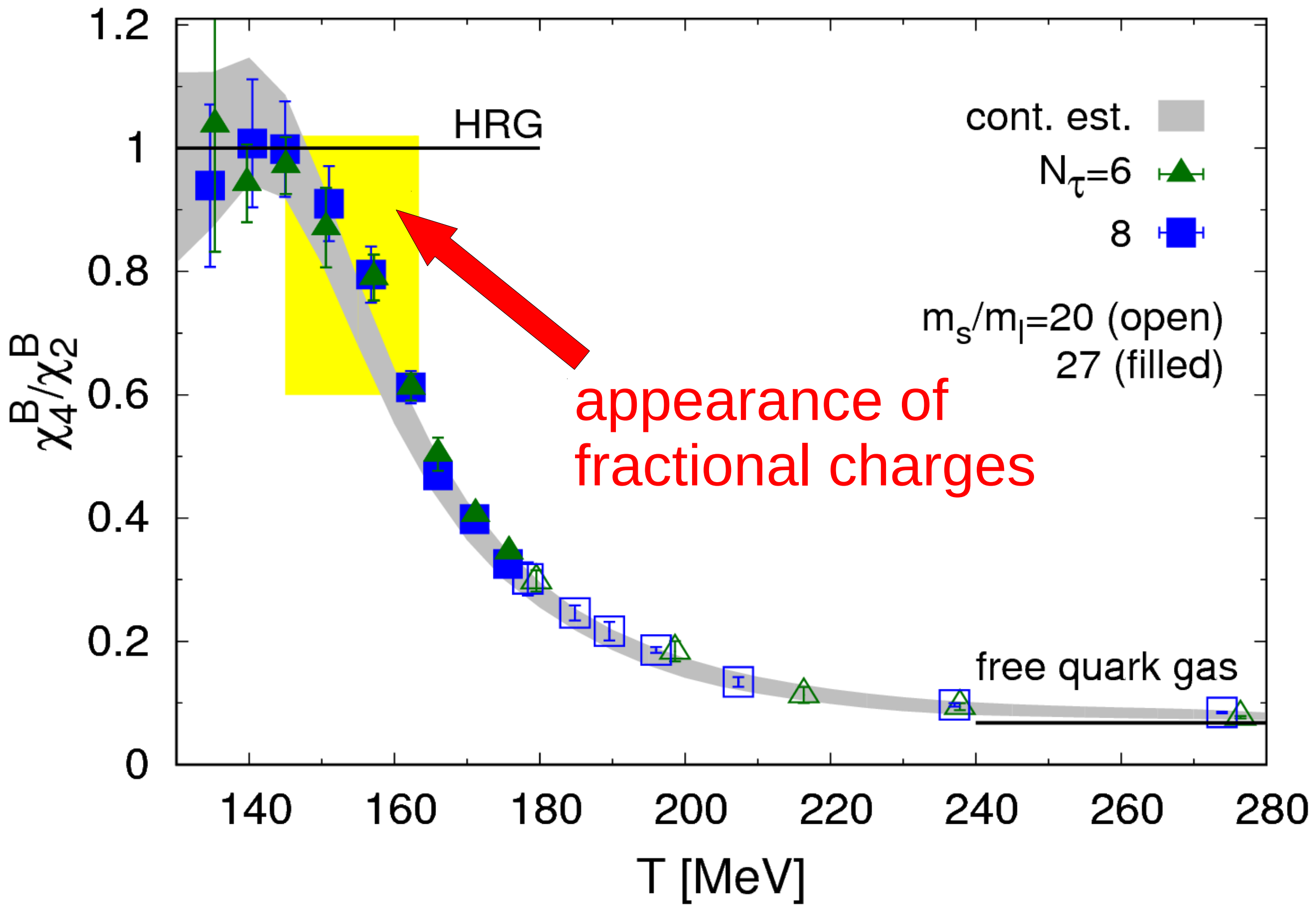
ratio of 4th and 2nd order cumulants:



BNL-Bielefeld-CCNU:
Phys. Rev. Lett. 111, 082301 (2013)
Phys. Lett. B737, 210 (2014)

Net baryon-number fluctuations

ratio of 4th and 2nd order cumulants:



BNL-Bielefeld-CCNU:
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Net baryon-number fluctuations

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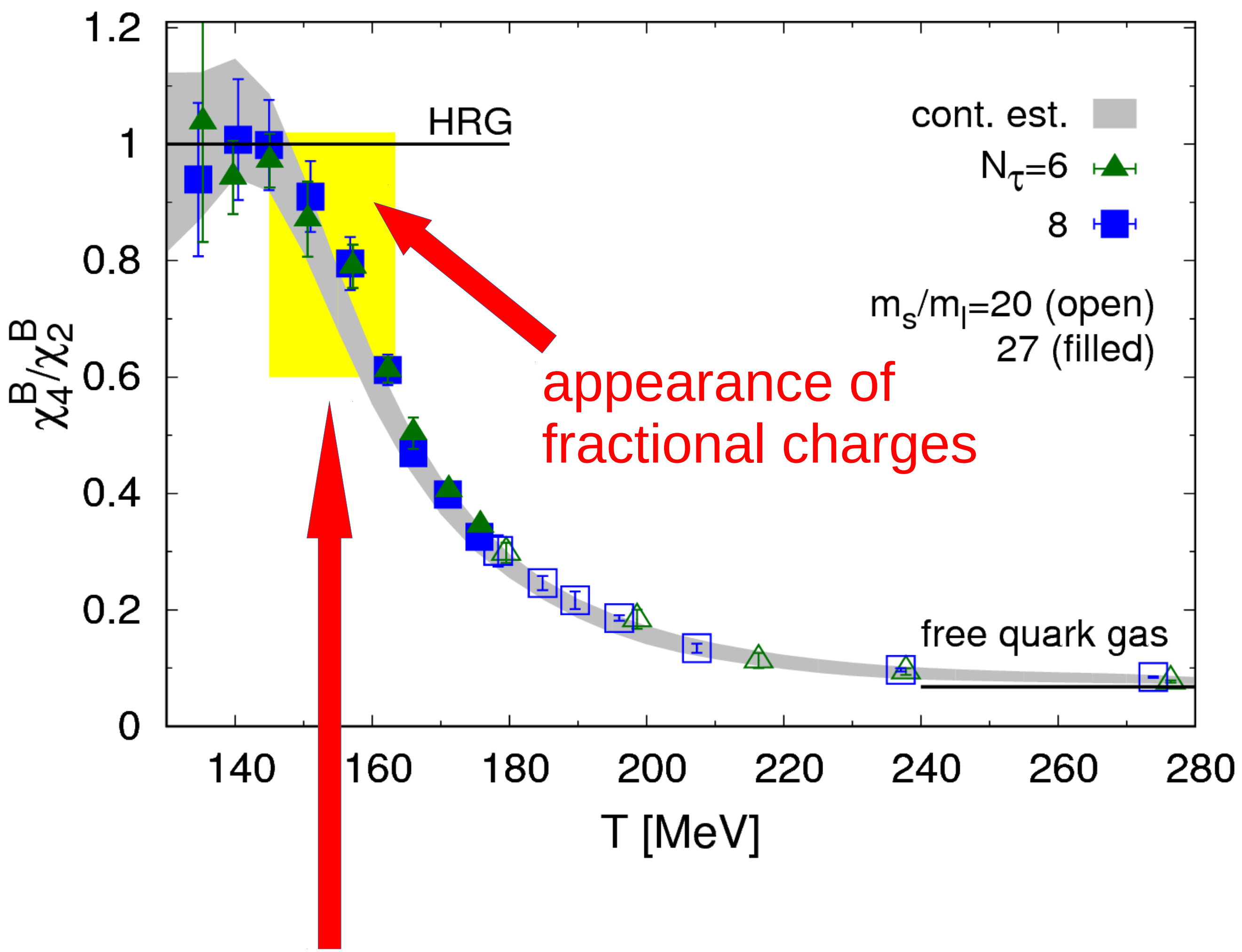
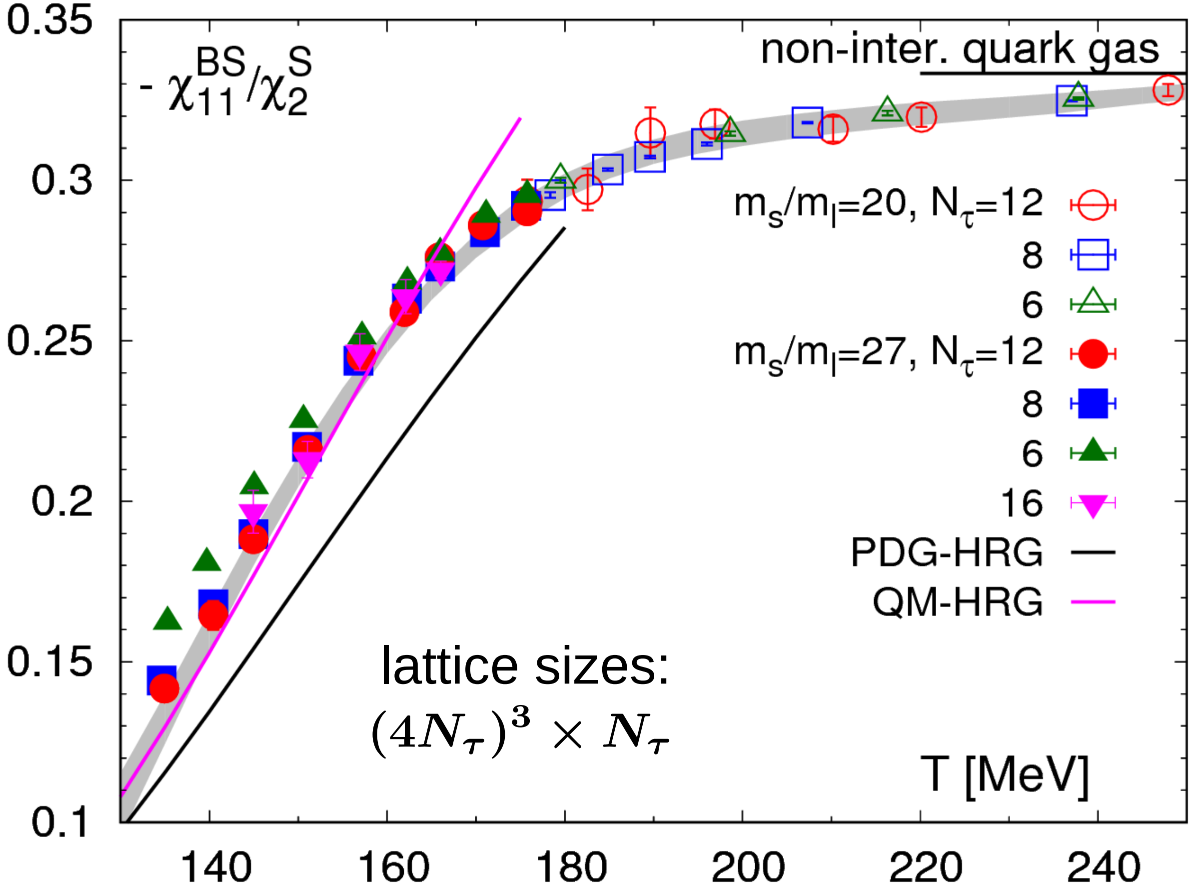


transition temperature

$$T = 154(9)\mathrm{MeV} \simeq 3 \times 10^{12}\ {}^oC$$

BNL-Bielefeld-CCNU:
Phys. Rev. Lett. 111, 082301 (2013)
Phys. Lett. B737, 210 (2014)

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

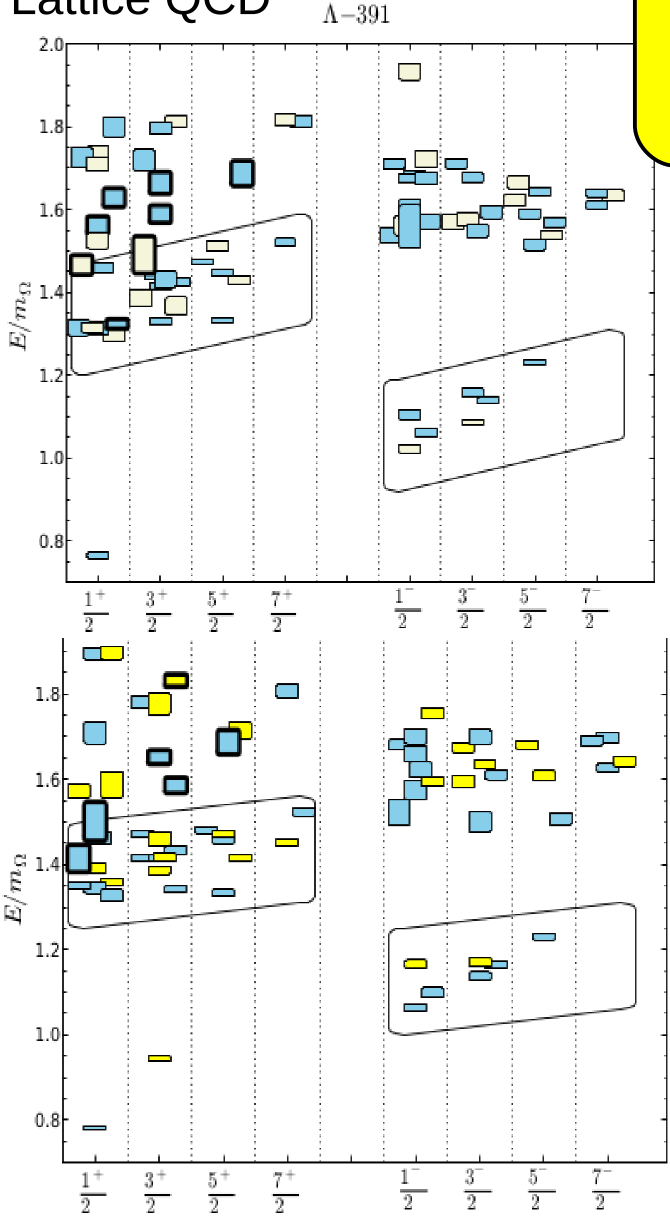


evidence for experimentally not yet observed strange baryons?

PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

Probing the hadron spectrum using QCD thermodynamics

Lattice QCD



$$P_{tot} = \sum P_h$$

h=all hadrons

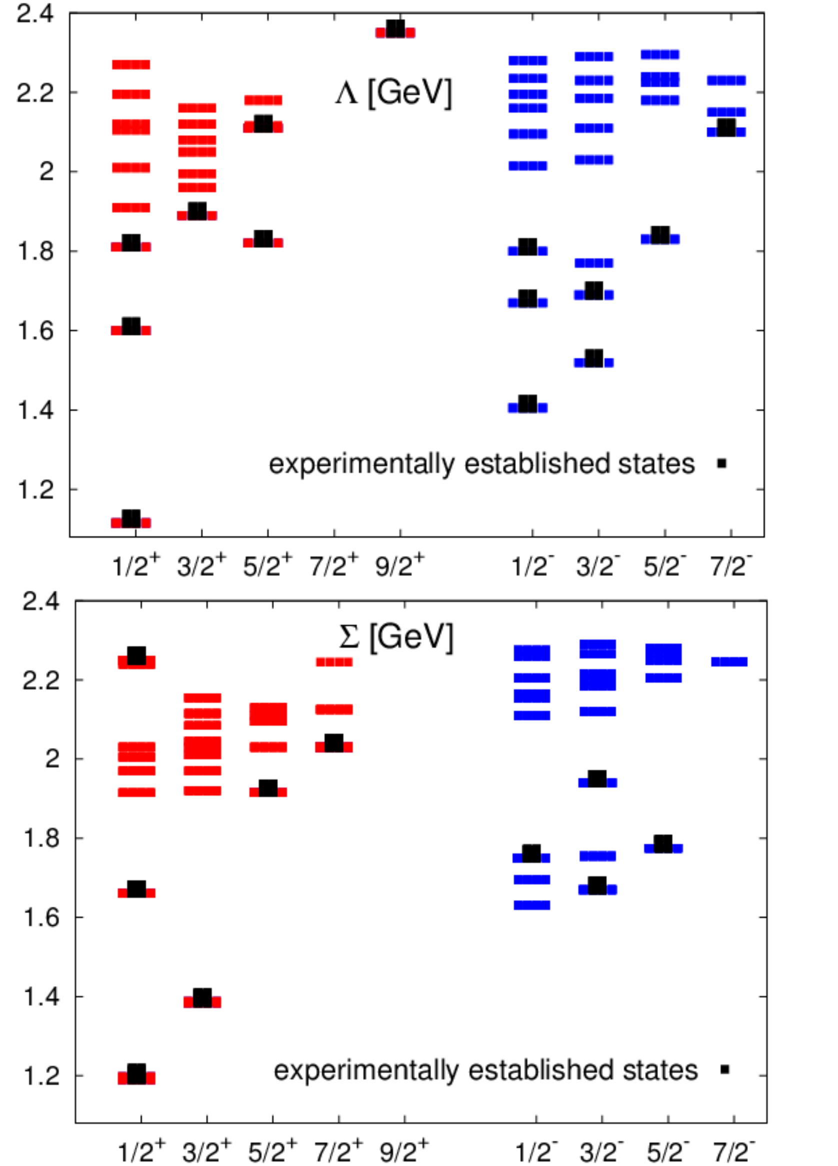
strange baryons

more strangeness

=

larger fluct.

Quark Model

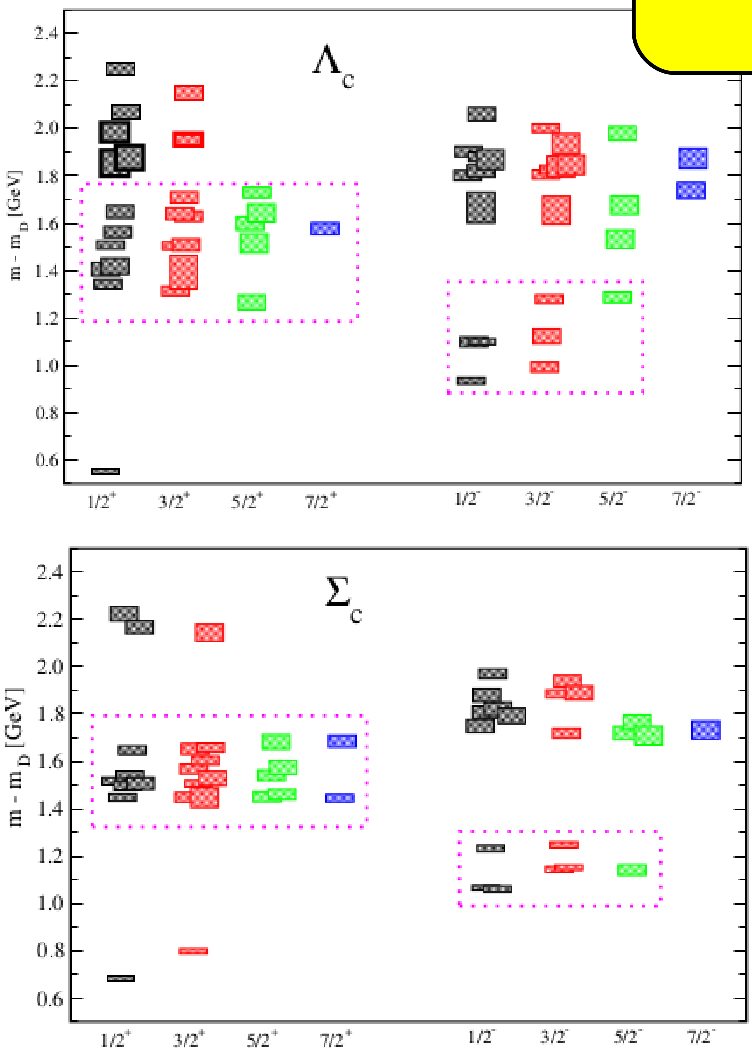


R. Edwards et.al., Phys. Rev D87, 054506 (2013)

S. Capstick, N. Isgur, Phys. Rev. D34, 2809 (1986)

Probing the hadron spectrum using QCD thermodynamics

Lattice QCD



$$P_{tot} = \sum_{\text{h=all hadrons}} P_h$$

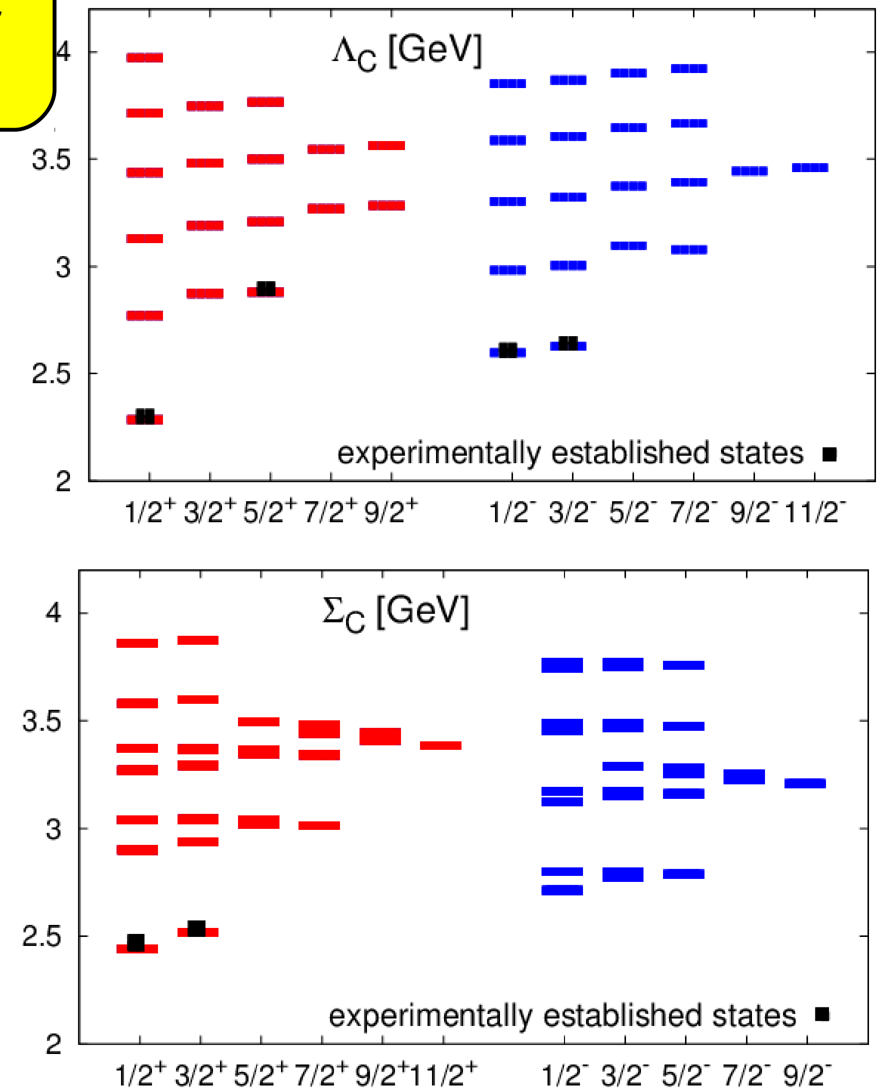
charmed baryons

more charm

=

larger fluct.

Quark Model



M. Padmanath et.al., arXiv:1311.4806

D. Ebert et. al., Eur. Phys. J. C66, 197 (2010); Phys. Rev. D84, 014025 (2011)

Correlations and Fluctuations: HRG vs. LQCD

– construct QCD observables that would project onto specific quantum numbers, if QCD = HRG

E.g.: HRG pressure:

$$\frac{P}{T^4} = \sum_{m \in mesons} \ln Z_m^b(T,V,\mu) + \sum_{m \in baryons} \ln Z_m^f(T,V,\mu)$$

HRG charmed-charged-baryon density:

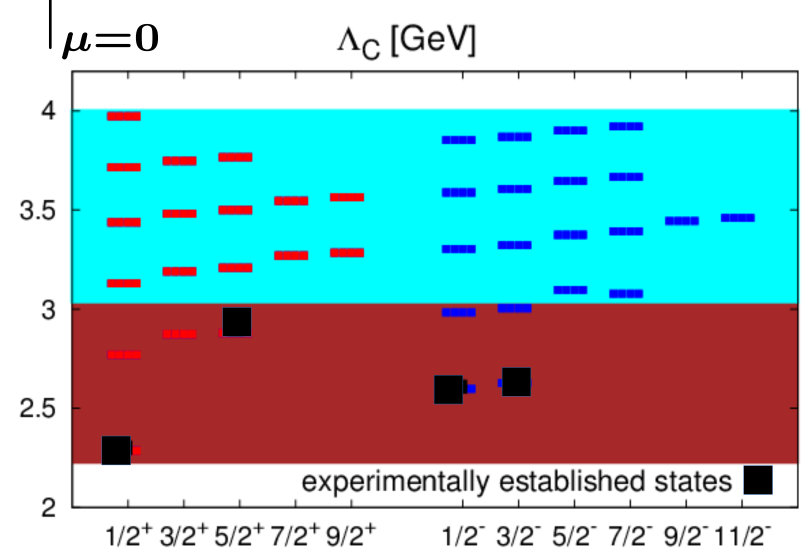
$$\chi_{211}^{BQC} = \sum_{\substack{m \in QC- \\ baryons}} \left. \frac{\partial^4 \ln Z_m^f(T,V,\mu)}{\partial \mu_B^2 \partial \mu_Q \partial \mu_C} \right|_{\mu=0}$$

sum "knows" about spectrum

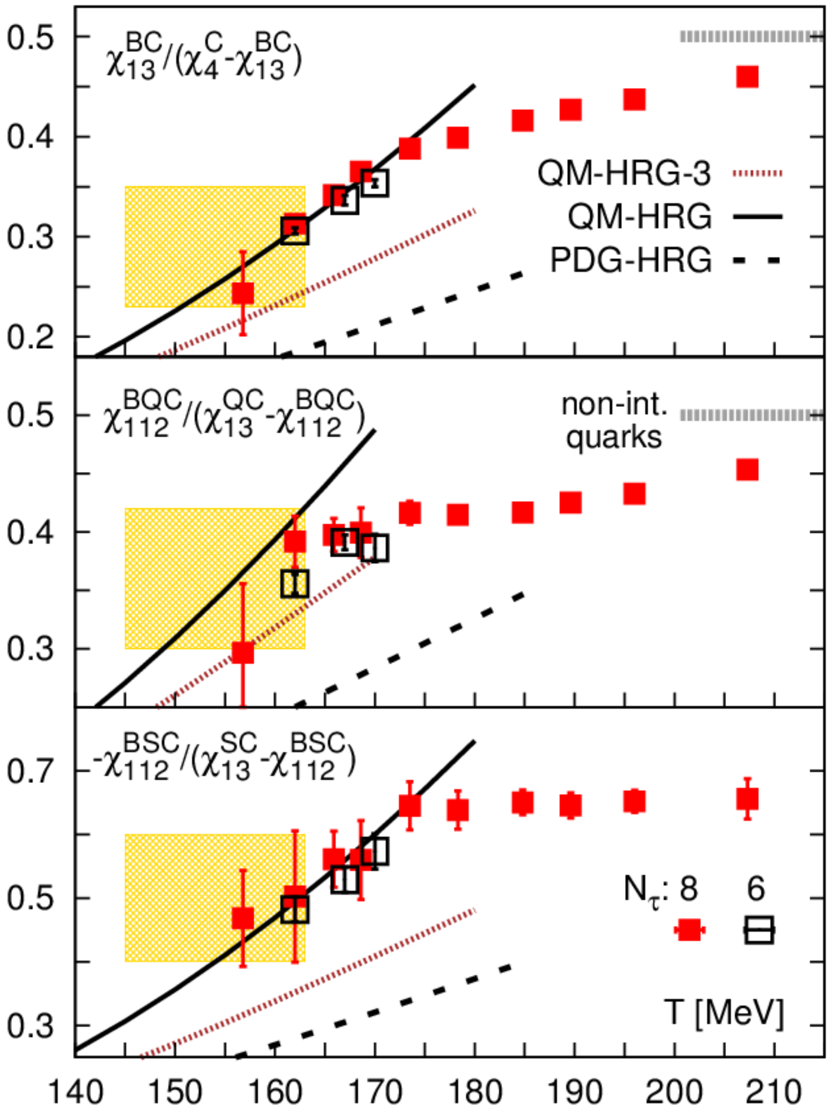
+ QM-HRG

QM-HRG-3

PDG-HRG



Evidence for many charmed baryons in thermodynamics



A. Bazavov et al., Phys.Lett. B737 (2014) 210

close to Tc charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PDG-HRG)

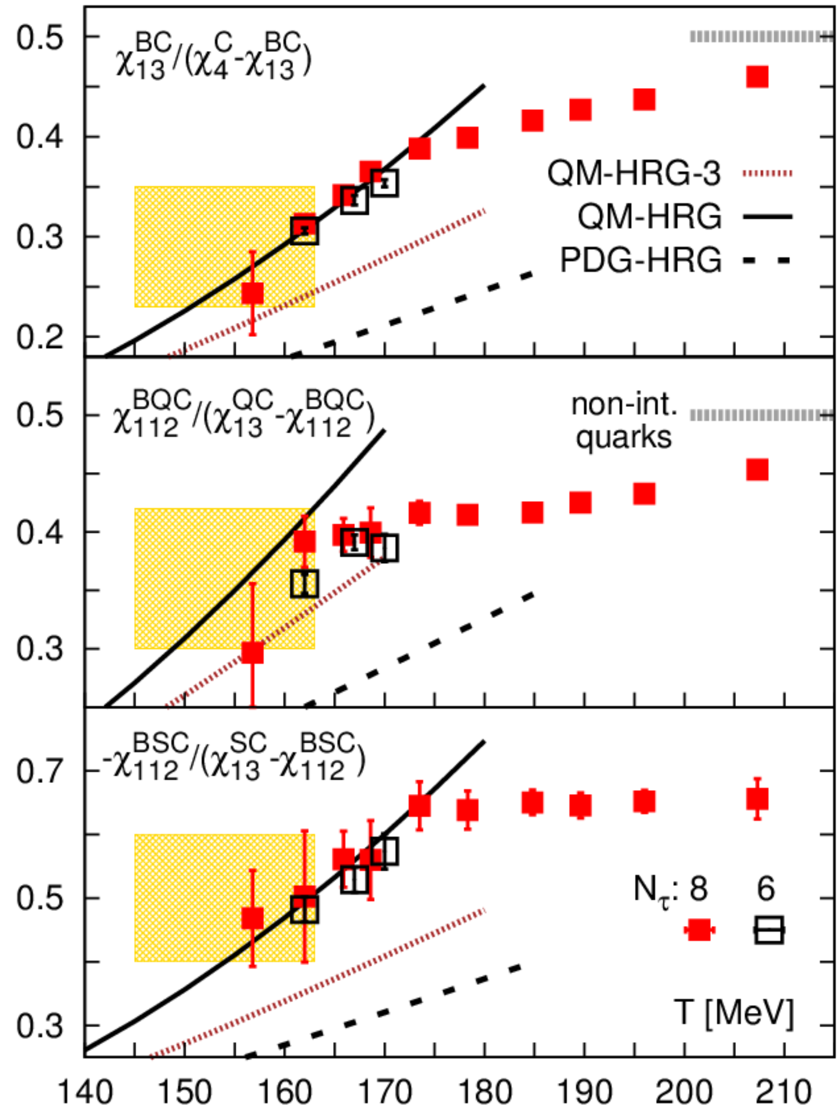
all charmed baryons/mesons

charged charmed baryons/mesons

strange charmed baryons/mesons

including resonances predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

Evidence for many charmed baryons in thermodynamics



A. Bazavov et al., Phys.Lett. B737 (2014) 210

close to Tc charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PD

observation of
5 new charmed baryons
by LHCb
arXiv:1703.04639

all charmed bary

charged charmed baryons/mesons

strange charmed baryons/mesons

including resonances predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

Thank you for your attention and the

many interested/interesting questions

you asked during the lectures and the breaks

Bielefeld University

Brookhaven National Laboratory