National Nuclear Physics Summer School
MIT, Cambridge, MA
July 18-29 2016

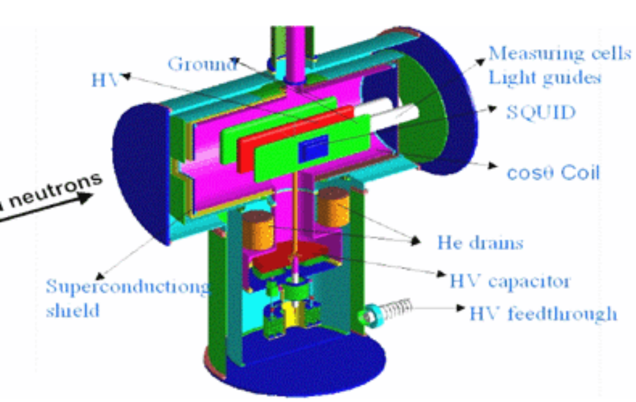
Fundamental Symmetries - I

Vincenzo Cirigliano
Los Alamos National Laboratory

Goal of these lectures

- Introduce the field of nuclear physics dubbed "Fundamental Symmetries"

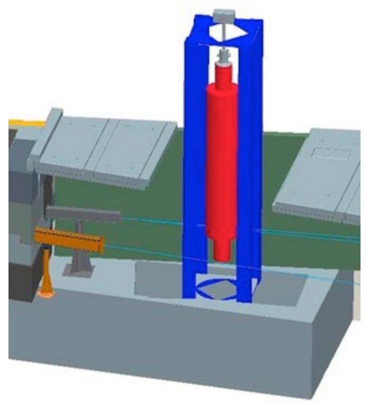
Precision measurements and symmetry tests that aim to challenge the Standard Model (SM) of electroweak interactions and probe/discover its possible extensions (BSM)

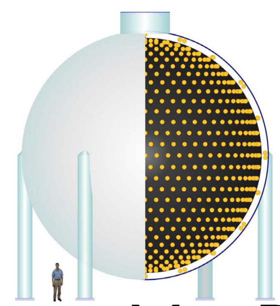


Neutron EDM

nEDM

Nab

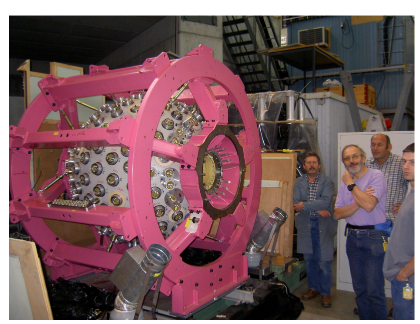


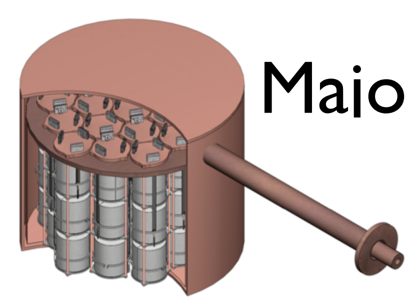


MiniBoone

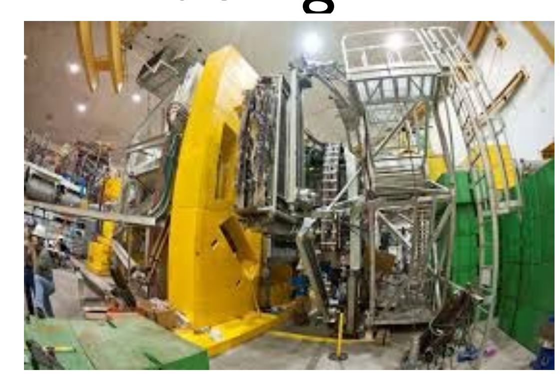
muon g-2

PEN





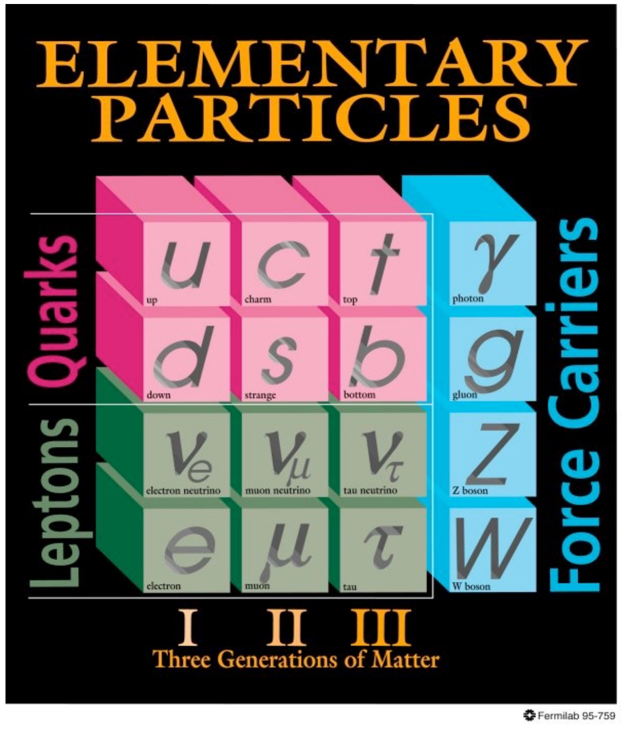
Majorana



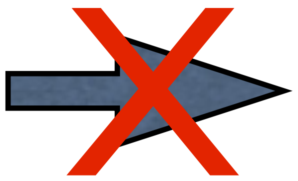
Qweak

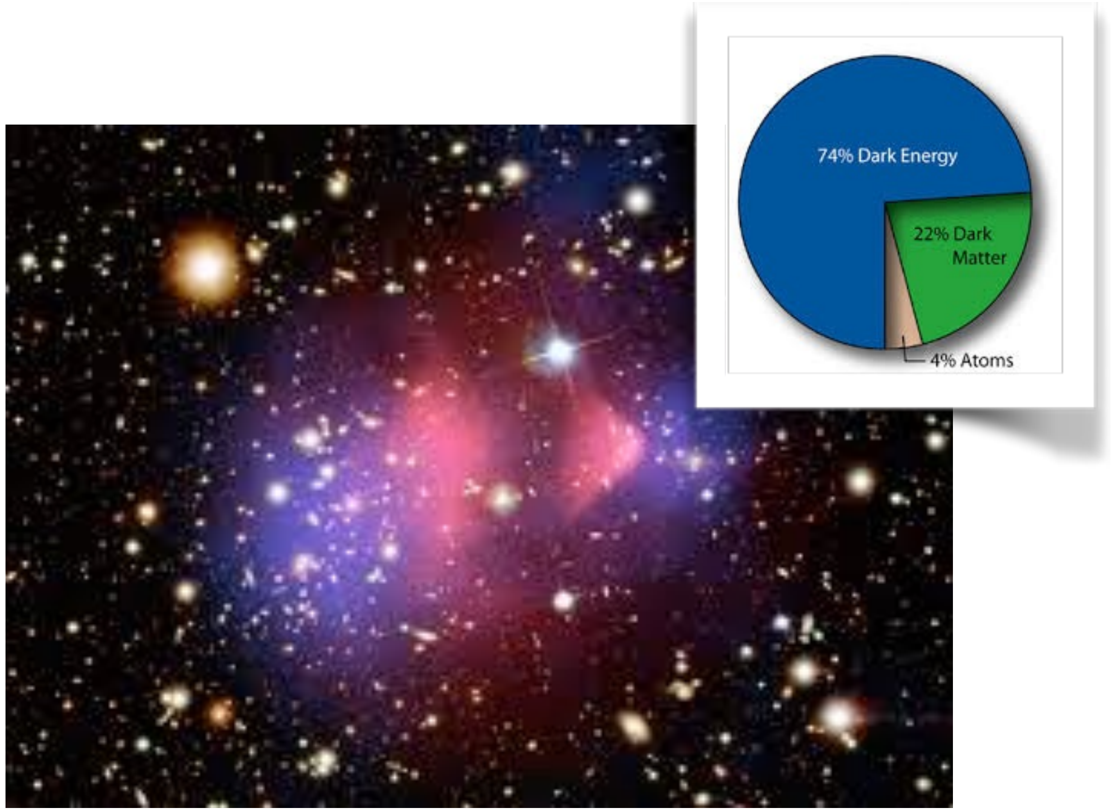
. . .

Fundamental Symmetries: why bother?



+ Higgs boson





No Matter, no Dark Matter, no Dark Energy

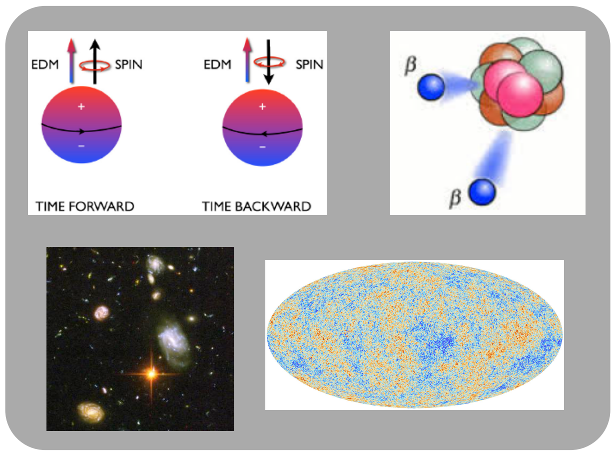
While remarkably successful in explaining phenomena over a wide range of energies, the SM has major shortcomings

Nuclear physics and "The new SM"

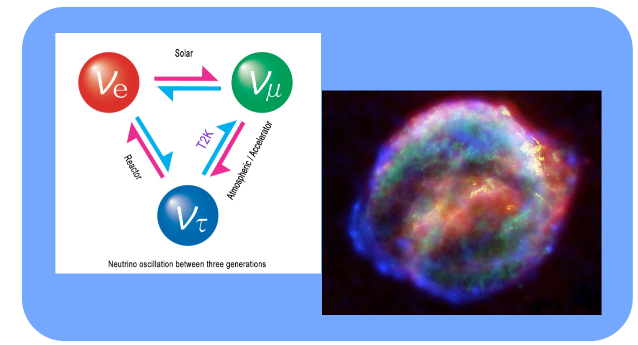
Nuclear physics plays an important role in searching for the new SM

EDMs, 0νββ, KATRIN, …

CPV ν oscillations (DUNE, …)

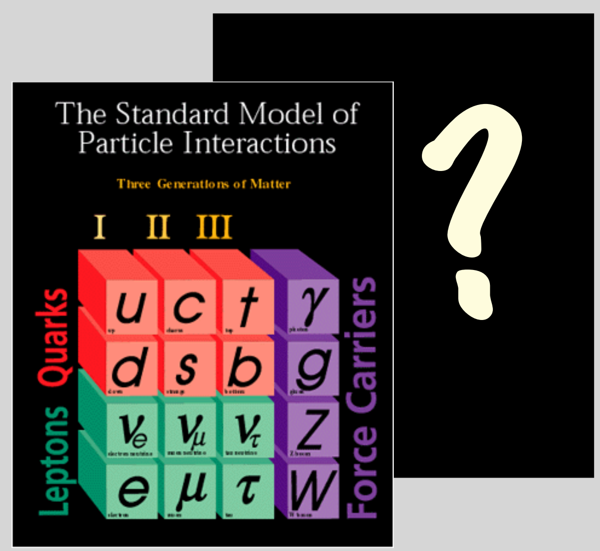


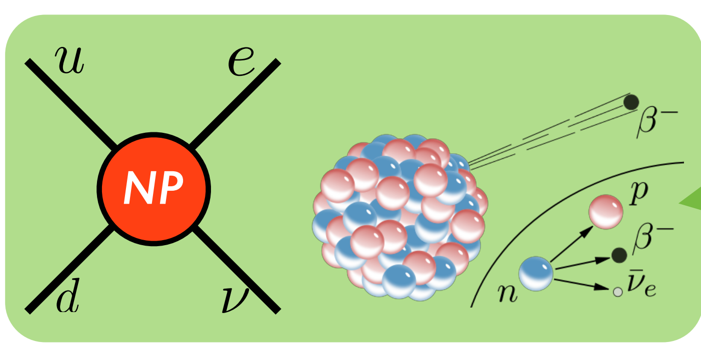
Broken symmetries (CP and L) and the Origin of Matter



Nature and properties of neutrinos, and their impact on the Cosmos

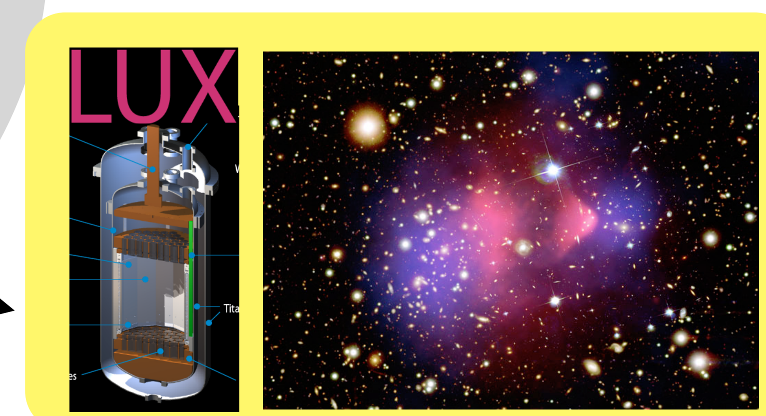
New Standard Model





Precision Measurements probe New Particles and Interactions

β-decays, APV, PVES, … g-2



The Nuclear Physics of Dark Matter

Dark γ, Z, …

Direct detection

Nuclear physics and "The new SM"

- High impact "fundamental symmetry" experiments come with their set of challenges (high precision, low backgrounds, …)

- Challenge for theory: want to extract information on new physics by using hadrons and nuclei as "laboratory"

- Interpretation of experimental results (positive or null!) requires interface with nucleon and nuclear structure

Overarching theme: multi-scale problem

BSM scale (TeV?)

↓

Nucleon scale (chiral EFT, Lattice QCD)

↓

Nuclear scale (nuclear structure)

Flow of the lectures

- Review symmetry and symmetry breaking
- Introduce the Standard Model and its symmetries
- Beyond the SM: an effective theory perspective and overview
- Discuss a number of "worked examples"
    - Precision measurements: charged current (beta decays); neutral current (PVES); muon g-2, ..
    - Symmetry tests: CP (T) violation and EDMs; Lepton Flavor and Lepton Number violation

Flow of the lectures

- Review symmetry and symmetry breaking
- Introduce the Standard Model and its symmetries

1.5 lectures

- Beyond the SM: an effective theory perspective and overview
- Discuss a number of "worked examples"
  - Precision measurements: charged current (beta decays); neutral current (PVES); muon g-2, ..
  - Symmetry tests: CP (T) violation and EDMs; Lepton Flavor and Lepton Number violation

1.5 lectures

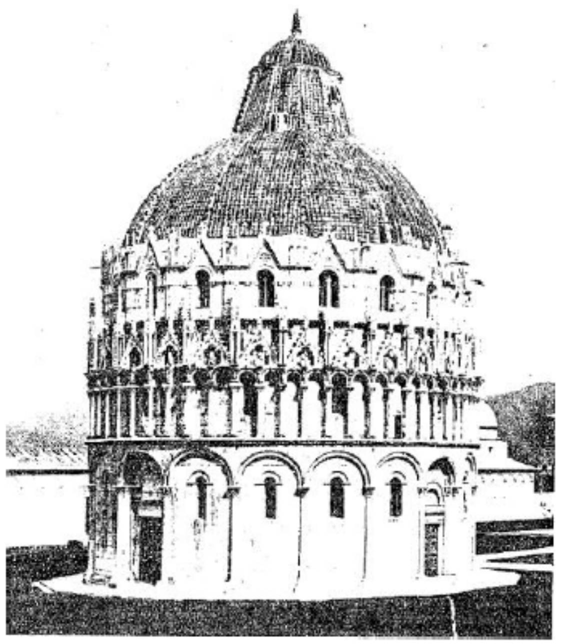
Symmetry
and
symmetry breaking

What is symmetry?

- "A thing** is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before" (Feynman paraphrasing Weyl)

  **An object or a *physical law*

Translational symmetry



Rotational symmetry

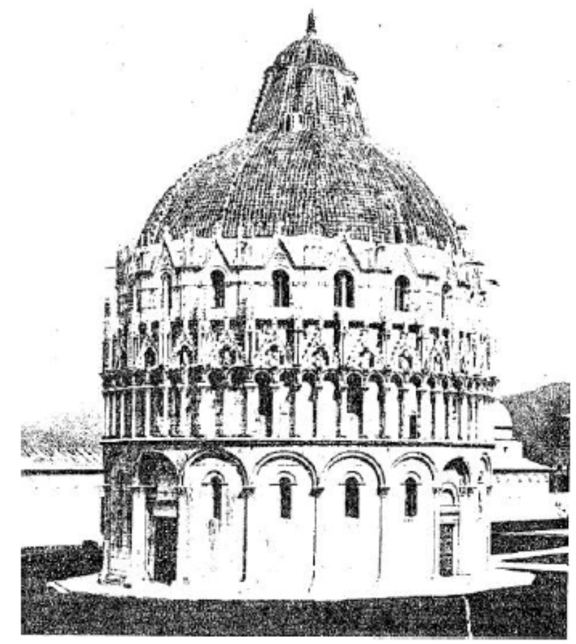
Images from H. Weyl, "Symmetry". Princeton University Press, 1952

What is symmetry?

- "A thing** is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before" (Feynman paraphrasing Weyl)

  **An object or a *physical law*

Translational symmetry



Rotational symmetry

Images from H. Weyl, "Symmetry". Princeton University Press, 1952

- "A *symmetry transformation* is a change in our point of view that does not change the results of possible experiments" (Weinberg)

What is symmetry?

- A transformation of the dynamical variables that leaves the action unchanged (equations of motion invariant)

$$q(t) \to q'(t) = R[q(t)]$$

$$\int_D dt\, \mathcal{L}[q(t), \dot{q}(t)] = \int_D dt\, \mathcal{L}[q'(t), \dot{q}'(t)]$$

$$\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_i} = \frac{\delta \mathcal{L}}{\delta q_i}$$

What is symmetry?

- A transformation of the dynamical variables that leaves the action unchanged (equations of motion invariant)

$$x \to x' \qquad \phi(x) \to \phi'(x') = R\phi(x)$$

$$\int_D d^4x\, \mathcal{L}[\phi(x), \partial_\mu \phi(x)] = \int_{D'} d^4x'\, \mathcal{L}[\phi'(x'), \partial_\mu \phi'(x')]$$

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))} = \frac{\delta \mathcal{L}}{\partial \phi_i(x)}$$

What is symmetry?

- A transformation of the dynamical variables that leaves the action unchanged (equations of motion invariant)

$$x \to x' \qquad \phi(x) \to \phi'(x') = R\phi(x)$$

$$\int_D d^4x \, \mathcal{L}[\phi(x), \partial_\mu \phi(x)] = \int_{D'} d^4x' \, \mathcal{L}[\phi'(x'), \partial_\mu \phi'(x')]$$

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))} = \frac{\delta \mathcal{L}}{\partial \phi_i(x)}$$

- Symmetry transformations have mathematical "group" structure: composition rule, existence of identity and inverse transformation

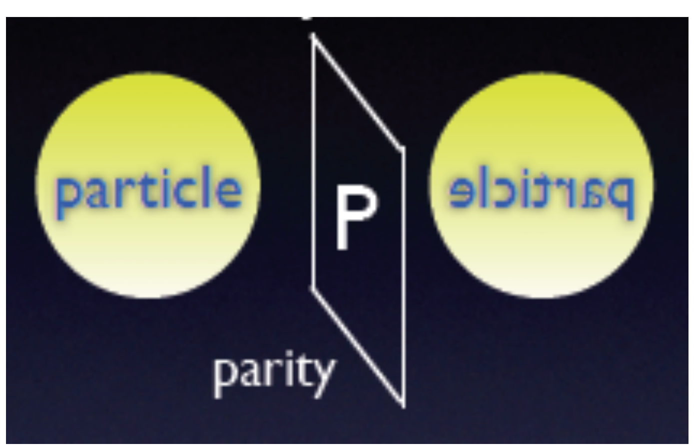
Examples of symmetries

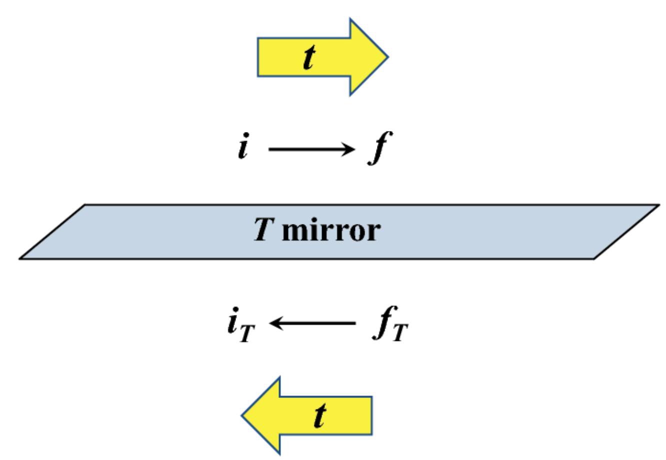
- Space-time
  - Continuous (translations, rotations, boosts: Poincare')

$$x \to x' = \Lambda x - a \qquad \Lambda: \quad t^2 - \mathbf{x}^2 = t'^2 - \mathbf{x}'^2$$

  - Discrete (Parity, Time-reversal)

$$t' = t \quad \mathbf{x}' = -\mathbf{x} \qquad\qquad t' = -t \quad \mathbf{x}' = \mathbf{x}$$





  - Local (general coordinate transformations)

Examples of symmetries

- "Internal"
  - Continuous

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$$

Dirac matrices

$$\psi(x) \;\rightarrow\; e^{i\epsilon}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

Examples of symmetries

- "Internal"
  - Continuous

$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ Dirac matrices

$$\psi(x) \rightarrow e^{i\epsilon}\,\psi(x) \qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

$$\begin{pmatrix} n \\ p \end{pmatrix} \rightarrow e^{i\epsilon^a\,\sigma^a/2}\begin{pmatrix} n \\ p \end{pmatrix}$$

SU(2) - isospin

(if $m_n = m_p$)

Examples of symmetries

- "Internal"
  - Continuous

$$\psi(x) \;\rightarrow\; e^{i\epsilon}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

  - Discrete: charge conjugation, …

$$\phi(x) \rightarrow -\phi(x) \qquad\qquad \mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi^2)$$

  - Local (gauge)

$$\psi(x) \;\rightarrow\; e^{i\epsilon(x)}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

?

Examples of symmetries

- "Internal"
  - Continuous

$$\psi(x) \;\to\; e^{i\epsilon}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

  - Discrete: charge conjugation, …

$$\phi(x) \to -\phi(x) \qquad\qquad \mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi^2)$$

  - Local (gauge)

$$\psi(x) \;\to\; e^{i\epsilon(x)}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

Leftover piece: $-\bar{\psi}\gamma_\mu\psi\,\partial^\mu\epsilon$

Examples of symmetries

- "Internal"
  - Continuous

$$\psi(x) \;\rightarrow\; e^{i\epsilon}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

  - Discrete: charge conjugation, …

$$\phi(x) \rightarrow -\phi(x) \qquad\qquad \mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi^2)$$

  - Local (gauge)

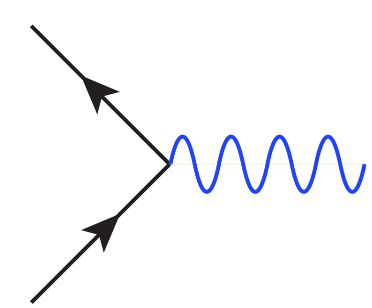
$$\psi(x) \;\rightarrow\; e^{i\epsilon(x)}\,\psi(x)$$

$$A^\mu \;\rightarrow\; A^\mu + \frac{1}{g}\partial^\mu\epsilon$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

$$+gA^\mu\bar{\psi}\gamma_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

U(1)



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

- In Quantum Mechanics
  - Symmetries represented by (anti)-unitary operators U_S (Wigner)
  - U_S commutes with Hamiltonian $[U_S, H] = 0$
  - Classification of the states of the system, selection rules, …

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

- Continuous symmetries imply conservation laws

$$\psi(x) \to e^{i\epsilon}\,\psi(x)$$

Symmetry	Conservation law
Time translation	Energy
Space translation	Momentum
Rotation	Angular momentum
U(1) phase	Electric charge
…	…

Emmy Noether

Implications of symmetry

- If a state is realized in nature, its “transformed” is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

- Continuous symmetries imply conservation laws

$$\psi(x) \to e^{i\epsilon}\,\psi(x)$$

Symmetry	Conservation law
Time translation	Energy
Space translation	Momentum
Rotation	Angular momentum
U(1) phase	#particles - #anti-particles
…	…

Emmy Noether

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

- Continuous symmetries imply conservation laws

$$\begin{aligned} x^\mu &\rightarrow x^\mu + \delta x^\mu \\ \phi(x) &\rightarrow \phi(x) + \delta\phi(x) \end{aligned}$$

$$\begin{aligned} \delta x^\mu &= \epsilon^a A^\mu_a \\ \delta\phi(x) &= \epsilon^a (M_a \phi - A^\mu_a \partial_\mu \phi) \end{aligned}$$

$$\frac{d}{dt}\int d^3x J^0_a(x) = 0$$

$$\partial_\mu J^\mu_a = 0$$

$$J^\mu_a = -\frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))}\,\frac{\delta\phi_i}{\delta\epsilon^a} - \mathcal{L}\,\frac{\delta x^\mu}{\delta\epsilon^a}$$

Emmy Noether

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

- Continuous symmetries imply conservation laws

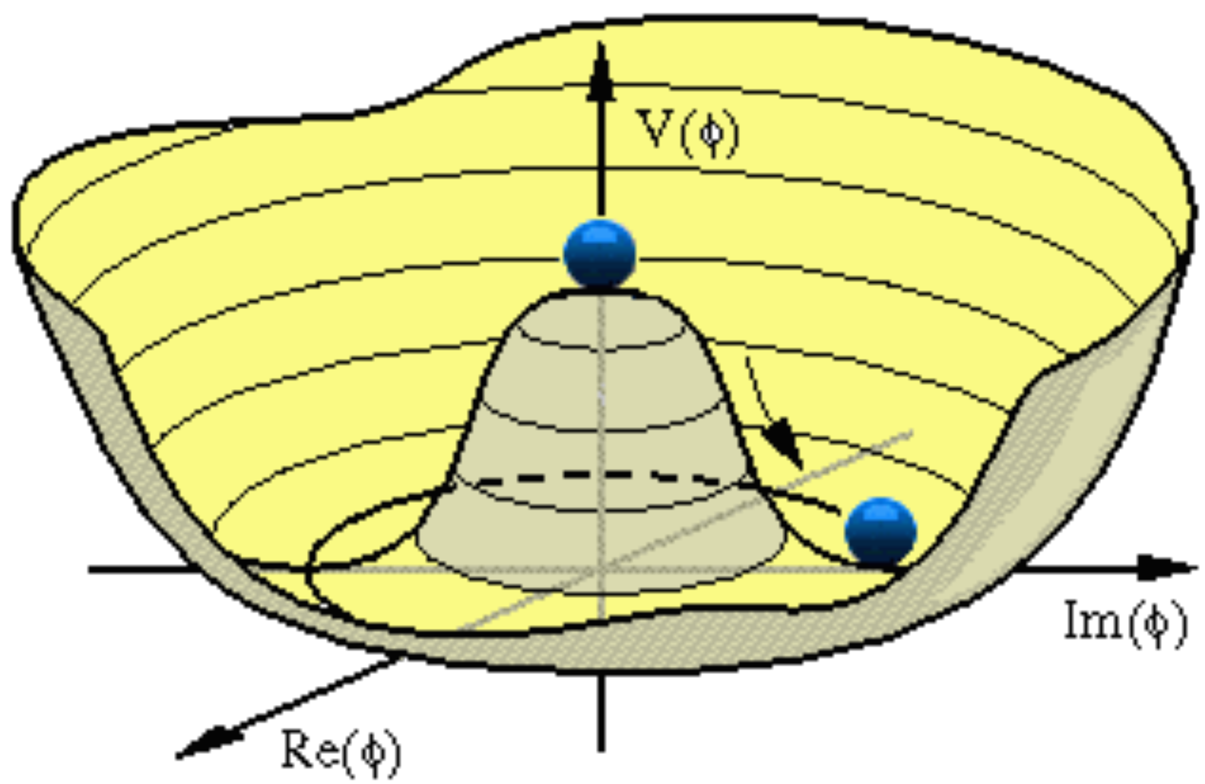
- Symmetry principles strongly constrain or even dictate the form of the laws of physics
    - General relativity
    - …
    - Gauge theories

Symmetry breaking

- Three known mechanisms
  - Explicit symmetry breaking
    - Symmetry is approximate; still very useful (e.g. isospin)
  - Spontaneous symmetry breaking
    - Equations of motion invariant, but ground state is not
  - Anomalous (quantum mechanical) symmetry breaking
    - Classical invariance but no symmetry at QM level

Spontaneous symmetry breaking

- Action is invariant, but ground state is not!
- Continuous symmetry: degenerate physically equivalent minima
- Excitations along the valley of minima → massless states in the spectrum (Goldstone Bosons)



- Many examples of Goldstone bosons in physics: phonons (sound waves) in solids; spin waves in magnets; pions in QCD

Anomalous symmetry breaking

- Action is invariant, but path-integral measure is not!

$$\int [d\psi][d\bar{\psi}]\ e^{iS[\psi,\bar{\psi}]}$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

Anomalous symmetry breaking

- Action is invariant, but path-integral measure is not!

$$\int [d\psi][d\bar{\psi}]\; e^{iS[\psi,\bar{\psi}]}$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

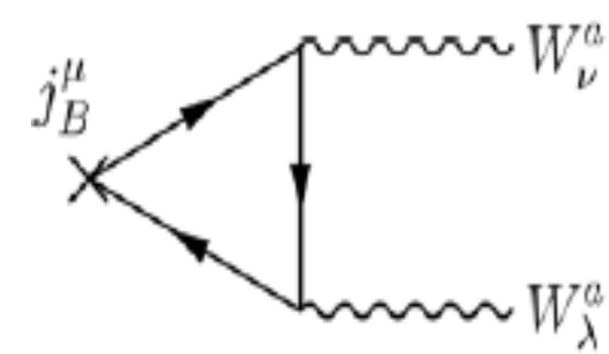
$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

- Important examples: trace (scale invariance) and chiral anomalies

- Baryon (B) and Lepton (L) number are anomalous in the SM

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right)$$



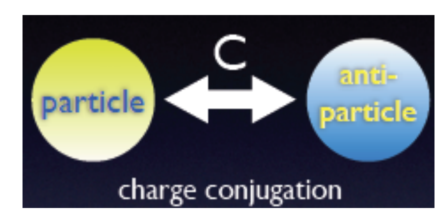
- Only B-L is conserved; B+L is violated; negligible at zero temperature

Symmetry breaking and the origin of matter

- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

Sakharov '67

1. B (baryon number) violation
   - To depart from initial (post inflation) B=0
2. C and CP violation $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
   - To distinguish baryon and anti-baryon production



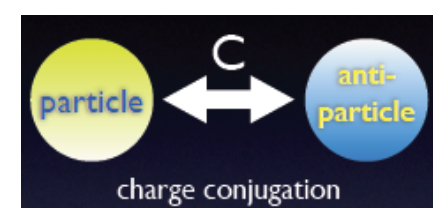
3. Departure from thermal equilibrium
   - <B(t)>=<B(0)>=0 in equilibrium

Symmetry breaking and the origin of matter

- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

Sakharov '67

1. B (baryon number) violation
   - To depart from initial (post inflation) B=0
2. C and CP violation $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
   - To distinguish baryon and anti-baryon production



3. Departure from thermal equilibrium
   - <B(t)>=<B(0)>=0 in equilibrium

Symmetry breaking and the origin of matter

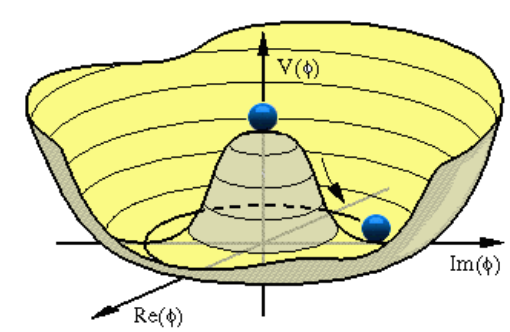
- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

- In weak-scale baryogenesis scenarios (T~100 GeV), the ingredients are tied to all known mechanisms of symmetry breaking:

Sakharov '67

1. B (baryon number) violation — anomalous
2. C and CP violation — explicit
3. Departure from thermal equilibrium — spontaneous (symmetry restoration at hight T: 1st order phase transition?)

$\langle\phi\rangle \neq 0 \Rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$



More on gauge symmetry

- Classical electrodynamics: $A_\mu \rightarrow A_\mu + \partial_\mu \varphi$ does not change **E** and **B**

> This gauge invariance is, of course, an artificial one, similar to that which we could obtain by introducing into our equations the location of a ghost. The equations must then be invariant with respect to changes of coordinates of that ghost. One does not see, in fact, what good the introduction of the coordinate of the ghost does.

E.Wigner

More on gauge symmetry

- Classical electrodynamics: $A_\mu \to A_\mu + \partial_\mu \varphi$ does not change **E** and **B**

> This gauge invariance is, of course, an artificial one, similar to that which we could obtain by introducing into our equations the location of a ghost. The equations must then be invariant with respect to changes of coordinates of that ghost. One does not see, in fact, what good the introduction of the coordinate of the ghost does.

E.Wigner

- Dramatic paradigm shift in the 60's and 70's: gauge invariance requires the existence of massless spin-1 particles (the gauge bosons)
- Successful description of strong and electroweak interactions

> "Symmetry dictates dynamics"

C. N. Yang

> "We now suspect that all fundamental symmetries are gauge symmetries"

D. Gross

Non abelian gauge symmetry

- Recall U(1) (abelian) example

$$\psi(x) \rightarrow e^{i\epsilon(x)}\psi(x)$$

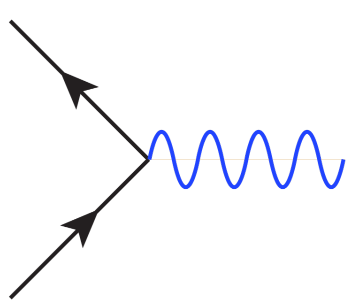
$$A^\mu \rightarrow A^\mu + \frac{1}{g}\partial^\mu\epsilon$$

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi + g\,\bar{\psi}\gamma_\mu\,A^\mu\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Form of the interaction:

$$\mathcal{L}_{\rm int} = g\,A_\mu\,J^\mu$$



$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

conserved current associated with global U(1)

Non abelian gauge symmetry

- Generalize to non-abelian group G (e.g. SU(2), SU(3), …). $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \end{pmatrix}$

$$\psi(x) \;\rightarrow\; U(x)\,\psi(x) \qquad U(x) = e^{i\epsilon^a(x)T^a} \qquad [T^a, T^b] = i f^{abc} T^c$$

- Invariant dynamics if introduce new vector fields $A_\mu = A_\mu^a\, T^a$ transforming as

$$A^\mu \;\rightarrow\; U\, A^\mu\, U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi \,+\, g\,\bar{\psi}\gamma^\mu\, T^a A_\mu^a\,\psi \,-\, \frac{1}{2}\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \qquad F_{\mu\nu} \rightarrow U\, F_{\mu\nu}\, U^\dagger$$

Non abelian gauge symmetry

- Generalize to non-abelian group G (e.g. SU(2), SU(3), ...). $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdots \end{pmatrix}$

$$\psi(x) \rightarrow U(x)\,\psi(x) \qquad U(x) = e^{i\epsilon^a(x)T^a} \qquad [T^a, T^b] = i f^{abc} T^c$$

- Invariant dynamics if introduce new vector fields $A_\mu = A_\mu^a T^a$ transforming as

$$A^\mu \rightarrow U\,A^\mu\,U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger$$

$$\mathcal{L} = \bar\psi\,(i\gamma_\mu\partial^\mu - m)\,\psi + g\,\bar\psi\gamma^\mu\,T^a A_\mu^a\,\psi - \frac{1}{2}\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})$$

$$\bar\psi\,i\gamma^\mu D_\mu\,\psi \qquad D_\mu \equiv \partial_\mu - igT^a A_\mu^a$$

covariant derivative

Non abelian gauge symmetry

- Generalize to non-abelian group G (e.g. SU(2), SU(3), …). $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \end{pmatrix}$

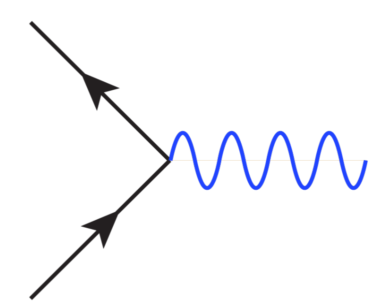
$$\psi(x) \;\to\; U(x)\,\psi(x) \qquad U(x) = e^{i\epsilon^a(x) T^a} \qquad [T^a, T^b] = i f^{abc} T^c$$

- Invariant dynamics if introduce new vector fields $A_\mu = A_\mu^a\, T^a$ transforming as

$$A^\mu \;\to\; U\,A^\mu\,U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger$$

- Form of the interaction:

$$\mathcal{L}_{\rm int} \;=\; g\,A_\mu^a\,J^{\mu,a} \qquad J^{\mu,a} = \bar\psi\gamma^\mu T^a\psi$$



conserved currents associated with global G symmetry

Spontaneously broken gauge symmetry

- Abelian Higgs model: complex scalar field coupled to EM field

$$\phi(x) \rightarrow e^{i\epsilon(x)}\phi(x)$$

$$A^\mu \;\rightarrow\; A^\mu - \frac{1}{e}\partial^\mu \epsilon$$

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^* \phi)^2$$

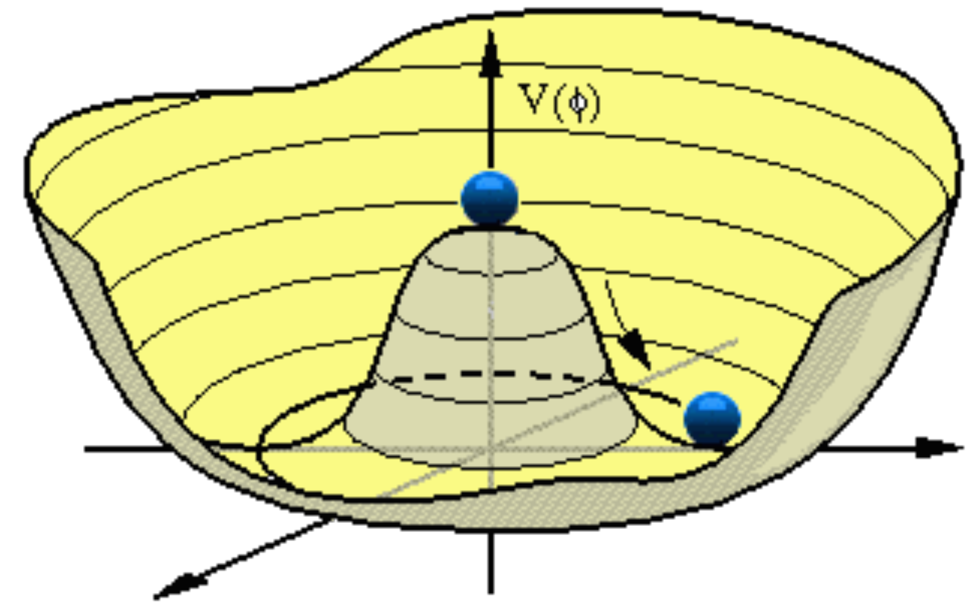
$$D_\mu = \partial_\mu + ieA_\mu$$

$\mu^2 < 0 \qquad |\langle \phi \rangle| = 0$ QED of charged scalar boson

$\mu^2 > 0 \qquad |\langle \phi \rangle| = |\phi_0| = \left(\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}$ U(1) spontaneously broken

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)} \left(\phi_0 + \frac{\beta(x)}{\sqrt{2}}\right)$$



$$\phi_0 = \frac{\mu}{\sqrt{\lambda}}$$

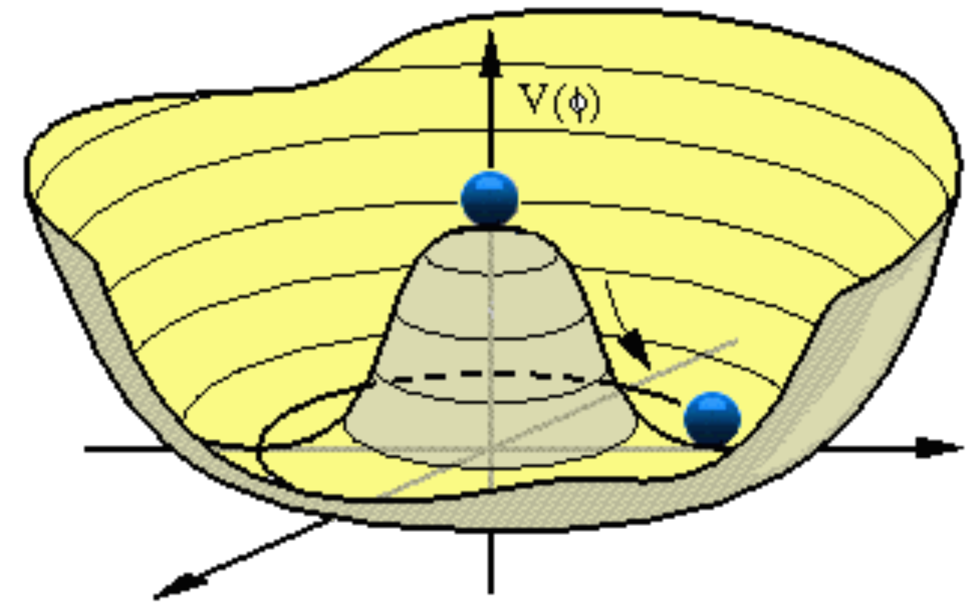
$$V(\phi) = -\frac{1}{2}\frac{\mu^4}{\lambda} + \mu^2\beta^2(x) + O(\beta^3(x))$$

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\beta)^2 + e^2\left(\phi_0 + \frac{\beta(x)}{\sqrt{2}}\right)^2 (A_\mu - \partial_\mu\alpha)^2$$

- $\beta(x)$ describes massive scalar field $m_\beta^2 = 2\lambda\phi_0^2$
- $\alpha(x)$ can be removed from the theory by a gauge transformation

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)} \left(\phi_0 + \frac{\beta(x)}{\sqrt{2}}\right)$$



$$\phi_0 = \frac{\mu}{\sqrt{\lambda}}$$

$$V(\phi) = -\frac{1}{2}\frac{\mu^4}{\lambda} + \mu^2\beta^2(x) + O(\beta^3(x))$$

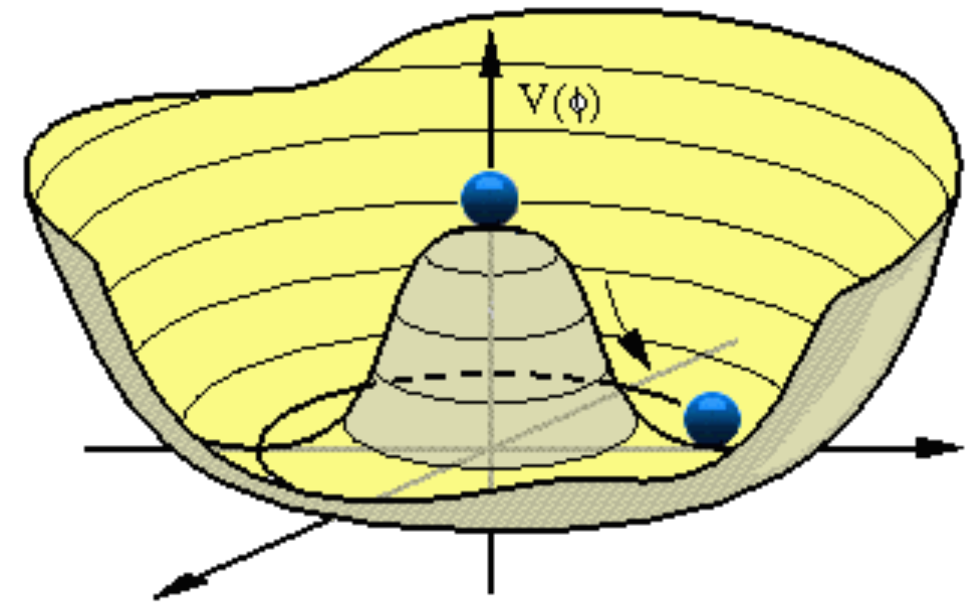
$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\beta)^2 + e^2\left(\phi_0^2 + \sqrt{2}\phi_0\beta(x) + \frac{\beta^2(x)}{2}\right) A_\mu A^\mu$$

- β(x) describes massive scalar field $m_\beta^2 = 2\lambda\phi_0^2$
- α(x) can be removed from the theory by a gauge transformation
- Photon has acquired mass $m_A^2 = 2e^2\phi_0^2$

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)} \left(\phi_0 + \frac{\beta(x)}{\sqrt{2}} \right)$$

$$\phi_0 = \frac{\mu}{\sqrt{\lambda}}$$



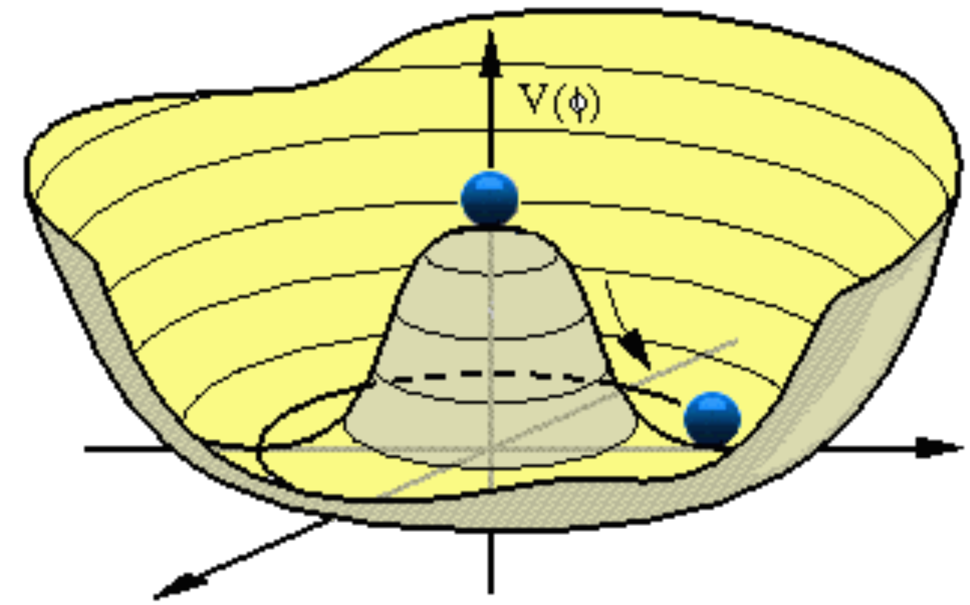
$$V(\phi) = -\frac{1}{2}\frac{\mu^4}{\lambda} + \mu^2\beta^2(x) + O(\beta^3(x))$$

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\beta)^2 + e^2\left(\phi_0^2 + \sqrt{2}\phi_0\beta(x) + \frac{\beta^2(x)}{2}\right) A_\mu A^\mu$$

- Count degrees of freedom:
  - Massless vector (2) + complex scalar (2) = 4
  - Massive vector (3) + real scalar (1) = 4

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)} \left(\phi_0 + \frac{\beta(x)}{\sqrt{2}} \right)$$



$$V(\phi) = -\frac{1}{2}\frac{\mu^4}{\lambda} + \mu^2\beta^2(x) + O(\beta^3(x))$$

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\beta)^2 + e^2 \left(\phi_0^2 + \sqrt{2}\phi_0\beta(x) + \frac{\beta^2(x)}{2} \right) A_\mu A^\mu$$

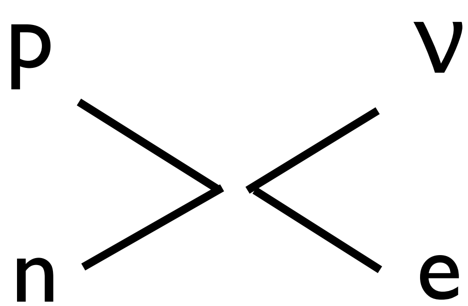
- Higgs phenomenon holds beyond U(1) model: in a gauge theory with SSB, Goldstone modes appear as longitudinal polarization of *massive* spin-1 gauge bosons

The Standard Model and its symmetries

The making of the Standard Model

(theory-centric and simplified perspective)

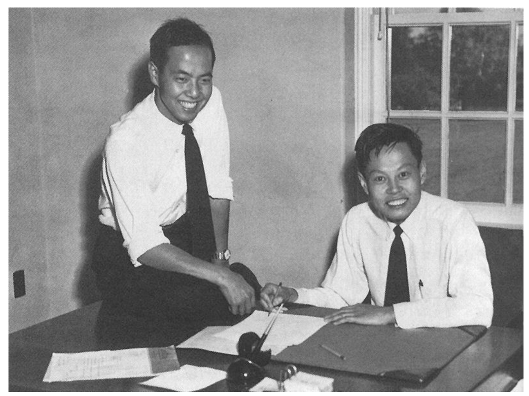
Fermi, 1934

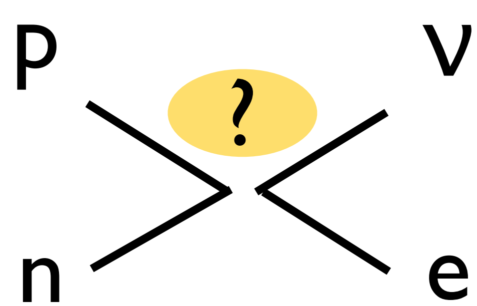


Current-current,
parity conserving

Fermi scale:
$G_F^{-1/2} \sim 200$ GeV

Lee and Yang, 1956





Parity conserving:
VV, AA, SS, TT ...
Parity violating: VA, SP, ...

Marshak & Sudarshan,
Feynman & Gell-Mann 1958

It's (V-A)*(V-A) !!

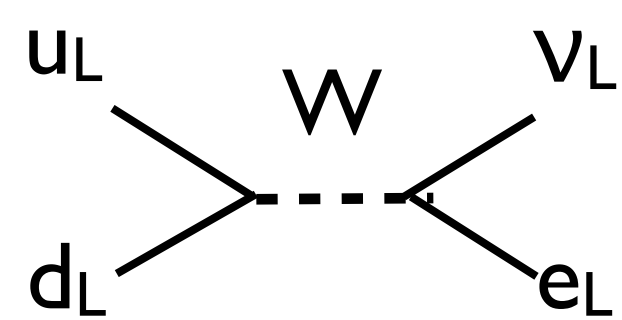
"V-A was the key"
S.Weinberg

Glashow,
Salam,
Weinberg

Sheldon Lee Glashow

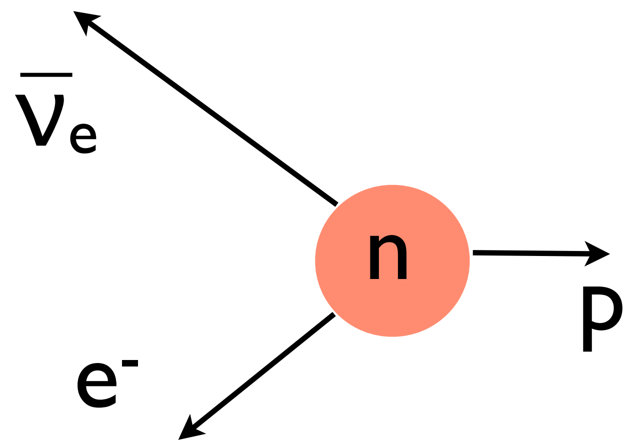
Abdus Salam

Steven Weinberg



Embed in non-abelian chiral gauge theory,
predict neutral currents

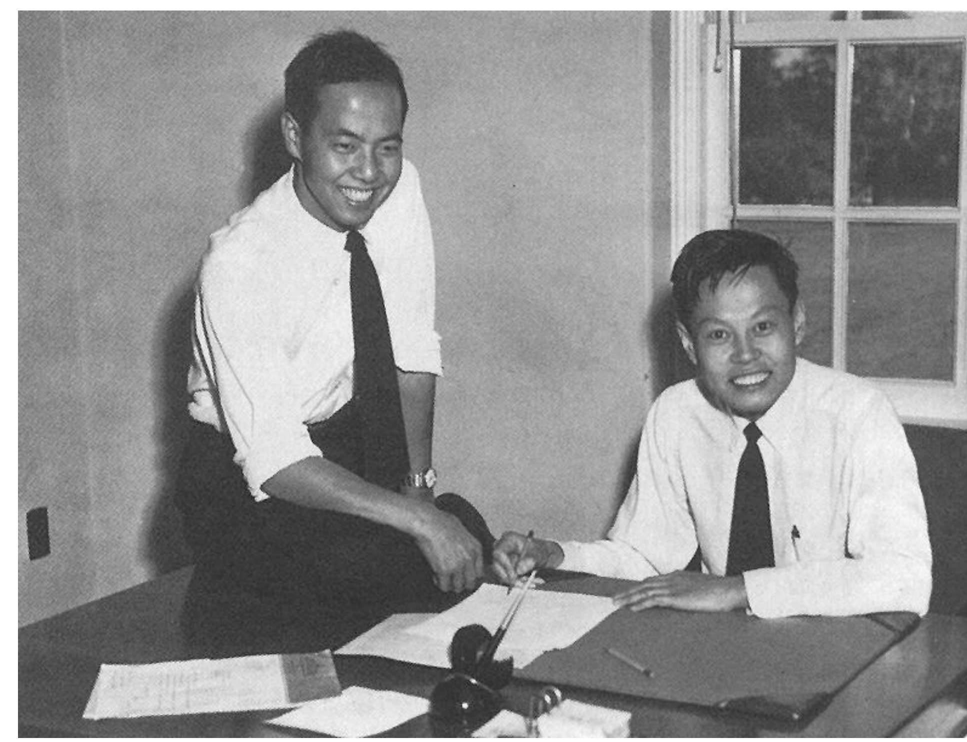
From Fermi theory to the V-A theory of β decay



Fermi

1934

Lee and Yang



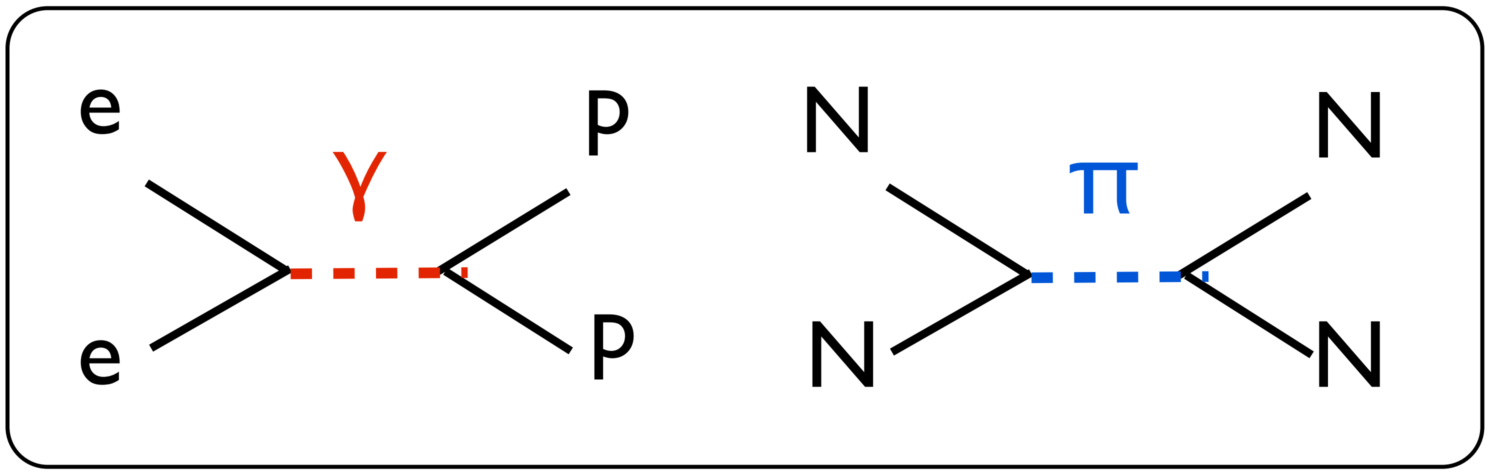
1956

An example of effective field theory “ante litteram”

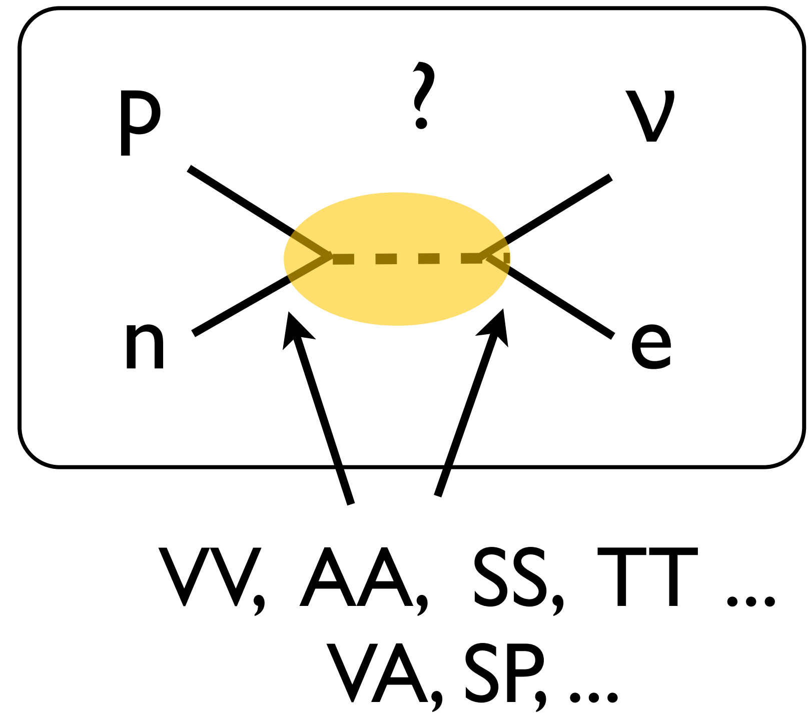
EFT approach to β decay

- Simplified picture

"SM"



"BSM"



★ "Standard Model" (E~GeV): QED + strong interactions

★ Neutron ($n \to p\, e\, \nu_e$), pion, and muon decay are rare processes

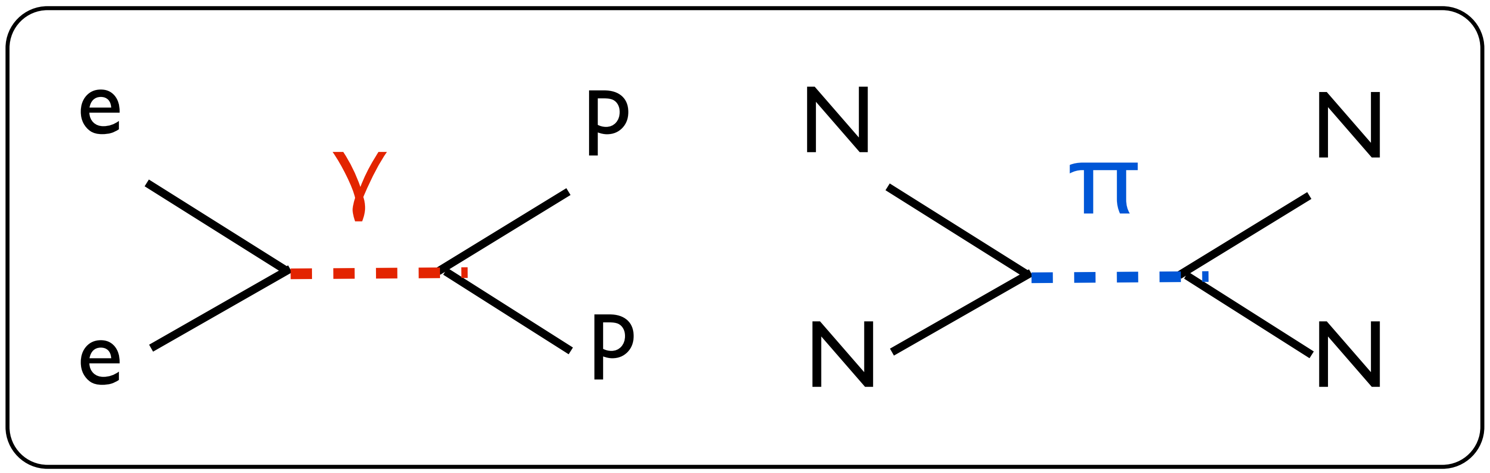
★ "New physics" mediating weak processes originates at $\Lambda_W >> 1\text{GeV}$

★ Describe the new physics through L_{eff}

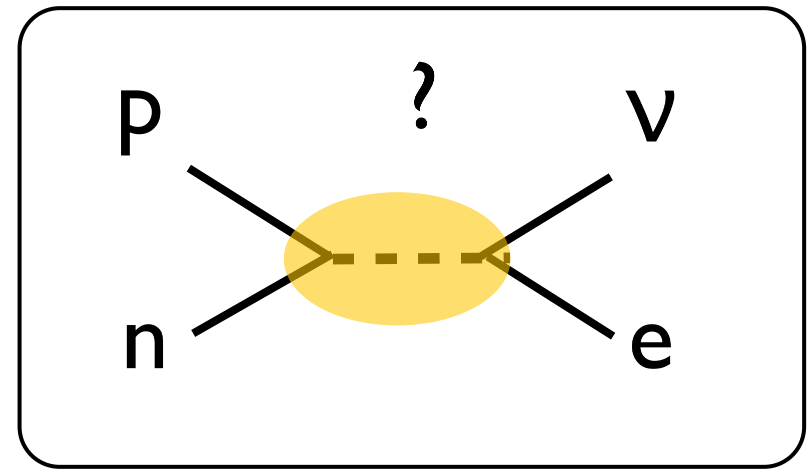
EFT approach to β decay

- Simplified picture

"SM"



"BSM"



- Identify ingredients for EFT description:

massless spin 1/2 with in principle both helicity states

★ Degrees of freedom: n, p, e, $(\nu_e)_{L/R} = (1 \pm \gamma_5)/2\ \nu_e$

★ Symmetries: Lorentz, $U(1)_{EM}$ gauge invariance, P,C,T (?)

★ Power counting in E/Λ_W: non-derivative 4-fermion interactions

- Most general interaction involves product of fermion bilinears

Dimensionless coefficients

$$\mathcal{L}_{\rm eff} \supset \frac{c_{12}}{\Lambda_W^2} \quad \bar{p}\,\Gamma_1\, n \quad \bar{e}\,\Gamma_2\,\nu_e$$

Scale of weak interactions

Operators of mass dimension 6
(recall [Ψ] = $m^{3/2}$)
that conserve electric charge

Dirac structures:

$$\Gamma_i \;=\; I,\; \gamma_5,\; \gamma_\mu,\; \gamma_\mu\gamma_5,\; \sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$$

S P V A T

- Most general interaction involves product of fermion bilinears
- Impose Lorentz invariance: $\mathcal{L}_{\rm eff} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \; \bar{e}\gamma^\mu (C_V + C_V' \gamma_5)\nu_e \; + \; \bar{p}\gamma_\mu\gamma_5 n \; \bar{e}\gamma^\mu\gamma_5 (C_A + C_A' \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \; \bar{e}(C_S + C_S' \gamma_5)\nu_e \; + \; \bar{p}\gamma_5 n \; \bar{e}\gamma_5 (C_P + C_P' \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2}\,\bar{p}\sigma_{\mu\nu} n \; \bar{e}\sigma^{\mu\nu} (C_T + C_T' \gamma_5)\nu_e \; + \; \text{h.c.}$$

$C_i \sim (1/\Lambda_W)^2$

- Most general interaction involves product of fermion bilinears
- Impose Lorentz invariance: $\mathcal{L}_{\rm eff} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \;\; \bar{e}\gamma^\mu (C_V + C_V' \gamma_5)\nu_e \;+\; \bar{p}\gamma_\mu\gamma_5 n \;\; \bar{e}\gamma^\mu\gamma_5 (C_A + C_A' \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \;\; \bar{e}(C_S + C_S' \gamma_5)\nu_e \;+\; \bar{p}\gamma_5 n \;\; \bar{e}\gamma_5 (C_P + C_P' \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2}\,\bar{p}\sigma_{\mu\nu} n \;\; \bar{e}\sigma^{\mu\nu} (C_T + C_T' \gamma_5)\nu_e \;+\; \text{h.c.}$$

$C_i \sim (1/\Lambda_W)^2$

- What about discrete symmetries P, C, T ?

Interlude on discrete symmetries

- Parity $\quad \mathbf{x} \to -\mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to \mathbf{s}$
  - Implemented by unitary operator $P\psi(\mathbf{x}) = \psi(-\mathbf{x})$
  - If [H,P] = 0, P cannot change in a reaction; expectation values of P-odd operators vanish

- Time reversal $\quad t \to -t \qquad \mathbf{x} \to \mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to -\mathbf{s}$
  - Implemented by anti-unitary operator $T\psi(\mathbf{x}) = U\,\psi^*(\mathbf{x})$: U flips the spin
  - If H is real in coordinate representation, T is a good symmetry ([T,H]=0)

- Charge conjugation $\quad |p\rangle \leftrightarrow |\bar{p}\rangle$
  - Particles that coincide with antiparticles are eigenstates of C, e.g. $C|\gamma\rangle = -|\gamma\rangle$
  - C-invariance ([C,H]=0) → C cannot change in a reaction:

Interlude on discrete symmetries

- Parity $\quad \mathbf{x} \to -\mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to \mathbf{s}$
    - Implemented by unitary operator $P\psi(\mathbf{x}) = \psi(-\mathbf{x})$
    - If [H,P] = 0, P cannot change in a reaction; expectation values of P-odd operators vanish
- Time reversal $\quad t \to -t \qquad \mathbf{x} \to \mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to -\mathbf{s}$
    - Implemented by <u>anti-unitary operator</u> $T\psi(\mathbf{x}) = U\,\psi^*(\mathbf{x})$: U flips the spin
    - If H is real in coordinate representation, T is a good symmetry ([T,H]=0)

- Charge conjugation $\quad |p\rangle \leftrightarrow |\bar{p}\rangle$
    - Particles that coincide with antiparticles are eigenstates of C, e.g. $C|\gamma\rangle = -|\gamma\rangle$
    - C-invariance ([C,H]=0) → C cannot change in a reaction:

Interlude on discrete symmetries

- Parity $\quad \mathbf{x} \to -\mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to \mathbf{s}$
    - Implemented by unitary operator $P\psi(\mathbf{x}) = \psi(-\mathbf{x})$
    - If [H,P] = 0, P cannot change in a reaction; expectation values of P-odd operators vanish

CPT theorem: hermitian & Lorentz invariant Lagrangian transforms as

$$\mathcal{L}(x) \;\to\; \mathcal{L}(-x)$$

CPT invariance! CP violation is equivalent to T violation

- Charge conjugation $\quad |p\rangle \leftrightarrow |\bar{p}\rangle$
    - Particles that coincide with antiparticles are eigenstates of C, e.g. $C|\gamma\rangle = -|\gamma\rangle$
    - C-invariance ([C,H]=0) → C cannot change in a reaction:

Back to $\mathcal{L}_{\text{eff}}$ for β decays

- Impose Lorentz invariance: $\mathcal{L}_{\text{eff}} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \;\bar{e}\gamma^\mu (C_V + C'_V\gamma_5)\nu_e \;+\; \bar{p}\gamma_\mu\gamma_5 n \;\bar{e}\gamma^\mu\gamma_5 (C_A + C'_A\gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \;\bar{e}(C_S + C'_S\gamma_5)\nu_e \;+\; \bar{p}\gamma_5 n \;\bar{e}\gamma_5 (C_P + C'_P\gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2}\bar{p}\sigma_{\mu\nu} n \;\bar{e}\sigma^{\mu\nu} (C_T + C'_T\gamma_5)\nu_e \;+\; \text{h.c.}$$

- Transformation properties of fermion bilinears

Bilinear	P	C	T	CP	CPT
$\overline{\psi}_1\psi_2$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_2\psi_1$
$\overline{\psi}_1\gamma_5\psi_2$	$-\overline{\psi}_1\gamma_5\psi_2$	$\overline{\psi}_2\gamma_5\psi_1$	$-\overline{\psi}_1\gamma_5\psi_2$	$-\overline{\psi}_2\gamma_5\psi_1$	$\overline{\psi}_2\gamma_5\psi_1$
$\overline{\psi}_1\gamma_\mu\psi_2$	$\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma_\mu\psi_1$	$\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma^\mu\psi_1$	$-\overline{\psi}_2\gamma_\mu\psi_1$
$\overline{\psi}_1\gamma_\mu\gamma_5\psi_2$	$-\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$	$\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi}_2\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$
$\overline{\psi}_1\sigma_{\mu\nu}\psi_2$	$\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma_{\mu\nu}\psi_1$	$-\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma^{\mu\nu}\psi_1$	$\overline{\psi}_2\sigma_{\mu\nu}\psi_1$

Back to $\mathcal{L}_{\text{eff}}$ for β decays

- Impose Lorentz invariance: $\mathcal{L}_{\text{eff}} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \ \bar{e}\gamma^\mu (C_V + C'_V \gamma_5)\nu_e \ + \ \bar{p}\gamma_\mu\gamma_5 n \ \bar{e}\gamma^\mu\gamma_5 (C_A + C'_A \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \ \bar{e}(C_S + C'_S \gamma_5)\nu_e \ + \ \bar{p}\gamma_5 n \ \bar{e}\gamma_5 (C_P + C'_P \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2}\bar{p}\sigma_{\mu\nu}n \ \bar{e}\sigma^{\mu\nu}(C_T + C'_T \gamma_5)\nu_e \ + \ \text{h.c.}$$

- P-invariance ⇔ $C_i' = 0$ or $C_i = 0$
- C-invariance ⇔ C_i real, C_i' imaginary (up to overall phase)
- T-invariance ⇔ C_i, C_i' real (up to overall phase)

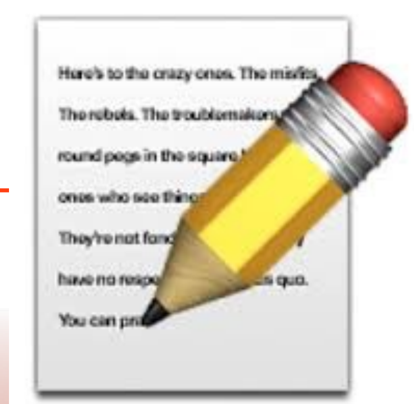
Back to $\mathcal{L}_{\text{eff}}$ for β decays

- Impose Lorentz invariance: $\mathcal{L}_{\text{eff}} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T$

$$-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \; \bar{e}\gamma^\mu (C_V + C_V' \gamma_5)\nu_e + \bar{p}\gamma_\mu\gamma_5 n \; \bar{e}\gamma^\mu\gamma_5 (C_A + C_A' \gamma_5)\nu_e$$

$$-\mathcal{L}_{S,P} = \bar{p}n \; \bar{e}(C_S + C_S' \gamma_5)\nu_e + \bar{p}\gamma_5 n \; \bar{e}\gamma_5 (C_P + C_P' \gamma_5)\nu_e + \text{h.c.}$$

$$-\mathcal{L}_T = \frac{1}{2} \bar{p}\sigma_{\mu\nu} n \; \bar{e}\sigma^{\mu\nu} (C_T + C_T' \gamma_5)\nu_e + \text{h.c.}$$



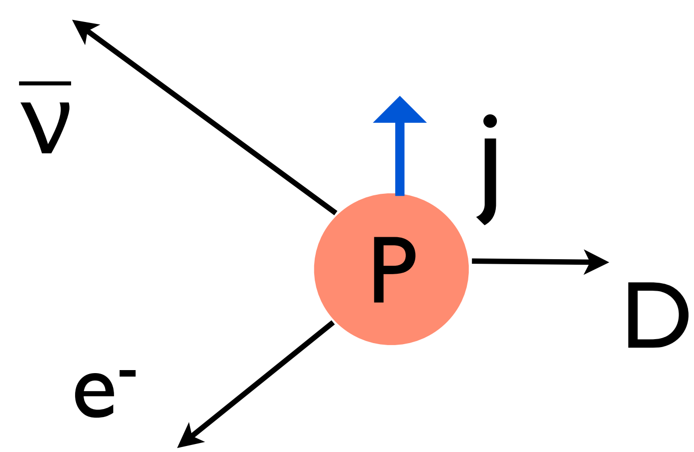
- P-invariance ⇔ $C_i' = 0$ or $C_i = 0$
- C-invariance ⇔ C_i real, C_i' imaginary (up to overall phase)
- T-invariance ⇔ C_i, C_i' real (up to overall phase)

Phenomenology with $\mathcal{L}_{\text{eff}}$

- Various structures imply different pattern of decay correlations

Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e)\left\{1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \frac{\langle\vec{J}\rangle}{|\vec{J}|}\cdot\left[\,A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots\right]\right\}$$



Phenomenology with $\mathcal{L}_{\rm eff}$

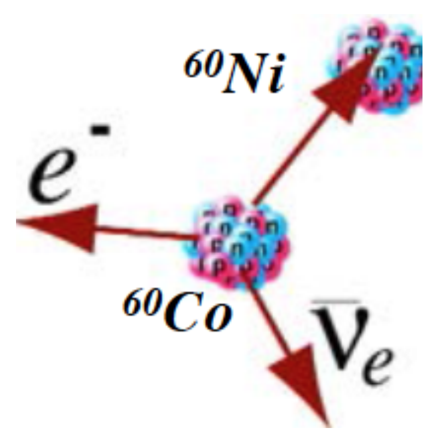
- Various structures imply different pattern of decay correlations

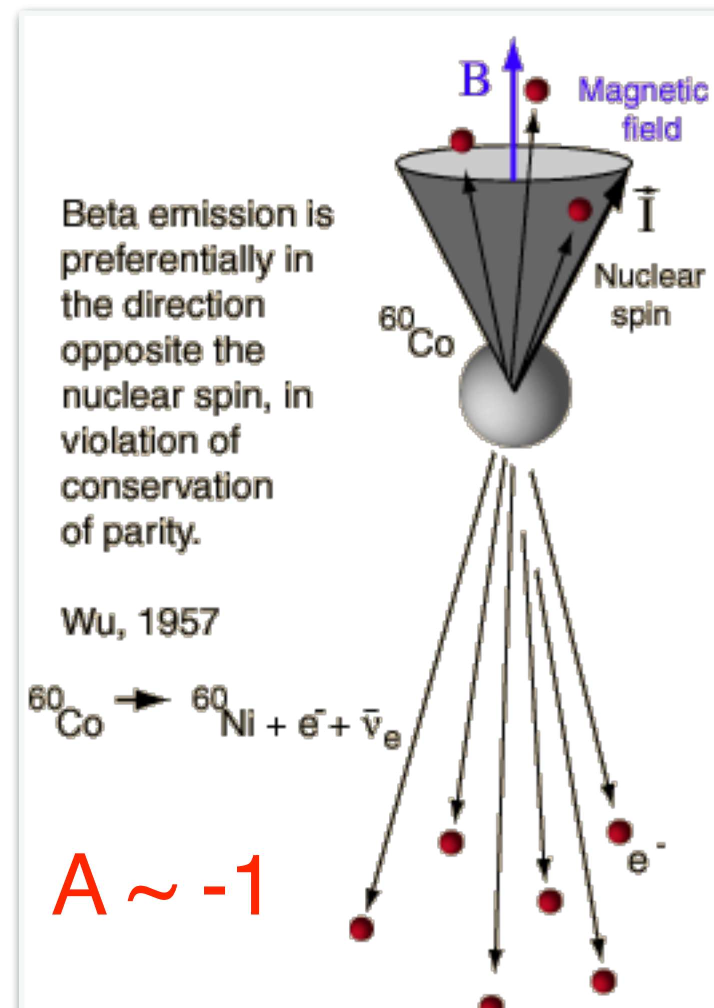
Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e) \left\{ 1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \frac{\langle \vec{J} \rangle}{|\vec{J}|} \cdot \left[A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots \right] \right\}$$

- Discovery of parity violation!
  Non-zero A implies P-violation:

$$\vec{J} \to \vec{J}$$
$$\vec{p}_e \to -\vec{p}_e$$





Phenomenology with $\mathcal{L}_{\text{eff}}$

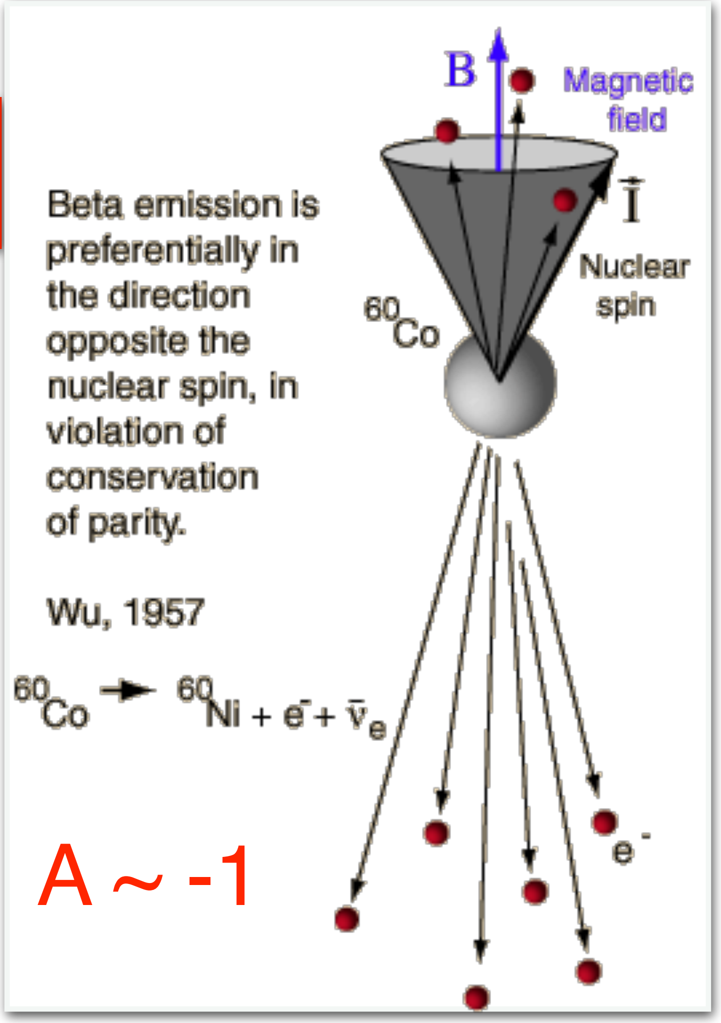
- Various structures imply different pattern of decay correlations

Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e)\left\{1 + b\frac{m_e}{E_e} + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \frac{\langle\vec{J}\rangle}{|\vec{J}|}\cdot\left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + \cdots\right]\right\}$$

- Information on electron helicity $h_e = \hat{J}_e\cdot\hat{p}_e$
  - $\Delta J_{nucl} = 1 \rightarrow$ e (and ν) spin aligned with J_{nucl}
  - Therefore $d\Gamma \propto [1 + (v_e/c)A\,h_e]$

$$\langle h_e\rangle = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} = A\,v_e/c = -v_e/c$$



Phenomenology with $\mathcal{L}_{\text{eff}}$

- Various structures imply different pattern of decay correlations

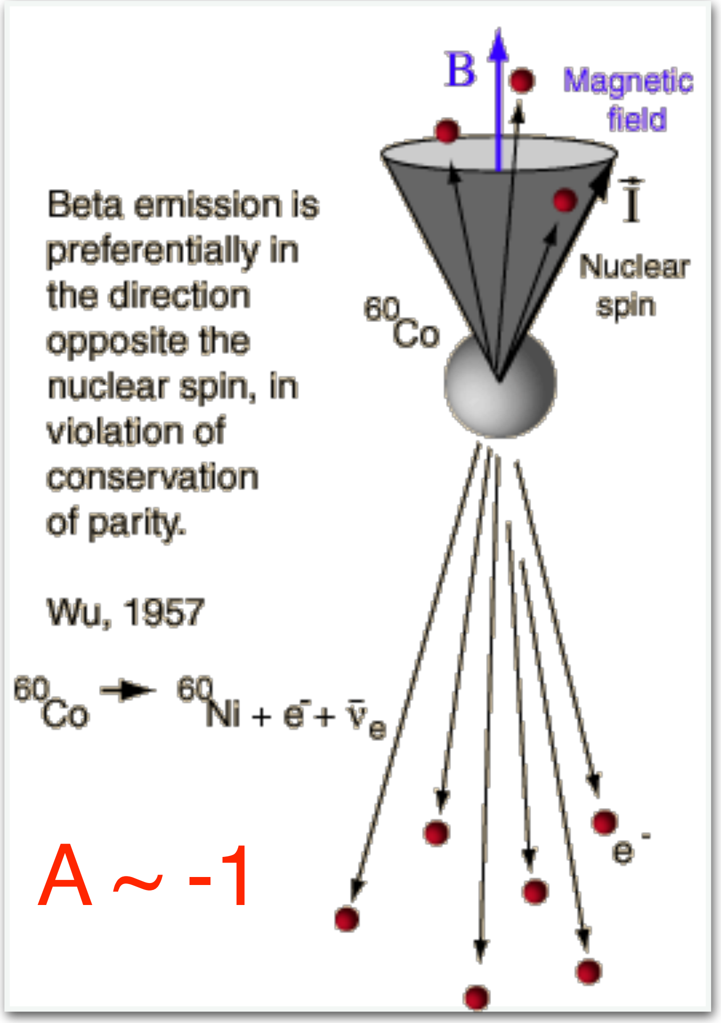
Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e) \left\{ 1 + b \frac{m_e}{E_e} + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \frac{\langle \vec{J} \rangle}{|\vec{J}|} \cdot \left[A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} + \cdots \right] \right\}$$

- Information on neutrino helicity from electron-capture reactions

$$e N_J \to \nu N'_{J'}$$

  - Exploit correlation between neutrino helicity and nuclear polarization
  - Find $h_\nu = -1$



Phenomenology with $\mathcal{L}_{\text{eff}}$

- Various structures imply different pattern of decay correlations

Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e) \left\{ 1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \frac{\langle \vec{J} \rangle}{|\vec{J}|} \cdot \left[A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots \right] \right\}$$

- In summary, only left-handed leptons (right-handed antilepton) participate in weak interactions

$$h(e^-) = -v/c \qquad h(\nu) = -1$$
$$h(e^+) = +v/c \qquad h(\bar{\nu}) = +1$$

- Described by $(\nu_e)_L = (1 - \gamma_5)/2\ \nu_e$ and $e_L = (1 - \gamma_5)/2\ e$

P (and C) are maximally violated

Phenomenology with $\mathcal{L}_{\text{eff}}$

- Various structures imply different pattern of decay correlations

Jackson-Treiman-Wyld 1957

$$d\Gamma \propto F(E_e) \left\{ 1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \frac{\langle \vec{J} \rangle}{|\vec{J}|} \cdot \left[A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots \right] \right\}$$

- Another example: e-ν correlation in Fermi β^+ decay (a(V)=+1, a(S)=−1)

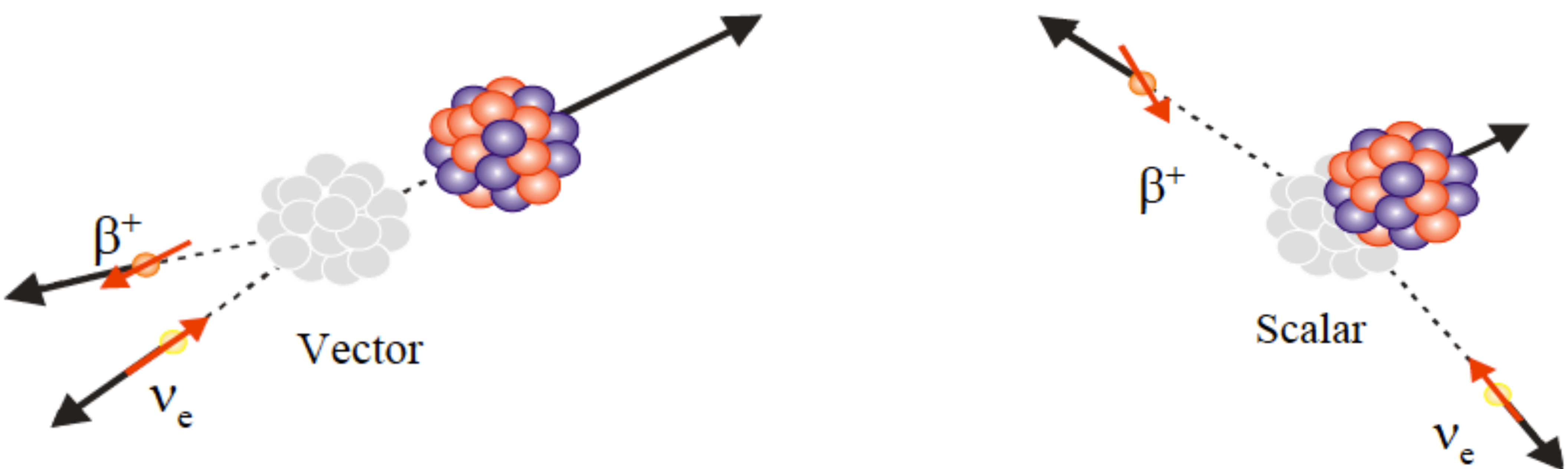


Figure from Nathal Severijns

Phenomenology with $\mathcal{L}_{\text{eff}}$

- Experimental information on β-decays (rates, correlations) ⇒

$$C_V \equiv \frac{1}{\Lambda_W^2} \qquad \Lambda_W \sim 350\,\text{GeV}$$

$$C_A \sim C_V$$

$$C_V = C'_V \quad C_A = C'_A$$

$$C_{S,P,T}/C_V,\; C'_{S,P,T}/C_V \;\leq\; \text{few } \%$$

- Weak decays probe scales of O(100 GeV) >> $m_{n,p}$!!
- P (and C) maximally violated; chiral nature of the weak couplings**
- Information on nature of underlying force mediators ($\Lambda_{S,T} \geq$ TeV)

- ** CP still conserved in this framework

The V-A theory

- By 1958 it became clear that a "universal" theory of weak interaction accounting for μ and β decays and μ capture had the V-A structure

$$\mathcal{L}_{\text{eff}} = 2\sqrt{2} G_F \, J_\mu^- J^{\mu+}$$

$$G_F^{-1/2} \simeq 250 \text{ GeV}$$

$$J_\mu^+ = \bar{p}\gamma_\mu \frac{1-\gamma_5}{2} n + \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e \qquad J_\mu^+ = (J_\mu^-)^\dagger$$

The V-A theory

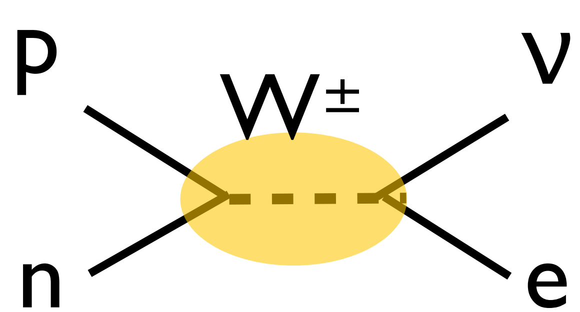
- By 1958 it became clear that a "universal" theory of weak interaction accounting for μ and β decays and μ capture had the V-A structure

$$\mathcal{L}_{\rm eff} = 2\sqrt{2} G_F \, J_\mu^- J^{\mu +} \qquad G_F^{-1/2} \simeq 250 \text{ GeV}$$

$$J_\mu^+ = \bar{p}\gamma_\mu \frac{1-\gamma_5}{2} n + \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e \qquad J_\mu^+ = (J_\mu^-)^\dagger$$

- Non-unitary behavior at high energy & non-renormalizable*: what is the underlying theory?

- By analogy with QED, it was conjectured that this interaction results from the exchange of a massive spin-1 vector boson



$$\mathcal{L}_{\rm int} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^+ + h.c. \longrightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

From the V-A theory to a gauge theory

Marshak & Sudarshan,
Feynman & Gell-Mann 1958

Glashow,
Salam,
Weinberg

Sheldon Lee Glashow

Abdus Salam

Steven Weinberg

An exercise in model building

Reference: R. Barbieri, lectures on "The Standard Model of Electroweak Interactions",
Proceedings of the 2nd European School in High Energy Physics, Sorrento, Italy, 1994. Vol. 1,

From the V-A theory to a gauge theory

- Only known way to have a consistent (=UV finite) theory of massive vector bosons: make them gauge bosons of SSB gauge symmetry

- To identify the gauge symmetry group, match interaction suggested by phenomenology (V-A structures)…

$$\mathcal{L}_{\rm int} = \frac{g}{\sqrt{2}} W^+_\mu \, J^+_\mu \; + \; h.c. \qquad J^+_\mu = \bar{p}\gamma_\mu \frac{1-\gamma_5}{2} n \; + \; \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e \qquad J^+_\mu = (J^-_\mu)^\dagger$$

- … to the general form of (non-abelian) gauge interaction

$$\mathcal{L}_{\rm int} \; = \; g \, A^a_\mu \, J^{\mu,a} \qquad J^{\mu,a} = \bar{\psi}\gamma^\mu T^a \psi$$

- First re-write the current in terms of L-handed fermion "doublets"

$$J_\mu^+ = \bar{p}\gamma_\mu \frac{1-\gamma_5}{2} n \;+\; \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e \quad \longrightarrow \quad J_\mu^+ = \bar{N}_L \gamma_\mu \frac{\sigma^+}{2} N_L \;+\; \bar{L}_L \gamma_\mu \frac{\sigma^+}{2} L_L$$

$$N_L = \frac{1-\gamma_5}{2}\begin{pmatrix} p \\ n \end{pmatrix} \qquad L_L = \frac{1-\gamma_5}{2}\begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \sigma^\pm = \sigma_1 \pm i\sigma_2$$

$$\frac{\sigma^+}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\psi_{L,R} = \frac{1 \mp \gamma_5}{2}\psi$$

$\Psi_{L,R}$: chiral fields. For m=0,

Ψ_L : L-handed (h=-1) particles, R-handed anti-particles (h=+1)

Ψ_R : R-handed (h=+1) particles, L-handed anti-particles (h=-1)

- First re-write the current in terms of L-handed fermion "doublets"

$$J_\mu^+ = \bar{p}\gamma_\mu \frac{1-\gamma_5}{2} n \; + \; \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e \quad \longrightarrow \quad J_\mu^+ = \bar{N}_L \gamma_\mu \frac{\sigma^+}{2} N_L \; + \; \bar{L}_L \gamma_\mu \frac{\sigma^+}{2} L_L$$

$$N_L = \frac{1-\gamma_5}{2}\begin{pmatrix} p \\ n \end{pmatrix} \qquad L_L = \frac{1-\gamma_5}{2}\begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \sigma^\pm = \sigma_1 \pm i\sigma_2$$

- Identify $\sigma^\pm/2$ with generators $T^\pm$ of a non-abelian gauge group. Commutation relation gives diagonal generator, closed algebra!

$$[T^+, T^-] = \left[\frac{\sigma^+}{2}, \frac{\sigma^-}{2}\right] = \sigma^3 = 2T^3$$

$$\downarrow$$

- Group includes SU(2) and theory predicts interactions of the doublets with a neutral gauge boson (associated with T_3)

- Is the neutral gauge boson the photon? In other words, can we identify the T_3 generator with Q (electric charge)? No, because
  - The eigenvalues of Q and T_3 are different
  - Q acts on both L- and R-handed charged fermions, while $T_{3,\pm}$ act only on L-handed fermions (key from phenomenology)

- Is the neutral gauge boson the photon? In other words, can we identify the T_3 generator with Q (electric charge)?   No, because
  - The eigenvalues of Q and T_3 are different
  - Q acts on both L- and R-handed charged fermions, while $T_{3,\pm}$ act only on L-handed fermions (key from phenomenology)

- However, Q can be represented as $Q = T_3 + Y$, in terms of a new diagonal generator Y (hypercharge) that commutes with $T_{3,\pm}$
- Y acts on both L- and R-handed fields

$$N_L = \frac{1-\gamma_5}{2}\begin{pmatrix} p \\ n \end{pmatrix} \qquad Q = T_3 + \frac{1}{2} \qquad YN_L = \frac{1}{2}N_L$$

$$L_L = \frac{1-\gamma_5}{2}\begin{pmatrix} \nu \\ e \end{pmatrix} \qquad Q = T_3 - \frac{1}{2} \qquad YL_L = -\frac{1}{2}L_L$$

$$Yp_R = p_R \ , \ Yn_R = 0 \ , \ Ye_R = -e_R \ , \ Y\nu_R = 0.$$

- So we end up with $\{T_{3,\pm}, Y\}$ generators; minimal group is SU(2) x U(1):
    - unifies weak and electromagnetic interactions
    - predicts neutral current coupling to a new neutral gauge boson, distinct from the photon

Starting from the effective Fermi interaction,
uncovered a candidate gauge symmetry of nature!

- So we end up with $\{T_{3,\pm}, Y\}$ generators; minimal group is SU(2) x U(1):
    - unifies weak and electromagnetic interactions
    - predicts neutral current coupling to a new neutral gauge boson, distinct from the photon

- Complete picture: replace nucleons with quarks

$$N_L = \frac{1-\gamma_5}{2}\begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow \quad Q_L = \frac{1-\gamma_5}{2}\begin{pmatrix} u \\ d \end{pmatrix} \qquad YQ_L = \frac{1}{6}Q_L$$

- Assignments of Y are made empirically to match known charges

	u_L	d_L	(u_R)	(d_R)	ν_L	e_L	(e_R)
Y	$1/6$	$1/6$	$2/3$	$-1/3$	$-1/2$	$-1/2$	-1

- So we end up with $\{T_{3,\pm}, Y\}$ generators; minimal group is SU(2) x U(1):
    - unifies weak and electromagnetic interactions
    - predicts neutral current coupling to a new neutral gauge boson, distinct from the photon

- Gauge-Fermion Lagrangian

$$L^{(g)} = -\frac{1}{4}\mathrm{Tr}\mathbf{W}_{\mu\nu}\mathbf{W}_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + i\overline{\Psi}\not{D}\Psi$$

$$D_\mu = \partial_\mu - igW^i_\mu t^i - ig'YB_\mu$$

$$\Psi = \begin{pmatrix} Q_L \\ L_L \\ u_R \\ d_R \\ e_R \end{pmatrix}$$

Towards a realistic model

- Pure gauge Lagrangian is unrealistic ⇒ massless fermions and gauge bosons (no gauge-invariant mass term can be written)

- "Minimal Standard Model" solution: add a new scalar EW doublet, the Higgs
  - Couples to gauge bosons
  - Couples to fermion (Yukawa interaction)
  - Has self-coupling potential, leading to spontaneous breaking of the gauge symmetry
  - After SSB fermions and 3 out of 4 gauge bosons acquire mass

The Standard Model

- Gauge group: $SU(3)_c \times SU(2)_W \times U(1)_Y$

$$\psi'(x) = e^{ig_s \alpha_A(x) \frac{\lambda_A}{2}} \psi(x)$$

$$\psi'(x) = e^{ig\beta_a(x) \frac{\sigma_a}{2}} \psi(x)$$

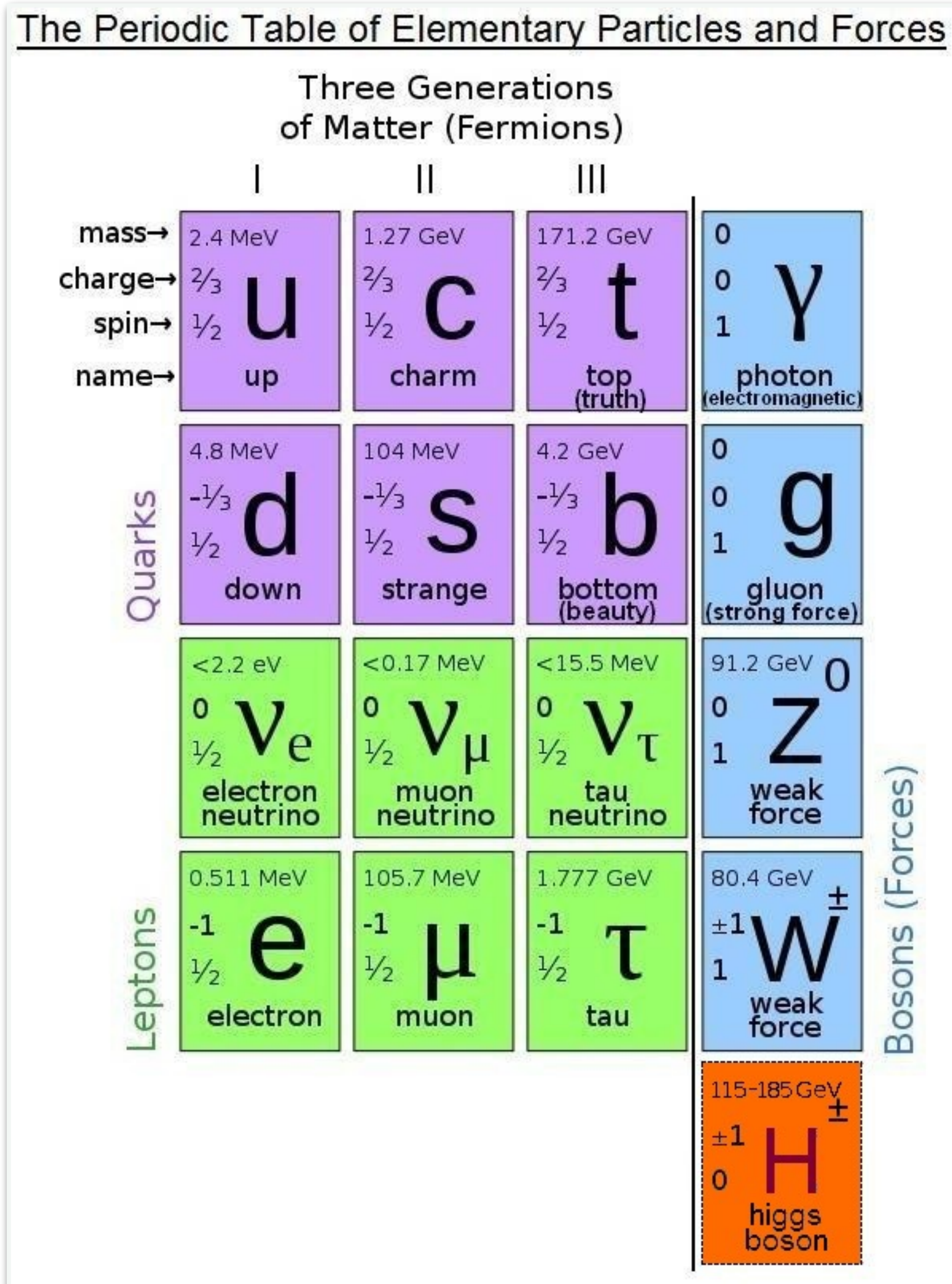
$$\psi'(x) = e^{ig'\gamma(x) Y} \psi(x)$$

Fundamental representation (color triplets and weak doublets)

- Notation for gauge group representations:

$$(\dim[SU(3)_c], \quad \dim[SU(2)_W], \quad Y)$$

- Building blocks



+

- Building blocks details: gauge fields

$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation

gluons:	$G^A_\mu, \quad A = 1 \cdots 8,$ $G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f_{ABC} G^B_\mu G^C_\nu$.	(8,1,0)
W bosons:	$W^I_\mu, \quad I = 1 \cdots 3,$ $W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \varepsilon_{IJK} W^J_\mu W^K_\nu$	(1,3,0)
B boson:	$B_\mu,$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.	(1,1,0)

Gauge transformation:

$$W^I_{\mu\nu} \frac{\sigma^I}{2} \longrightarrow V(x) \left[W^I_{\mu\nu} \frac{\sigma^I}{2} \right] V^\dagger(x)$$

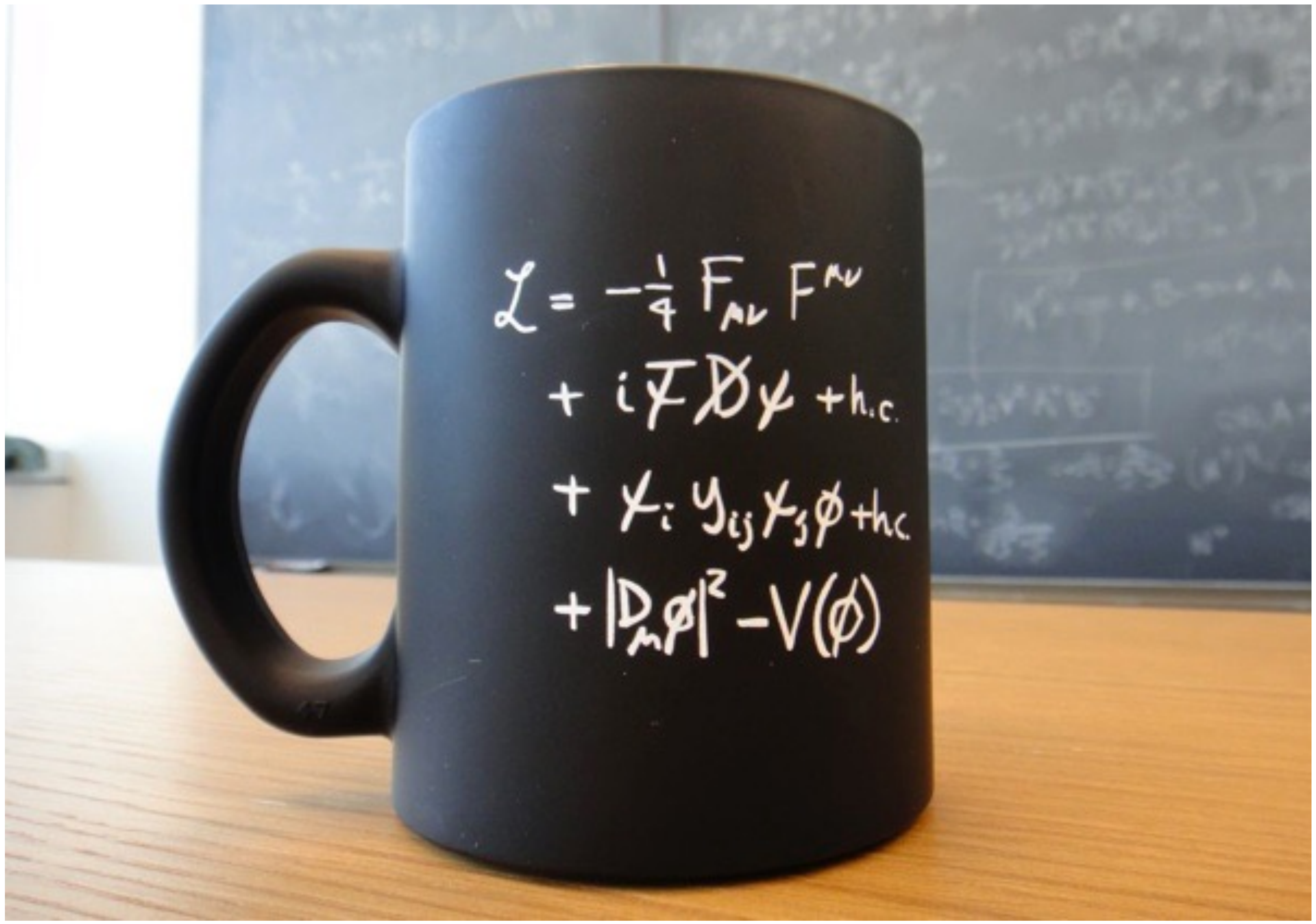
$$V(x) = e^{ig\beta_a(x)\frac{\sigma_a}{2}}$$

- Building blocks details: fermions and Higgs

	$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation	$SU(2)_W$ transformation
$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1,2,-1/2)	$l \to V_{SU(2)}\, l$
$e = e_R$	(1,1,-1)	
$q^i = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	(3,2,1/6)	$q \to V_{SU(2)}\, q$
$u^i = u^i_R$	(3,1,2/3)	
$d^i = d^i_R$	(3,1,-1/3)	
$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	(1,2,1/2)	$\varphi \to V_{SU(2)}\, \varphi$
$\tilde{\varphi} = \epsilon\, \varphi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}$	(1,2,-1/2)	$\tilde{\varphi} \to V_{SU(2)}\, \tilde{\varphi}$

$\epsilon = i\sigma_2$

- SM Lagrangian:



- SM Lagrangian: $$\mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$+ i\bar{\ell}\not{D}\ell \; + \; i\bar{e}\not{D}e \; + \; i\bar{q}\not{D}q \; + \; i\bar{u}\not{D}u \; + \; i\bar{d}\not{D}d$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\varphi)^\dagger(D^\mu\varphi) - \lambda(\varphi^\dagger\varphi - v^2)^2$$

$$\mathcal{L}_{\text{Yukawa}} = Y_e\,\bar{\ell}e\varphi \; + \; Y_d\,\bar{q}d\varphi \; + \; Y_u\,\bar{q}u\tilde{\varphi} \; + \; \text{h.c.}$$

EWSB →

$$\langle\varphi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle\tilde{\varphi}\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$v = 174\,\text{GeV}$$

- Covariant derivative

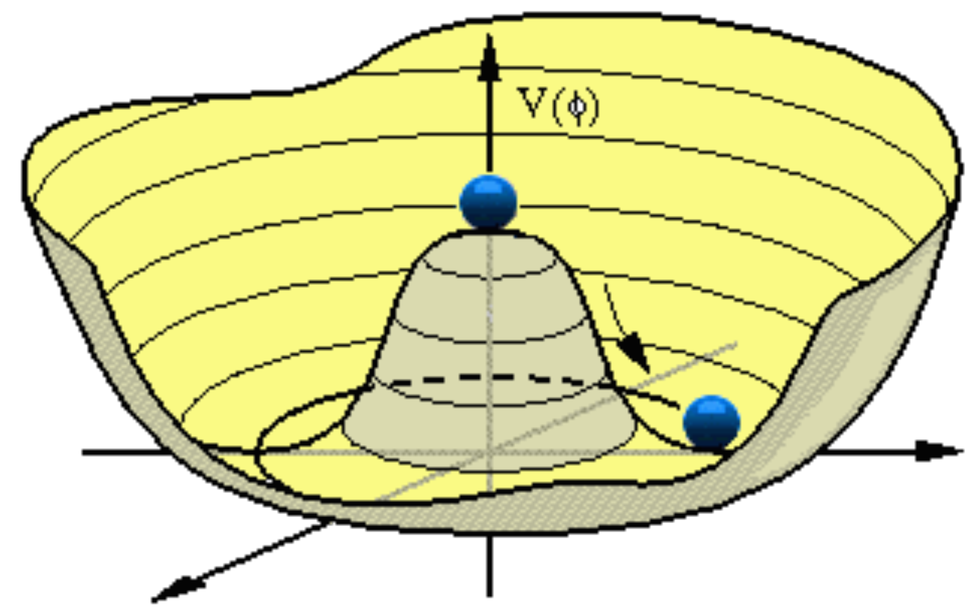
$$D_\mu = I\,\partial_\mu \; - \; ig_s\frac{\lambda^A}{2}G^A_\mu \; - \; ig\frac{\sigma^a}{2}W^a_\mu \; - \; ig'Y\,B_\mu$$

$SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$

- Expand around the minimum of the potential

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - \lambda(\varphi^\dagger \varphi - v^2)^2$$

$$\phi(x) = e^{i\pi_i(x)\sigma_i/v} \begin{pmatrix} 0 \\ v + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$$



- Generalization of the abelian Higgs model discussed in detail earlier on

- $Q = T_3 + Y$ annihilates the vacuum → unbroken $U(1)_{EM}$. Photon remains massless, other gauge bosons ($W^\pm$, Z) acquire mass

$SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$

- Expand around the minimum of the potential

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\varphi)^\dagger(D^\mu\varphi) - \lambda(\varphi^\dagger\varphi - v^2)^2$$

$$\phi(x) = e^{i\pi_i(x)\sigma_i/v}\begin{pmatrix} 0 \\ v + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$$

$$[m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}m_Z^2 Z_\mu Z_\mu](1 + \frac{1}{\sqrt{2}v}h)^2$$

Neutral scalar h couples to $W^\pm$ Z proportionally to their mass squared

$$W_\mu^\pm = 1/\sqrt{2}(W_\mu^1 \pm W_\mu^2) \qquad m_W = gv/\sqrt{2}$$

$$Z_\mu = \cos\theta W_\mu^3 - \sin\theta B_\mu \qquad m_Z = \frac{\sqrt{g^2 + g'^2}}{2}v$$

$$A_\mu = \sin\theta W_\mu^3 + \cos\theta B_\mu \qquad \theta = \arctan\frac{g'}{g}$$

SU(2)W x U(1)Y → U(1)EM

- Expand around the minimum of the potential

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\varphi)^\dagger(D^\mu\varphi) - \lambda(\varphi^\dagger\varphi - v^2)^2$$

$$\phi(x) = e^{i\pi_i(x)\sigma_i/v}\begin{pmatrix} 0 \\ v + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$$

$$[m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}m_Z^2 Z_\mu Z_\mu](1 + \frac{1}{\sqrt{2}v}h)^2$$

Neutral scalar h couples to W± Z proportionally to their mass squared

$$G_F^{-1} = 2\sqrt{2}v^2$$

$$m_W = m_Z \cos\theta$$

$$e = g \sin\theta$$

$SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$

- Expand around the minimum of the potential

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - \lambda(\varphi^\dagger \varphi - v^2)^2$$

$$\phi(x) = e^{i\pi_i(x)\sigma_i/v} \begin{pmatrix} 0 \\ v + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$$

$$[m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu](1 + \frac{1}{\sqrt{2}v} h)^2$$

$$\frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2 - \sqrt{\frac{\lambda}{2}} m_h h^3 - \frac{\lambda}{4} h^4$$

$$m_h = 2\sqrt{\lambda} v$$

Higgs mass controlled by v~174 GeV and Higgs self-coupling

$$G_F^{-1} = 2\sqrt{2} v^2$$

Fermion-Higgs sector: $\mathcal{L}_{\text{Yukawa}}$

$$\mathcal{L}_{\text{Yukawa}} = \bar{e}_L Y_e e_R \left(v + \frac{h}{\sqrt{2}}\right) + \bar{d}_L Y_d d_R \left(v + \frac{h}{\sqrt{2}}\right) + \bar{u}_L Y_d u_R \left(v + \frac{h}{\sqrt{2}}\right) + \text{h.c.}$$

- Fermion mass matrices (i=1,2,3) diagonalized by bi-unitary transf.

$$Y_f = V_{f_L}^{\dagger}\, Y_f^{\text{diag}}\, V_{f_R} \qquad f = e, d, u \qquad \longrightarrow \qquad m_{f,i} = v \left(Y_f^{\text{diag}}\right)_{ii}$$

- Higgs coupling to fermions is flavor diagonal and proportional to mass

$$\mathcal{L}_{\text{Yukawa}} = \sum_{f=e,d,u} m_f\, \bar{f} f \left(1 + \frac{h}{\sqrt{2}v}\right)$$

$$f = f_L + f_R$$

Fermion-gauge sector: $\mathcal{L}_{int} = g\, A_\mu{}^a\, J^{\mu,a}$

- Neutral current

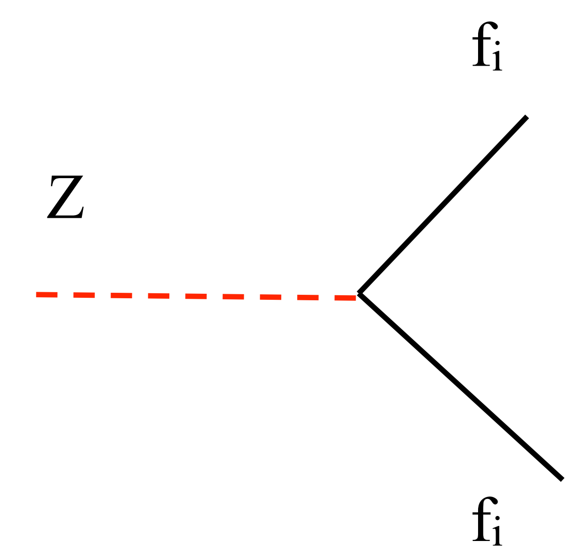
$$\mathcal{L}_{\rm int} = -\frac{g}{2\cos\theta}\, Z^\mu\, \bar\psi_f \left(g_V^{(f)}\gamma_\mu - g_A^{(f)}\gamma_\mu\gamma_5 \right) \psi_f$$

$$\theta = \arctan\frac{g'}{g}$$

$$e = g\sin\theta,$$

$$g_V^{(f)} = T_3^{(f)} - 2\sin^2\theta\, Q^{(f)} \qquad g_A^{(f)} = T_3^{(f)}$$

- Flavor diagonal
- Both V and A: expect P-violation in NC processes
- Both L- and R-handed particles interact (as long as $Q \neq 0$)



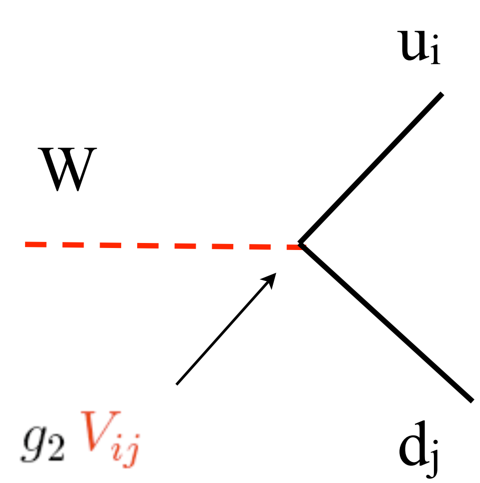
Fermion-gauge sector: $\mathcal{L}_{int} = g\, A_\mu^a\, J^{\mu,a}$

- Charged current:

$$\frac{g}{\sqrt{2}} W^+\ \bar{u}_L \gamma_\mu d_L \ \rightarrow\ \frac{g}{\sqrt{2}} W^+\ \bar{u}'_L\, V_{\rm CKM}\, \gamma_\mu\, d'_L$$

$$V_{\rm CKM} = V_{u_L} V_{d_L}^\dagger$$

Physically observable mismatch in the transformation of u_L and d_L needed to diagonalize quark masses



$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Maksawa matrix

- CKM matrix is unitary:
  - 9 real parameters, but redefinition of quark phases reduces physical parameters to 4: 3 mixing angles and 1 phase

$$V_{ij} \;\longrightarrow\; V_{ij}\; e^{i((\phi_d)_j - (\phi_u)_i)}$$

5 independent parameters (phase differences)

- Irreducible phase implies CP violation:

$$g_2 V_{ij}\, W_\mu^+\, \bar{u}_L^i \gamma^\mu d_L^j \;+\; g_2 V_{ij}^*\, W_\mu^-\, \bar{d}_L^j \gamma^\mu u_L^i$$

$$\Big\downarrow \quad \text{CP transformation}$$

$$g_2 V_{ij}\, W_\mu^-\, \bar{d}_L^j \gamma^\mu u_L^i \;+\; g_2 V_{ij}^*\, W_\mu^+\, \bar{u}_L^i \gamma^\mu d_L^j$$

- CKM matrix and m_q govern the pattern of flavor and CPV in the SM

- CKM matrix is unitary:
  - 9 real parameters, but redefinition of quark phases reduces physical parameters to 4: 3 mixing angles and 1 phase

$$V_{ij} \;\rightarrow\; V_{ij}\; e^{i((\phi_d)_j - (\phi_u)_i)}$$

5 independent parameters (phase differences)

- Irreducible phase implies CP violation:

$$g_2 V_{ij}\, W^+_\mu\, \bar{u}^i_L \gamma^\mu d^j_L \;+\; g_2 V^*_{ij}\, W^-_\mu\, \bar{d}^j_L \gamma^\mu u^i_L$$

$$\downarrow \quad \text{CP transformation}$$

$$g_2 V_{ij}\, W^-_\mu\, \bar{d}^j_L \gamma^\mu u^i_L \;+\; g_2 V^*_{ij}\, W^+_\mu\, \bar{u}^i_L \gamma^\mu d^j_L$$

- CKM matrix and m_q govern the pattern of flavor and CPV in the SM

Symmetries of the Standard Model

- Now pause and take stock of what is the fate of symmetries in the SM (besides Poincare', which is built in)

  - Gauge symmetry is hidden (spontaneously broken)

  - Global (flavor) symmetries: all explicitly broken** except for U(1) associated with B, L, and L_α (individual lepton families)
  - Impact of anomalies: only B-L is conserved (but no worries at T=0)

  - P, C maximally violated by Weak interactions

  - CP (and T): violated by CKM (and QCD theta term)

** Approximate SU(2) and SU(3) vector and axial symmetries of QCD play key role in strong interactions

Symmetries of the Standard Model

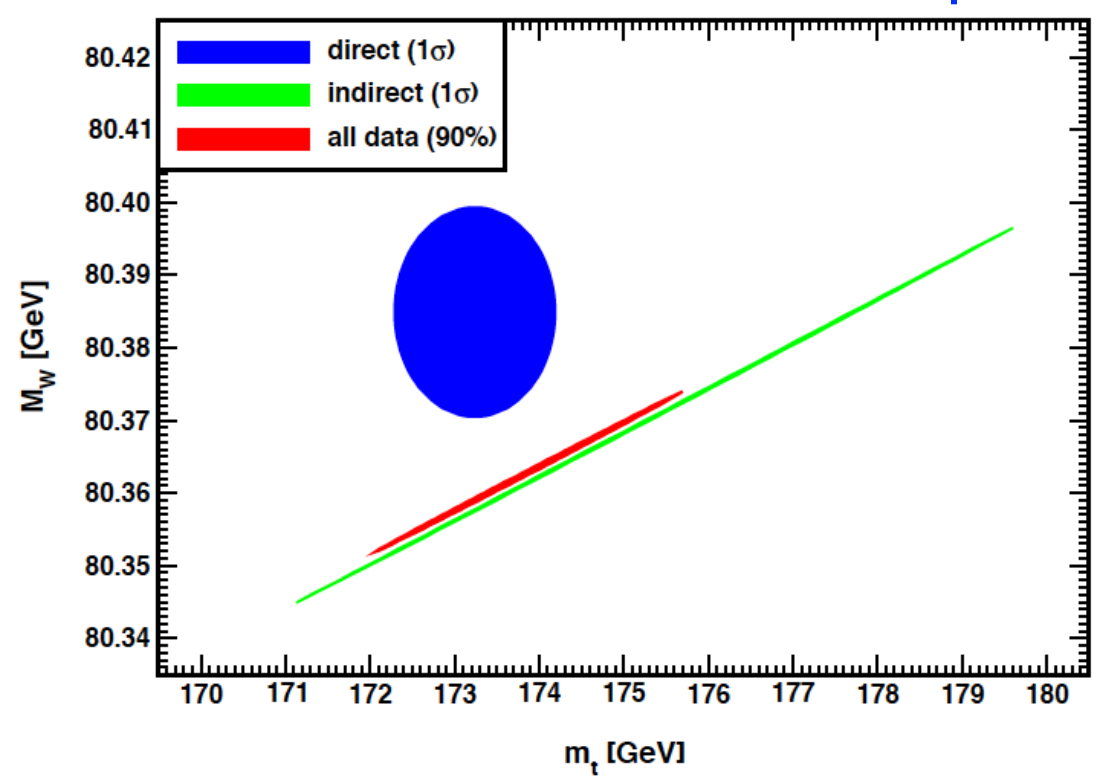
- Most symmetries are broken
- However, SM displays approximate discrete (C, P, T) and global symmetries (flavor, B, L) observed in nature
- Not an input in the model, rather an outcome that depends on the assigned gauge quantum numbers (+ renormalizability = keep only dim4 operators)

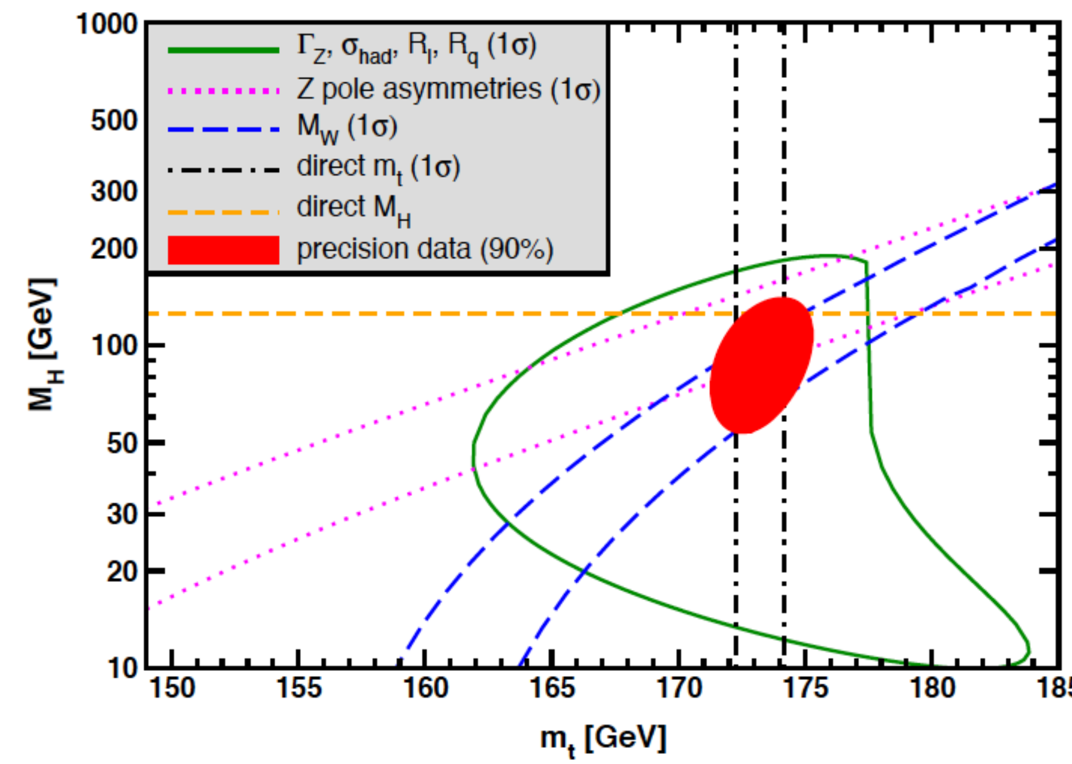
Status of the Standard Model

- Standard Model tested at the quantum (loop) level in both electroweak and flavor sector

- Precision EW tests are at the 0.1% level. Examples of global fits:

Note the vertical scale in this plot



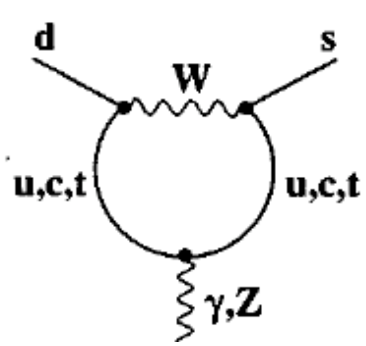


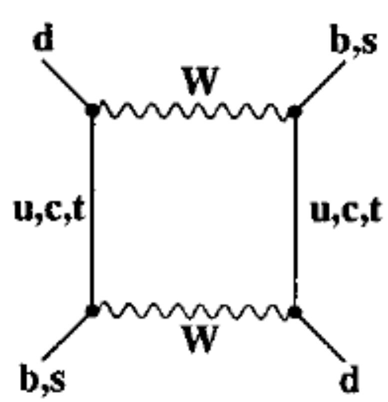
- A few "tensions" and "anomalies": g-2, … (will discuss it later on)

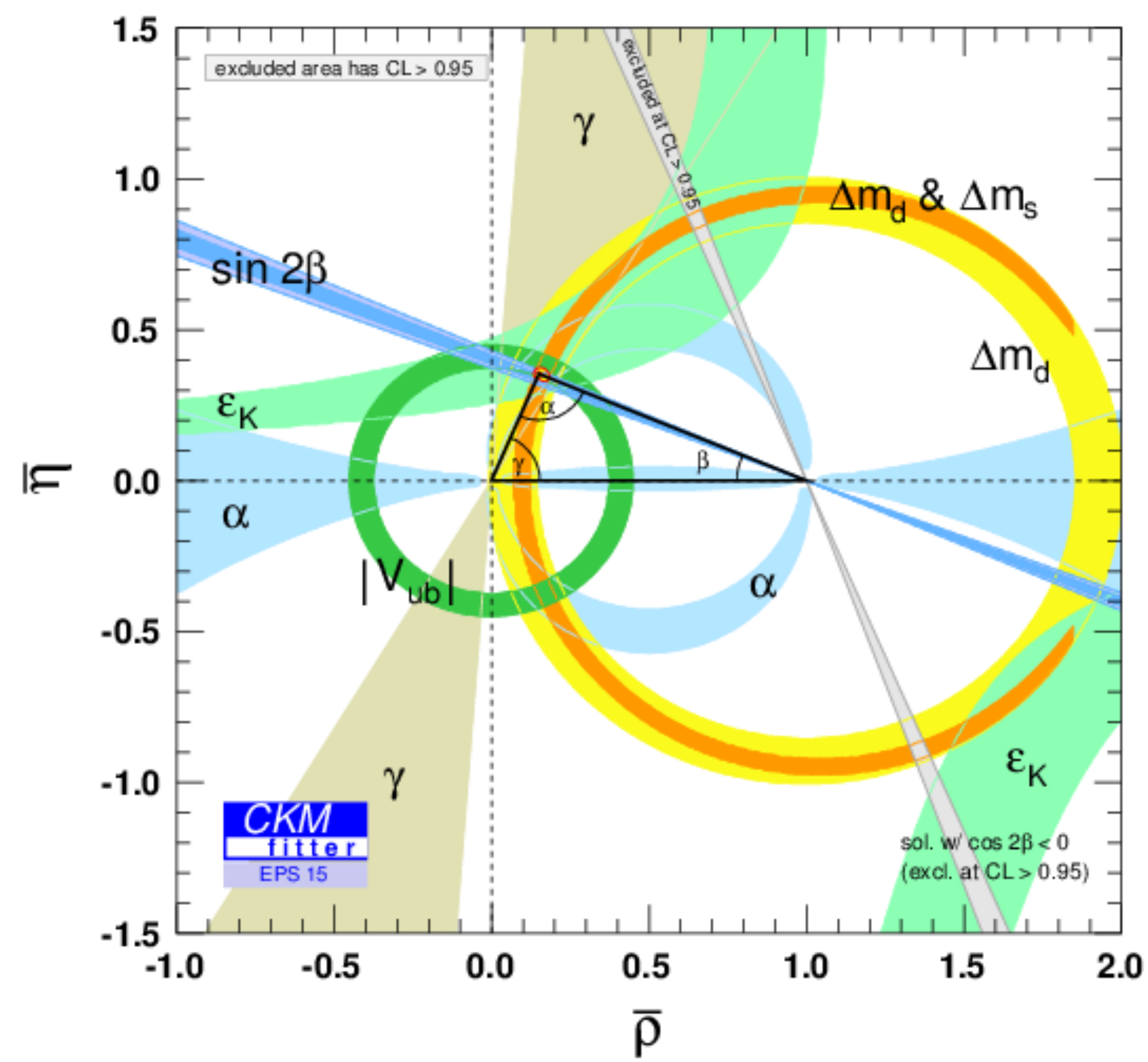
Status of the Standard Model

- Flavor physics and CP violation: K, B, D meson physics well described by CKM matrix, in terms of 3 mixing angles and a phase!

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



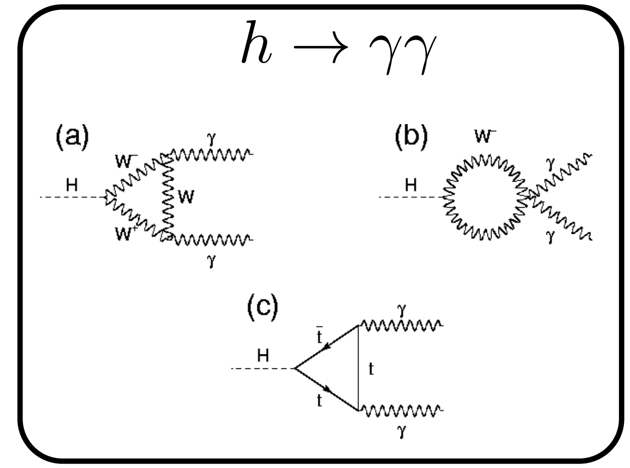


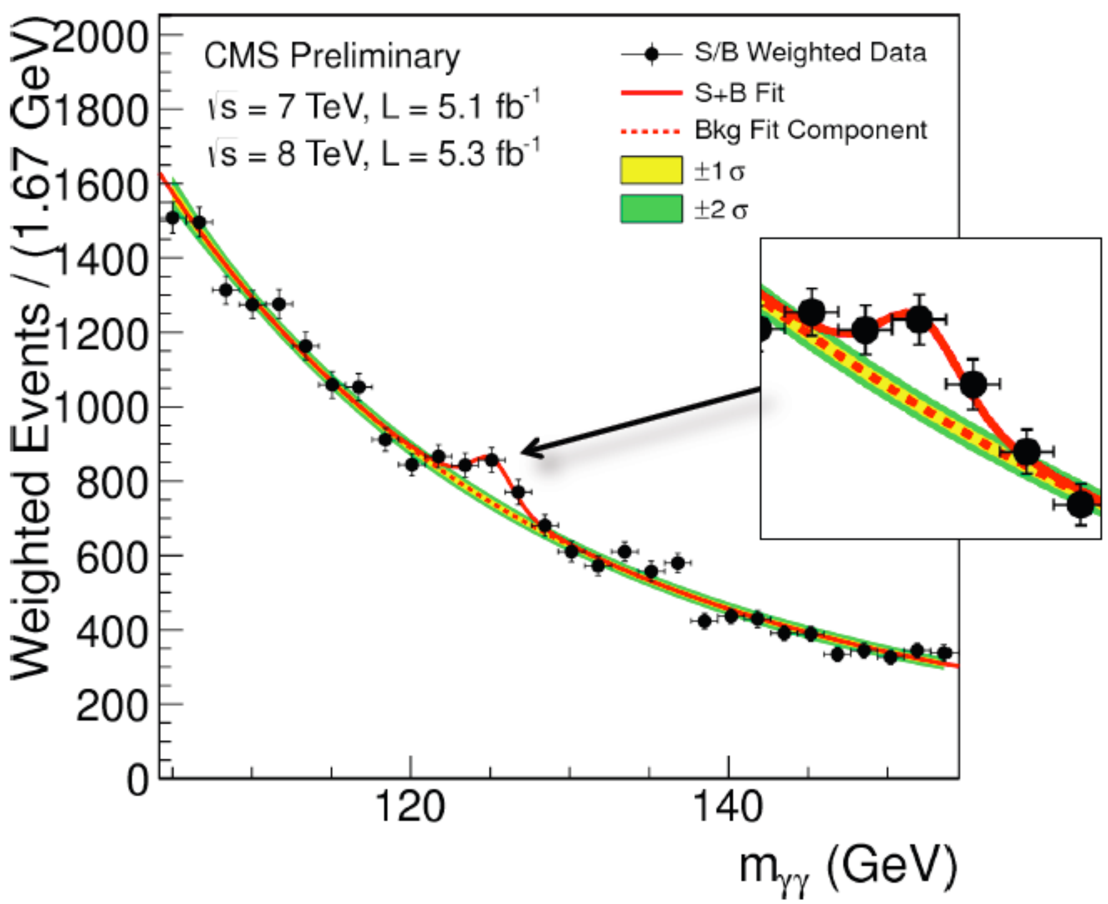


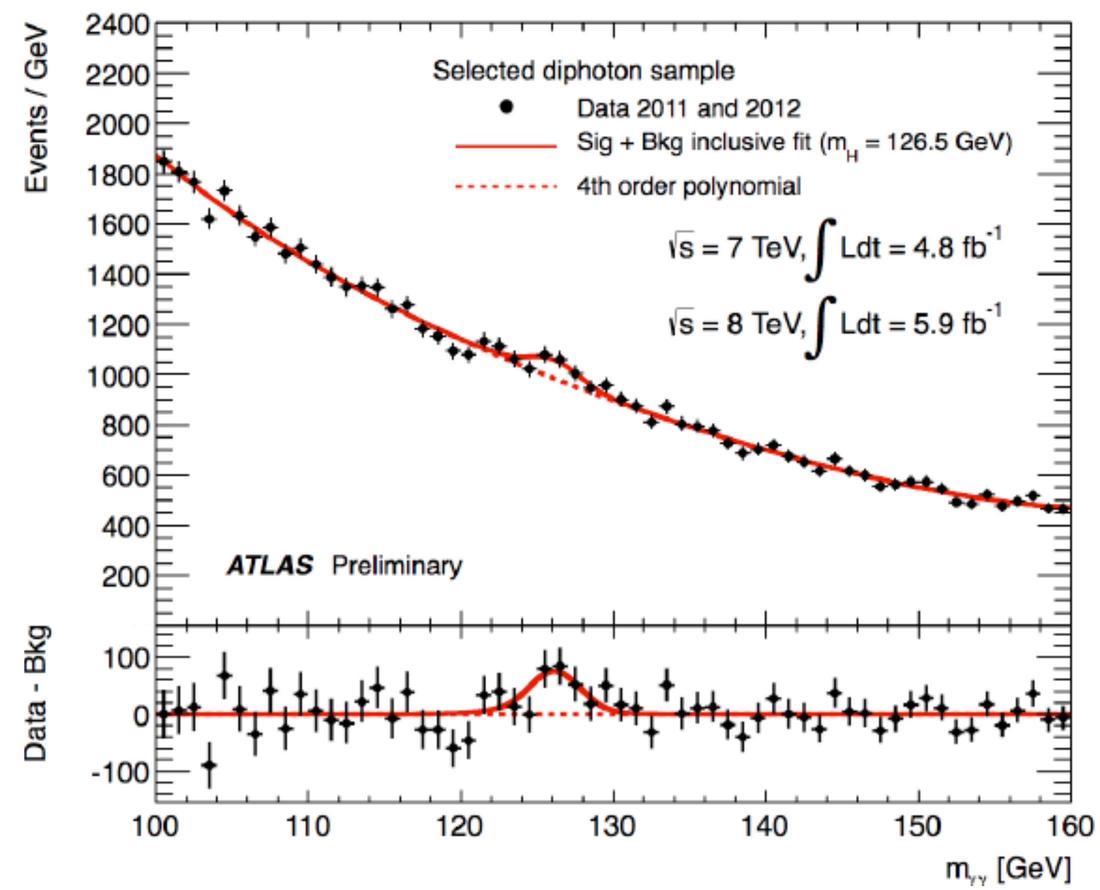
- Some recent "anomalies" in B decays

Status of the Standard Model

- Higgs boson: discovered in H →γγ mode

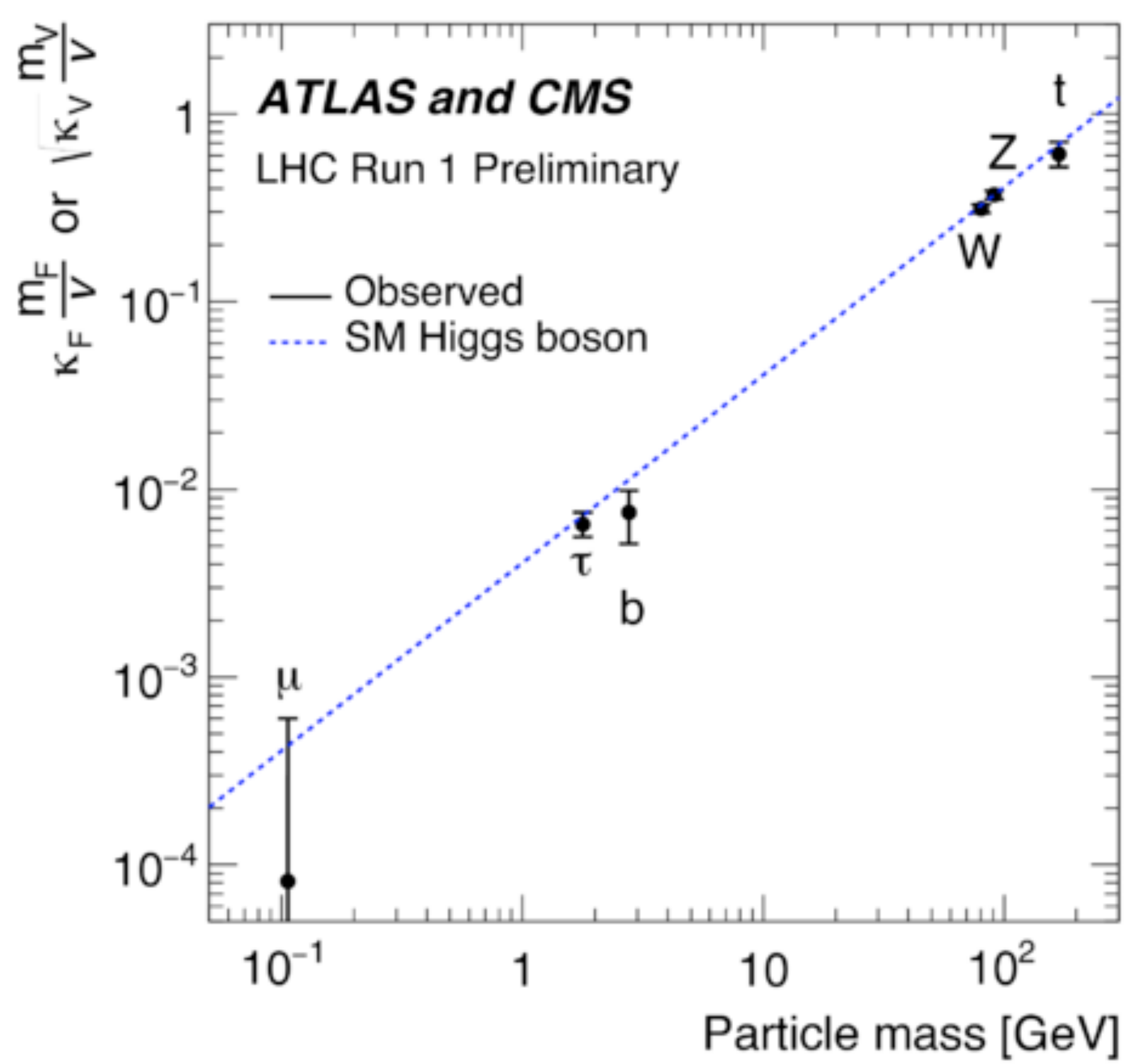






Status of the Standard Model

- Higgs boson: discovered in H →γγ mode
- So far Higgs properties are compatible with the Standard Model



- Couplings to W, Z, γ,g and t, b, τ known at 20-30% level
- But couplings to light flavors much less constrained
- Still room for deviations: is this the SM Higgs? Key question at LHC Run 2 & important target for low energy experiments