Nuclear Astrophysics

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Lecture 1
Basic notions in low energy nuclear physics and dense matter physics.

Lecture 2
Supernovae, neutron stars, and compact object merges in the multi-messenger era.

INSTITUTE for NUCLEAR THEORY

Lectures delivered at the National Nuclear Physics Summer School, Boulder, 2017

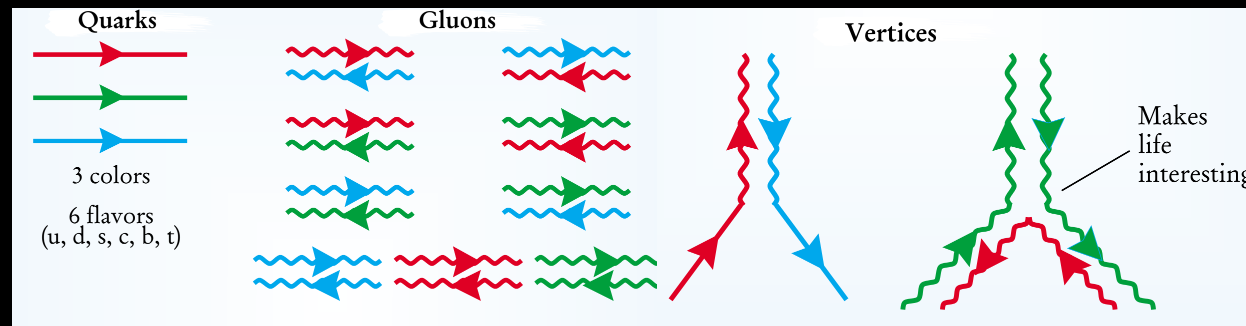
Nuclear Interactions

QCD (Lagrangian) is simple is write down

$$D_\mu \equiv \partial_\mu + i t^a A^a_\mu$$

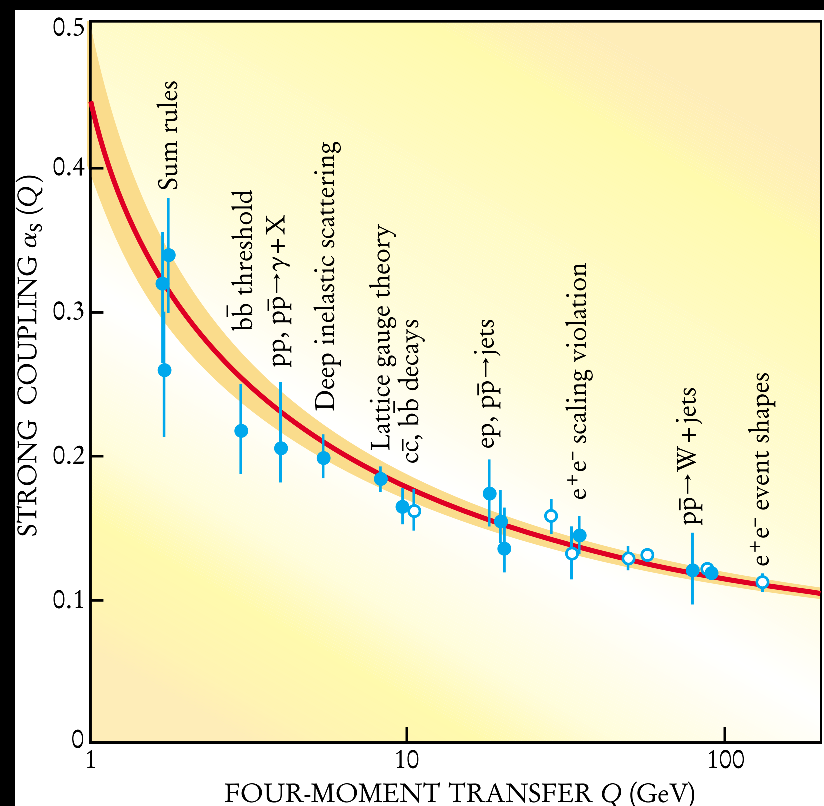
$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

$$G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$$



but is difficult to solve at low energy.
It gets simpler at high energy
(asymptotic freedom).

F. Wilczek, Physics Today (2000)



The low energy QCD vacuum is non-perturbative:

- It confines quarks to color singlet states.
- Spontaneously breaks chiral symmetry.

Nuclear Interactions

- Baryons and mesons are the relevant low energy degrees of freedom at low energy. Interactions between them are strong, complex, and short-range.

- Pions are special. They are the Goldstone bosons associated with chiral symmetry breaking and provide the longest range force between nucleons.

- Other mesons are significantly heavier. It is not very useful to single them out as mediators of the strong interaction between composite color singlet states.

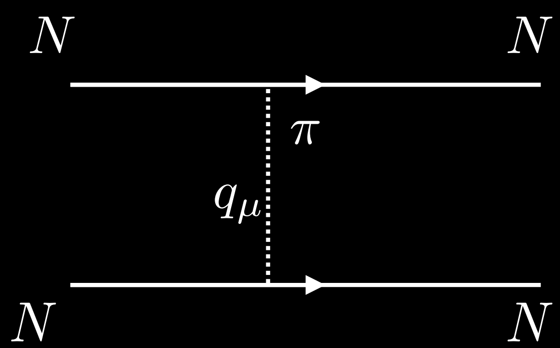
- How then can we write down a theory of strong interactions between nucleons at low energy ?

Potential Models | Effective Field Theories (EFT)

Nucleon-Nucleon Potentials

One-pion exchange:

$$\mathcal{L}_{NN\pi} = \psi_N^\dagger \left(i\partial_t - \frac{\nabla^2}{2M_N} \right) \psi_N \; - \frac{g_a}{f_\pi} \; \psi_N^\dagger \tau^a \sigma \cdot \nabla \pi^a \psi_N$$



$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \frac{\sigma_1 \cdot \mathbf{q} \; \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2}$$

In coordinate space the potential is

$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \left[S_{12} \; \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{3} \; \sigma_1 \cdot \sigma_2 \; \delta^3(r) \right]$$

Potential depends on spin and iso-spin.

It has a tensor component: $S_{12} = 3(\sigma_1 \cdot \hat{r}_1) \; (\sigma_2 \cdot \hat{r}_2) - \sigma_1 \cdot \sigma_2$

It is singular: $V(r \to 0) \approx \frac{1}{r^3}$

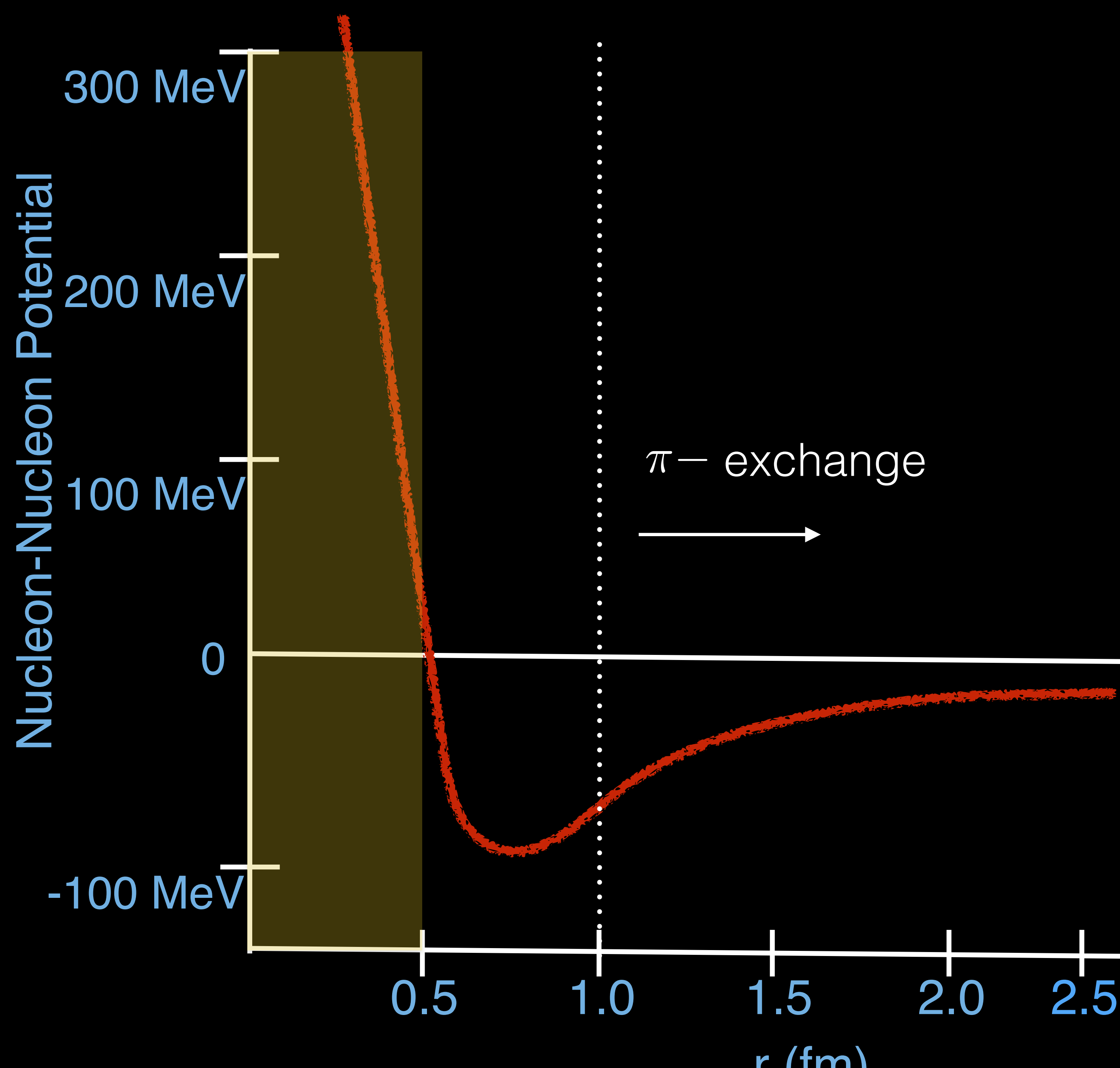
Nuclear Forces at Short Distances

They are essential even at low energy.

Are constrained by nucleon-nucleon scattering data (phase shifts).

Models favor strong repulsion. (hard-core)

Range of these forces is comparable to the intrinsic size of the nucleon.



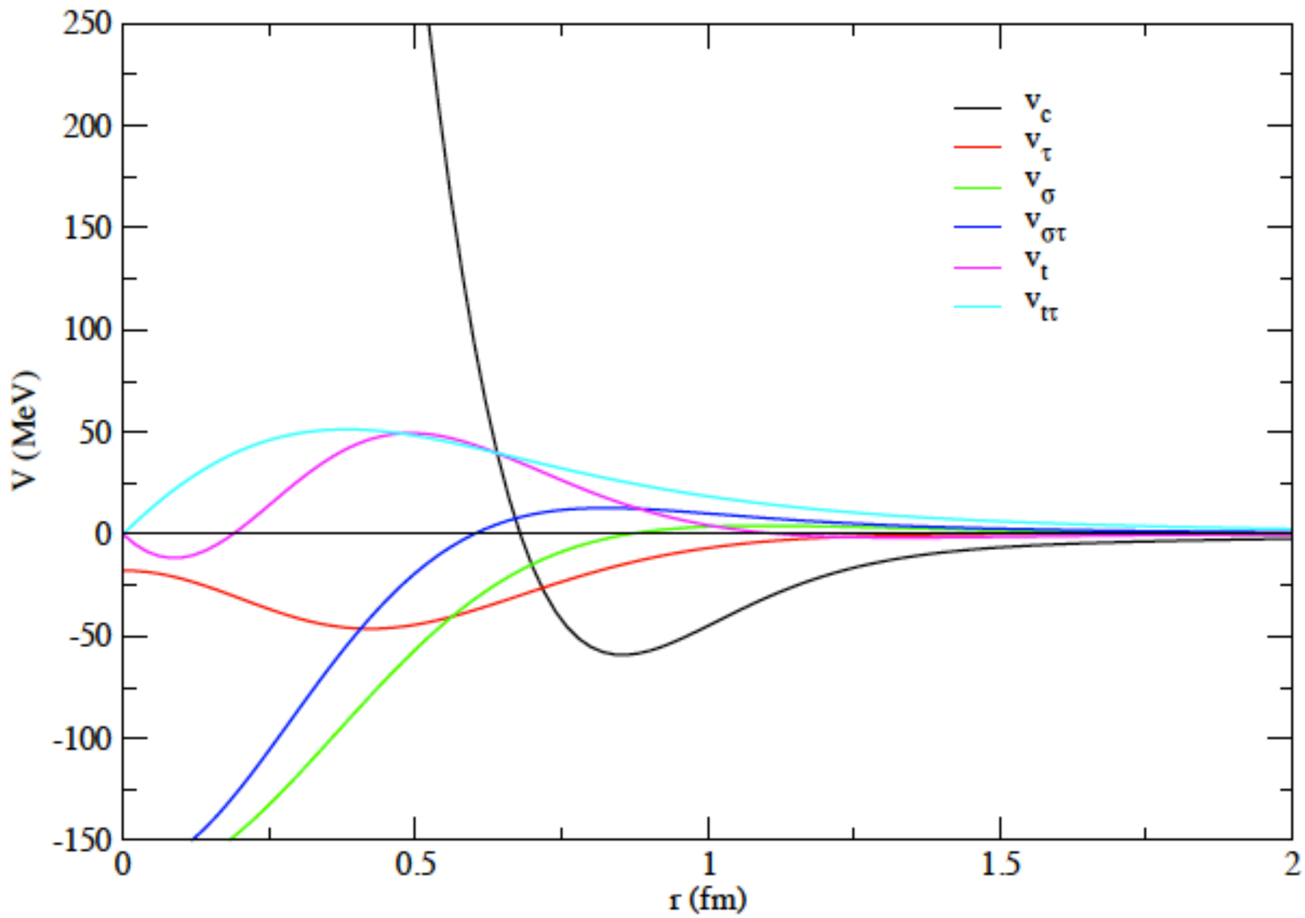
A Realistic Potential Model

$$V_{ij} = \sum_p v_p(r_{ij})\, O^p_{ij}$$

Intricate spin, isospin and tensor structure.

$$\begin{aligned} O^p_{ij} = &\ [1, \sigma_i\cdot\sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i\cdot\sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \\ &+ [1, \sigma_i\cdot\sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i\cdot\sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes \tau_i\cdot\tau_j \\ &+ [1, \sigma_i\cdot\sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes T_{ij} \\ &+ [1, \sigma_i\cdot\sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes (\tau_i + \tau_j)_z \end{aligned}$$

$$S_{ij} = 3\sigma_i\cdot\hat{r}_{ij}\sigma_j\cdot\hat{r}_{ij} - \sigma_i\cdot\sigma_j \qquad T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i\cdot\tau_j$$



Potential is Neither Unique Nor Observable (in QM)

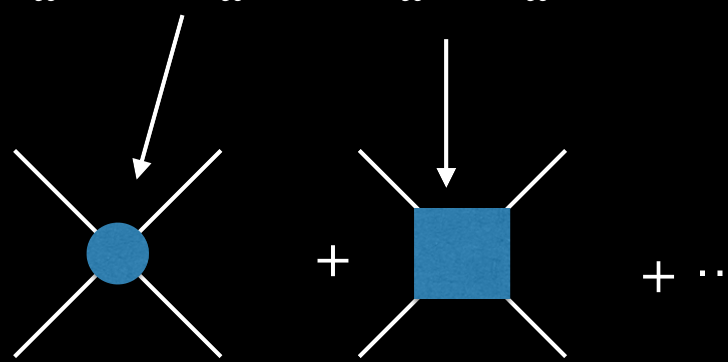
Potential Models: Relies on a set of (reasonable) assumptions about the short distance behavior to solve the Schrödinger equation and fit observables.

Effective Field Theory: Relies on a separation of scales to Taylor expand potential in powers of momenta or inverse radial separation. Coefficients of the expansion are determined by fitting to observables.

A simple (heuristic) EFT example:

$$V_\omega(q) = \frac{g_\omega^2}{q^2 + m_\omega^2} \approx \frac{g_\omega^2}{m_\omega^2} - \frac{g_\omega^2}{m_\omega^2}\,\frac{q^2}{m_\omega^2} + \ldots$$

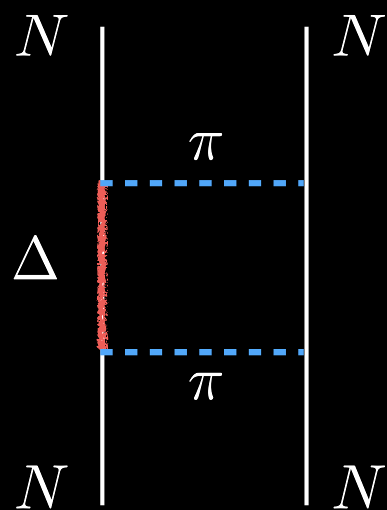
Exchange of heavy bosons at low energy cannot be resolved.

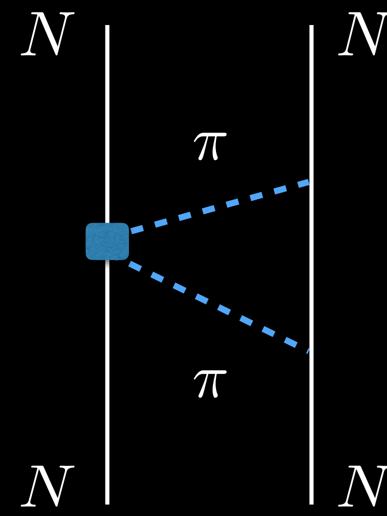


When several heavy particles may be exchanged, or when the underlying mechanism is unknown, the general expansion is

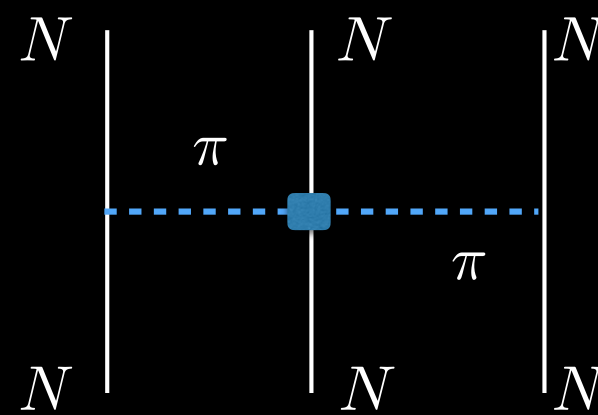
$$V_{\text{short}}(q) = C_0 + C_2\,\frac{q^2}{\Lambda^2} + \ldots$$

Nucleons are composite with internal excitations

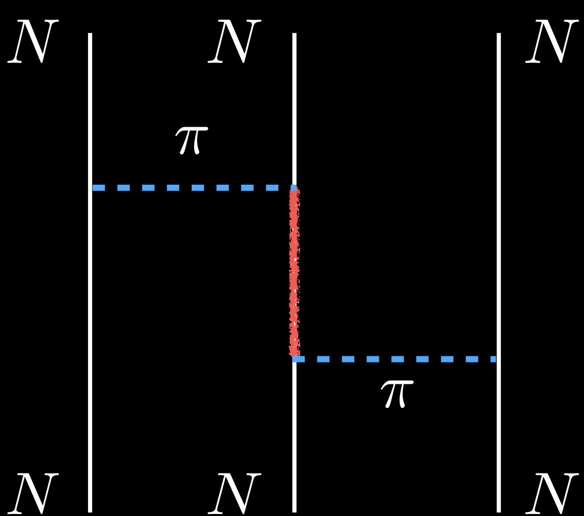


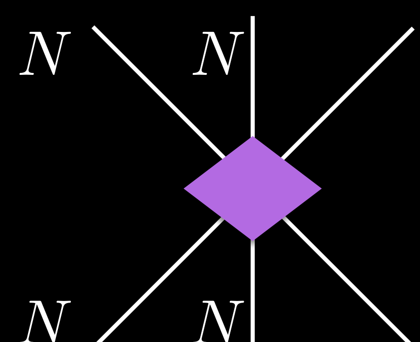


$$m_\Delta - m_N \simeq 1232 - 939 \text{ MeV} \approx 300 \text{ MeV}$$



There are three and many-body forces:



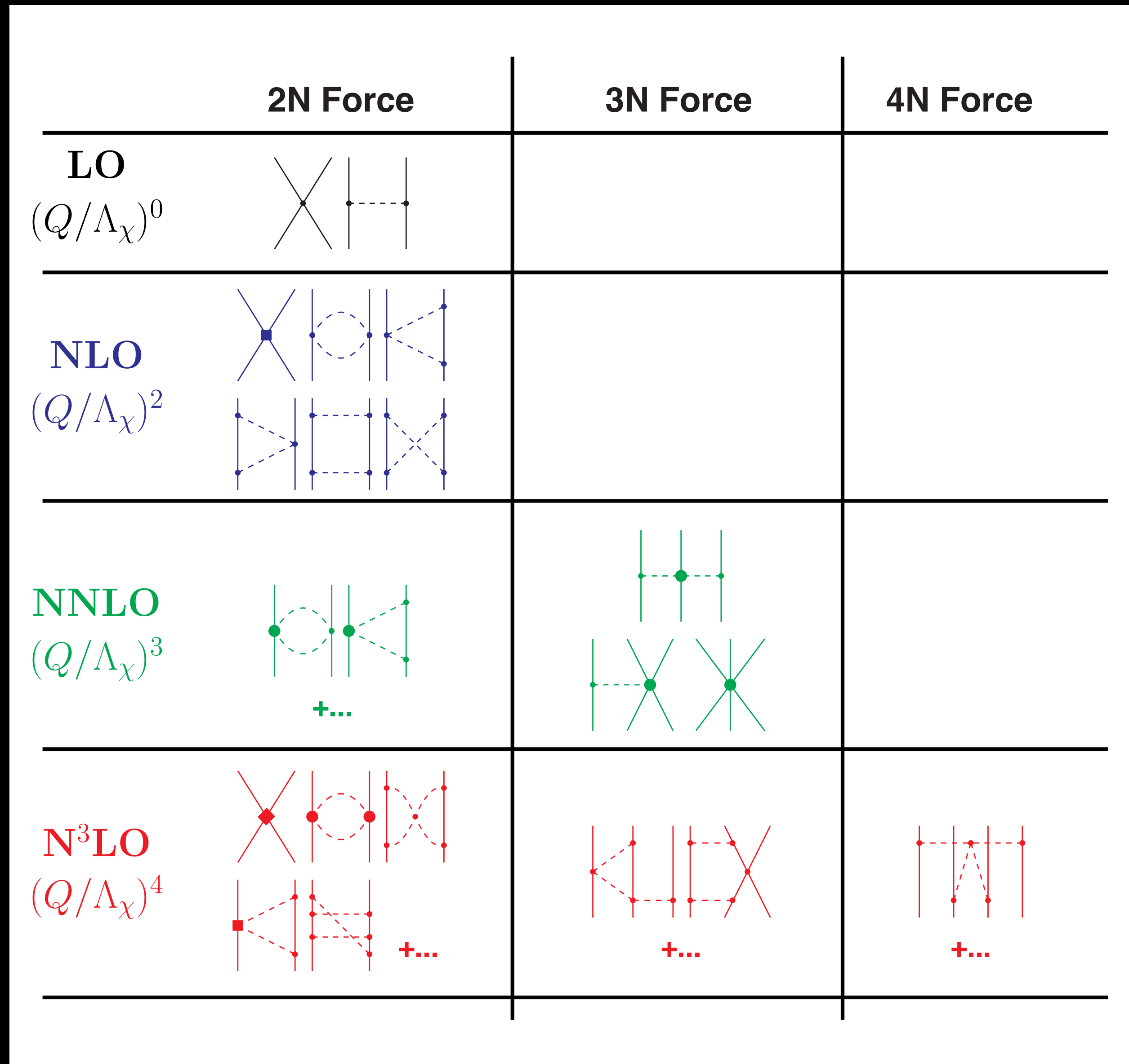


Chiral EFT

Systematic approach to low energy nuclear interactions.

Expectation is that the expansion will remain valid up to nuclear density.

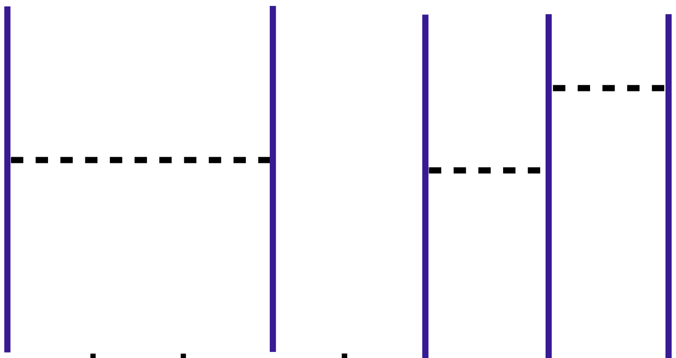
Consistent treatment of two, three and many-body forces.

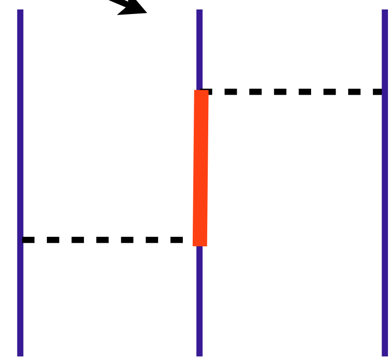


Weinberg (1990), Ordonez, Ray, van Kolck (1996), Kaplan, Savage, Wise (1996), Epelbaum, Meissner, Gloeckle (1999), Machleidt (2001) …

Ground State Energy

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$





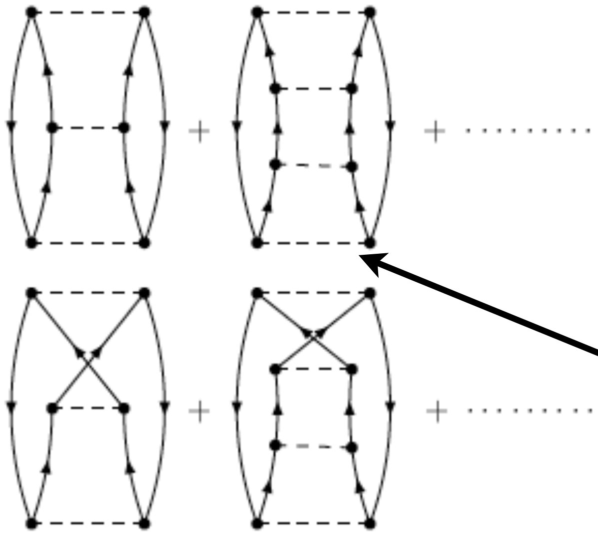
two-body nucleon-nucleon potential is well constrained by scattering data.

three-neutron potential is constrained by light nuclei.

Quantum Many-Body Theory:
Quantum Monte Carlo
Diagrammatic Methods
(perturbation theory)

$E(\rho_n, \rho_p)$: Energy per particle

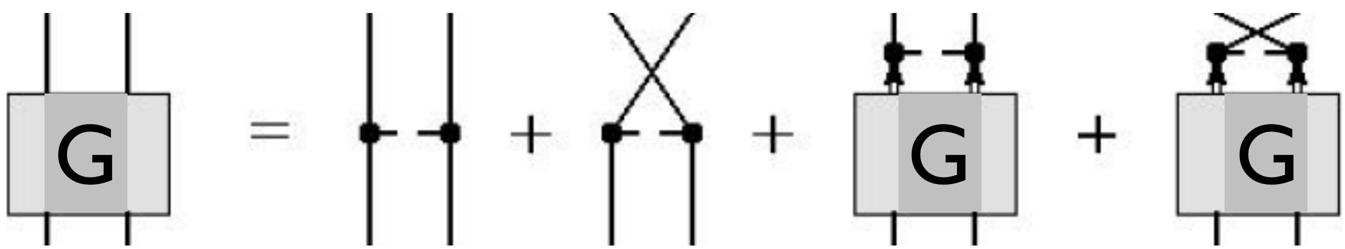
Diagrammatic Methods



Sum certain classes of Feynman diagrams to capture non-perturbative aspects.

nucleon-nucleon interaction

Eg. Bruckner or G-matrix Theory:



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | v | k_3 k_4 \rangle +$$

$$+ \sum_{k_3' k_4'} \langle k_1 k_2 | v | k_3' k_4' \rangle \frac{(1 - \Theta_F(k_3'))(1 - \Theta_F(k_4'))}{\omega - e_{k_3'} - e_{k_4'}} \langle k_3' k_4' | G(\omega) | k_3 k_4 \rangle$$

Quantum Monte Carlo

Variational Monte Carlo: $$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Greens Function Monte Carlo:

$$\begin{aligned} \Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V &= \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n \\ \Psi(\tau \to \infty) &= a_0 \psi_0 \end{aligned}$$

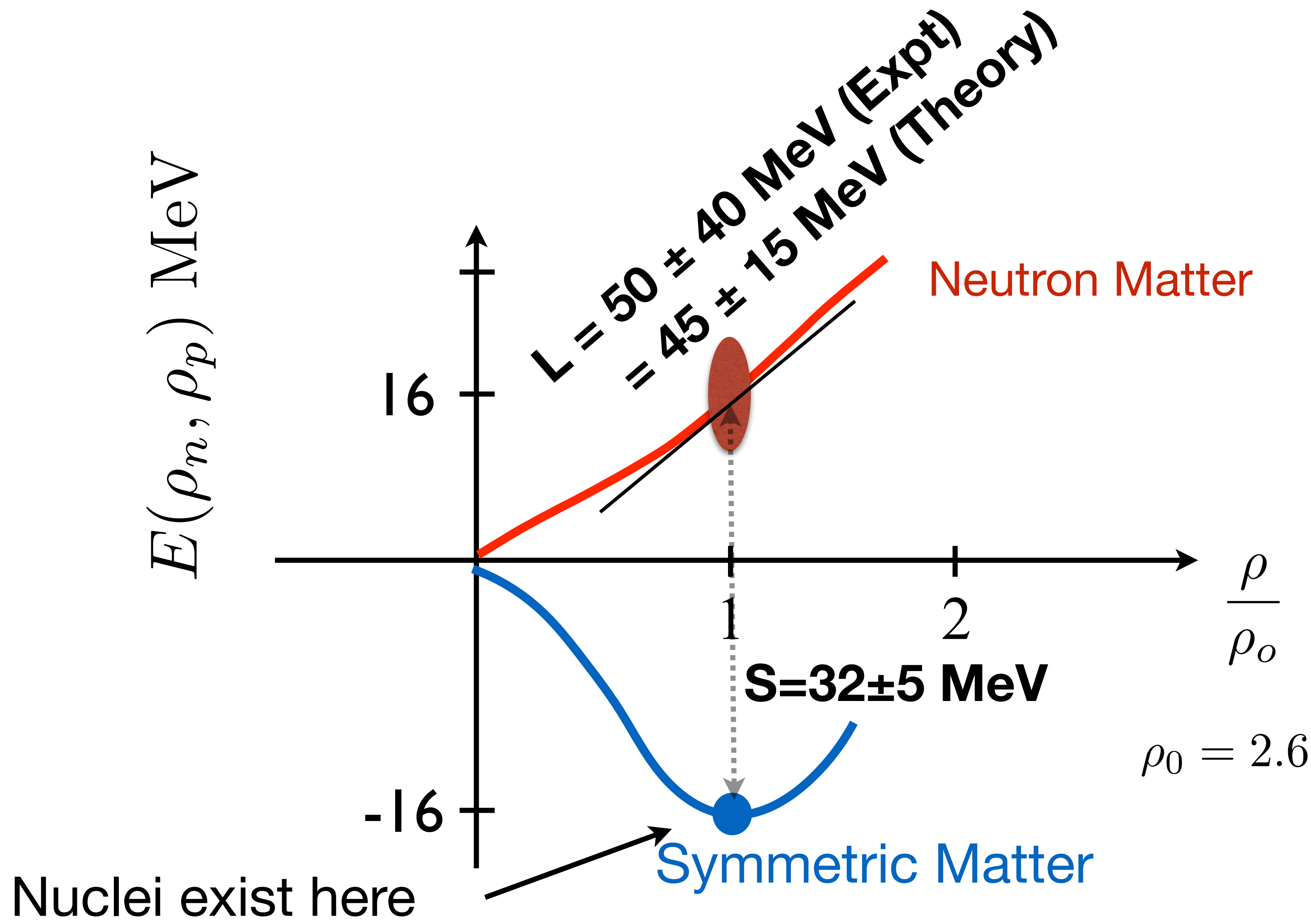
- Evolve particle coordinates.
- MC kinetic terms.
- Explicitly compute potential.

$$\Psi(\mathbf{R}_n, \tau) = \int G(\mathbf{R}_n, \mathbf{R}_{n-1}) \cdots G(\mathbf{R}_1, \mathbf{R}_0) \Psi_V(\mathbf{R}_0) d\mathbf{R}_{n-1} \cdots d\mathbf{R}_0$$

Fermion sign problem - limits GFMC

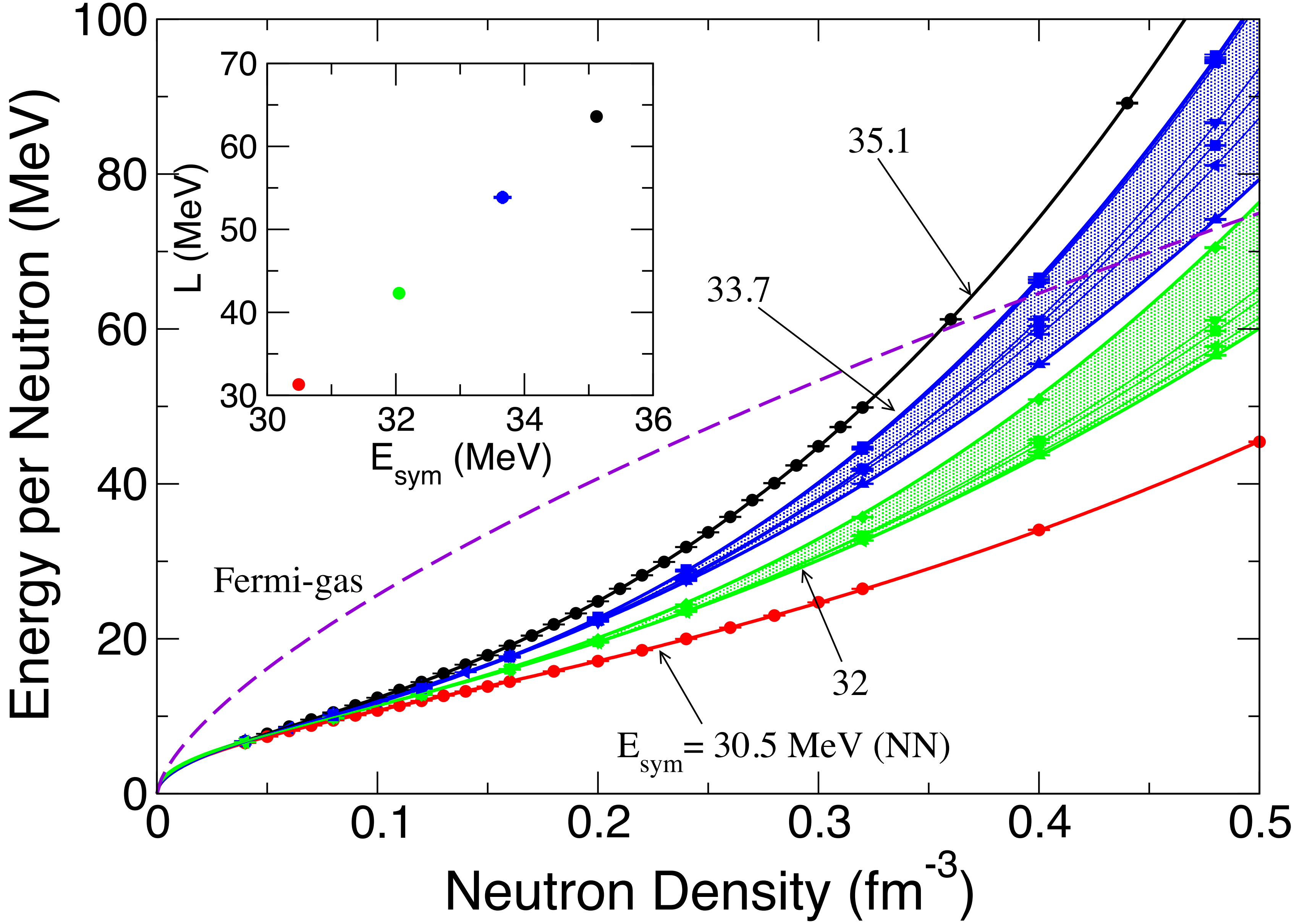
Energy of Uniform Matter: Nucleons in a Large Box

Given a Hamiltonian and a many-body theory we can calculate the energy of N neutrons + M protons in a box.



$\rho_0 = 2.6 \times 10^{14}\ \mathrm{g/cm^3}$

Neutron Matter & 3N Forces



Gandolfi, Carlson, Reddy (2010)

Nuclear Saturation, (A)symmetry Energy & Neutron Matter

Symmetric matter has zero pressure and is self-bound at a characteristic density $n_0 \approx 0.16\ \mathrm{fm}^{-3}$

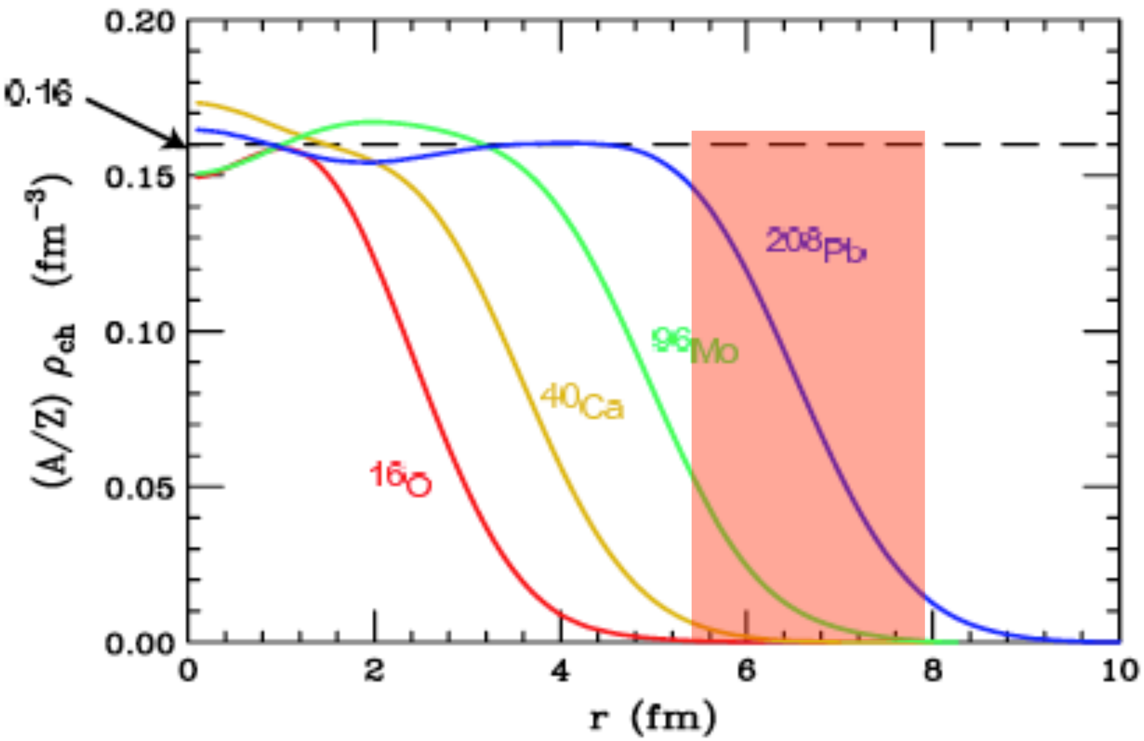
Energy per particle of symmetric matter is about -16 MeV.

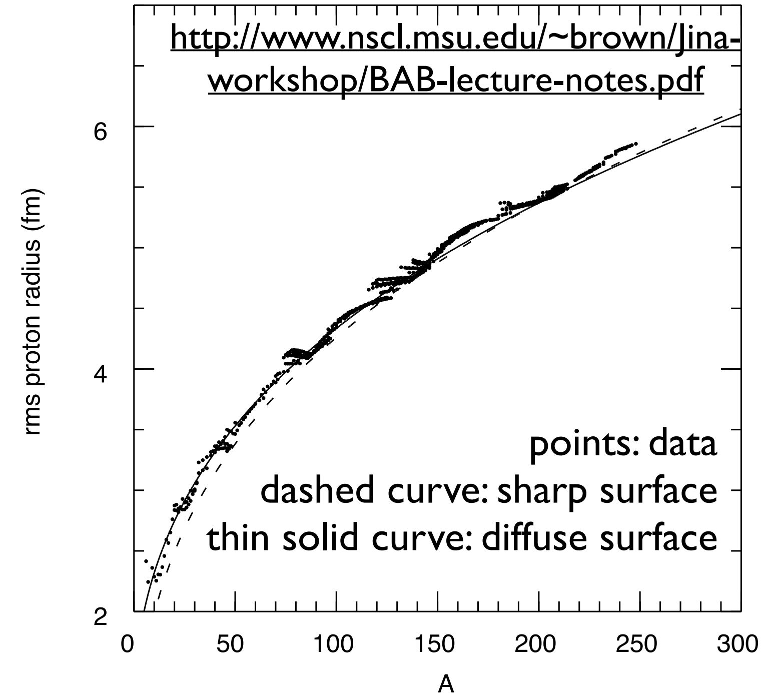
Its costs energy to make matter asymmetric.

Kinetic (Fermi) energy and potential energy costs are comparable. Total cost at saturation is about 30 MeV.

It is possible to calculate the energy of pure neutron matter up to about twice nuclear saturation density. Errors due to uncertainties in nuclear Hamiltonian (especially three-body forces) grows rapidly with density.

Nuclei as drops of nuclear matter





Nuclear saturation density $n_0 \approx 0.16\ \mathrm{fm}^{-3}$
Volume energy/nucleon ≈ -16 MeV
Symmetry energy/nucleon ≈ 30 MeV
Surface tension ≈ 1 MeV/fm²
Coulomb energy ≈ 0.86 $(Z^2/A^{1/3})$ MeV

$$Z = \frac{4\pi}{3}\ \rho_{\rm ch}\ R^3$$

$$A = \frac{4\pi}{3}\ \frac{A}{Z}\ \rho_{\rm ch}\ R^3$$

$$\simeq \frac{4\pi\ 0.16}{3}\ \left(\frac{R}{\rm fm}\right)^3$$

Prblm 1.1: Derive the above expression for the Coulomb energy and surface tension.

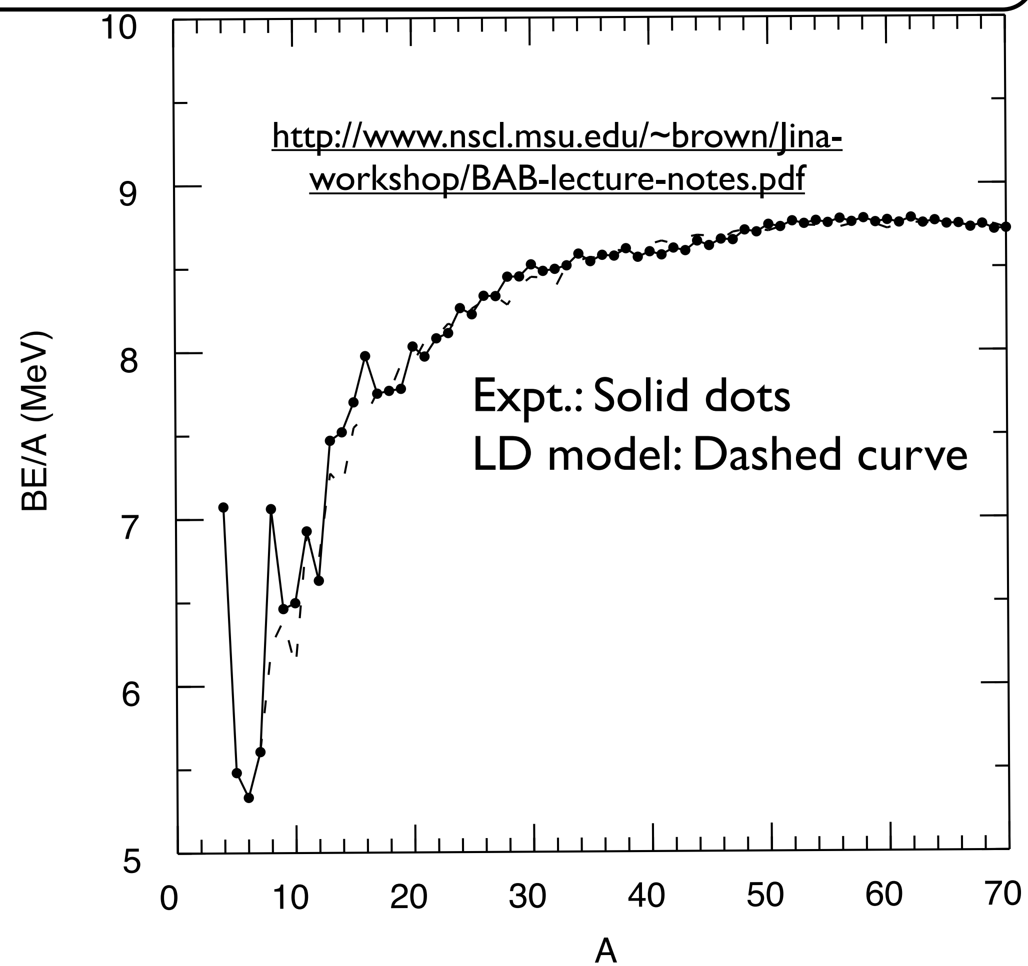
Liquid drop model:

Binding Energy:

$$\mathrm{BE}(A,Z) = \alpha_{\mathrm{bulk}}\ A - \alpha_{\mathrm{sym}}\ \frac{(N-Z)^2}{A} - \alpha_S\ A^{2/3} - \alpha_C\ \frac{Z^2}{A^{1/3}}$$

$$\alpha_{\mathrm{bulk}} = 15.49\ \mathrm{MeV}$$
$$\alpha_{\mathrm{sym}} = 22.6\ \mathrm{MeV}$$
$$\alpha_{\mathrm{S}} = 17.23\ \mathrm{MeV}$$
$$\alpha_C = 0.697\ \mathrm{MeV}$$

The website: http://128.208.190.199/~intuser/ld.html allows you to modify the liquid drop model and a refit parameters:



Question 1.1: Why does the liquid drop model of the nucleus provide a fair description of nuclei ?

Surface and Coulomb Energies

$$E_S = 4\pi\sigma \; R^2 \qquad E_C = \frac{3}{5}\alpha_{\rm em}\;\frac{Z^2}{R} = \frac{3}{5}x_p^2\alpha_{\rm em}\;\frac{A^2}{R}$$

Constant density : $R = r_0 A^{1/3}$ $\qquad r_0 = \left(\frac{3}{4\pi\; n_0}\right)^{1/3} \simeq 1.14 \text{ fm}$

$$\frac{E_S}{A} = 4\pi\; r_0^2\;\sigma\;\; A^{-1/3} \qquad \frac{E_C}{A} = \frac{3\;\alpha_{\rm em}\;x_p^2}{5\;r_0}\;A^{2/3}$$

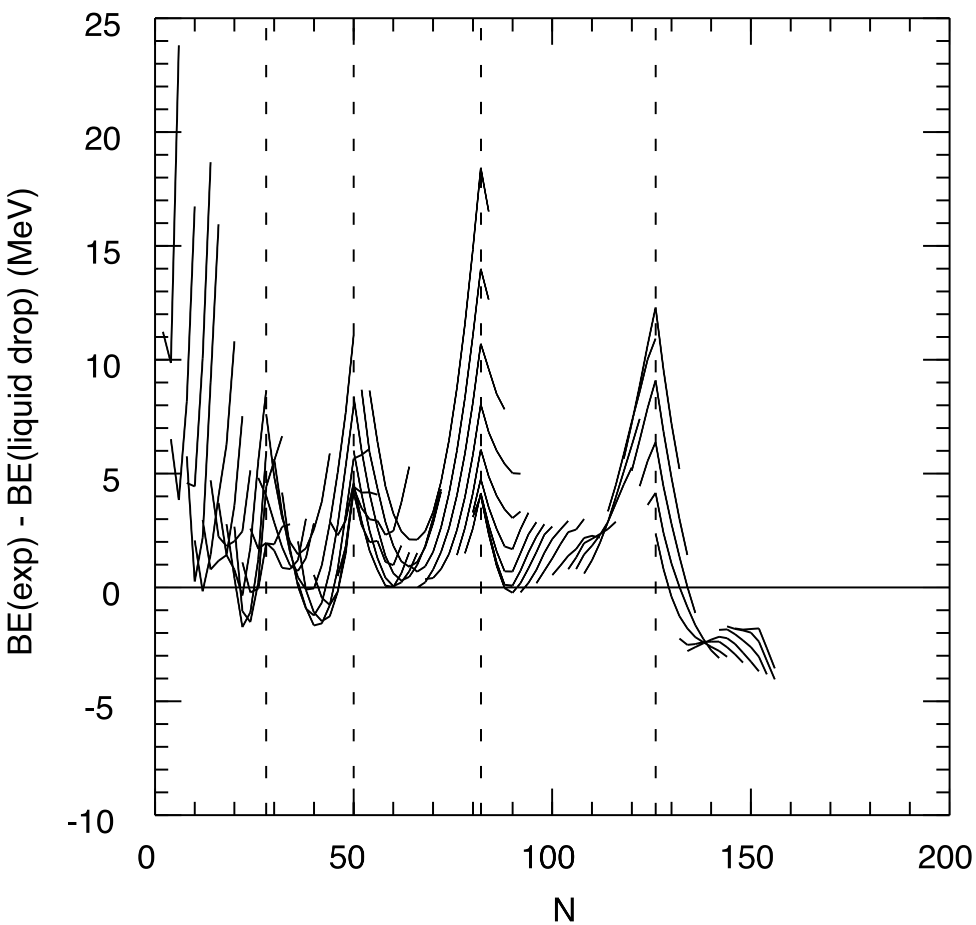
At the minimum : $$\frac{2}{3}E_C - \frac{1}{3}E_S = 0$$

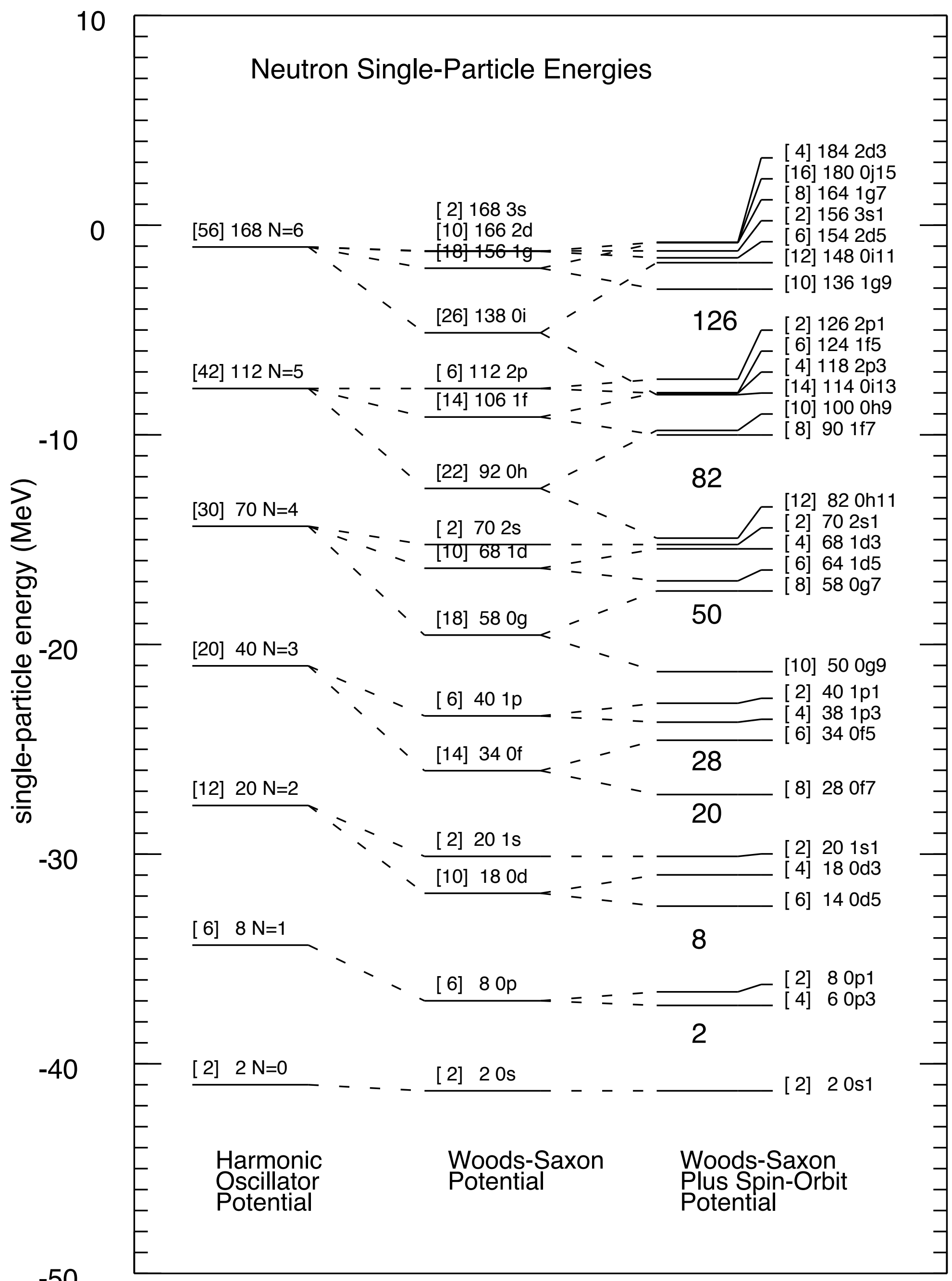
or : $$E_S = 2E_C$$

Prblm 1.2 : Show that the most favored nucleus has $$A \simeq \left(\frac{r_0}{1.2 \text{ fm}}\right)^3 \left(\frac{\sigma}{1 \text{ MeV/fm}^2}\right)\frac{4\pi}{x_p^2}$$

Shell Structure - Magic numbers

Spin-orbit force is strong in nuclei.

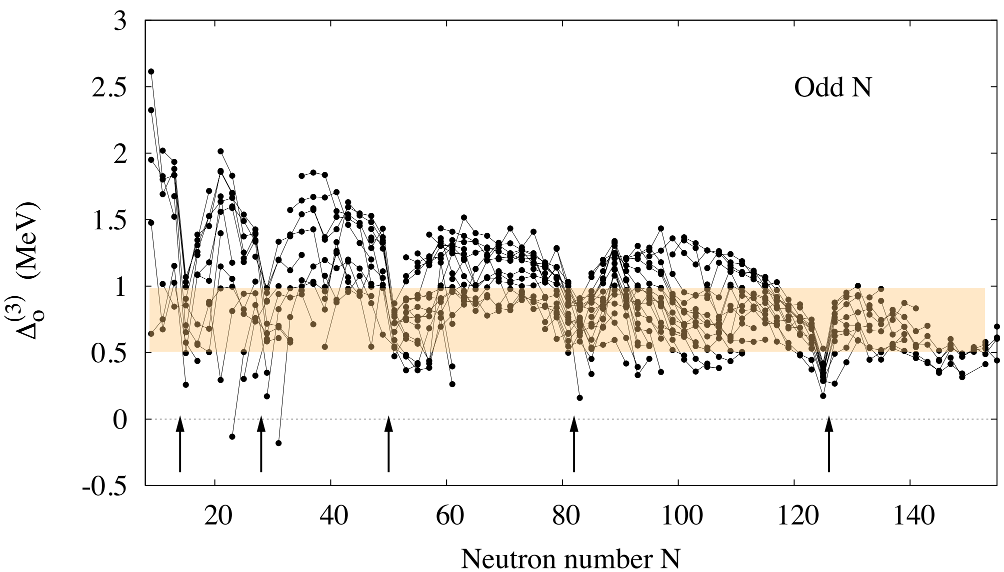




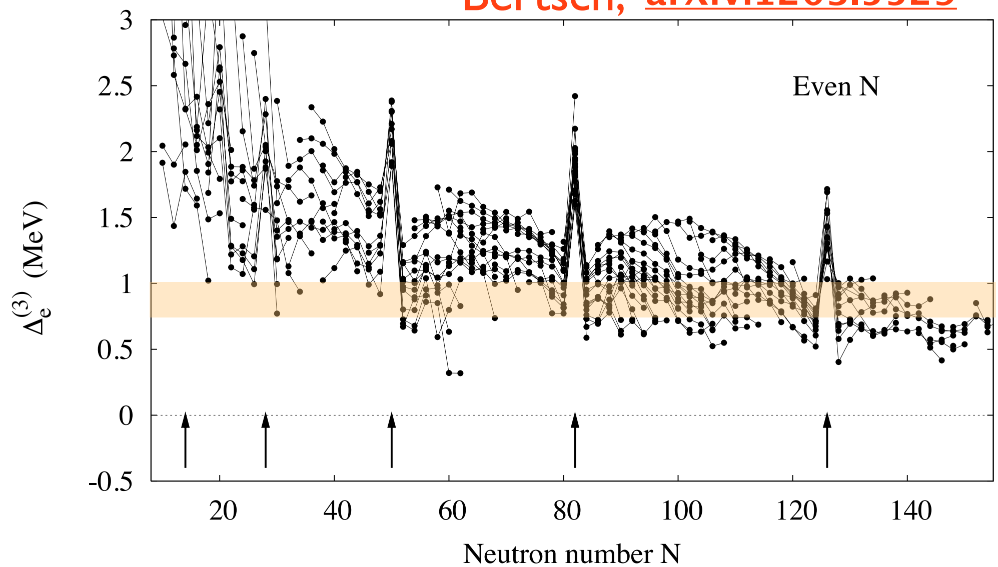
Figures: http://www.nscl.msu.edu/~brown/Jina-workshop/BAB-lecture-notes.pdf

Pairing

Bertsch, arXiv:1203.5529



$$\Delta^{(3)}_{o,Z}(N) = \frac{1}{2}(E_b(Z,N+1) - 2E_b(Z,N) + E_b(Z,N-1))$$



$$\Delta^{(3)}_{e,Z}(N) = -\frac{1}{2}(E_b(Z,N+1) - 2E_b(Z,N) + E_b(Z,N-1))$$

Systems with odd number of neutrons or protons have lower relative binding energy.

There is a gap in the single particle spectrum.

Pairing is ubiquitous in Fermi systems we shall return this latter.

Pairing is Ubiquitous

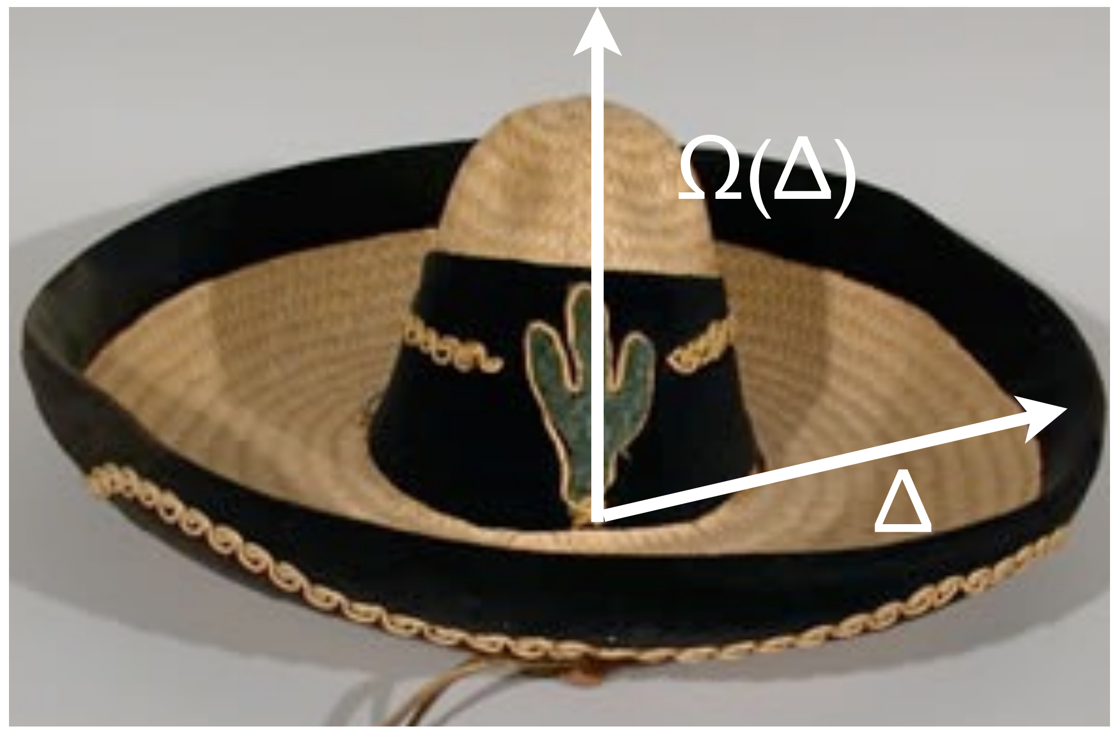
$$H = \sum_{k,s=\uparrow,\downarrow} (\frac{k^2}{2m} - \mu)\ a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

$$\Delta \propto \mu\ \exp\left(\frac{-1}{gN(0)}\right)$$

Cooper Pair condensation results in superfluidity and superconductivity:

- Energy-gap for fermions
- New collective excitations (Goldstone modes)



$$E(p) = \sqrt{(\frac{p^2}{2m} - \mu)^2 + \Delta^2} \qquad \omega_{\text{phonon}} = v_s\ q$$