# Neutrino Theory

1] Introduction ; $\nu$ masses

2] $\nu$ oscillations

3] $\nu$ interactions

## Main phylosophy:

- build on basic knowledge (QFT, Standard Model)
- provide tools for self study
- omitted : review material that can be easily be learned independently

## References:

- E. Akhmedov, lecture notes hep-ph/0001264
- Kim & Giunti , textbook, Oxford
- Fukugita & Yanagida , textbook, ed. Springer
- Misc. books by G. Raffelt, J. Bahcall, k. Zuber, ....

# Conventions, etc.

natural units: $c = \hbar = 1$

$1$ = ONE (not SEVEN)

$7$ = SEVEN (not ONE)

$n$ = cursive N

$m$ = cursive M

▶ = work out the steps (suggested exercise)

# Lecture 1 – Introduction, $\nu$ masses

1) Intro: a) Sources of $\nu$

b) Status and prospects of $\nu$ physics

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2) Massive $\nu$

- $\nu$ in S.M.

- Dirac $\nu$, Majorana $\nu$, See-Saw mechanism

- The $\nu$ mixing matrix

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# Introduction : $\nu$ sources

$$e^{\overset{(+)}{-}} + (A,Z) \longrightarrow (A, Z \overset{(+)}{-} 1) + \overset{(-)}{\nu_e}$$

$$(A,Z) \longrightarrow (A, Z \overset{(+)}{-} 1) + \overset{(-)}{\nu_e} + e^{\overset{(-)}{+}}$$

Sun, stars

supernovae

Early universe

Natural radioactivity

reactors

β-beams

$$\gamma^* \longrightarrow \nu \bar{\nu}$$

Early universe

stars, supernovae

accretion disks

$$p + p \longrightarrow \pi^+ + p + n$$

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

accelerators

astrophysical jets

cosmic rays,

atmosphere

# $\nu$ physics today: status

- Discovery of $\nu$ flavor oscillations
  - $\nu$ are massive,
  - mass-flavor mixing, flavor non-conservation

- Parameter measurement:

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \qquad \text{(mass}^2\text{ splittings)}$$

$$U_{\alpha j} \quad \begin{array}{l} \alpha = e, \mu, \tau \\ j = 1, 2, 3 \end{array} \qquad \text{(flavor-mass mixing)}$$

## The flavor - mass mixing :

$$U = U(\vartheta_{12}, \vartheta_{23}, \vartheta_{13}, \delta, \varphi_1, \varphi_2)$$

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix}.$$

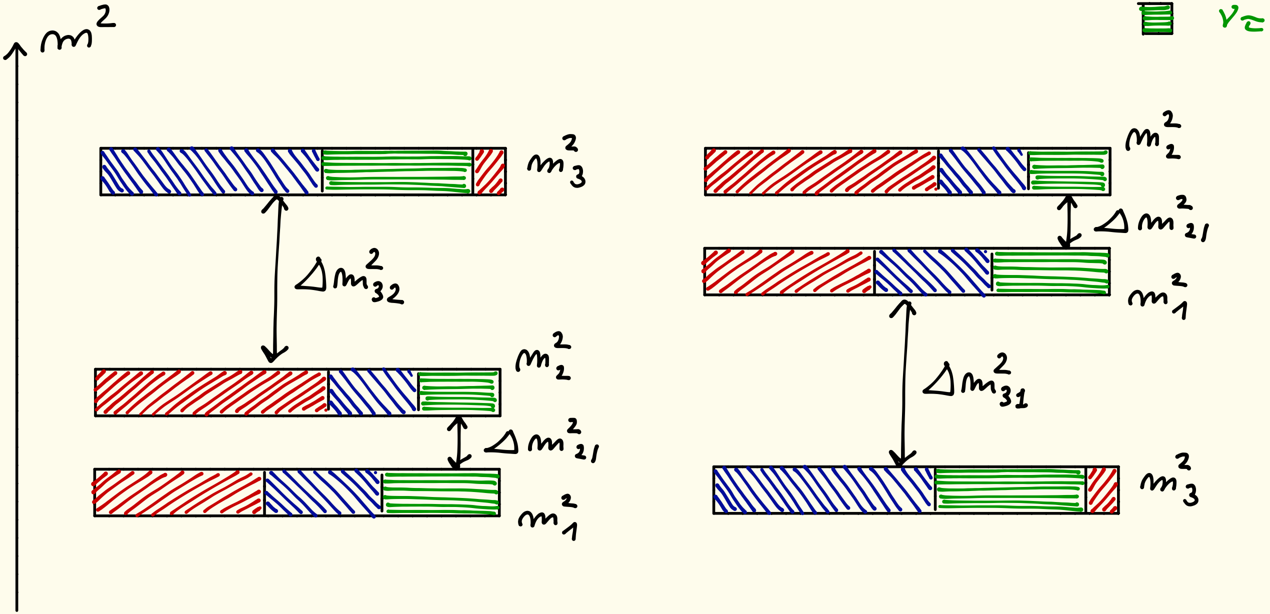
( Majorana phases , $\varphi_1, \varphi_2 = 0$ here)

$c_{ij} = \cos\vartheta_{ij}$ ,

$s_{ij} = \sin\vartheta_{ij}$

# The mass spectrum: possibilities

🟥 $\nu_e$
🟦 $\nu_\mu$
🟩 $\nu_\tau$

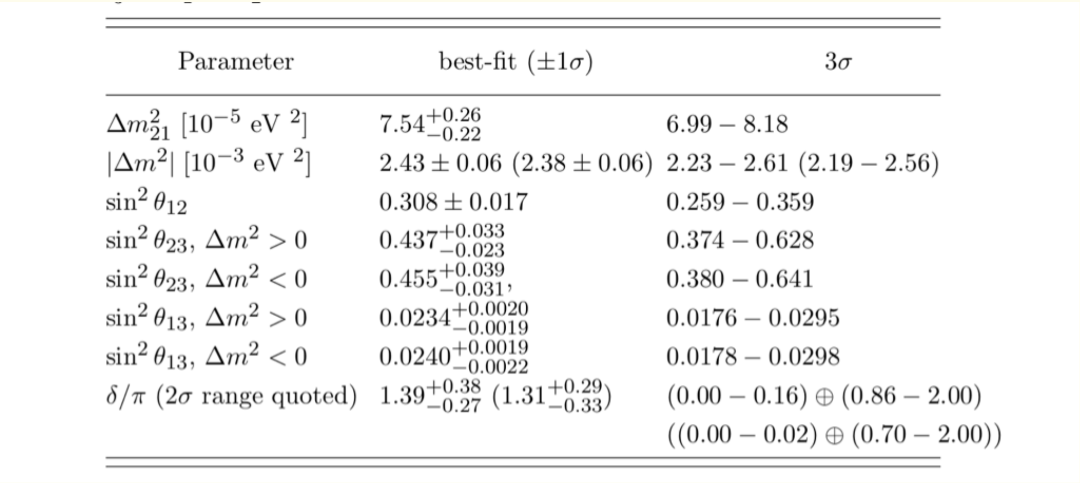


Normal hierarchy ← OR → Inverted hierarchy

$m_1 < m_2 < m_3$ ? $m_1 < m_2 < m_3$

Measurements / constraints :

(from PDG)



| Parameter | best-fit ($\pm 1\sigma$) | $3\sigma$ |
|---|---|---|
| $\Delta m^2_{21}$ $[10^{-5}$ eV $^2]$ | $7.54^{+0.26}_{-0.22}$ | $6.99 - 8.18$ |
| $\lvert\Delta m^2\rvert$ $[10^{-3}$ eV $^2]$ | $2.43 \pm 0.06$ $(2.38 \pm 0.06)$ | $2.23 - 2.61$ $(2.19 - 2.56)$ |
| $\sin^2\theta_{12}$ | $0.308 \pm 0.017$ | $0.259 - 0.359$ |
| $\sin^2\theta_{23}$, $\Delta m^2 > 0$ | $0.437^{+0.033}_{-0.023}$ | $0.374 - 0.628$ |
| $\sin^2\theta_{23}$, $\Delta m^2 < 0$ | $0.455^{+0.039}_{-0.031}$, | $0.380 - 0.641$ |
| $\sin^2\theta_{13}$, $\Delta m^2 > 0$ | $0.0234^{+0.0020}_{-0.0019}$ | $0.0176 - 0.0295$ |
| $\sin^2\theta_{13}$, $\Delta m^2 < 0$ | $0.0240^{+0.0019}_{-0.0022}$ | $0.0178 - 0.0298$ |
| $\delta/\pi$ ($2\sigma$ range quoted) | $1.39^{+0.38}_{-0.27}$ $(1.31^{+0.29}_{-0.33})$ | $(0.00 - 0.16) \oplus (0.86 - 2.00)$ |
| | | $((0.00 - 0.02) \oplus (0.70 - 2.00))$ |

$$\Delta m^2 = (\Delta m^2_{31} + \Delta m^2_{32})/2 \quad , \quad \Delta m^2 \begin{cases} > 0 & \text{normal hierarchy} \\ < 0 & \text{inverted hierarchy} \end{cases}$$

$$\begin{cases} \theta_{12}\ ,\ \theta_{23} & \text{"large"} \quad (\sim \pi/4) \\ \theta_{13} & \text{"small"} \quad (\ll \pi/4) \end{cases}$$

# Next goals:

- mass hierarchy
- $\delta$ (Phase of $U_{e3}$) → CP symmetry violation?
- $\nu$ mass ($m_\nu \lesssim eV$)
- Dirac or Majorana (are neutrinos their own antiparticle?)
- $\nu$ and physics beyond SM
- $\nu$ in cosmology
- $\nu$ astronomy

# Massive $\nu$: spin 1/2 fermion fields

- recall : Dirac equation

$$\boxed{(i\partial\!\!\!/ - m)\psi = 0}$$

$\psi$ 4-component vector, 4 degrees of freedom

- recall: chirality and chiral fields:

$$P_L = \frac{1-\gamma^5}{2} \quad , \quad P_R = \frac{1+\gamma^5}{2}$$

$$P_L\psi = \psi_L \quad , \quad P_R\psi = \psi_R \quad \longrightarrow \quad \boxed{\psi = \psi_L + \psi_R}$$

# $\nu$ in the Standard Model

- brief recap: SM lepton content, $SU(2)_L$ structure

| | $I$ | $I_3$ | $Y$ | $Q$ |
|---|---|---|---|---|
| $L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ | $1/2$ | $1/2$<br>$-1/2$ | $-1$ | $0$<br>$-1$ |
| $e_R$ | $0$ | $0$ | $-2$ | $-1$ |

$$Q = I_3 + Y/2$$

# The SM: leptonic weak charged current

- Basis:

$$\ell'_L = \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \ell'_R = \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \nu'_L = \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$$

quantum numbers as in table, but not definite mass

- Weak charged current:

$$\boxed{j^{\rho}_{W,L} = 2\, \overline{\nu}'_L \gamma^{\rho} \ell'_L} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}'_{\alpha L} \gamma^{\rho} \ell'_{\alpha L}$$

(Notation: $\ell'_{eL} = e'_L$, etc.)

# Mass terms in the SM:

generic:

$$\mathcal{L}_m \propto m\bar{\psi}\psi = m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R)$$

$$= m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

# Higgs-lepton Yukawa couplings: $(\alpha, \beta = e, \mu, \tau)$

$$\mathcal{L}_{H,L} = - \sum_{\alpha,\beta} Y'^{\ell}_{\alpha\beta} \, \overline{L}'_{\alpha L} \, \phi \, \ell'_{\beta R} + h.c.$$

Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$(v = \langle \phi^0 \rangle)$$

In matrix form:

$$\boxed{\mathcal{L}_{H,L} = - \frac{v+H}{\sqrt{2}} \left[ \overline{\ell'_L} \, Y'^{\ell} \, \ell'_R \right] + h.c.}$$

↳ no $\nu$ mass term! (No $\nu_R$)

# Extending the SM: $\nu$ masses and mixing

## 1) Dirac $\nu$:

- add $\nu_R$ singlet ⟶ $\bar{\nu}_L \nu_R$ terms
- price: unnaturally small Yukawa coupling ($m_\nu \lesssim 10^{-6}\, m_e$)

$$\mathcal{L}_{H,L} = -\frac{v+H}{\sqrt{2}} \left[ \overline{\ell'_L}\, Y'^{\ell}\, \ell'_R + \overline{\nu'_L}\, Y'^{\nu}\, \nu'_R \right] + h.c.$$

$$j^{\rho}_{W,L} = 2\, \overline{\nu'_L}\, \gamma^{\rho}\, \ell'_L \quad (\text{unchanged})$$

## Find mass eigenstates : bi-unitary transformations

$$V_L^{\nu\dagger} Y'^{\nu} V_R^{\nu} = Y^{\nu} \text{ (diagonal)} \qquad V_L^{\ell\dagger} Y'^{\ell} V_R^{\ell} = Y^{\ell} \text{ (diagonal)}$$

$$n_L = V_L^{\nu\dagger} \nu_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} \qquad \ell_L = V_L^{\ell\dagger} \ell_L' = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

$$n_R = V_R^{\nu\dagger} \nu_R' = \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} \qquad \ell_R = V_R^{\ell\dagger} \ell_R' = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$\hookrightarrow \quad \mathcal{L}_{L,H} = -\frac{v+H}{\sqrt{2}} \left[ \bar{\ell}_L Y^{\ell} \ell_R + \bar{n}_L Y^{\nu} n_R \right] + h.c.$$

# Mass eigenstates: Dirac fermions

$$\mathcal{L}_{L,H} = -\frac{v+H}{\sqrt{2}}\left[\bar{\ell}_L Y^{\ell} \ell_R + \bar{n}_L Y^{\nu} n_R\right] + h.c.$$

$$= \cdots\cdots - \frac{v}{\sqrt{2}}\left(\bar{n}_L + \bar{n}_R\right) Y^{\nu} \left(n_L + n_R\right) =$$

$$= \cdots\cdots - \sum_{j=1}^{3} \underbrace{\frac{v}{\sqrt{2}} Y^{\nu}_{jj}}_{m_j} \underbrace{\left(\bar{\nu}_{jL} + \bar{\nu}_{jR}\right)}_{\bar{\nu}_j} \underbrace{\left(\nu_{jL} + \nu_{jR}\right)}_{\nu_j}$$

Dirac fermion: 4 degrees of freedom (L and R independent)

# Weak current in mass basis : $\nu$ flavor definition

$$j^{\rho}_{W,L} = 2\,\bar{\nu}'_L\,\gamma^{\rho}\,\ell'_L = 2\,\bar{m}_L\,V_L^{\nu +}\,\gamma^{\rho}\,V_L^{\ell}\,\ell_L = 2\,\underbrace{\bar{m}_L\,V_L^{\nu +}\,V_L^{\ell}}_{\equiv\,\bar{\nu}_L}\,\gamma^{\rho}\,\ell_L$$

$$\equiv 2\,\bar{\nu}_L\,\gamma^{\rho}\,\ell_L$$

→ definition of $\nu$ flavor states:

$$\boxed{\nu_L \equiv V_L^{\ell +}\,V_L^{\nu}\,m_L = U\,m_L}$$

$$\boxed{U = V_L^{\ell +}\,V_L^{\nu}}$$

# The flavor - mass mixing matrix

$$\boxed{\quad \nu_L \equiv V_L^{\ell +} V_L^{\nu}\, n_L = U\, n_L \quad}$$

$$\boxed{\quad U = V_L^{\ell +} V_L^{\nu} \quad}$$

- $U$ depends only on "left" matrices, $V_L^{\ell}$, $V_L^{\nu}$

  ↳ due to chiral structure of weak interaction !

- charged leptons $l_{\alpha L}$ are <u>defined</u> as mass eigenstates (mass is measured directly)

  ↳ $\nu$ flavor state $\nu_{\alpha L}$ is defined as coupling only to $l_{\alpha L}$ lepton

## Parameterization (L subscript omitted)

$$U : \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Three angles, 1 measurable phase (Dirac) : $\theta_{12}, \theta_{13}, \theta_{23}, \delta$

Useful : $U = V_{23} W_{13} V_{12}$

$$V_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad W_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \qquad V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{13} \end{pmatrix}$$

$$c_{ij} = \cos \theta_{ij} \qquad s_{ij} = \sin \theta_{ij}$$

Full expression: MNS (or PMNS) matrix:

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix} .$$

PMNS = Pontecorvo, Maki, Nakgawa, Sakata

## 2) Majorana $\nu$: the Majorana mass term

### Introduction to Majorana fermions:

- Recall particle - antiparticle conjugation:

$$C = i\gamma^2\gamma^0$$

$$\underset{\substack{\uparrow \\ \text{particle}}}{\psi} \longrightarrow \underset{\substack{\uparrow \\ \text{antiparticle}}}{\psi^c} = C\,\overline{\psi}^T = i\gamma^2\psi^* .$$

- Useful properties:

▶ $C^\dagger = C^T = C^{-1} = -C$ , $C\gamma^\mu C^{-1} = -\gamma^{\mu T}$

▶ $(\psi^c)^c = \psi$ $\overline{\psi^c} = \psi^T C$ , $\bar{\psi}_1 \psi_2^c = \overline{\psi_2^c}\,\psi_1$ , $\bar{\psi}_1 A \psi_2 = \overline{\psi_2^c}\,(C A^T C^{-1})\,\psi_1^c$

Note: $\psi \longrightarrow \psi^c$ flips all charges and chirality:

▶ $(\psi_L)^c \equiv \psi_R^c$ , $(\psi_R)^c \equiv \psi_L^c$

## Majorana fermion:

$$\psi = \chi_L + \eta (\chi_L)^c = \chi_L + \eta \chi_R^c$$

$\eta = e^{i\varphi}$

(generic phase)

## Properties:

- only one independent chiral field : 2 degrees of freedom

- particle = antiparticle (up to a phase) : $\psi^c = \eta^* \psi$

$\downarrow$

must be neutral : $\nu$ only candidate in SM !

## Majorana mass term vs. Dirac:

$$\mathcal{L}_M \propto m\,\overline{(\nu_L)^c}\,\nu_L + h.c. \qquad\qquad \mathcal{L}_D \propto m\,\underbrace{\overline{\nu}_L\,\nu_R}_{\text{independent}} + h.c.$$

$$= m\,\underbrace{\overline{\nu^c_R}\,\nu_L}_{\text{not independent}} + h.c. = m\,\nu_L^T C^\dagger \nu_L + h.c.$$

- $\mathcal{L}_M$ violates $U(1)$ symmetries by 2 units

  ↳ lepton number violation!

$$\nu_L \longrightarrow e^{i\varphi}\nu_L \quad : \quad \mathcal{L}_M \longrightarrow e^{i2\varphi}\mathcal{L}_M$$

# Majorana masses and new physics

- a $\overline{\nu_L}(\nu_L)^c$ term not possible in SM

- can arise from dimension $d \geq 5$ operators from new physics at high energy scale

- New physics can also give heavy $\nu_R$

# Dirac + Majorana lagrangian:

- $\nu_L$ , $\nu_R$ , all possible mass terms. For 1 generation:

$$\mathcal{L}_{M+D} = \frac{1}{2}\underbrace{m_L \nu_L^T C^\dagger \nu_L}_{\text{Majorana}} - \underbrace{m_D \bar{\nu}_R \nu_L}_{\text{Dirac}} + \underbrace{\frac{1}{2} m_R \nu_R^T C^\dagger \nu_R}_{\text{Majorana}} + h.c.$$

## Rewrite in matrix form:

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}$$

▶ $$\boxed{\mathcal{L}_{M+D} = \frac{1}{2} n_L^T C^\dagger M n_L + h.c.}$$

$$\boxed{M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}}$$ (real matrix)

## Diagonalize M :

$$\begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}$$

$$\boxed{\begin{cases} \chi_{1L} = \cos\vartheta\, \nu_L - \sin\vartheta\, \nu_L^c \\ \chi_{2L} = \sin\vartheta\, \nu_L + \cos\vartheta\, \nu_L^c \end{cases}}$$

$$\boxed{\tan 2\vartheta = \frac{2 m_D}{m_R - m_L}}$$

$$\boxed{m_{1,2} = \frac{m_R + m_L}{2} \mp \sqrt{\left(\frac{m_R - m_L}{2}\right)^2 + m_D^2}}$$

(can be negative!)

## Mass eigenstates :

$$\mathcal{L}_{M+D} = \frac{1}{2}\left(m_1 \chi_{1L}^T C^+ \chi_{1L} + m_2 \chi_{2L}^T C^+ \chi_{2L}\right) + h.c.$$

▶ $$= -\left(|m_1| \overline{\chi}_1 \chi_1 + |m_2| \overline{\chi}_2 \chi_2\right)$$

$|m_1|, |m_2| > 0$
physical masses

## Physical, massive fields:

$$\begin{cases} \chi_1 = \chi_{1L} + \eta_1 (\chi_{1L})^c \\ \chi_2 = \chi_{2L} + \eta_2 (\chi_{2L})^c \end{cases}$$

▶ $$\eta_i = \begin{cases} 1 & \text{if } m_i > 0 \\ -1 & \text{if } m_i < 0 \end{cases}$$

▶ Satisfy Majorana condition: $(\chi_i)^c = \chi_i$ (up to a phase)

# Conclusion :

- Dirac + Majorana $\Longrightarrow$ 2 Majorana particles

  ↳ check: count degrees of freedom (4 d.o.f.)

- Physical masses are positive (as should be)

- Dirac limit: for $m_L = m_R = 0$ $\longrightarrow$ 1 Dirac particle

  (4 d.o.f.)

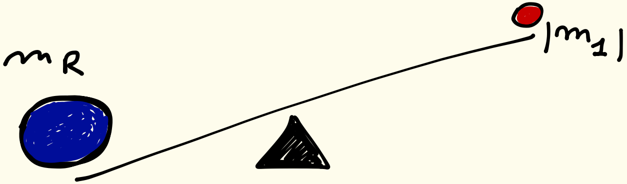
## The simplest See-Saw : $m_L = 0$ , $m_D << m_R$

▶ $\theta \sim \frac{m_D}{m_R} << 1$

$$\begin{cases} |m_1| \sim \frac{m_D^2}{m_R} << m_D \\ |m_2| \sim m_R \end{cases}$$

$$\begin{cases} \chi_1 \text{ is } \underline{\text{light}}, \text{ mostly } \underline{\text{active}} \; : \; \chi_{1L} \simeq \nu_L \\ \chi_2 \text{ is } \underline{\text{heavy}}, \text{ mostly } \underline{\text{sterile}} \; \phantom{:} \; \chi_{2L} \simeq (\nu_R)^c \end{cases}$$

→ **Naturalness**: New physics scale $M \sim m_R$ causes small $\nu$ masses!



# Generalization to n generations:

$$\mathcal{L}_{M+D} = \frac{1}{2}\, n_L^T\, C^{\dagger} M\, n_L + h.c.$$

$$n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \vdots \\ \nu_{nL} \\ (\nu_{1R})^c \\ (\nu_{2R})^c \\ \vdots \\ (\nu_{kR})^c \end{pmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right] \leftarrow n \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right] \leftarrow k \end{array}$$

$$\overset{n \times n}{\phantom{x}}$$

$$M = \begin{pmatrix} \boxed{m_L} & m_D \\ m_D^T & \boxed{m_R} \end{pmatrix} \leftarrow k \times k$$

- Diagonalization yelds $\left.\begin{array}{l} n \text{ light} \\ k \text{ heavy} \end{array}\right\}$ Majorana fields

# Mixing matrix for Majorana neutrinos:

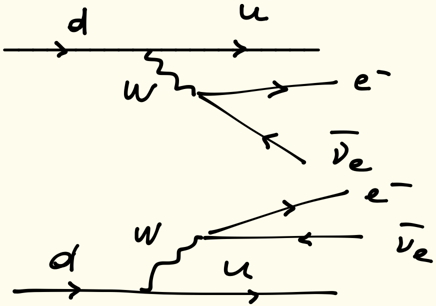
Same as Dirac, except for 2 extra observable phases:

$$U = V_{23}\, W_{13}\, V_{12}\, \underbrace{D}_{\uparrow} \quad , \qquad D = \begin{pmatrix} e^{-i\varphi_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\varphi_2} \end{pmatrix}$$

# Majorana phenomenology: neutrino-less double beta decay

- Lepton number-conserving (observed!)

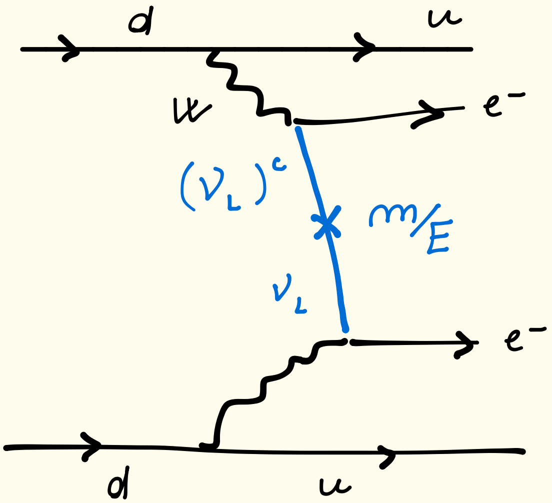
$$(A,Z) \longrightarrow (A, Z \pm 2) + 2e^{\mp} + 2\overset{(-)}{\nu_e}$$



- Lepton number-violating ($\Delta L = \pm 2$, not observed)

$$(A,Z) \longrightarrow (A, Z \pm 2) + 2e^{\mp} \implies \text{requires Majorana } \nu$$

↳ path to discovery!



- Chirality non-conservation: mediated by mass term

↳ path to measure m!

- Rate $\propto \left| \sum_i U_{ei}^2 m_i \right|^2$ → depends on Majorana phases!