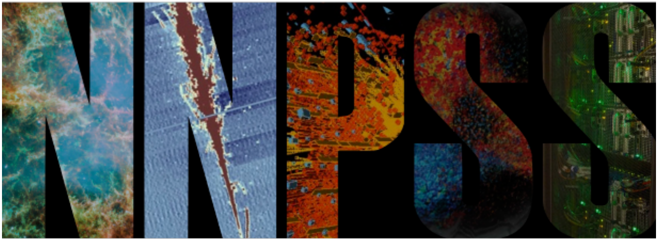
Nuclear structure III: Nuclear and neutron matter

Stefano Gandolfi

Los Alamos National Laboratory (LANL)



National Nuclear Physics Summer School
Massachusetts Institute of Technology (MIT)
July 18-29, 2016

Fundamental questions in nuclear physics

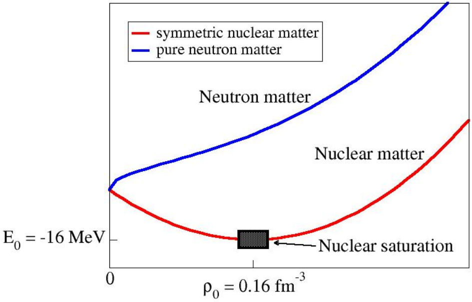
Physics of nuclei:

- How do nucleons interact?
- How are nuclei formed? How can their properties be so different for different A?
- What's the nature of closed shell numbers, and what's their evolution for neutron rich nuclei?
- What is the equation of state of dense matter?
- Can we describe simultaneously 2, 3, and many-body nuclei?

Uniform nuclear and neutron matter

What is **nuclear** matter? Easy, an infinite system of nucleons!

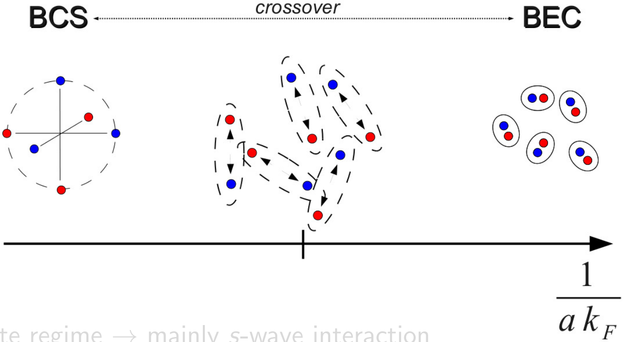
- Infinite systems
- Symmetric nuclear matter: equal protons and neutrons
- Pure neutron matter: only neutrons
- W/o Coulomb: homogeneous
- Nuclear matter saturates (heavy nuclei, "bulk")
- Neutron matter positive pressure
- Properties of infinite matter important to constrain energy density functionals



Low-density neutron matter
and **Cold atoms**

"Low-density" means $\rho << \rho_0$

Ultracold Fermi atoms



- Dilute regime → mainly *s*-wave interaction
- T fraction of T_F → $T \sim 0$
- Experimentally tunable interaction
- Crossover from weakly interacting Fermions (paired) to weakly repulsive Bosons (molecules)

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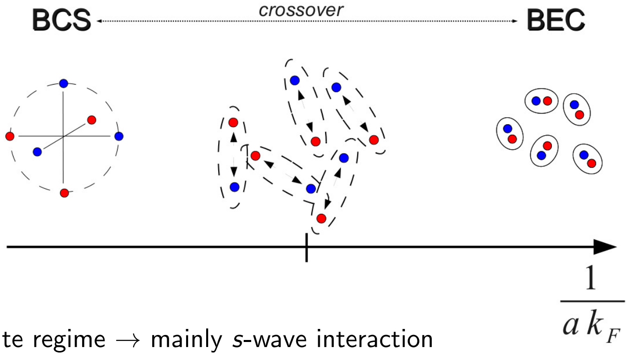

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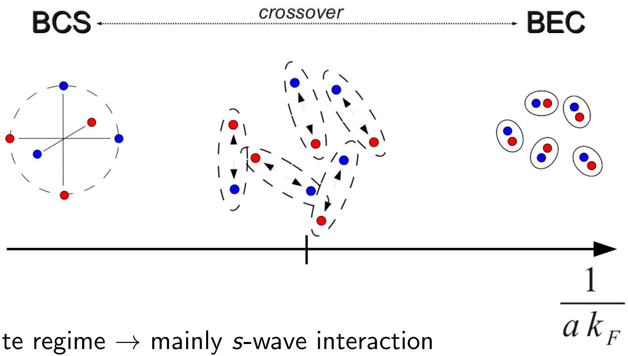

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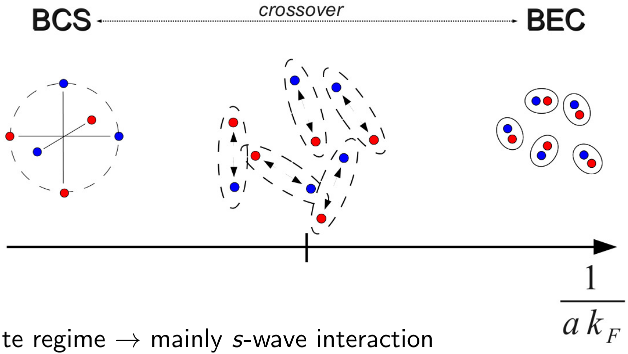

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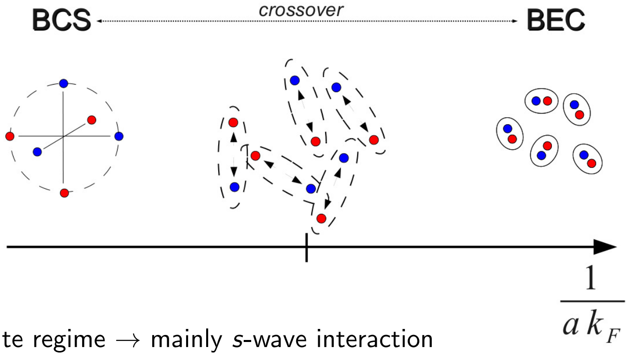

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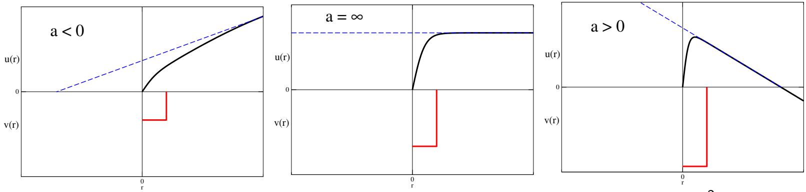
Scattering length

Two-body system with attractive interaction:



No bound states | Bound state with E_b=0 | $E_b \sim \frac{\hbar^2}{2m\,a^2}$

Very Low Density Neutron Matter: cold atoms

Low density neutron matter → unitary limit:

$$r_{eff} \ll r_0 \ll |a| \,, \quad r_{eff} = 0 \,, \; |a| = \infty$$

Only one scale: $\rightarrow E = \xi E_{FG}$

- NN scattering length is large and negative, $a = -18.5$ fm
- NN effective range is small, $r_{eff} = 2.7$ fm

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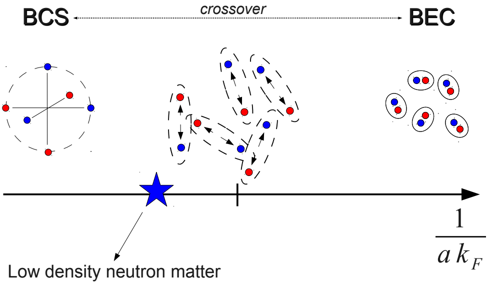
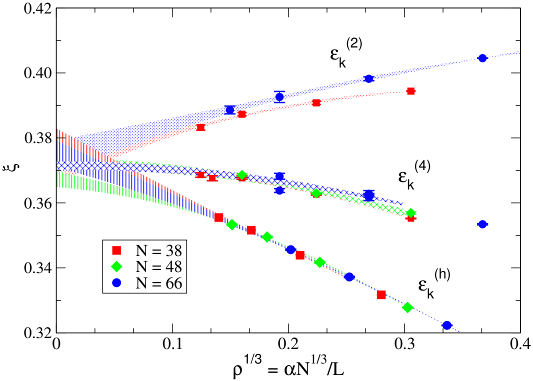
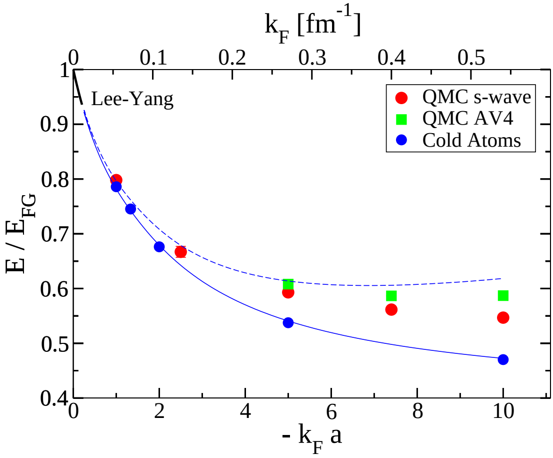


Exact calculation of ξ using AFMC:



$\xi = 0.372(5)$ Carlson, Gandolfi, Schmidt, Zhang, PRA 84, 061602 (2011)
$\xi = 0.376(5)$ Ku, Sommer, Cheuk, Zwierlein, Science 335, 563 (2012)

Validation of Quantum Monte Carlo calculations



Carlson, Gandolfi, Gezerlis, PTEP 01A209 (2012).

Ultracold atoms very useful for nuclear physics!

BCS and Fermi superfluidity

BCS (Bardeen-Cooper-Schrieffer) pairs: an arbitrarily small attraction between Fermions can cause a paired state of particles and the system has a lower energy than the normal gas.

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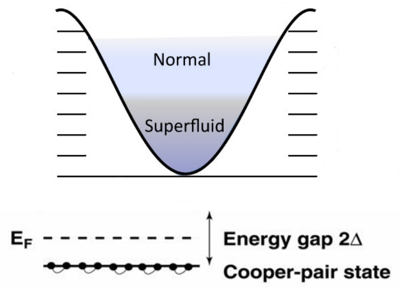
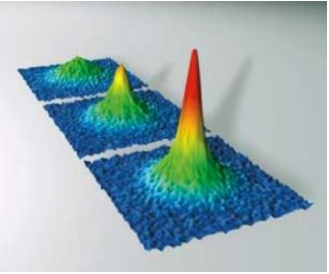
The (condensate) pairs may then form a superfluid:





The pairing gap is basically the energy needed to "break" a pair, and then excite the system to its normal energy.

Pairing gap and neutron stars

The pairing gap is fundamental for the cooling of neutron stars.

Neutron star crust made of nuclei arranged on a lattice surrounded by a gas of neutrons.

Specific heat suppressed by superfluidity (similarly to the superconducting mechanism).

Cooling dependent to the pairing gap!

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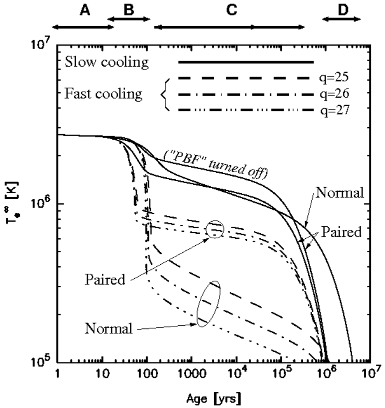
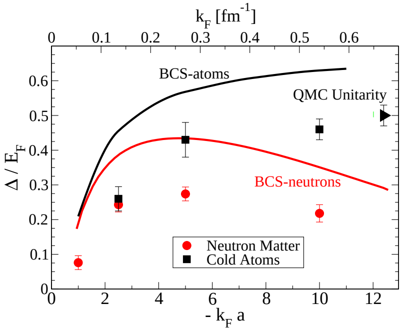


D. Page (2012)

Dilute neutron matter

The pairing gap is the energy cost to excite one particle from a BCS (collective) state.

Pairing gap of low–density neutron matter vs cold atoms:



Gezerlis, Carlson (2008)

Cold atoms results confirmed by experiments!

Dense neutron matter

"Dense" means $\rho \sim (0.5\text{—few times})\rho_0$

Fermi gas (1/2)

Non interacting two-components Fermi gas (non-relativistic):

$$\frac{E}{N}\left(k_F\right) = \frac{3}{5}\frac{\hbar^2}{2m}k_F^2 = E_{FG}\left(k_F\right) \ ,$$

where $k_F = (3\pi^2\rho)^{1/3}$.

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For a system made of neutrons and protons, define:

$$\rho = \rho_n + \rho_p \, , \qquad \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \, ,$$

Useful relations:

$$\rho_p = \frac{1-\alpha}{2}\rho \, , \qquad \rho_n = \frac{1+\alpha}{2}\rho \, ,$$

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Useful relations:

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The nuclear matter energy is given by:

$$\frac{E}{A}(\rho, x) = \frac{N}{A}\frac{E}{N}\left(k_F^{(n)}\right) + \frac{Z}{A}\frac{E}{Z}\left(k_F^{(p)}\right) = \frac{3}{5}\frac{\hbar^2}{2m}k_F^2\frac{1}{2}\left[(1+\alpha)^{5/3} + (1-\alpha)^{5/3}\right]$$
$$= f\left(\alpha\right)E_{FG}\left(k_F\right) \ .$$

Fermi gas (2/2)

For small asymmetries, $\alpha \approx 0$, the function $f(\alpha)$ can be expanded

$$f(\alpha) = 1 + \frac{5}{9}\alpha^2 + \frac{5}{243}\alpha^4 + \ldots\,,$$

And thus the equation of state is given by:

$$\frac{E}{A}(\rho, x) = \frac{3}{5}E_{FG}(\rho) + \frac{5}{9}E_{FG}(\rho)\alpha^2 + \cdots = E_{SNM} + \alpha^2 S(\rho) + \alpha^4 S_4(\rho) + \ldots\,,$$

where E_{SNM} is the energy of symmetric nuclear matter ($\alpha = 0$) and $S(\rho)$ is the symmetry energy given by

$$S(\rho) = \frac{1}{2}\frac{\partial^2}{\partial\alpha^2}\left[E(\rho, x)\right]_{\alpha=0} \simeq E_{PNM}(\rho) - E_{SNM}(\rho)\,,$$

and E_{PNM} is the energy of pure neutron matter ($\alpha = 1$).

Symmetry energy

Around density ρ_0 nuclear matter saturates, thus

$$\frac{\partial E_{SNM}(\rho)}{\partial \rho} = 0\Big|_{\rho=\rho_0}$$

and we can expand as

$$E_{SNM} = E_0 + \alpha \left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \beta \left(\frac{\rho - \rho_0}{\rho_0}\right)^3 + \dots ,$$

Pure neutron matter instead does not saturate, thus also linear power in ρ is fine.

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Then, around ρ_0 we can expand:

$$E_{sym} = S_0 + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \dots ,$$

where L is the slope of the symmetry energy, and K_{sym} is the symmetry compressibility.

Neutron matter equation of state

Why do we care?

- EOS of neutron matter gives the symmetry energy and its slope.
- Assume that NN is very good - fit scattering data with very high precision.
  Three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments.

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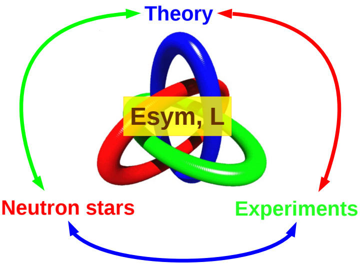

Neutron matter at nuclear densities

At nuclear densities neutron matter cannot be modeled as in the dilute regime. Nucleon-nucleon (and three-nucleon) interaction become very important.

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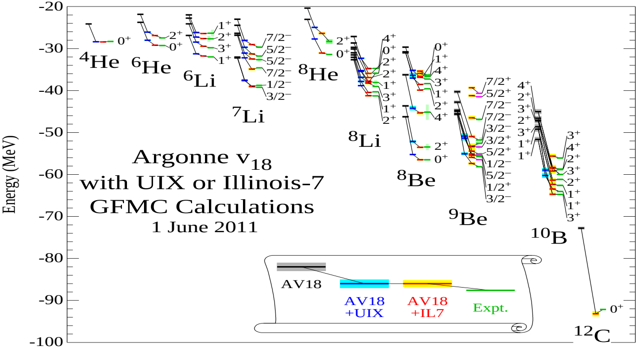
Let's start from Hamiltonians used for nuclei:



Carlson, *et al.*, RMP (2015)

Scattering data and neutron matter

How much can we trust the nucleon-nucleon interactions?

In a scattering event with energy E_{lab} two nucleons have

$$k \approx \sqrt{E_{lab}\, m/2}\,, \qquad \rightarrow k_F$$

that correspond to

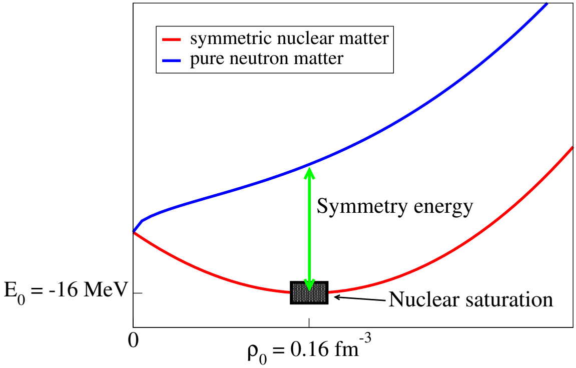
$$k_F \rightarrow \rho \approx \frac{(E_{lab}\, m/2)^{3/2}}{2\pi^2}\,.$$

E_{lab}=150 MeV corresponds to about 0.12 fm^{-3}.

E_{lab}=350 MeV to 0.44 fm^{-3}.

Argonne potentials useful to study dense matter above ρ_0=0.16 fm^{-3}, other (soft) interactions not clear

What is the Symmetry energy?



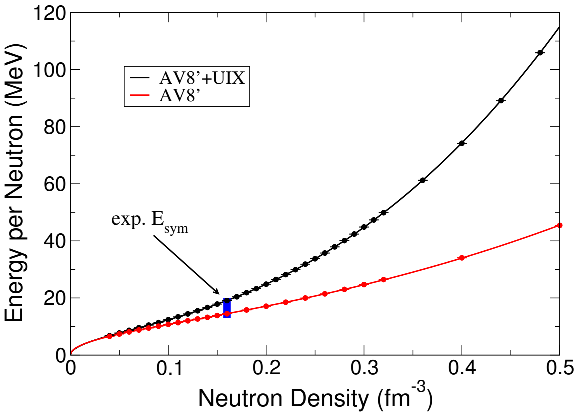
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV\,, \quad \rho_0 = 0.16 fm^{-3}\,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM

Neutron matter

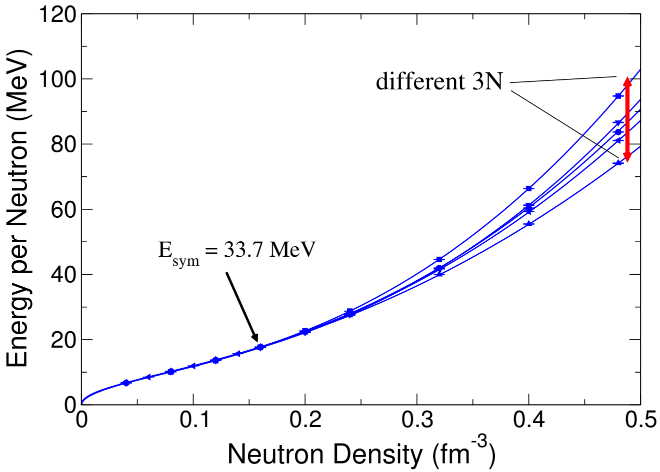
Equation of state of neutron matter using the AV8′+UIX Hamiltonian.



Incidentally these can be considered as "extremes" with respect to the measured E_{sym}.

Neutron matter

Three-neutron interaction uncertainty:



Neutron matter

Main sources of uncertainties:

- Experimental: $E_{\rm sym}$
- Theoretical: form of three-neutron interaction not totally understood

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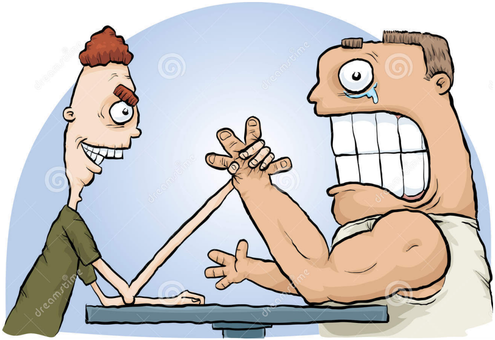
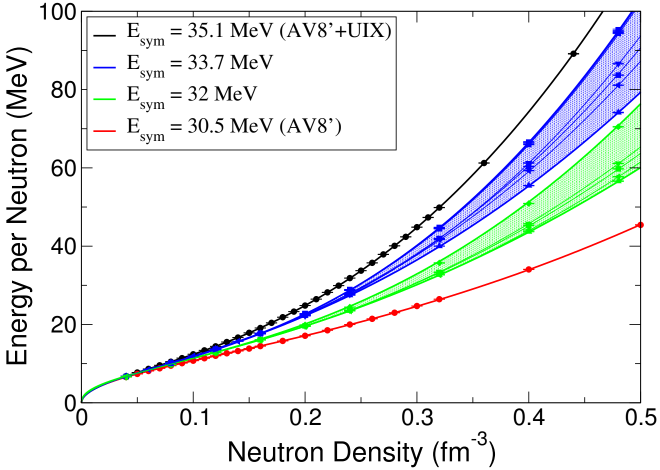


Which one dominates?

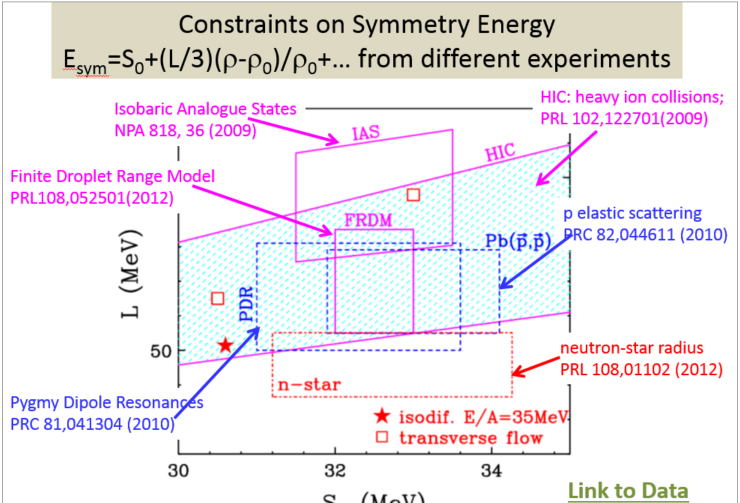
Neutron matter

Equation of state of neutron matter using Argonne forces:



Gandolfi, Carlson, Reddy, PRC (2012)

Many experimental efforts to measure E_{sym} (or S_0) and its slope L:

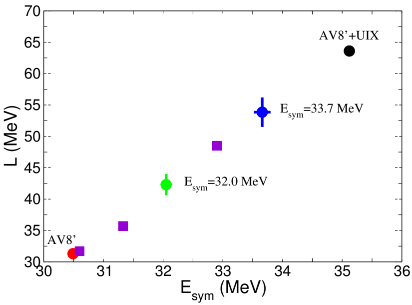


$$E_{sym} = S_o + \frac{L}{3}\left(\frac{\rho_B - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho_B - \rho_0}{\rho_0}\right)^2 + \ldots$$

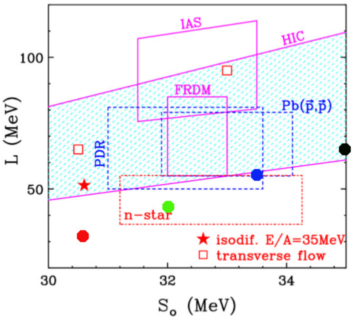
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



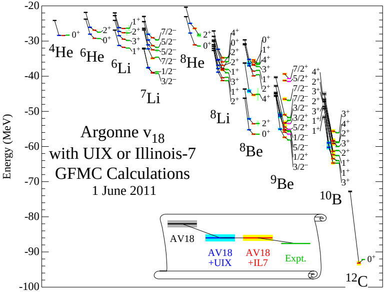
Gandolfi *et al.*, EPJ (2014)

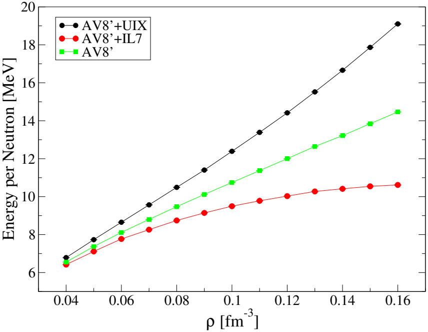


Tsang *et al.*, PRC (2012)

Very weak dependence to the model of 3N force for a given E_{sym}.
Knowing E_{sym} or L useful to constrain 3N! (within this model...)

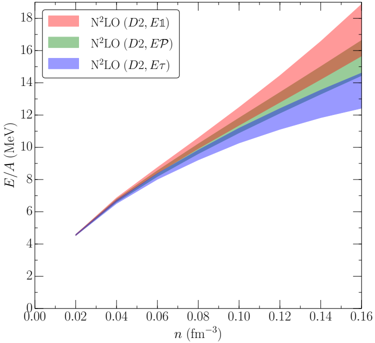
Neutron matter and the "puzzle" of the three-body force

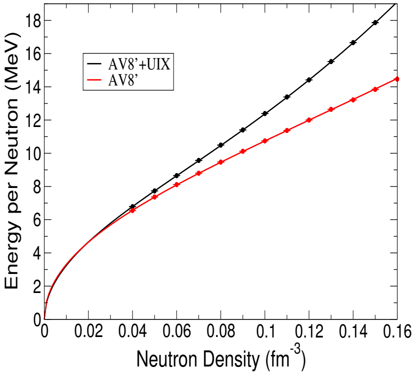




Note: AV8'+UIX and (almost) AV8' are stiff enough to support observed neutron stars, but AV8'+IL7 too soft. → How to reconcile with nuclei???

EOS of pure neutron matter at N2LO, R_0=1.0 fm.





Lynn, *et al.*, PRL (2016).

Note: **the above** (but not all) chiral Hamiltonian able to describe A=3,4,5 nuclei and neutron matter *reasonably*.

Summary of this lecture:

- Low-density neutron matter and cold atoms
- Superfluidity
- Dense matter: free Fermi gas
- Dense matter with interaction

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End for today...