Neutrino Physics

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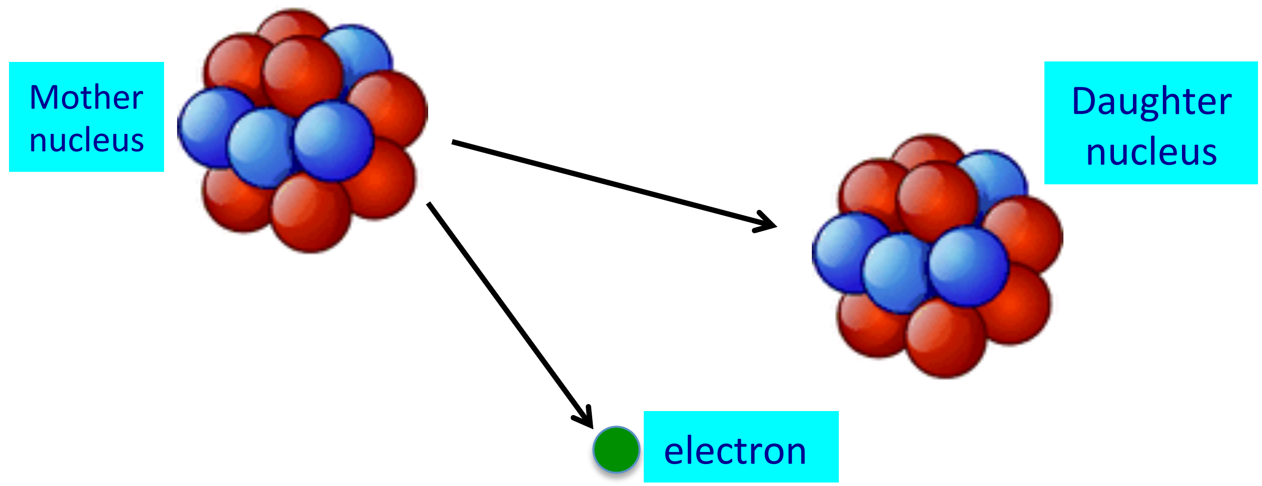
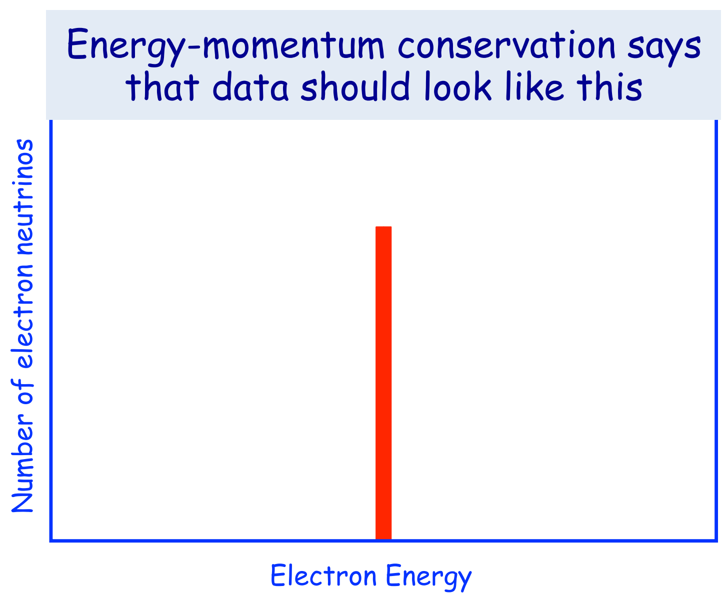
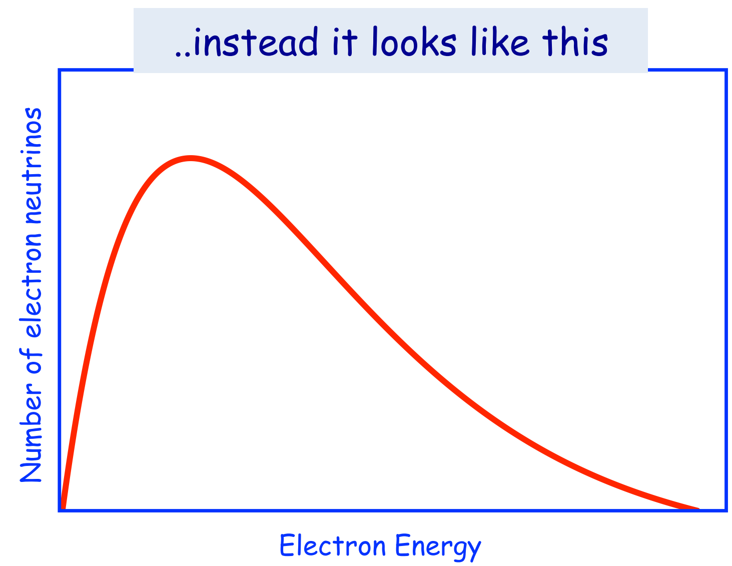
NNPSS 2017, Boulder

Lecture 1

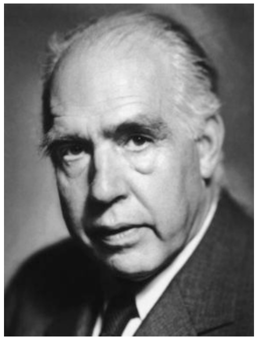
Neutrino came out of a puzzle about the radioactive decay in the early 1920's:



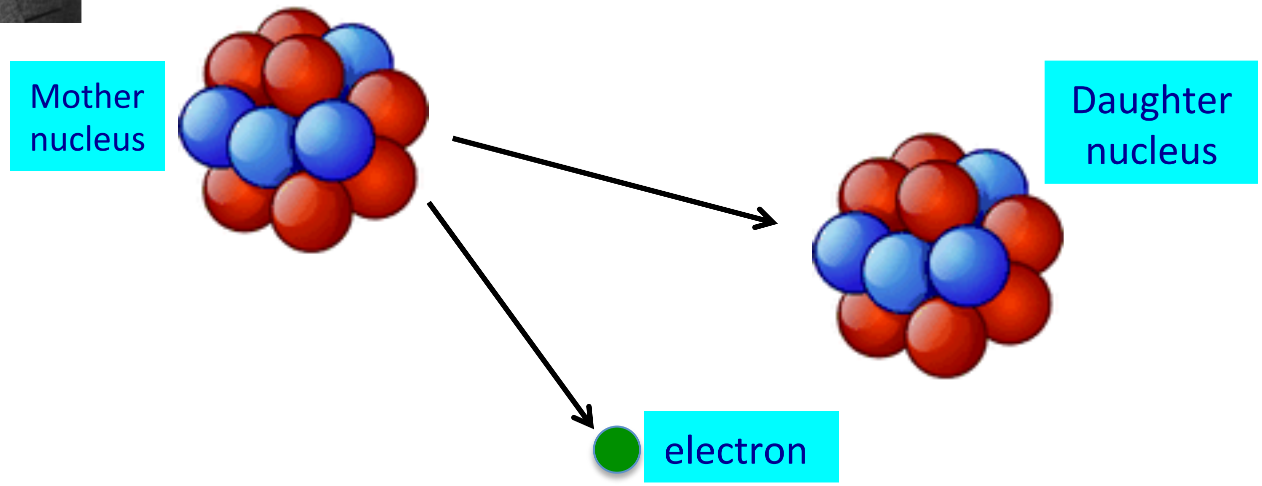




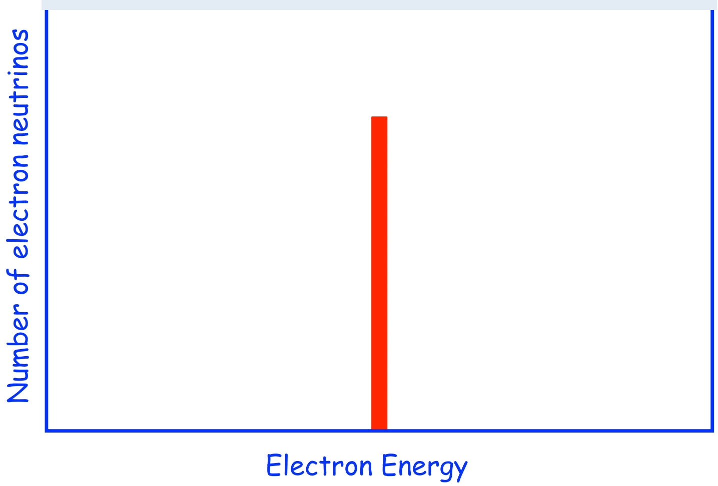
Niels Bohr



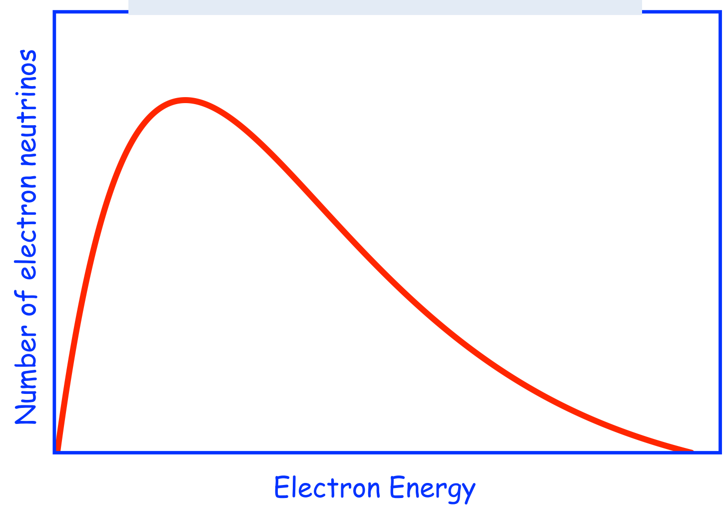
In radioactive decays energy-momentum conservation no longer holds!



Energy-momentum conservation says that data should look like this

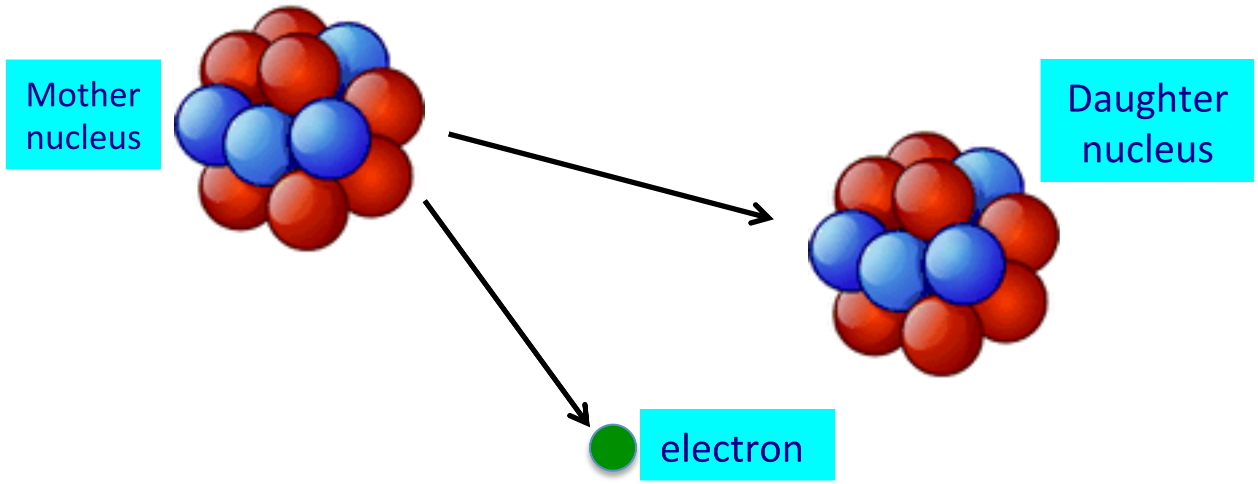


..instead it looks like this



Wolfgang Pauli

In this reaction there is a third particle produced that you cannot (yet) see!



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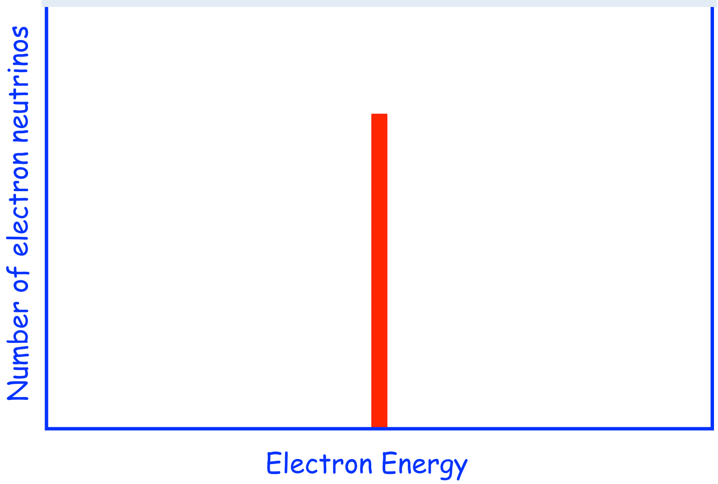

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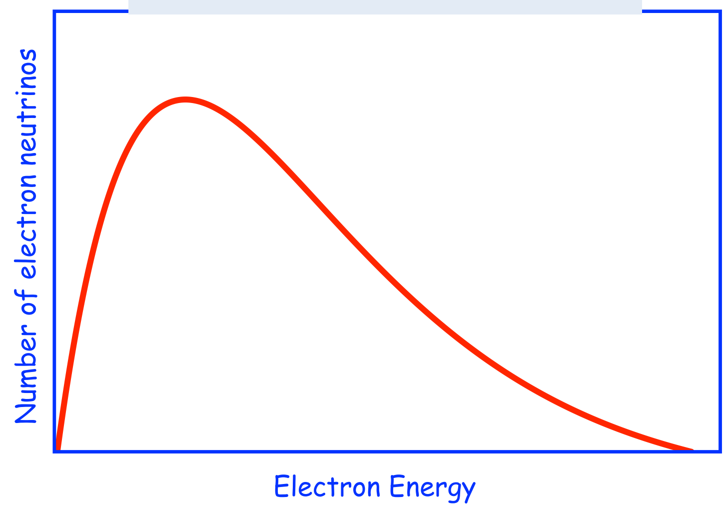

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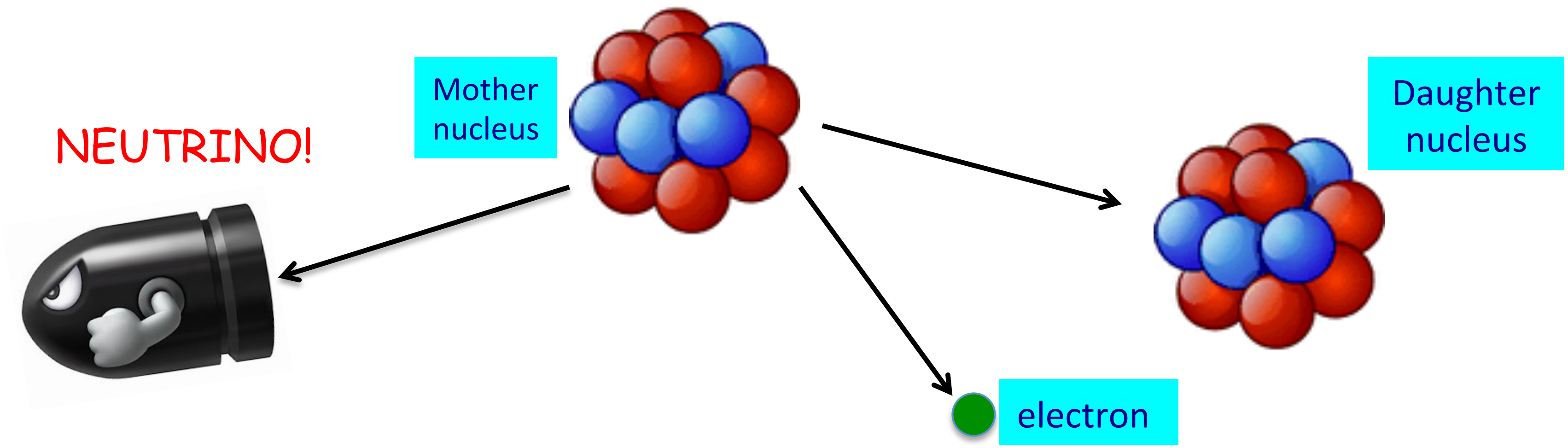

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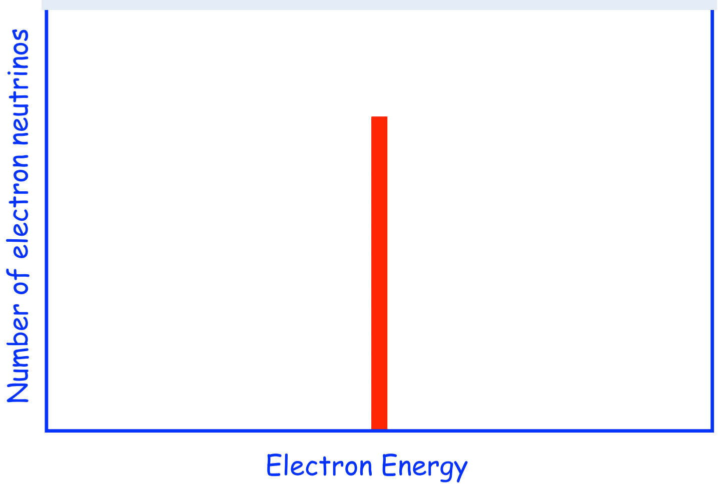

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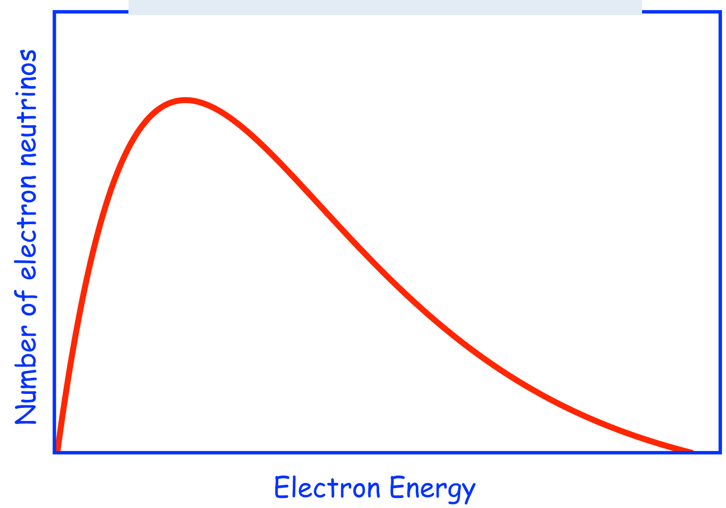
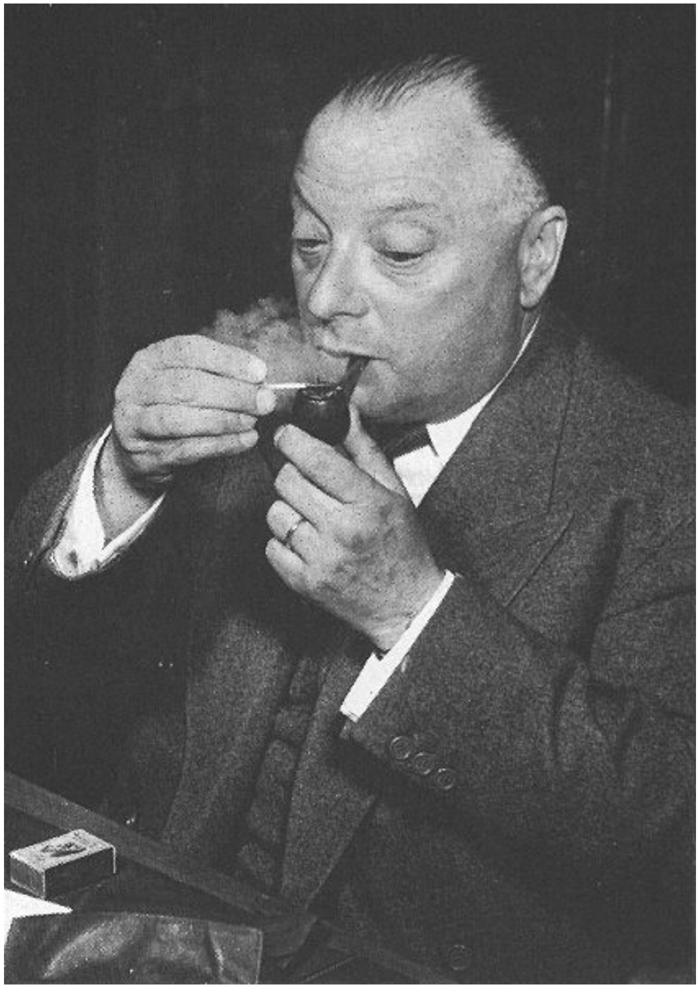




Wolfgang Pauli,
father of the neutrino
and Pauli exclusion
principle

Physicist goes to a ball

or

Mystery of Missing Energy

Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

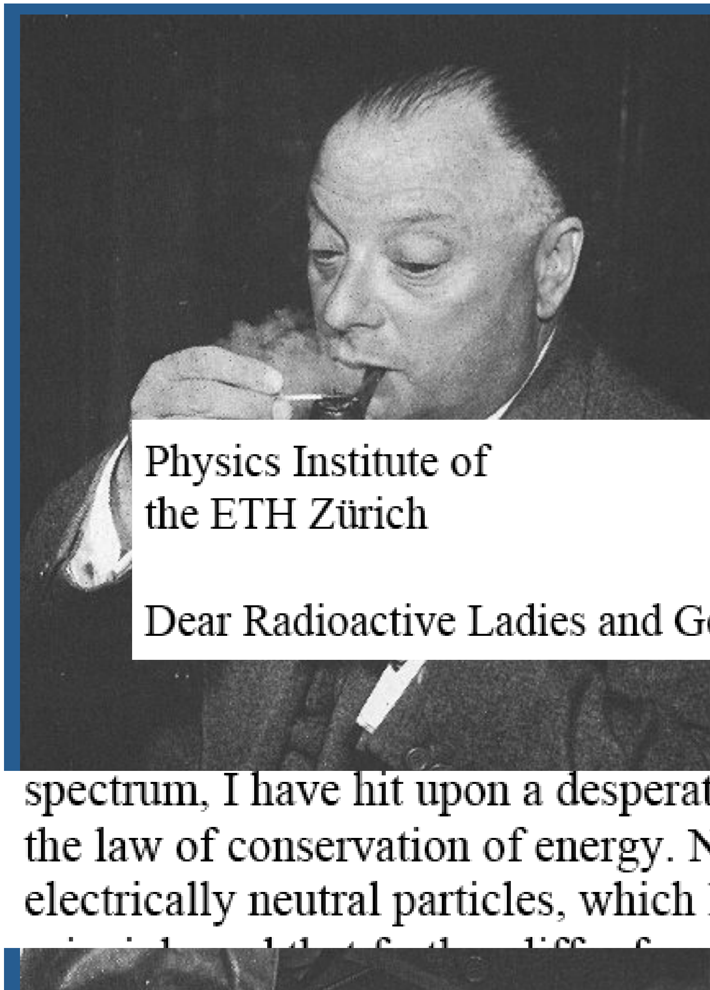
Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Physicist goes to a ball

or

ergy



Physics Institute of
the ETH Zürich

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

spectrum, I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion

Wolfgang Pauli

way of rescue. Thus, dear radioactive people, scrutinize and judge. - Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7. With my best regards to you, and also to Mr. Back, your humble

principle

Pauli

Fermi

Meyer

Majorana

Pontecorvo

Pauli

Fermi

Meyer

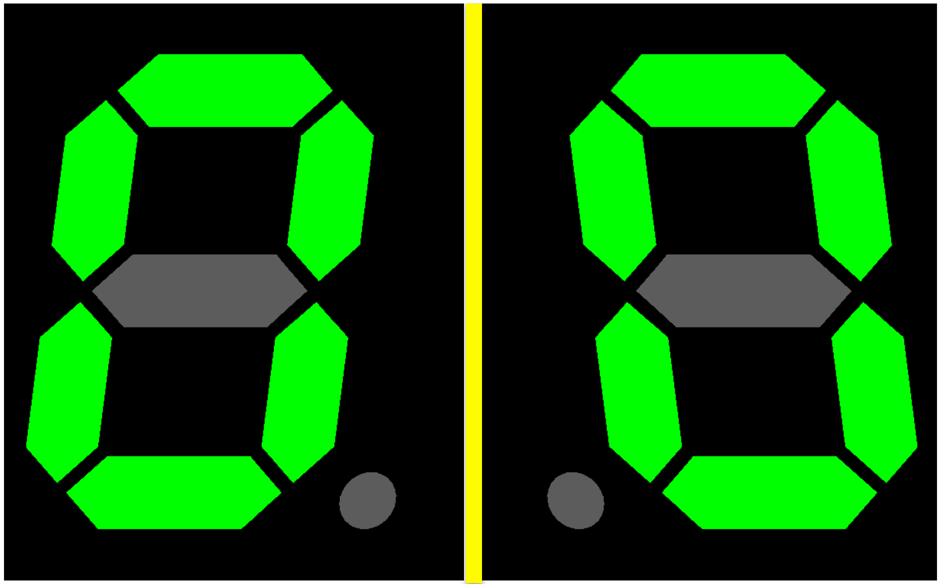
Majorana

Pontecorvo

PARITY (P)

If a process is permitted by the laws of physics, its mirror image is also permitted.

Not always true



$$\mathbf{r} \xrightarrow{P} -\mathbf{r}$$

$$\mathbf{p} = -i\hbar\nabla \xrightarrow{P} -\mathbf{p}$$

$$\mathbf{A} \xrightarrow{P} -\mathbf{A}$$

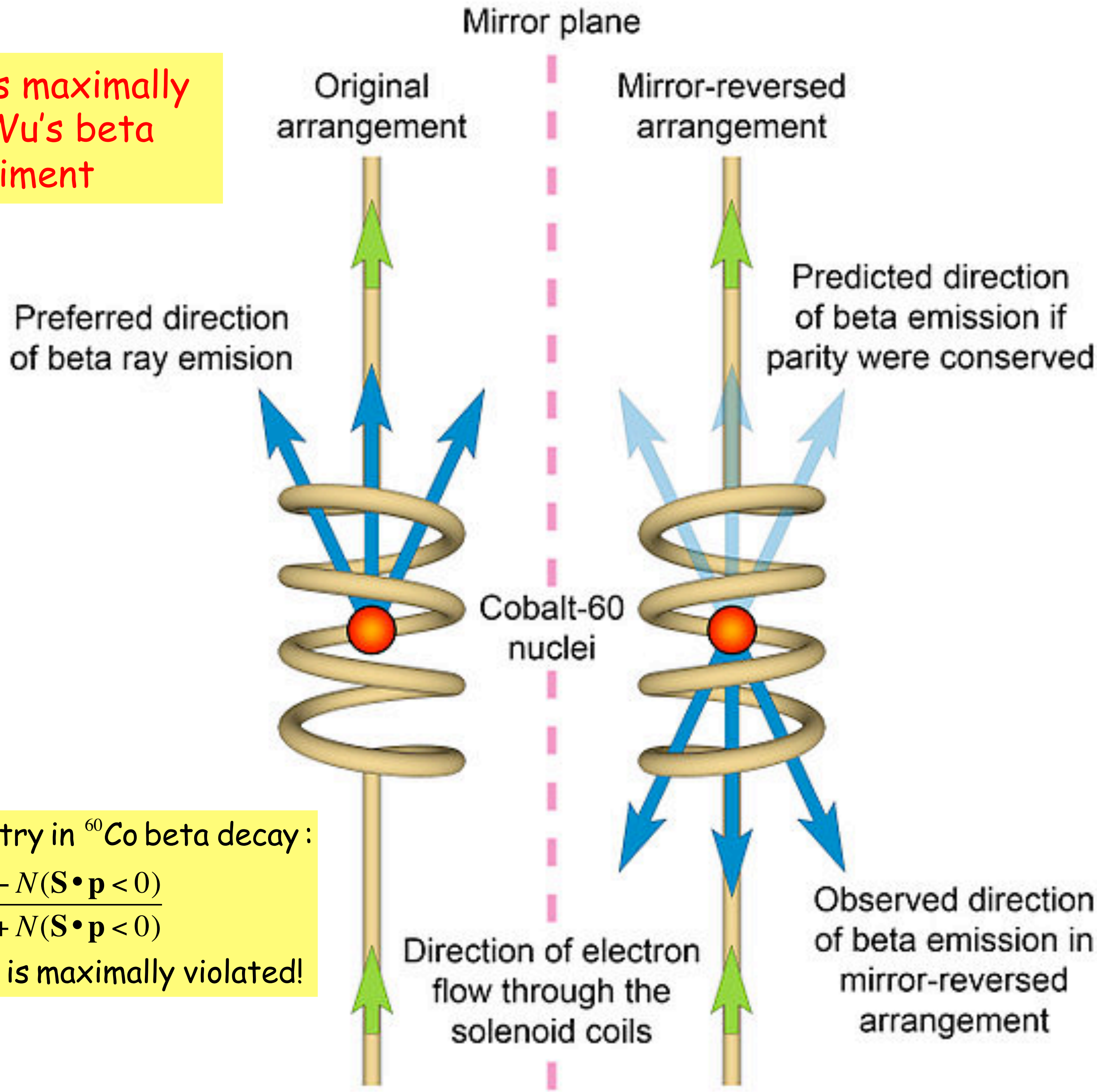
$$\underbrace{\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \xrightarrow{P} -\mathbf{E}}_{\text{vectors}}$$

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} \xrightarrow{P} \mathbf{L}$$

$$\mathbf{S} \xrightarrow{P} \mathbf{S}$$

$$\underbrace{\mathbf{B} = \nabla\times\mathbf{A} \xrightarrow{P} \mathbf{B}}_{\text{pseudo-vectors}}$$

Weak interactions maximally violate parity: Wu's beta decay experiment



Wu searched for asymmetry in ^{60}Co beta decay:

$$A = \frac{N(\mathbf{S}\bullet\mathbf{p} > 0) - N(\mathbf{S}\bullet\mathbf{p} < 0)}{N(\mathbf{S}\bullet\mathbf{p} > 0) + N(\mathbf{S}\bullet\mathbf{p} < 0)}$$

Large asymmetry: parity is maximally violated!

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \xrightarrow{P} i\gamma_\mu \partial'^\mu \psi' - m\psi' = 0$$

Assume $\psi'(-\mathbf{r},t) = S\psi(\mathbf{r},t) \Rightarrow \begin{cases} (i\gamma_0\partial^0 - i\gamma_i\partial^i)S\psi - mS\psi = 0 \\ i = 1,2,3 \end{cases}$

$$\Rightarrow \left\{ \begin{array}{ll} \gamma_0 S = S\gamma_0 & \\ \gamma_i S = -S\gamma_i & i = 1,2,3 \end{array} \right\} \Rightarrow S = \eta_P \gamma_0$$

Note that $|S|^2 = 1!$

$$V_i = \bar{\psi}\gamma_i\psi \xrightarrow{P} \psi^{\dagger'}\gamma_0\gamma_i\psi' = \psi^\dagger S^\dagger \gamma_0\gamma_i S\psi = \psi^\dagger\gamma_i\gamma_0\psi = -V_i$$

$$A_i = \bar{\psi}\gamma_i\gamma_5\psi \xrightarrow{P} \psi^{\dagger'}\gamma_0\gamma_i\gamma_5\psi' = \psi^\dagger S^\dagger \gamma_0\gamma_i\gamma_5 S\psi = -\psi^\dagger\gamma_i\gamma_0\gamma_5\psi = A_i$$

Chiral representation of Dirac matrices

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\psi = \begin{pmatrix} R \\ L \end{pmatrix} \Rightarrow \begin{cases} \psi_L = \frac{1}{2}\left(1-\gamma_5\right)\psi = \begin{pmatrix} 0 \\ L \end{pmatrix} \\ \psi_R = \frac{1}{2}\left(1+\gamma_5\right)\psi = \begin{pmatrix} R \\ 0 \end{pmatrix} \end{cases}$$

$$\gamma_0 \psi_L = \psi_R$$
$$\gamma_0 \psi_R = \psi_L$$

Right-eyed flounder (both eyes are on the right side)

Left-eyed flounder (both eyes are on the left side)

Different taste!

CHARGE CONJUGATION (C)

The laws of physics do not change if particles are replaced by antiparticles.

Not always true

$$\psi \xrightarrow{C} \psi^C = \eta_C C \overline{\psi}^T = \eta_C C \gamma_0^T \psi^*$$

$$-C = C^{-1} = C^T = C^\dagger$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$C^2 = -1$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \xrightarrow{\text{Hermitian conjugate}} i\partial^\mu \bar{\psi}\gamma_\mu + m\bar{\psi} = 0$$

$$\xrightarrow{\text{transpose}} i\gamma_\mu^T \partial^\mu \bar{\psi}^T + m\bar{\psi}^T = 0 \Rightarrow -iC^{-1}\gamma_\mu C\partial^\mu \bar{\psi}^T + m\bar{\psi}^T = 0$$

$$\Rightarrow -i\gamma_\mu \partial^\mu C\bar{\psi}^T + mC\bar{\psi}^T = 0 \quad \text{or} \quad i\gamma_\mu \partial^\mu \psi^C - m\psi^C = 0$$

$$\psi^C = \eta_C C\bar{\psi}^T = \eta_C C\gamma_0^T \psi^*$$

TIME-REVERSAL (T)

The laws of physics are the same whether time is running forward or backward

Not always true

$$\mathbf{r} \xrightarrow{T} \mathbf{r}$$

$$\mathbf{p} \xrightarrow{T} -\mathbf{p}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \xrightarrow{T} -\mathbf{L}$$

$$\mathbf{S} \xrightarrow{T} -\mathbf{S}$$

$$\mathbf{E} \xrightarrow{T} \mathbf{E}$$

$$\mathbf{B} \xrightarrow{T} -\mathbf{B}$$

$$\left.\begin{array}{l} \mathbf{E}^2 - \mathbf{B}^2 \xrightarrow{T} \mathbf{E}^2 - \mathbf{B}^2 \\ \mathbf{E} \bullet \mathbf{B} \xrightarrow{T} -\mathbf{E} \bullet \mathbf{B} \end{array}\right\} \text{Lorentz invariants}$$

Dipole moments: $H_{md} = -\vec{\mu} \bullet \mathbf{B} \quad \vec{\mu} = \mu \mathbf{S} \qquad H_{ed} = -\mathbf{d} \bullet \mathbf{E} \quad \mathbf{d} = d\mathbf{S}$

$$-\vec{\mu} \bullet \mathbf{B} \xrightarrow{T} -\vec{\mu} \bullet \mathbf{B} \qquad -\mathbf{d} \bullet \mathbf{E} \xrightarrow{T} \mathbf{d} \bullet \mathbf{E}$$

CPT Theorem

$$\psi(\mathbf{r},t) \xrightarrow{P} \gamma_0\psi(-\mathbf{r},t)$$

$$\xrightarrow{C} \gamma_0 C\gamma_0^T\psi^*(-\mathbf{r},t)$$

$$\xrightarrow{T} \gamma_5 \underbrace{C\gamma_0 C}_{\gamma_0^T}\gamma_0^T\psi(-\mathbf{r},-t) = \gamma_5\psi(-\mathbf{r},-t)$$

$$(i\gamma_\mu\partial^\mu - m)\gamma_5\psi(-\mathbf{r},-t) = 0 \Rightarrow (-i\gamma_\mu\partial^\mu - m)\gamma_5\psi(\mathbf{r},t) = 0$$

$$\Rightarrow \gamma_5(i\gamma_\mu\partial^\mu - m)\psi(\mathbf{r},t) = 0$$

Under rather general assumptions (locality, Lorentz invariance, etc.) physical laws remain the same under combined P, C, and T operations. The derivation above is an illustration of this theorem

The World according to Escher

P

C

T

start

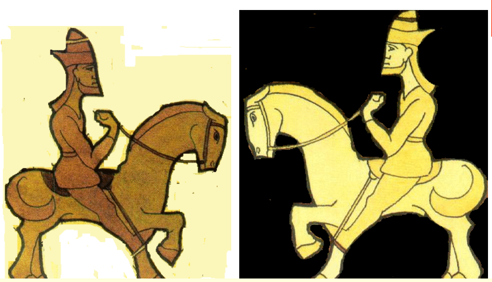
matter

anti-matter

identical to start

mirror image

time → ← time

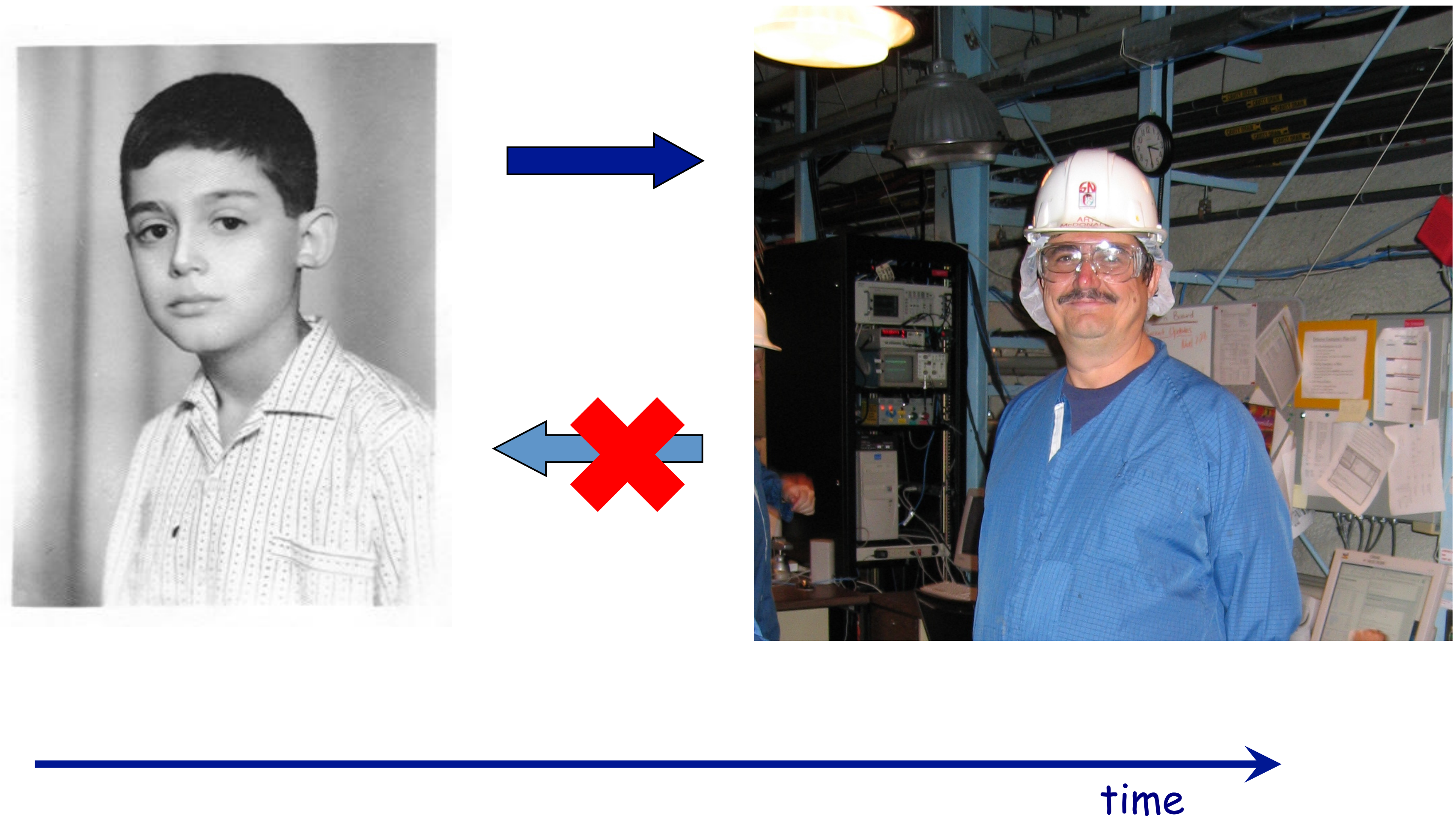


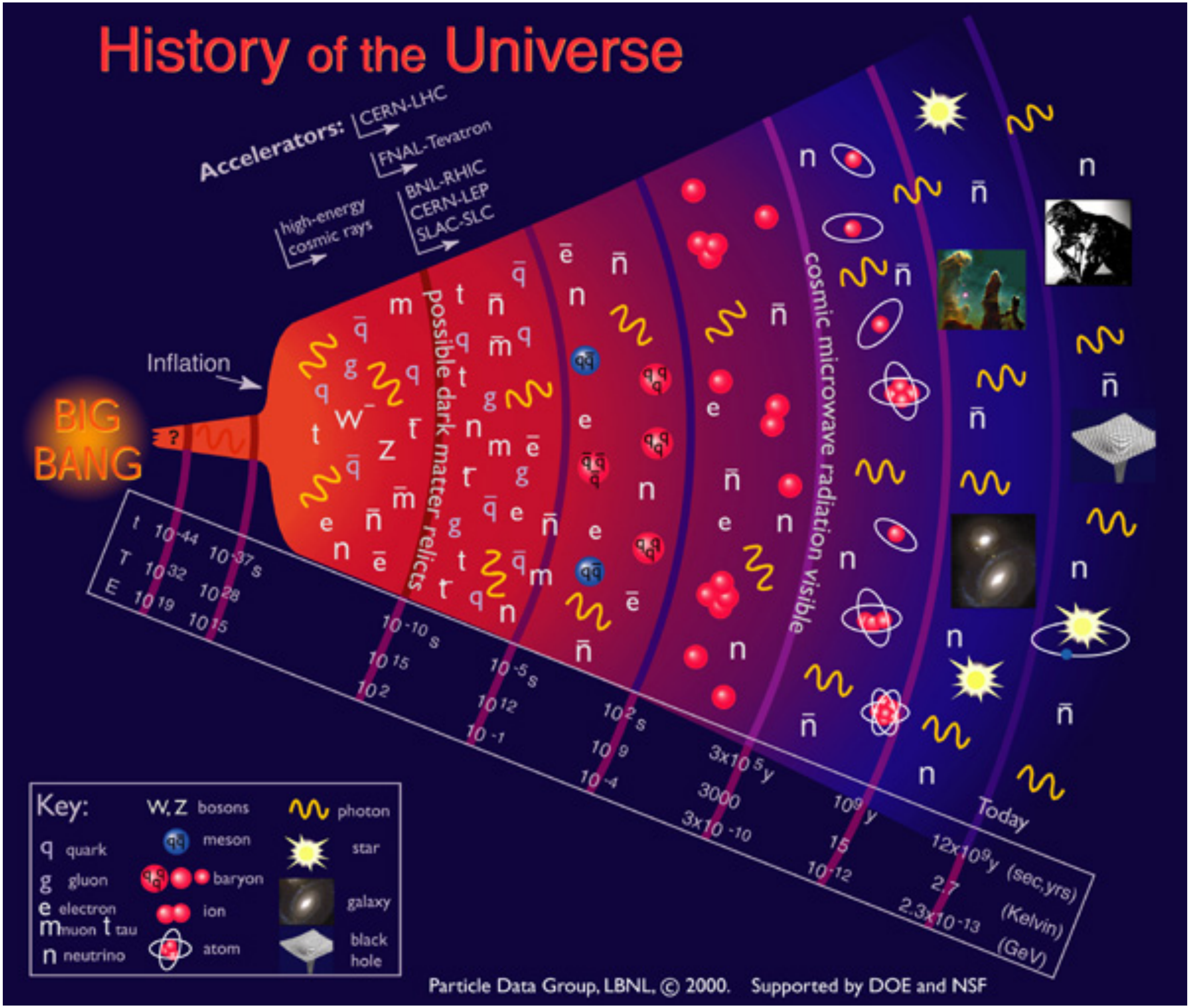
anti-particle **particle**

e^+ e^-

Time Reversal Violation can be measured at low energies

From H.W. Wilschut via K. Jungmann


time



History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relics
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

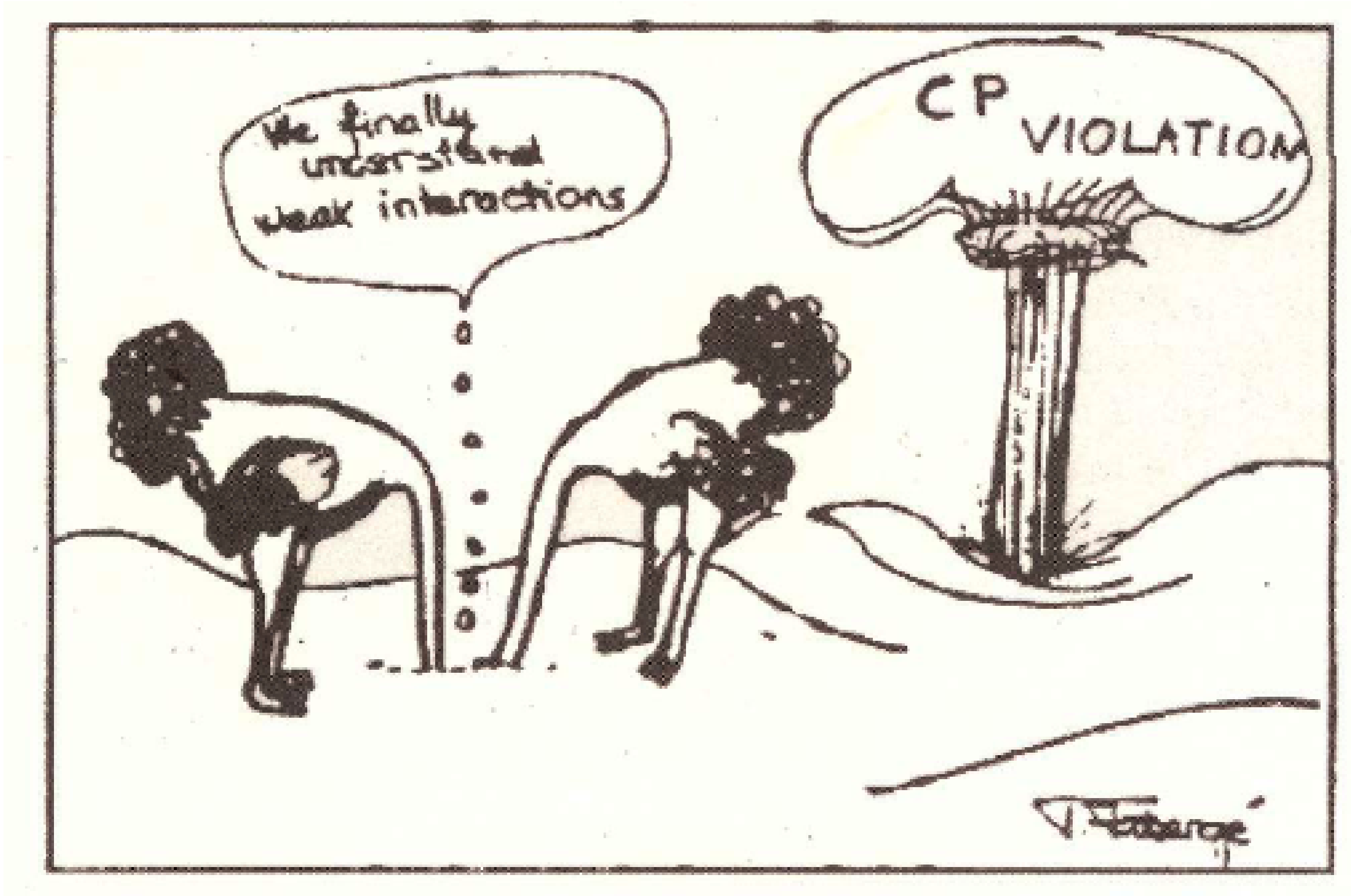
CP asymmetry at high temperatures was proposed by Okubo in 1958.

In 1967, Sakharov wrote:
“From S. Okubo’s effect at high temperature a coat is tailored for the Universe to fit its skewed shape”.

He proposed the conditions under which one can achieve matter-antimatter asymmetry via baryogenesis:

- C and CP-violation
- B violation
- Deviation from thermal equilibrium

In 1986 Fukugita & Yanagida explored conditions for leptogenesis. For example heavy partners of neutrinos in the see-saw mechanism must have been produced in the Early Universe. If their decays are CP-violating they may have given rise to particle-antiparticle asymmetry.



CP-violation is observed in neutral kaon decays, but what is seen there is not big enough to provide baryon-antibaryon asymmetry!

What is a particle?

Wigner: A particle is an irreducible representation of the Poincare group.

Recall the quantities Lorentz transformations leave invariant:

$$t^2 - \mathbf{r}^2 = \tau^2$$

$$E^2 - \mathbf{p}^2 = m^2$$

$$\mathbf{E}^2 - \mathbf{B}^2$$

$$\mathbf{E} \cdot \mathbf{B}$$

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Poincare group is the group including Lorentz boosts, translations and rotations.

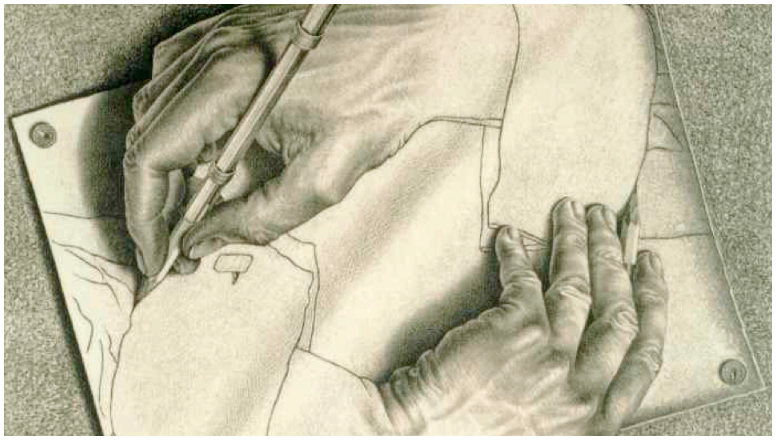
Next let us explore the concept of mass

What is mass?

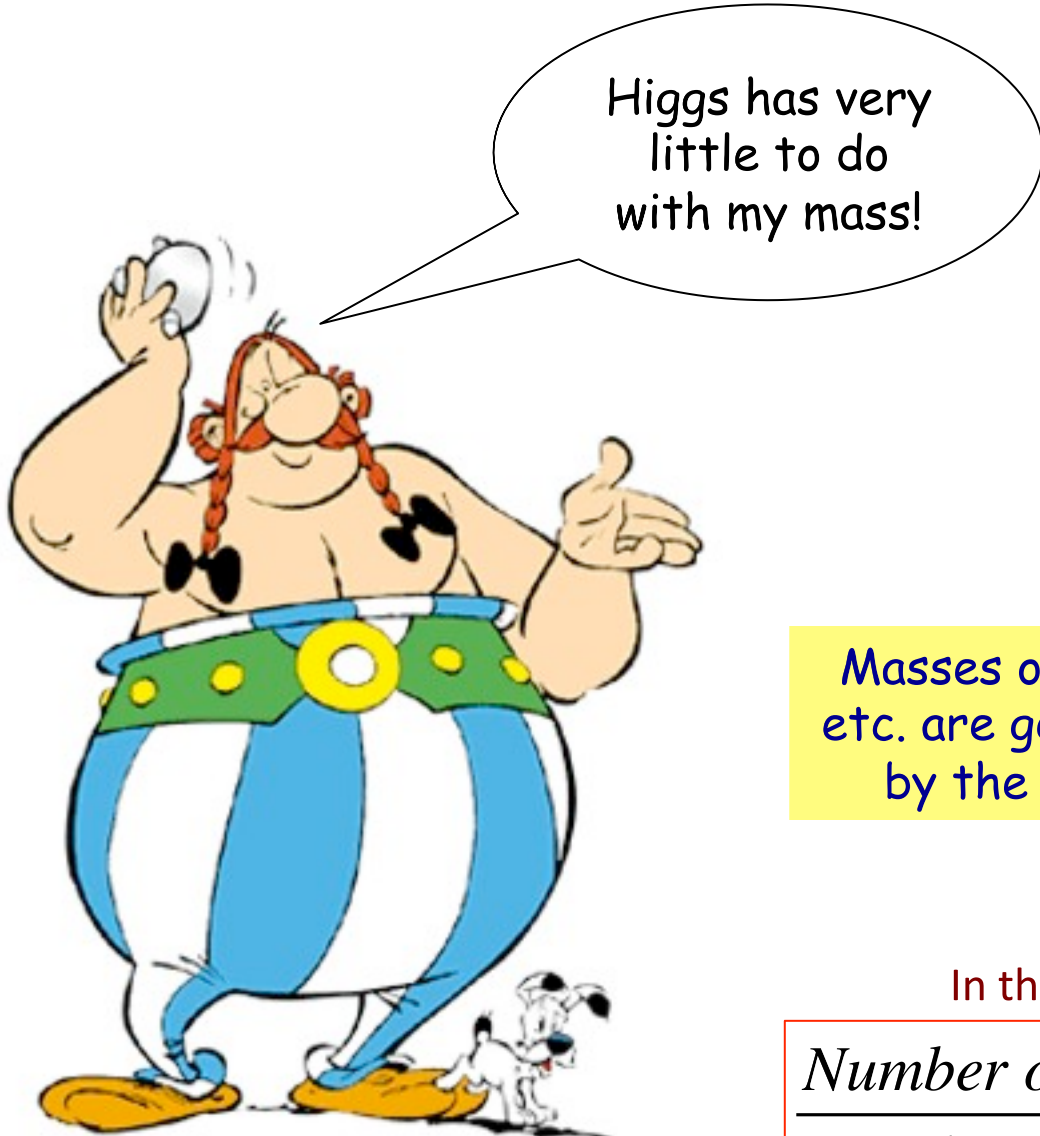
$$\Psi_L = \frac{1}{2}\left(1 - \gamma_5\right)\Psi$$

$$\Psi_R = \frac{1}{2}\left(1 + \gamma_5\right)\Psi$$

$$\mathcal{L} = m\bar{\Psi}\Psi = m\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L\right)$$



In the Standard Model all elementary masses possibly except those for neutrinos are generated by the Yukawa couplings of the Higgs.



Masses of protons, neutrons, etc. are generated dynamically by the QCD interactions!

In the Early Universe

$$\frac{Number\ of\ neutrons}{Number\ of\ protons} = \frac{e^{-m_n/T}}{e^{-m_p/T}}$$

Chiral representation of Dirac matrices

$$\gamma^0 = \beta = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Chirality: γ_5,

Helicity: $\dfrac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$ $\qquad \left[\gamma_5, \dfrac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\right] = 0$

Helicity and chirality commute, hence they can be simultaneously diagonalized!

Dirac equation in chiral representation: $i\gamma^{\mu}\partial_{\mu}\Psi - m_D\Psi = 0$

$$\Psi_D(\mathbf{x},t) = \sum_h \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E}} \left[b(p,h)u(p,h)e^{-ip\cdot x} + d^{\dagger}(p,h)v(p,h)e^{ip\cdot x} \right]$$

$$p^{\mu}\gamma_{\mu}u(p,h) = mu(p,h) \qquad p^{\mu}\gamma_{\mu}v(p,h) = -mv(p,h)$$

$$\gamma_5 n^{\mu}\gamma_{\mu}u(p,h) = hu(p,h) \qquad \gamma_5 n^{\mu}\gamma_{\mu}v(p,h) = hv(p,h)$$

$$h = \pm 1$$

$$n^{\mu} = \left(\frac{|\vec{p}|}{m}, \frac{E}{m}\frac{\vec{p}}{|\vec{p}|} \right)$$ (spin in the rest frame)

$$n \cdot p = 0$$

We choose $v(p,h) = u^C(p,h)$

Helicity eigenstates

$$\vec{p} = |\vec{p}|\left(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\right), \quad \vec{n} = \frac{\vec{p}}{|\vec{p}|}$$

$$\vec{\sigma}\cdot\vec{n}\,\varphi\left(\vec{n},\chi\right) = \chi\,\varphi\left(\vec{n},\chi\right), \qquad \chi = \pm 1$$

$$\varphi(\vec{n},\chi=+1) = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}, \qquad \varphi(\vec{n},\chi=-1) = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

$$\varphi^{\dagger}(\vec{n},\chi)\varphi(\vec{n},\chi') = \delta_{\chi\chi'} \qquad \varphi^{\dagger}(\vec{n},\chi)\,\vec{\sigma}\,\varphi(\vec{n},\chi) = \chi\vec{n}$$

$$\varphi(\vec{n},\chi)\xrightarrow[\vec{p}\to-\vec{p}]{}\varphi(-\vec{n},\chi) = -\chi e^{i\chi\phi}\varphi(\vec{n},-\chi)$$

Chiral representation of Dirac matrices

$$\gamma^0 = \beta = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Ultra-relativistic (massless) limit

$$u(p, h=+1) = -\sqrt{2E}\begin{pmatrix} \varphi(\vec{n}, \chi=+1) \\ 0 \end{pmatrix}, \quad u(p, h=-1) = \sqrt{2E}\begin{pmatrix} 0 \\ \varphi(\vec{n}, \chi=-1) \end{pmatrix}$$

$$v(p, h=-1) = \sqrt{2E}\begin{pmatrix} \varphi(\vec{n}, \chi=+1) \\ 0 \end{pmatrix}, \quad v(p, h=+1) = -\sqrt{2E}\begin{pmatrix} 0 \\ \varphi(\vec{n}, \chi=-1) \end{pmatrix}$$

Hence the total Hamiltonian, $H = \int d^3\vec{x}\,\mathrm{H}(x)$, with Dirac mass is

$$H = \sum_{\vec{p},h} \left\{ |\vec{p}| \left[b^\dagger_{ph} b_{ph} - d_{-ph} d^\dagger_{-ph} \right] - m_D \left[b^\dagger_{ph} d^\dagger_{-ph} - d_{-ph} b_{ph} \right] \right\}$$

This Hamiltonian can be diagonalized by the transformation

$$\begin{pmatrix} B_{ph} \\ D^\dagger_{-ph} \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} b_{ph} \\ d^\dagger_{-ph} \end{pmatrix}$$

$$H = \sum_{ph} \sqrt{\vec{p}^{\,2} + m_D^2} \left(B^\dagger_{ph} B_{ph} - D_{-ph} D^\dagger_{-ph} \right)$$

$$\cos 2\vartheta = \frac{|\vec{p}|}{\sqrt{\vec{p}^{\,2} + m_D^2}} \qquad \sin 2\vartheta = \frac{m_D}{\sqrt{\vec{p}^{\,2} + m_D^2}}$$

Chiral representation of Dirac matrices

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}$$

$$\psi = \begin{pmatrix} R \\ L \end{pmatrix} \Rightarrow \begin{cases} \psi_L = \frac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 \\ L \end{pmatrix} \\ \psi_R = \frac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} R \\ 0 \end{pmatrix} \end{cases}$$

$$\psi^C = \eta_C C \gamma_0^T \psi^* = \begin{pmatrix} i\sigma_2 L^* \\ -i\sigma_2 R^* \end{pmatrix} \Rightarrow \begin{cases} (\psi_L)^C \text{ is right-handed} \\ (\psi_R)^C \text{ is left-handed} \end{cases}$$

Majorana mass term:

$$m_M \left(\left(\Psi_L \right)^C \right)^\dagger \gamma_0 \, \Psi_L$$

- Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.

• Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W \times U(1)$.

• In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet (I_W =1/2) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet (I_W=0). Actually ν_R is not even in the Standard Model, but can be included in a minimally extended version.

SU(2) × U(1) Standard Model

Weak isospin

$$SU(2)_W : \; I^{(W)} = \frac{1}{2} \Rightarrow \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{matrix} I_3^{(W)} = +\frac{1}{2} \\ I_3^{(W)} = -\frac{1}{2} \end{matrix}$$

Weak singlets

$$I^{(W)} = 0 : \; \nu_R, \, e_R$$

Higgs Field sits in a weak doublet with $I_3^{(W)} = -\frac{1}{2}$.

• Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W x U(1)$.

• In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet (I_W =1/2) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet (I_W=0).

• A mass term connects left- and right-handed components. The usual Dirac mass term is $L = m\overline{\psi}\psi = m(\overline{\psi_L}\psi_R + \overline{\psi}_R\psi_L)$. But such a neutrino mass term requires a right-handed neutrino, hence it is not in the Standard Model.

• The right-handed component of the neutrino carries no weak isospin quantum numbers.

A very brief introduction to the effective field theories

A note on dimensional counting

• Lagrangian, $\mathcal{L}$, has dimensions of energy (or mass).

• $\mathcal{L} = \int d^3x\, L \Rightarrow$ Lagrangian density, L, has dimensions of energy/volume or M^4.

• Define the scaling dimension of x, [x] to be -1 $\Rightarrow$ scaling dimension of momentum (or mass) is [m] = +1 (recall that (p.x/h) is dimensionless and we take [h]=0).

• Clearly [L] = 4. This should be true for any Lagrangian density of any theory.

• Consider the mass term for fermions, $L_m = m\,\bar{\Psi}\Psi$. Then $[\bar{\Psi}\Psi] = 3$ or $[\Psi] = 3/2$.

• In the Standard Model the Higgs field vacuum expectation value gives the particle mass: $L = H\,\bar{\Psi}\Psi$. Hence [H] = 1.

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \underbrace{\text{terms which are higher order in fields}}_{\substack{\text{Consistent with the} \\ \text{symmetries of the system}}}$$

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E} \bullet \mathbf{B}\right)^2}_{\text{mass dimension 8}} + c_3 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)\left(\mathbf{E} \bullet \mathbf{B}\right)}_{\text{mass dimension 8}} \right]$$

These operators have mass dimension 8, so there should be a power of 4 here

Symmetries of the Electromagnetism

Lorentz Invariants: $\mathbf{E}^2 - \mathbf{B}^2$ and $\mathbf{E} \cdot \mathbf{B}$

$$\underbrace{\begin{array}{c} \mathbf{E} \to \mathbf{E} \\ \mathbf{B} \to -\mathbf{B} \\ \mathbf{E}^2 - \mathbf{B}^2 \to \mathbf{E}^2 - \mathbf{B}^2 \\ \mathbf{E} \cdot \mathbf{B} \to -\mathbf{E} \cdot \mathbf{B} \end{array}}_{Under\ time-reversal}$$

Since we want the Lagrangian density to be *invariant* under *both* Lorentz *and* time-reversal transformations we pick $\mathbf{E}^2 - \mathbf{B}^2$.

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E}\bullet\mathbf{B}\right)^2}_{\text{mass dimension 8}} + c_3 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)\left(\mathbf{E}\bullet\mathbf{B}\right)}_{\text{mass dimension 8}} \right]$$

Not good (not T-invariant)

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E} \bullet \mathbf{B}\right)^2}_{\text{mass dimension 8}}\right]$$

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}\,(\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45 m_e^4}\left[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7\,(\mathbf{E}\cdot\mathbf{B})^2\right]$$

Another example: Low-energy limit of weak interactions

$$L = \left(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \right)$$

Another example: Low-energy limit of weak interactions

$$L = \left(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \right)$$

$$L = \frac{g^2}{\Lambda^2} \left(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \right)$$

$$G_F = \frac{\sqrt{2} g^2}{8 M_W^2}$$

Using the Standard Model degrees of freedom one can parameterize the neutrino mass by a dimension 5 operator. (Recall that $I_3^W = 1/2$ for the ν_L and -1/2 for H_{SM}).

$$L = X_{\alpha\beta}\, H_{SM}\, H_{SM}\, \overline{\nu_{L\alpha}^{C}}\, \nu_{L\beta} / \Lambda$$

$$v^2 X_{\alpha\beta} / \Lambda = U\, m_\nu^{diagonal}\, U^T$$

This term is not renormalizable! It is the only dimension-five operator one can write using the Standard Model degrees of freedom. Hence the neutrino mass is the most accessible new physics beyond the Standard Model!

There are other ways to obtain neutrino mass:

$$L = H_{I=1} \overline{\nu_{L\alpha}{}^{c}} \nu_{L\beta}$$

Note: This Higgs is not in the Standard Model!

At lower energies, Beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

Majorana neutrino mass (unique)

Includes Majorana neutrino magnetic moment

Majorana mass term:

$$m_M \left(\left(\Psi_L \right)^C \right)^\dagger \beta \, \Psi_L$$

- Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.
- It is permitted by the weak-isospin invariance of the Standard Model.
- Neutrino mass terms are not included in the fundamental Lagrangian of the Standard Model. They arise from new physics. Of course it is possible to write down an *effective* Lagrangian for the neutrino mass in terms of only the Standard Model fields if you give up renormalizability.

In effective field theories at lower energies, beyond Standard Model physics is described by local operators

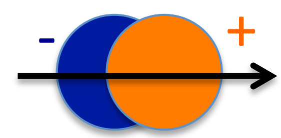
$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

Majorana neutrino mass (unique)

+

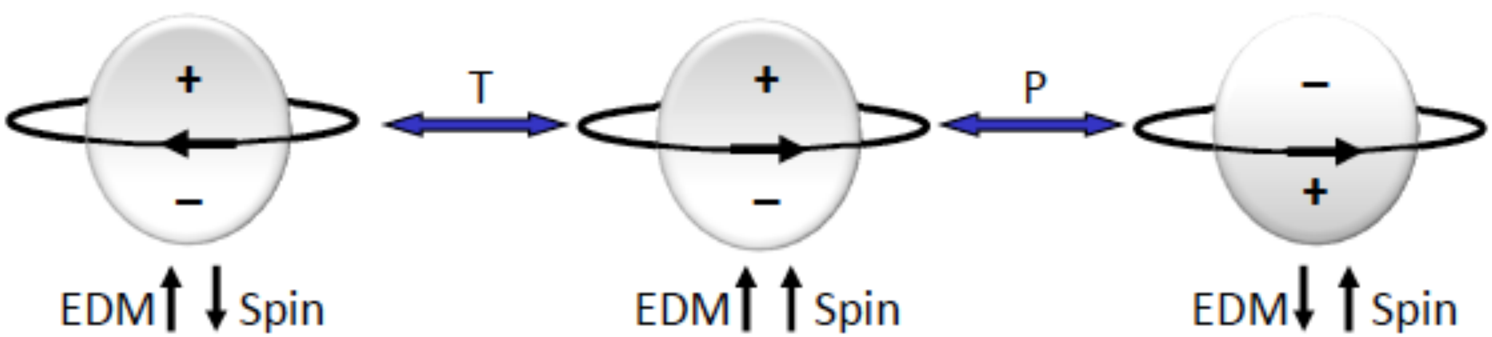
$$-\theta \frac{g_s^2}{32\pi^2} \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$H \approx -d\, \mathbf{J} \cdot \mathbf{E}$$



Electric dipole moment

$$d_i \propto \frac{m_i}{\Lambda^2} \sin \phi_{CP}$$



$$\Psi(\mathbf{x},t) = \sum_h \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E}} \left[b(p,h)u(p,h)e^{-ip\cdot x} + d^\dagger(p,h)v(p,h)e^{ip\cdot x} \right]$$

$$\Psi^C(\mathbf{x},t) = \sum_h \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E}} \left[b^\dagger(p,h)u^C(p,h)e^{+ip\cdot x} + d(p,h)v^C(p,h)e^{-ip\cdot x} \right]$$

Majorana Field: $\Psi_M^C = \Psi_M$

$$\Psi_M(\mathbf{x},t) = \sum_h \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E}} \left[b(p,h)u(p,h)e^{-ip\cdot x} + b^\dagger(p,h)v(p,h)e^{ip\cdot x} \right]$$

Note that the Majorana field can be written as

$$\Psi_M = \psi + \psi^C$$

Neutrino mixing

$$\left|\nu_{flavor}\right\rangle = T\left|\nu_{mass}\right\rangle$$

$$T = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{atmospheric\ neutrinos} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{reactor\ neutrinos} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{solar\ neutrinos}$$

$$c_{ij} = \cos\theta_{ij} \quad s_{ij} = \sin\theta_{ij}$$

$$P\left(\nu_e \to \nu_e\right) = 1 - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\left(\Delta_{31}L\right) + \sin^2\theta_{12}\sin^2\left(\Delta_{32}L\right)\right]$$
$$-\cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\left(\Delta_{21}L\right)$$

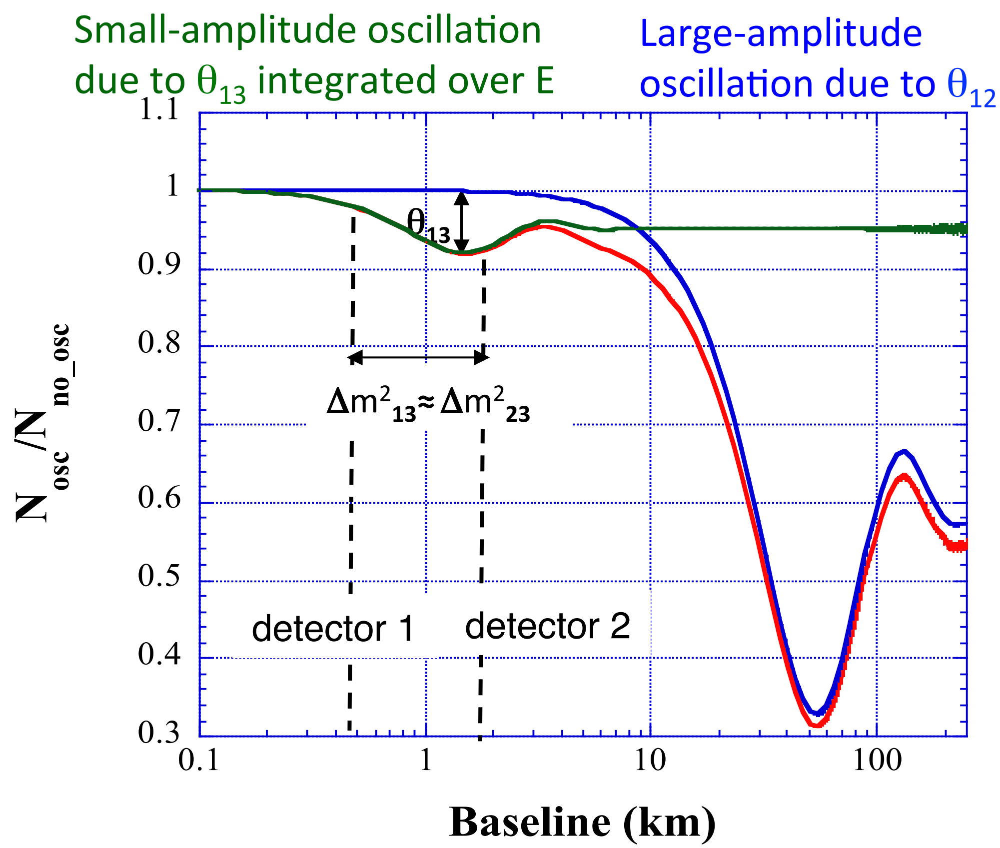
$$\Delta_{ij} = \frac{\delta m_{ij}^2}{4E_\nu} = \frac{m_i^2 - m_j^2}{4E_\nu}, \qquad \Delta_{32} = \Delta_{31} - \Delta_{21}$$

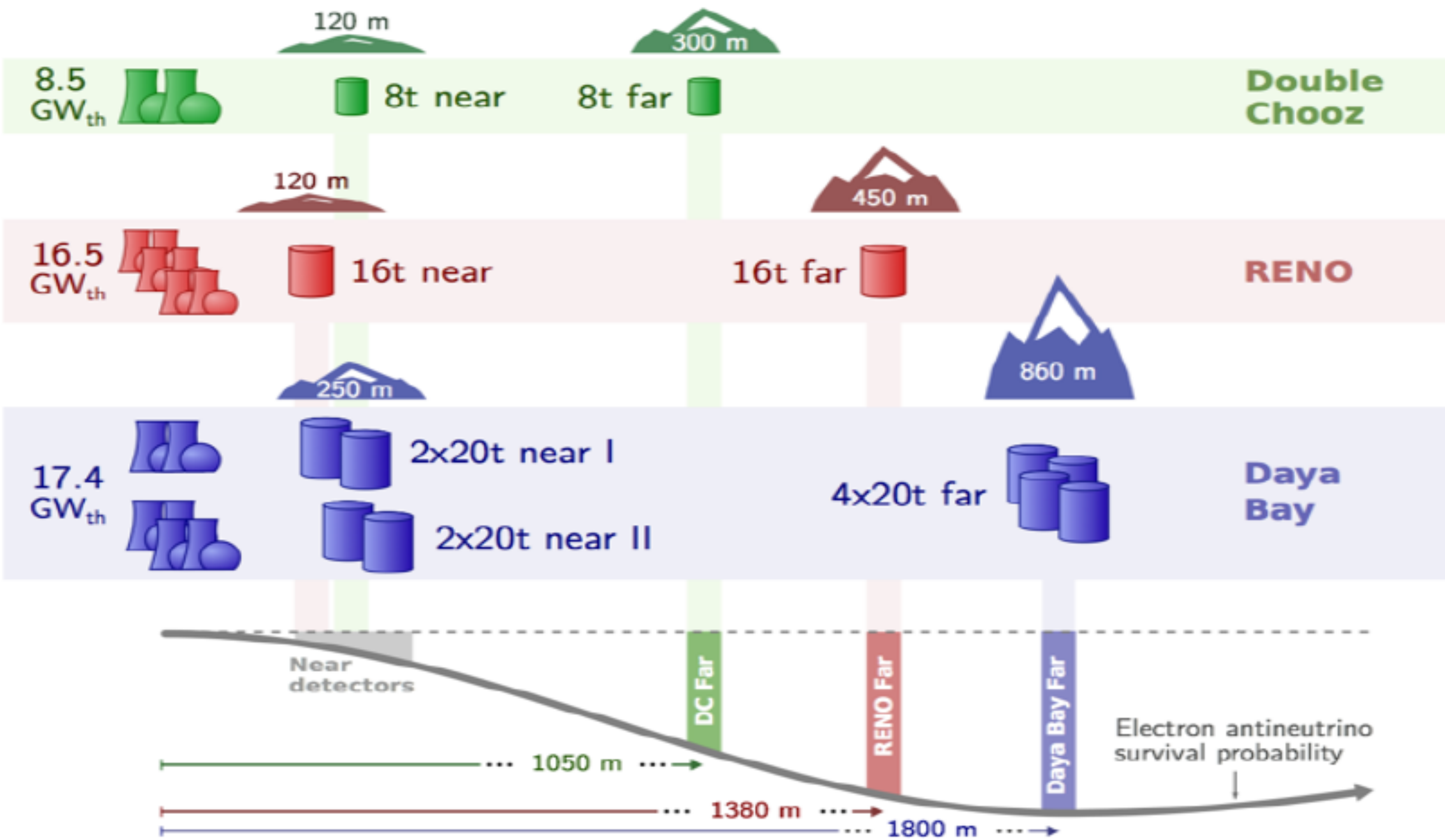
Measuring θ_{13} with Reactor Antineutrinos

Double Chooz
Daya Bay
RENO

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}{}^2 L}{4E_\nu}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}{}^2 L}{4E_\nu}\right)$$

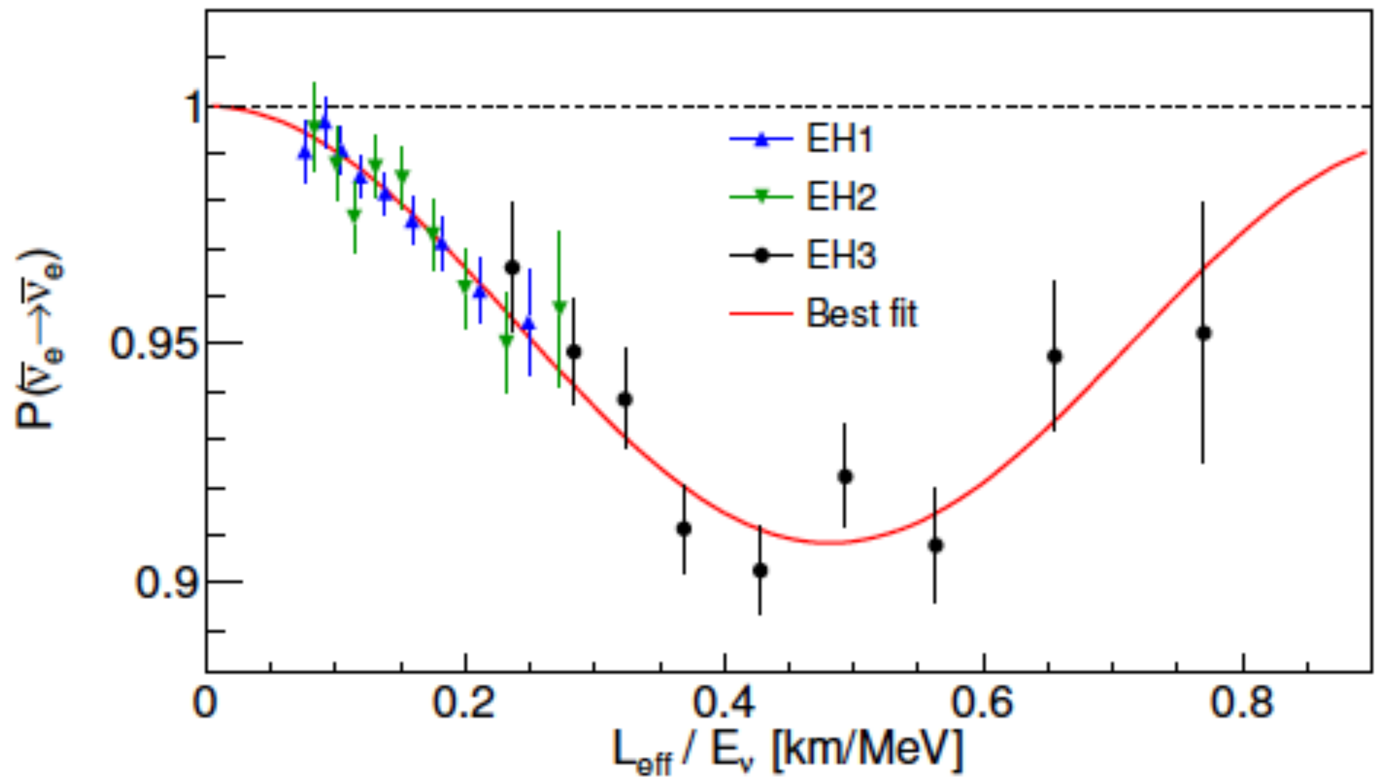
- Reactor neutrino energies are too low to produce muons. Hence this is an antineutrino disappearance experiment (also no matter effects).
- Measure ratio(s) of interaction rates in two or more detectors to cancel systematic errors.
- Those detectors will never be identical, hence one should try to control mass differences, detection efficiencies, etc.





120 m
300 m
8.5 GWth
8t near
8t far
Double Chooz
120 m
450 m
16.5 GWth
16t near
16t far
RENO
250 m
860 m
17.4 GWth
2x20t near I
2x20t near II
4x20t far
Daya Bay
Near detectors
DC Far
RENO Far
Daya Bay Far
Electron antineutrino survival probability
1050 m
1380 m
1800 m

Fig: S. Jetter



$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} \underbrace{\left[\cos^2\theta_{12}\sin^2(\Delta_{31}L) + \sin^2\theta_{12}\sin^2(\Delta_{32}L)\right]}_{\sin^2(\delta m^2_{ee}L/4E)}$$
$$- \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2(\Delta_{21}L)$$

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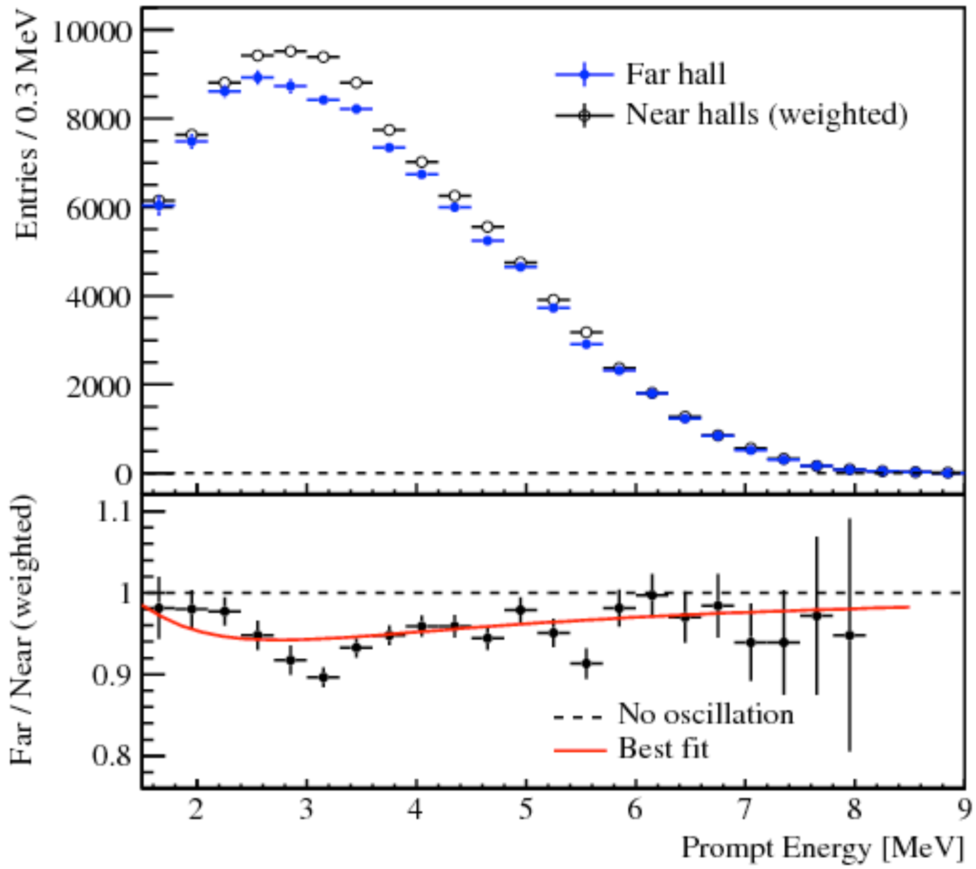


Neutron capture on hydrogen:

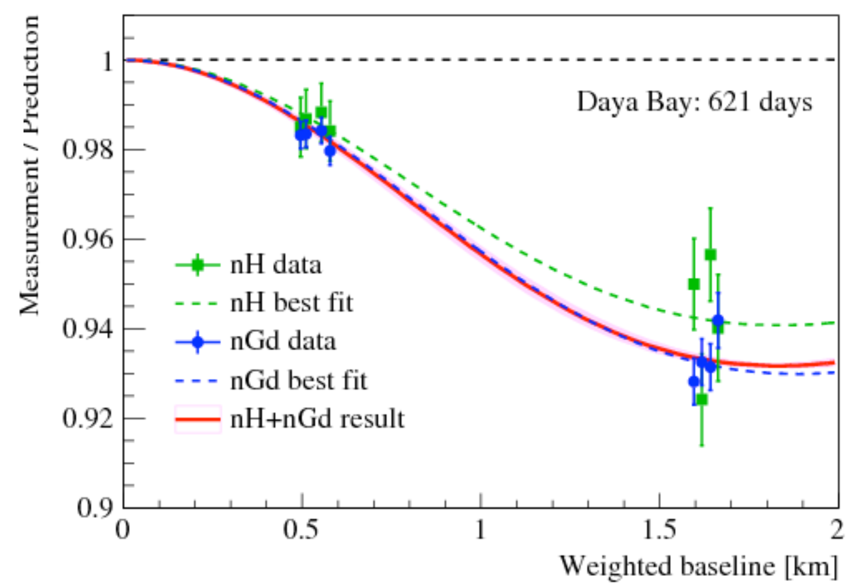
$$\sin^2 2\theta_{13} = 0.071 \pm 0.011$$

Neutron capture on Gd:

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

Simultaneous fit:

$$\sin^2 2\theta_{13} = 0.082 \pm 0.004$$



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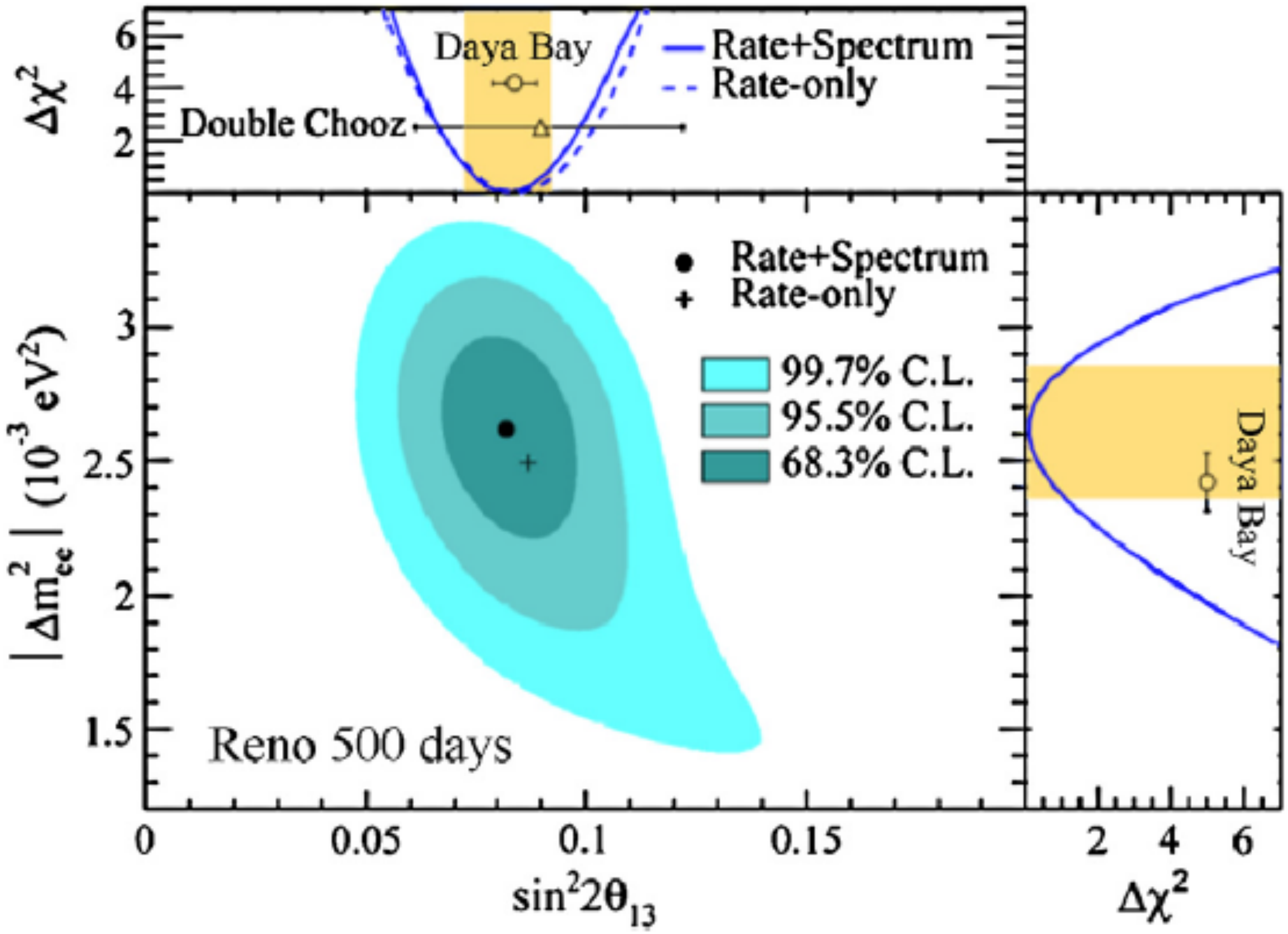

$$\sin^2 2\theta_{13} = 0.082 \pm 0.009 \text{ (stat.) } \pm 0.006 \text{ (syst.)}$$

$$\left|\delta m^2_{ee}\right| = \left[2.62^{+0.21}_{-0.23} \text{ (stat.)}^{+0.12}_{-0.13} \text{ (syst.)}\right] \times 10^{-3} \text{ eV}^2$$