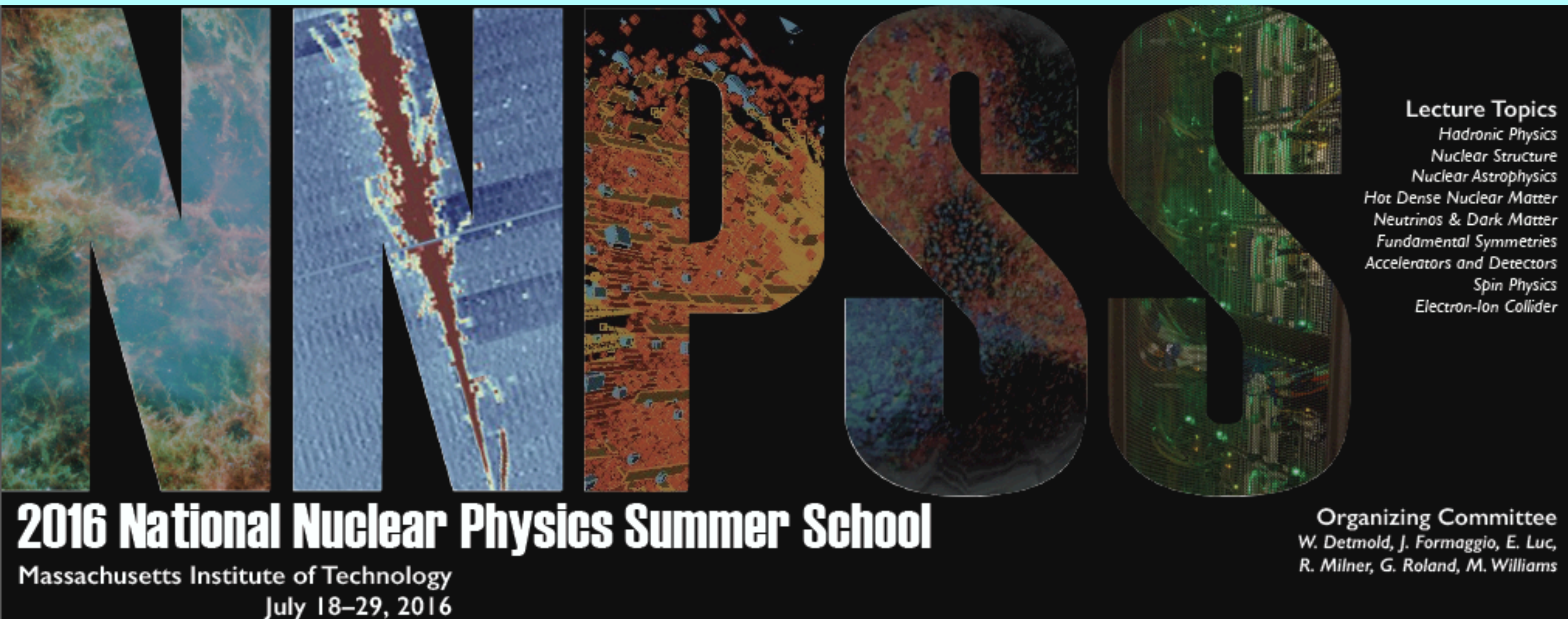
Hadron Structure

Jianwei Qiu

Brookhaven National Laboratory

Stony Brook University



The plan for my three lectures

❑ The Goal:

To understand the hadron structure in terms of QCD and its hadronic matrix elements of quark-gluon field operators,
to connect these matrix elements to physical observables, and
to calculate them from QCD (lattice QCD, inspired models, …)

❑ The outline:

Hadrons, partons (quarks and gluons),
and probes of hadron structure

One lecture

Parton Distribution Functions (PDFs) and
Transverse Momentum Dependent PDFs (TMDs)

One lecture

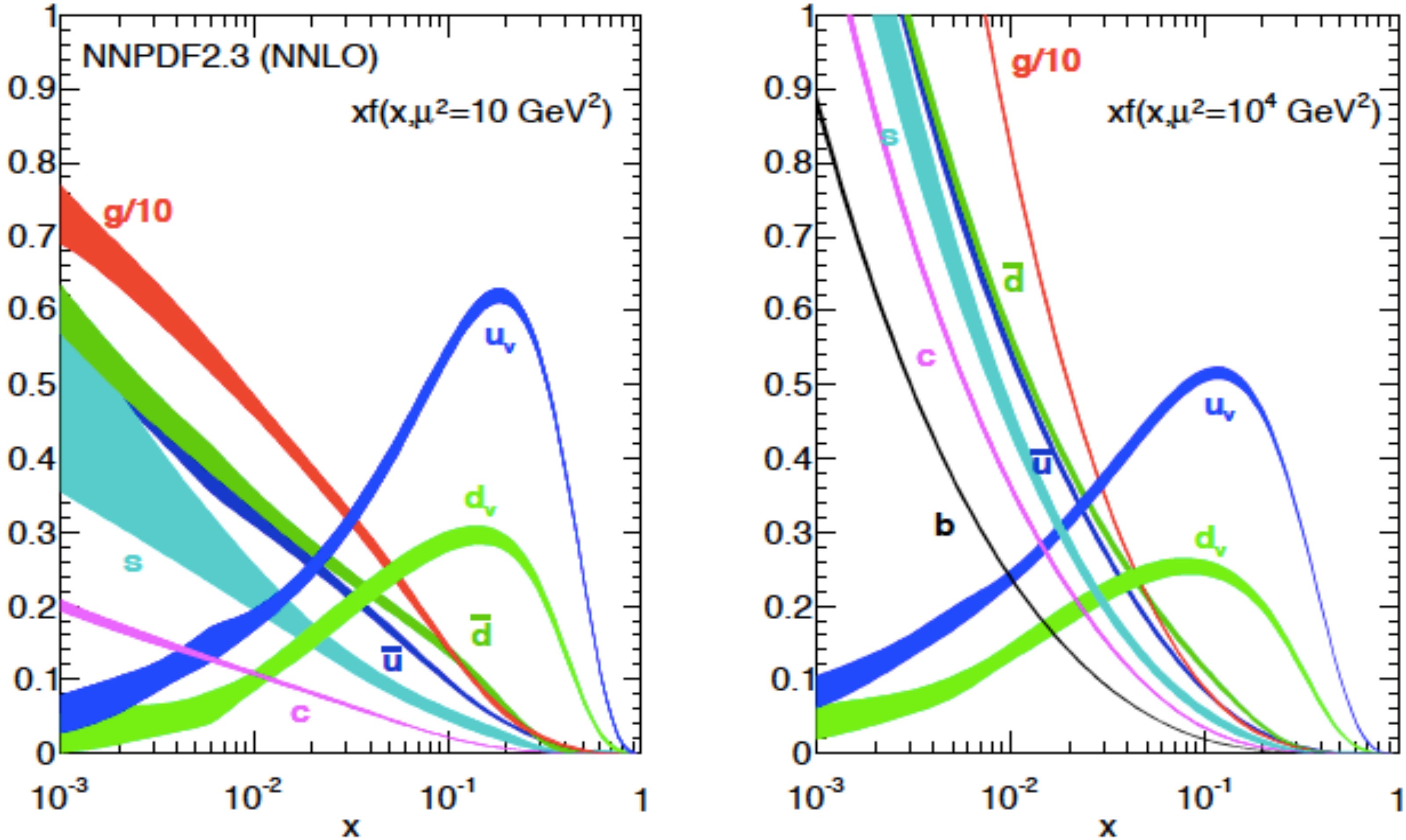
Generalized PDFs (GPDs) and multi-parton correlation functions

One lecture

*See also
lectures by Shepard on
"Hadron Spectroscopy",
and
lectures by Deshpande on
"Electron-Ion Collider"
and
lectures by Gandolfi on
"Nuclear Structure"
and
lectures by Aschenauer on
"Accelerators & detectors"*

PDFs of a spin-averaged proton

- Modern sets of PDFs @NNLO with uncertainties:



K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

Consistently fit almost all data with Q > 2GeV

Parton distribution functions (PDFs)

Does the factorization in DIS work for cross sections involving two or more hadrons?

How to extract PDFs from data?

What are the uncertainties?

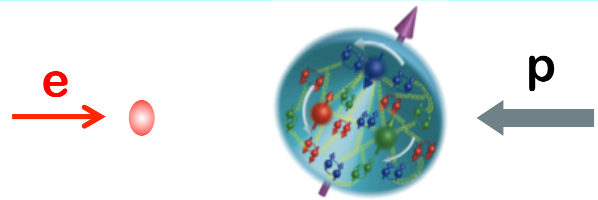
Can lattice QCD calculate PDFs?

What do we learn from the PDFs?

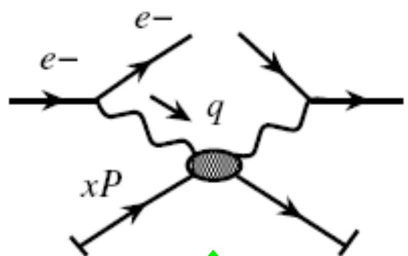
...

From one hadron to two hadrons

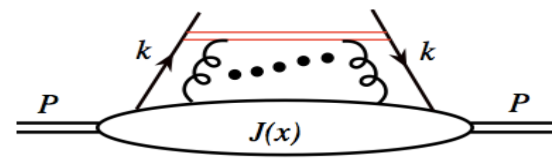
- **One hadron:**



$\sigma_{\text{tot}}^{\text{DIS}} \sim$



$\otimes$



$+ O\left(\dfrac{1}{QR}\right)$

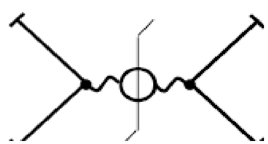
Hard-part
Probe

Parton-distribution
Structure

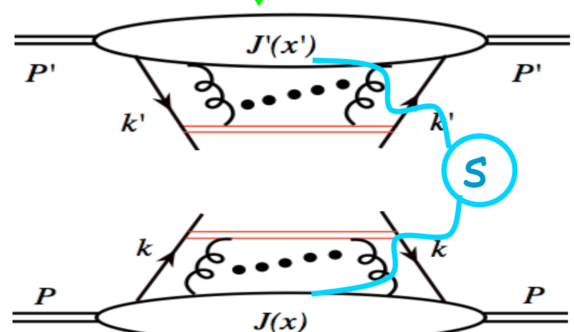
Power corrections
Approximation

- **Two hadrons:**

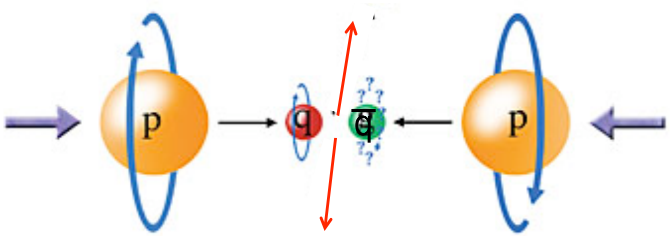
$\sigma_{\text{tot}}^{\text{DY}} \sim$



$\otimes$



$+ O\left(\dfrac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

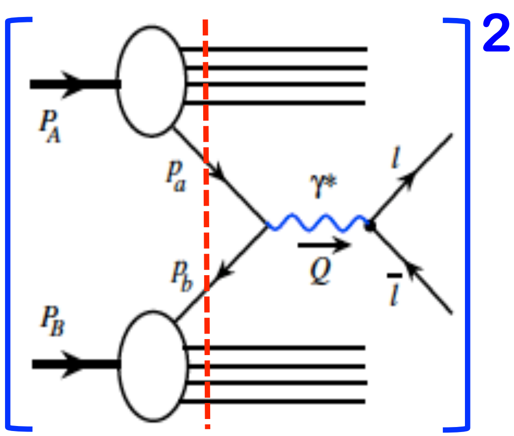
❑ Drell-Yan mechanism:

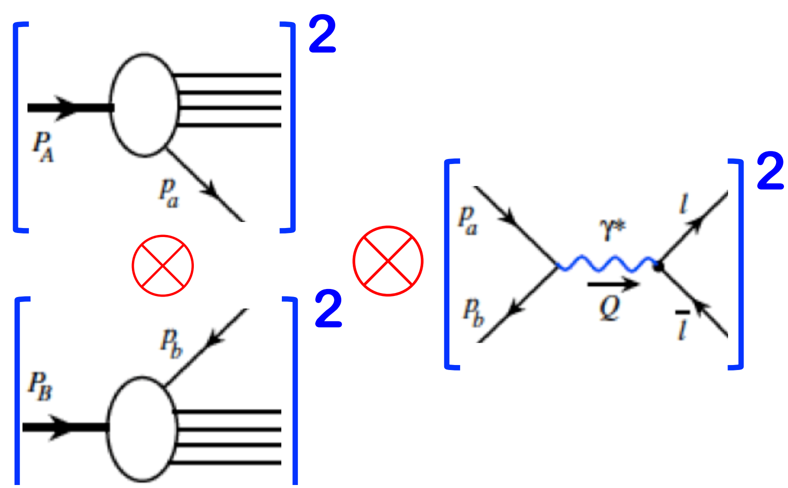
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X$$ **with** $$q^2 \equiv Q^2 \gg \Lambda_{\rm QCD}^2 \sim 1/{\rm fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

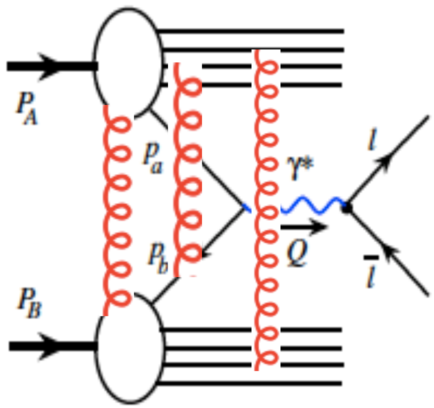
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}} e^{y} \qquad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

❑ Beyond the lowest order:

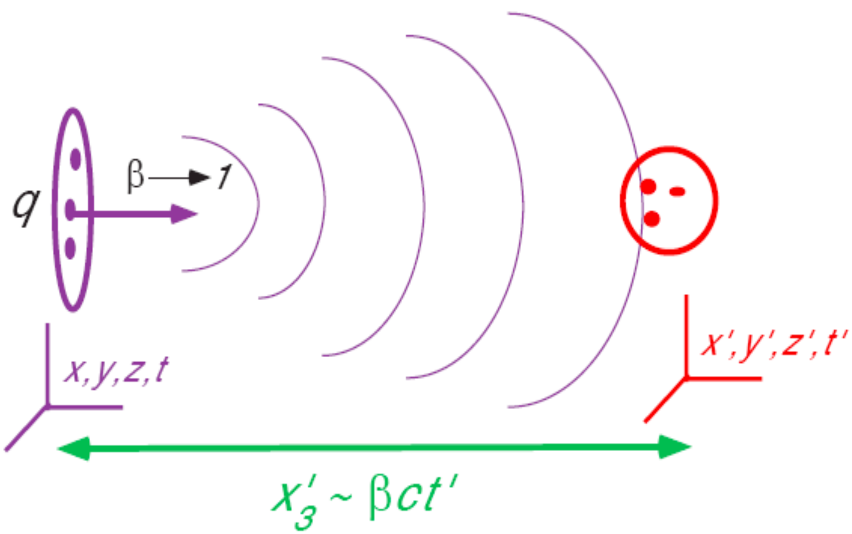


✧ Soft-gluon interaction takes place all the time

✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

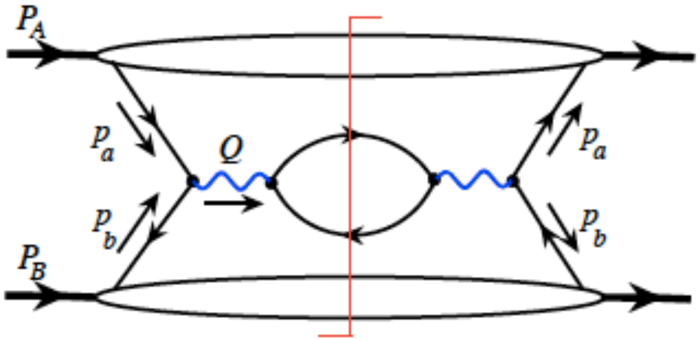
Drell-Yan process in QCD – factorization

❑ **Factorization – approximation:**

Collins, Soper, Sterman, 1988

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon}\frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

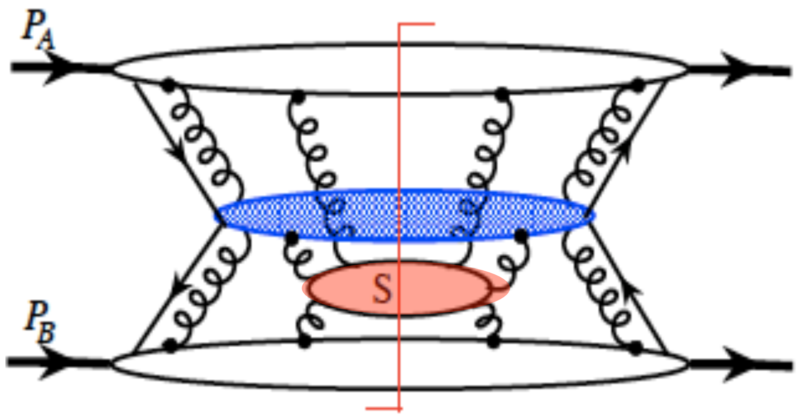
✧ Infrared safe of partonic parts:

Cancelation of IR behavior
Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD – factorization

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

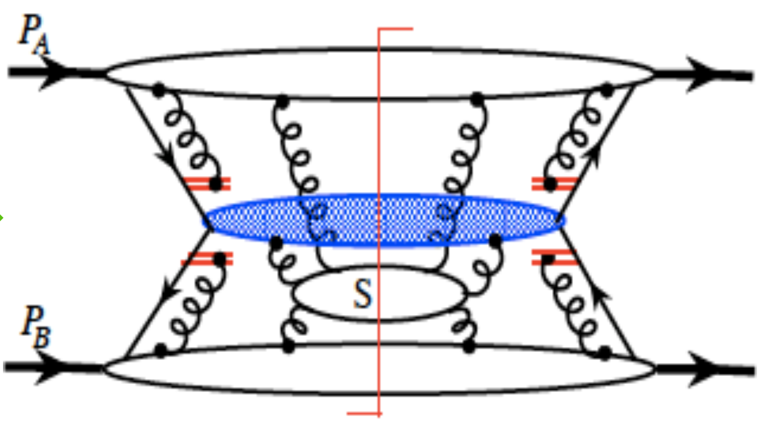
Collinear:
✧ lines collinear to A and B
✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

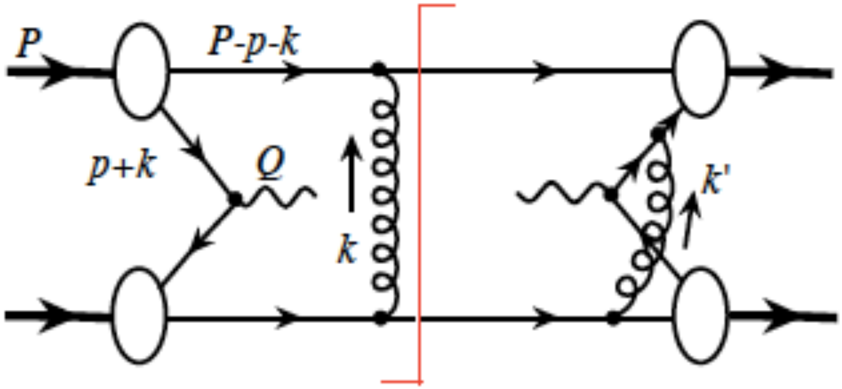
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ **Trouble with soft gluons:**



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

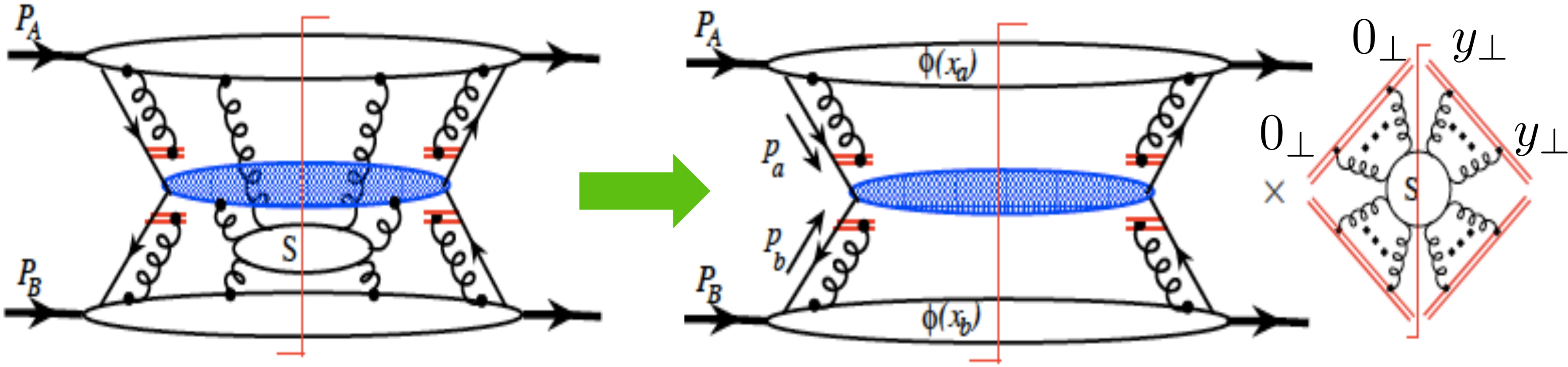
✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

- **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane – no more pinch poles

✧ Deform the $k^\pm$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines – gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

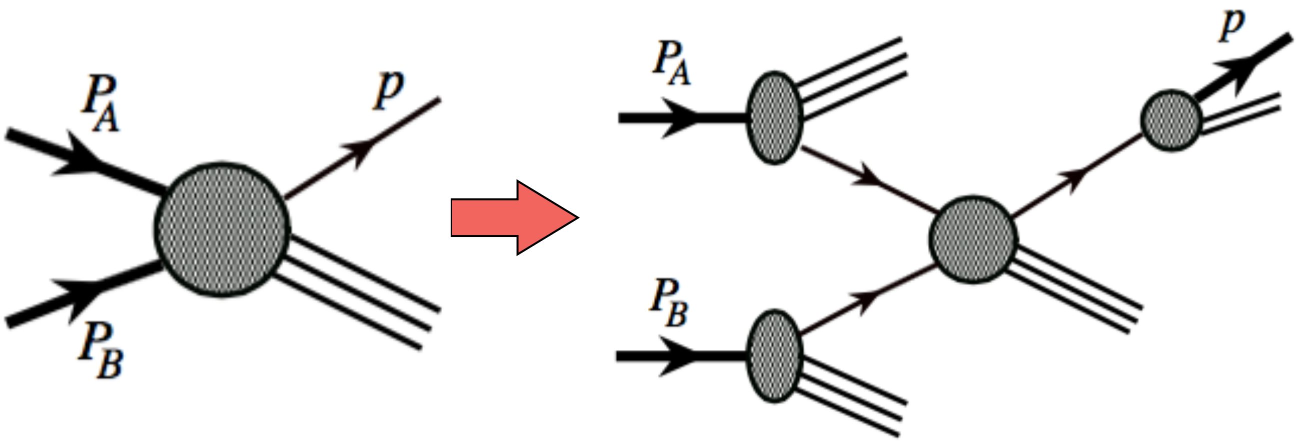
The factorization arguments are independent of the spin states of the colliding hadrons

➡ **same formula with polarized PDFs for γ^*,W/Z, H^0...**

Factorization for more than two hadrons

Nayak, Qiu, Sterman, 2006

- Factorization for high p_T single hadron:



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O $(1/P_T^2)$

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2) \otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

✧ **Fragmentation function:** $D_{c\to C}(z, \mu_F^2)$

✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Global QCD analyses – test of pQCD

- **Factorization for observables with identified hadrons:**
  - **One-hadron (DIS):**

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - **Two-hadrons (DY, Jets, W/Z, …) :**

$$\frac{d\sigma}{dydp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dydp_T^2} \otimes f'(x')$$

  - **DGLAP Evolution:**

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

- **Input for QCD Global analysis/fitting:**
  - **World data with "Q" > 2 GeV**
  - **PDFs at an input scale:** $\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$

Input scale ~ GeV

Fitting paramters

Global QCD analysis for PDFs

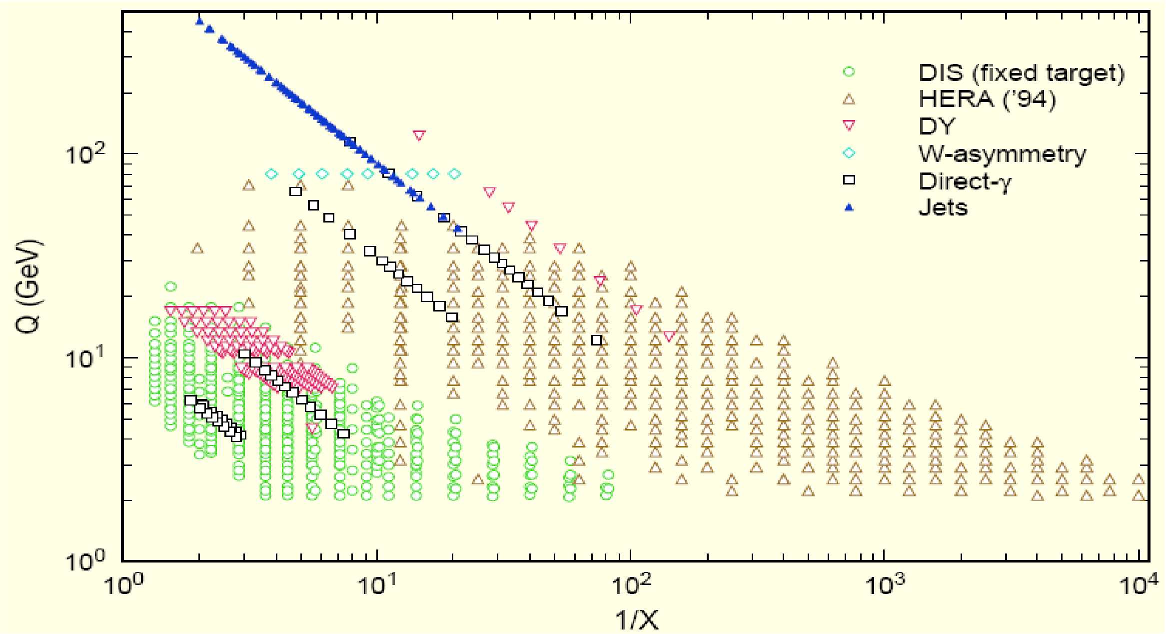
Input DPFs at Q_0

$\varphi_{f/h}\left(x,\{a_j\}\right)$

Vary $\{a_j\}$

Minimize Chi^2

DGLAP



$\varphi_{f/h}(x)$ **at $Q>Q_0$**

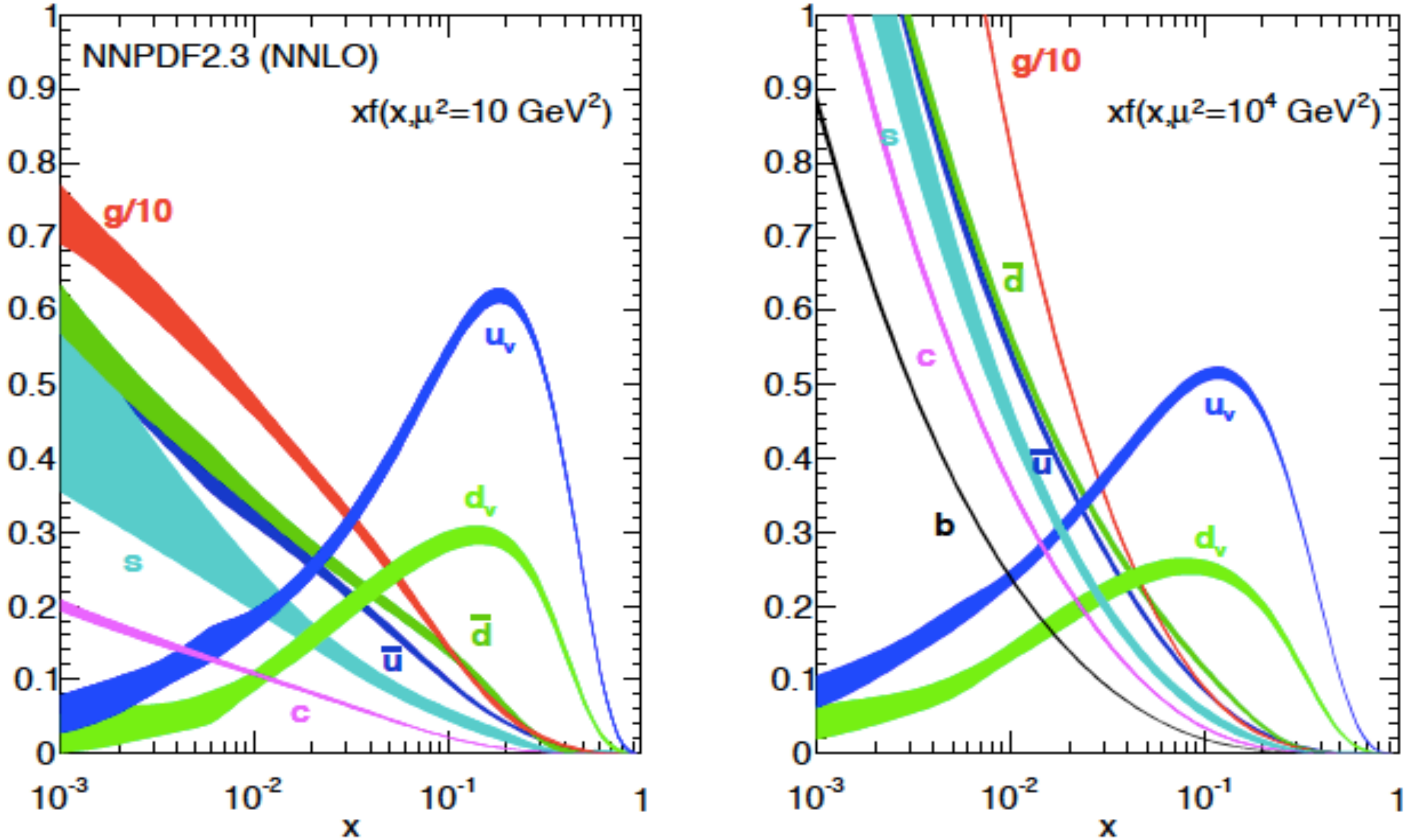
QCD calculation

Comparison with Data at various x and Q

Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

PDFs of a spin-averaged proton

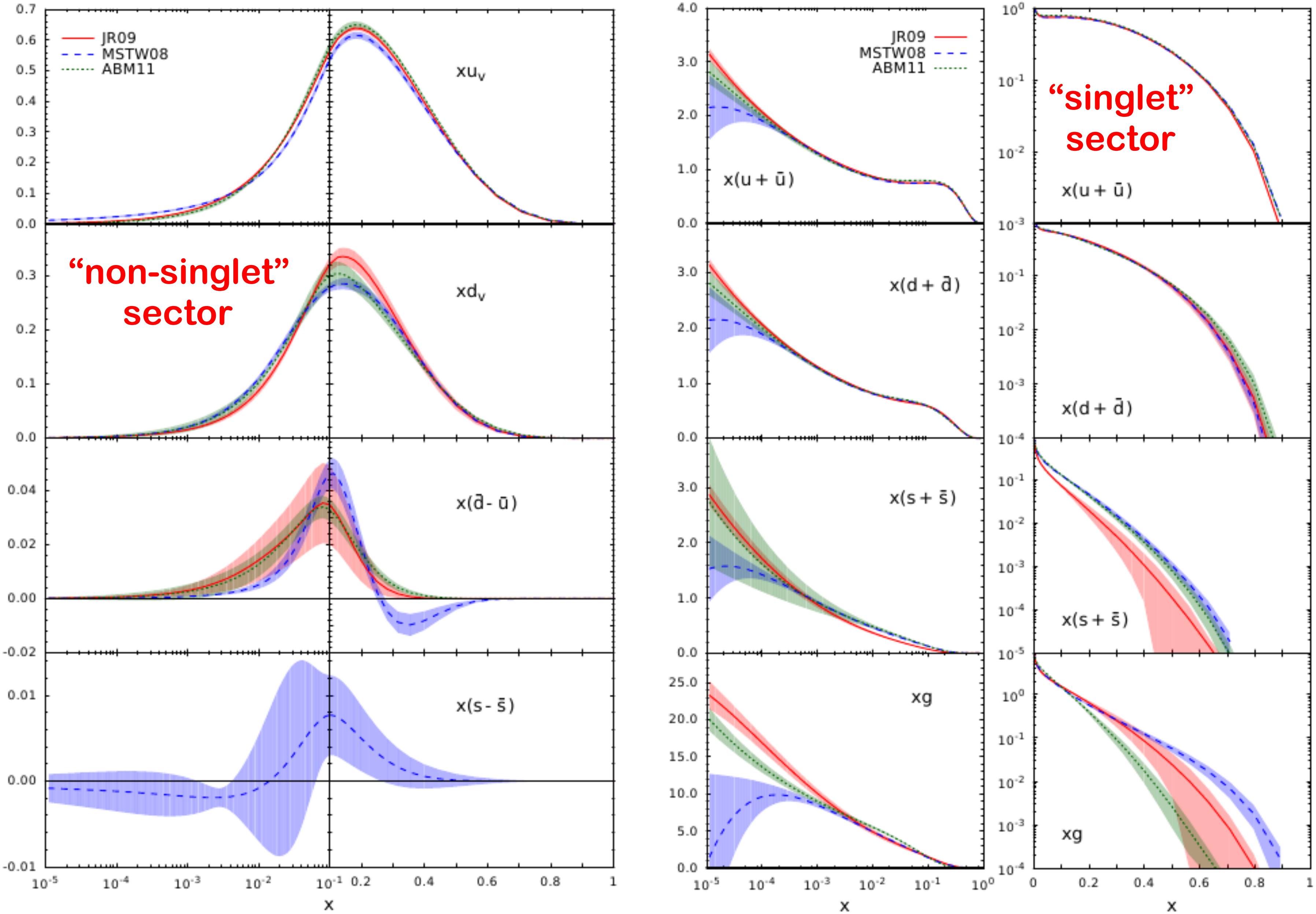
❑ **Modern sets of PDFs @NNLO with uncertainties:**



K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

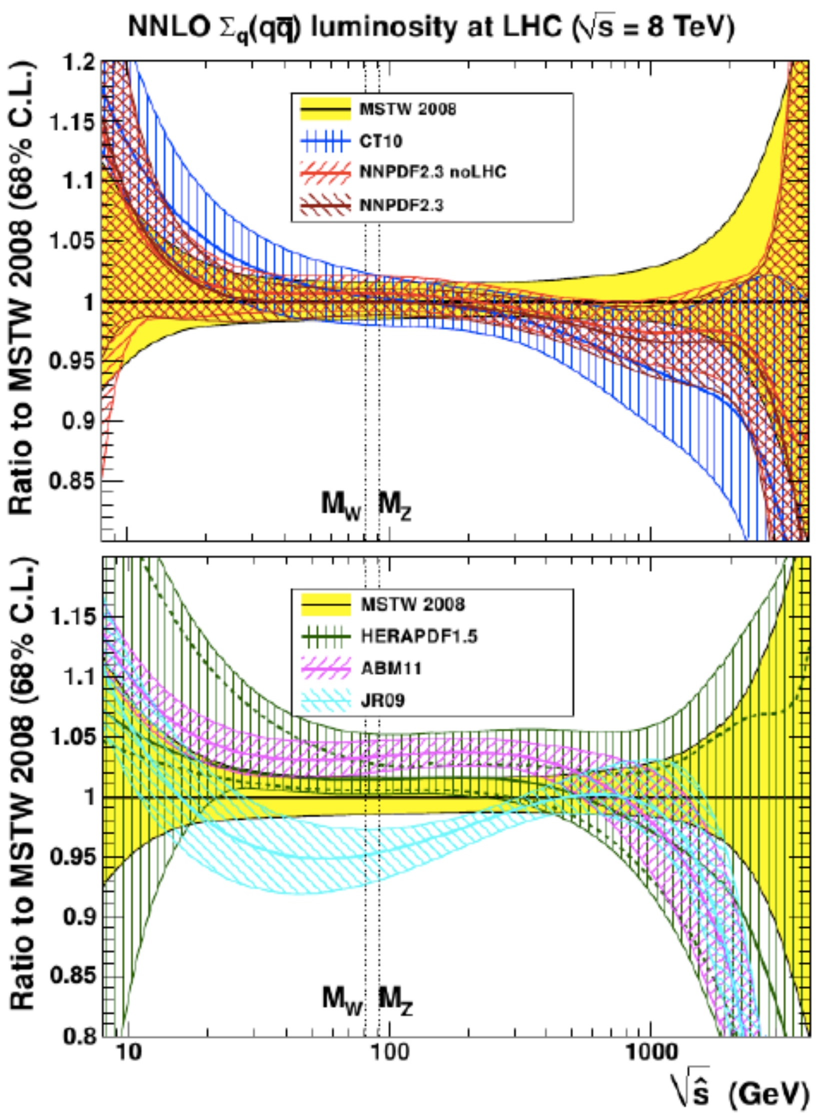
Consistently fit almost all data with Q > 2GeV

Uncertainties of PDFs

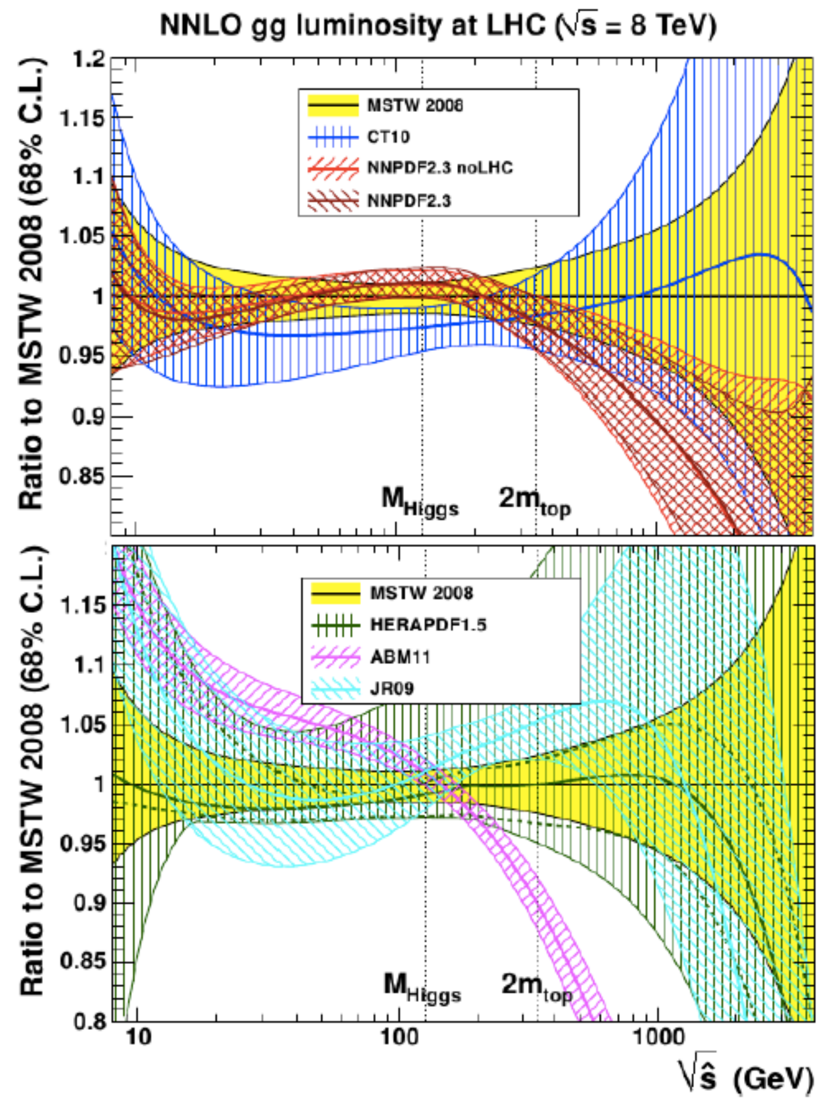


Partonic luminosities

q - qbar



g - g



PDFs at large x

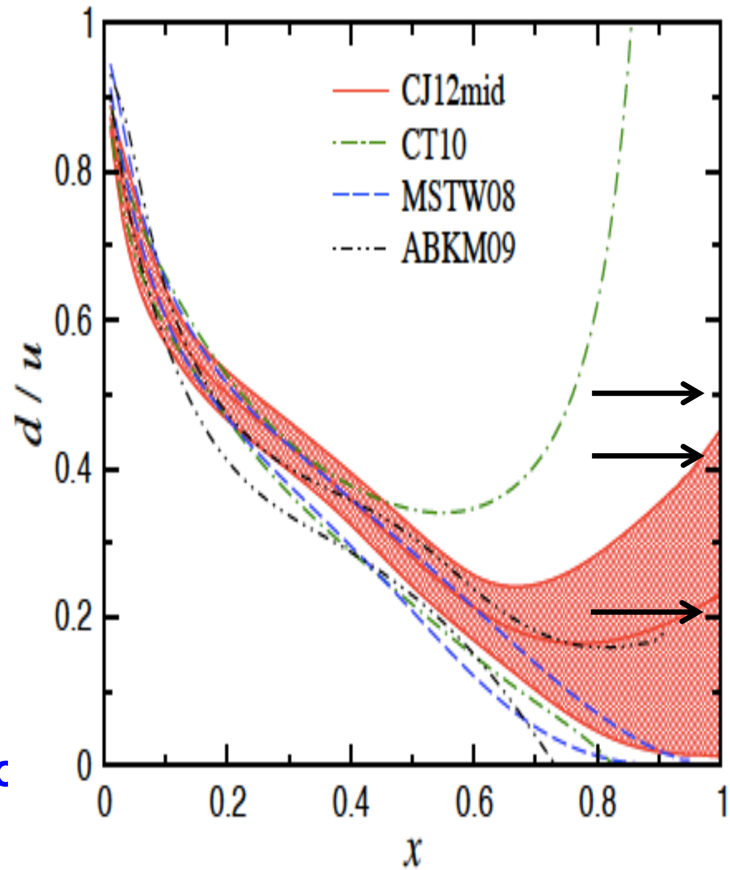
❑ **Testing ground for hadron structure at x → 1:**

✧ $d/u \to 1/2$ — **SU(6) Spin-flavor symmetry**

✧ $d/u \to 0$ — **Scalar diquark dominance**

✧ $d/u \to 1/5$ — **pQCD power counting**

✧ $$d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$$ — **Local quark-hadrc duality**



PDFs at large x

- Testing ground for hadron structure at x → 1:

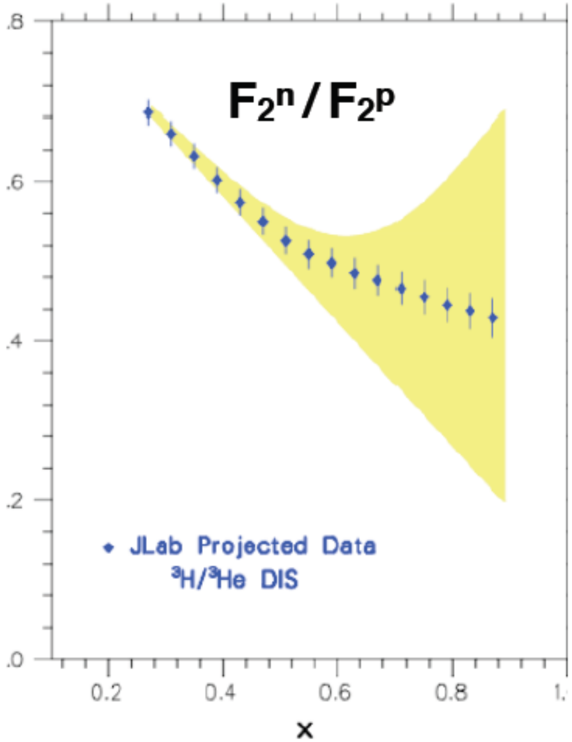
✧ $d/u \to 1/2$	SU(6) Spin-flavor symmetry	✧ $\Delta u/u \to 2/3$ $\Delta d/d \to -1/3$
✧ $d/u \to 0$	Scalar diquark dominance	✧ $\Delta u/u \to 1$ $\Delta d/d \to -1/3$
✧ $d/u \to 1/5$	pQCD power counting	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$
✧ $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$	Local quark-hadron duality	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$

Can lattice QCD help?

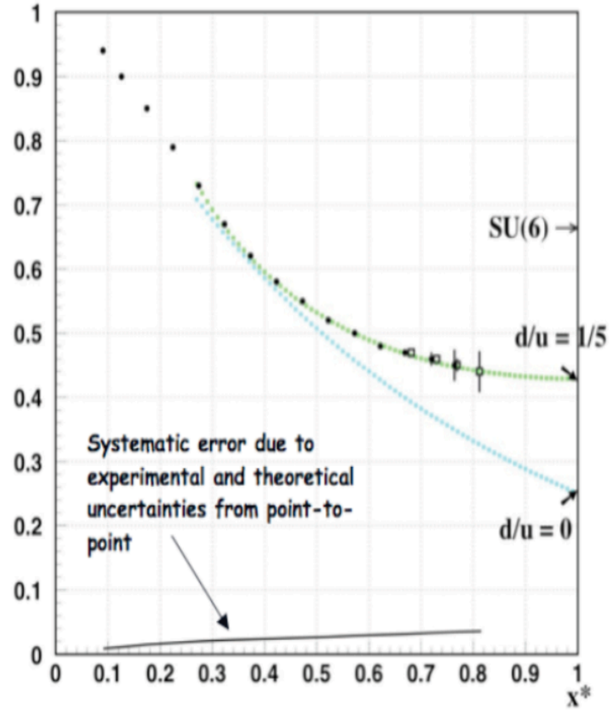
Future large-x experiments – JLab12

- ❑ **NSAC milestone HP14 (2018):**

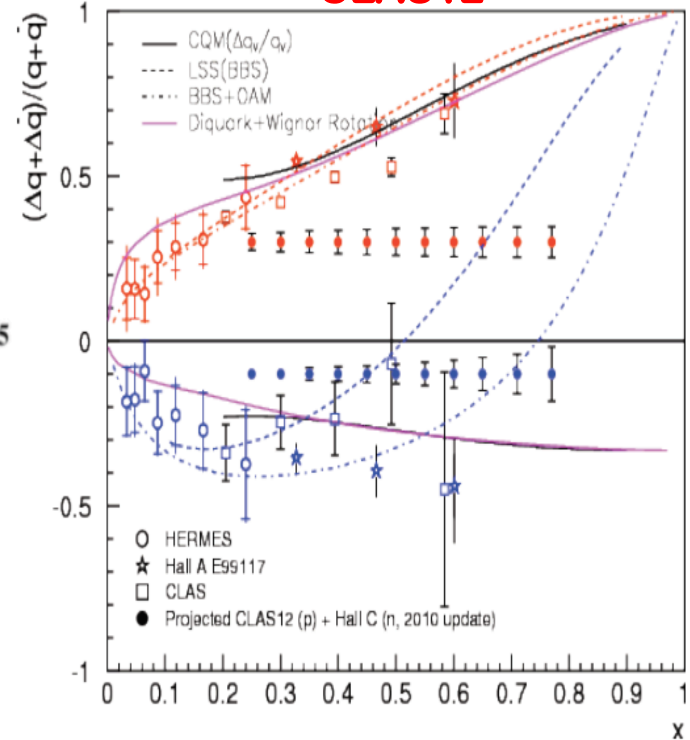
MARATHON



BONUS



CLAS12



Plus many more JLab experiments:

E12-06-110 (Hall C on ^3He), E12-06-122 (Hall A on ^3He),

E12-06-109 (CLAS on NH_3, ND_3), …

and Fermilab E906, …

Lattice calculations of hadron structure

❑ Quark distribution (spin-averaged):

$$q(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle \; + \text{UVCT}$$

- ✧ Matrix element of an operator of fields on the light-cone
- ✧ $\xi_\pm = (t \pm z)/\sqrt{2}$ - light-cone coordinate
- ✧ Time-dependent – cannot be calculated by lattice QCD

❑ Moments (spin-averaged):

$$q_n(\mu) = \int dx x^{n-1} q(x,\mu) = \frac{1}{P^{\mu_1}\ldots P^{\mu_n}} \langle P|O^{\mu_1\cdots\mu_n}|P\rangle \mathcal{Z}_O$$

- ✧ Defined with a local operator: $O^{\{\mu_1\cdots\mu_n\}} = \overline{\psi}(0)\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)$
- ✧ Calculable on lattice (in principle)
- ✧ But, in practice, higher moments are difficult to access

Lattice calculations of hadron structure

Ji, arXiv:1305.1539

❑ Quasi-quark distribution (spin-averaged):

$$\tilde{q}(\tilde{x},\mu,P_z)=\int\frac{d\delta z}{4\pi}e^{-i\tilde{x}P_z\delta z}\langle P|\overline{\psi}(\delta z)\gamma_z\exp\left\{-ig\int_0^{\delta z}d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- ✧ Field operators are separated in spatial z-direction
- ✧ No time dependence – calculable in lattice QCD
- ✧ At the limit, $P_z\to\infty$, normal quark-PDF is expected to recover

❑ Matching in Large Momentum Effective Theory (LMET):

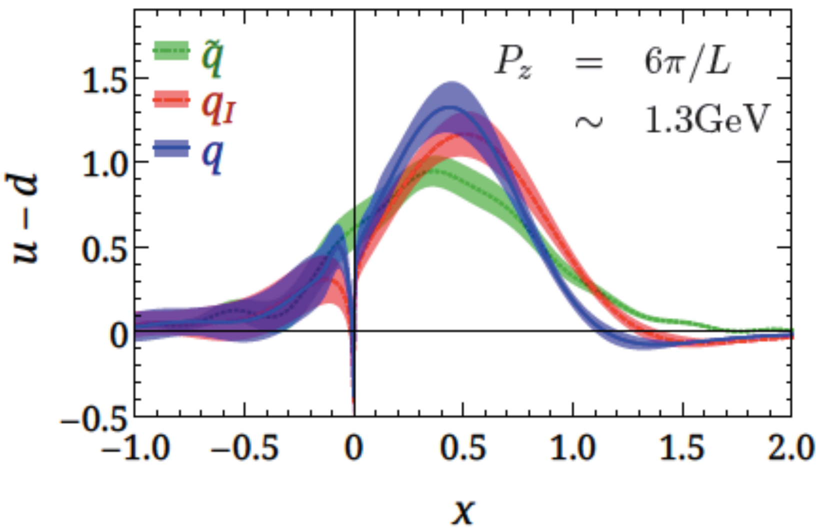
$$\tilde{q}(\tilde{x},\Lambda,P_z)=\int\frac{dy}{y}Z\left(\frac{\tilde{x}}{y},\frac{\Lambda}{P_z},\frac{\mu}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{p_z^2},\frac{M_N^2}{p_z^2}\right)$$

- ✧ Matching function, Z , can be perturbatively derived
- ✧ Large P_z is needed for small corrections
- ✧ UV power divergences of the quasi-PDFs

Lattice calculations of hadron structure

❑ **Two calculations in LMET approach:**

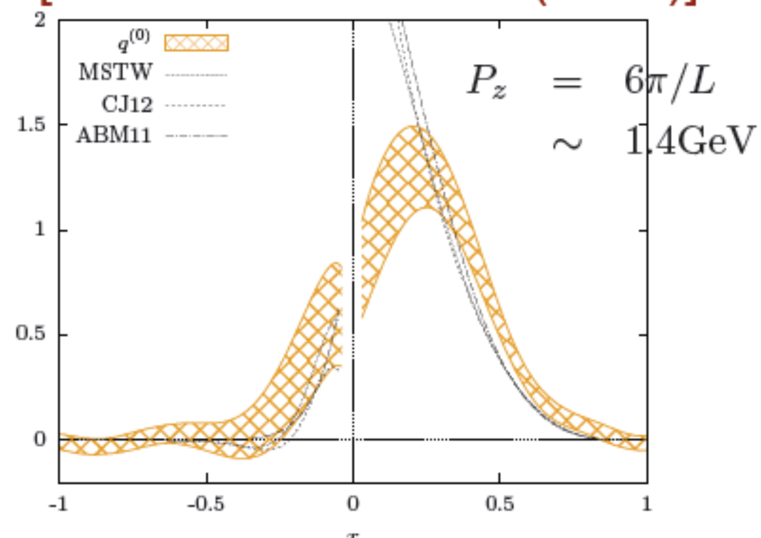
[J.-W. Chen et al. (2016)]



$24^3 \times 64, N_f = 2+1+1$ HISQ

$a \sim 0.12\text{fm}\ (1.6\text{GeV}), m_{\text{PS}} \sim 310\text{MeV}$

[C. Alexandrou et al. (2015)]



$32^3 \times 64, N_f = 2+1+1$ Twisted Mass

$a \sim 0.082\text{fm}\ (2.4\text{GeV}), m_{\text{PS}} \sim 370\text{MeV}$

- ✧ Exploratory study
- ✧ Two calculations look consistent with each other
- ✧ Matching between lattice and continuum is seemly omitted

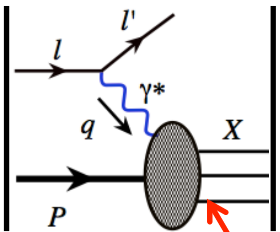
Lattice calculations of hadron structure

Ma, Qiu (2014)

- QCD collinear factorization approach:

Momentum transfer | CO Factorization | PDFs

$$\sigma^{\rm DIS}(x, Q^2; \sqrt{s}) \propto$$



$$\Big|\ldots\Big|^2 \approx \sum_f \mathcal{C}_f(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_i(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

Collision energy | Single hadron Matrix elements | Perturbative coefficients | Power corrections

✧ PDFs are UV and IR finite, absorb all perturbative CO divergence!

- Lattice calculable "cross section":

$$\tilde{q}(\tilde{x}, \tilde{\mu}^2, P_z)_{\rm ren} = \sum_f \tilde{\mathcal{C}}_f(\tilde{x}, \frac{\tilde{\mu}^2}{\mu^2}, P_z) \otimes f(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\tilde{\mu}^2}\right)$$

✧ With the correspondence:

$\mu \longleftrightarrow \mu$ (factorization scale)

$Q \longleftrightarrow \tilde{\mu}$ (resolution)

$\sqrt{s} \longleftrightarrow P_z$ (parameter)

Lattice calculations of hadron structure

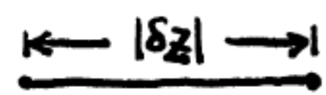
Ishikawa, Ma, Qiu, Yoshida (2016)

- ❑ **Renormalization – subtraction of power divergence:**
  - ✧ Power divergence makes the Lattice "cross section" ill-defined (no continuum limit on lattice)
  - ✧ Power divergence must be subtracted nonperturbatively
  - ✧ Found: *Power divergence of quasi-PDFs only from the Wilson line*
- ❑ **Renormalization of the Wilson line:**

$$W_{\mathcal{C}} = e^{\delta m \ell(\mathcal{C})} W_{\mathcal{C}}^{\rm ren}$$

- ✧ Well-known, e.g., Dotsenko, Vergeles, Arefeva, Craigie, Dorn, … ('80)
- ✧ δm : mass renormalization of a test particle moving along the path $\mathcal{C}$ contains all the power divergences
- ✧ Subtraction of the power divergence can be done nonperturbatively in coordinate space:

$$\tilde{O}(\delta z)_{\rm ren} = e^{-\delta m|\delta z|}\tilde{O}(\delta z)$$



Lattice calculations of hadron structure

Ishikawa, Ma, Qiu, Yoshida (2016)

❑ Subtracting the power divergence:

- ✧ Choice of δm (renormalization scheme) [M.U. Busch et al 2011]
- ✧ One possible way is to use static $Q\bar{Q}$ potential $V(R)$
- ✧ $V(R)$ is obtained from the Wilson loop:

$$W_{R\times T} \propto e^{-V(R)T} \quad (T \to \text{large})$$

- ✧ Renormalization of $V(R)$:

$$V^{\text{ren}}(R) = V(R) + 2\delta m$$

- ✧ Renormalization condition we take:

$$V^{\text{ren}}(R_0) = V_0 \longrightarrow \delta m = \frac{1}{2}(V_0 - V(R_0))$$

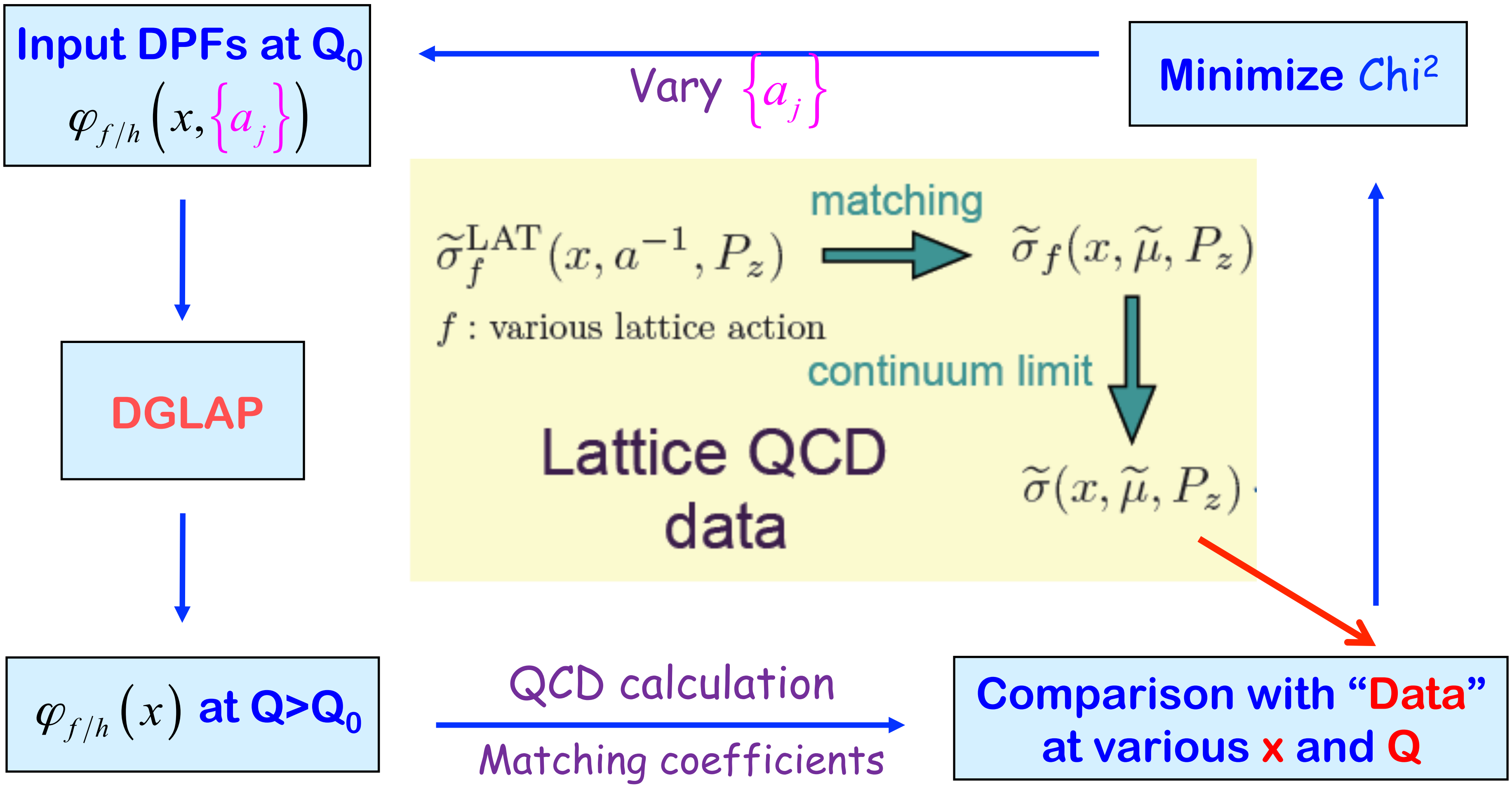
❑ Renormalized quasi-quark distribution:

$$\tilde{q}(\tilde{x}, \mu, P_z)_{\text{ren}} = \int \frac{d\delta z}{4\pi} e^{-i\tilde{x}P_z\delta z} e^{-\delta m|\delta z|} \langle P|\overline{\psi}(\delta z)\gamma_z \exp\left\{-ig\int_0^{\delta z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

Need more lattice "cross sections":

– Calculable, renormalizable, factorizable single hadron matrix elements

Global QCD analysis with lattice data



Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

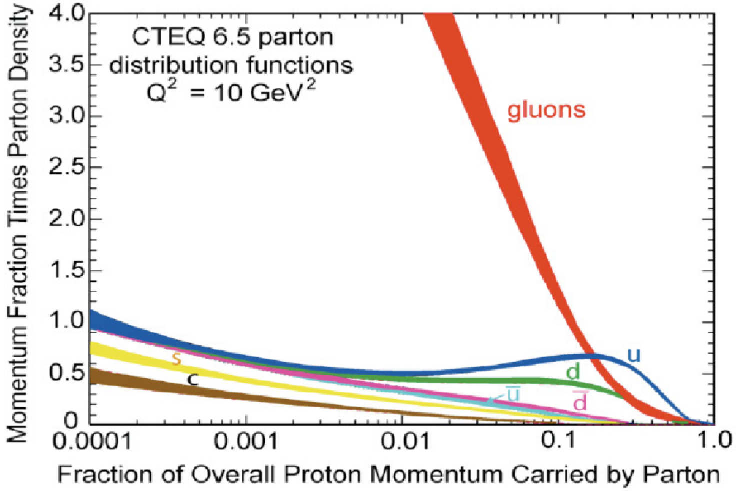
- **Tremendous potentials:**

The TMD Collaboration

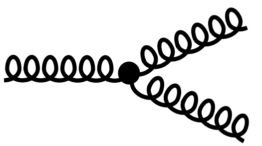
PDFs of proton, neutron, pion, …, TMDs, GPDs, … – the TMD Collaboration

Run away gluon density at small x?

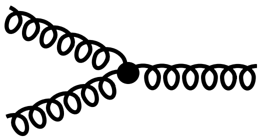
❑ HERA discovery:



What causes the low-x rise?
gluon radiation
– non-linear gluon interaction



What tames the low-x rise?
gluon recombination
– non-linear gluon interaction



❑ QCD vs. QED:

QCD – gluon in a proton:

$$Q^2 \frac{d}{dQ^2} xG(x,Q^2) \approx \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dx'}{x'} x' G(x', Q^2)$$

QED – photon in a positronium:

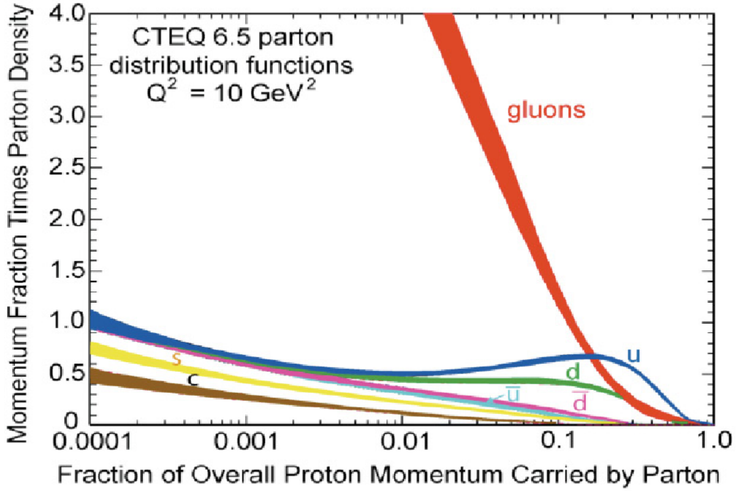
$$Q^2 \frac{d}{dQ^2} x\phi_\gamma(x,Q^2) \approx \frac{\alpha_{em}}{\pi} \left[-\frac{2}{3} x\phi_\gamma(x,Q^2) + \int_x^1 \frac{dx'}{x'} x' [\phi_{e^+}(x',Q^2) + \phi_{e^-}(x',Q^2)] \right]$$

✧ At very small-x, proton is "black", positronium is still transparent!

✧ Recombination of large numbers of glue could lead to saturation phenomena

Run away gluon density at small x?

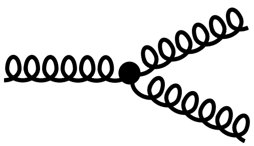
❑ HERA discovery:



What causes the low-x rise?

gluon radiation

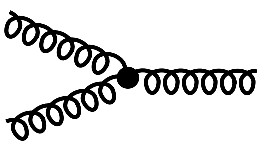
– non-linear gluon interaction



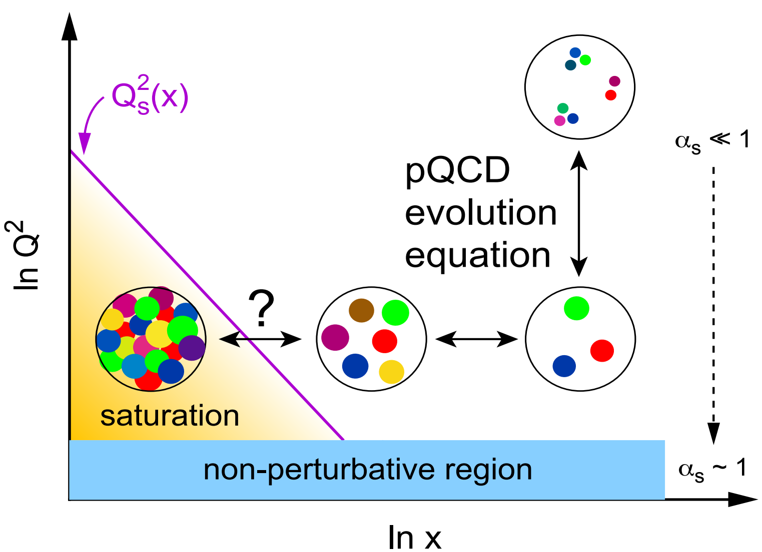
What tames the low-x rise?

gluon recombination

– non-linear gluon interaction

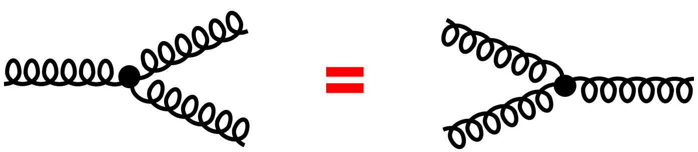


❑ Particle vs. wave feature:



Gluon saturation – Color Glass Condensate

Radiation = Recombination



at Q_s

Leading to a collective gluonic system?

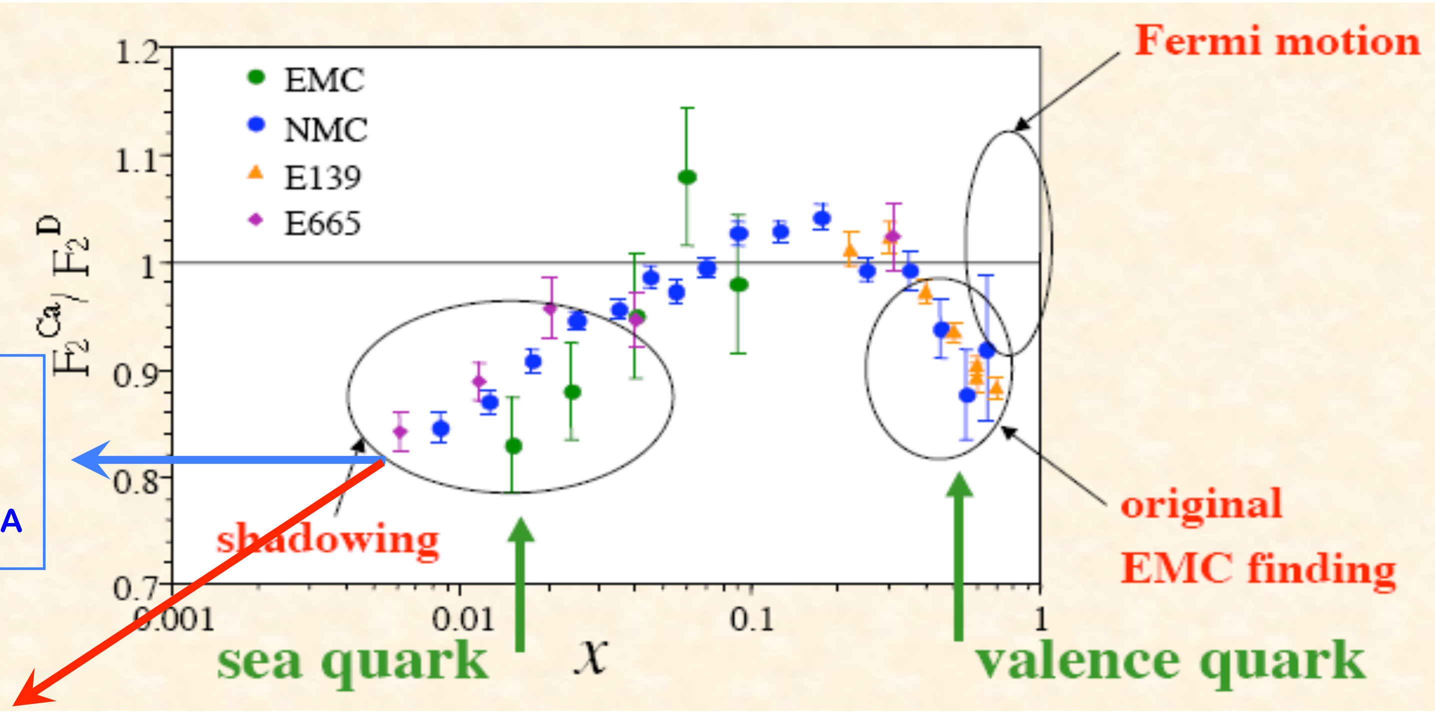
with a universal property?

new effective theory QCD – CGC?

An "easiest" measurement at EIC

❑ Ratio of F_2: EMC effect, Shadowing and Saturation:

Saturation in R_{F2}
=
NO saturation in F_2^A



❑ Questions:

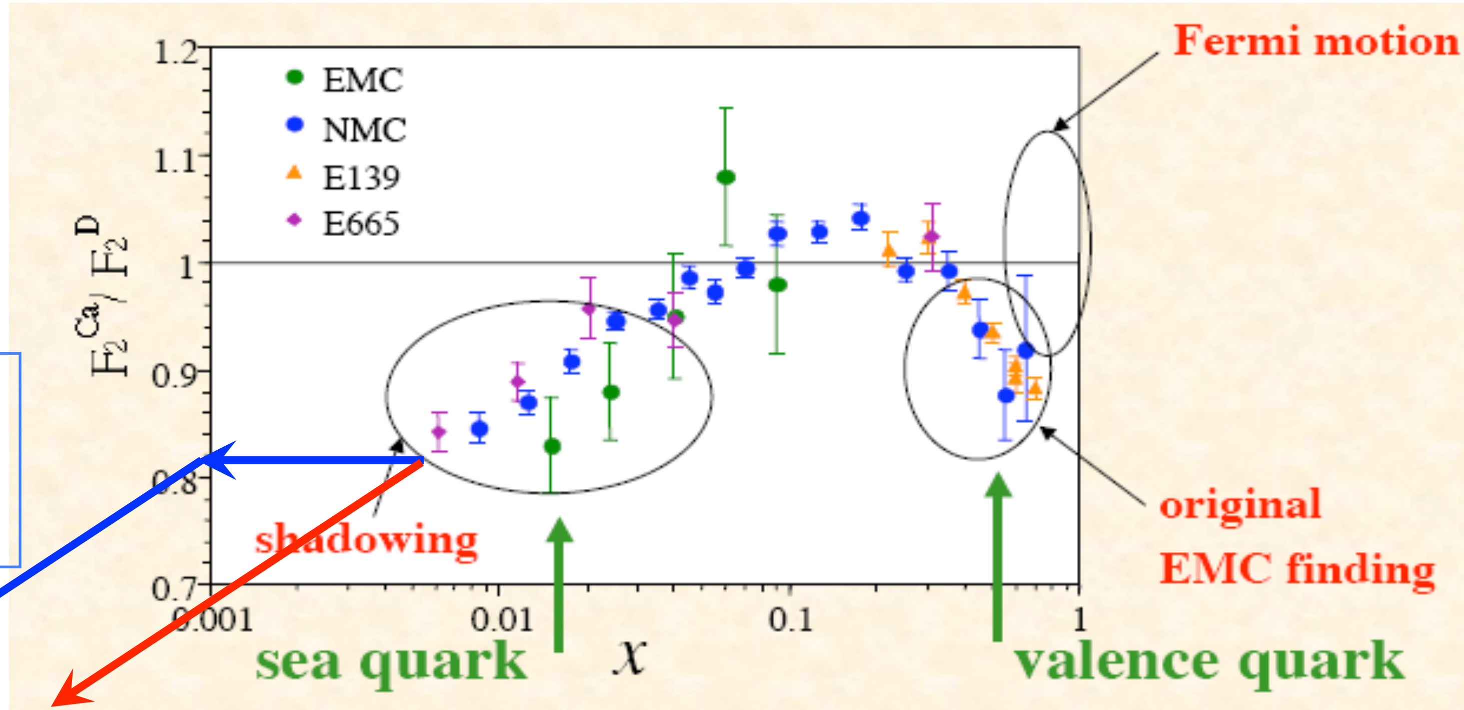
Will the suppression/shadowing continue fall as x decreases?

Could nucleus behaves as a large proton at small-x?

Range of color correlation – could impact the center of neutron stars!

An “easiest” measurement at EIC

❑ EMC effect, Shadowing and Saturation:



Continuously fall = saturation in F_2^A

❑ Questions:

Will the suppression/shadowing continue fall as x decreases?

Could nucleus behaves as a large proton at small-x?

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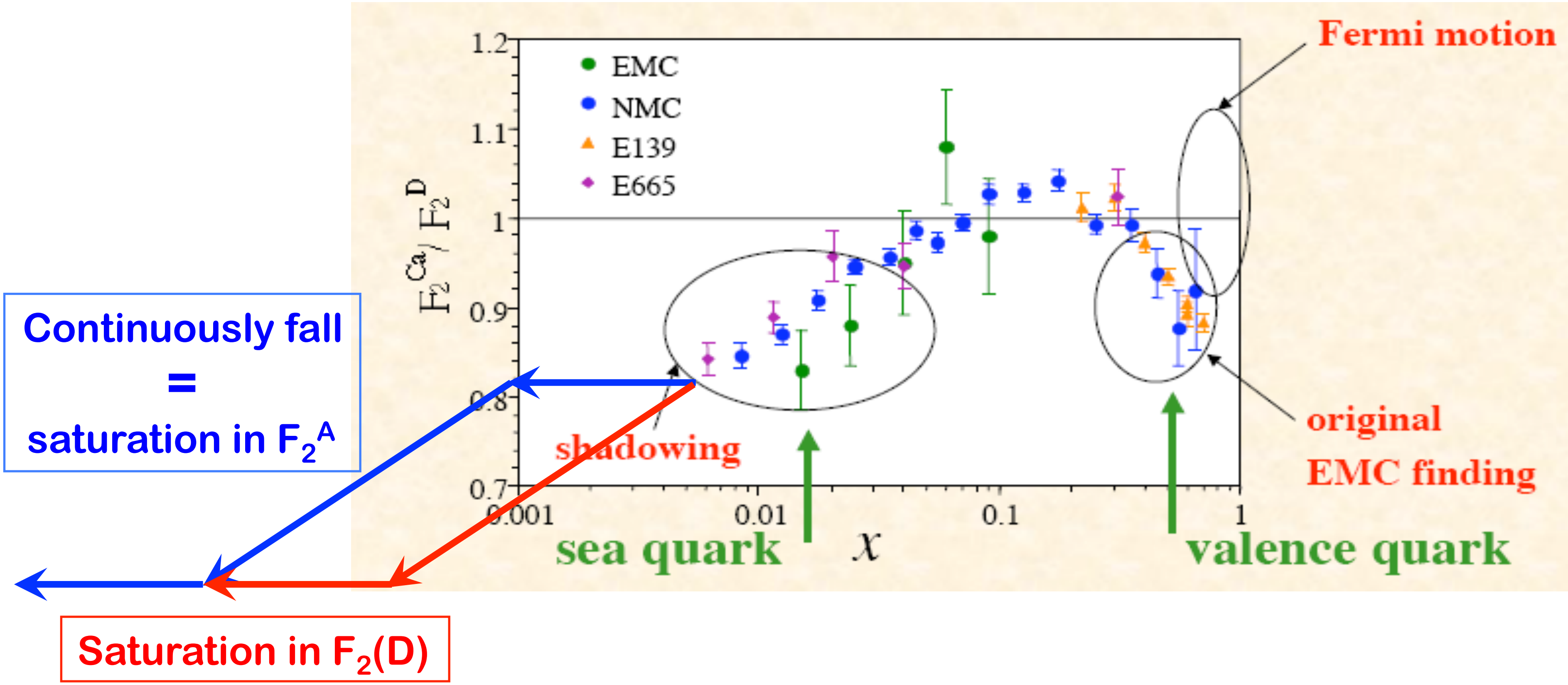
An “easiest” measurement at EIC

❑ EMC effect, Shadowing and Saturation:



❑ Questions:

Will the suppression/shadowing continue fall as x decreases?

Could nucleus behaves as a large proton at small-x?

Range of color correlation – could impact the center of neutron stars!

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

- **Cross section:**

  Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma_{AB}^{(2)}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma_{AB}^{(3)}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma_{AB}^{(4)}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$

– Positive definite

- **Asymmetries or difference of cross sections:**

  – Not necessary positive!

  - both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - one beam polarized A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Basics for spin observables

Factorized cross section:

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

Parity and Time-reversal invariance:

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

IF:

$$\langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

or $$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

Operators lead to the "+" sign ⟹ spin-averaged cross sections

Operators lead to the "-" sign ⟹ spin asymmetries

Example:

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

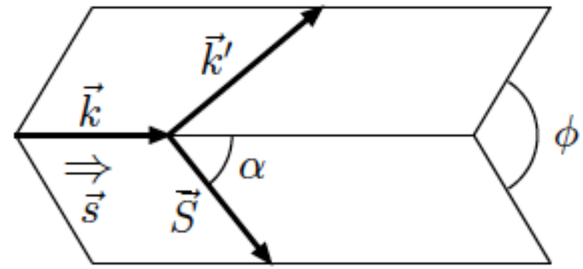
$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \to h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Polarized deep inelastic scattering

❑ **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) - \mathcal{W}^{\mu\nu}(P,q,-S)$$



✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$ **,**
and lepton helicity λ

✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times\left\{\cos\alpha\left\{\left[1-\frac{y}{2}-\frac{m^2x^2y^2}{Q^2}\right] g_1(x,Q^2) - \frac{2m^2x^2y}{Q^2}\,g_2(x,Q^2)\right\}\right.$$

$$\left. -\sin\alpha\cos\phi\,\frac{2mx}{Q}\sqrt{\left(1-y-\frac{m^2x^2y^2}{Q^2}\right)}\left(\frac{y}{2}\,g_1(x,Q^2) + g_2(x,Q^2)\right)\right\}$$

✧ **Spin orientation:**

$$\alpha = 0 : \Rightarrow \quad g_1$$

$$\alpha = \pi/2 : \Rightarrow \quad y\,g_1 + 2\,g_2$$, suppressed m/Q

Polarized deep inelastic scattering

❑ Spin asymmetries – measured experimentally:

✧ **Longitudinal polarization –** $\alpha = 0$

Known function

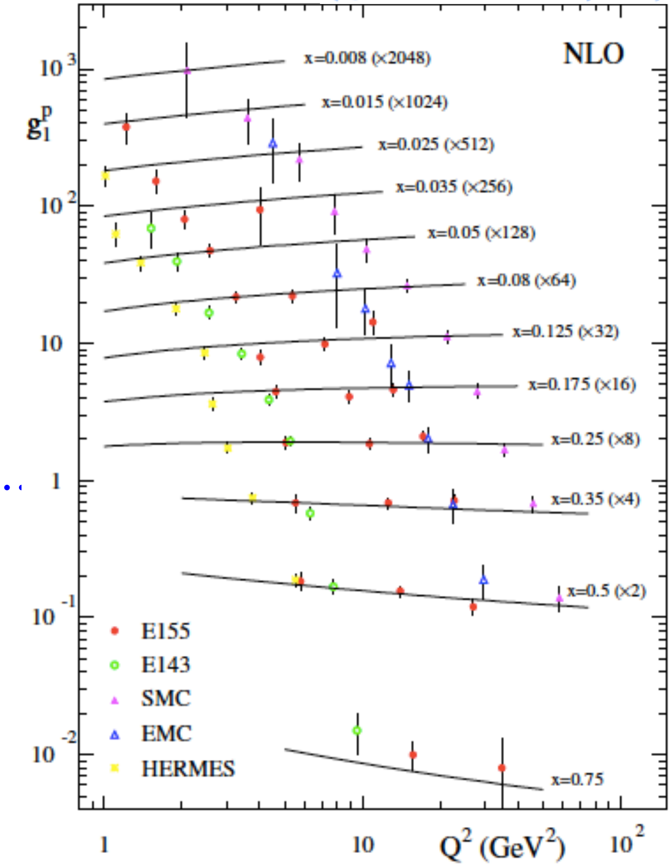
$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

$$(\, y = 1 - E'/E \,)$$

✧ **So far only "fixed target" experiments:**

CERN: EMC, SMC, COMPASS
SLAC: E80, E130, E142, E143, E154
DESY: HERMES
JLab: Hall A,B,C with many experiments

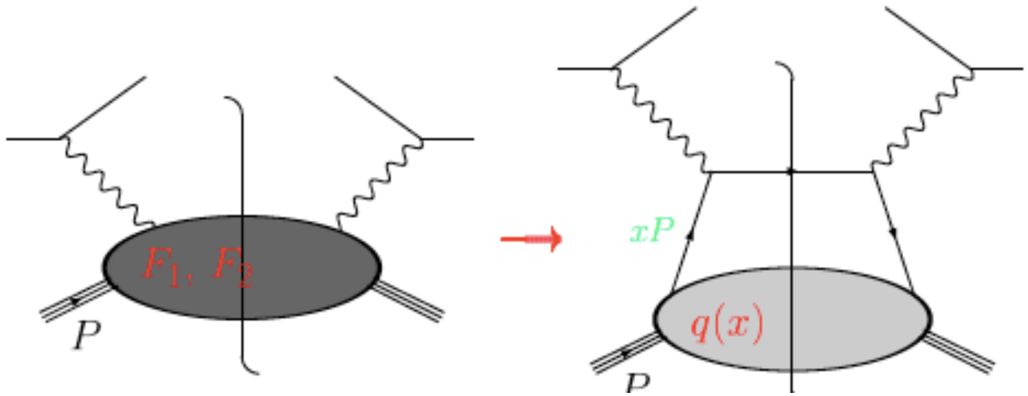
with various polarized targets: $p,\ d,\ {}^3\mathrm{He},\ldots$



✧ **Future: EIC**

Polarized deep inelastic scattering

❑ Parton model results – LO QCD:



✧ **Structure functions:**

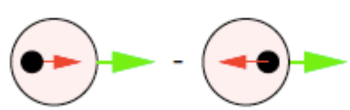
$$F_1(x) \;=\; \frac{1}{2}\sum_q e_q^2\left[q(x)+\bar{q}(x)\right]$$

$$g_1(x) \;=\; \frac{1}{2}\sum e_q^2\left[\Delta q(x)+\Delta\bar{q}(x)\right]$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u+\Delta\bar{u}\right)+\frac{1}{9}\left(\Delta d+\Delta\bar{d}+\Delta s+\Delta\bar{s}\right)\right]$$

✧ **Polarized quark distribution:**

$$\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)$$



Information on nucleon's spin structure

Flavor separation?

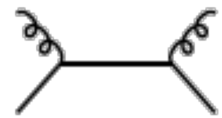
RHIC Measurements on ΔG

- **Physical channels sensitive to ΔG:**

$$\vec{p} + \vec{p} \to \pi + X$$
$$\vec{p} + \vec{p} \to \text{jet} + X$$

$$\vec{g}\vec{g} \to gg$$

$$\vec{q}\vec{g} \to qg$$

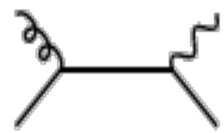


Pion or jet production

high rates

$$\vec{p} + \vec{p} \to \gamma + X$$
$$\vec{p} + \vec{p} \to \gamma + \text{jet} + X$$

$$\vec{q}\vec{g} \to \gamma q$$



Direct photon production

low rates

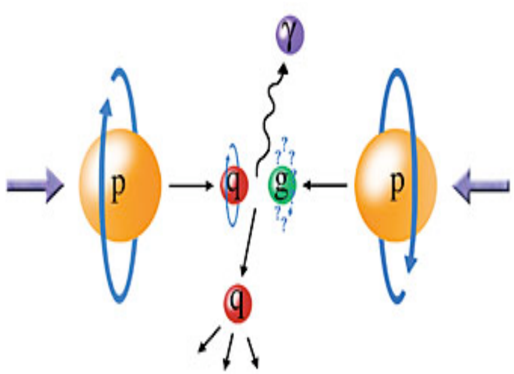
$$\vec{p} + \vec{p} \to D + X$$
$$\vec{p} + \vec{p} \to B + X$$

$$\vec{g}\vec{g} \to c\bar{c}$$
$$\vec{g}\vec{g} \to b\bar{b}$$

Heavy-flavour production

separated vertex detection required

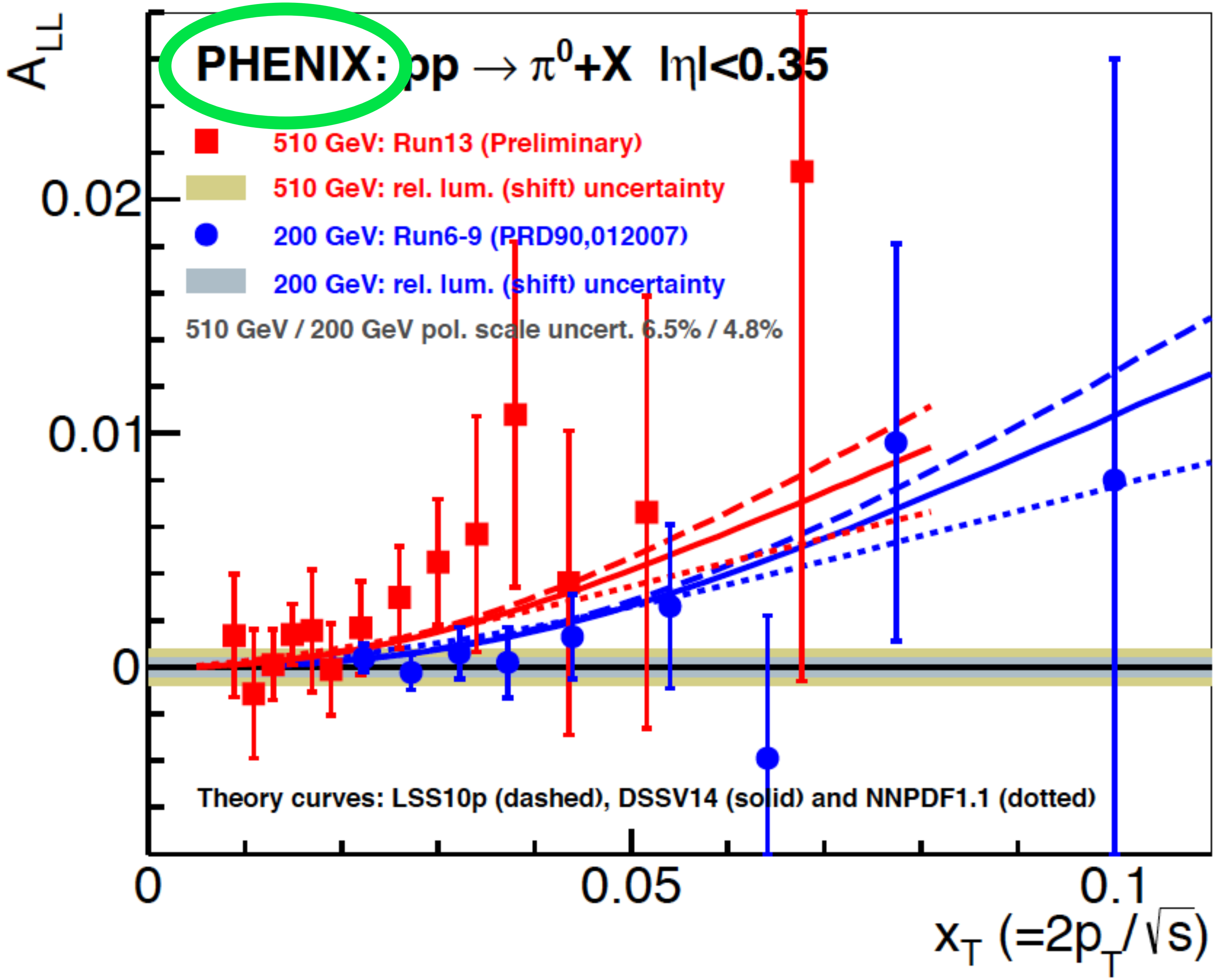
- **Collinear QCD factorization:**



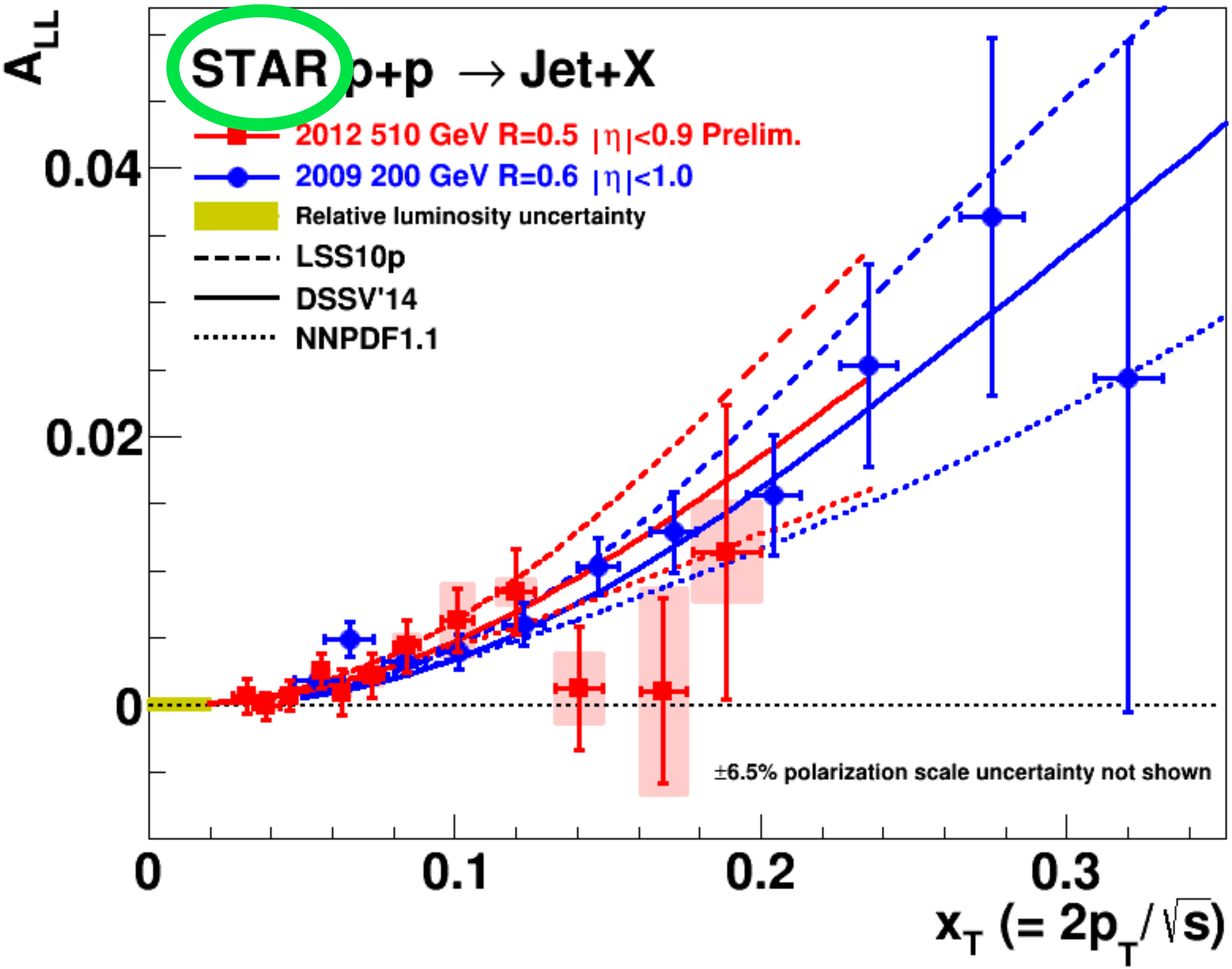
Experiments measure cross sections,
And asymmetries, not helicity distributions!

QCD global analyses to extract the best helicity districutions at NLO, to calculate helicity contribution to proton spin

RHIC Measurements on ΔG

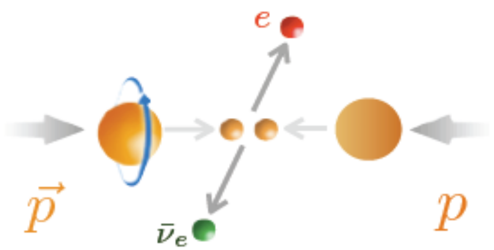


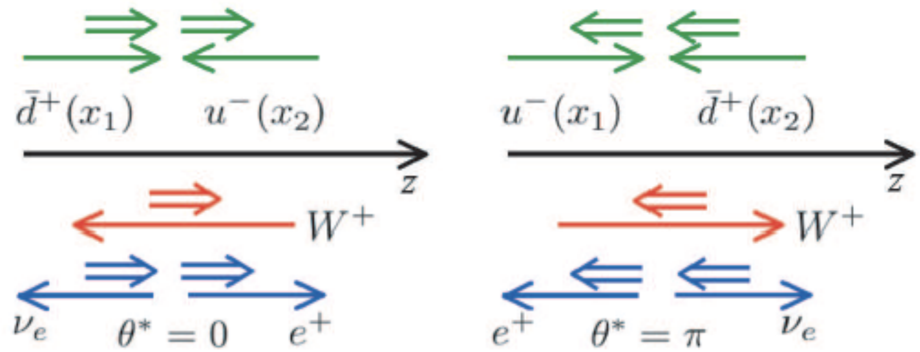
RHIC Measurements on ΔG



RHIC measurements of Δq and $\Delta \bar{q}$

- **W's are left-handed:**





- **Flavor separation:**

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta \bar{d}(x_1) u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1) u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta \bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

- **Complications:**

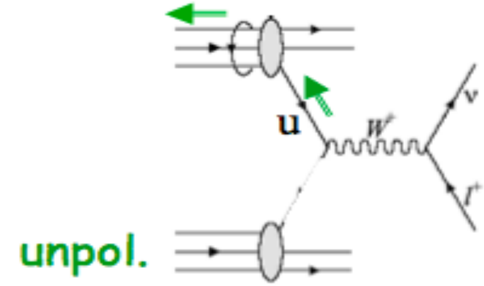
High order, W's p_T-distribution at low p_T

Sea quark polarization – RHIC W program

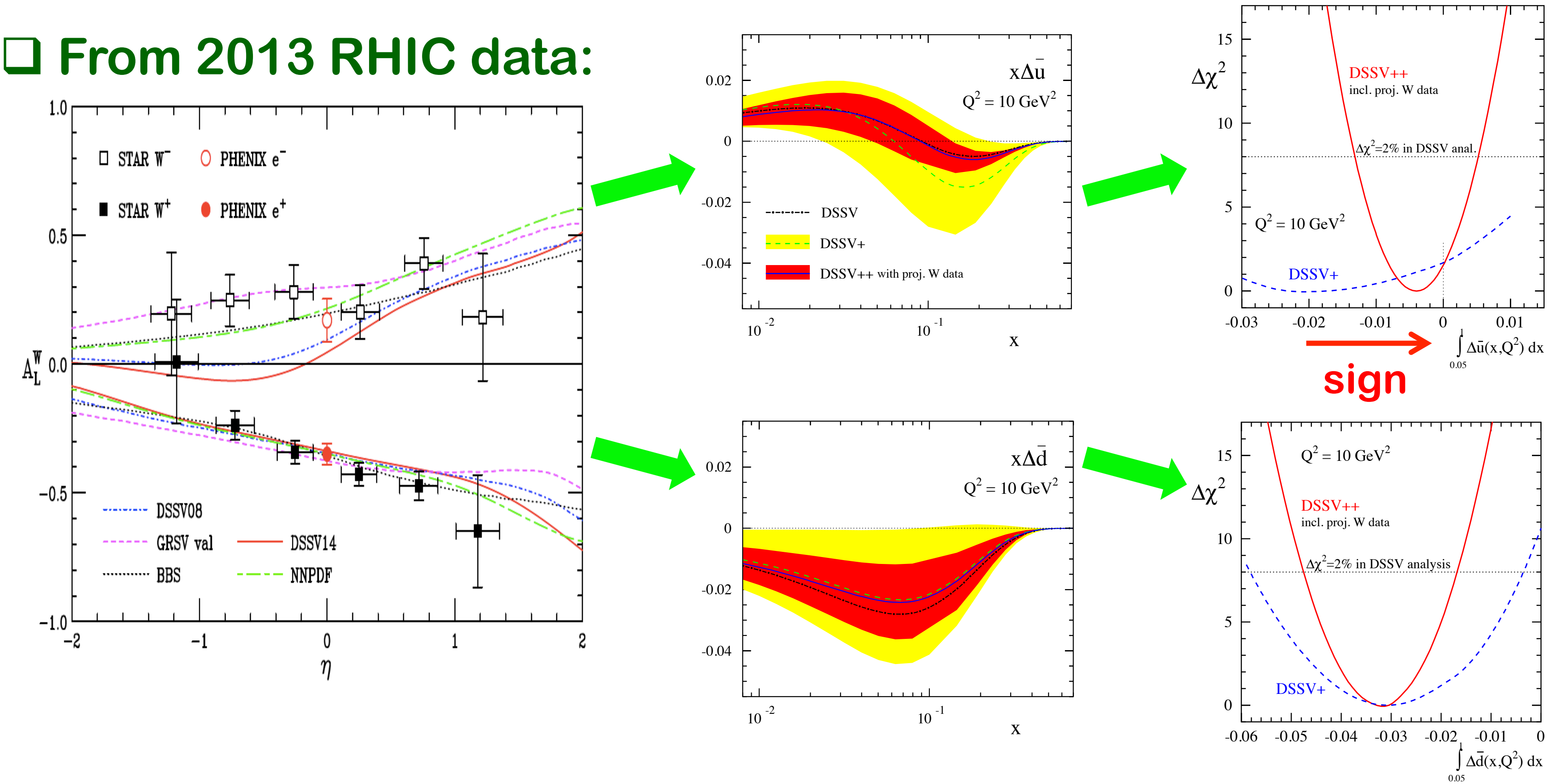
❑ **Single longitudinal spin asymmetries:**

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s)$$

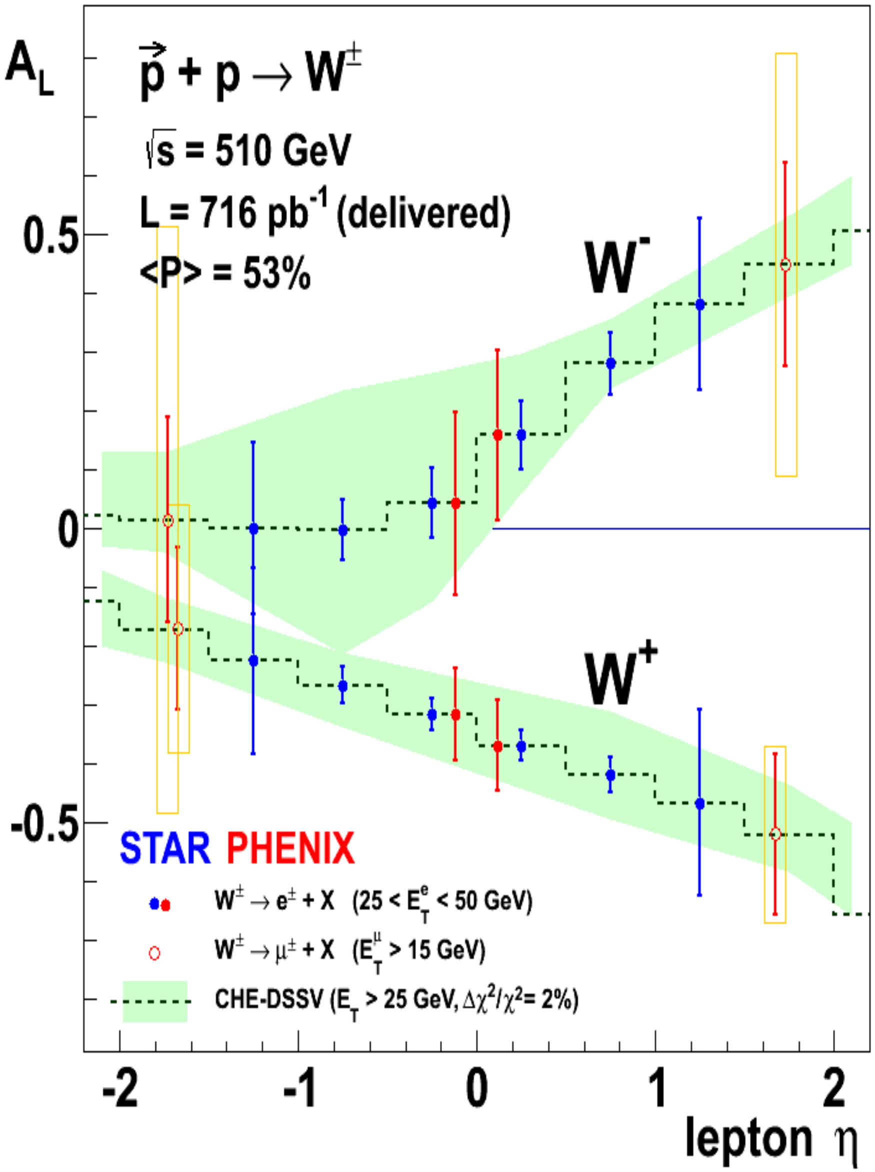
Parity violating weak interaction

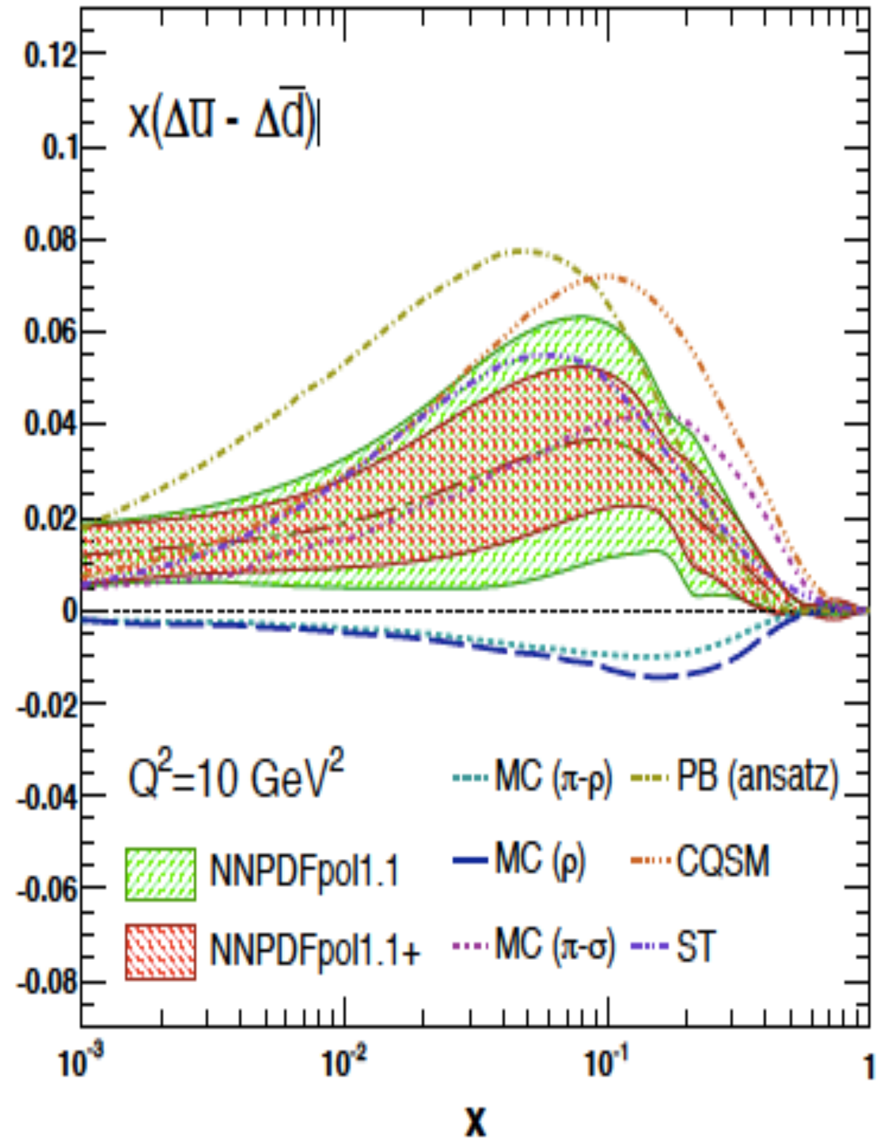


❑ **From 2013 RHIC data:**



Projected future W asymmetries



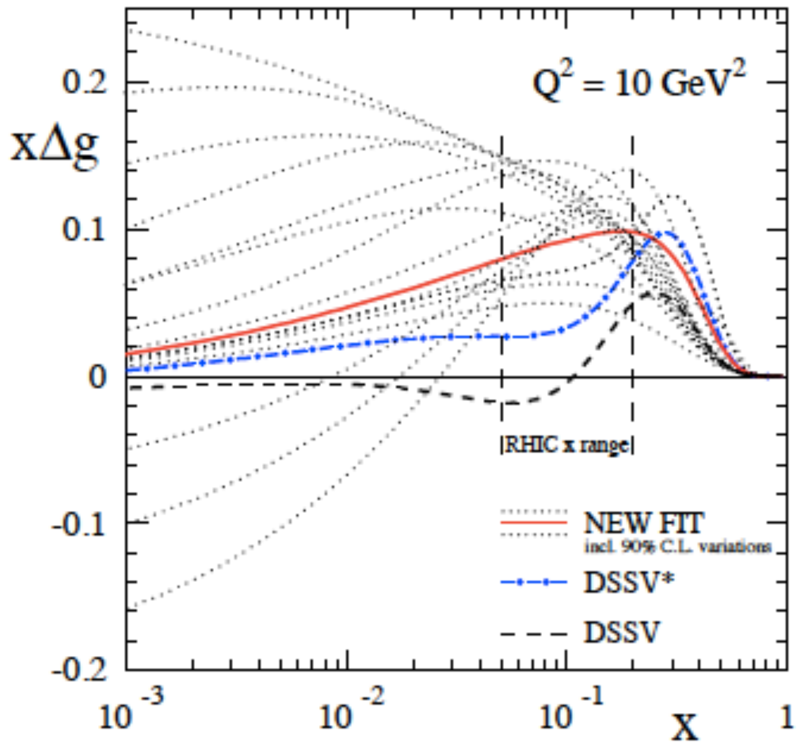


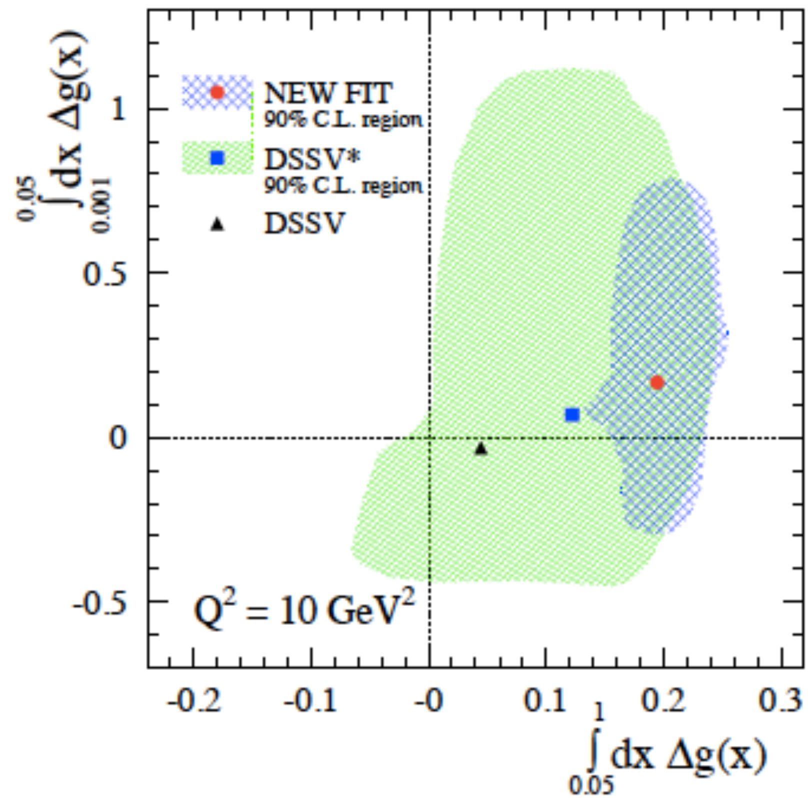
Global QCD analysis of helicity PDFs

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014) 012001

results featured in Sci. Am., Phys. World, ...

- Impact on gluon helicity:



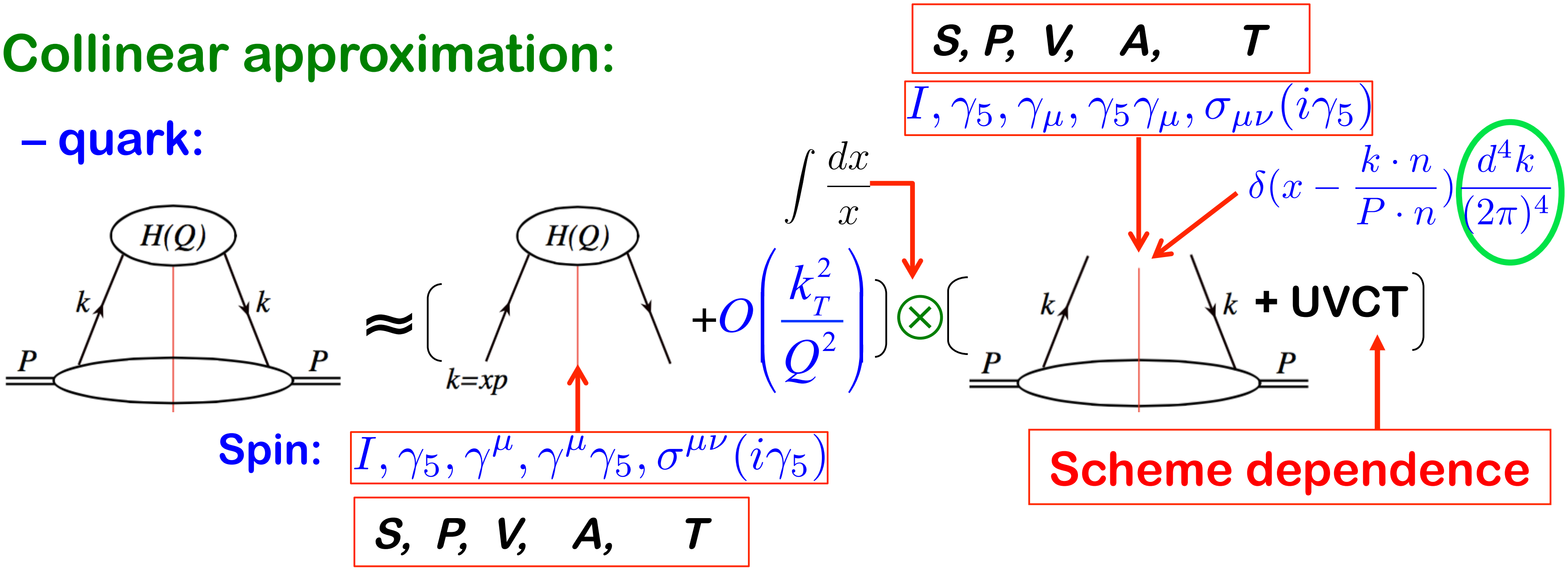


- ✧ Red line is the new fit
- ✧ Dotted lines = other fits with 90% C.L.

- ✧ 90% C.L. areas
- ✧ Leads ΔG to a positive #

The other leading power PDFs

- **Collinear approximation:**
  - **quark:**



- **Leading power hard parts in *p*:**

$$\frac{1}{2}\gamma\cdot p(V),\ \frac{1}{2}\gamma_5\gamma\cdot p(A),\ \frac{1}{2}\gamma\cdot p\gamma_\perp^\alpha\gamma_5(T) \longleftrightarrow$$ **4 – spin states of the "quark-pair"**

Non-flip, longitudinally flip, transversely flip

- **Leading power distributions:**

$$\frac{\gamma\cdot n}{2p\cdot n}(V),\ \frac{\gamma_5\gamma\cdot n}{2p\cdot n}(A),\ \frac{\gamma\cdot n\gamma_\perp^\alpha\gamma_5}{2p\cdot n}(T) \longleftrightarrow q(x,Q),\ \Delta q(x,Q),\ h_1(x,Q)$$

Unpolarized PDF, Helicity/Polarized PDF, Transversity distribution

Transversity Distributions

❑ Transversity:

Jaffe and Ji, 1991

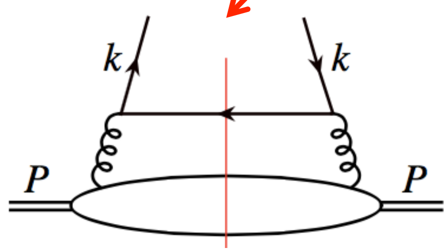
$$h_1(x) = \frac{1}{\sqrt{2}p^+}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\perp|\psi_+^\dagger(0)\gamma_\perp\gamma_5\psi_+(\lambda n)|PS_\perp\rangle \text{ + UVCT}$$

with $\psi_\pm = P_\pm \psi$ and $P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$

❑ Unique for the quarks:

No mixing with gluons!

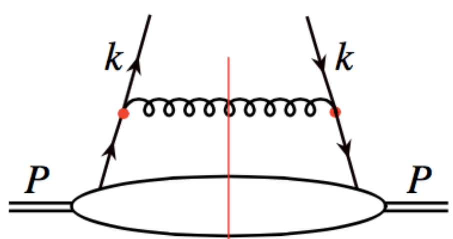
$\gamma\cdot n\gamma_\perp\gamma_5$ ← Even # of γ's



= 0

No mixing with PDFs, helicity distributions

❑ Perturbatively UV and CO divergent:



+ wave function renormalization

$$\Delta_T P_{qq}^{(0)}(x) = C_F\left[\frac{2x}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

→ "DGLAP" evolution kernels

NLO - Vogelsang, 1998

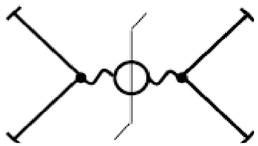
Connection to physical observables

- **Need two-chiral odd distributions – two hadrons:**

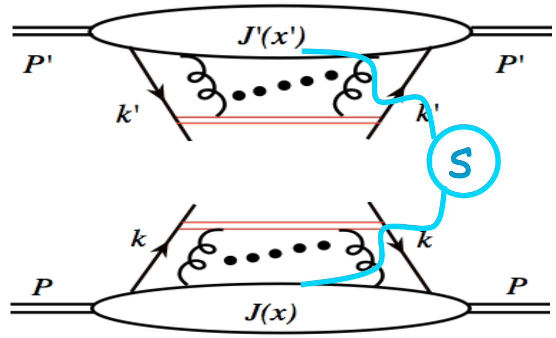
Soper, Ralston, 1978
Jaffe, Ji, 1991, 1992

– **Drell-Yan:**

$$\sigma^{\mathrm{DY}}_{\mathrm{tot}} \sim$$



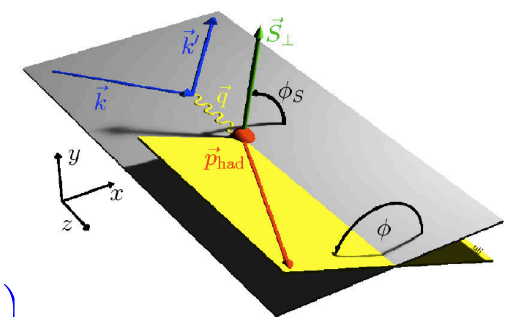
$$\otimes$$



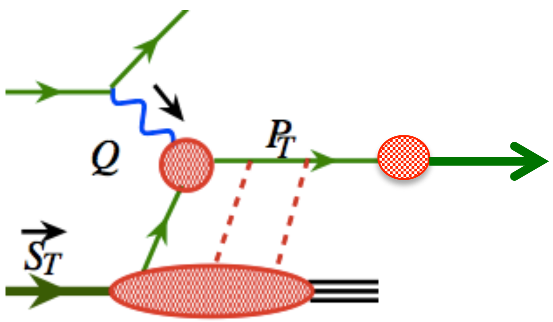
$$+\, O\left(\frac{1}{QR}\right)$$

$$\sim \; h_1(x) \otimes h_1(x')$$

Predictive power: Universal Transversity



– **SIDIS:**



$$\sim \; h_1(x) \otimes D_{\mathrm{Collins}}(z)$$

$$\sim \; A_T^{\sin(\phi+\phi_s)}$$

- **Caution:**

Transversity extracted depends on the “scheme” or UVCT

Cross section is always positive!

Like PDFs (helicity distributions), transversity does not have to be positive

Soffer's inequality

- Relation between quark distributions:

$$h_1(x) \leq \frac{1}{2}\left[q(x) + \Delta q(x)\right] = q^+(x)$$

Derived by using the positivity constraint of
quark + nucleon -> quark + nucleon
forward scattering helicity amplitudes

Cautions:

- ✧ Quark field of the Transversity distribution is NOT on-shell
- ✧ Quark + nucleon -> quark + nucleon
  forward scattering amplitude is perturbatively divergent

- Testing vs using as a constraint:

It is important to test this inequality, rather than using it as a constraint for fitting the transversity

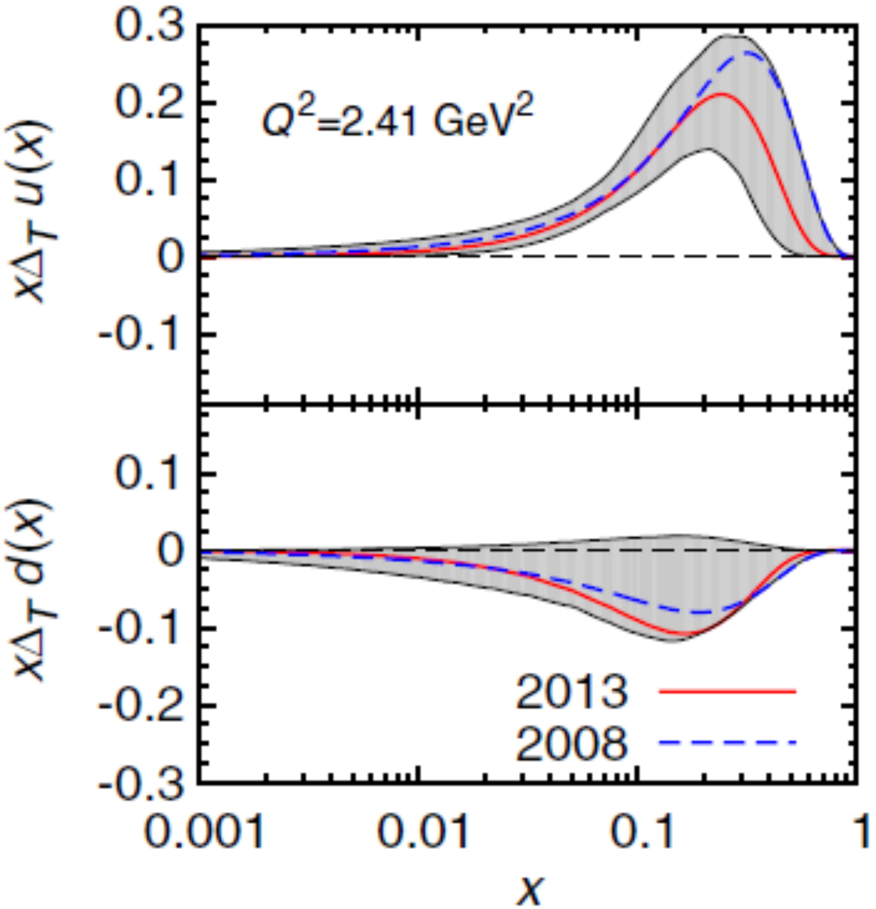
Perturbatively calculated evolution kernels seem to be consistent with the inequality – the scale dependence

Extraction of Transversity

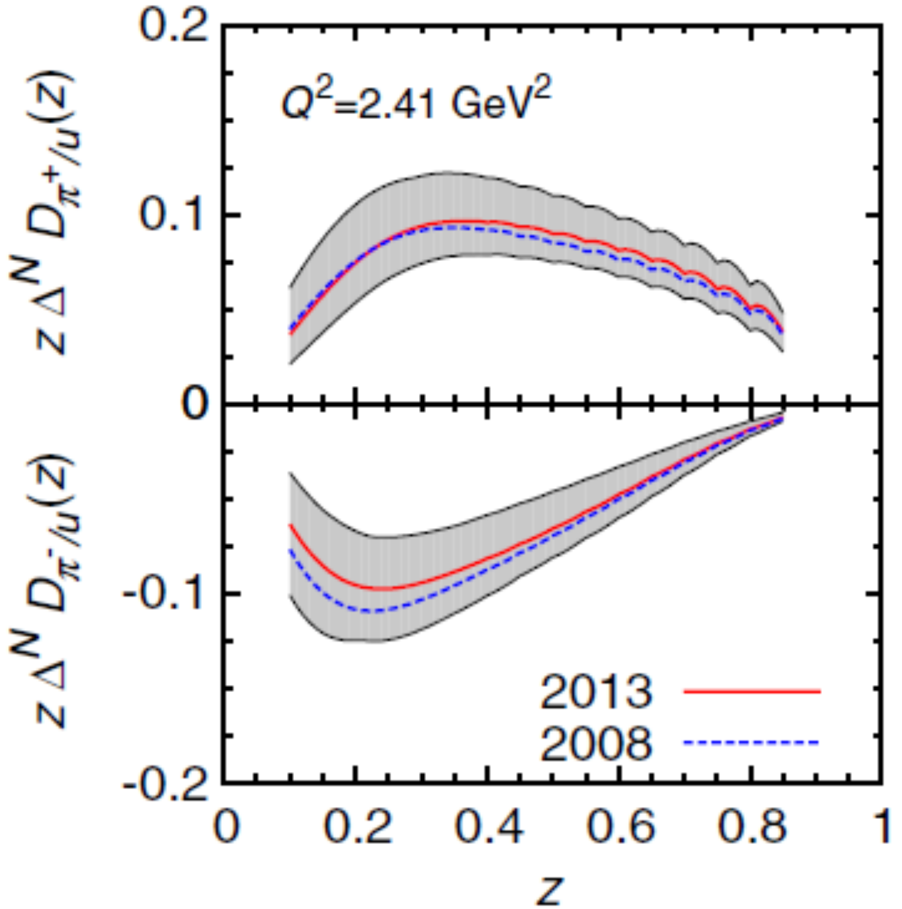
- Transversity and Collins function:

Anselmino et al.,
PRD 87, 094019 (2013)

Transversity



Collins function

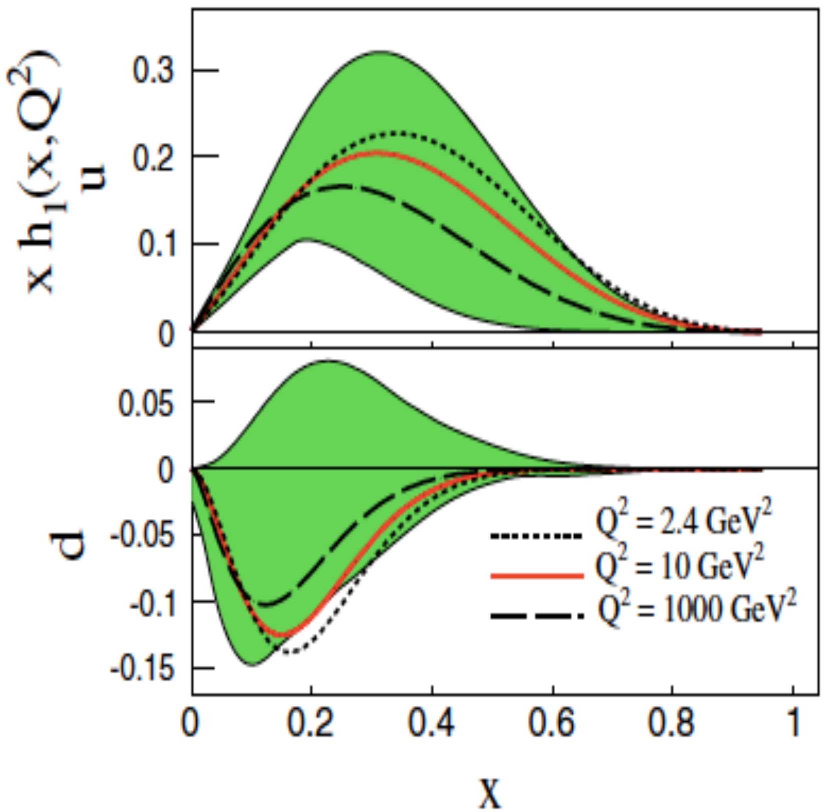


Extraction of Transversity

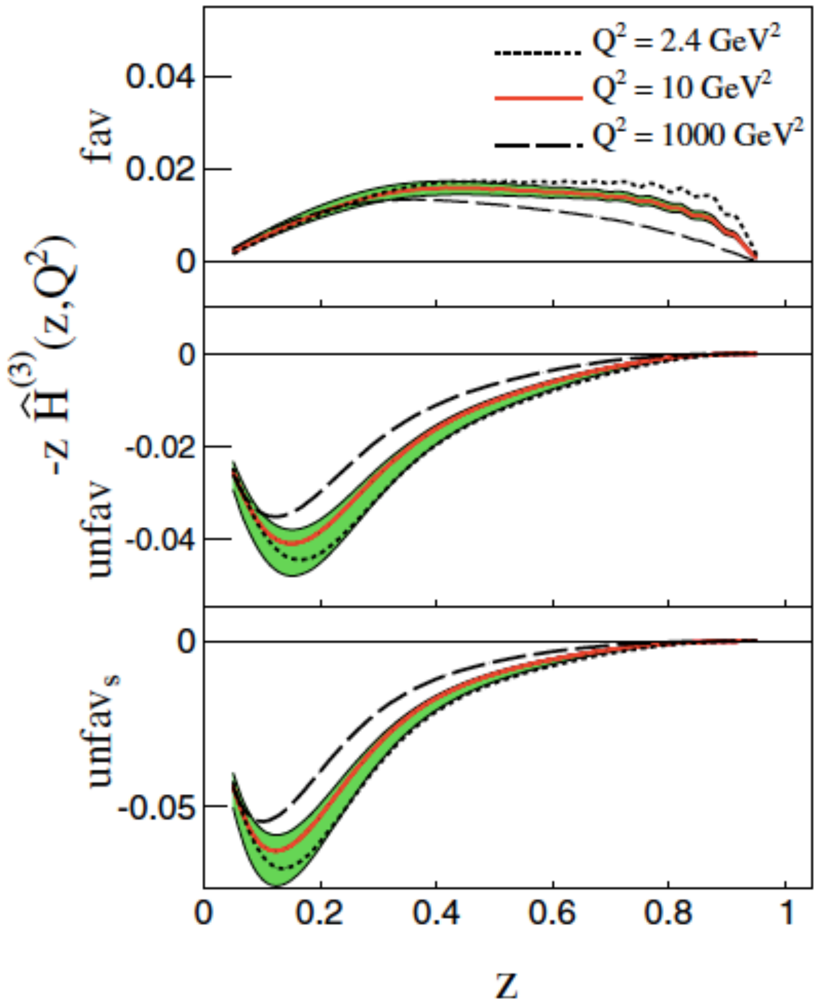
Kang et al, PRD, 2016

- ❑ **Transversity and Collins function:**

Transversity



Collins function

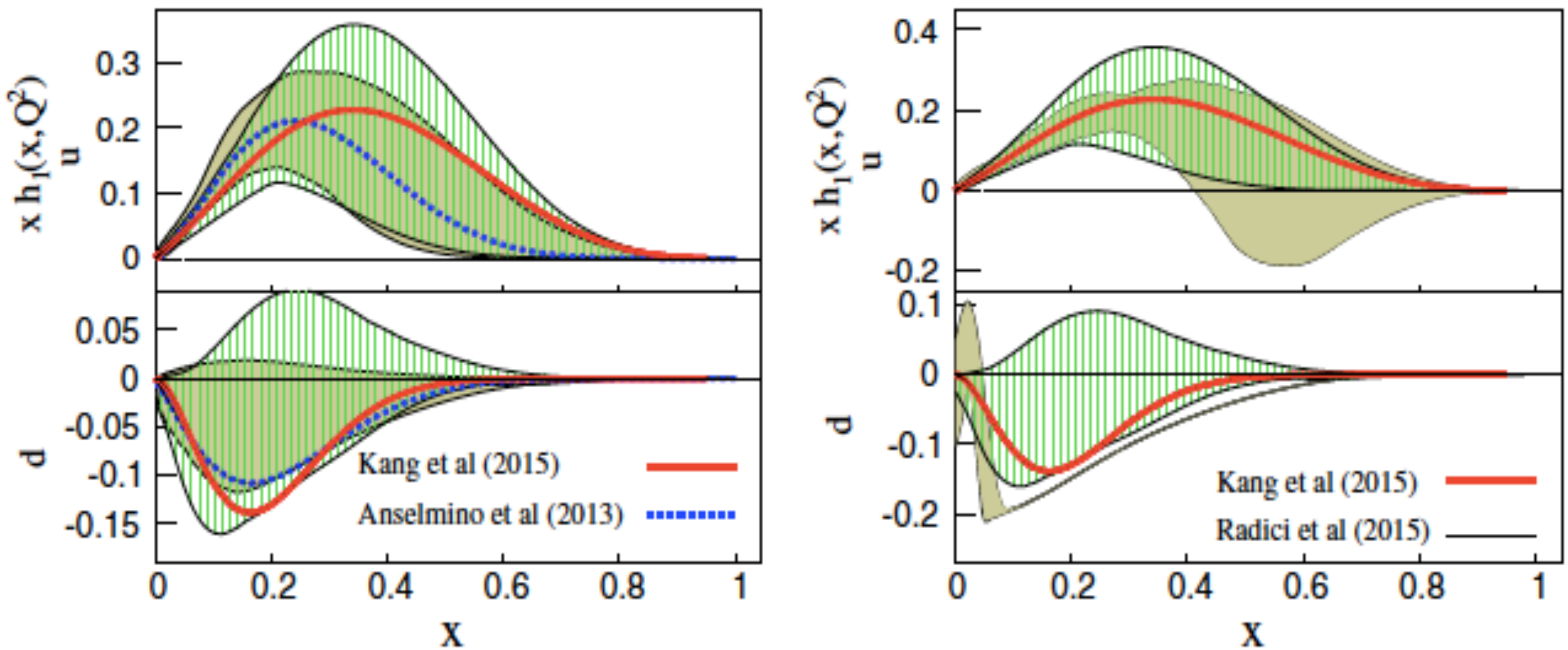


Extraction of Transversity

- Transversity comparison:

Anselmino et al.,
PRD 87, 094019 (2013)

Kang et al, PRD, 2016



✧ Consistent in overall shape and sign, but, different in details

✧ Large uncertainties!

- Future:

JLab12, Compass, EIC; Transverse polarized Drell-Yan?

Summary of lecture two

- QCD is consistent with all existing data from lepton-hadron and hadron-hadron collisions with unpolarized, as well as polarized beams, when there is a large momentum transfer

- From QCD global fits, we have a good idea on the quark and gluon PDFs, as well as their helicity distributions

- Transversity distribution and its moment (tensor charge) are fundamental QCD quantities, we start to know them

- But, EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, parton confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, …

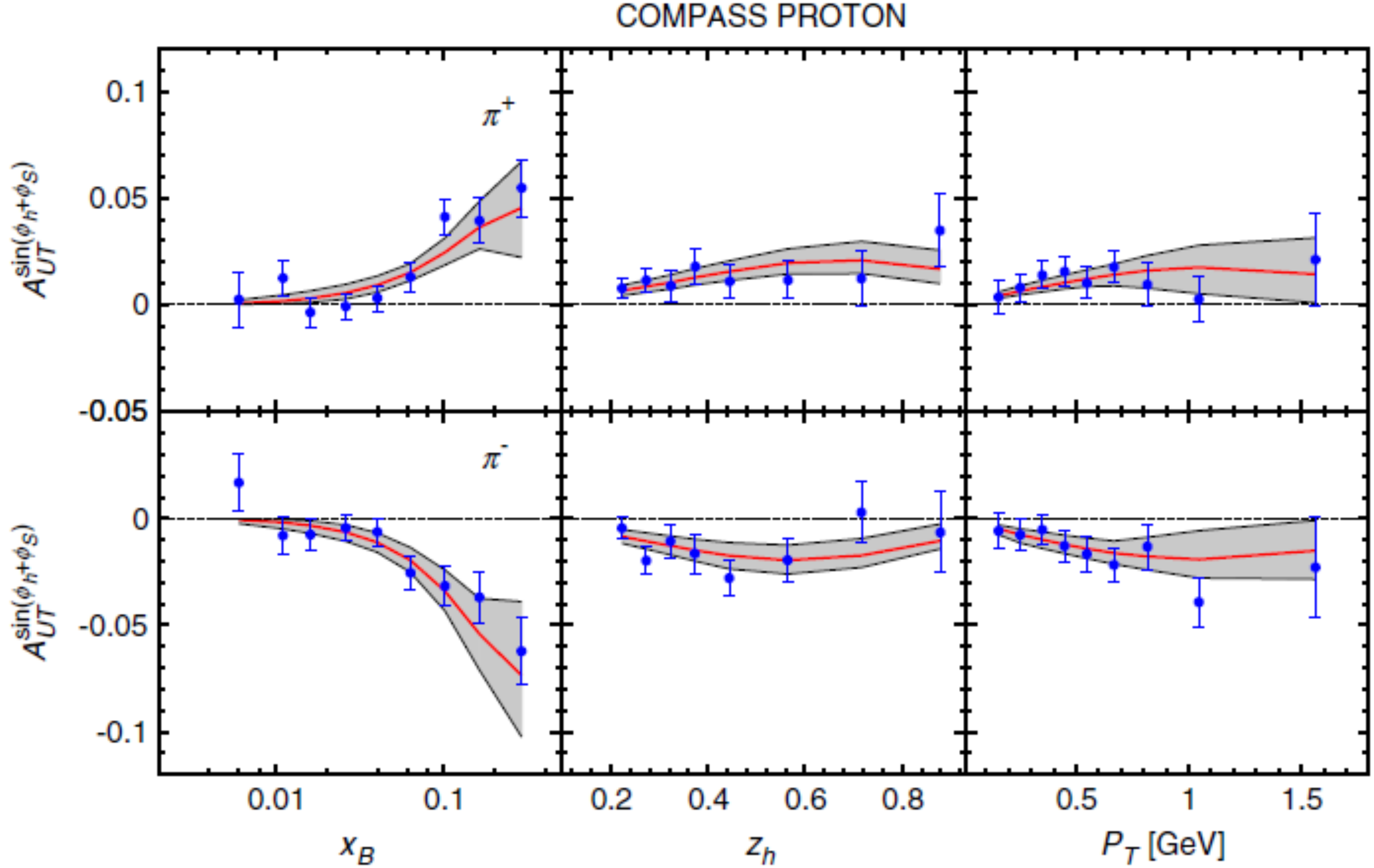
Thanks!

Backup slides

Extraction of Transversity

- SIDIS – mixed with Collins function:

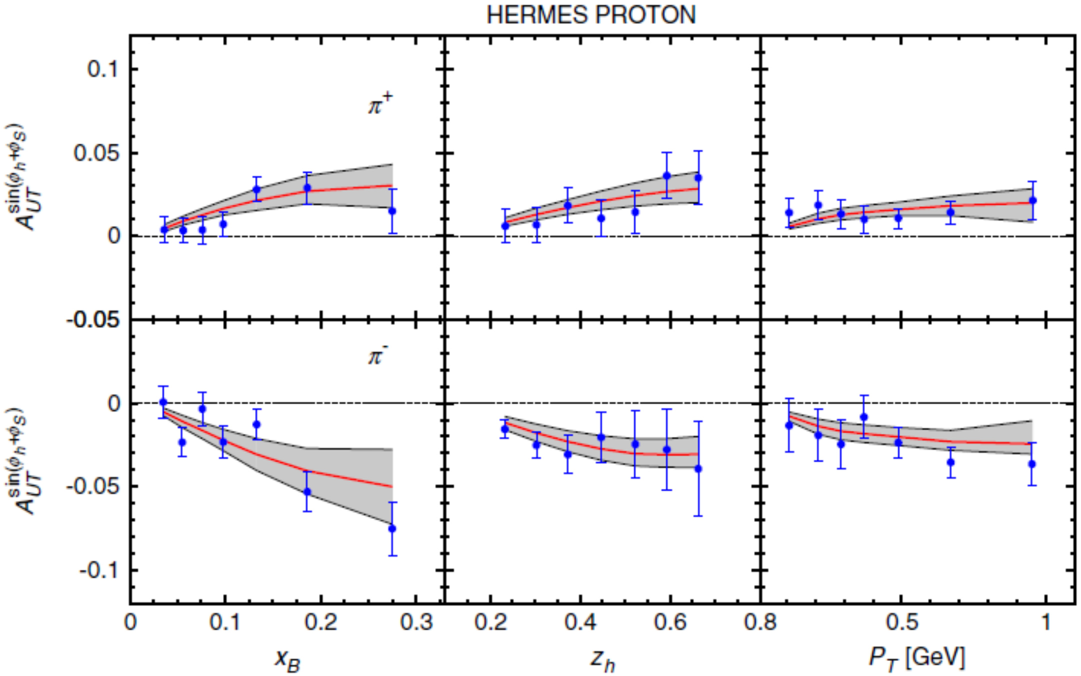
Anselmino et al., PRD 87, 094019 (2013)



Extraction of Transversity

Anselmino et al., PRD 87, 094019 (2013)

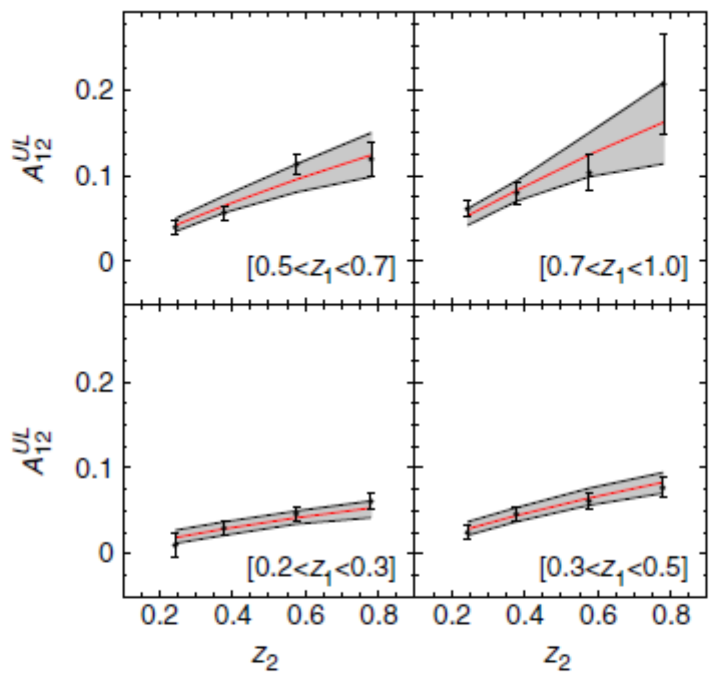
- SIDIS – mixed with Collins function:



HERMES (eP)

- e+e- – Collins functions:

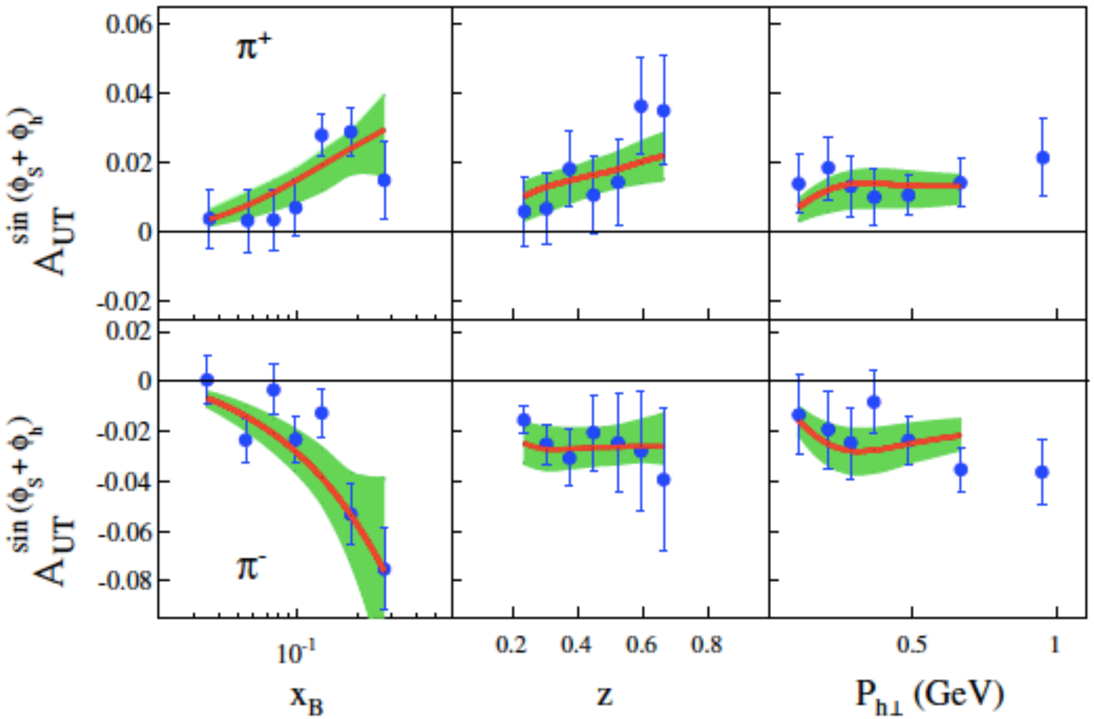
Belle (e+e-)



Extraction of Transversity

Kang et al, PRD, 2016

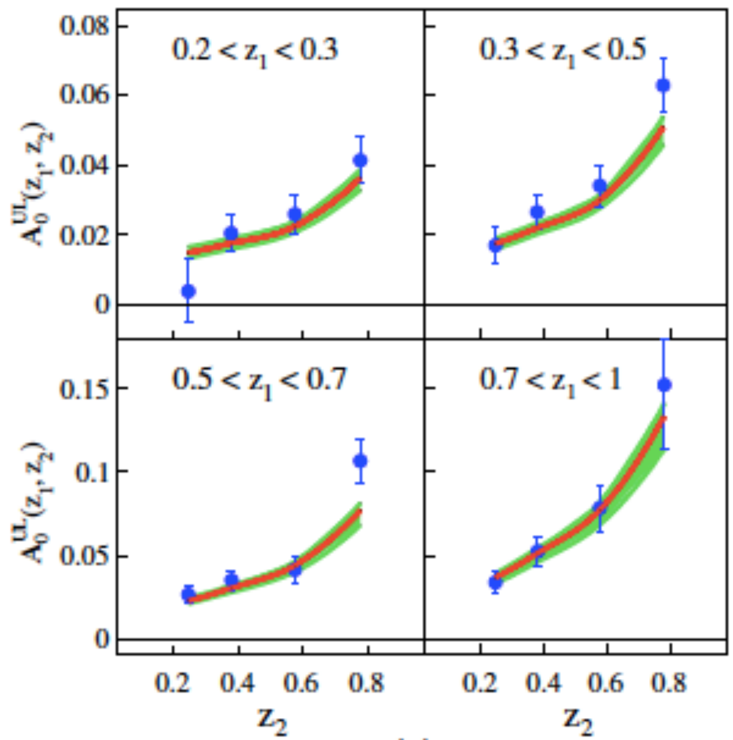
- SIDIS – mixed with Collins function:



← HERMES (eP)

- e+e- – Collins functions:

Belle (e+e-) →



Tensor charge

- **Definition:**

$$\delta q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]dx$$

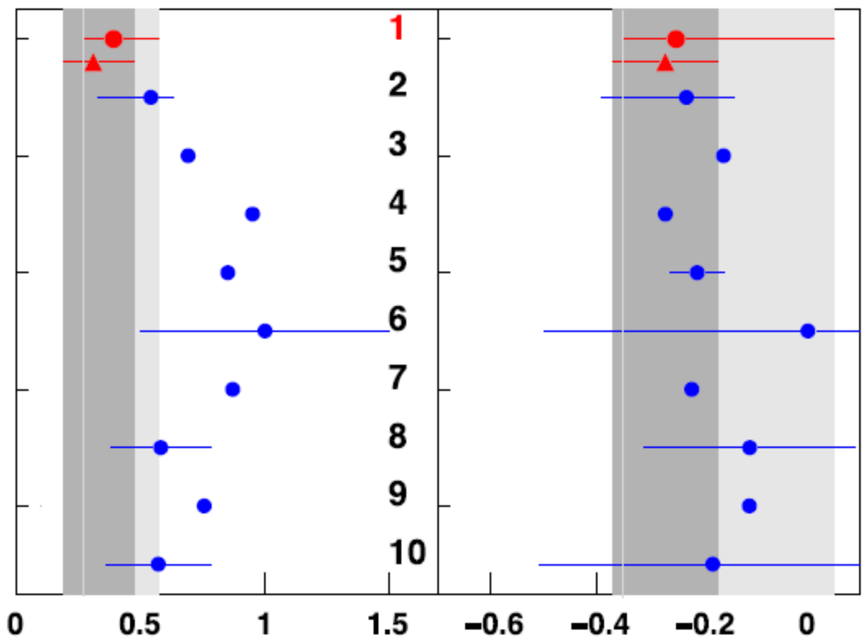
Moment – matrix elements of local operators
– fundamental QCD quantity – calculable on lattice or using models

- **Extraction:**

Anselmino et al.,
PRD 87, 094019 (2013)

● $\delta u = 0.39^{+0.18}_{-0.12}$ ● $\delta d = -0.25^{+0.30}_{-0.10}$

▲ $\delta u = 0.31^{+0.16}_{-0.12}$ ▲ $\delta d = -0.27^{+0.10}_{-0.10}$



✧ **Extracted from global fits**

by using two different parameterizations for Collins FF)

✧ **Predictions from various models (including LQCD)**

✧ **Tensor charges are expected to be smaller than axial charge**

$$\Delta u = 0.787 \quad \Delta d = -0.319$$

Tensor charge

- **Definition:**

$$\delta q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]dx$$

Moment – matrix elements of local operators
– fundamental QCD quantity – calculable on lattice or using models

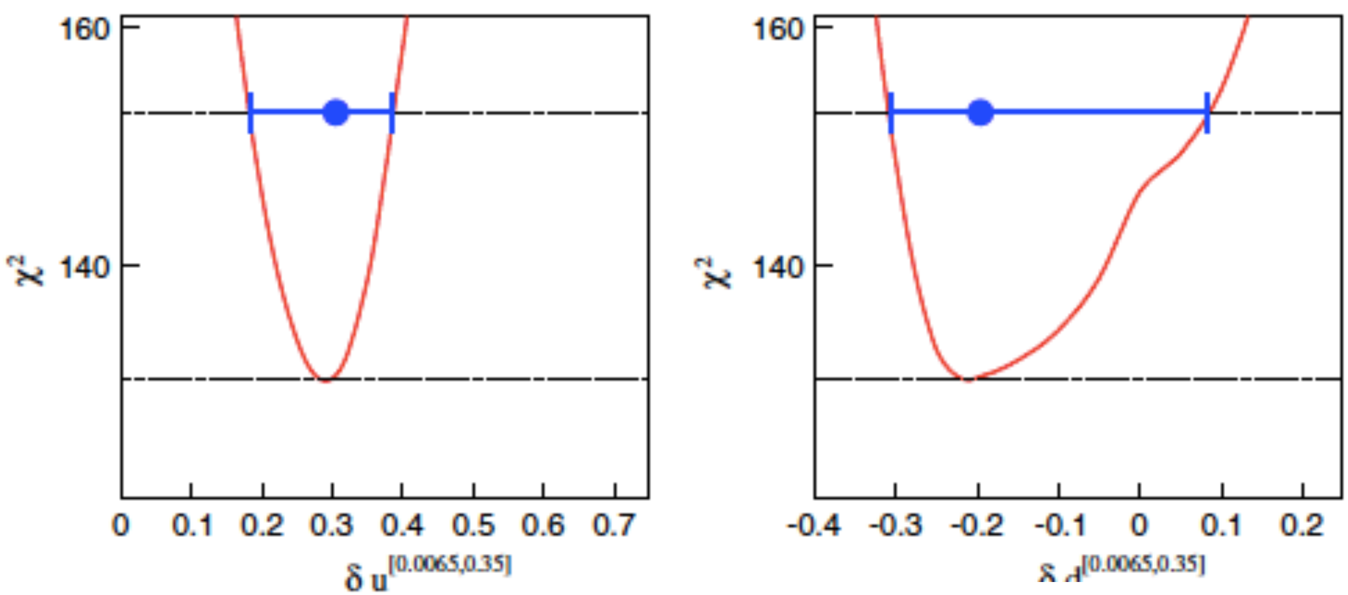
- **Extraction from global fits :**

Kang et al, PRD, 2016

$$\delta q^{[x_{\min}, x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} dx h_1^q(x, Q^2)$$

$$\delta u^{[0.0065,0.35]} = +0.30^{+0.08}_{-0.12}$$

$$\delta d^{[0.0065,0.35]} = -0.20^{+0.28}_{-0.11}$$



$Q^2 = 10 \text{ GeV}^2$

90% C.L.

$\Delta u = 0.787 \quad \Delta d = -0.319$

QCD and hadrons