Hadron Spectroscopy
Lecture 3
Light Quark Mesons

National Nuclear Physics Summer School
at MIT

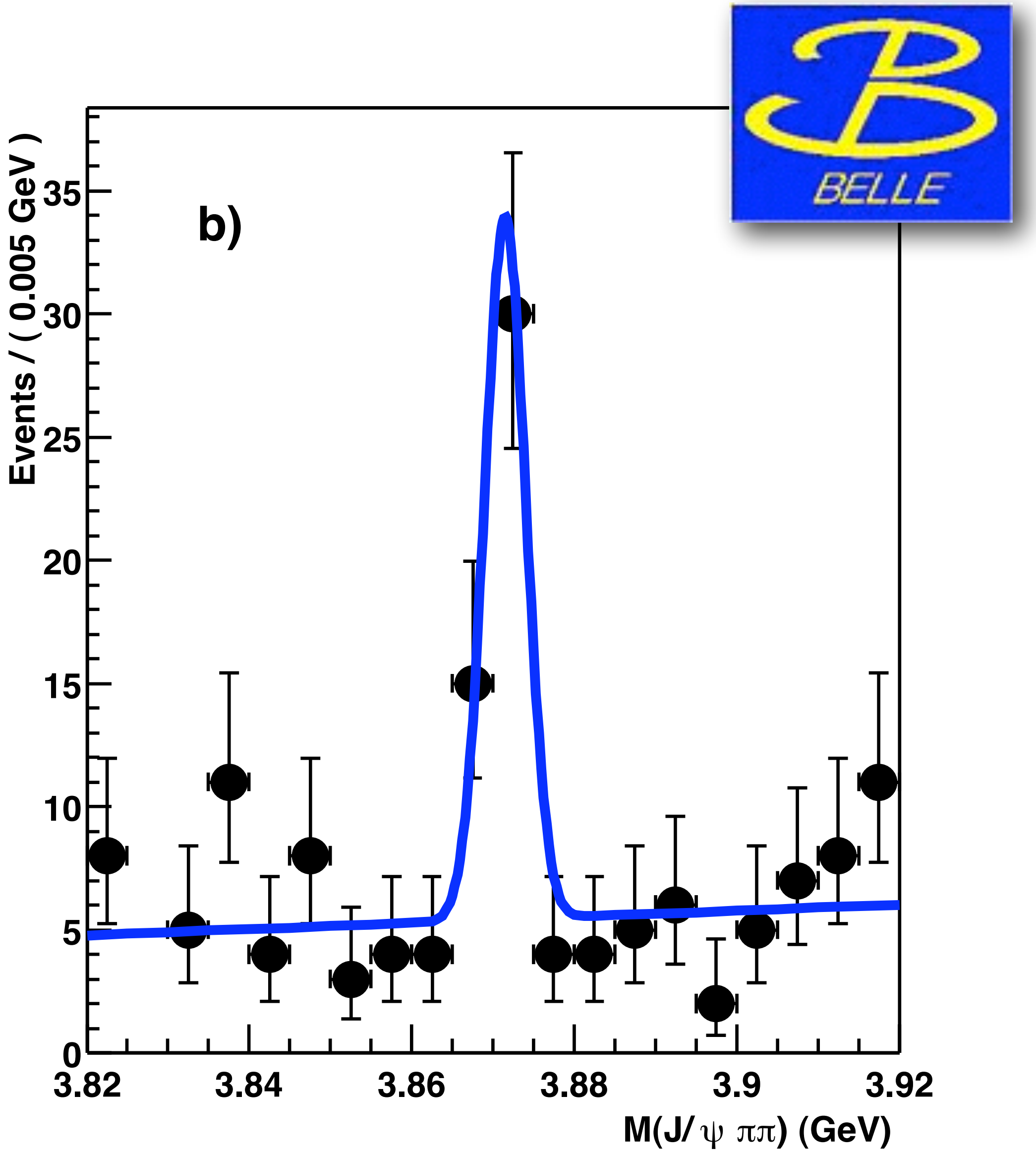
Matthew Shepherd
Indiana University

Outline

1. Overview and Motivation
2. Spectroscopy of Heavy Quark Systems
3. Spectroscopy of Light Quark Systems
    3.1. Amplitude analysis: a tool for light quark spectroscopy
    3.2. Selected recent results in light quark spectroscopy
4. (Very brief!) Summary and Outlook: Present and Future Facilities

Spectroscopy Tools

- Detector measures four-momenta of particles
- Many discoveries made by plotting invariant mass and looking for peaks (“bump hunting”)
  - works best for narrow peaks (~10 MeV)
- For light quark mesons want to measure
  - mass and width of broad (~200 MeV) resonances
  - quantum numbers of resonances
- Need more than just magnitude of four-momentum
  - angular distributions also relevant!

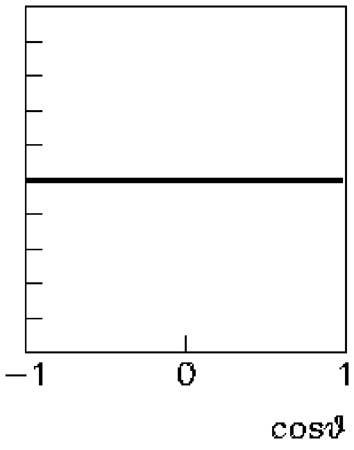


PRL 91, 262001 (2003)

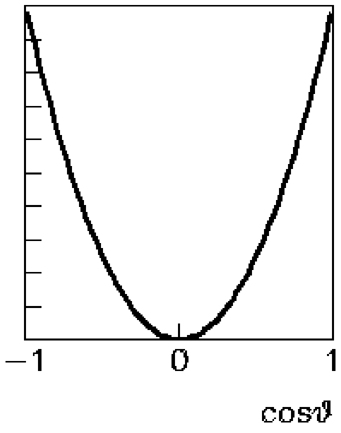
(1223 citations as of this morning, the most cited Belle collaboration paper by a > 400 citation margin)

An Example: Measuring Spin

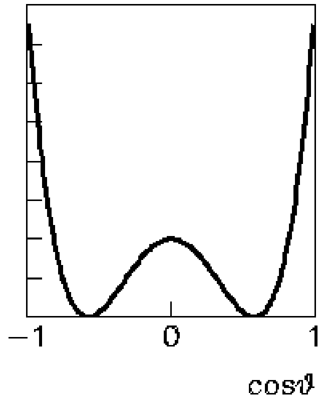
$$\left|Y_0^0\right|^2$$



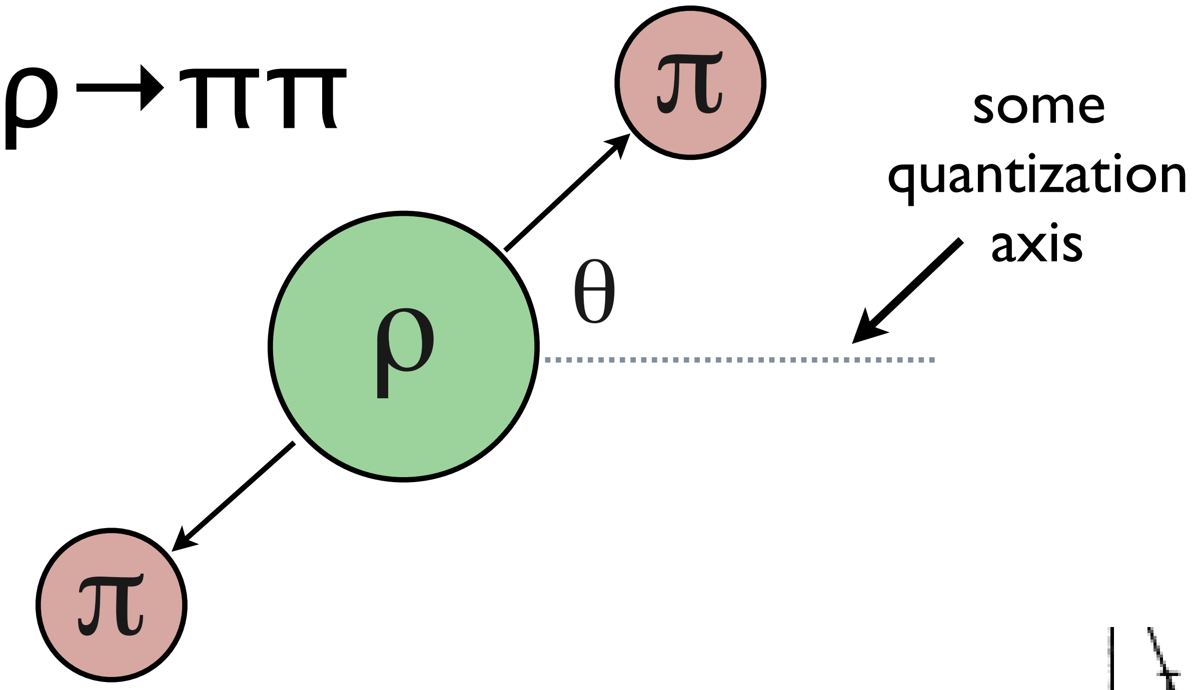
$$\left|Y_1^0\right|^2$$



$$\left|Y_2^0\right|^2$$



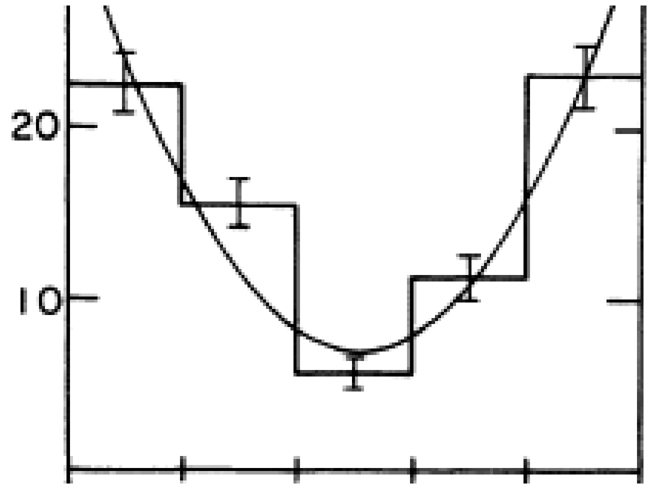
$\rho \rightarrow \pi\pi$

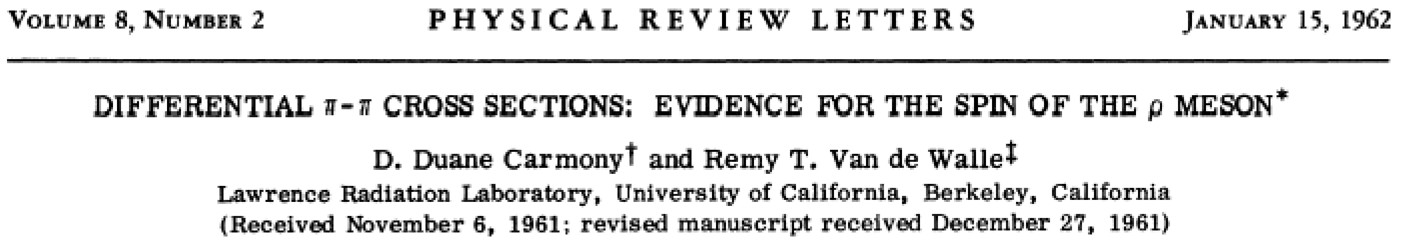


some quantization axis

Pions are spinless so spin of ρ is carried in the orbital angular momentum of the two pions.

From data conclude J = 1





VOLUME 8, NUMBER 2 PHYSICAL REVIEW LETTERS JANUARY 15, 1962

DIFFERENTIAL π-π CROSS SECTIONS: EVIDENCE FOR THE SPIN OF THE ρ MESON*

D. Duane Carmony† and Remy T. Van de Walle‡
Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received November 6, 1961; revised manuscript received December 27, 1961)

Amplitude Analysis

- historically called partial wave analysis (PWA), but only a special subset of analyses are really partial wave expansions
- needs two very different but rather complicated ingredients
  - Experimental/Technical: multidimensional unbinned likelihood fit that correctly deals with detector acceptance
    - high performance computing is essential for practical fits
  - Theoretical: a physics model with free parameters that describes the experimental data

Maximum Likelihood

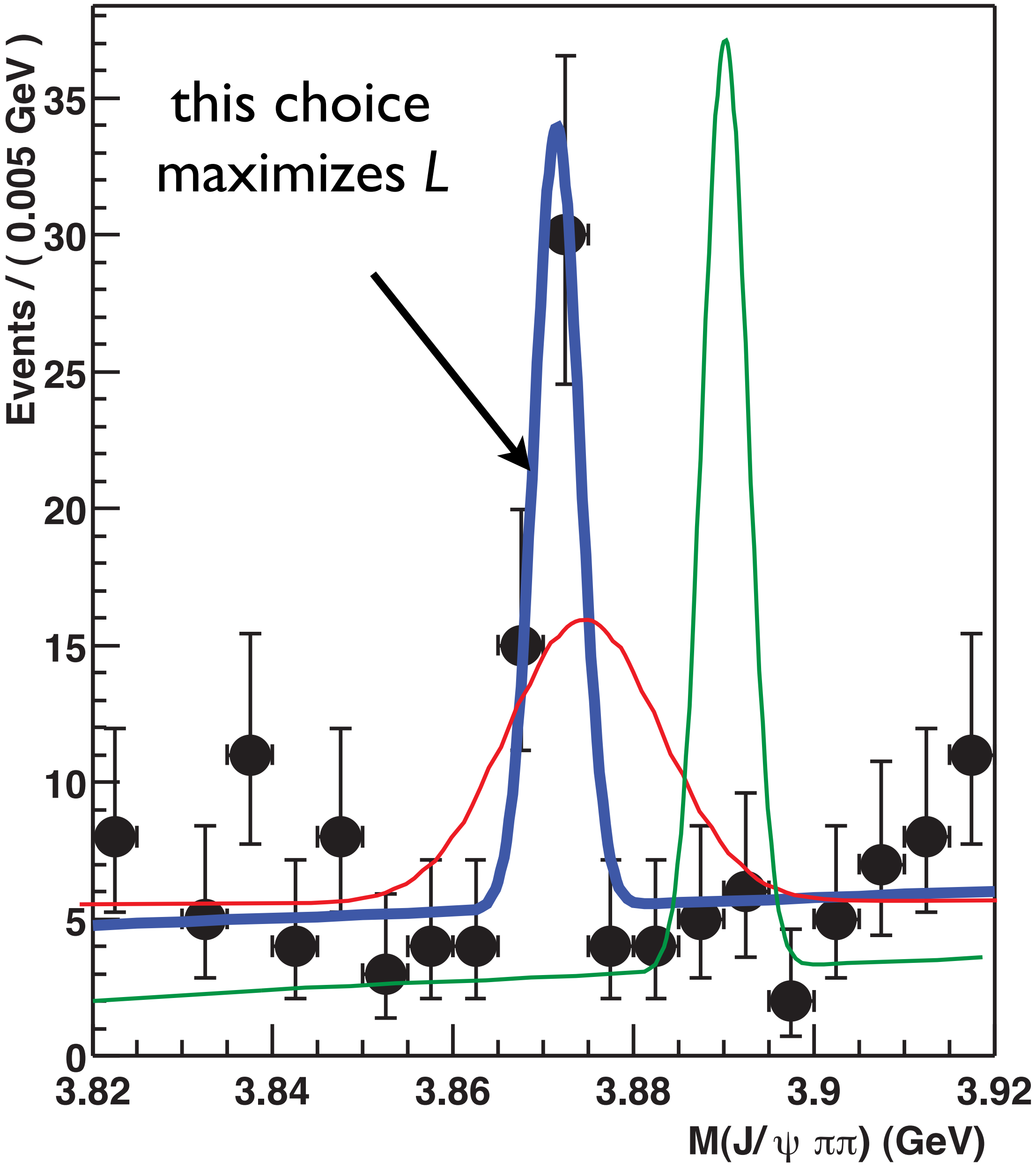
- Amplitude analysis is built around the (extended) maximum likelihood method
- Start with a model that contains free parameters (θ) and predicts the probability of having an event with a particular set of kinematic variables x (angles, invariant mass, etc.)

$$\mathcal{P}(\vec{x};\vec{\theta})$$

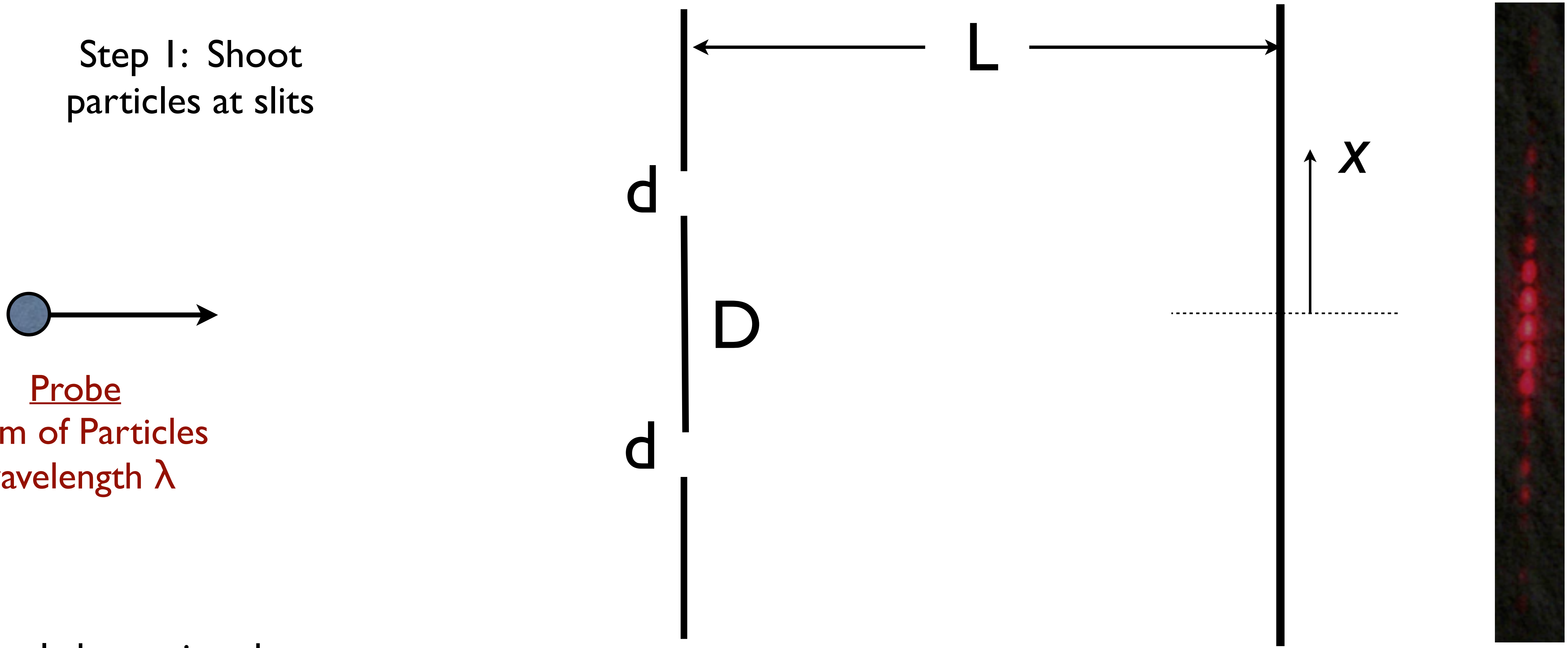
- Vary the free parameters to maximize the probability for the entire data set

$$\mathcal{L} = \prod_{i=1}^{N_{\text{events}}} \mathcal{P}(\vec{x}_i;\vec{\theta})$$

Example: 1D in *x*



Experiment Application



Step 2: For each particle record location *x* where it was detected

Step 1: Shoot particles at slits

<u>Probe</u>
Beam of Particles
wavelength λ

<u>Goal</u>: determine the values of *d* and *D*

<u>Physical System Under Study</u>
Two Slits: width d, separation D

<u>Detector</u>
Measures location x_i
for each arriving particle

The Fit Procedure

- Our "theoretical model" that parametrizes the intensity of the particles in the detector is given by

$$I(x) = I_0 \left(\frac{\sin(d\pi x/\lambda L)}{d\pi x/\lambda L} \right)^2 \cos^2(2D\pi x/\lambda L)$$

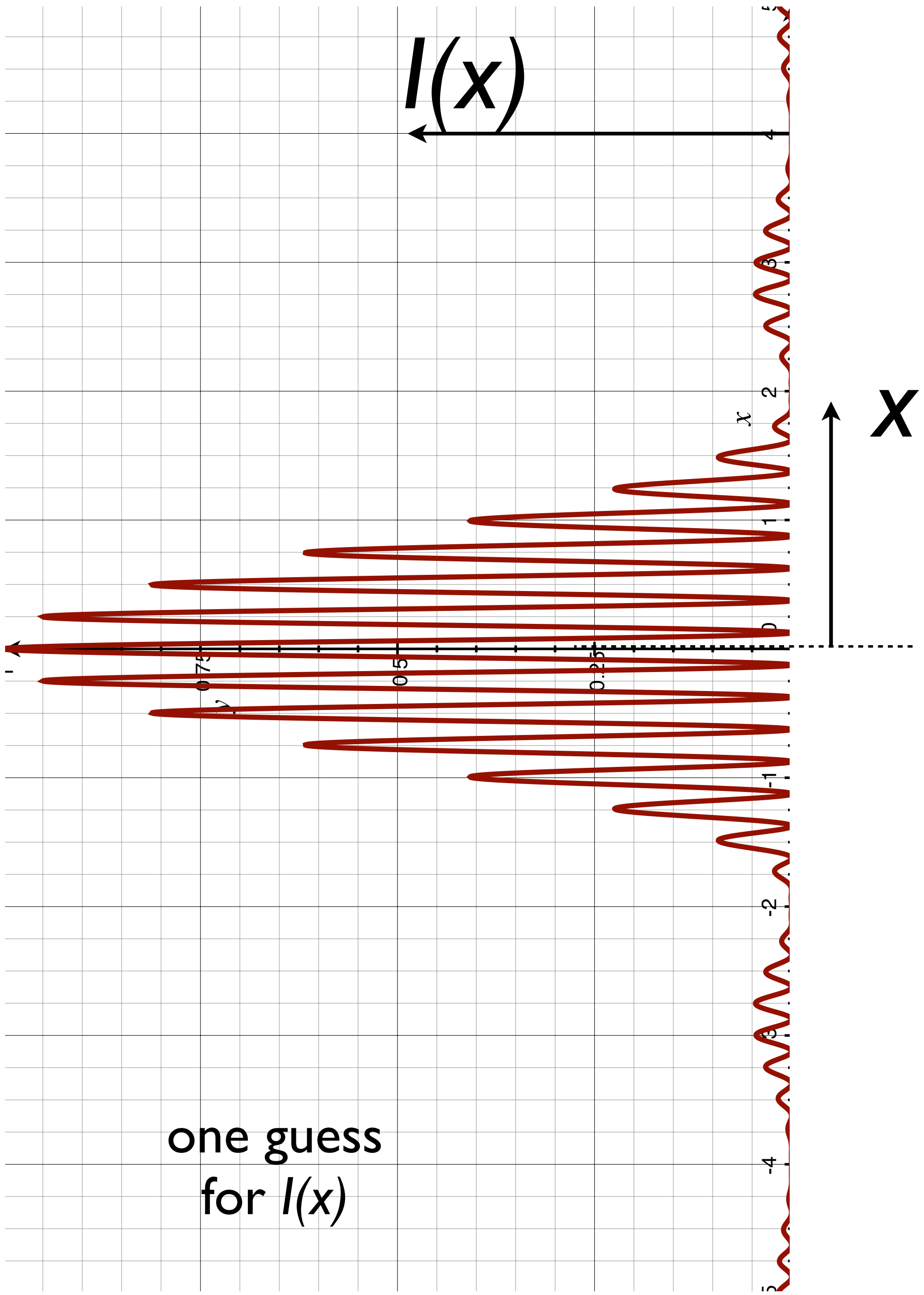
- Start with a guess for values for *d* and *D*
- Convert *I*(*x*) into a properly normalized PDF -- multiple techniques are available for evaluating the integral

$$\mathcal{P}(x) = \frac{I(x)}{\int_{x_{\min}}^{x_{\max}} I(x)dx}$$

- Compute the likelihood by taking the product over all detected events

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(x_i)$$

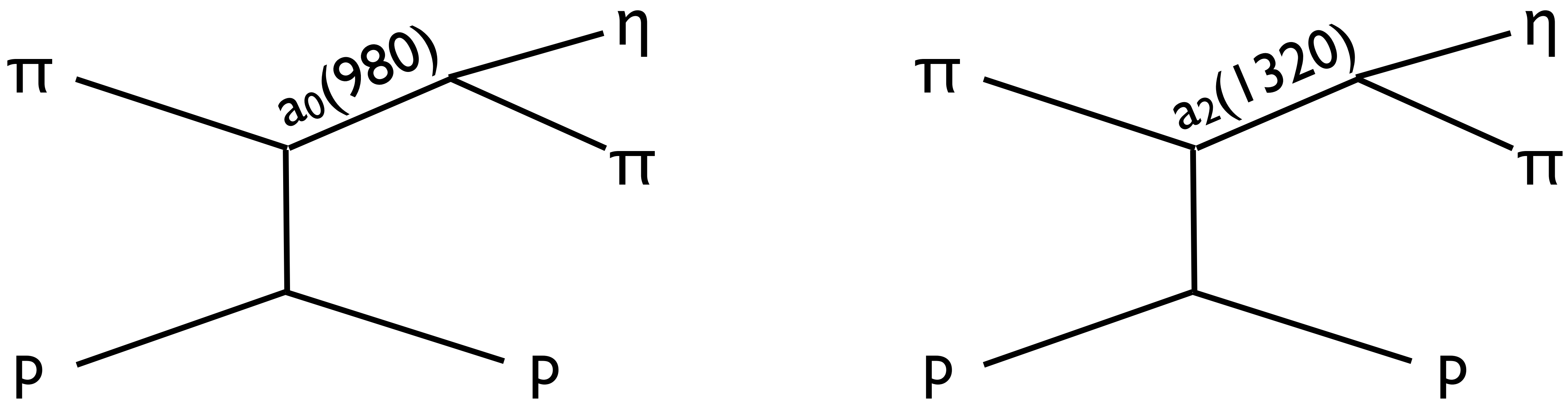
- Iterate with a new choice of *d* and *D* until the likelihood is maximized



one guess
for *I*(*x*)

Connecting to Spectroscopy

- Suppose we have πp→ηπp, we can draw two (of many) possible diagrams



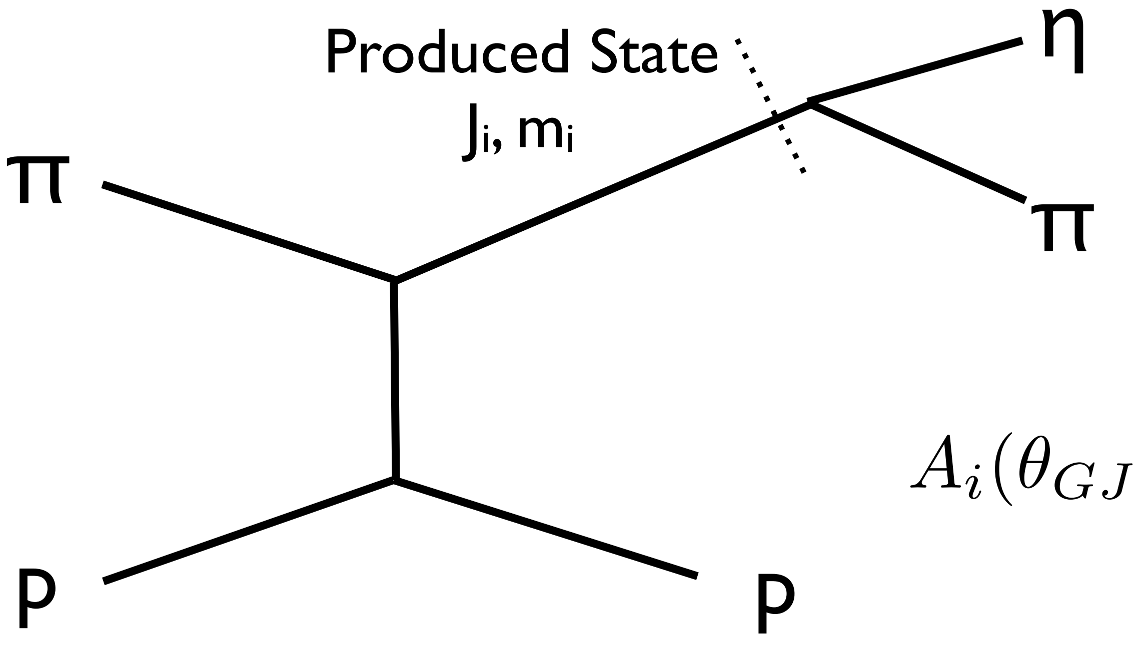
- Each of these can be related to an independent quantum mechanical *amplitude*
- Given any single event with fixed kinematic variables we do not know which process occurred -- they are indistinguishable

Amplitude Structure

production amplitude

V_i
unknown complex
fit parameter

magnitude and phase
of amplitude



decay amplitude

$A_i(\vec{x})$
complex *function* of
the final state observables,
$\vec{x}$ is a location in multi-body
phase space

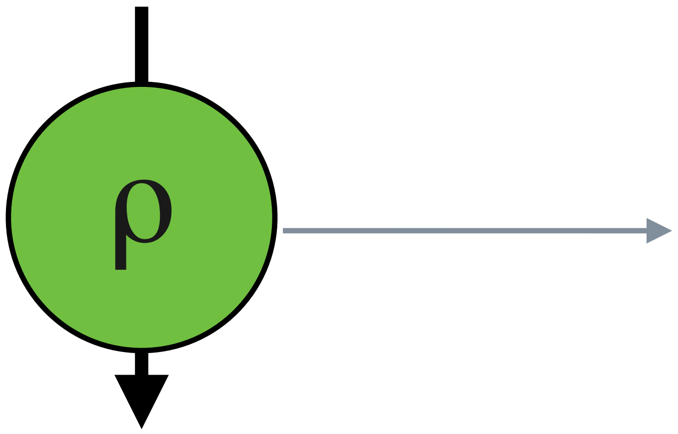
"known"

$$A_i(\theta_{GJ}, \phi_{GJ}) \propto D^{J_i}_{m_i,0}(\theta_{GJ}, \phi_{GJ}, 0)$$

I is intensity:
number of
events per unit
phase space

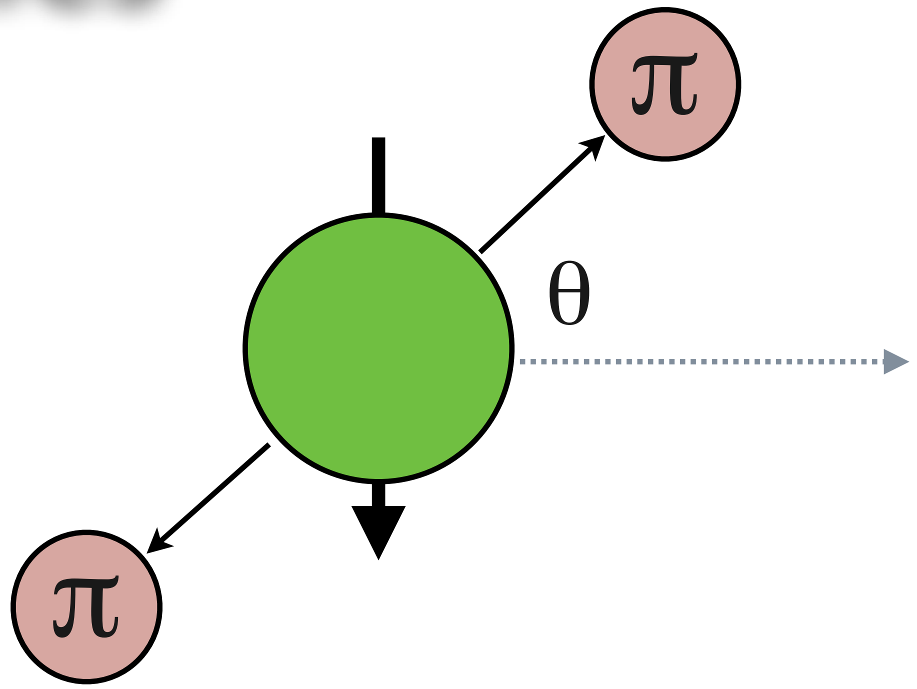
$$I(\vec{x}) = \frac{dN}{d\vec{x}} = \left| \sum_{\alpha}^{N_{\text{amps}}} V_\alpha A_\alpha(\vec{x}) \right|^2 = \sum_{\alpha,\beta}^{N_{\text{amps}}} V_\alpha V_\beta^* A_\alpha(\vec{x}) A_\beta(\vec{x})^*$$

Kinematics



initial ρ configuration
in the helicity frame

J=1 m=0



spinless pions
each have helicity zero
helicity sum is zero
in this frame: m’ = 0

Rotation between frames given by

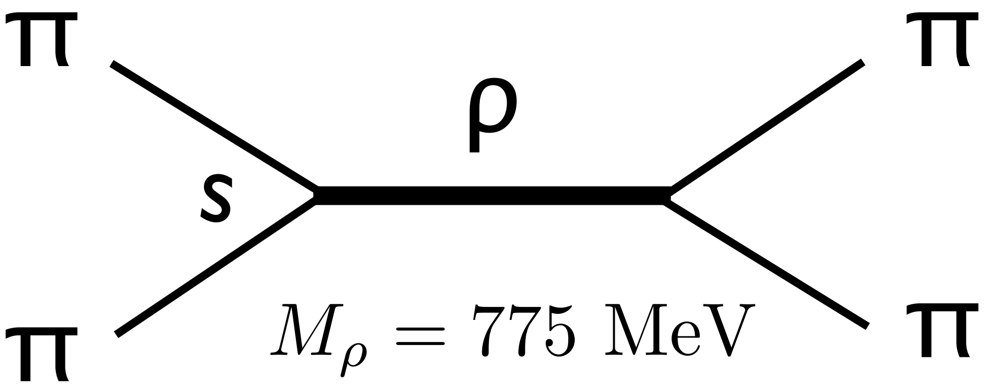
$$D^J_{m',m}(\alpha,\beta,\gamma) = e^{-im'\alpha} d^J_{m',m}(\beta) e^{-im\gamma}$$

$$d^J_{m',m} = (-1)^{m-m'} d^J_{m,m'} = d^J_{-m,-m'}$$

For the sketch above: α = γ = 0 and β = θ

$$d^1_{0,0}(\theta) = \cos\theta \qquad d^1_{1,0}(\theta) = \frac{-\sin\theta}{\sqrt{2}}$$

(Simple) Dynamics

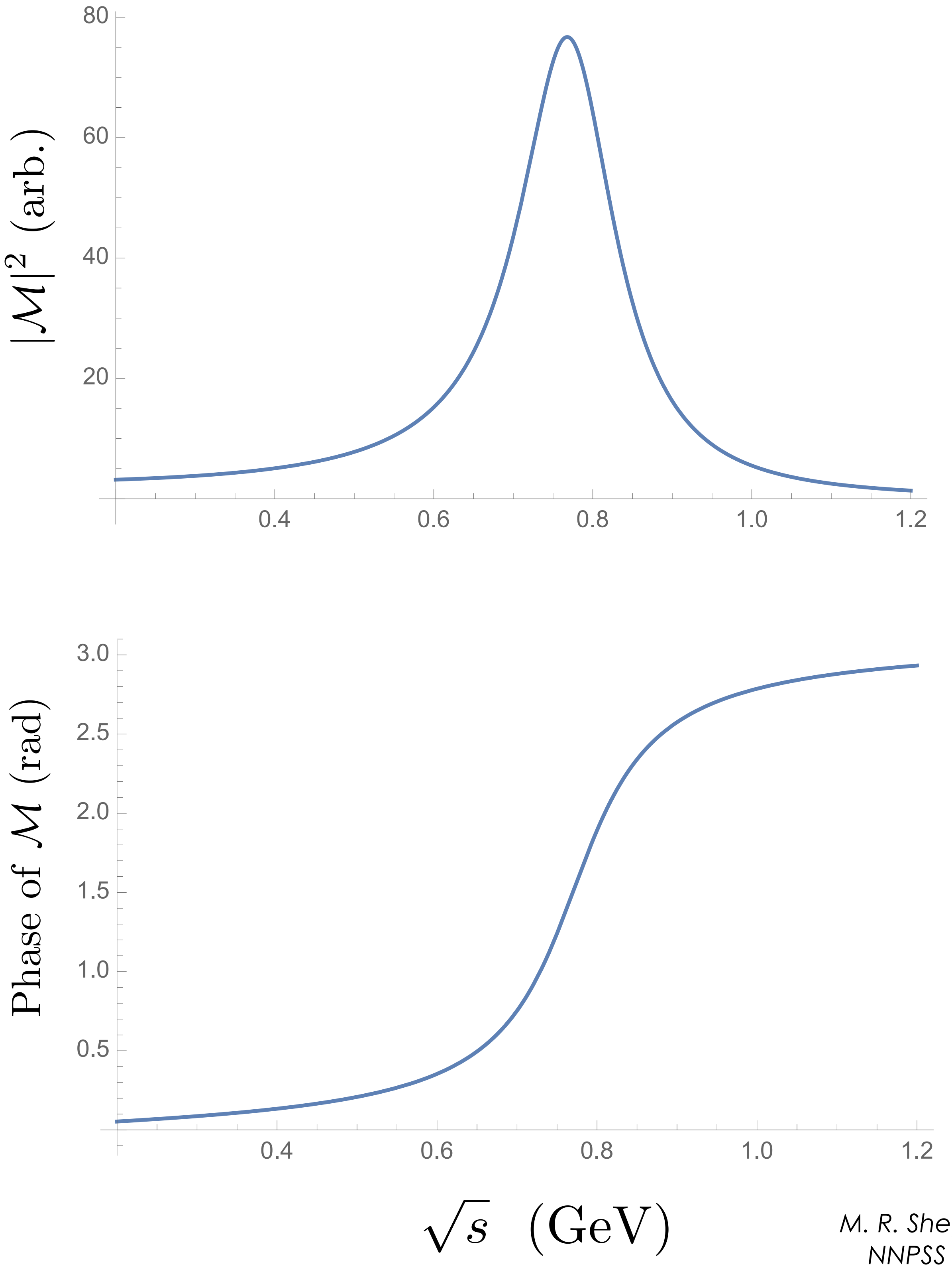


$$M_\rho = 775 \text{ MeV}$$
$$\Gamma_\rho = 148 \text{ MeV}$$

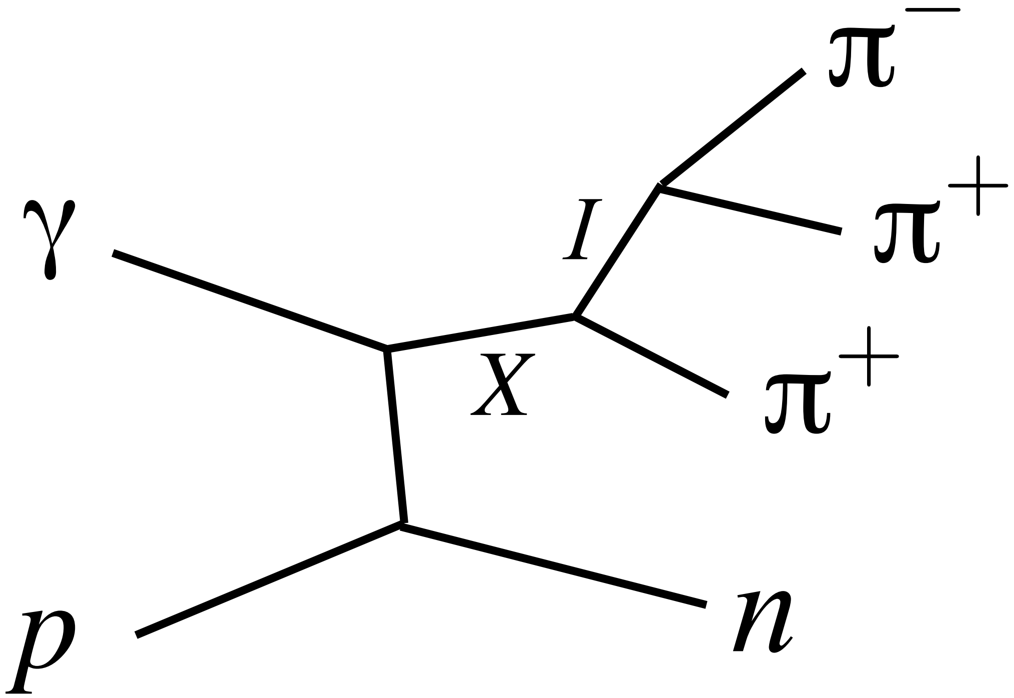
$$\mathcal{M} \propto \frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

$$\text{pole}: \quad \sqrt{s} = M - i\Gamma/2$$

Expt.: $s = [M(\pi\pi)]^2$ (real)



A more complex example…



Dynamical Assumptions:

- helicity transfer to X: $\lambda_\gamma=\lambda_X$
- exchange, lower vertex is distributed like e^{-5t}
- 1 → 2 decay chain “isobar model”
- resonances described by BreitWigner shapes

Factorize amplitude:

Put isospin CJ coefs in here

$$A \propto \sum_{\text{perms.}} BW_X(M_X, k) BW_I(M_I, q) F_{x,y}(\theta_X, \phi_X, \theta_I, \phi_I, k, q)$$

(permute the two identical pions)

k and q are breakup momenta of resonance and isobar, respectively

subscript denotes polarization of initial photon

… "just kinematics"

angles

$$f(\lambda_\gamma,\theta_X,\phi_X,\theta_I,\phi_I,k,q) = \sum_{m_X=-L_X}^{L_X}\sum_{\lambda_I=-J_I}^{J_I} Y_{L_X}^{m_X}(\theta_X,\phi_X)Y_{J_I}^{\lambda_I}(\theta_I,\phi_I)\times$$

angular mom. CJ
for resonance decay

isospin CJ for
isobar decay

isospin CJ for
resonance decay

$$\langle J_I\lambda_I L_X m_X|J_X\underset{\uparrow}{\lambda_\gamma}\rangle\langle 1Q_0 1Q_1|I_I(Q_0+Q_1)\rangle\langle I_I(Q_0+Q_1)1Q_2|I_X(Q_0+Q_1+Q_2)\rangle\times$$

in general, this is λ_X and there is a sum over λ_X,
but we assume pure helicity transfer

Q_i are charges of
particles 0,1,2

$$\left(\delta_{\lambda_\gamma,1}+\delta_{\lambda_\gamma,-1}P_X(-1)^{(J_X+1)}\right)k^{L_X}q^{J_I}$$

P_X is the parity of the resonance

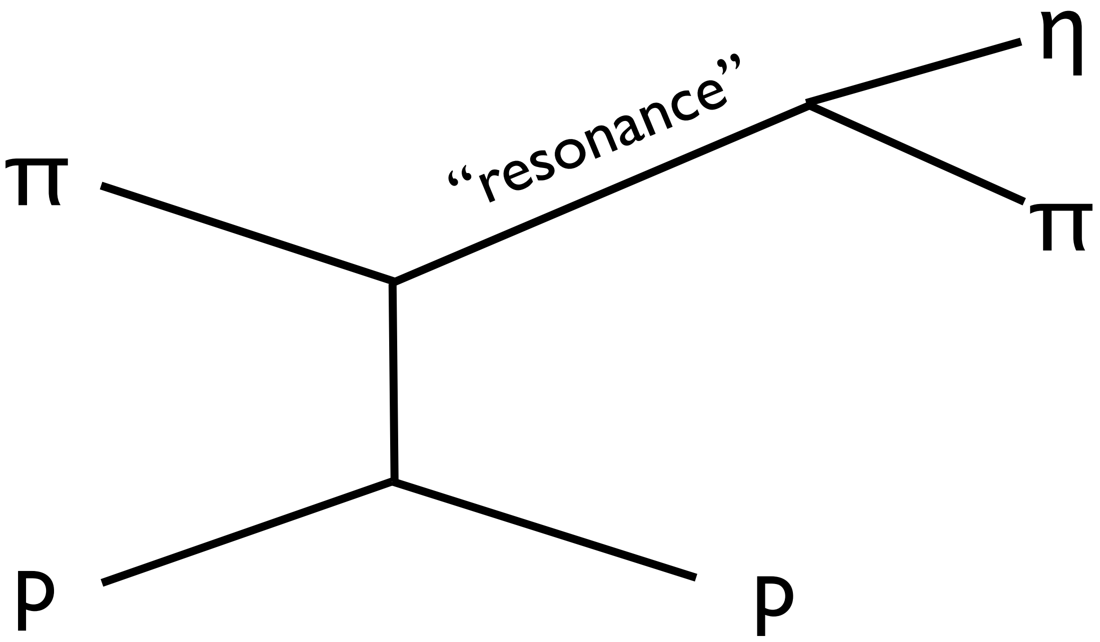
...rewrite in linear polarization basis:

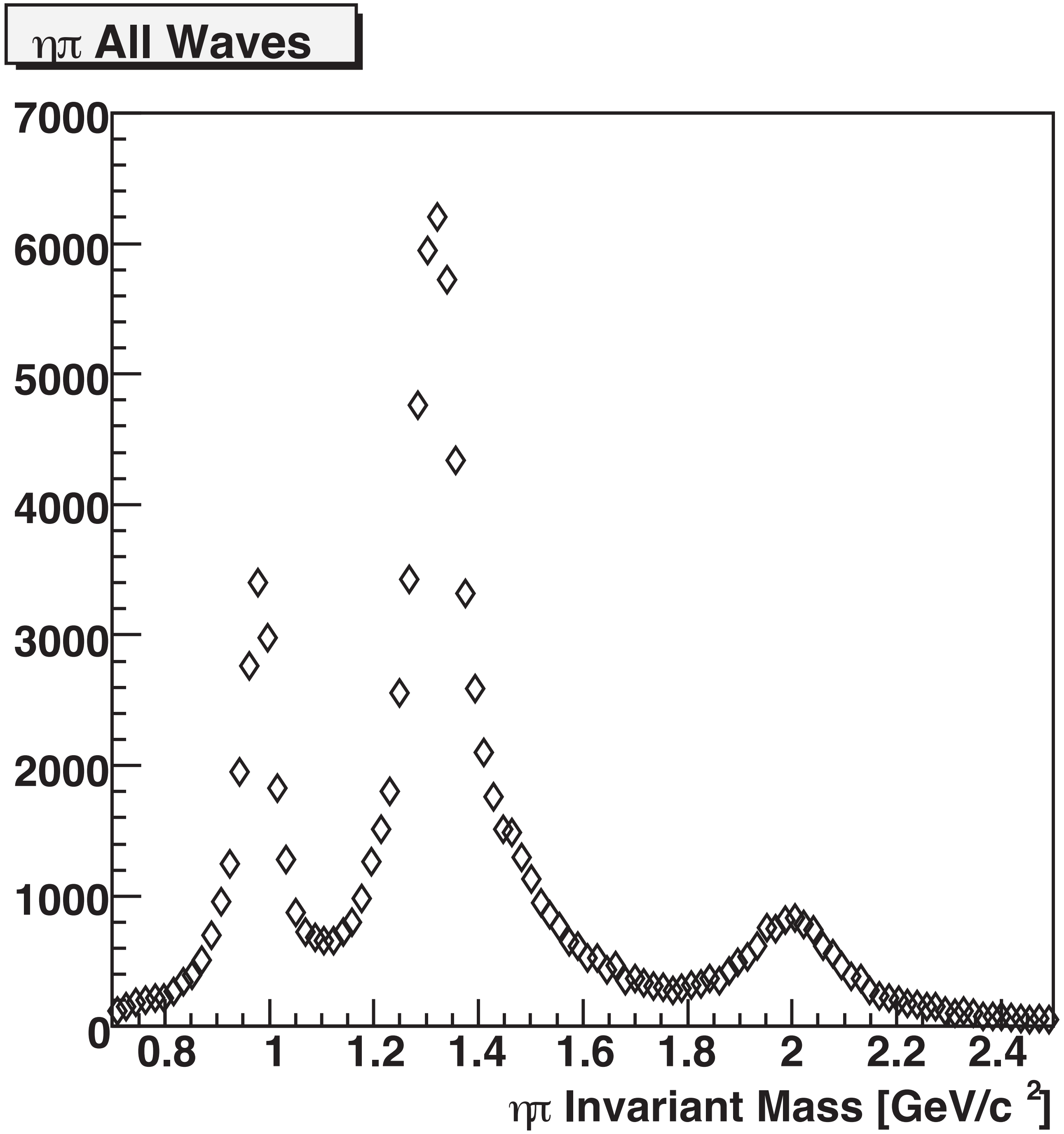
$$F_x(\theta_X,\phi_X,\theta_I,\phi_I,k,q) = \frac{1}{\sqrt{2}}\big(f(-1,\theta_X,\phi_X,\theta_I,\phi_I,k,q) - f(1,\theta_X,\phi_X,\theta_I,\phi_I,k,q)\big)$$

$$F_y(\theta_X,\phi_X,\theta_I,\phi_I,k,q) = \frac{i}{\sqrt{2}}\big(f(-1,\theta_X,\phi_X,\theta_I,\phi_I,k,q) + f(1,\theta_X,\phi_X,\theta_I,\phi_I,k,q)\big)$$

In Practice

- Fake data: $\pi^- p \rightarrow \eta\pi^- p$
- How many resonances?
- What are their spins?

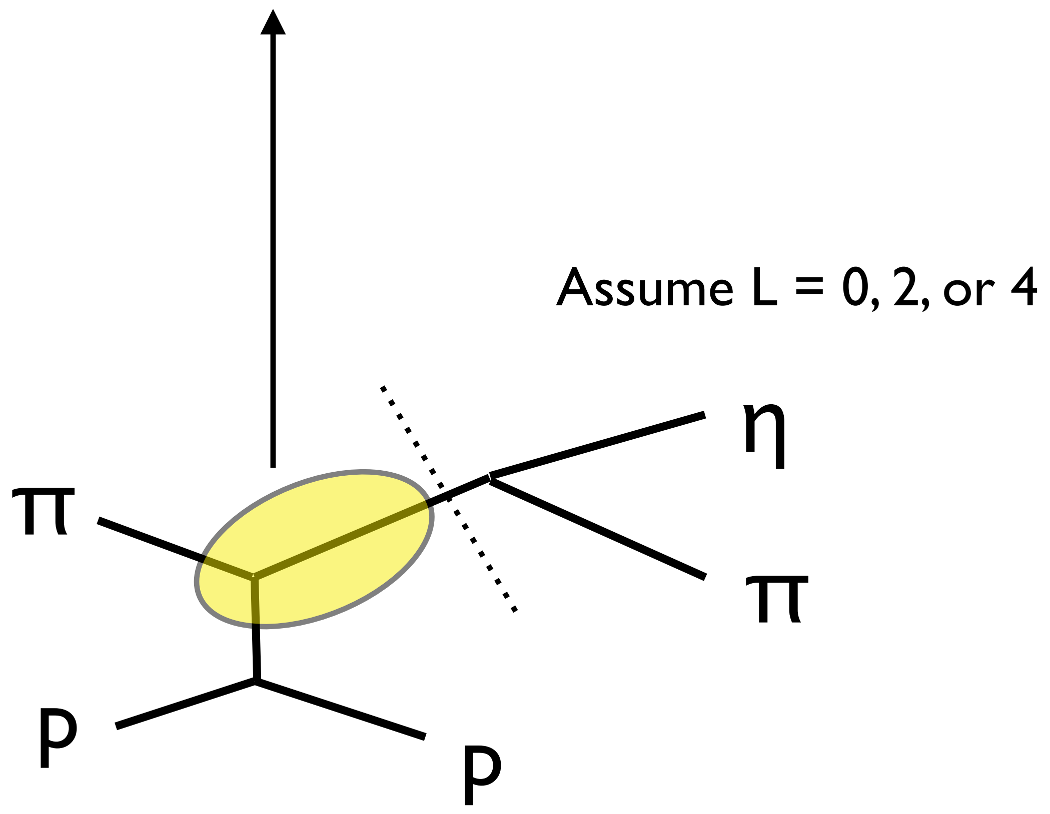


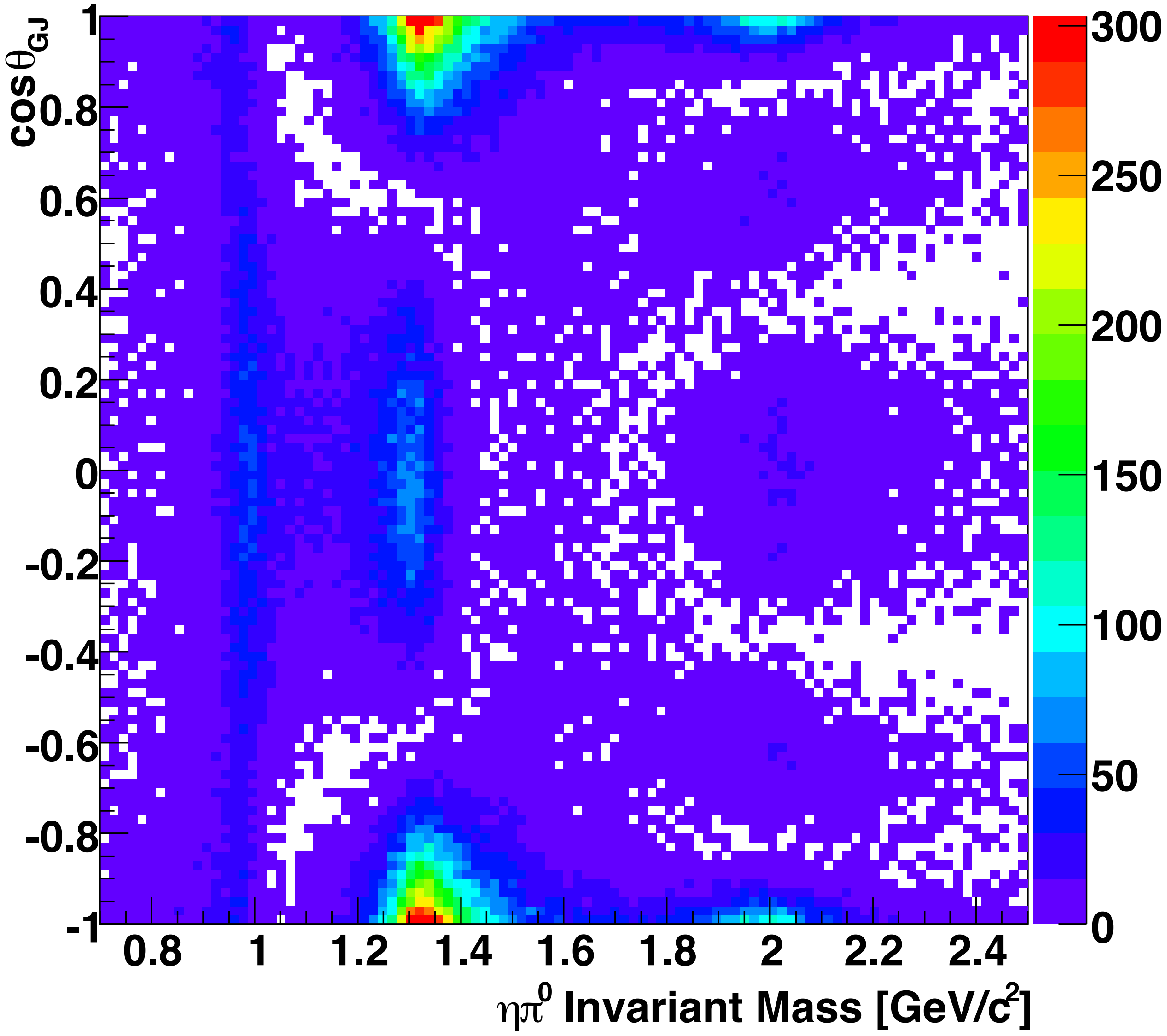


Incorporating Mass Dependence

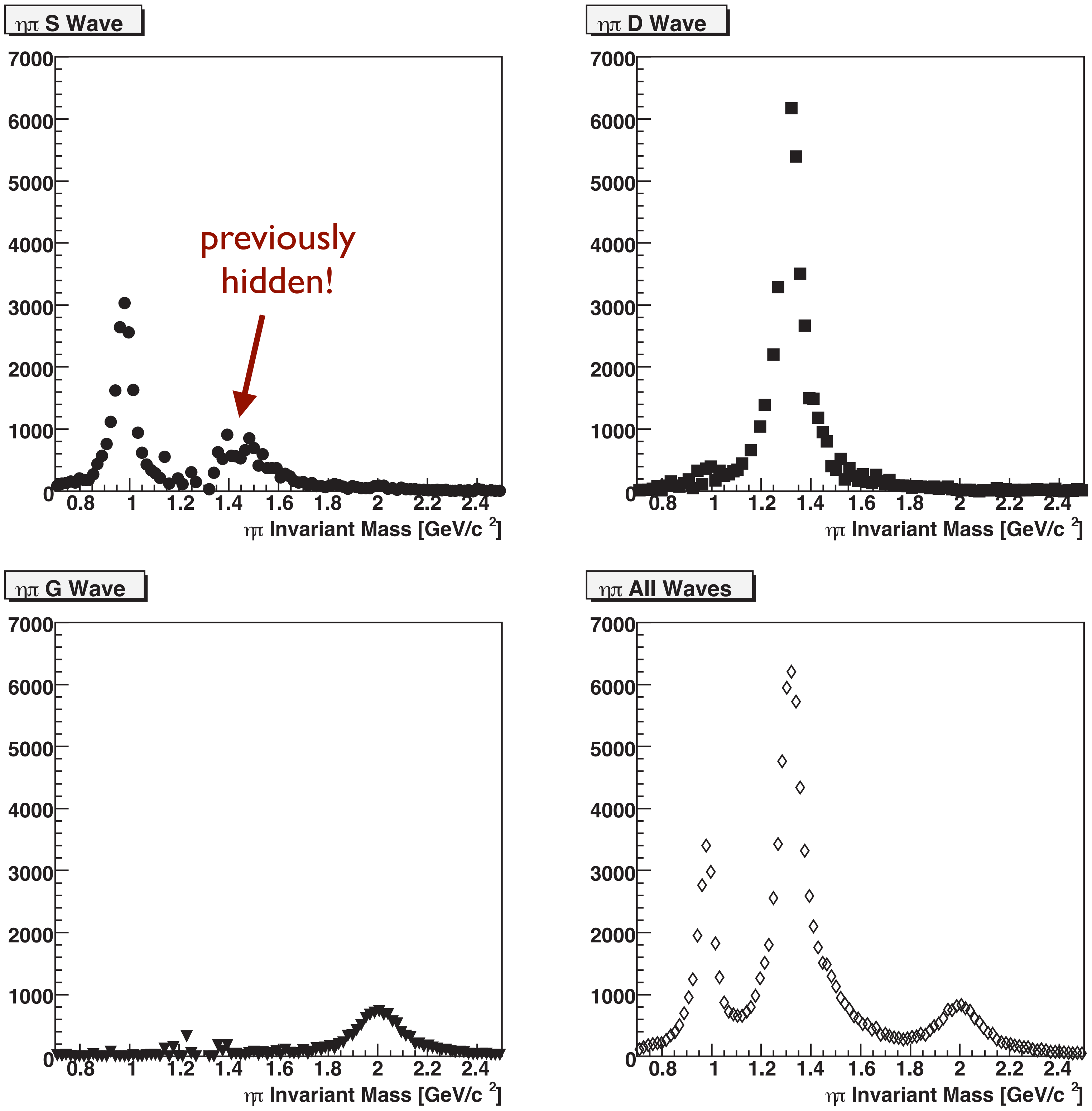
s: invariant mass squared of ηπ

$$V(s) = \frac{V_0}{s - M^2 + iM\Gamma}$$





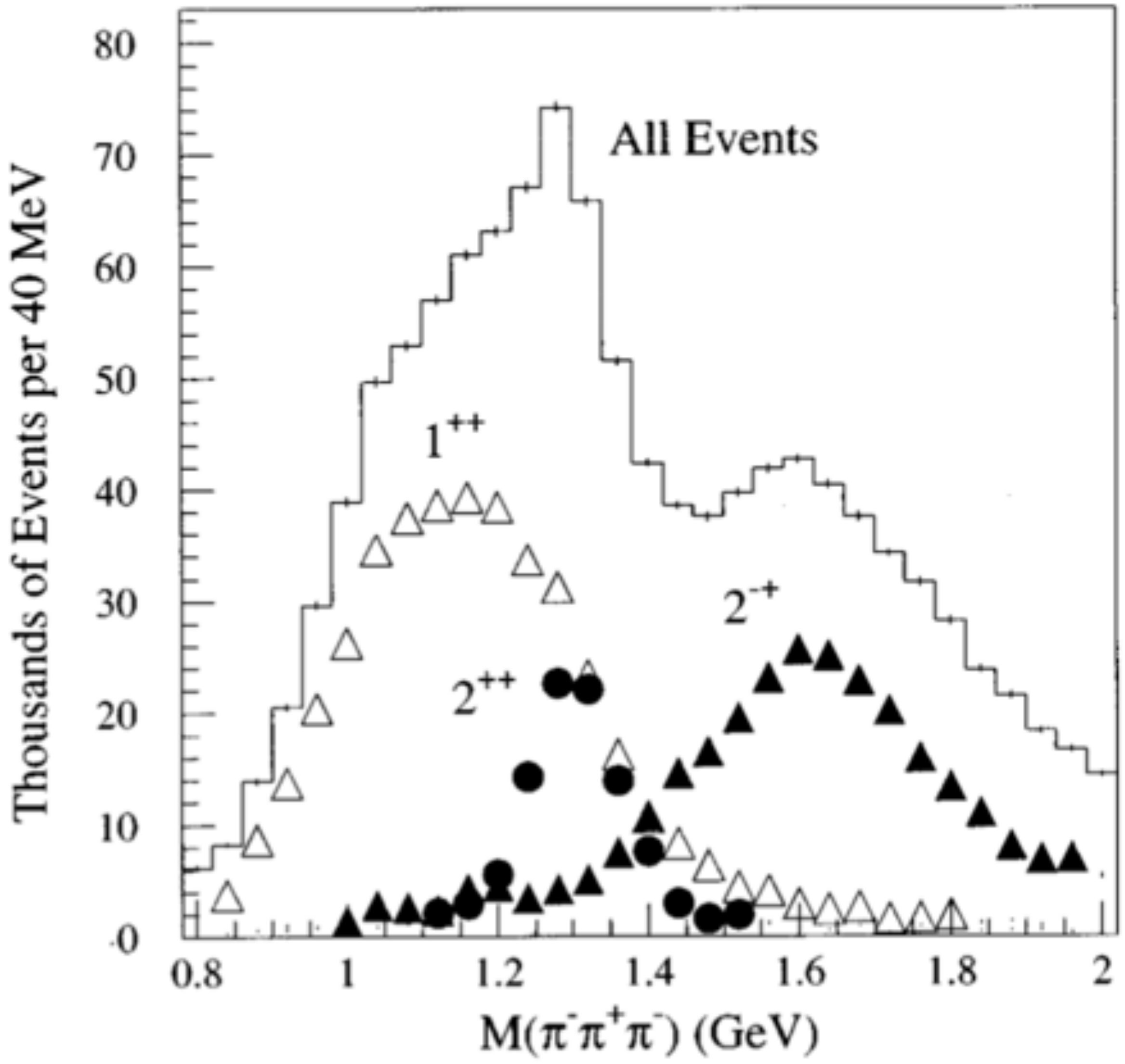
Partial Wave Decomposition

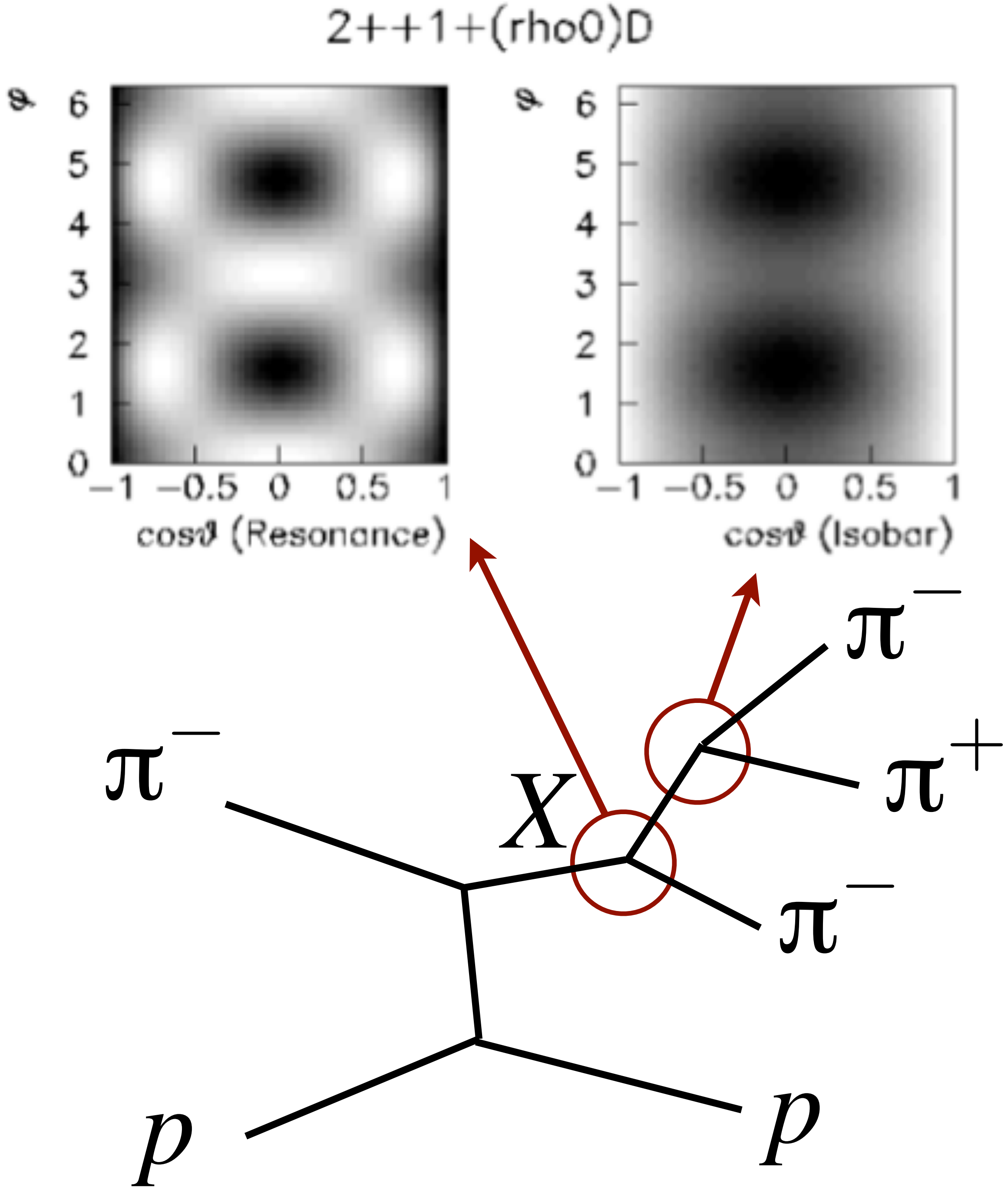


previously hidden!

Another Example

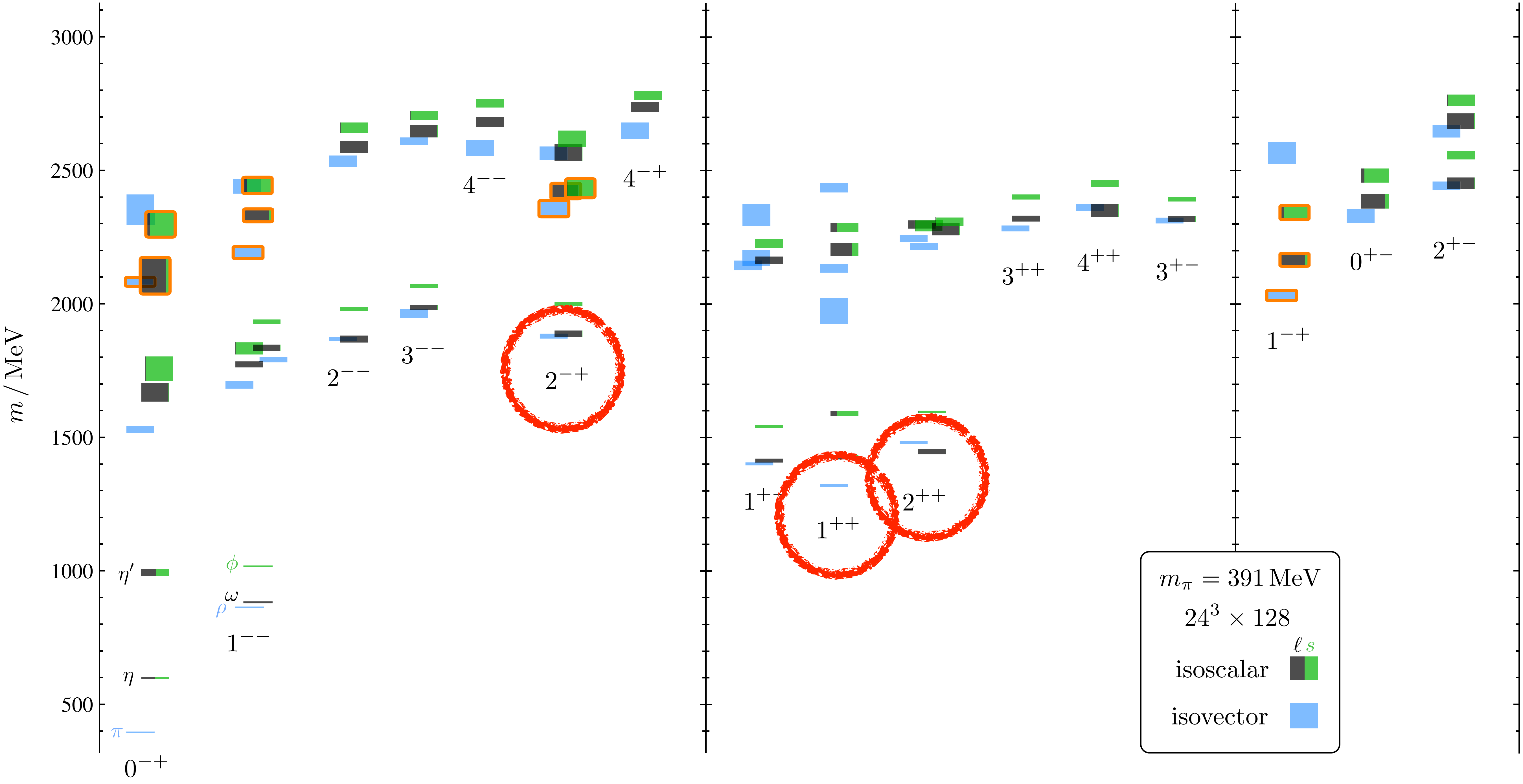
from pion production: $\pi^-p \to \pi^-\pi^-\pi^+p$ at 18 GeV/c





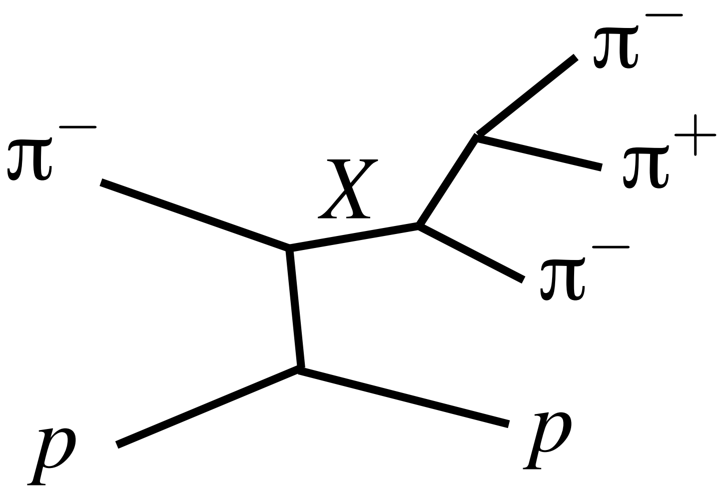
Compare with LQCD

Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)

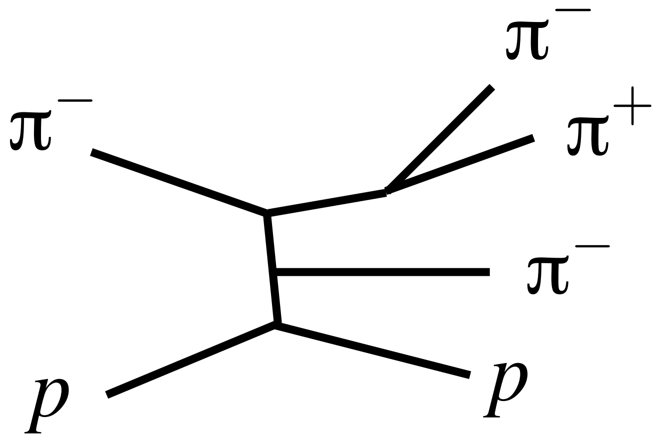


Practical Problems

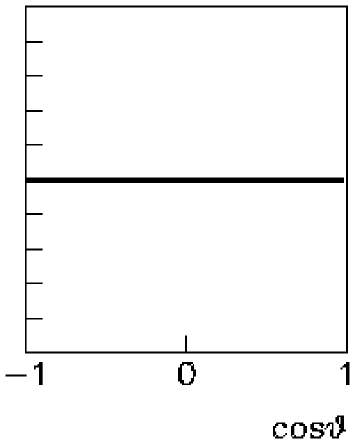
- Features of real detectors:
  - Gaps in acceptance which could make it hard to distinguish between amplitudes
  - Poorly understood efficiencies which could mimic variations in intensity
- Need a complete set of amplitudes that is suitable for "projecting" the dominant/interesting physics



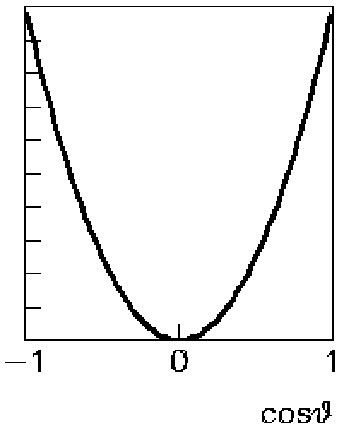
…or



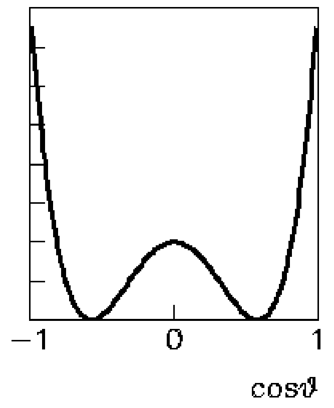
$$\left|Y_0^0\right|^2$$



$$\left|Y_1^0\right|^2$$



$$\left|Y_2^0\right|^2$$



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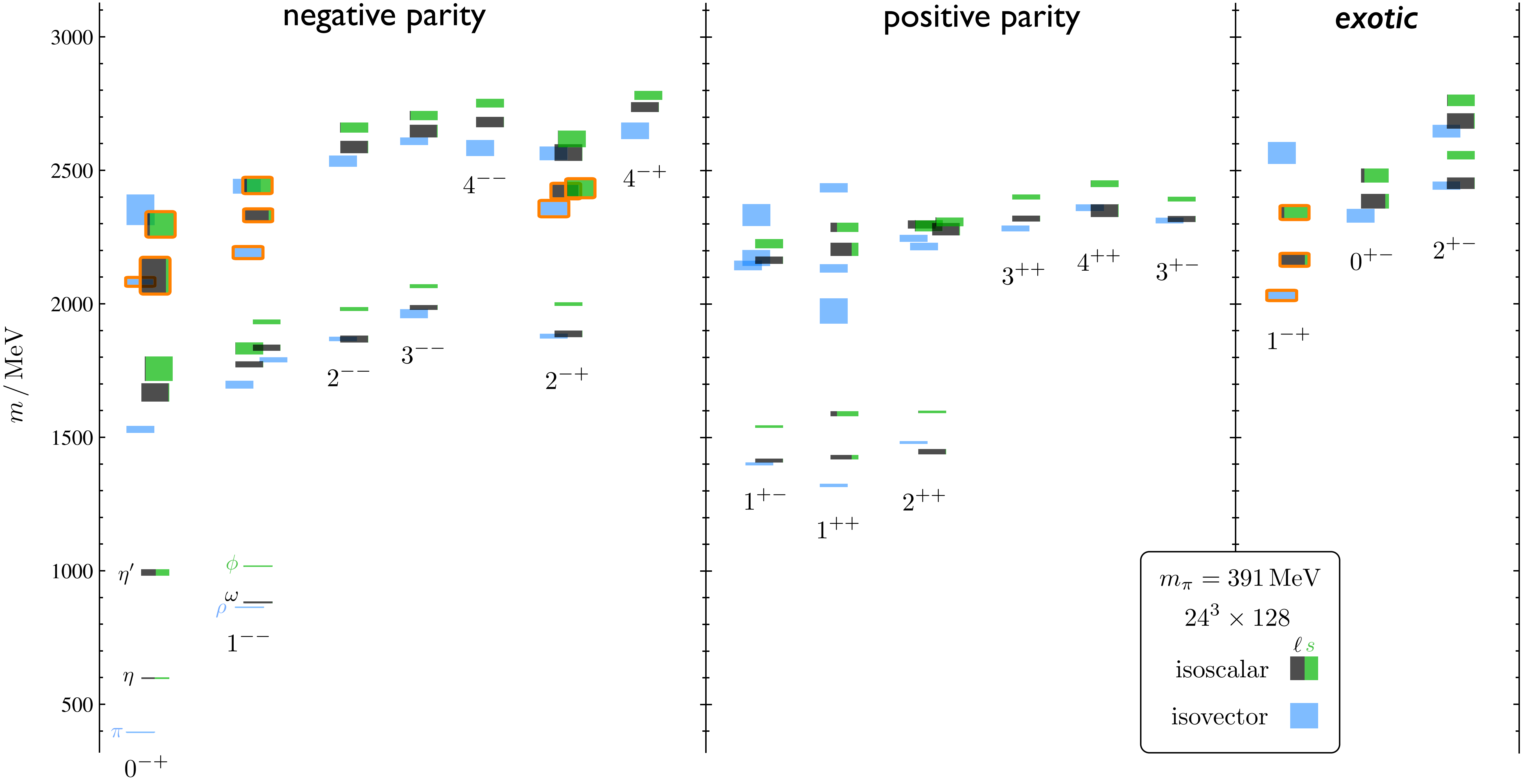
Light Quark Mesons from Lattice QCD

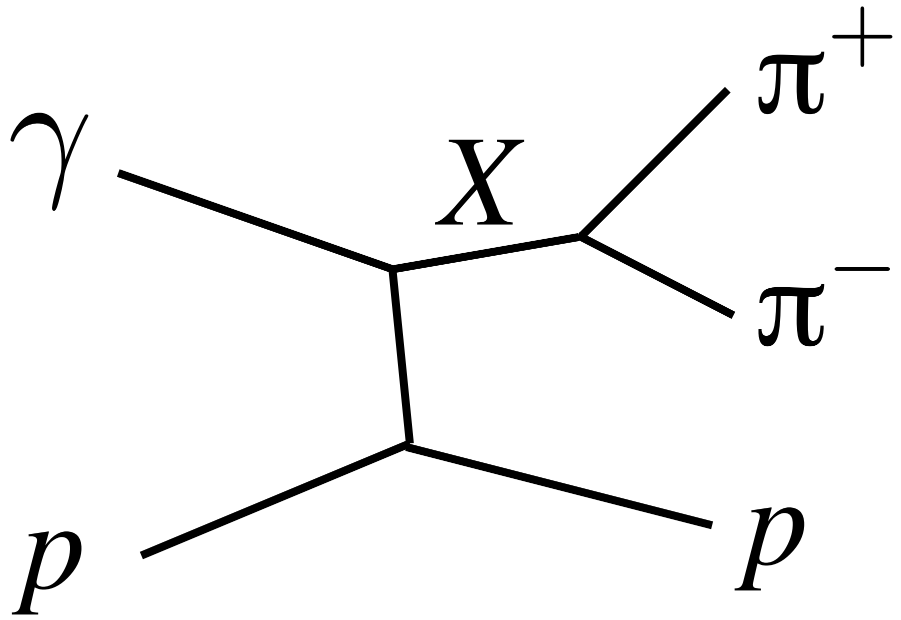
Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)



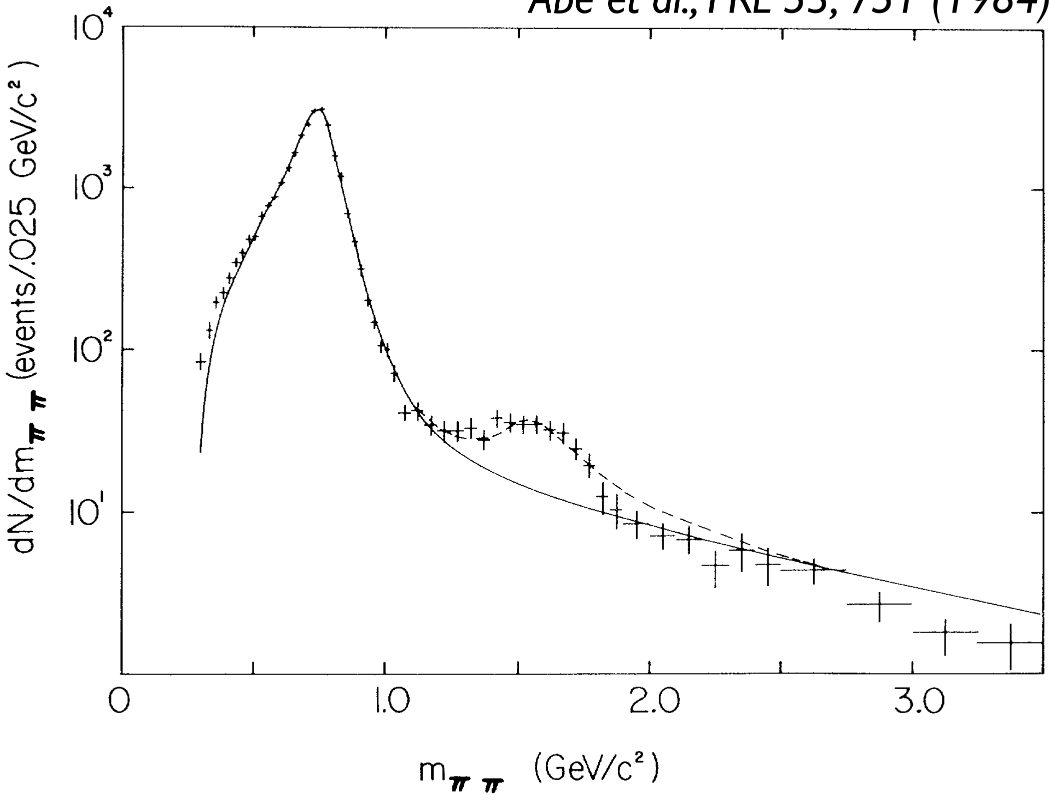
DEPARTMENT OF PHYSICS
INDIANA UNIVERSITY
College of Arts and Sciences
Bloomington

Access to Light Quark Mesons

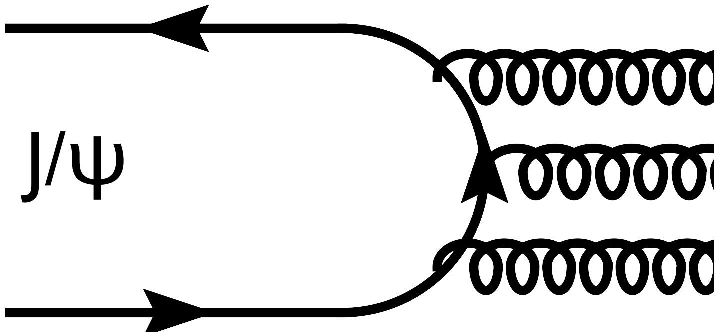
Direct Production



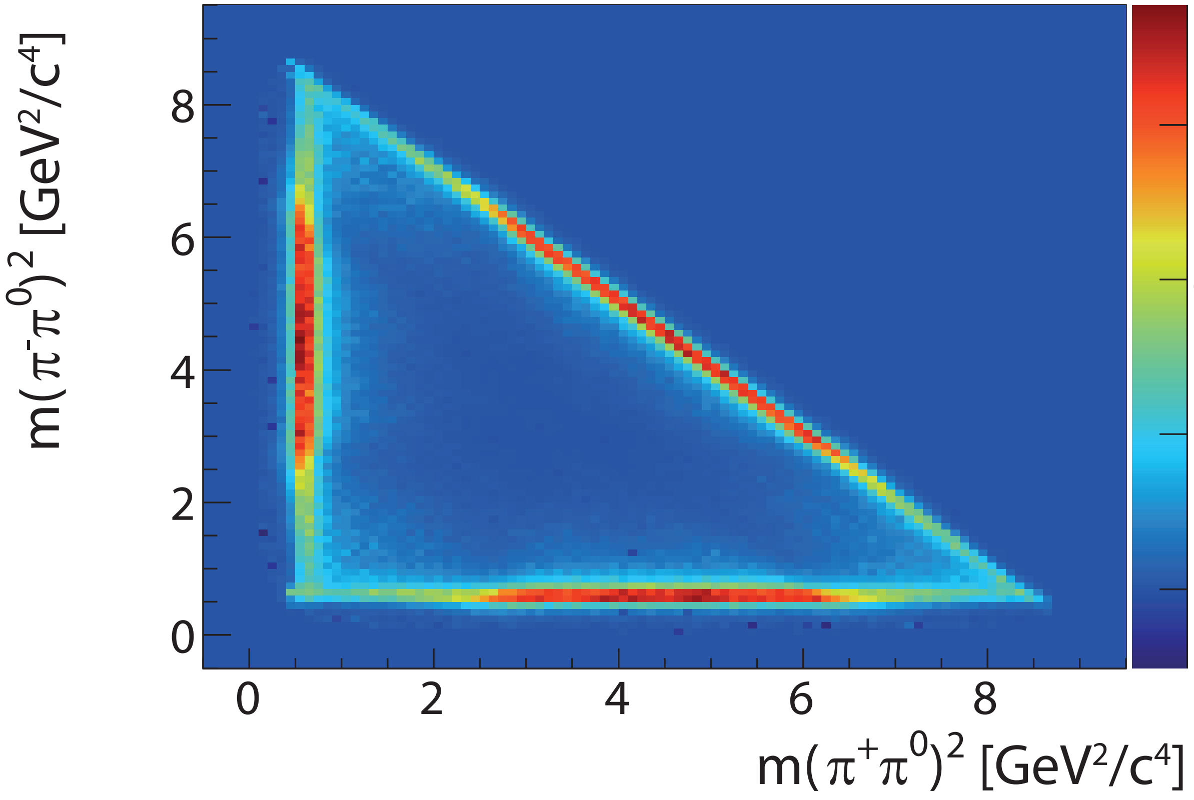
Abe et al., PRL 53, 751 (1984)



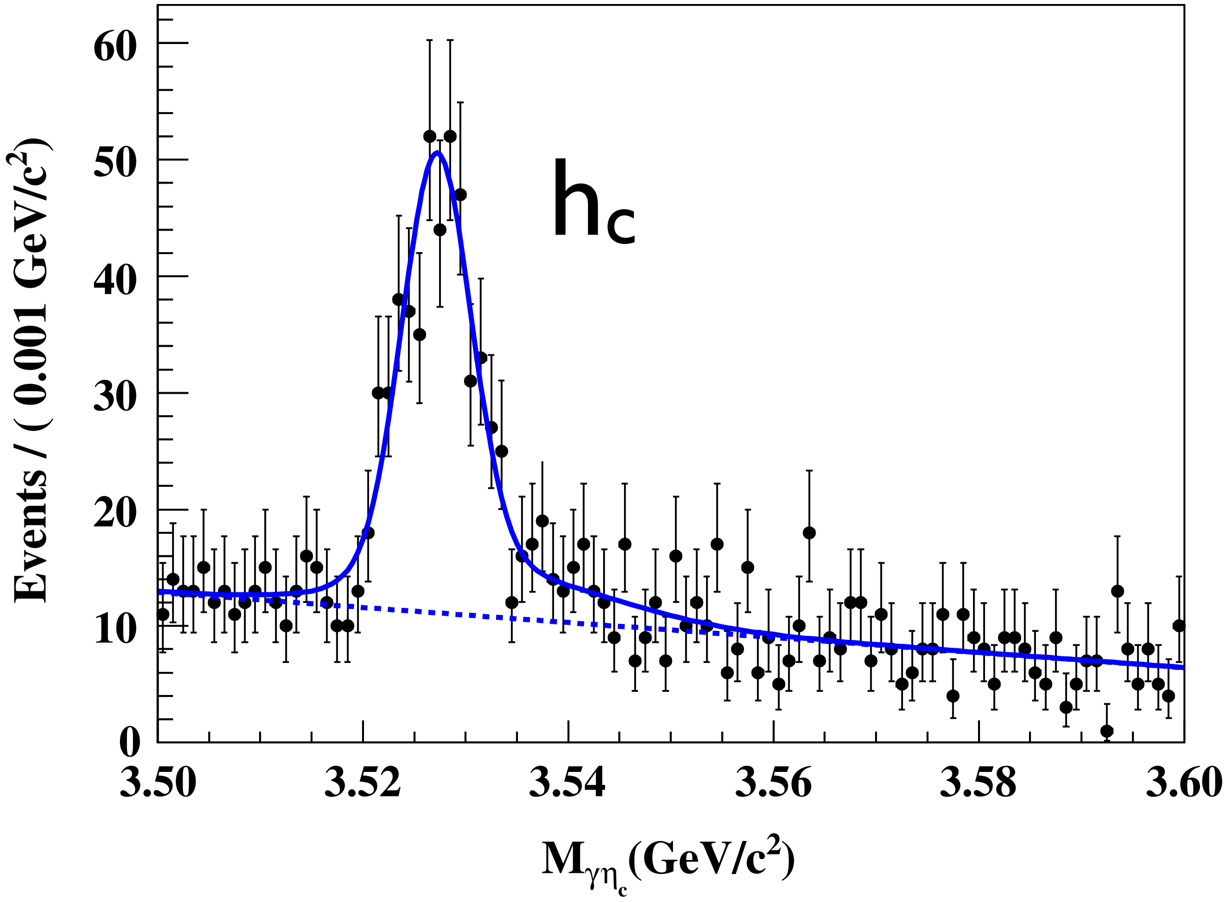
Decays of Heavier States

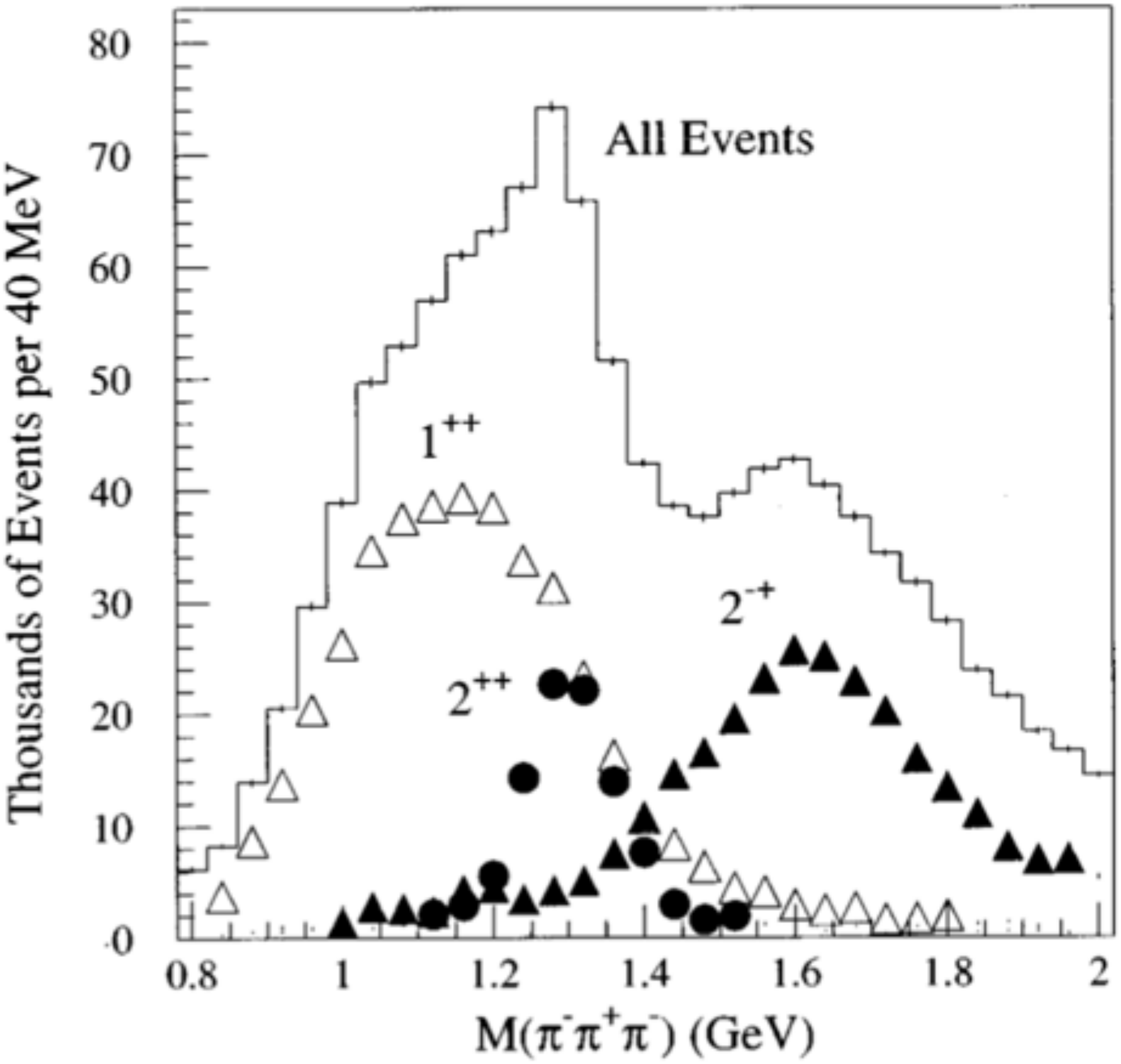


BESIII, PLB 710, 594 (2012)



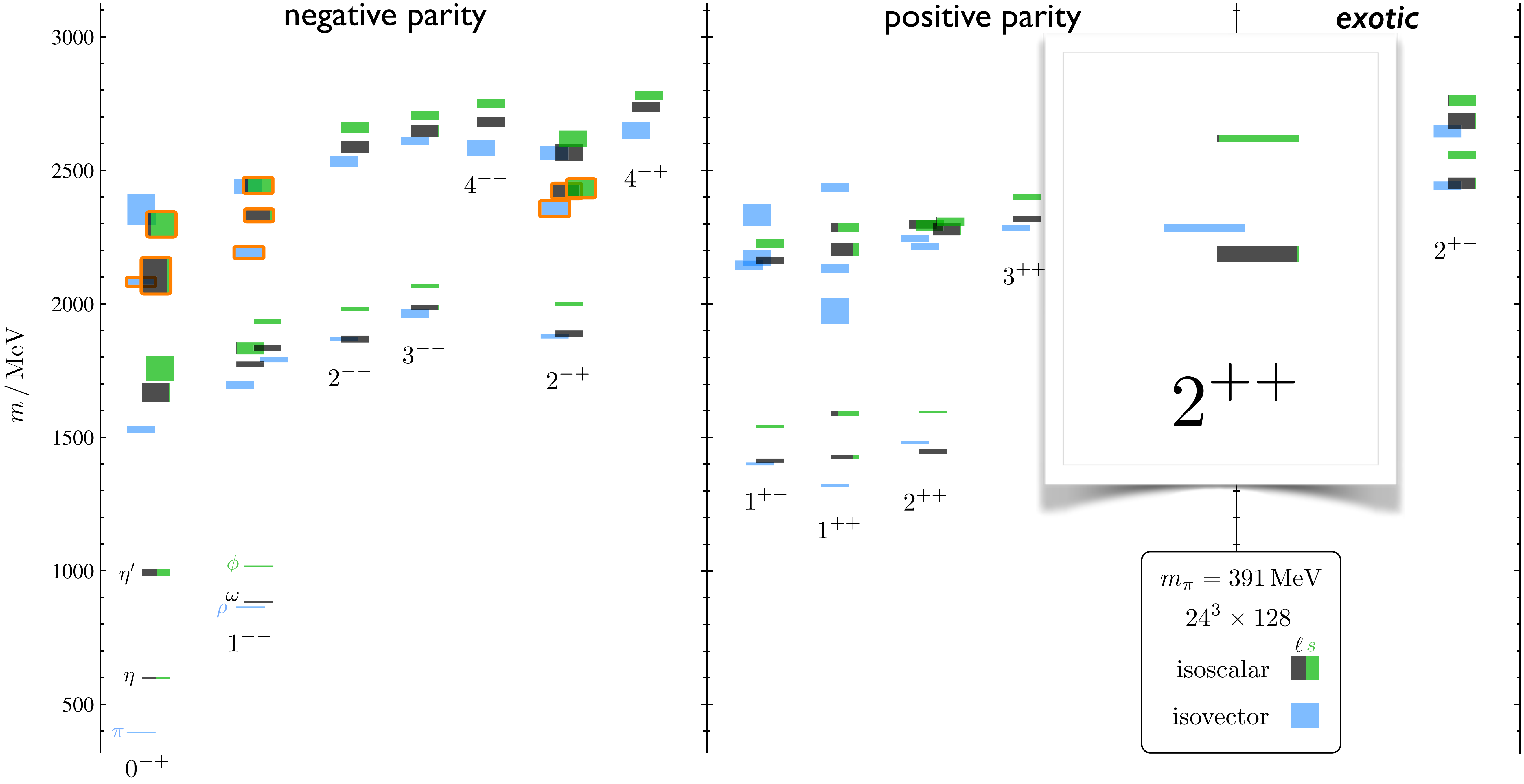
“Features” of the Light Quark States





Light Quark Mesons from Lattice QCD

Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)

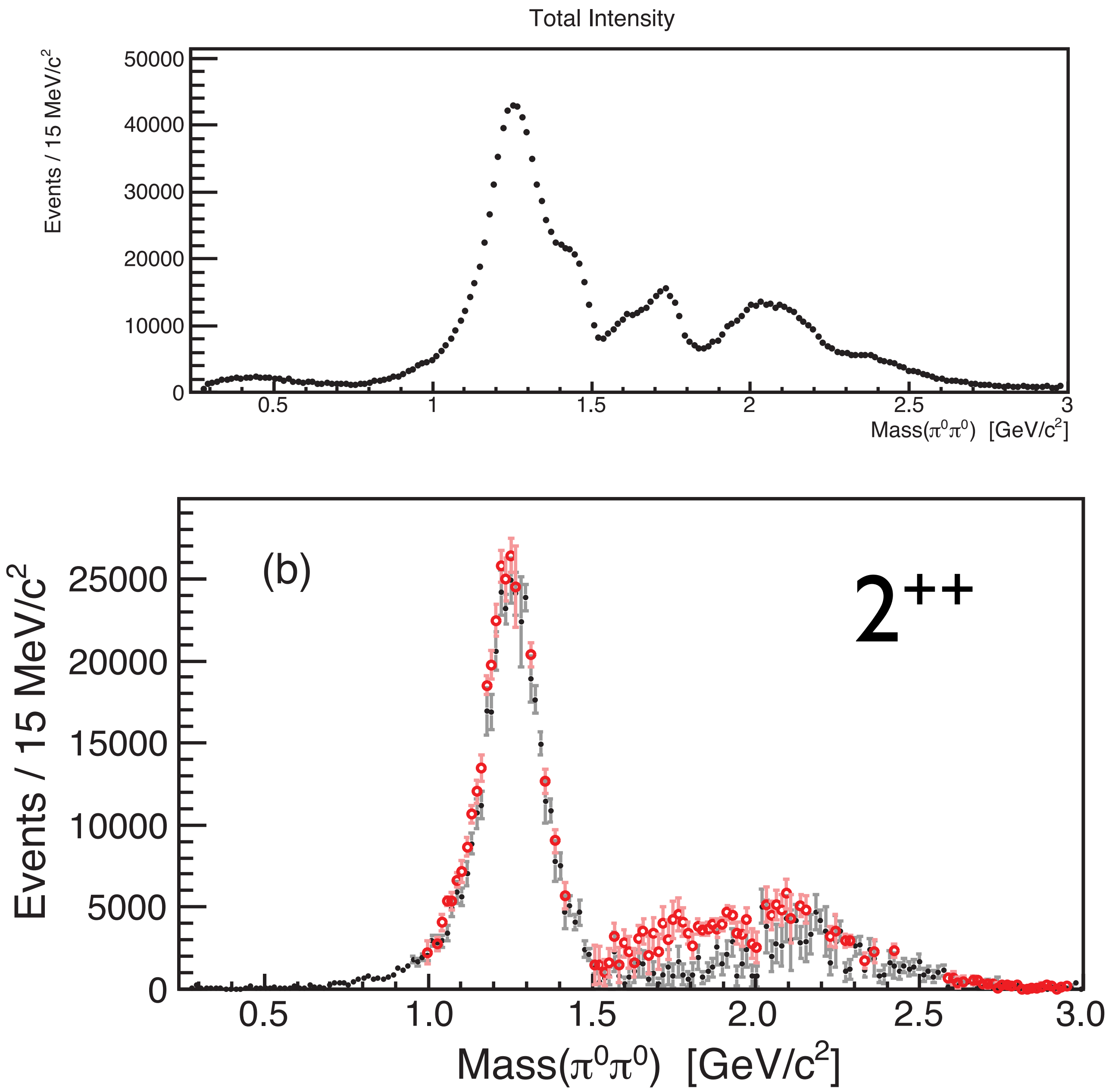


$f_2(1270)$ and $f_2'(1525)$

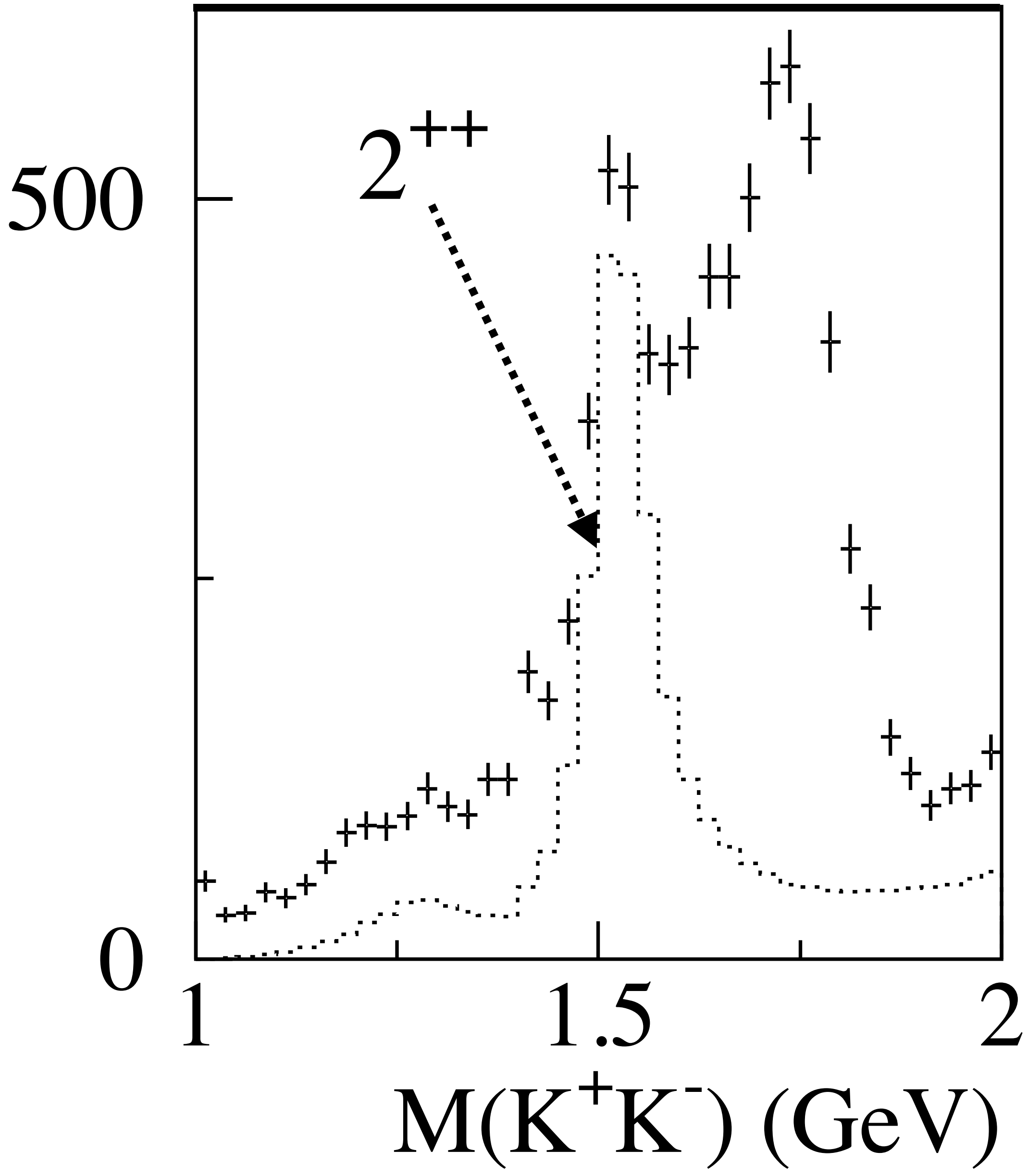
PRD 92, 052003 (2015)

PRD 68, 052003 (2003)

$J/\psi \to \gamma \pi^0 \pi^0$

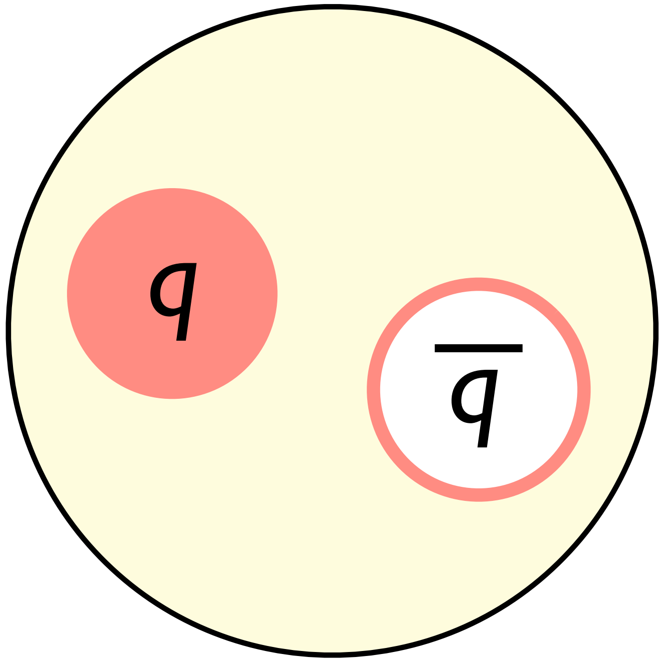


$J/\psi \to \gamma K^+ K^-$



Hybrid Mesons

color singlet
quark anti-quark

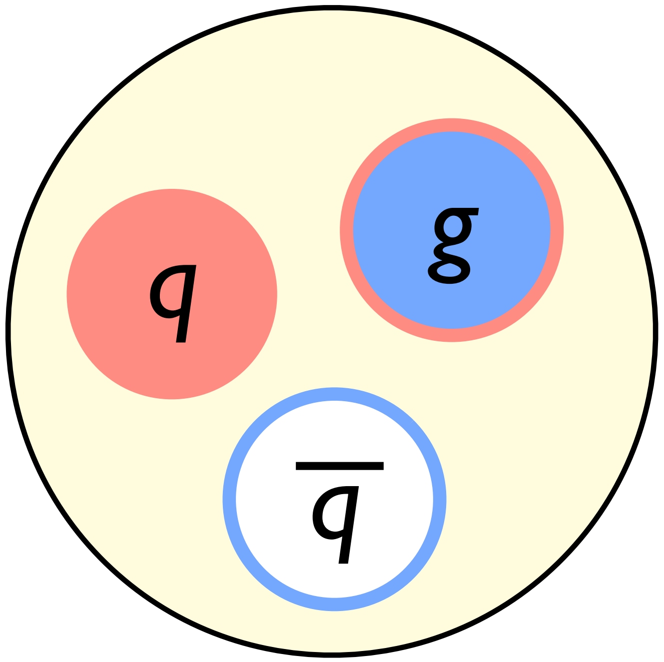


$J = L + S \quad P = (-1)^{L+1} \quad C = (-1)^{L+S}$

Allowed J^{PC}: $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots$
Forbidden J^{PC}: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

“constituent gluon”
$(J^{PC})_g = 1^{+-}$
mass ≈ 1.0-1.5 GeV

color-octet
$q\bar{q}$ pair



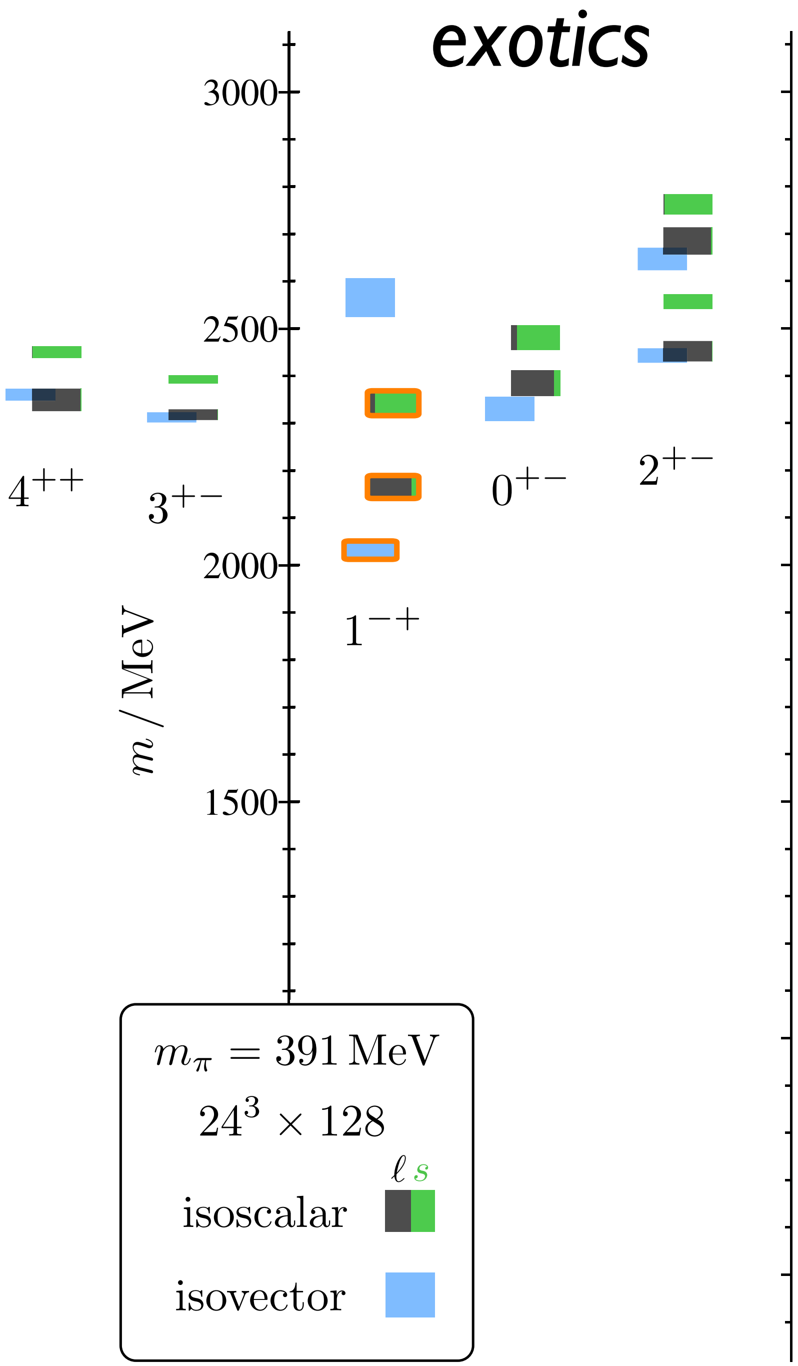
Lightest Hybrids

	$S_{q\bar{q}} = 1$	$S_{q\bar{q}} = 0$
J^{PC}:	$0^{-+}, 1^{-+}, 2^{-+}$	1^{--}

“exotic hybrid”

Exotic Hybrids

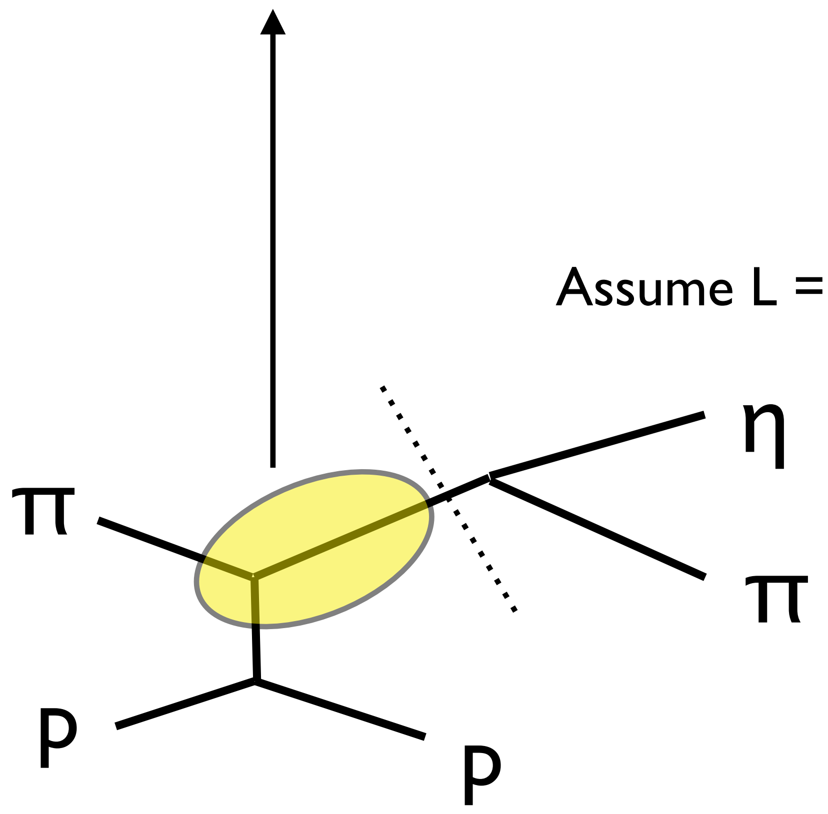
- Production of a heavy quark system with exotic J^{PC} seems challenging
- No evidence of exotic J^{PC} states in charmonium or bottomonium
- What about light quarks?



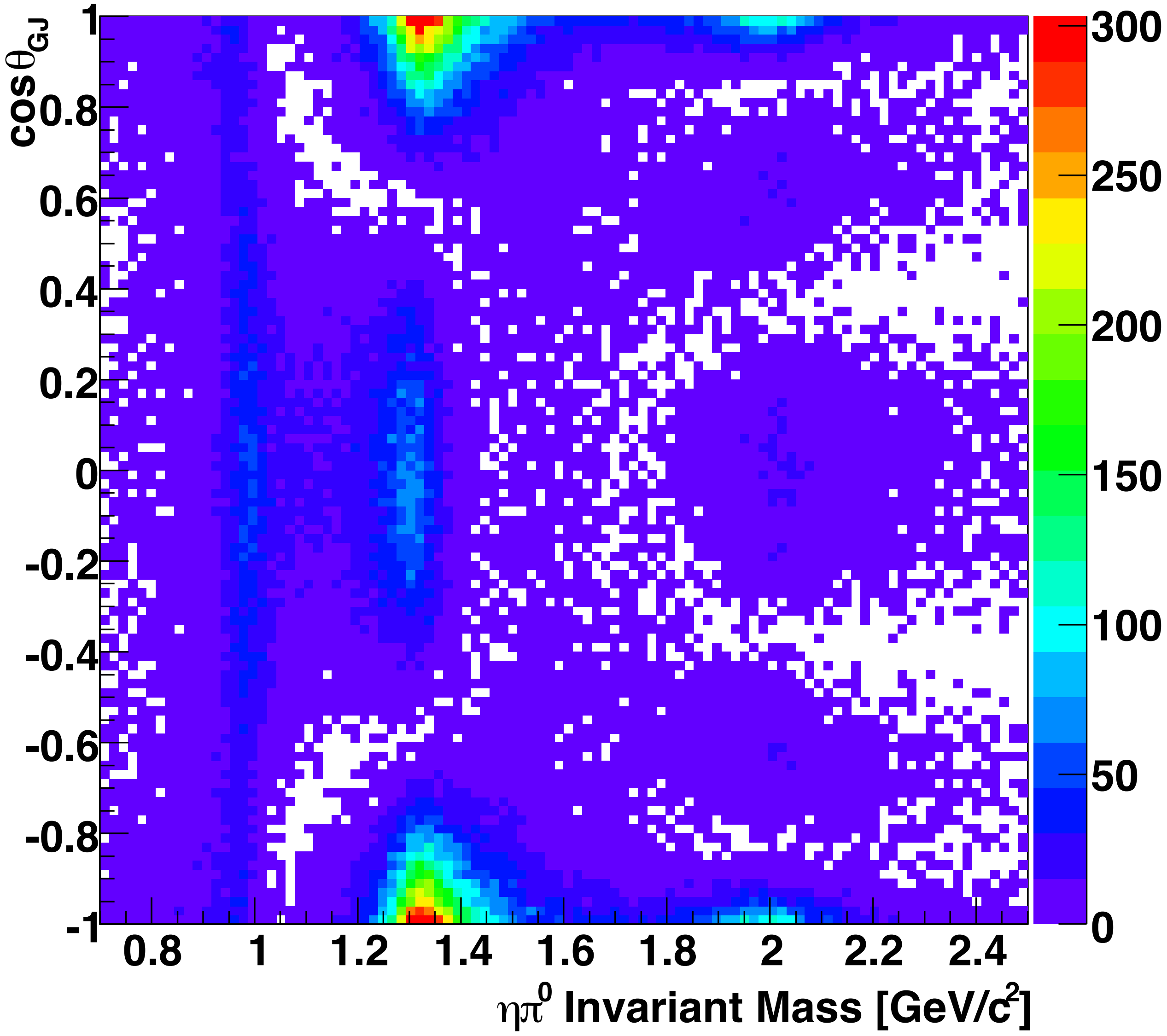
πp→ηπp fake data

s: invariant mass squared of ηπ

$$V(s) = \frac{V_0}{s - M^2 + iM\Gamma}$$



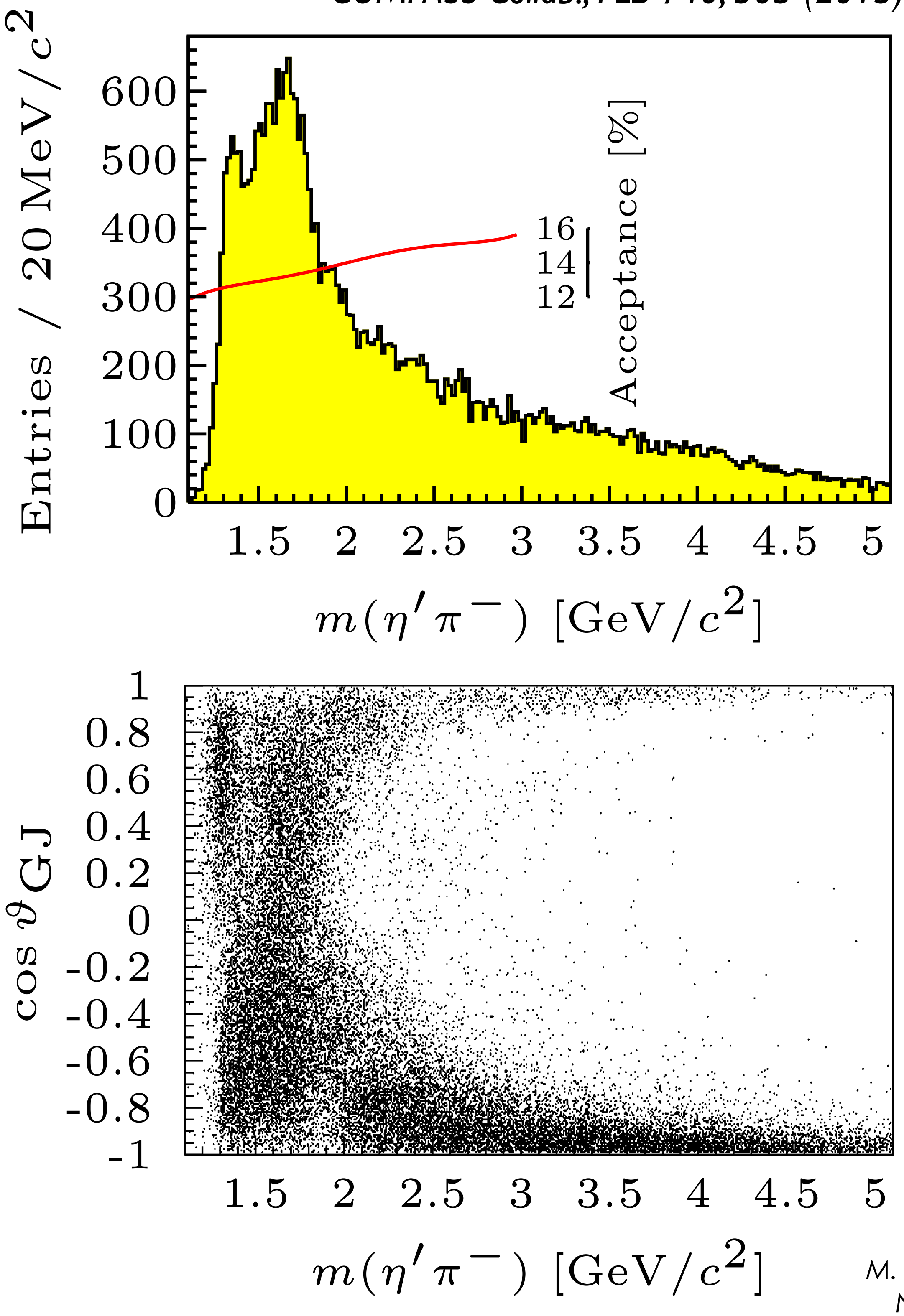
Assume L = 0, 2, or 4



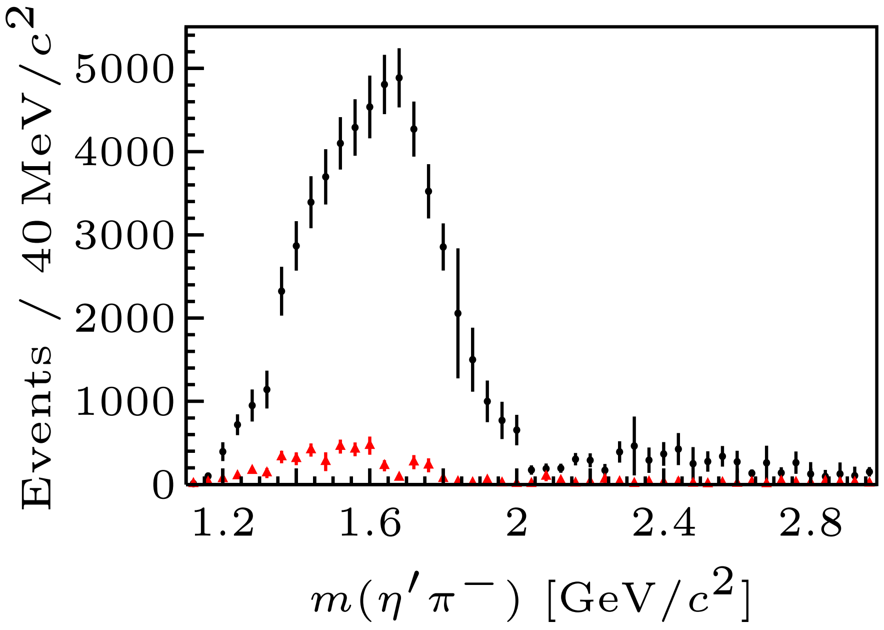
$\pi^- p \rightarrow \eta' \pi^- p$

- Data collected by COMPASS using a 190 GeV pion beam
- $\eta' \pi^-$ in a P-wave: L=1
  - parity: -
  - G: -
  - isospin: 1
  - J^{PC} of neutral isovector is 1^{-+} (exotic!)

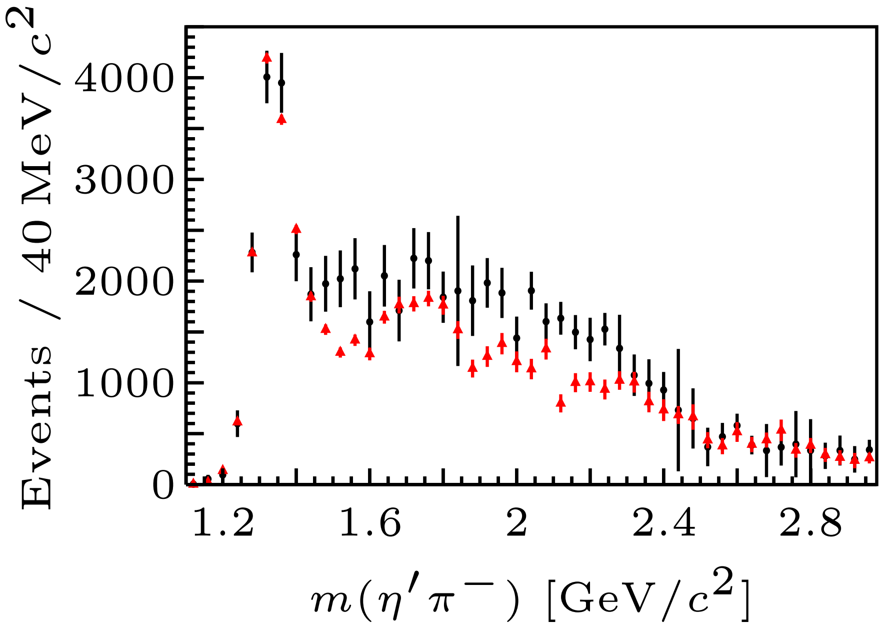
COMPASS Collab., PLB 740, 303 (2015)



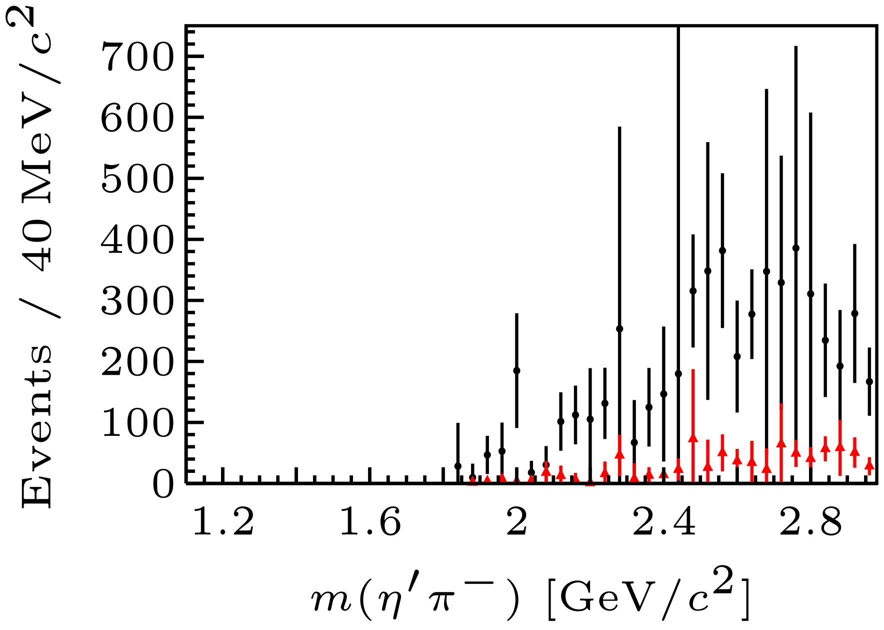
$\pi^- p \to \eta^{(\prime)} \pi^- p$



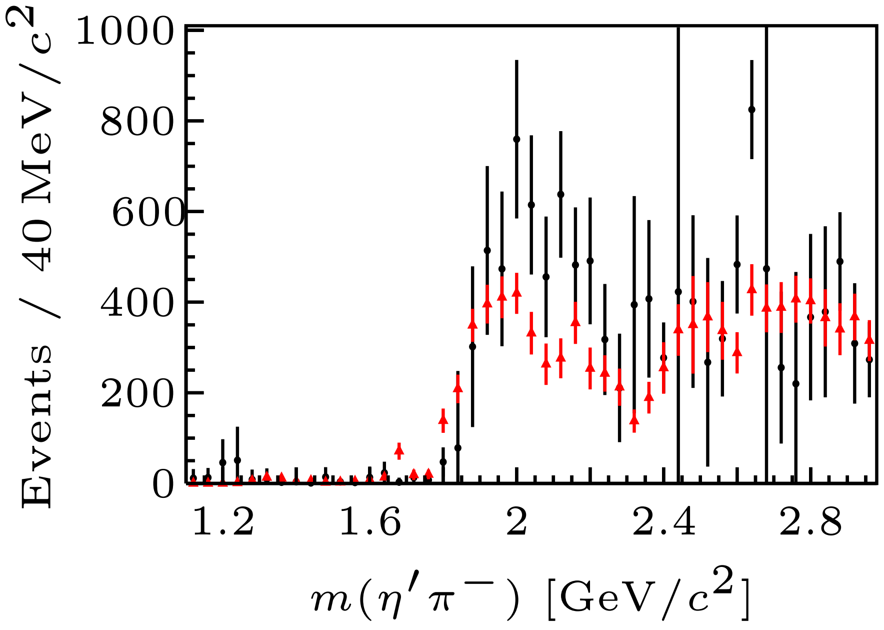
(a) P-wave, $L = 1$



(b) D-wave, $L = 2$



(c) F-wave, $L = 3$

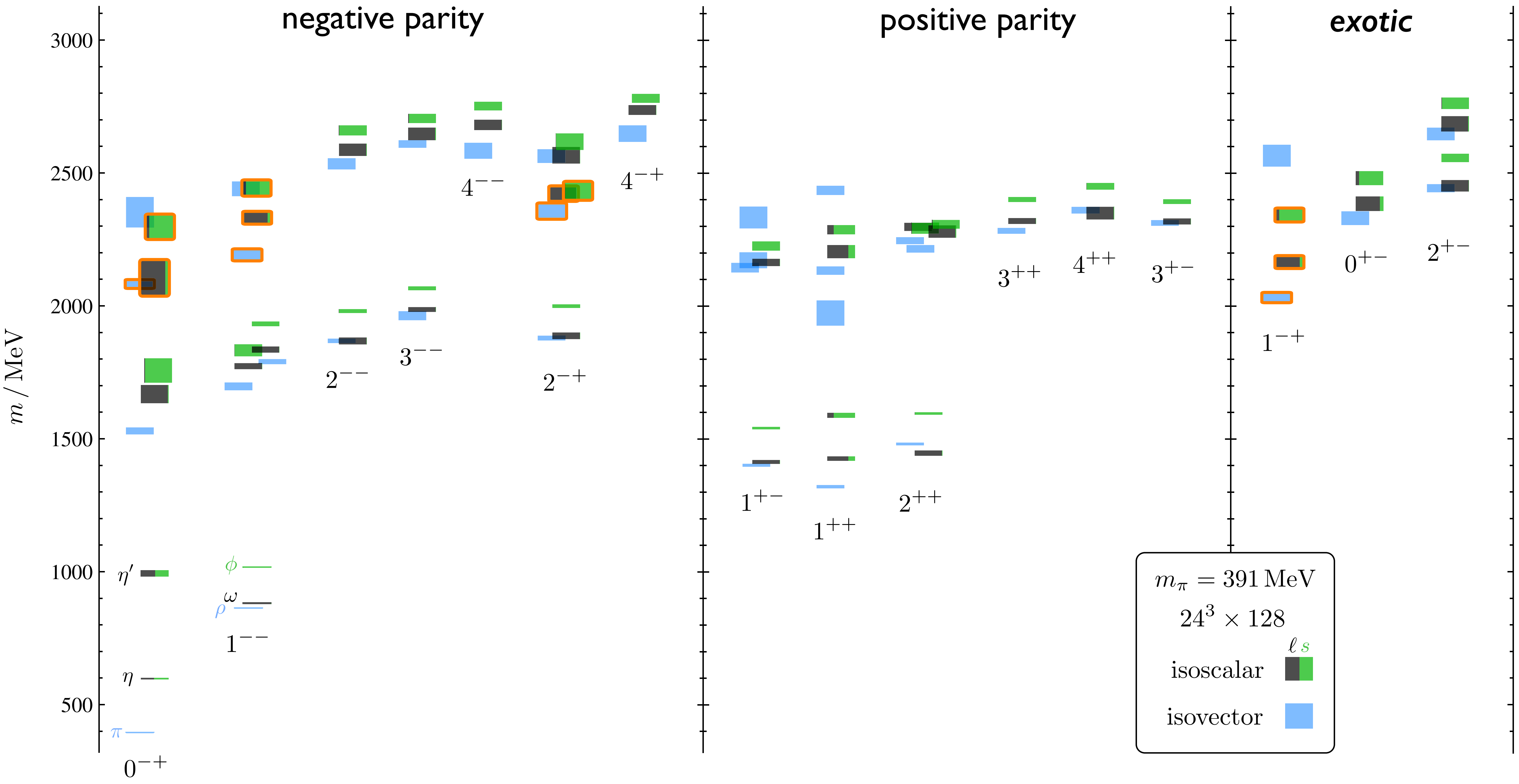


(d) G-wave, $L = 4$

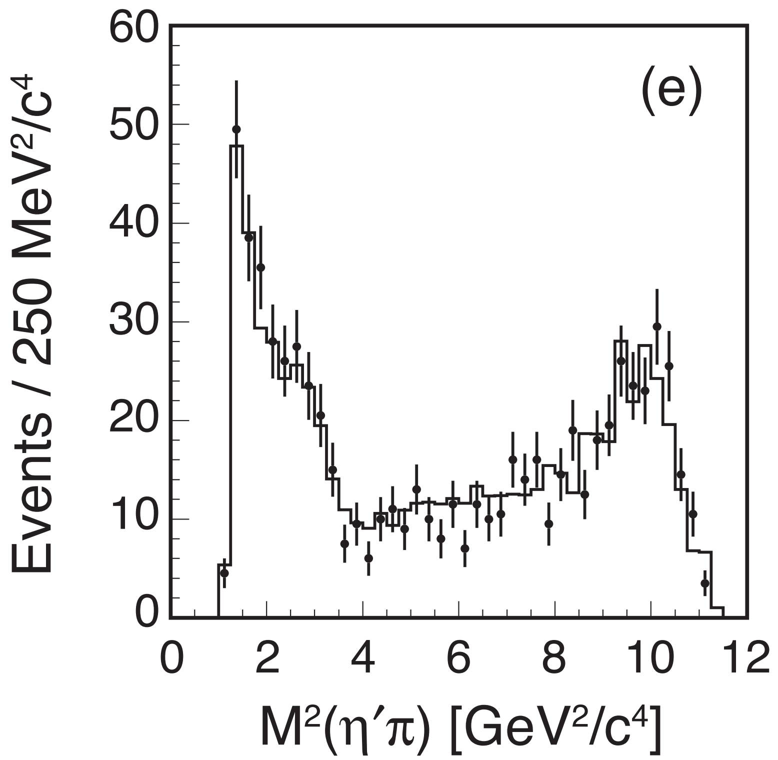
COMPASS Collab., PLB 740, 303 (2015)

Light Quark Mesons from Lattice QCD

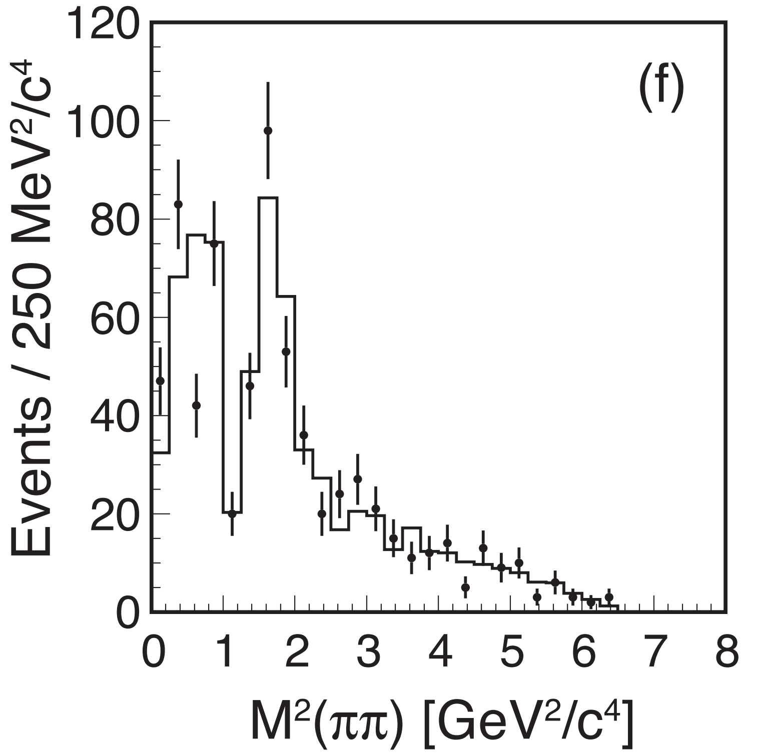
Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)

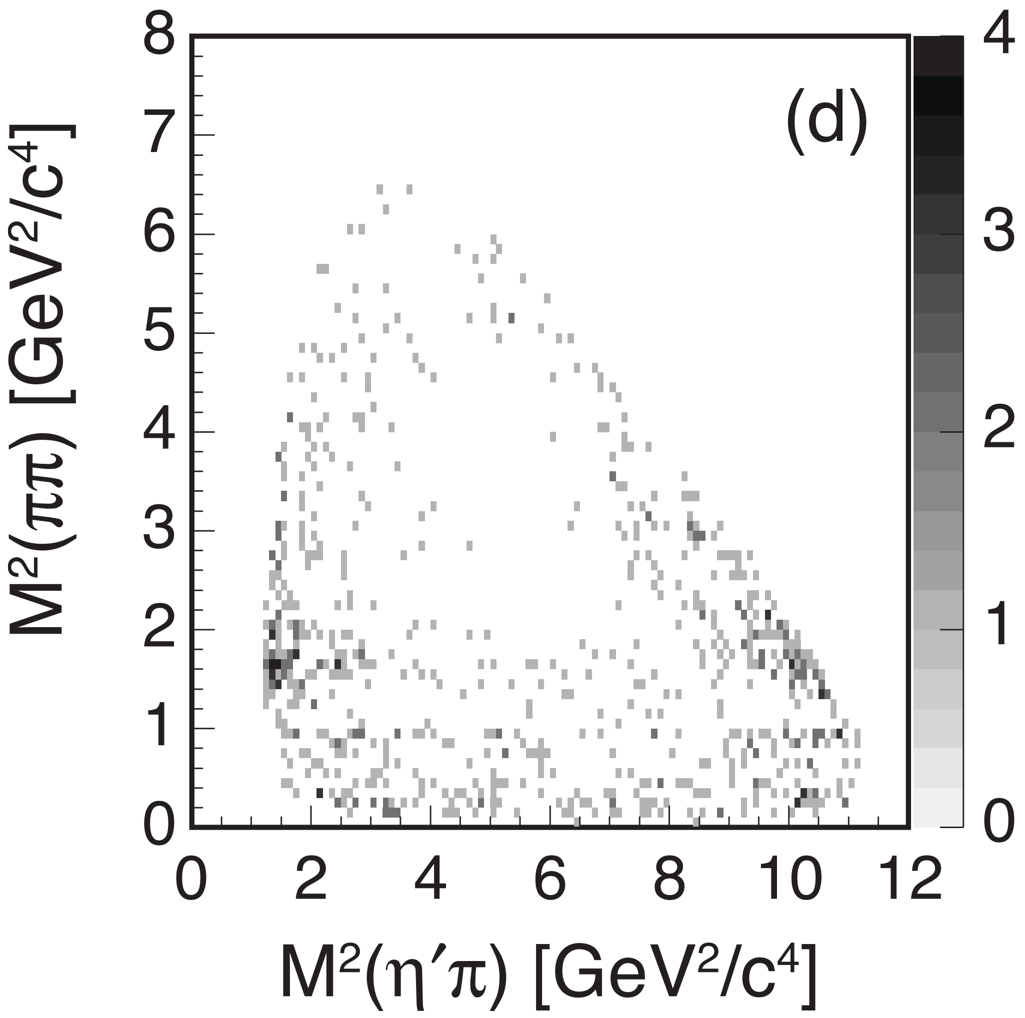


$\chi_{c1} \to \eta'\pi\pi$



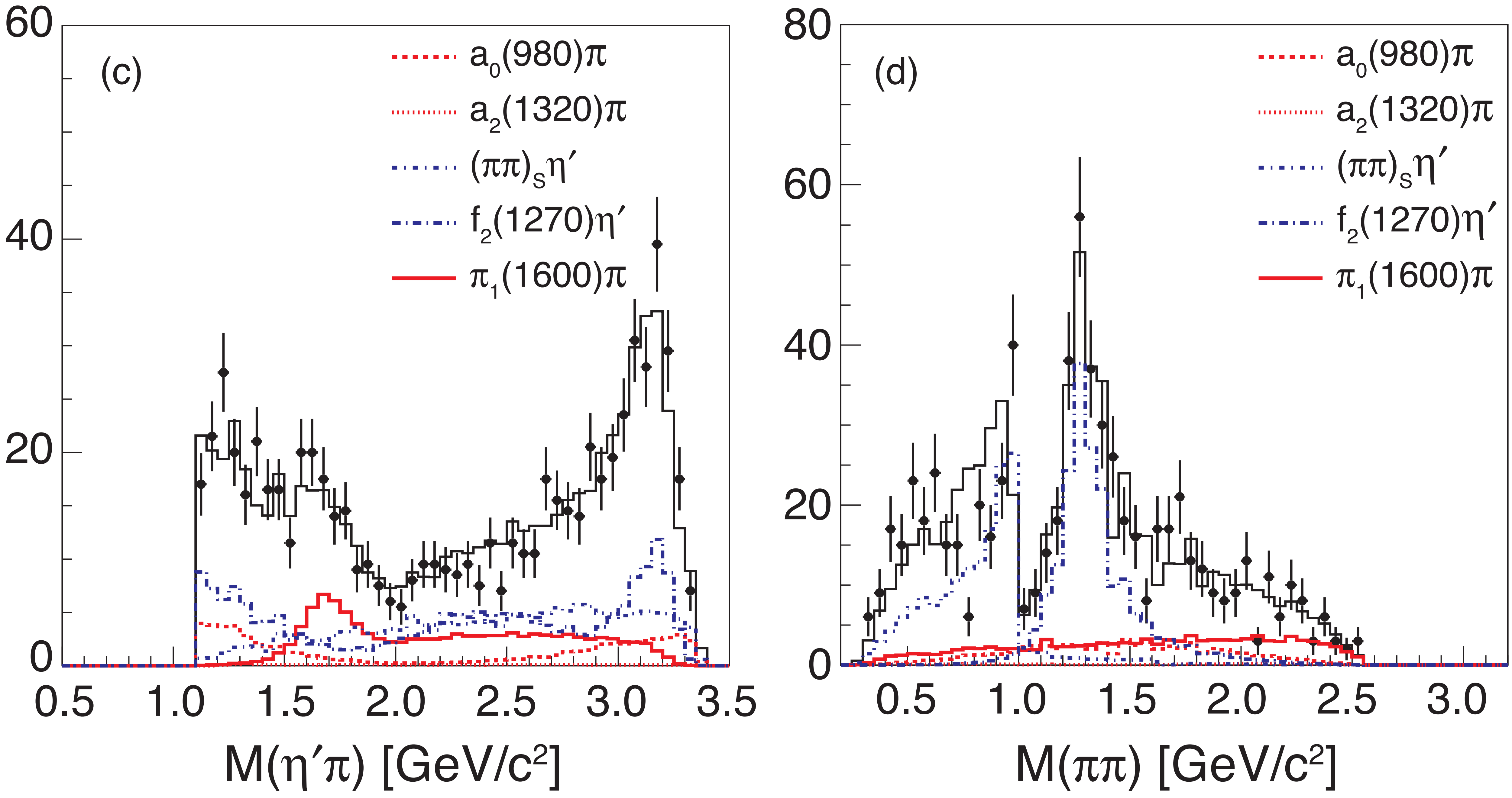
χ_{c1} Decay Mode	L	Isobar J^{PC}
$a_0\pi;\ a_0 \to \eta^{(\prime)}\pi$	P	0^{++}
$\pi_1\pi;\ \pi_1 \to \eta^{(\prime)}\pi$	S, D	1^{-+}
$a_2\pi;\ a_2 \to \eta^{(\prime)}\pi$	P, F	2^{++}
$a_4\pi;\ a_4 \to \eta^{(\prime)}\pi$	F, H	4^{++}
$f_0\eta^{(\prime)};\ f_0 \to \pi\pi$	P	0^{++}
$f_2\eta^{(\prime)};\ f_2 \to \pi\pi$	P, F	2^{++}
$f_4\eta^{(\prime)};\ f_4 \to \pi\pi$	F, H	4^{++}





CLEO Collaboration, PRD 84, 112009 (2011)

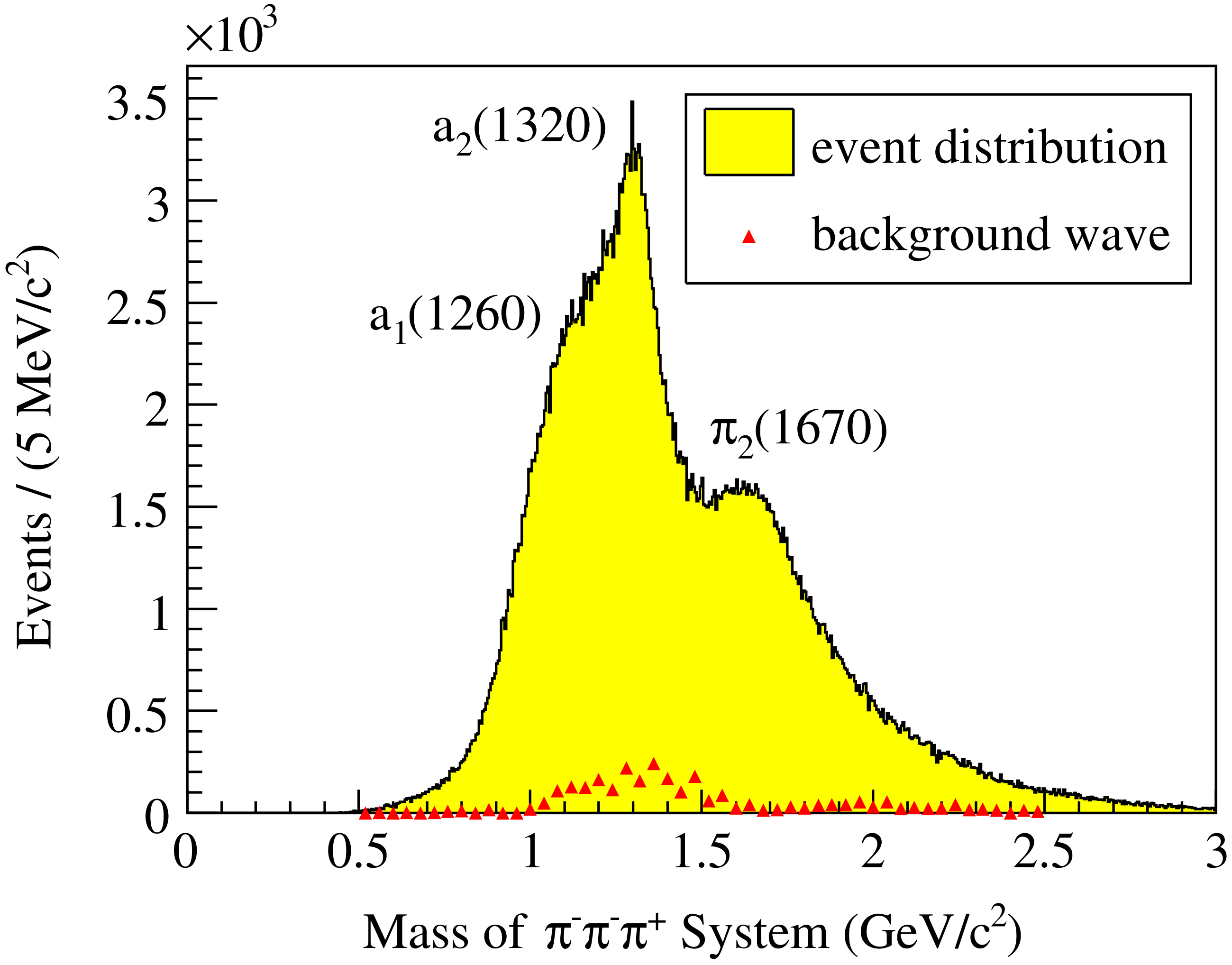
$\chi_{c1} \rightarrow \eta'\pi\pi$



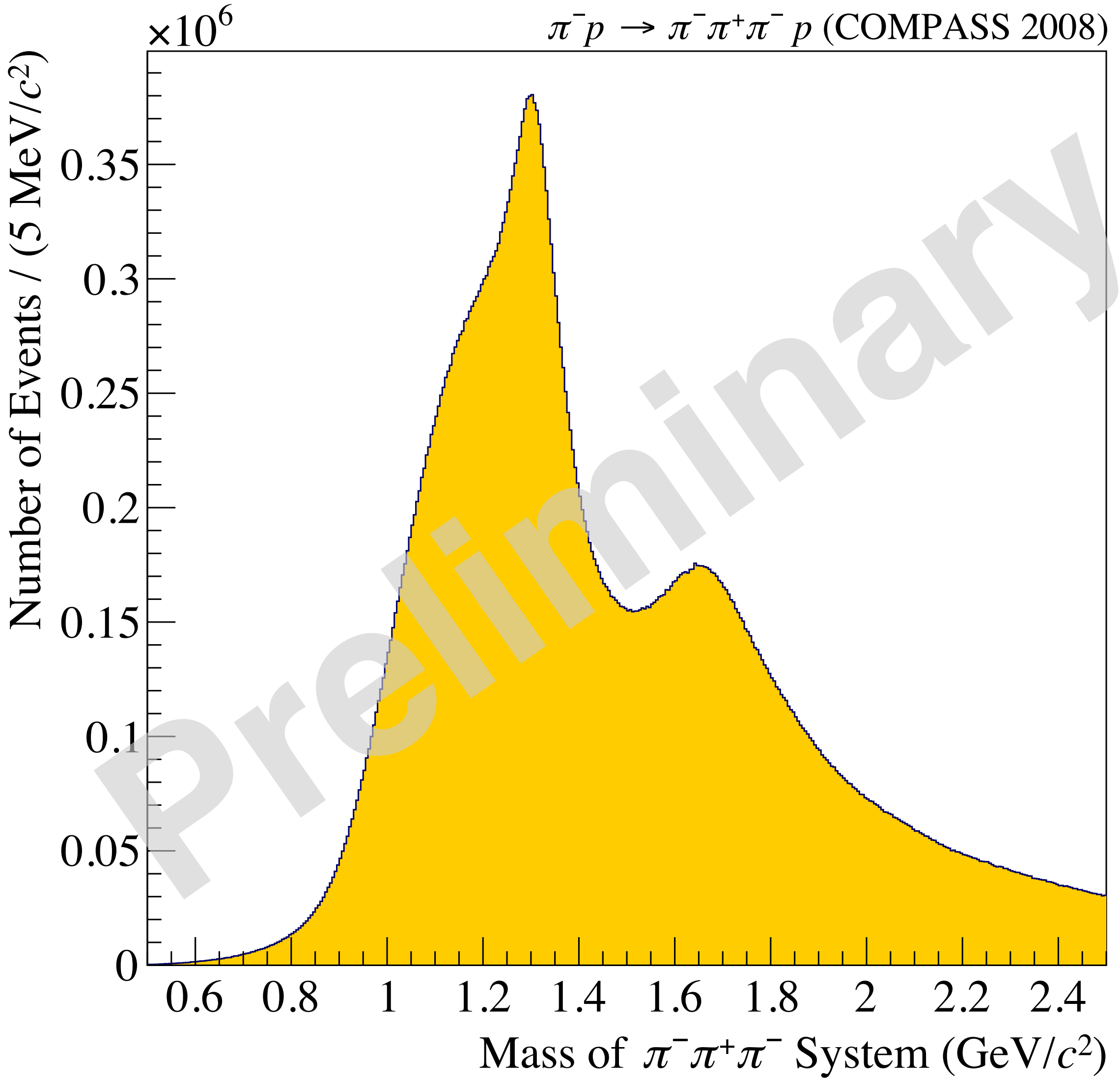
π beam data from COMPASS

$\pi^-\pi^-\pi^+$ from 190 GeV π on Pb

$\pi^-\pi^-\pi^+$ from 190 GeV π on p



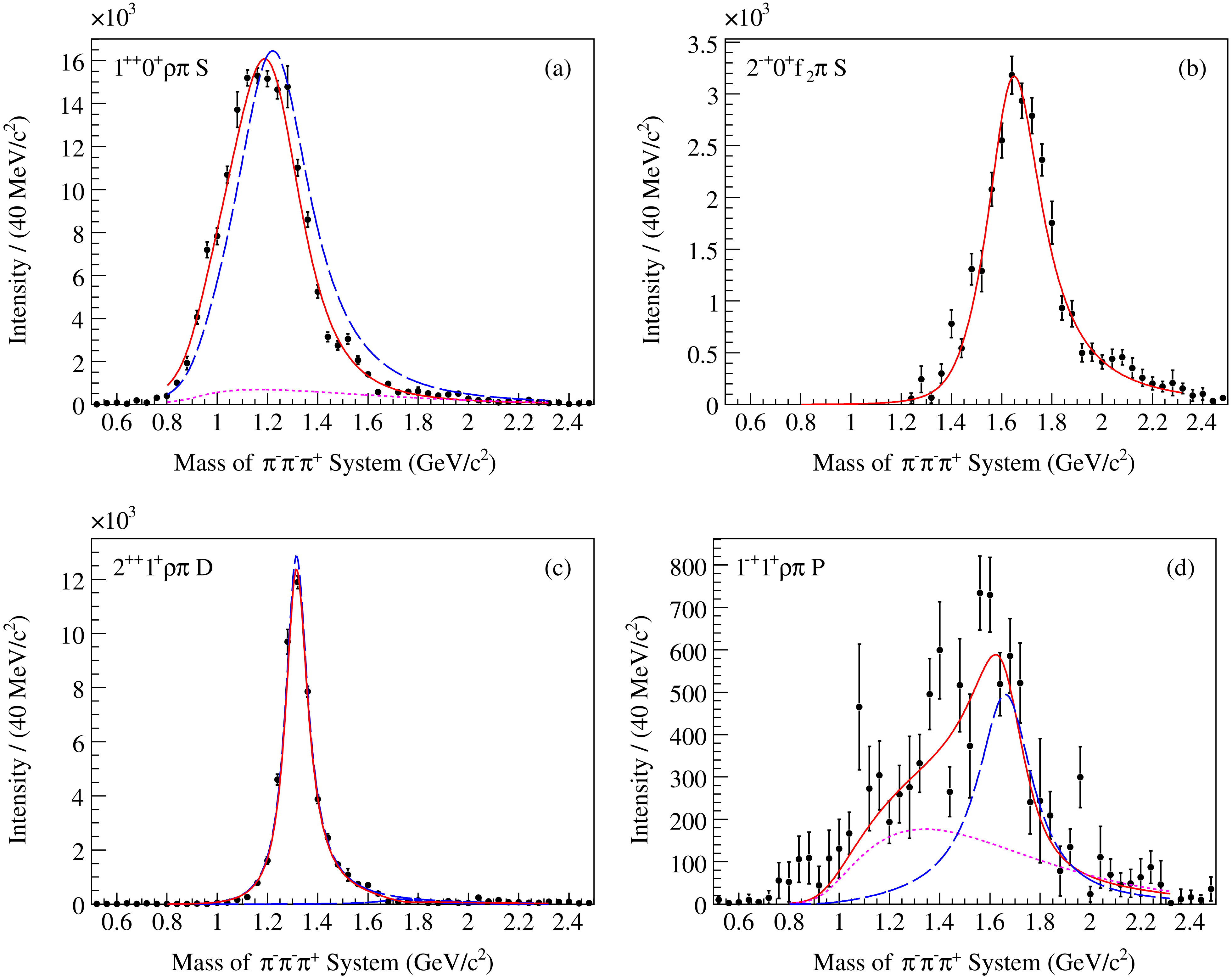
COMPASS Collab., PRL 104, 241803 (2010)



F. Krinner, POS (Bormio 2014), 031

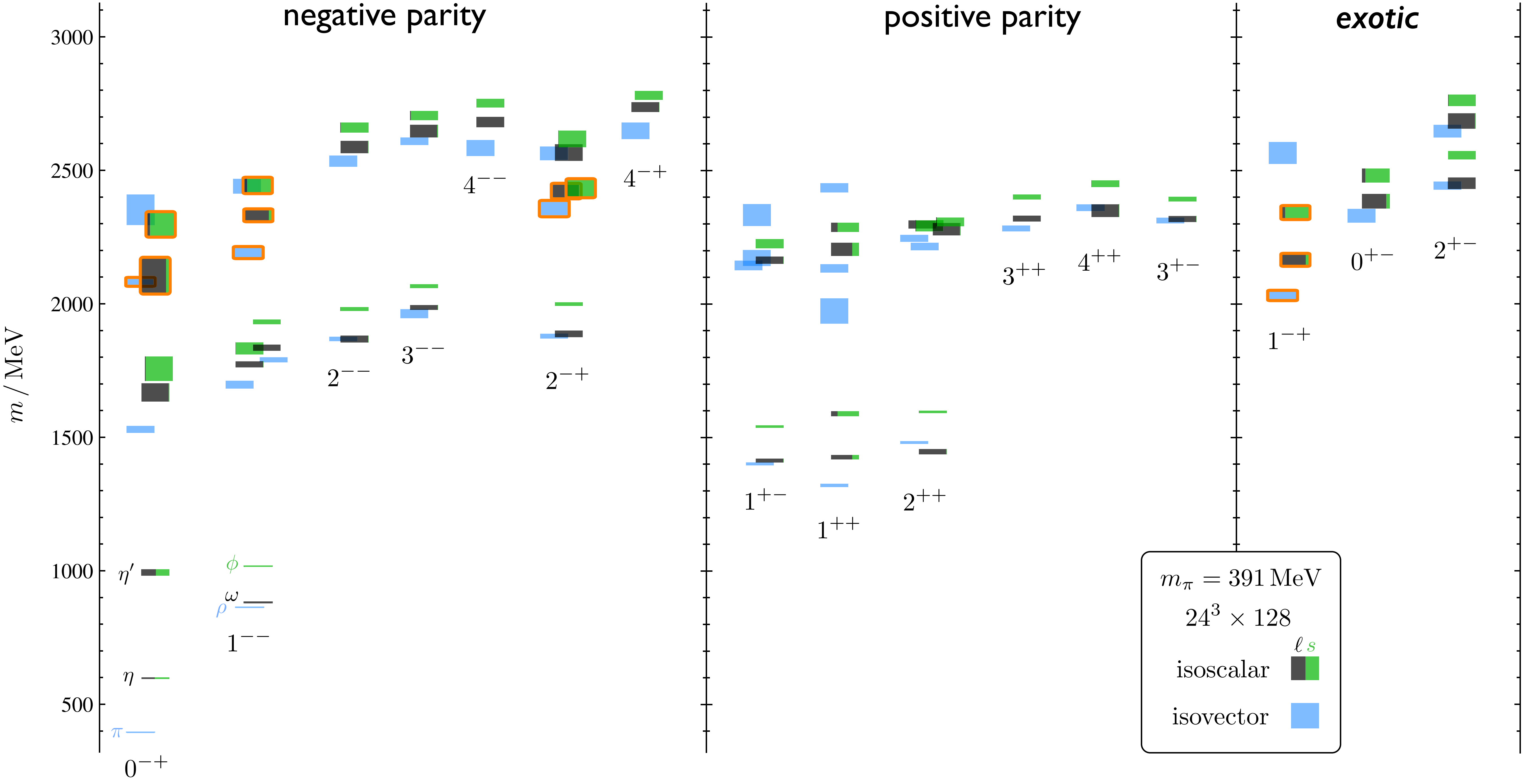
COMPASS Collab., PRL 104, 241803 (2010)

Key Amplitudes



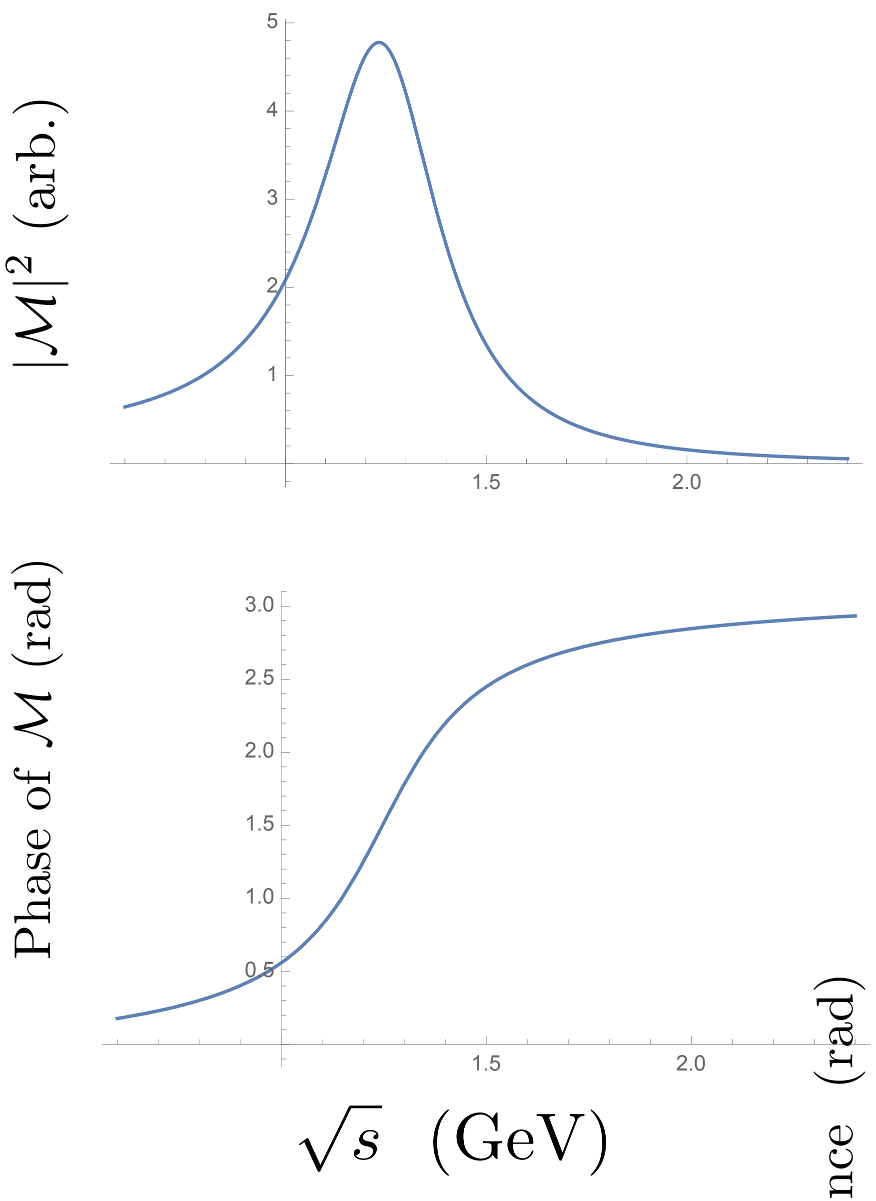
Light Quark Mesons from Lattice QCD

Dudek, Edwards, Guo, and Thomas, PRD 88, 094505 (2013)



Expectation

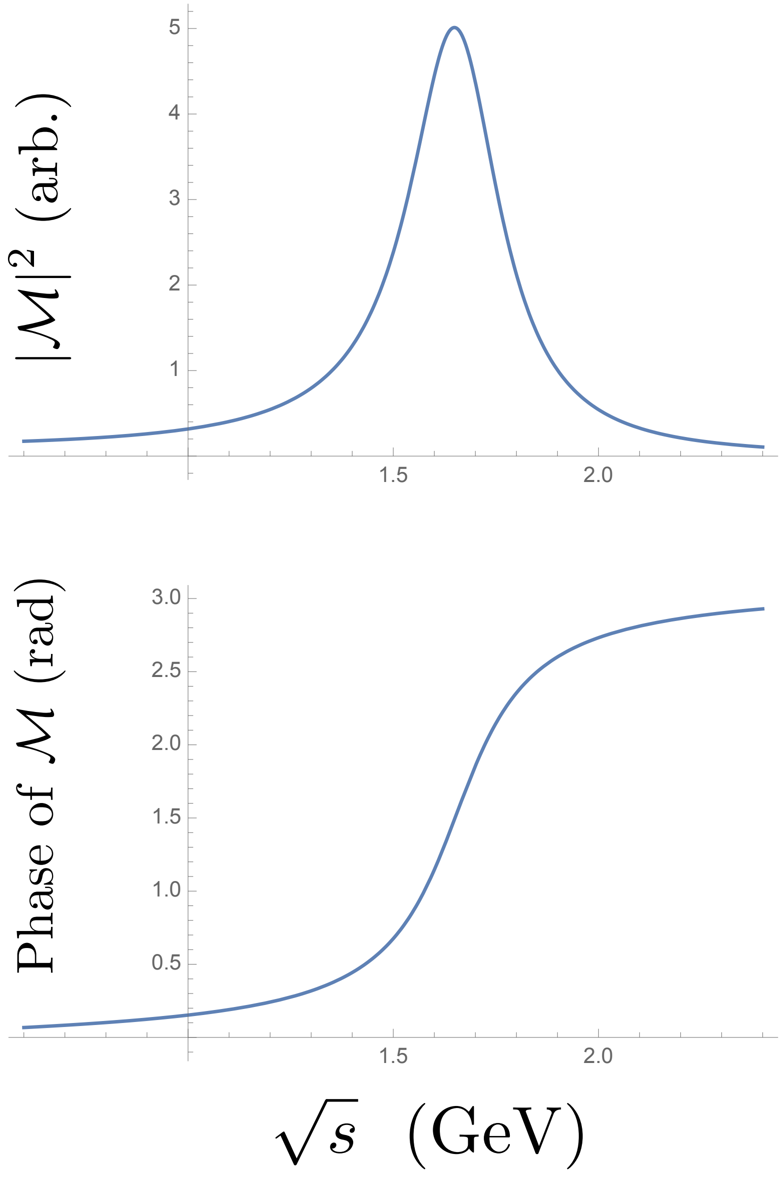
$M = 1.26\ \mathrm{GeV}$
$\Gamma = 0.37\ \mathrm{GeV}$

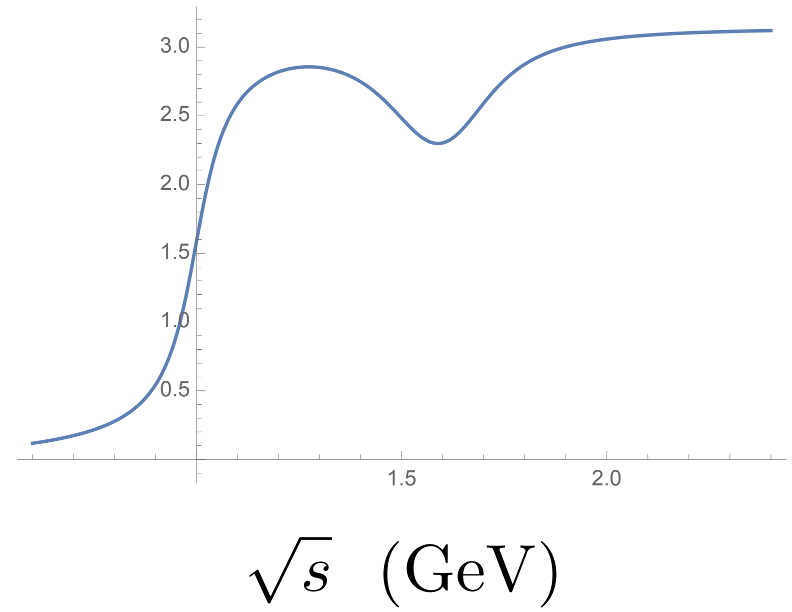


using:

$$\mathcal{M} \propto \frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

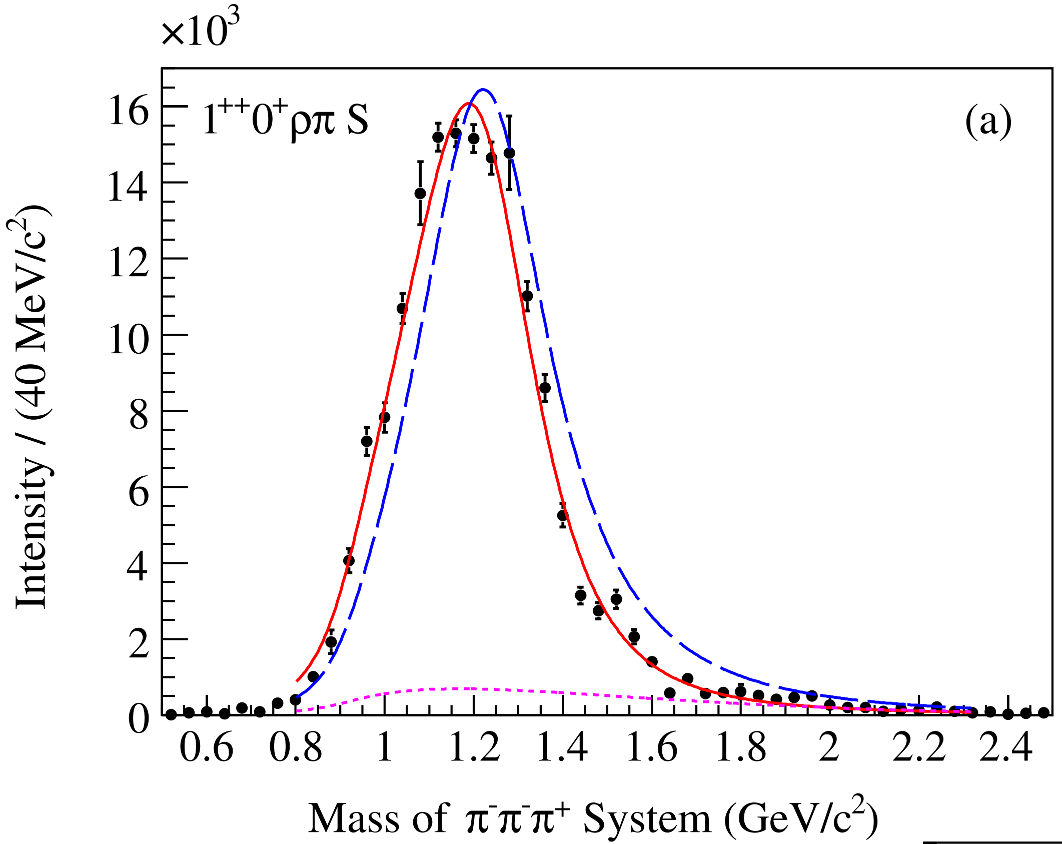
$M = 1.66\ \mathrm{GeV}$
$\Gamma = 0.27\ \mathrm{GeV}$

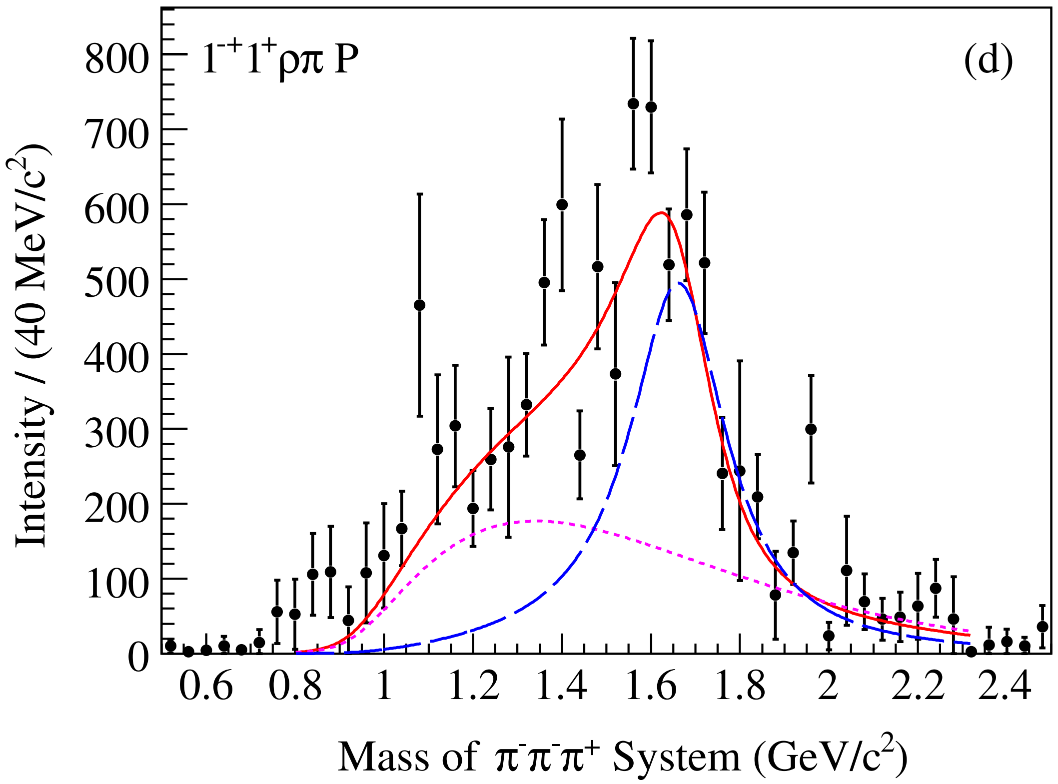


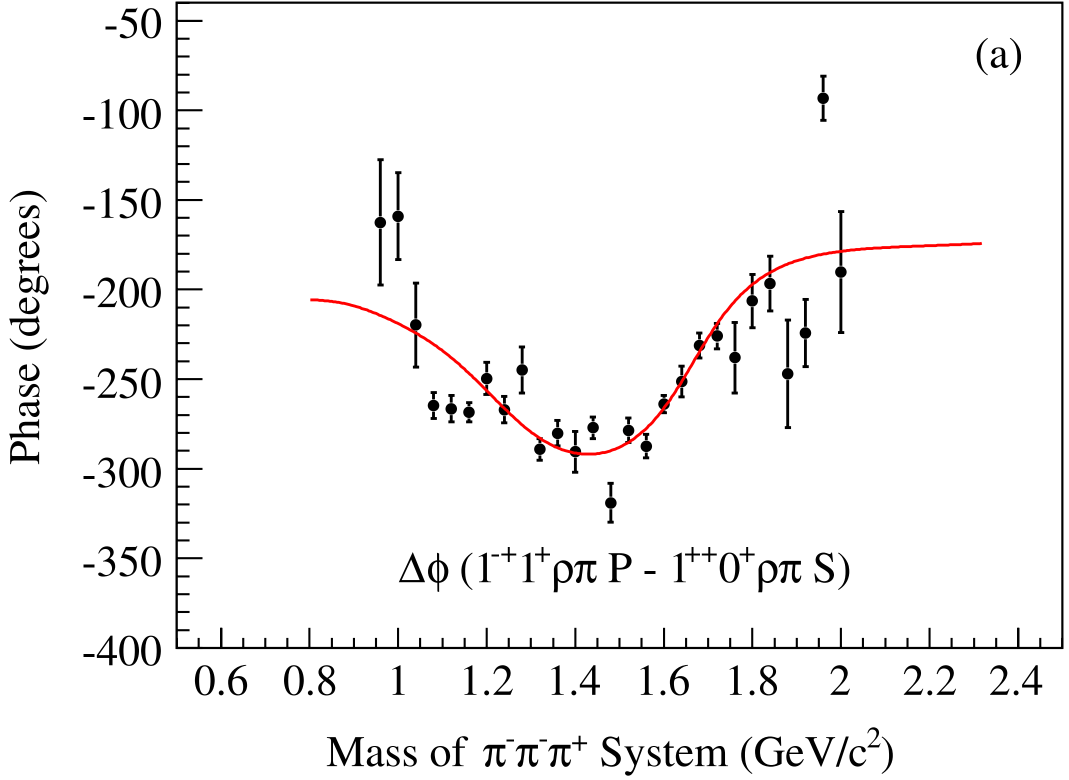


COMPASS Collab., PRL 104, 241803 (2010)

Interferometry







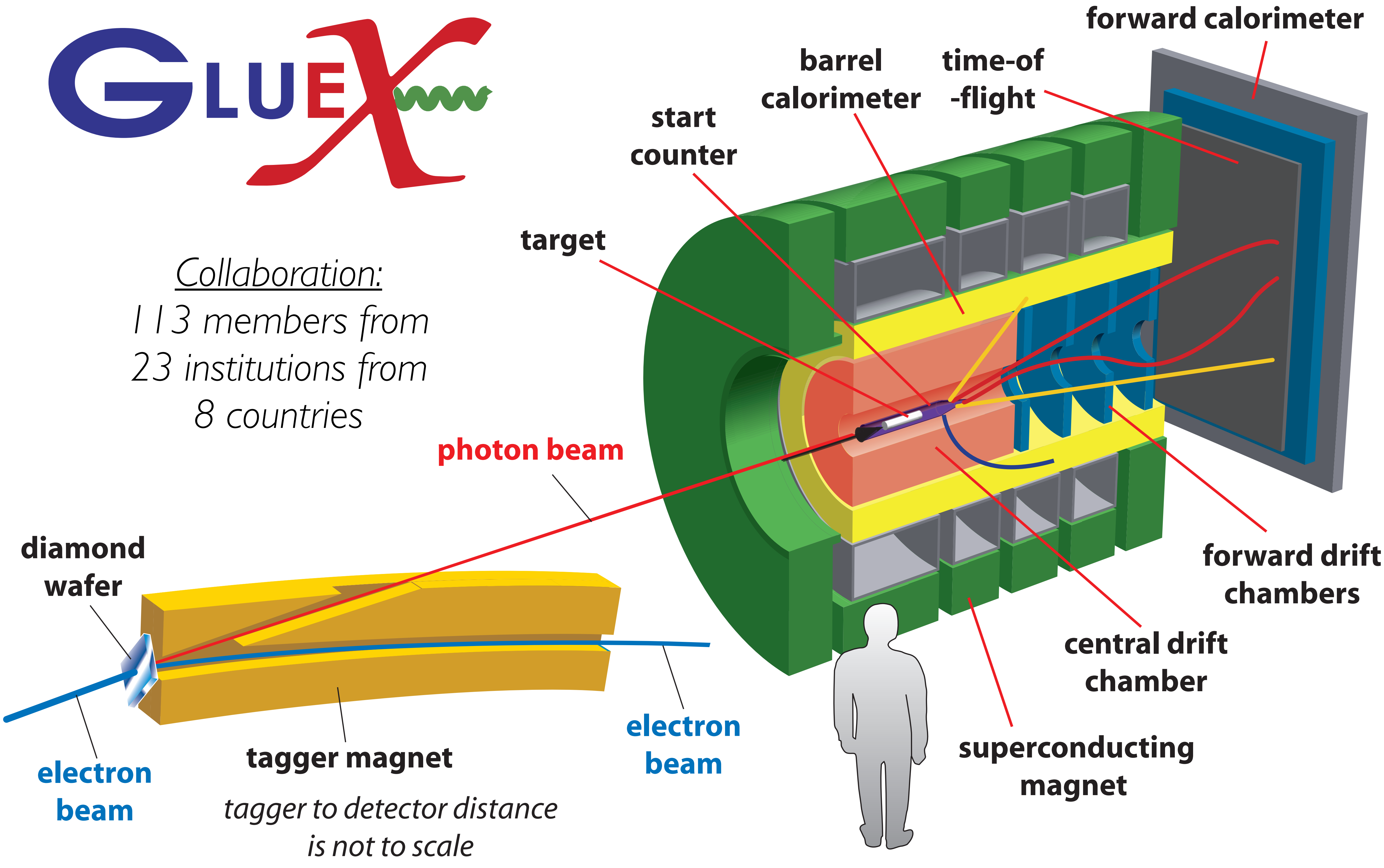
Comments

- Very interesting results to date in the search for exotic mesons
- Most studied is the lightest isovector hybrid, $J^{PC}=1^{-+}$ the π_1
- Much theoretical discussion about modeling production and final state interactions
  - Experimental data is now statistically precise enough and computational tools are now good enough to systematically explore models… and that is where the emphasis is now
- Need a spectrum of states to conclusively establish the existence of hybrids (exotic and non-exotic) - patterns are much more important than the idea of a single smoking gun

Meson Spectroscopy: A Global Approach

	hadron probes		electromagnetic probes	
	completed/analysis	ongoing/future	ongoing/future	completed/analysis
colliding beam	CDF II DØ	ATLAS CMS LHCb	BESIII Belle II	BABAR BELLE
fixed target	COMPASS	p̄anda	GLUEX CLAS12	clas CEBAF Large Acceptance Spectrometer

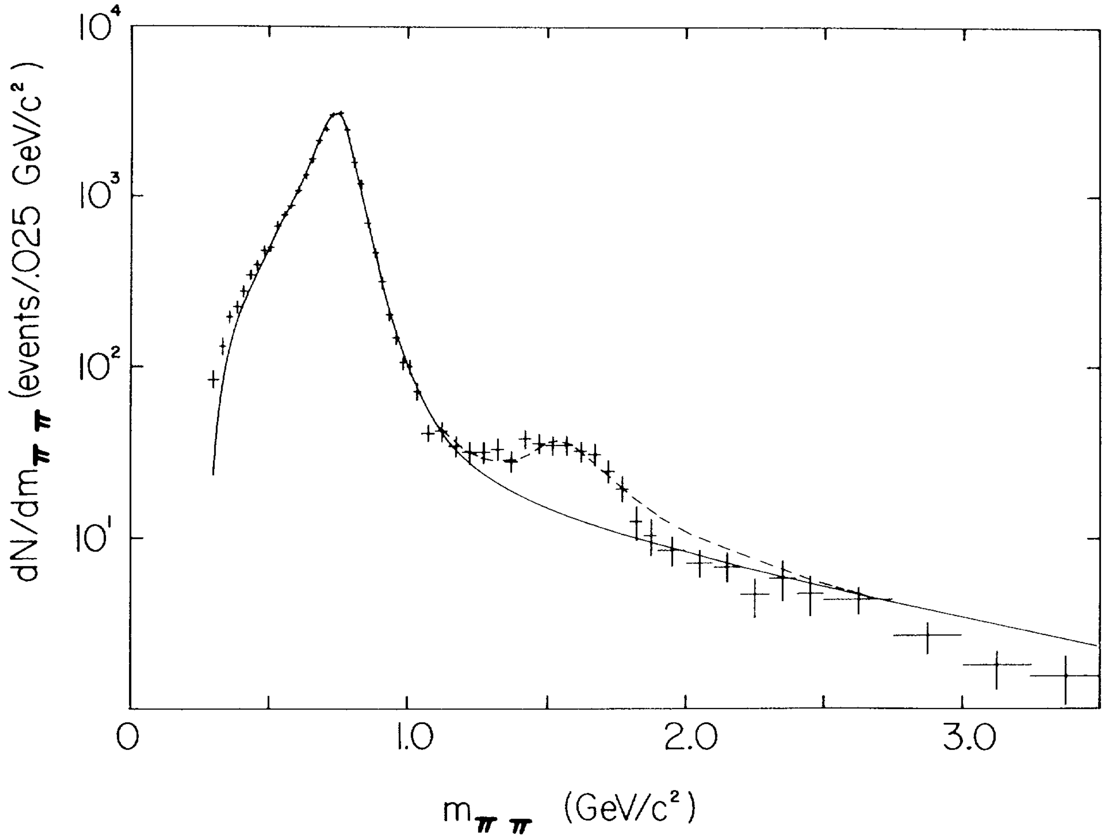
Collaboration:
113 members from
23 institutions from
8 countries



tagger to detector distance is not to scale

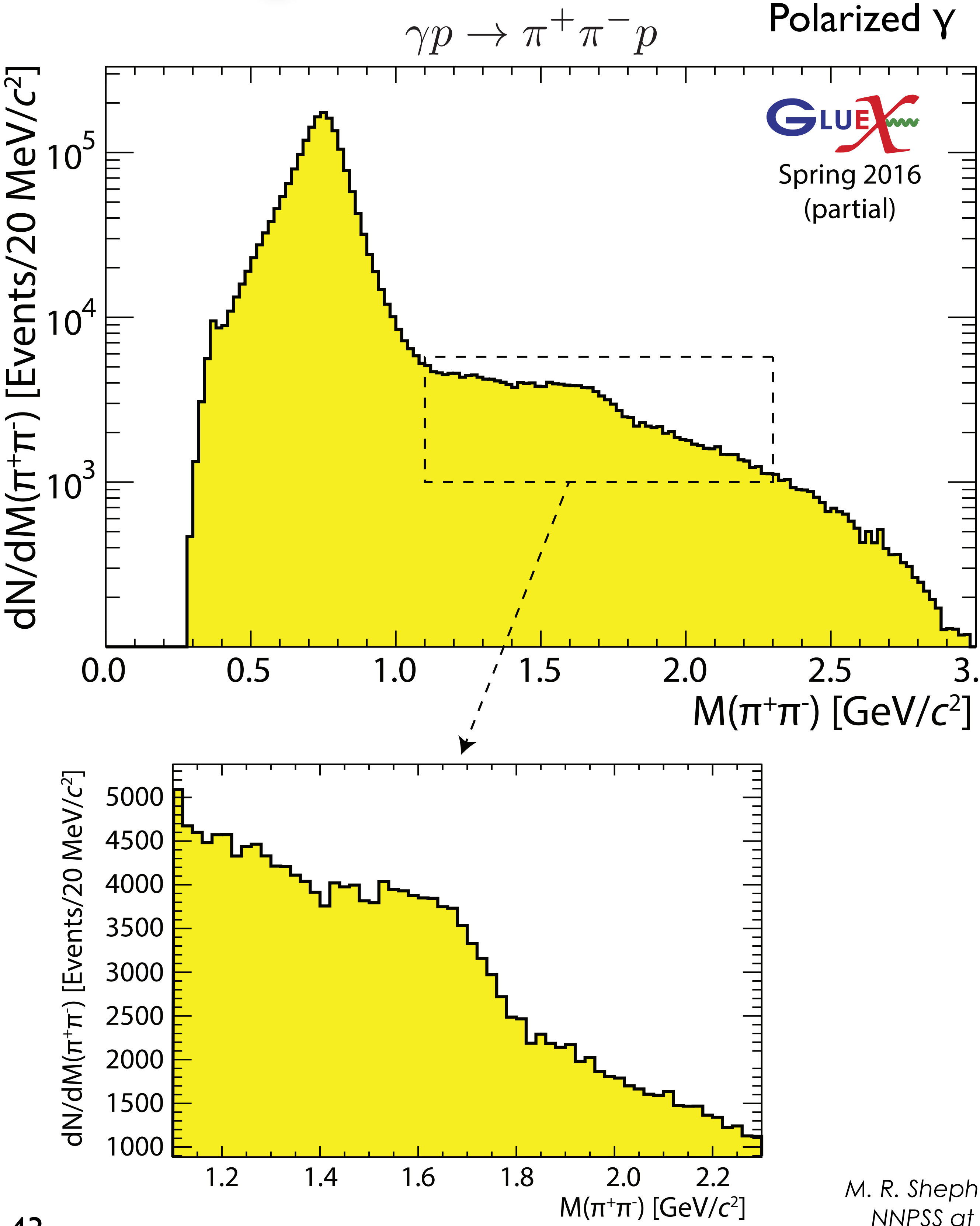
$\gamma p \rightarrow \pi^+\pi^- p$

SLAC Hybrid Facility Photon Collaboration
20 GeV Polarized γ



K. Abe *et al.*, PRL 53, 751 (1984)

8.4 - 9.0 GeV
Polarized γ



Final Thoughts

- Key issue (besides better quantifying our understanding of QCD): understand why certain types of hadrons seem to emerge much more often from QCD than others
- A very active field: unresolved puzzles in charmonium that didn't exist three years ago
- A bright outlook: continued study of the heavy quark spectrum and new experiments and data in the light quark sector
- An exciting time for graduate students to get involved in the field