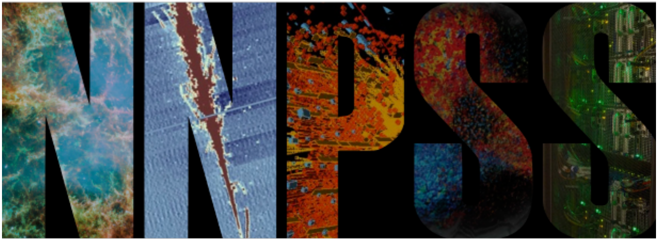
Nuclear structure I:
Introduction and nuclear interactions

Stefano Gandolfi

Los Alamos National Laboratory (LANL)



National Nuclear Physics Summer School
Massachusetts Institute of Technology (MIT)
July 18-29, 2016

The Nuclear Landscape and the Big Questions (NAS report)

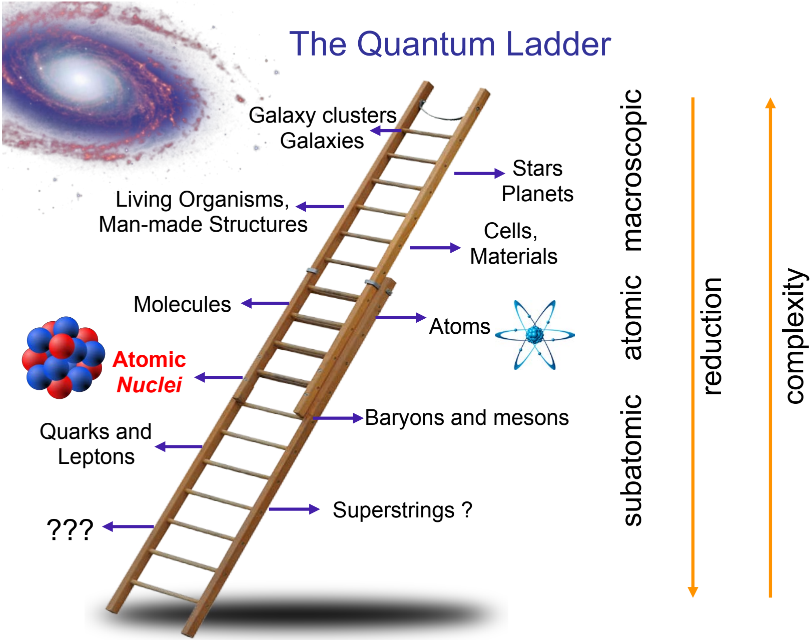
- How did visible matter come into being and how does it evolve? (origin of nuclei and atoms)
- How does subatomic matter organize itself and what phenomena emerge? (self-organization)
- Are the fundamental interactions that are basic to the structure of matter fully understood?
- How can the knowledge and technological progress provided by nuclear physics best be used to benefit society?

where the action is…

The Nuclear Landscape

- QCD transition (color singlets formed): 10 ms after Big Bang (13.8 billion years ago)
- D, ^{3,4}He, ^{7}Be/^{7}Li formed 3-50 min after Big Bang
- Other nuclei born later in heavy stars and supernovae

The Quantum Ladder



Galaxy clusters
Galaxies

Stars
Planets

Living Organisms,
Man-made Structures

Cells,
Materials

Molecules

Atoms

Atomic
Nuclei

Baryons and mesons

Quarks and
Leptons

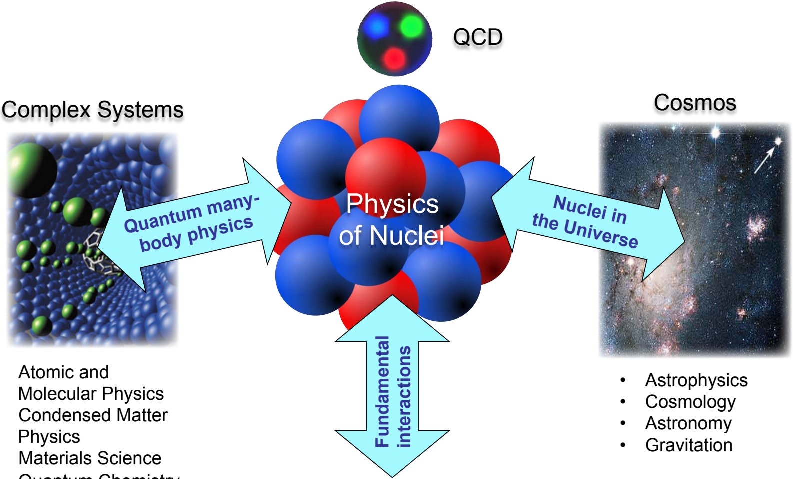
Superstrings ?

???

subatomic atomic macroscopic

reduction

complexity



QCD

Complex Systems

Physics of Nuclei

Cosmos

Quantum many-body physics

Nuclei in the Universe

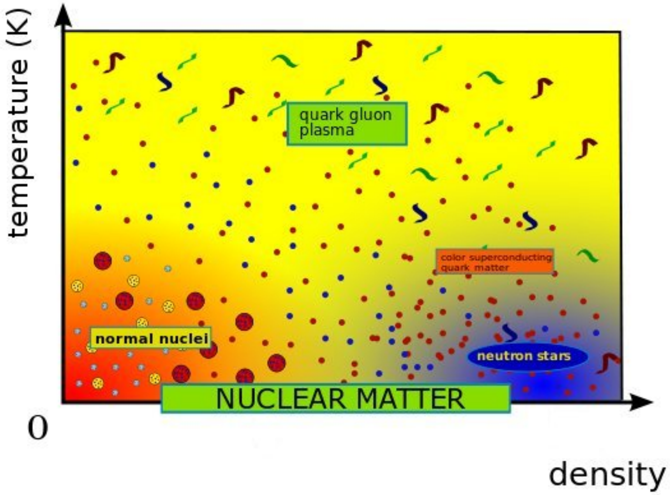
Fundamental interactions

- Atomic and Molecular Physics
- Condensed Matter Physics
- Materials Science
- Quantum Chemistry

- Particle Physics
- Cosmology

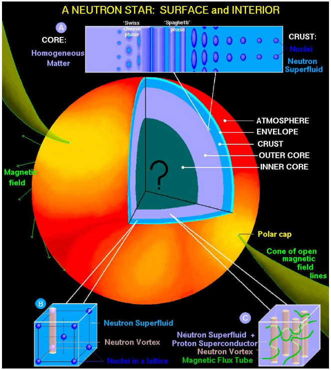
- Astrophysics
- Cosmology
- Astronomy
- Gravitation

The big picture of the microscopic world



Neutron stars

Neutron star is a wonderful natural laboratory, rich of physics!



D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates? ...?

Fundamental questions in nuclear physics

Physics of nuclei:

- How do nucleons interact?
- How are nuclei formed? How can their properties be so different for different A?
- What's the nature of closed shell numbers, and what's their evolution for neutron rich nuclei?
- What is the equation of state of dense matter?
- Can we describe simultaneously 2, 3, and many-body nuclei?

Nuclear astrophysics:

- What's the relation between nuclear physics and neutron stars?
- What are the composition and properties of neutron stars?
- How do supernovae explode?
- How are heavy elements formed?

Very incomplete list... Many questions will arise during these lectures!

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Fundamental questions in nuclear physics

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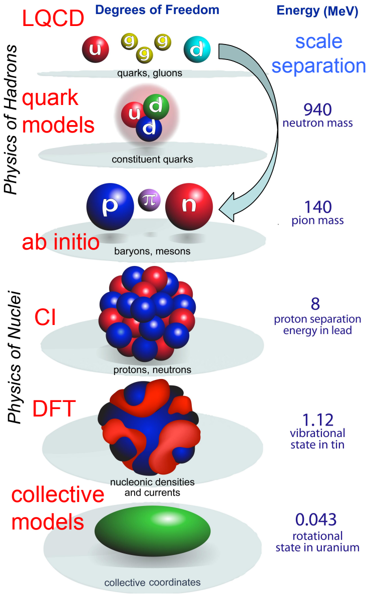
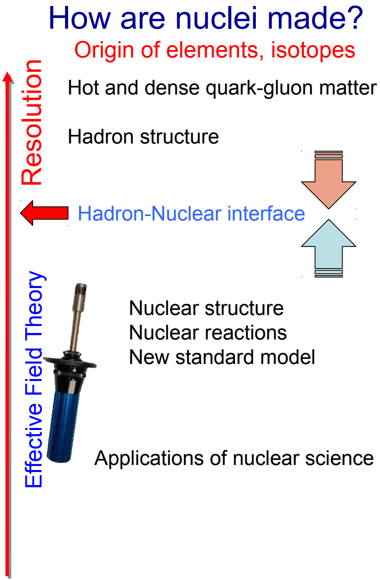
How do we describe nuclear systems? Degrees of freedom?





How are nuclei made?

Origin of elements, isotopes

Hot and dense quark-gluon matter

Hadron structure

Hadron-Nuclear interface

Nuclear structure
Nuclear reactions
New standard model

Applications of nuclear science

Tom Banks: ”only a fool would imagine that one should try to understand the properties of waves in the ocean in terms of Feynman-diagram calculations in the standard model, even if the latter understanding is possible ’in principle’.”

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Weinberg's Laws of Progress in Theoretical Physics
From: "Asymptotic Realms of Physics" (ed. by Guth, Huang, Jaffe, MIT Press, 1983). Third Law: "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

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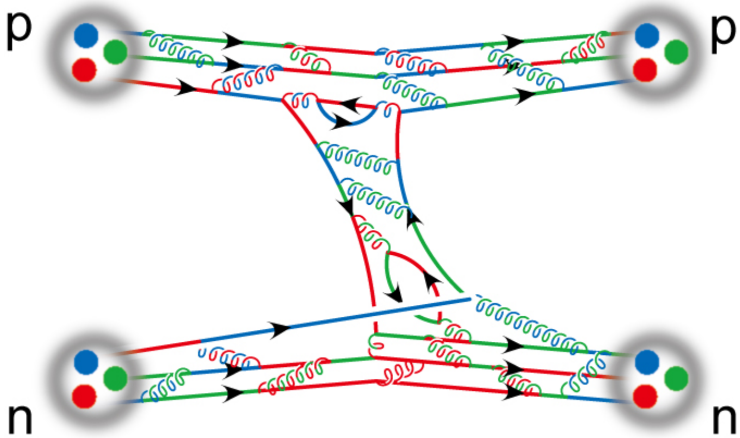
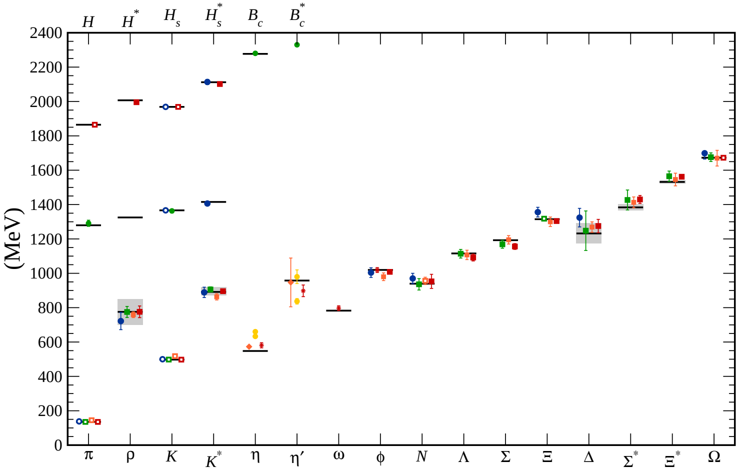
Quantum chromodynamics (QCD) is **THE** theory.



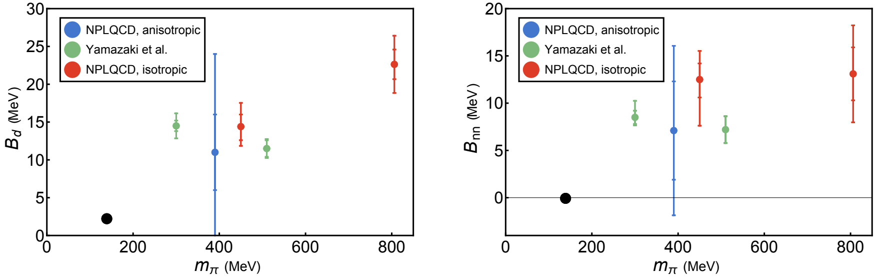
this (unrealistic) picture is already complicated. Calculations even more!

Lattice QCD calculations of single hadron mass spectrum, A. S. Kronfeld, arXiv:1209.3468.



Great predictive power, excellent agreement with experiments!

Nucleon-nucleon binding energy from lattice QCD as a function of m_π

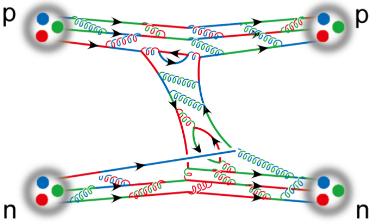


Deuteron binding energy Di-neutron binding energy

K. Orginos, *et. al*, Phys. Rev. D 92, 114512 (2015).

The problem is the sign problem.

One possible solution: change the degrees of freedom!



The goal: use effective nucleon dof's systematically.

- Seek model independence and theory error estimates
- Future: Use lattice QCD to match via "low-energy constants"
- Need quark dof's at higher densities or at high momentum transfers, where phase transitions happen

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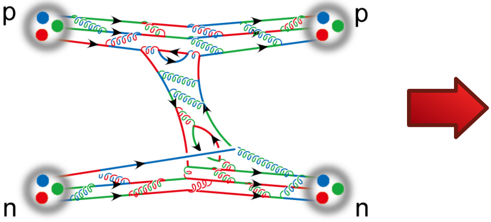

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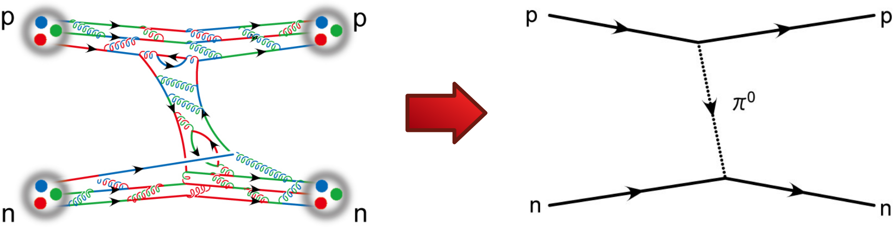

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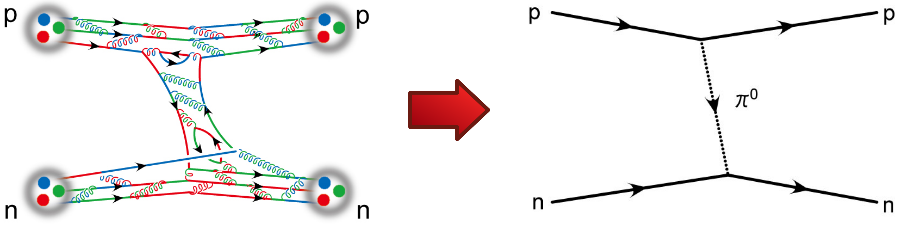

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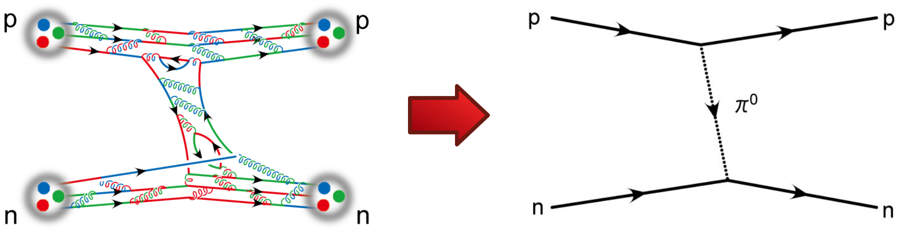

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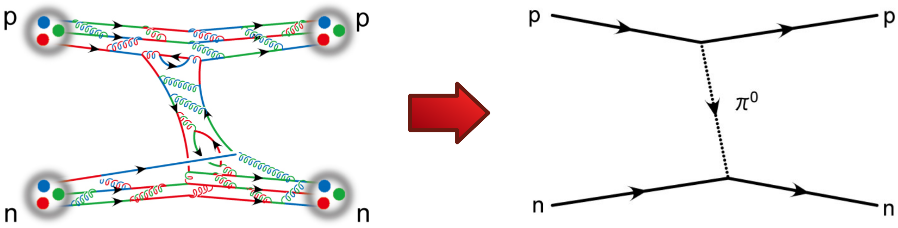

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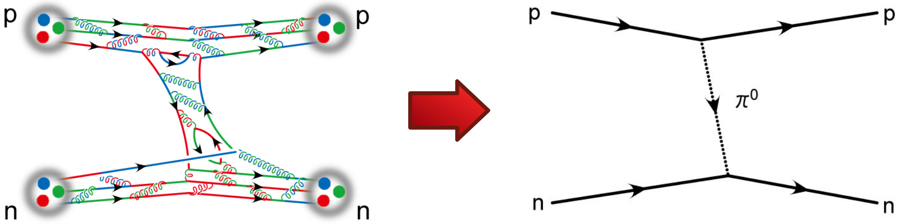
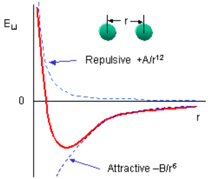
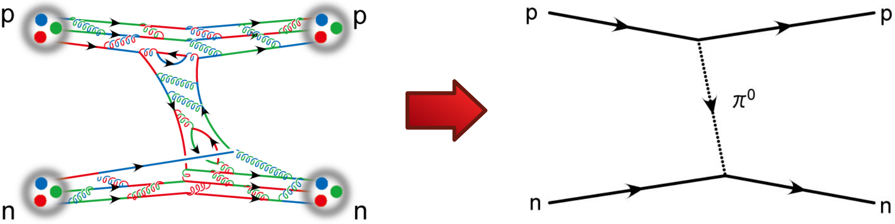


- Nucleon-nucleon interaction not fundamental (c.f. Lennard-Jones for noble gases)
- Range $\sim 1/m_\pi \sim 1.4$ fm vs nucleon rms ~ 0.9 fm
- Nucleon's wave functions overlap
- Nucleon's composite objects: expect three- and many-body forces important





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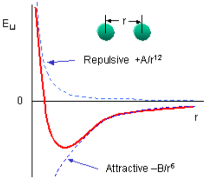

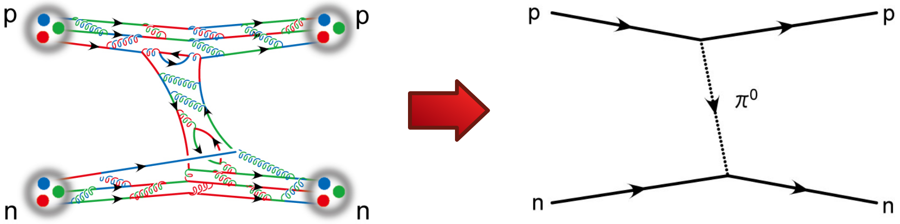

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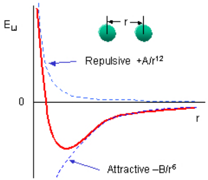

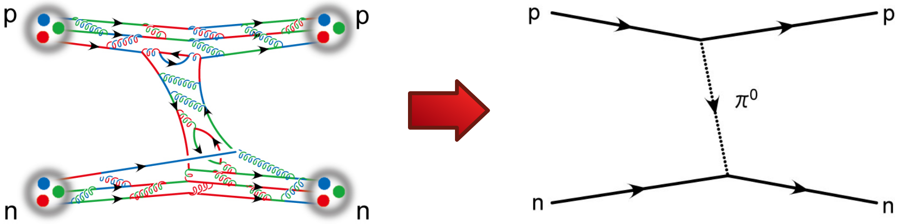

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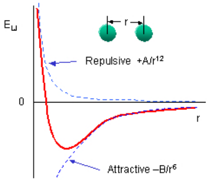

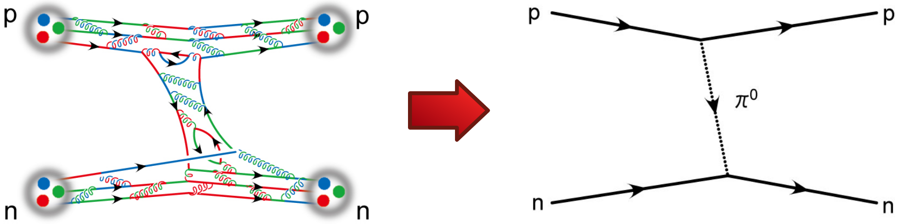

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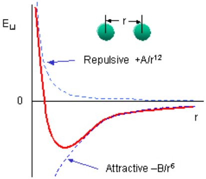

But, many nucleon-nucleon data available!

Let's describe the (many-body) system using a non-relativistic Hamiltonian. The d.o.f. are nucleons, described as interacting point-like particles:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

- The kinetic energy can corrected to account for the proton vs neutron mass difference
- v_{ij} is an effective two-nucleon potential including the strong interaction and Coulomb force (with corrections due to the spin of nucleons, form factors, etc.)
- V_{ijk} is a three-nucleon force, whose role **and need** will be clear later
- $+\ldots$ can include anything missing (four- five- ...-body forces)
- Assumption: all the nucleon's form factors, their excitations, and other properties can be included in the potentials, and this description is valid until *nucleons overlap too much* (that means reasonably low densities and momenta), i.e. their structure don't change much.

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Nucleon-nucleon interaction

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The force between nucleons strongly depends upon the spin and the isospin.

Nucleon-nucleon interaction

Let's consider the isospin (for the spin is the same story):

$$T = 0 \qquad \left\{ \begin{array}{ll} T_z = 0 & \frac{1}{\sqrt{2}}\left(|np\rangle - |pn\rangle\right) \end{array} \right.$$

$$T = 1 \qquad \left\{ \begin{array}{ll} T_z = 1 & |pp\rangle \\ T_z = 0 & \frac{1}{\sqrt{2}}\left(|np\rangle + |pn\rangle\right) \\ T_z = -1 & |nn\rangle \end{array} \right.$$

Nucleon-nucleon interaction

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$$T = 0 \qquad \left\{ \begin{array}{ll} T_z = 0 & \frac{1}{\sqrt{2}}\left(|np\rangle - |pn\rangle\right) \end{array} \right.$$

$$T = 1 \qquad \left\{ \begin{array}{ll} T_z = 1 & |pp\rangle \\ T_z = 0 & \frac{1}{\sqrt{2}}\left(|np\rangle + |pn\rangle\right) \\ T_z = -1 & |nn\rangle \end{array} \right.$$

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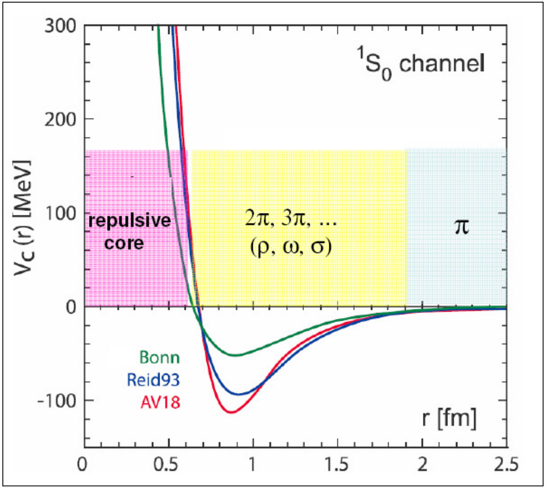
In general:

$$V_{NN} = V_{S=0,T=0} + V_{S=1,T=0} + V_{S=0,T=1} + V_{S=1,T=1}$$

Note: V_{ST} have different contributions in different relative angular momenta!

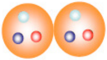
Traditional approach (credit D. Furnsthal, T. Papenbrock)



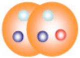
From T. Hatsuda (Oslo 2008)

One-pion exchange
by Yukawa (1935)

Multi-pions
by Taketani (1951)



Repulsive core
by Jastrow (1951)



Example: Argonne v_{18} (Wiringa, Stoks, Schiavilla, PRC (1995)) includes an electromagnetic, one-pion exchange (long), and intermediate and a short range part

$$v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum_{p} v_p(r_{ij}) O_{ij}^{p}$$

The operators depend on relative states of the two nucleons.

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There are charge independent (CI):

$$O_{ij}^{\mathrm{CI}} = \left[1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L}\cdot\mathbf{S})^2\right] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

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And charge dependent (CD) and charge symmetry breaking (CSB) terms:

$$O_{ij}^{\mathrm{CD}} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}] \otimes T_{ij}\,, \qquad\qquad O_{ij}^{\mathrm{CSB}} = \tau_{z_i} + \tau_{z_j}\ .$$

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where $S_{ij} = 3\boldsymbol{\sigma}_i \cdot \hat{r}_{ij} \boldsymbol{\sigma}_j \cdot \hat{r}_{ij} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ is the **tensor**

$\mathbf{L}_{ij} = \frac{1}{2i}(\mathbf{r}_i - \mathbf{r}_j) \times (\boldsymbol{\nabla}_i - \boldsymbol{\nabla}_j)$ is the **relative angular momentum**

$\mathbf{S}_{ij} = \frac{1}{2}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)$ is the **total spin** of the pair

and $T_{ij} = 3\tau_{z_i}\tau_{z_j} - \boldsymbol{\tau}_\mathbf{i} \cdot \boldsymbol{\tau}_\mathbf{j}$ is the **isotensor** operator.

Argonne v_{18} (Wiringa, Stoks, Schiavilla, PRC (1995))

For example, the long-range part (one pion exchange) has the form

$$v^{5,6} \sim [Y(m_\pi r_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T(m_\pi r_{ij})S_{ij}] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

where

$Y(x) = \frac{e^{-x}}{x}\xi(r)$ is the Yukawa function,

$T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y(x)\xi(r)$ is the tensor function,

and $\xi(r) = 1 - \exp(-cr^2)$ is a cutoff.

The intermediate part has similar form:

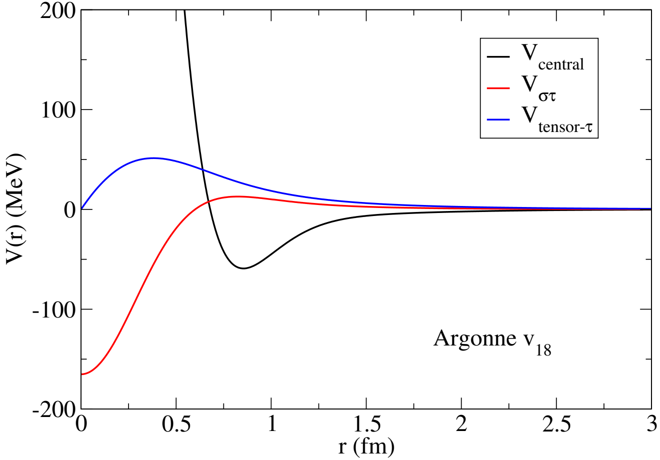
$$v_{ij}^I = \sum_{p=1}^{18} I^p T^2(\mu r_{ij}) O_{ij}^p$$

and the short range is

$$v_{ij}^S = \sum_{p=1}^{18} \left[P^p + Q^p r + R^p r^2\right] W(r) O_{ij}^p$$

where $W(r)$ is a Wood-Saxon potential, and there are 42 free parameters I^p, P^p, Q^p and R^p.

Argonne v_{18} (Wiringa, Stoks, Schiavilla, PRC (1995))



Example: radial functions v_1, v_4 and v_6 that multiply respectively the operators 1, $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$, and $S_{ij} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$.

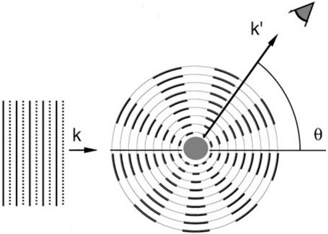
Two slides on scattering theory (1/2)

At **large distances**

before the scattering, $\psi \sim e^{ikz}$

after the scattering, $\psi \sim e^{ikz} + \frac{f(\theta)}{r} e^{ikr}$

(let's assume that $k \sim k'$)



$f(\theta)$ is the scattering amplitude, and it is directly related to the differential cross-section:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

For a central potential $f(\theta)$ can be expanded as

$$f(\theta) = \frac{1}{2ik} \sum_l (2l+1) \left(e^{2i\delta_l} - 1\right) P_l(\cos\theta)$$

where δ_l are the **phase shifts**, and

$$\sigma_{\rm tot} = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l(k)$$

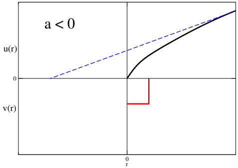
Two slides on scattering theory (2/2)

In the limit of slow particles (low momenta)

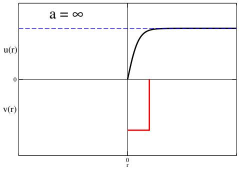
$$k \cot \delta(k) \approx -\frac{1}{a} + \frac{1}{2} r_e k^2 + \dots$$

where a is the **scattering length** and r_e the **effective range**.

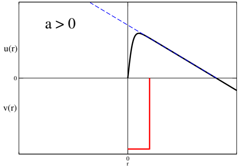
Very schematically (but pedagogical...), let's consider a two-body system with attractive interaction:



No bound states



Bound state with E_b=0

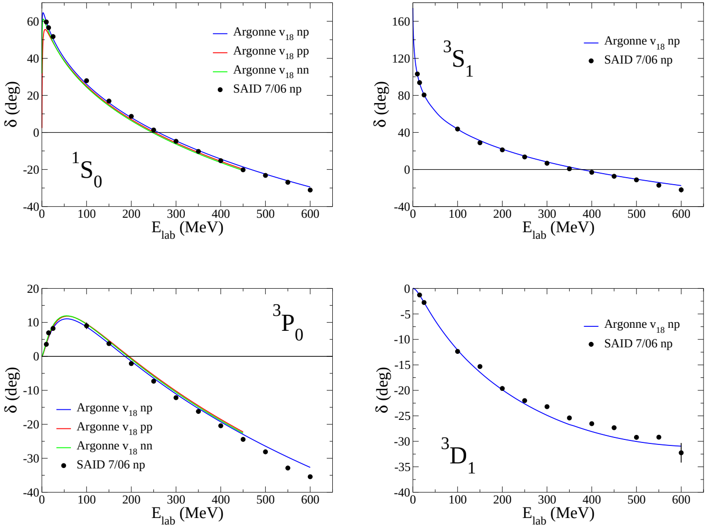


$E_b \sim \frac{\hbar^2}{2m\, a^2}$

The nucleon-nucleon ${}^3S_1$ channel (deuteron) has $a \approx 5.5$ fm and is slightly bound. The ${}^1S_0$ (two neutrons) has $a \approx -18$ fm and is slightly unbound.

Argonne v_{18} (Wiringa, Stoks, Schiavilla, PRC (1995))

The parameters are fit to nucleon-nucleon scattering data up to lab energies of 350 MeV with very high precision, $\chi^2 \sim 1$. Phase shifts:



There are also other (simpler) versions of Argonne potentials, $AV8'$, $AV6'$, …, but also others like those of the Nijmegen group, CD-Bonn potentials, and many others.

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Another more recent approach, consists in developing nucleon-nucleon interactions within the framework of chiral effective field theory.

Chiral EFT interactions

Main idea of EFT: identify scales of the problem that are different, and expand in the ratio.

⇒ “Ideal” systematic improvements possible!

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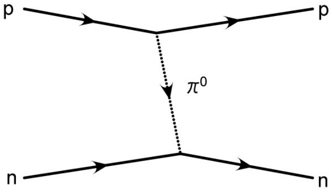

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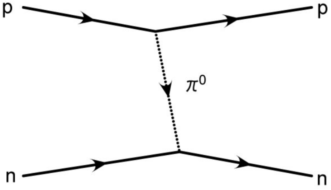

Pretend that $m_\pi(\sim 140 MeV) \to 0$ (*soft* scale) and $m_N(\sim 939 MeV) \to \infty$ (*hard* scale).

In the low-energy (low-momentum) limit, we can expand the interaction in powers of $(Q/\Lambda)^\nu$, where $Q \sim$*soft* scale, and $\Lambda \sim$*hard* scale.

Chiral EFT interactions

One possible power counting (Weinberg) ν in $(Q/\Lambda)^\nu$ is given by

$$\nu = -4 + 2N + 2L + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

where
N=nucleons involved in the process,
L=pion loops,
V_i=vertices of type i,
d_i=derivatives and or insertions of m_π,
n_i=nucleonic fields operators.

Note:

- Adding one nucleon increases one order in Q/Λ
- Expect many-body forces! "Natural" expectation that $V_2 \gg V_3 \gg V_4 \ldots$
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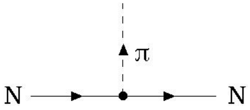
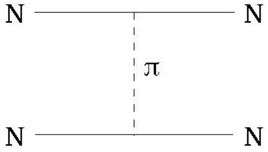
Chiral EFT interactions

Example (I), one-pion exchange ($N = 2$, $L = 0$):

Two identical vertices: $V_1{=}2$, $d_1{=}1$, $n_1{=}2$,

$\rightarrow \nu = 0$ (LO)





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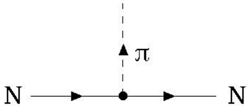

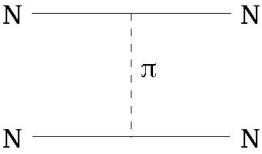

Example (II), contact interactions ($N = 2,\ L = 0$):

One vertex: V_1=1, d_1=0, n_1=4,

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Chiral EFT interactions

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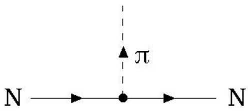

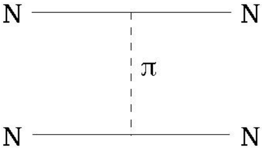

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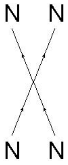
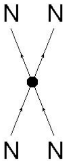
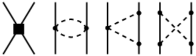
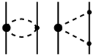
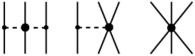
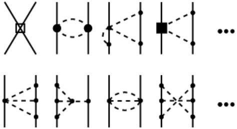
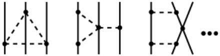
$\rightarrow \nu = 2$ (NLO)





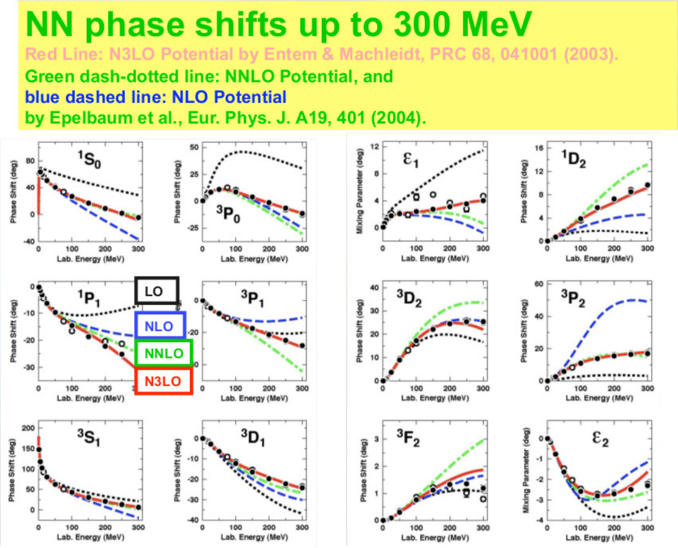
Chiral EFT interactions

So at each order, draw all the possible diagrams, and that's it!

	2N force	**3N force**	**4N force**
LO		—	—
NLO		—	—
N²LO			—
N³LO			

Several versions available on the market up to N^{3}LO (maybe higher).

Chiral EFT interactions



NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).

Green dash-dotted line: NNLO Potential, and

blue dashed line: NLO Potential

by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

R. Machleidt, NTSE 2013

Chiral EFT interactions

Other possible expansions are possible, i.e. pionless theory, delta-full, ...

In the same way also electroweak currents and other operators can be constructed that are consistent with the Hamiltonian.

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- Introduction: nuclear physics in a big contest
- Questions in nuclear physics
- Degrees of freedom
- Nucleon-nucleon interactions

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End for today...