Lattice QCD for hadron and nuclear physics: scattering and many-particle systems

Sinéad M. Ryan
Trinity College Dublin

National Nuclear Physics Summer School, MIT July 2016

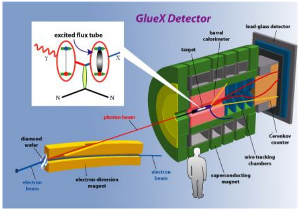
Plan

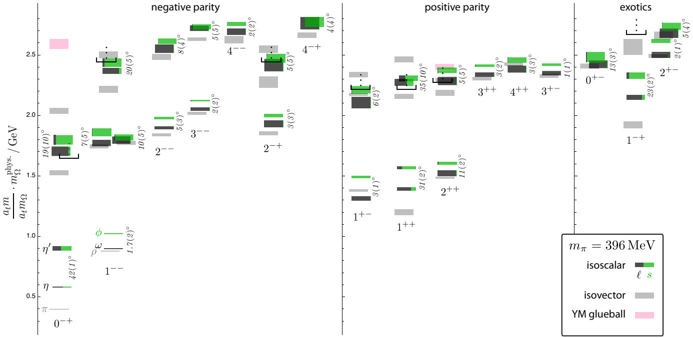
- A look at the spectrum of single-particle states
- Resonances
- Two particles in a box
- Accessing resonance information from finite volume calculations
- Toy models
- Recent simulations
- Spectrosopy at finite temperature
- Summary

The spectrum of light states

- Using the technology we have discussed, the spectra of mesons and baryons can be determined precisely.

On the menu:





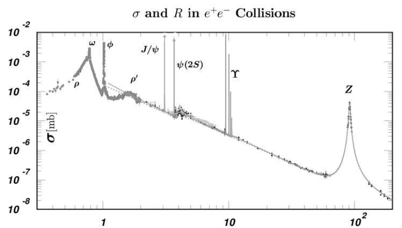
Is this everything?

Resonances and scattering states

- We have assumed that all particles in the spectrum are stable
- Many (the majority) are not.
- A resonance is a state that forms eg when colliding two particles and then decays quickly to scattering states.
- They respect conservation laws: if isospin of the colliding particles is 3/2, resonance must have isospin 3/2 (Δ resonance)
- Usually indicated by a sharp peak in a cross section as a function of c.o.m. energy of the collision.
- Can lattice qcd distinguish resonances and scattering states?

Resonances in $e^+e^- \to$ hadrons

$$\sigma_{ee\to\mathrm{hadrons}}(E_{ee})$$

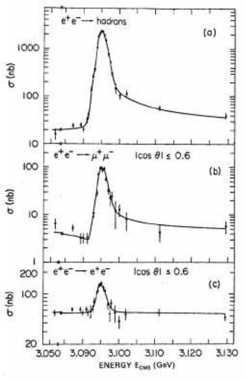


- Note the more or less sharp resonances on a comparably flat "continuum", coming from $e^+e^- \to q\bar{q}$

(We will discuss this in more detail!)

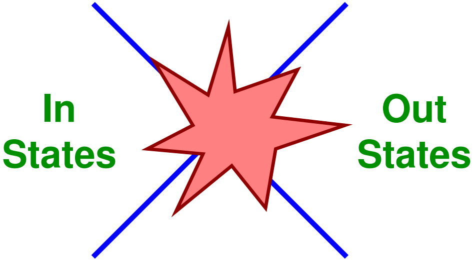
- They are (apart from the Z) all related to $q\bar{q}$-bound states.

Zoom into J/Ψ



- Note: Here width around 3 MeV completely determined by detector ($\Gamma_{J/\Psi} = 87$ keV)

Maiani-Testa no-go theorem

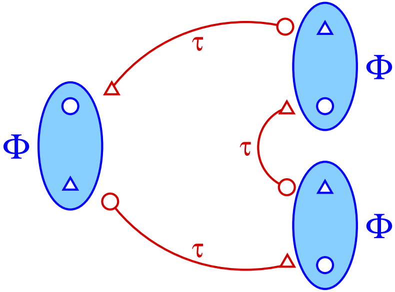


- Importance sampling Monte-carlo simulations rely on a path integral with positive definite probability measure: Euclidean space
- Maiani-Testa: scattering matrix (S-matrix) elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold)

Michael 1989 and Maiani, Testa (1990)

Maiani-Testa (2)

- Can understand this since:
  - Minkowski space: S-matrix elements complex functions above kinematic thresholds
  - Euclidean space: S-matrix elements are real for all kinematics - phase information lost
- Lattice simulations with dynamical fermions admit strong decays eg for light-enough up and down dynamical quarks $\rho \to \pi\pi$



Maiani-Testa (3)

- Can the lattice get around the no-go theorem to extract the masses and widths of such unstable particles?
- Yes - use the finite volume.
- Computations done in a periodic box
  - momenta quantised
  - discrete energy spectrum of stationary states → single hadron, 2 hadron ...
  - scattering phase shifts → resonance masses, widths deduced from finite-box spectrum
    - B. DeWitt, PR 103, (1956) - sphere
    - M. Lüscher, NPB (1991) - cube
  - Two-particle states and resonances identified by examining the behaviour of energies in finite volume

For elastic two-body resonances (Lüscher): $M_1 M_2 \rightarrow R \rightarrow M_1 M_2$

⟶ Volume dependence of energy spectrum
⟶ Phase shift in infinite volume
⟶ Mass and width of resonance - parameterising the phase shift e.g. with Breit Wigner.

Note: This is a rapidly developing field. I will add some refs for recent work or see Lattice Conf talks.

Particles in a box

- Spatial lattice of extent L with periodic boundary conditions
- Allowed momenta are quantized: $p = \frac{2\pi}{L}(n_x, n_y, n_z)$ with $n_i \in \{0, 1, 2, \ldots L-1\}$
- Energy spectrum is a set of **discrete** levels, classified by p: Allowed energies of a particle of mass m

$$E = \sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 N^2} \quad \text{with } N^2 = n_x^2 + n_y^2 + n_z^2$$

- Can make states with **zero total momentum** from pairs of hadrons with momenta $p, -p$.
- "Density of states" **increases** with energy since there are more ways to make a particular value of N^2 e.g. $\{3, 0, 0\}$ and $\{2, 2, 1\} \rightarrow N^2 = 9$

Avoided level crossings

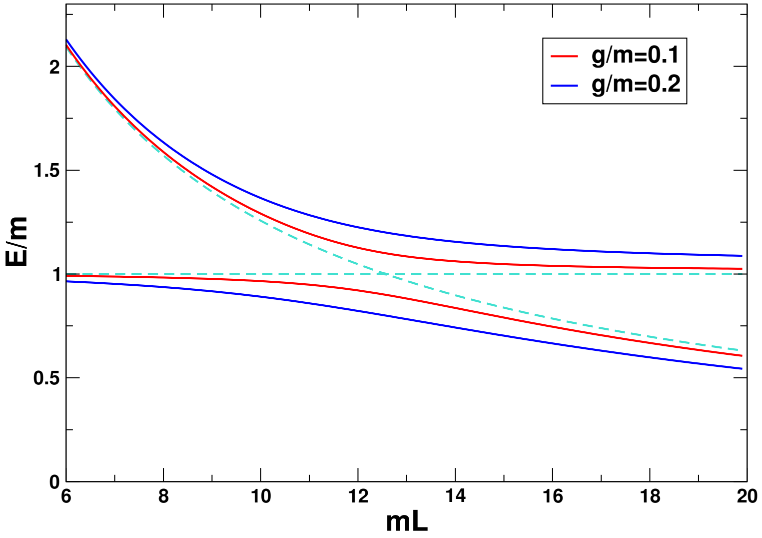
- Consider a toy model with two states (a resonance and a two-particle decay mode) in a box of side-length L
- Write a mixing hamiltonian:

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$

- Now the energy eigenvalues of this hamiltonian are given by

$$E_{\pm} = \frac{(m + \frac{4\pi}{L}) \pm \sqrt{(m - \frac{4\pi}{L})^2 + 4g^2}}{2}$$

Avoided level crossings



Avoided level crossings

- **Ground-state** smoothly changes from resonance to two-particle state
- Need a large box. This example, levels cross at $mL = 4\pi \approx 12.6$
- Example: $m = 1$ GeV state, decaying to two massless pions - avoided level crossing is at $L = 2.5$fm.
- If the decay product pions have $m_\pi = 300$ MeV, this increases to $L = 3.1$fm

Lüscher's method

- Relates the spectrum in a finite box to the scattering phase shift (and so resonance properties)

Lüscher's formula

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

and $\cot\phi$ a known function (containing a generalised zeta function).

- The idea dates from the quenched era. To use it in a dynamical simulation need energy levels at extraordinary precision. This is why it has taken a while ....

An alternative approach called the *potential method* by HALQCD is also in use [PRL99 (2007), 022001] - less robust, certainly less widely used.

Lüscher's method

- Z_{00} is a generalised Zeta function:

$$Z_{js}(1,q^2) = \sum_{n \in Z^3} \frac{r^j Y_{js}(\theta,\phi)}{(n^2 - q^2)^s}$$

[Commun.Math.Phys.105:153-188,1986.]

- With the phase shift, and for a well-defined resonance, can fit a Breit-Wigner to extract the **resonance width** and **mass**.

$$\delta(p) \approx \tan^{-1}\left(\frac{4p^2 + 4m_\pi^2 - m_\sigma^2}{m_\sigma \Gamma \sigma}\right)$$

Lüscher (3): considering $\rho \longrightarrow \pi\pi$

- For non-interacting pions, the energy levels of a 2 pion system in a periodic box of length L are

$$E = 2\sqrt{m_\pi^2 + p^2} \quad p = 2\pi|\vec{n}|/L$$

  and $\vec{n}$ has components $n_i \in \mathbb{N}$.

- In the interacting case the energy levels are shifted

$$E = 2\sqrt{m_\pi^2 + p^2} \quad p = (2\pi/L)q$$

  where q is no longer constrained to orginate from a quantised momentum mode.

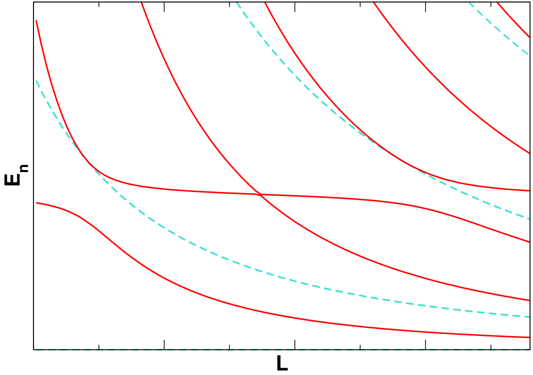
- In the presence of the interaction, energy eigenvalues deviate from the noninteracting case
- These deviations contain the information on the underlying strong interaction - yielding resonance information via Luscher formulism.

Schrödinger equation

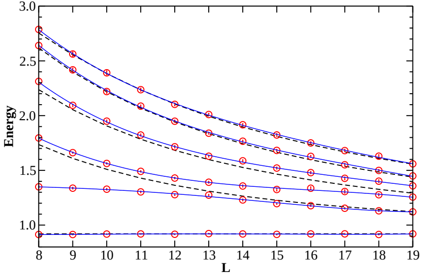
Exercise: find the phase shift for a 1-d potential

$$V(x) = V_0\delta(x - a) + V_0\delta(x + a)$$

- Now compute the spectrum in a finite box and use Lüscher's method to compare the two



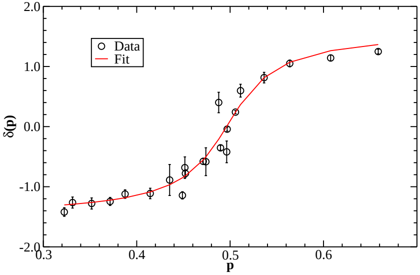
Test: O(4) Sigma model



M.Peardon and P. Giudice

Spectrum of $O(4)$ model in broken phase

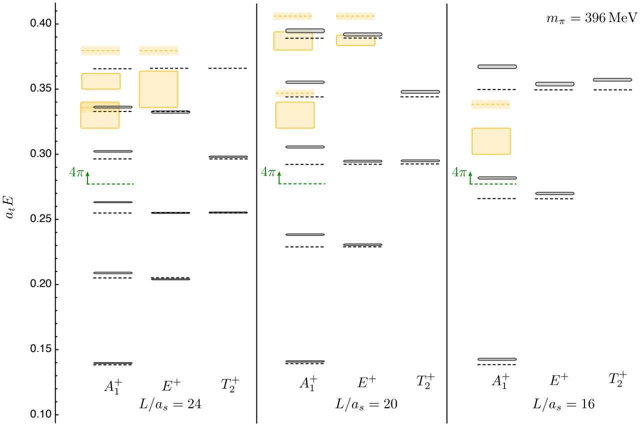
Phase shift inferred from Lüscher's method



$\rho \longrightarrow \pi\pi$

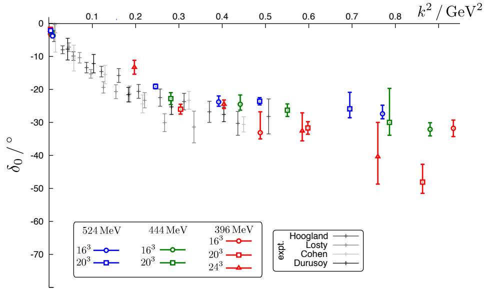
- Start with an "easy" system, $I = 2\pi\pi$ and test methods there. Interaction not strong enough to form a resonance, but is weak and repulsive.
- In the real world want to study $rho \to \pi\pi$ in isospin $I = 1$
- This involves disconnected diagrams which is already a complication - although in principle doable.
- $I = 1$ case is now studied (distillation has helped a lot here)

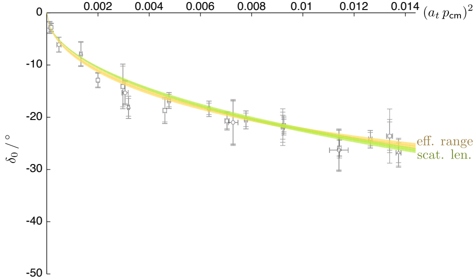
I=2 $\pi\pi$ SCATTERING

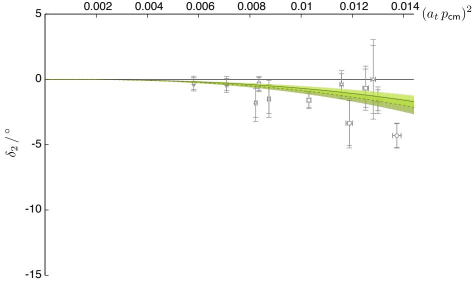


- Resolve shifts in masses away from non-interacting values
- Orange boxes: possible $\pi\pi^*$ scattering states
- Dashed lines: non-interacting pion pairs

I=2 $\pi\pi$ scattering - not physical pions



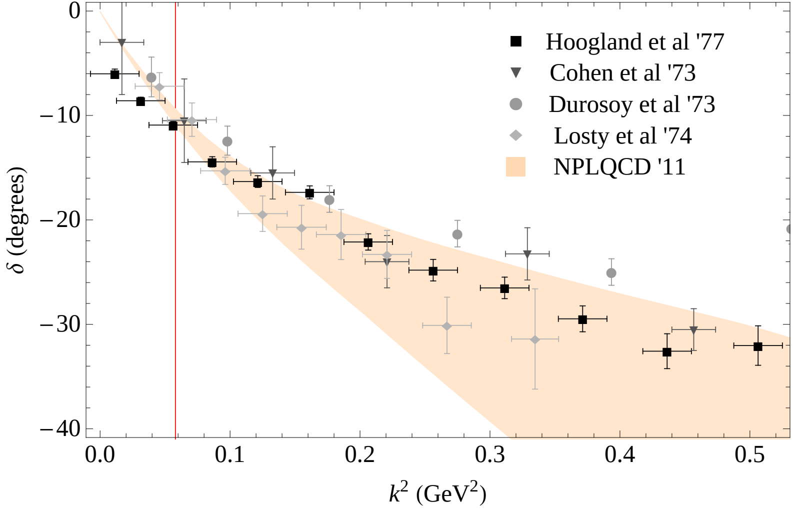




HadSpec - Dudek et al, 1011.6352 & 1203.6041

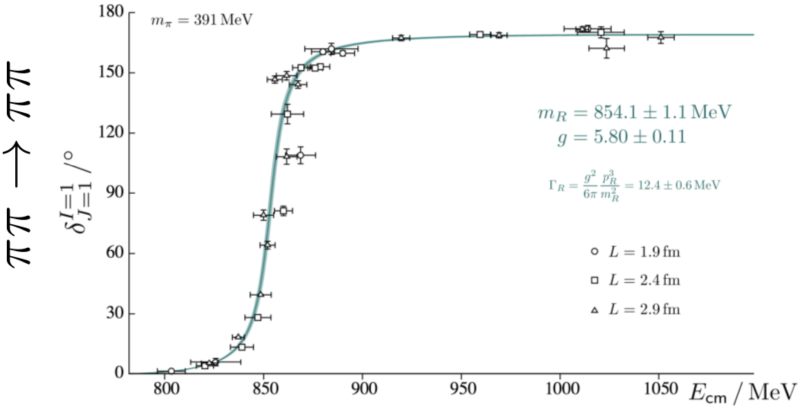
- Non-resonant scattering in S-wave and D-waves.

TOWARDS PHYSICAL PIONS IN I=2 $\pi\pi$.



NPLQCD, 1107.5023. Combines chiral perturbation theory and lattice results at $m_\pi \sim 396 MeV$ to predict phase shift at the physical pion mass.

$\pi\pi$ IN I=1



$$m_\pi = 391 \text{ MeV}$$

from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505

The inelastic threshold

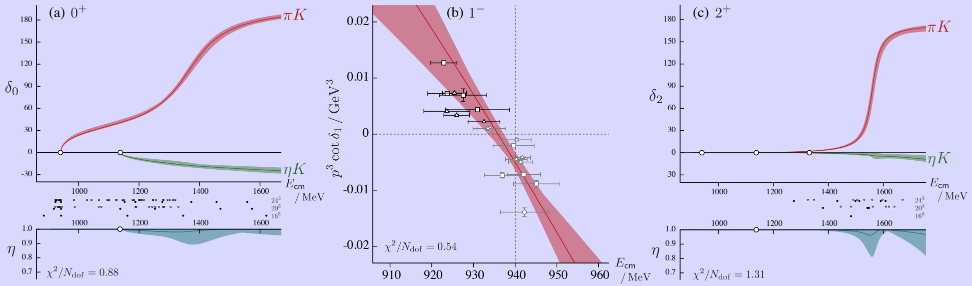
- Lüscher's method is based on **elastic** scattering.
- Since m_π is small, most resonances are above this threshold
- It will be crucial to ensure we have a comprehensive **basis of operators that create multi-hadron states**.
- Going beyond elastic. The method is generalised for: moving frames; non-identical particles; multiple two-particle channels, particles with spin, by many authors.

$$\det\left[t^{-1}(E) + i\rho(E) - M(E, L)\right] = 0$$

  relates the scattering t matrix to the discrete spectrum of states in finite volume (coupled channels).
- The precision and robustness of some numerical implementations is now very impressive. *[See e.g. talks at Lattice 2015 & 2016]*
- First coupled-channel resonance in a lattice calculation

$\pi K \to \eta K$ by D. Wilson et al 1406.4158 and 1507.02599

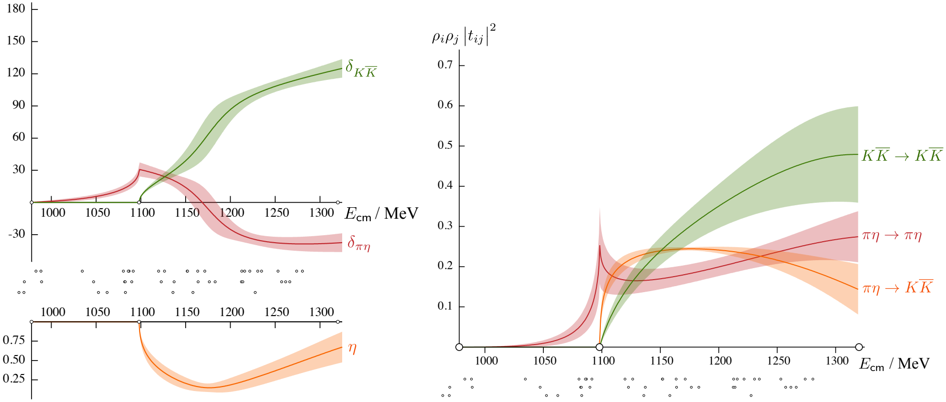
Recent (and very recent) results



$\pi K \to \eta K$ by D. Wilson et al 1406.4158 and 1507.02599

More coupled channels!

Is a_0 a $q\bar{q}$ state or dominated by a $K\bar{K}$ molecular configuration?



HSC, Wilson et al 1602.05122

- Phase shifts, inelasticity and amplitudes (for $m_\pi \sim 400$MeV)
- Find an S-wave resonance in a two-coupled channel region - $\pi\eta$, $K\bar{K}$, includes limited 3-channel scattering - $(\pi\eta, K\bar{K}, \pi\eta')$. Resonance pole has large coupling to $K\bar{K}$.

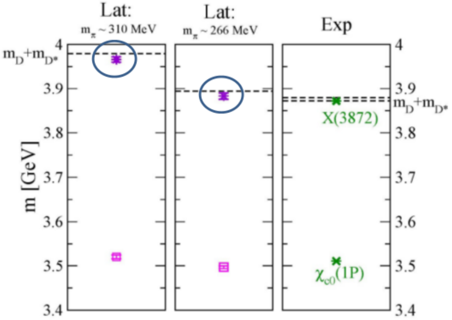
H-Dibaryon

- A bound 6-quark state (udsuds) first proposed by Jaffe (1977) in MIT bag-model - at 81MeV below $\Lambda\Lambda$ threshold.
- Lattice calculations [NPLQCD, HALQCD] find H-dibaryon bound but at quark masses larger than physical pion.
- Extracting resonance parameters from $\Lambda\Lambda$
- A linear chiral extrapolation does not discriminate between bound/unbound at the physical pion mass. Does suggest a state in I=0, J=0, s=-2 ($\Lambda\Lambda$) that is just bound/unbound.

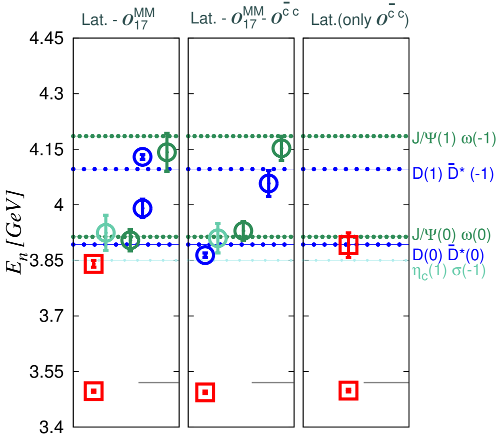
More work to be done for good understanding

X(3872) - A FIRST LOOK

Prelovsek & Leskovec 1307.5172



Padmanath, Lang, Prelovsek 1503.03257



ground state: $\chi_{c1}(1P)$
$D\bar{D}^*$ scattering mx: pole just below thr.
Threshold $\sim m_{u,d}$ *and* m_c *discretisation?*

X(3872) not found if $c\bar{c}$ not in basis.

Also results from Lee et al 1411.1389
Within 1MeV of $D^0\bar{D}^0{}^*$, 8MeV of D^+D^* thresholds: isospin breaking effects important?

Z_c^+

An “exotic” hadron i.e. does not fit in the quark model picture.

There are a number of exploratory calculations on the lattice.

Challenges:

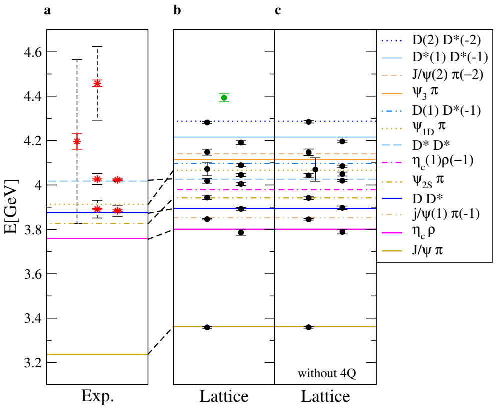
- The Z_c^+ (and most of the XYZ states) lies above several thresholds and so decay to several two-meson final states
- requires a coupled-channel analysis for a rigorous treatment
- on a lattice the number of relevant coupled-channels is large for high energies.

State of the art in coupled-channel analysis:

- Lüscher: $K\pi$, $K\eta$ [HSC 2014,2015]
- HALQCD: Z_c [preliminary results]

Z_c^+ - First look on the lattice

Prelovsek, Lang, Leskovec, Mohler: 1405.7615



- 13 expected 2-meson e'states found (black)
- no additional state below 4.2GeV
- no Z_c^+ candidate below 4.2GeV

Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318]

Why no eigenstate for Zc? Is Z_c^+ a coupled channel effect? What can other groups say? Work needed!

Many other states being investigated

Tetraquarks:

- Double charm tetraquarks ($J^P = 1^+, I = 0$) by HALQCD [PLB712 (2012)]
  - attractive potential, no bound tetraquark state
- Charm tetraquarks: variational method with DD^*, $D^* D^*$ and tetraquark operators finds no candidate.

Y(4140)

- Ozaki and Sasaki [1211.5512] - no resonant Y(4140) structure found
- Padmanath, Lang, Prelovsek [1503.03257] considered operators: $c\bar{c}$, $(\bar{c}s)(\bar{s}c)$, $(\bar{c}c)(\bar{s}s)$, $[\bar{c}\bar{s}][cs]$ in $J^P = 1^+$. Expected 2-particle states found and χ_{c1}, $X(3872)$ **not** Y(4140).

⋮

See Prelovsek @ Charm2015 for more

Exploratory studies of scattering/many-body systems

Characterised by:

- New methods (developed/applied in last 5 years)
  - algorithmic: distillation allows access to all elements of propagators *and* construction of sophisticated basis of operators.
  - theoretical: spin-identification; construction of multi-hadron operators etc
- Generally high statistics, improved actions etc - results can be very precise.
- Systematic errors not all controlled in exploratory studies: e.g. no continuum extrapolation, relatively heavy pions ...

A different frontier: finite temperature and density QCD

Spectroscopy at finite temperature

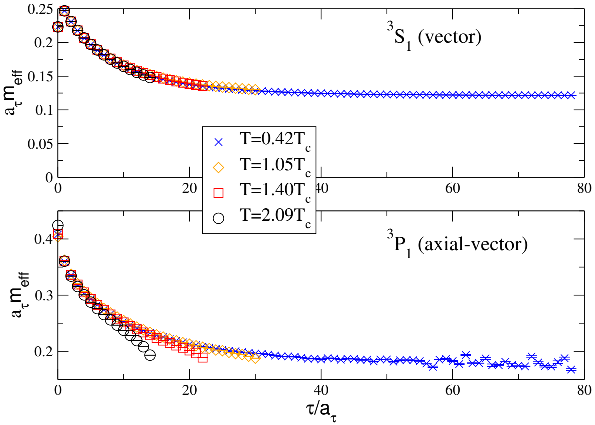
- We have heard about finite temp QCD.
- One avenue of investigations: States made from heavy quarks are expected to act as a probe of dynamics of the QGP
- There are interesting results coming from RHIC and CERN for the melting and suppresion of such states.
- Can lattice say anything? It is a challenge!

- (Without details here) the thermal correlator is

$$C(\tau) \sim \int_0^\infty d\omega \rho(\omega, T) K(\omega, \tau, T), \quad (p = 0).$$

- $C(\tau)$ sampled discretely but ρ has values for continuous ω
- An ill-posed problem!
- Maximum entropy methods (MEM) can be used but can be unstable and model-dependent
- New ideas needed!

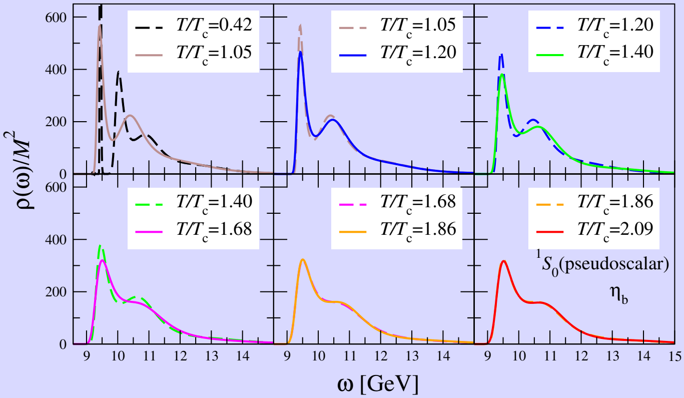
Effective masses at finite T



- Anisotropic lattices $\xi = 6$, $N_f = 2$, $a_s = 0.15$fm
- Note P wave behaviour at $T > T_c$
- Appears to rule out pure exponential decay at high T for P waves.

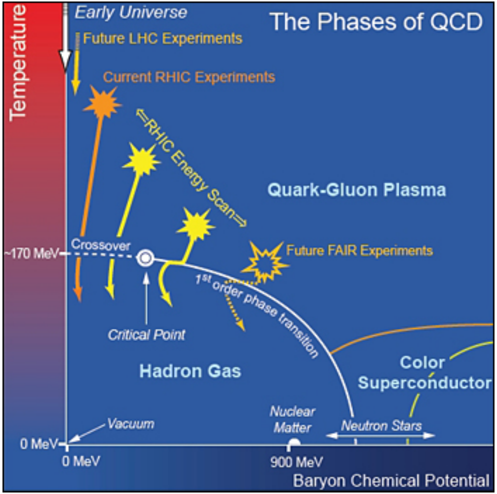
Results: maximum entropy analysis

η_b MEM



- 1S survives to highest T examined.
- excited states not discernable at $1.4 \lesssim T/T_c \lesssim 1.68 \Rightarrow$ melting or suppression?

Finite density QCD



- Would like to explore in finite μ with lattice qcd
- $\mu \neq 0 \Rightarrow M$ not γ_5 hermitian.
- No longer have a (positive) probability weight for Monte Carlo simulations - **sign problem**
- There are "work-arounds" but no solutions (yet!)

Executive Summary & Outlook

- There is much that I did not cover in these lectures
- I chose to focus on methods, new and old, for the "basic" building blocks of spectroscopy
- ... and described their successful applications as well as some pitfalls
- Lattice hadron/nuclear physics is moving rapidly at the moment as new techniques emerge
- Many challenges remain e.g. no general framework for extracting scattering amplitudes involving more than two hadrons. Clever ideas needed!
- There will be lots more experimental data in the near future and to keep pace will be challenging

Thanks for listening and enjoy the rest of the school!