Low-Energy Nuclear Experiments
Lecture 3: ‘Probing’ Wavefunctions

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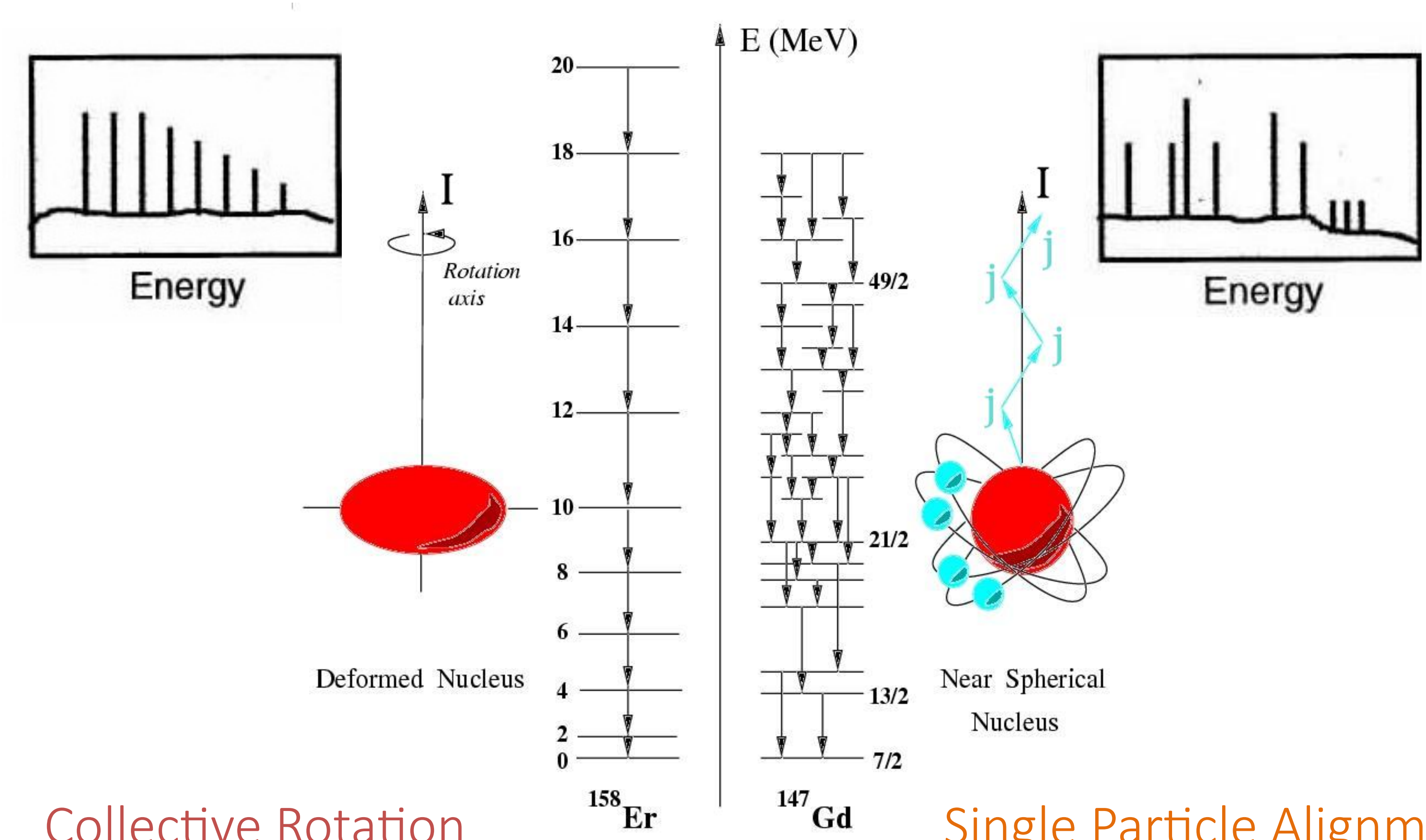
The (Third) Plan

- Investigating level schemes
  - Decay spectroscopy
- Details of nuclear wavefunctions
  - Single particle ‘occupancies’ and spectroscopy with nuclear reactions
  - Excited state lifetimes and transition probabilities
- Example – planning an experiment
  - What, where, why?

Level schemes – collective vs. single particle

Level Schemes Contain Structural Information

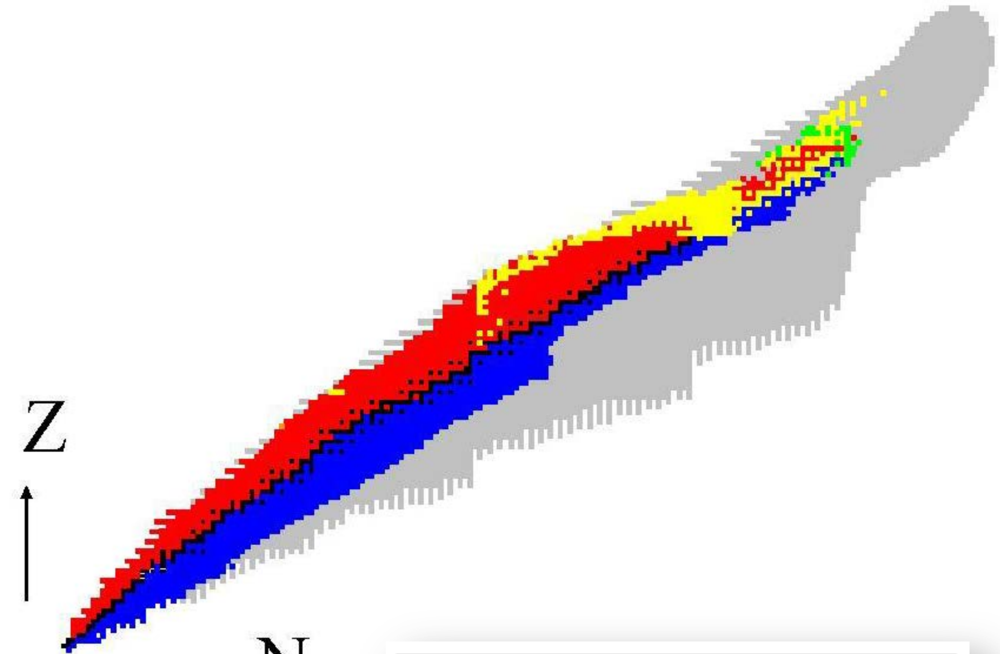


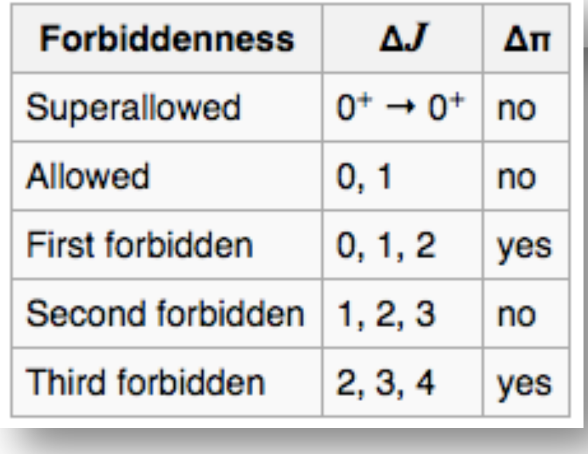
Collective Rotation

Single Particle Alignment

Back to β decay…

- The majority of nuclides on the chart decay via β^+ or β^- decay
  - n --> p + β^- + ν_e
  - p --> n + β^+ + $\bar{\nu}_e$
- We can consider β-decay (and other decays) as a tool to populate excited states in daughter nuclei, but with a unique selectivity

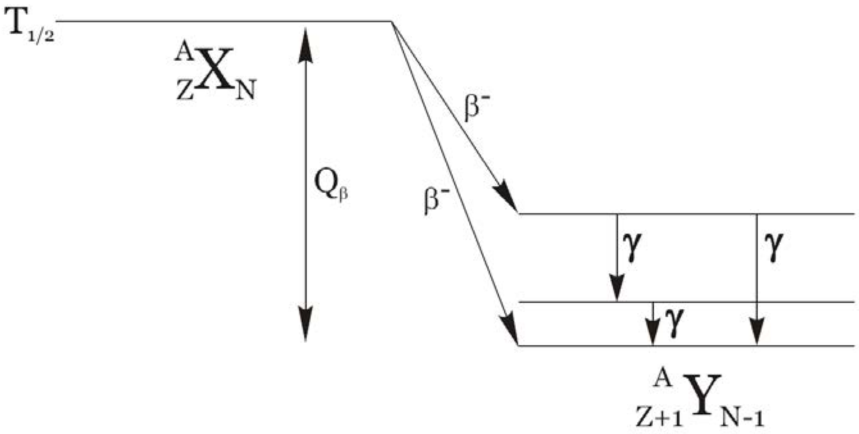




Forbiddenness	ΔJ	Δπ
Superallowed	$0^+ \to 0^+$	no
Allowed	0, 1	no
First forbidden	0, 1, 2	yes
Second forbidden	1, 2, 3	no
Third forbidden	2, 3, 4	yes

Implantation β decay spectroscopy

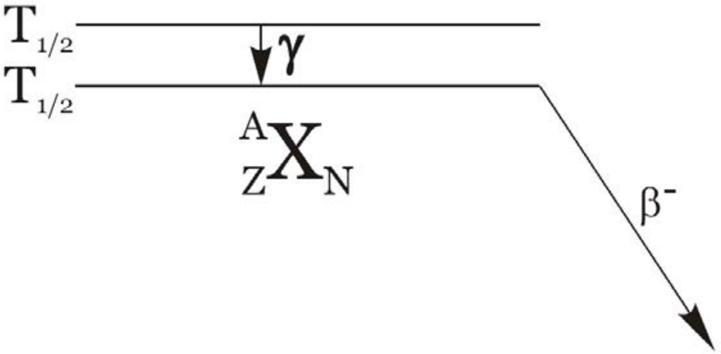
β-Delayed Gamma Spectroscopy



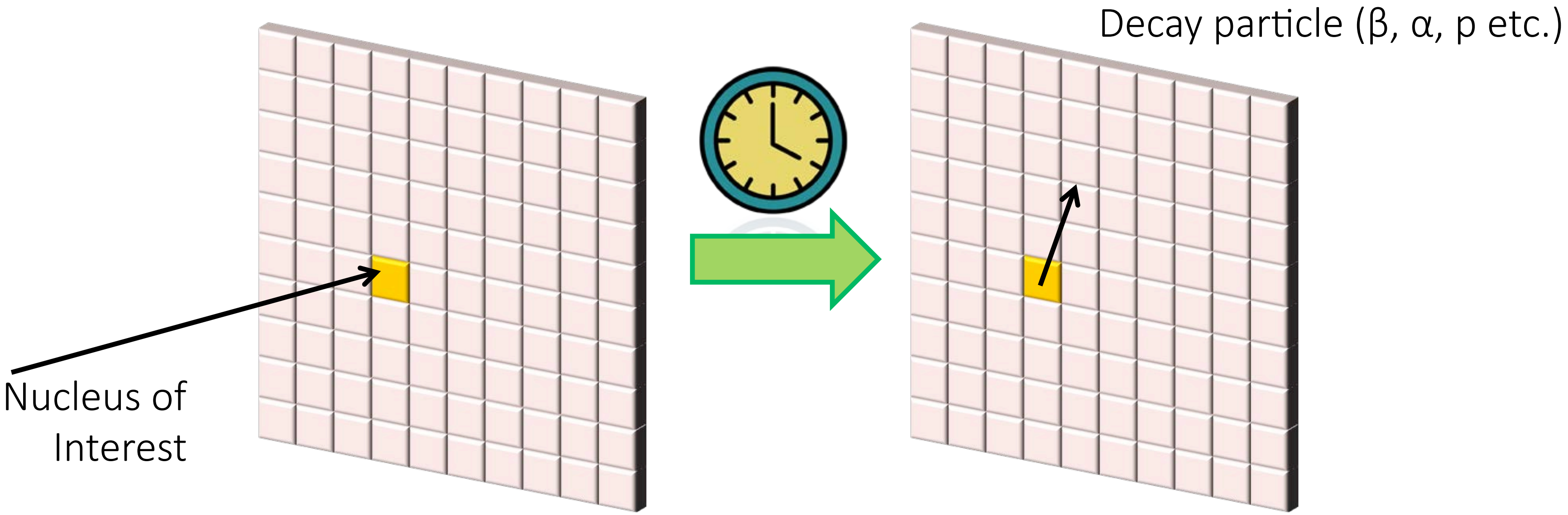
- gamma rays following decay events provide information on low-level structure of daughter nuclei

Isomeric Decay

- depending on the production mechanism, nuclei may be produced in long-lived excited states (isomeric states)
- a TAC for implantation-gamma provides the possibility for isomer lifetime determination, if you look for gammas following an implantation

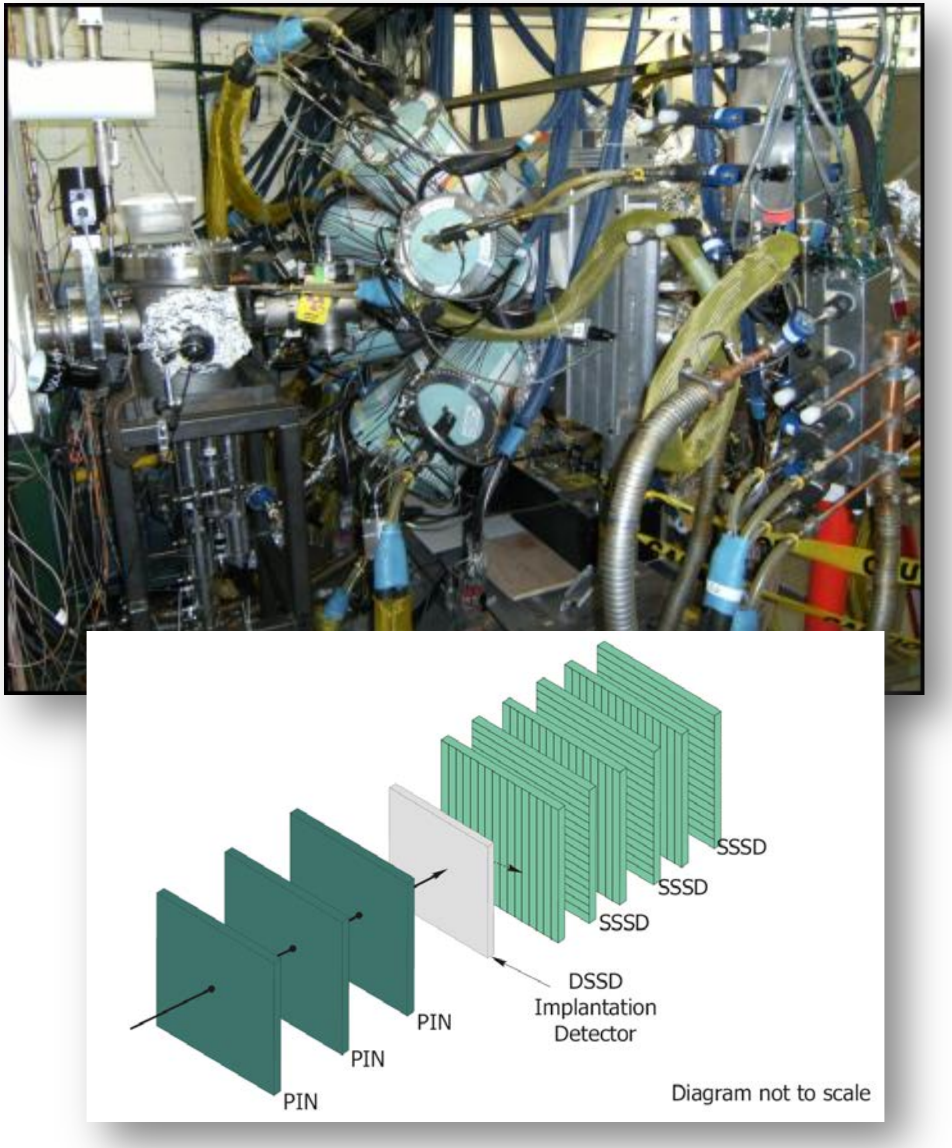


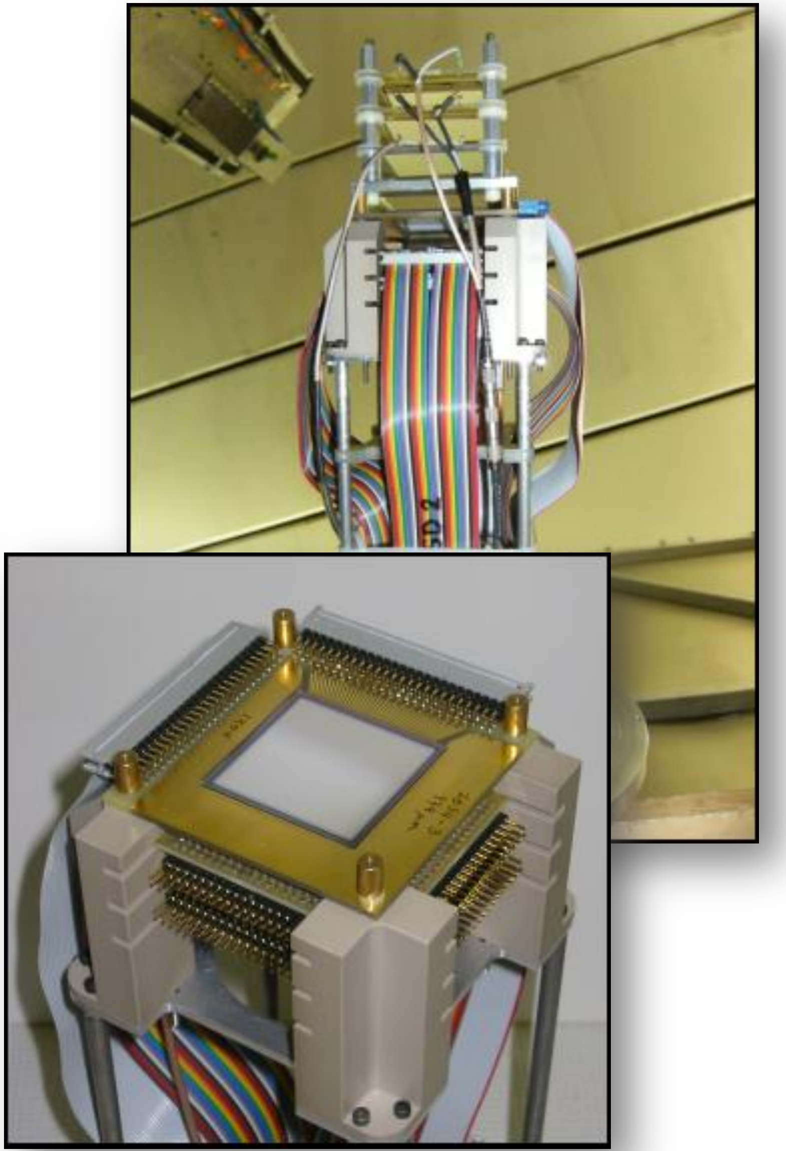
Implant-decay correlation technique



The use of highly-segmented detectors (usually Si) allows temporal and spatial correlations between implanted nuclei, and their subsequent decays → detect the implant and the decay to obtain half-lives and information on levels in the daughter relative to the parent ground state

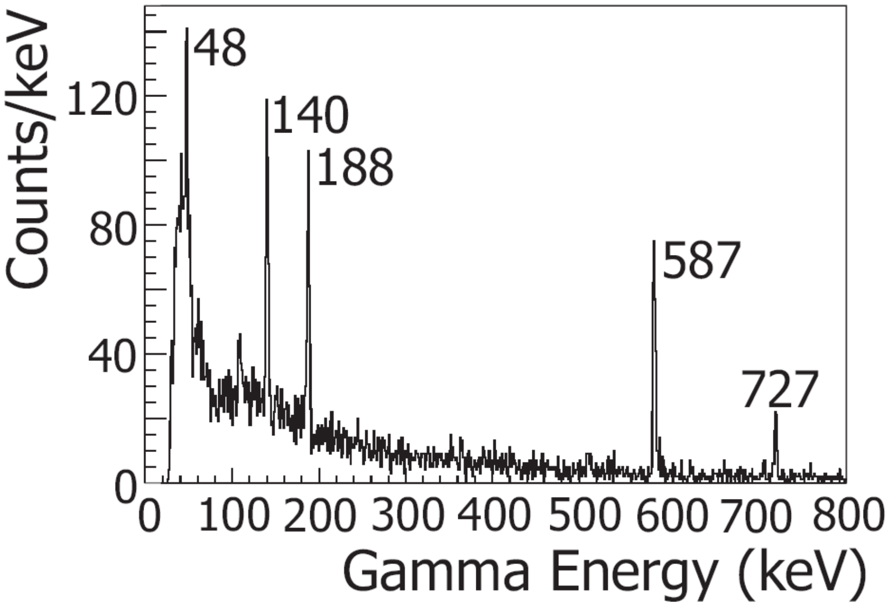
β-decay spectroscopy set-up: NSCL

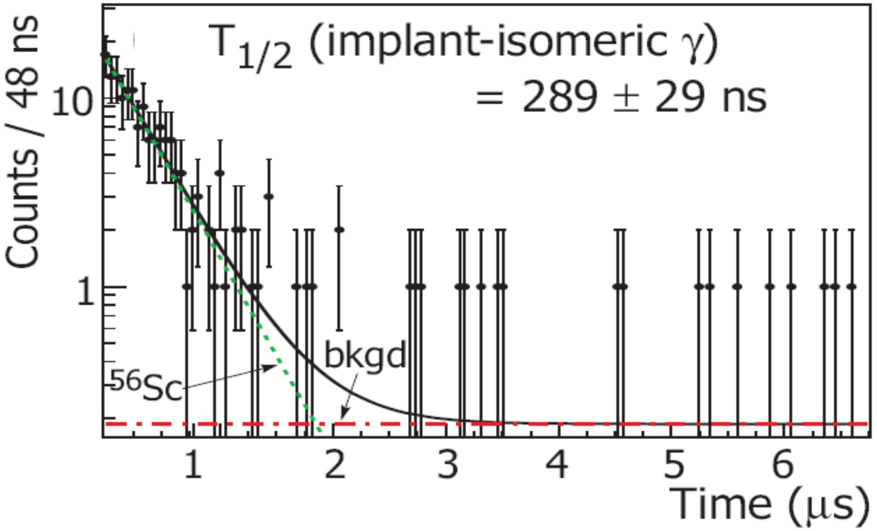




β-decay spectroscopy: complex example

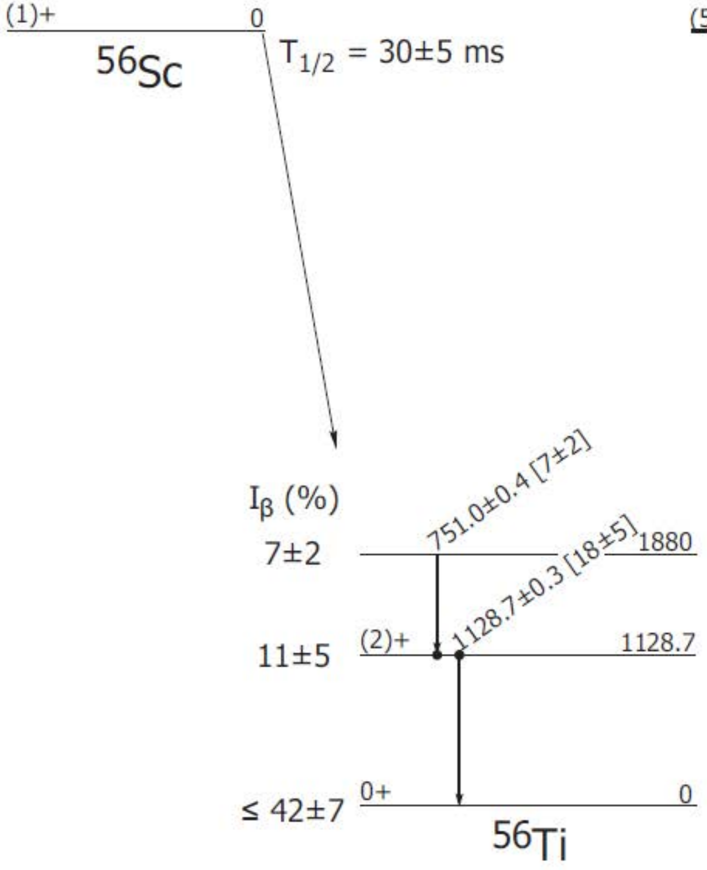
A. Look at the gamma-rays in coincidence with the nucleus of interest (^{56}Sc) implantations – by fitting half-lives of the isomer, and through gamma-gamma correlations, build up a level scheme, and can get relative spin-parities for the states in ^{56}Sc

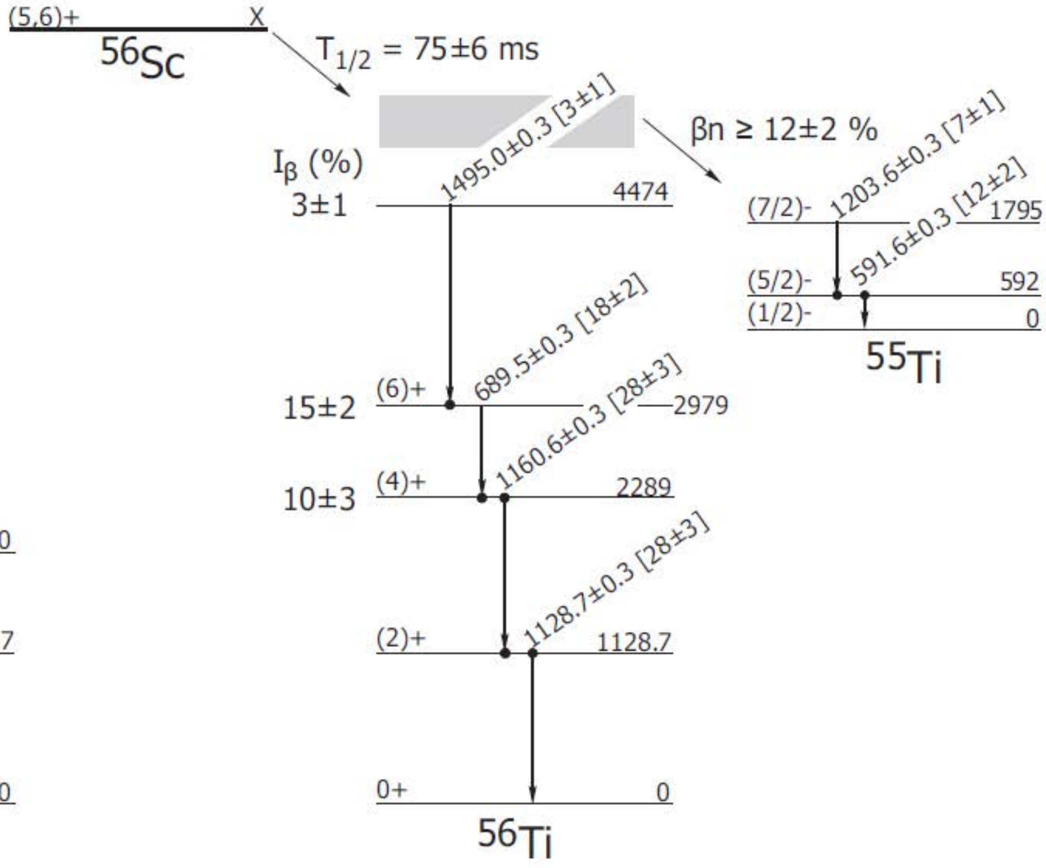




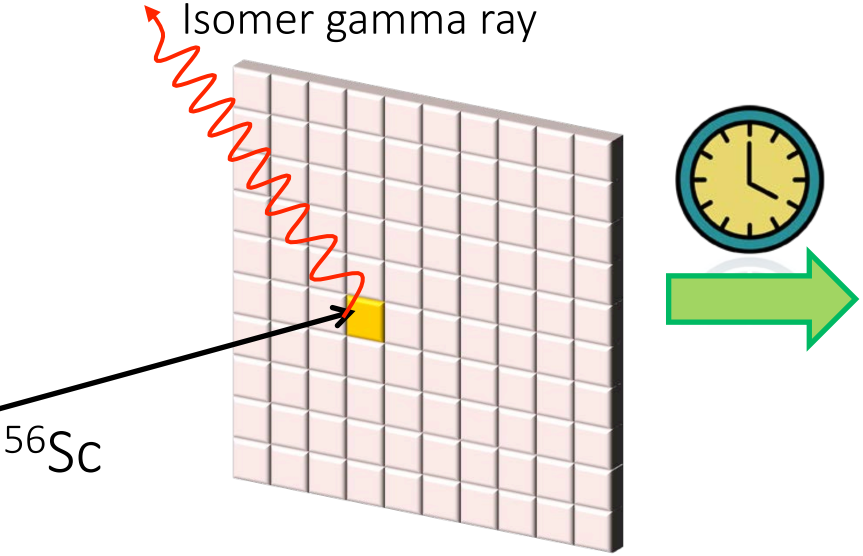
HLC et al., PRC 82, 014311 (2010).

β-decay spectroscopy: complex example

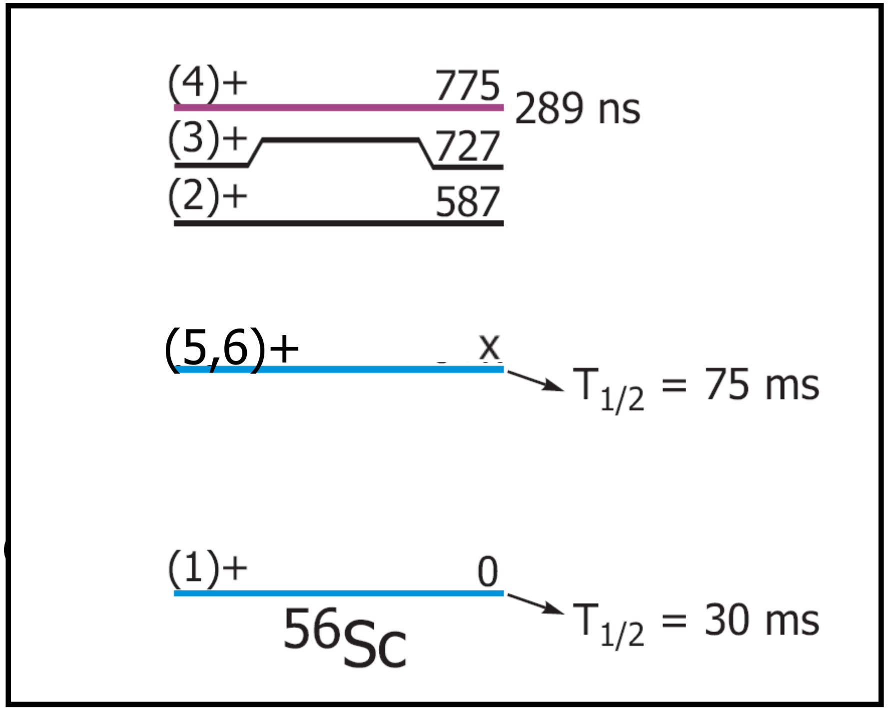




β-decay spectroscopy: complex example



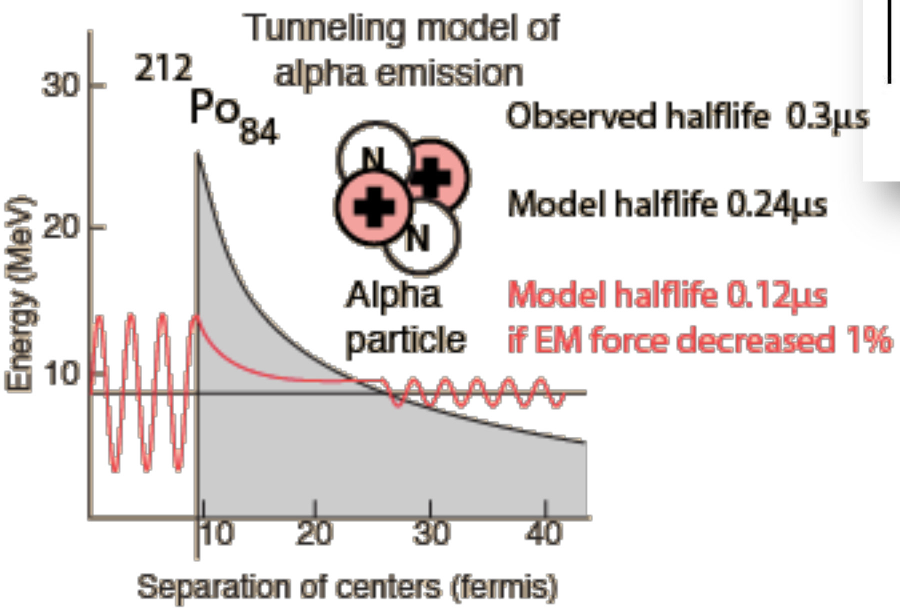
Tagged as ‘isomeric’ implantation

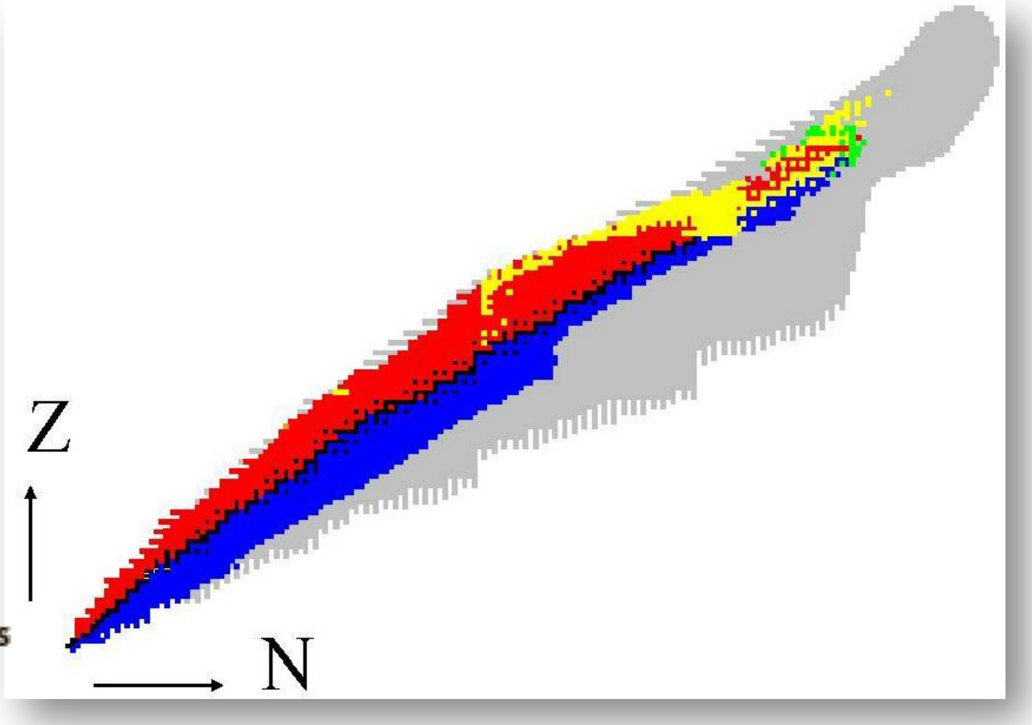


D. Gate on implantations that came in coincidence with isomer gamma-rays and look at half-life → determine which state the isomer populates, and fix the spin/parity

Alpha decay

- α decay occurs only in heavier systems on the nuclear chart
- Alpha decay however probes different aspects of the nuclear forces

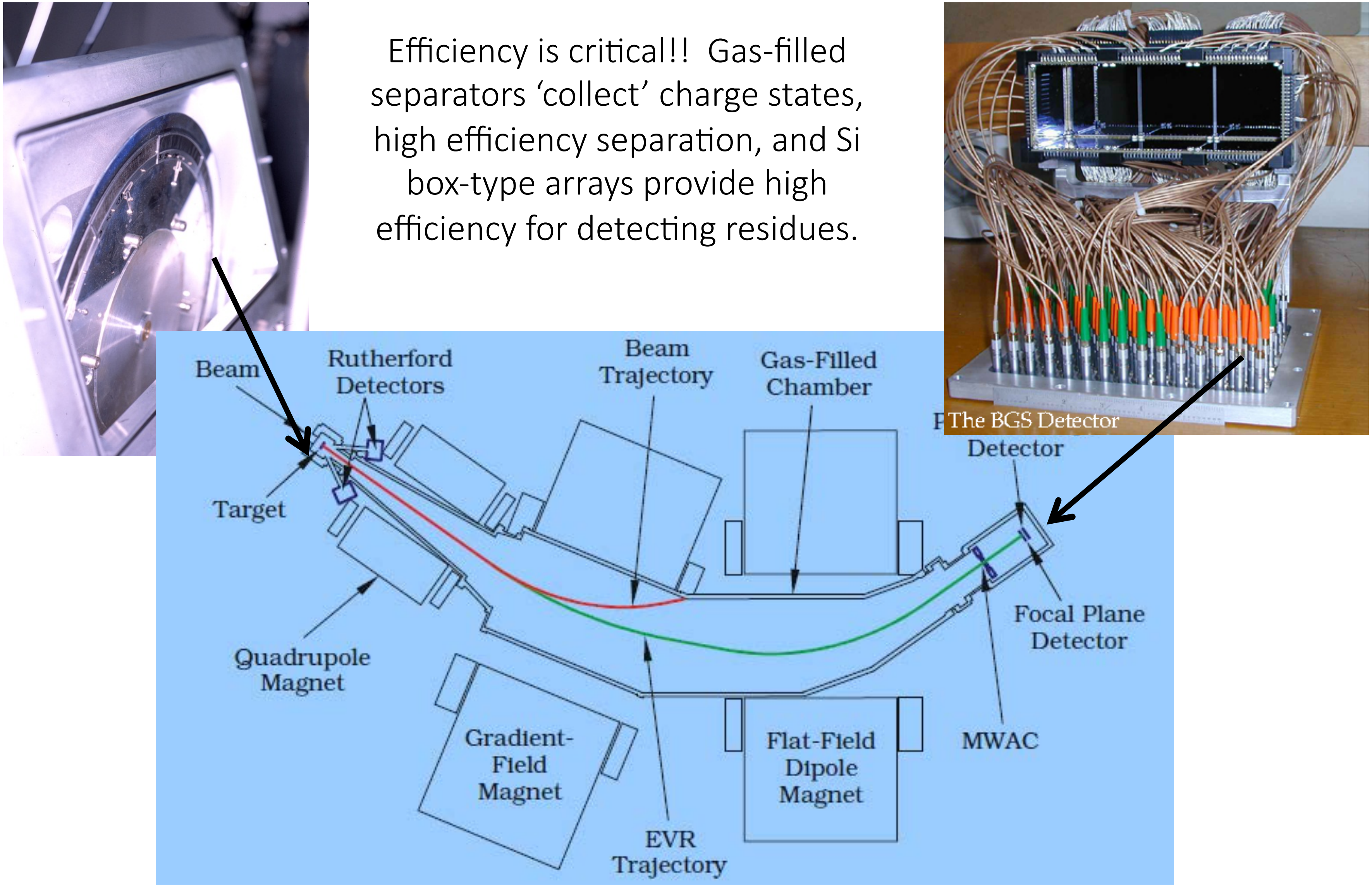




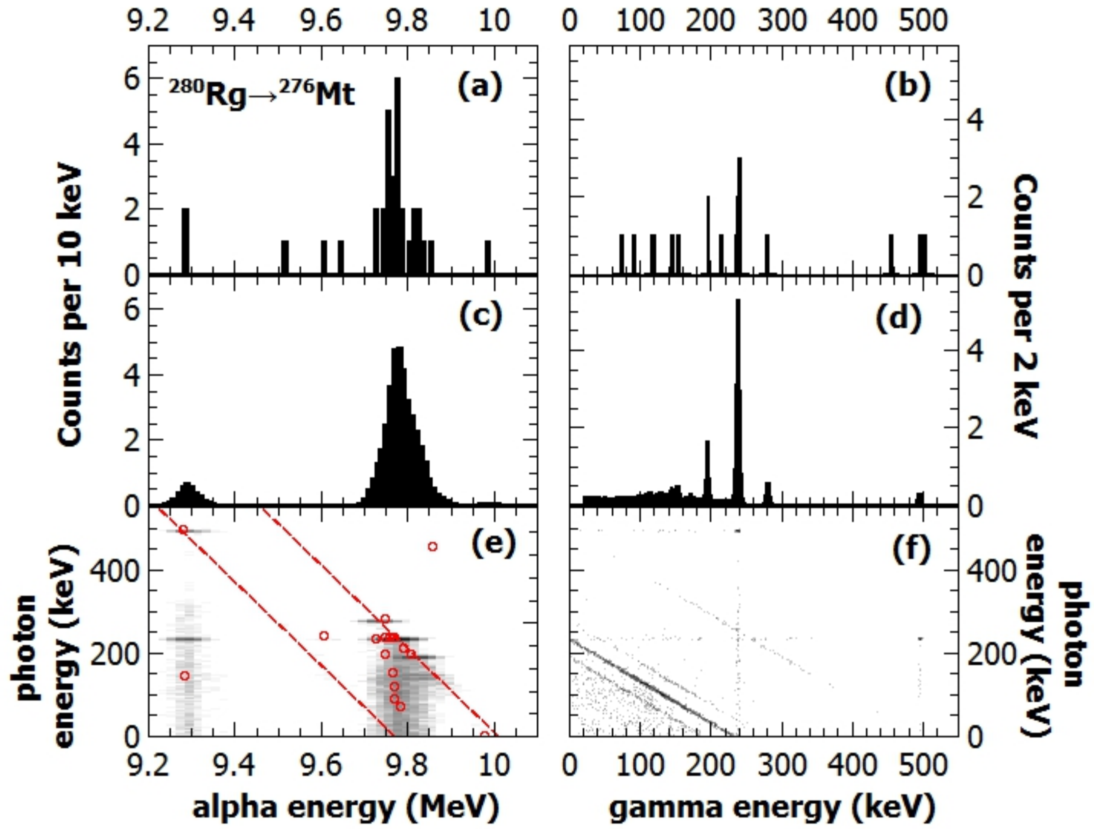
- Different selectivity in the process --> favour low L alpha emission

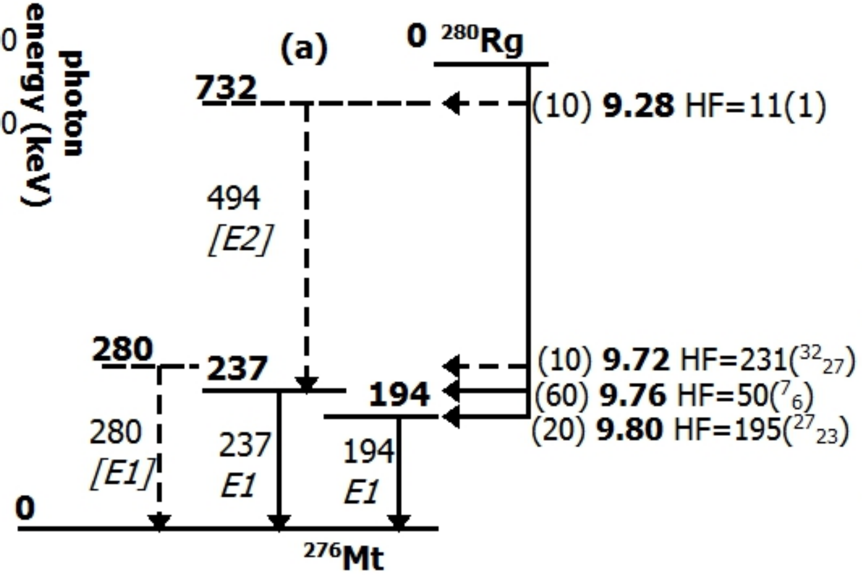
Spectroscopy of heavy elements

Efficiency is critical!! Gas-filled separators 'collect' charge states, high efficiency separation, and Si box-type arrays provide high efficiency for detecting residues.



Spectroscopy from element 115





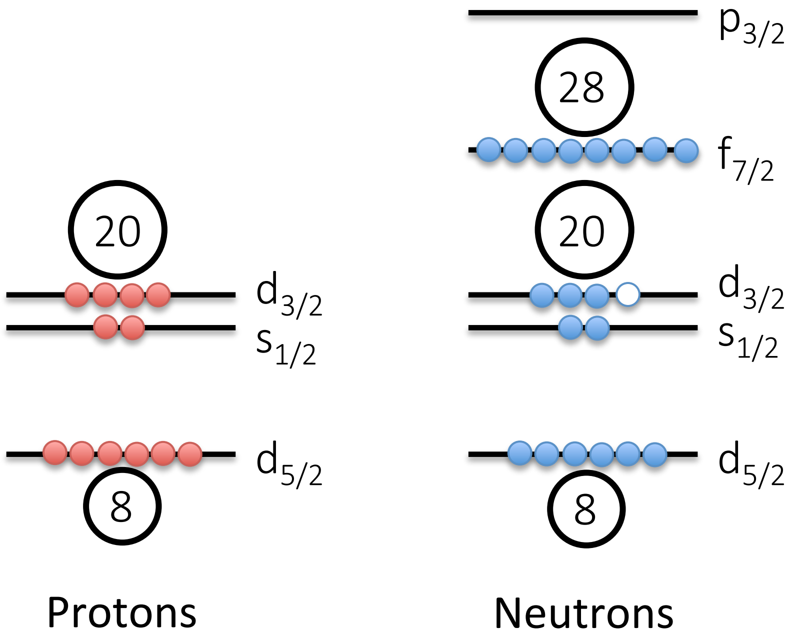
J. M. Gates *et al.*, submitted (2015).

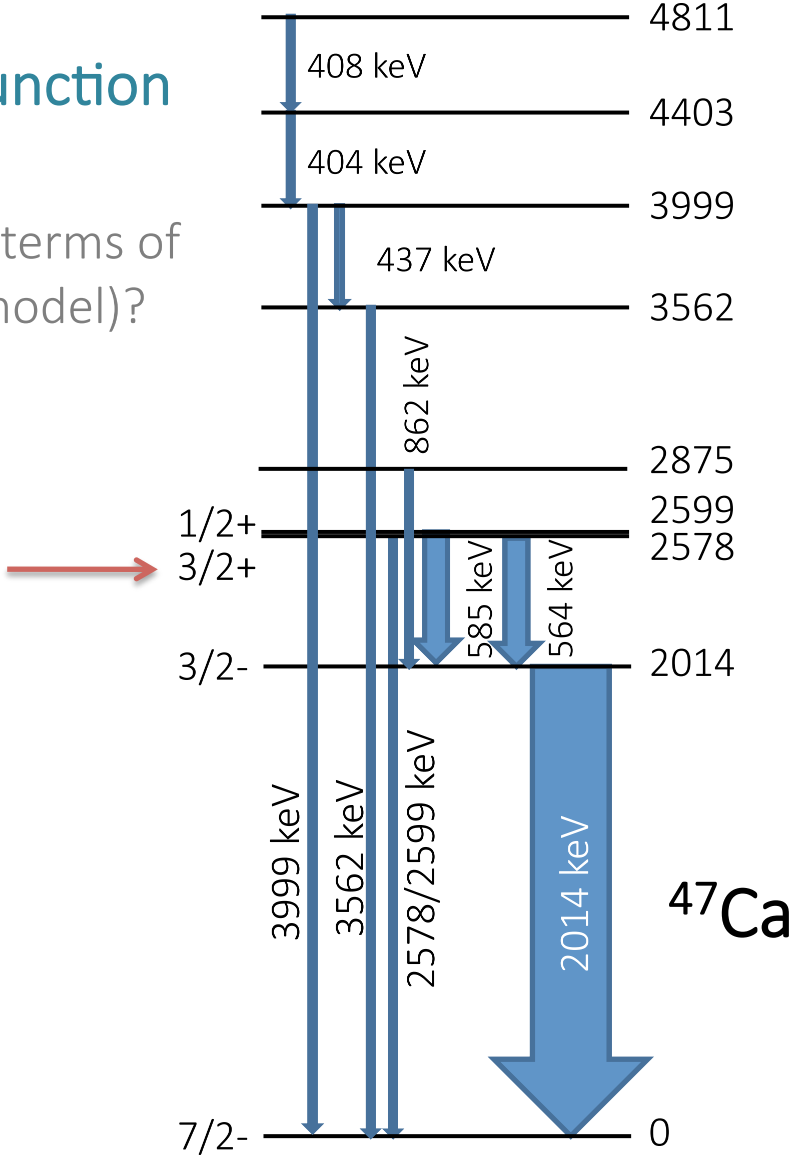
Probing wavefunctions

Beyond excitation energies and spins?

Can we probe the details of the wavefunction 'directly'?

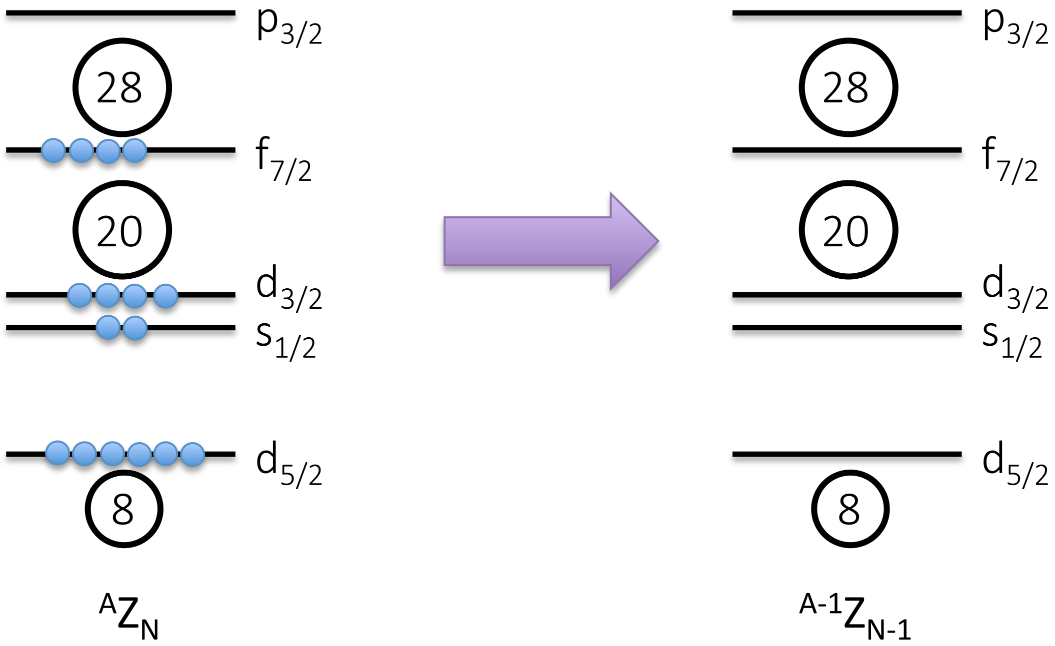
Is there a way to tell where the particles are in terms of single-particle states (even within a specific model)?





Direct nucleon removal (or addition)

- Information regarding the 'occupancy' of single-particle states can be investigated within a model framework
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout



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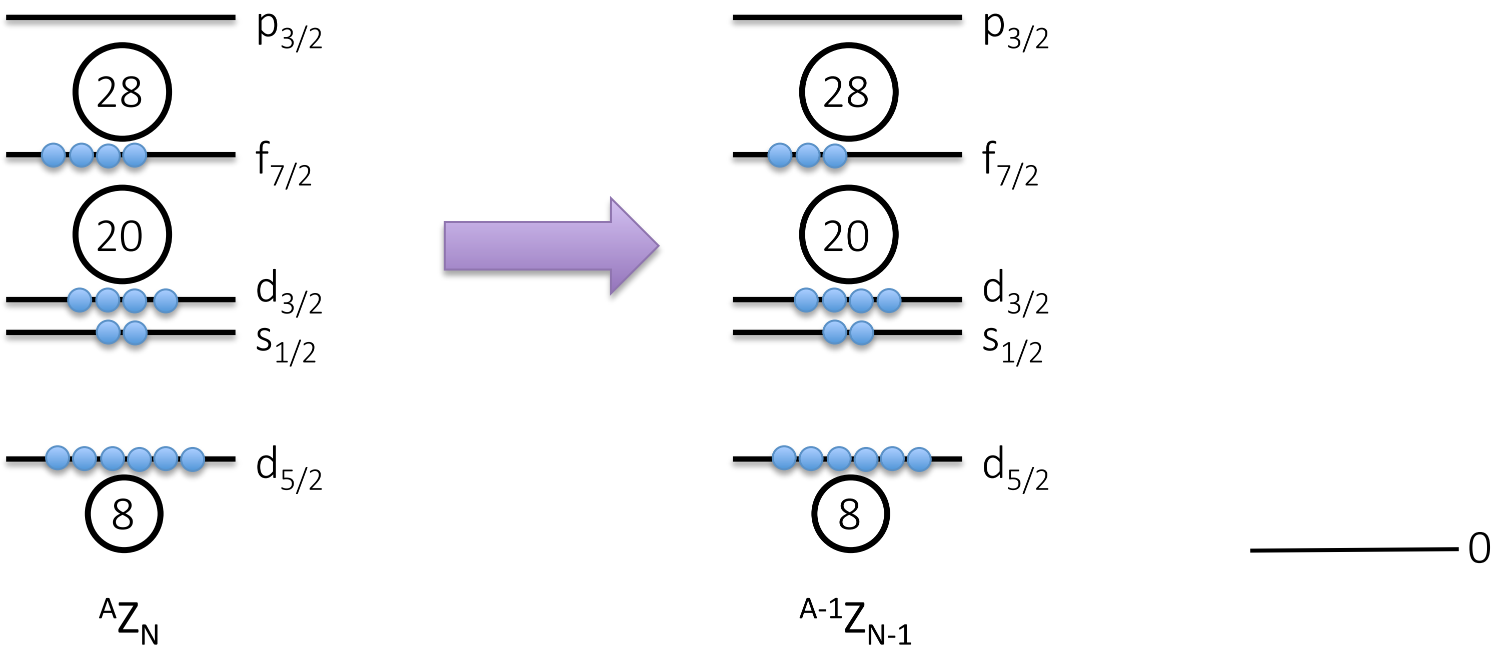

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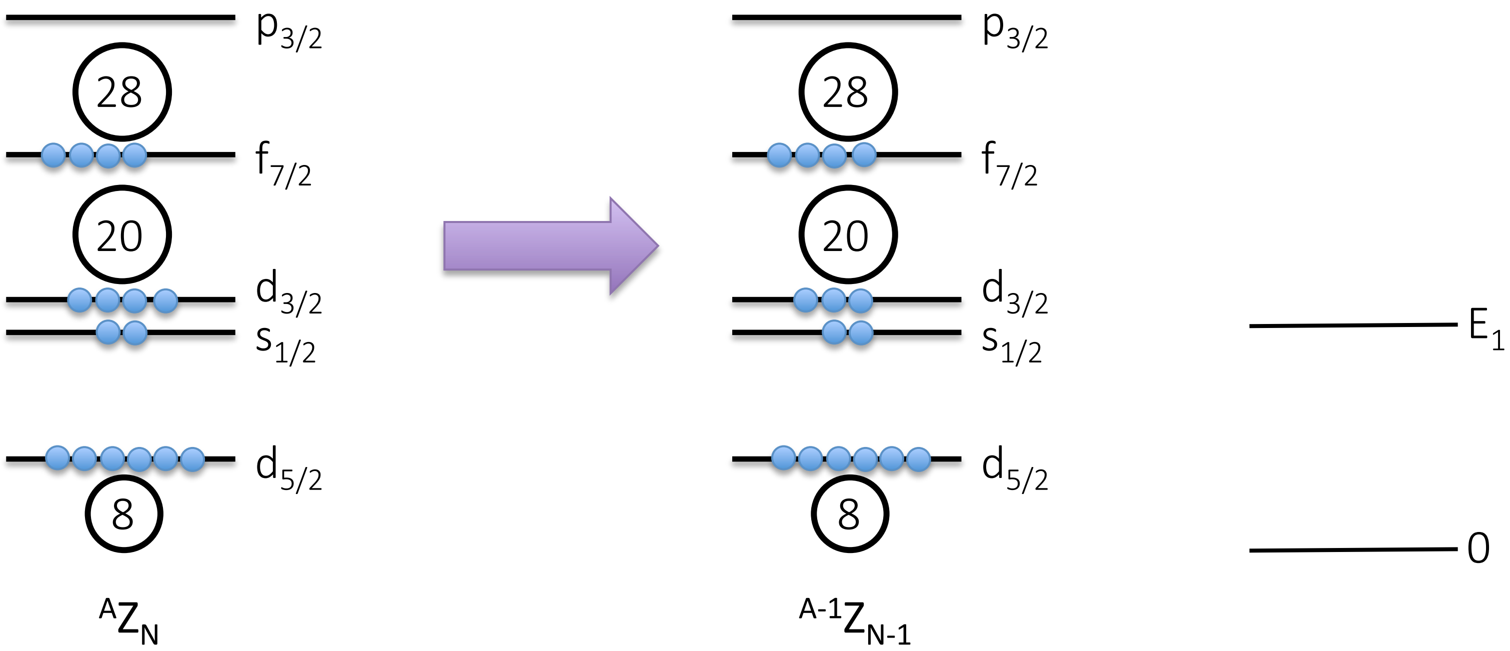

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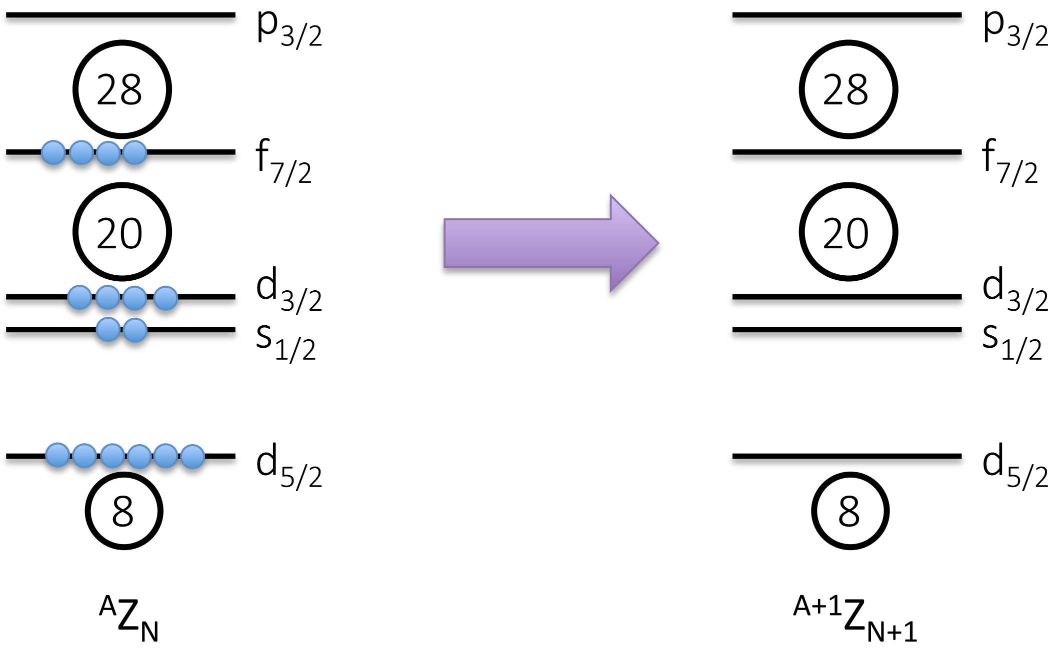
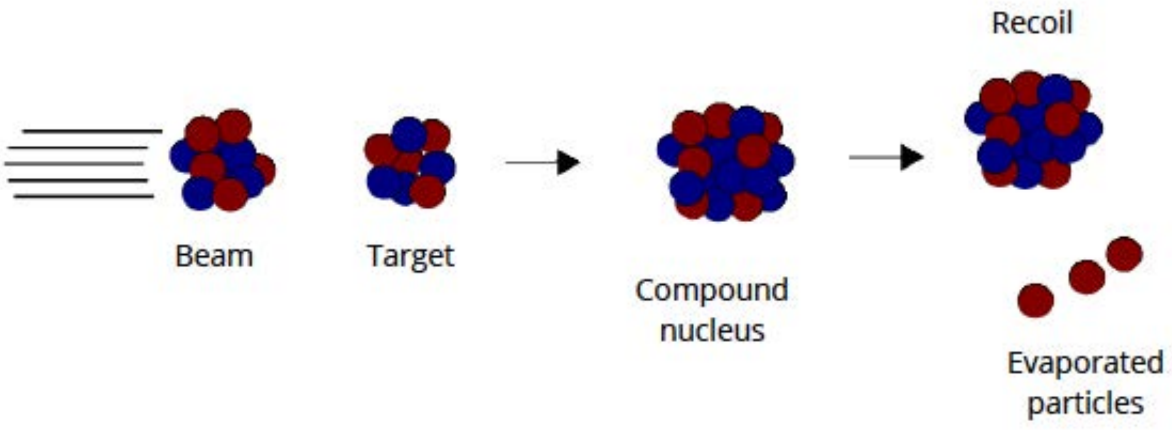


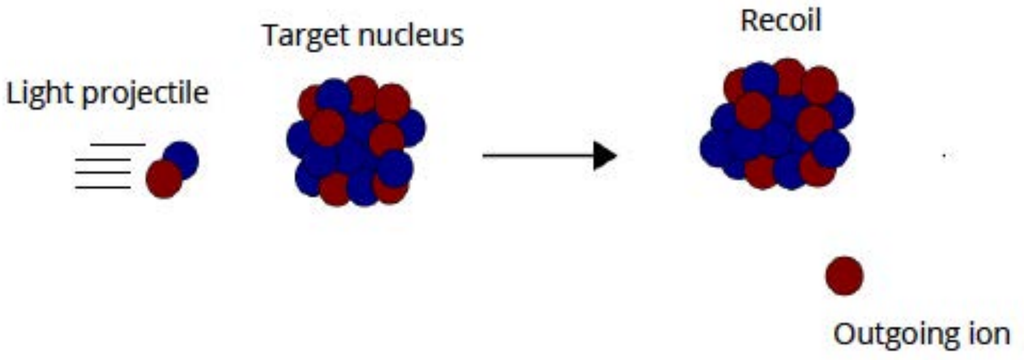
Selectivity of the reaction mechanism

- Knockout / nucleon removal
- Fusion – evaporation
- Transfer
- Deep inelastic
- Scattering (elastic / inelastic)
- Capture

Fusion evaporation vs. direct transfer

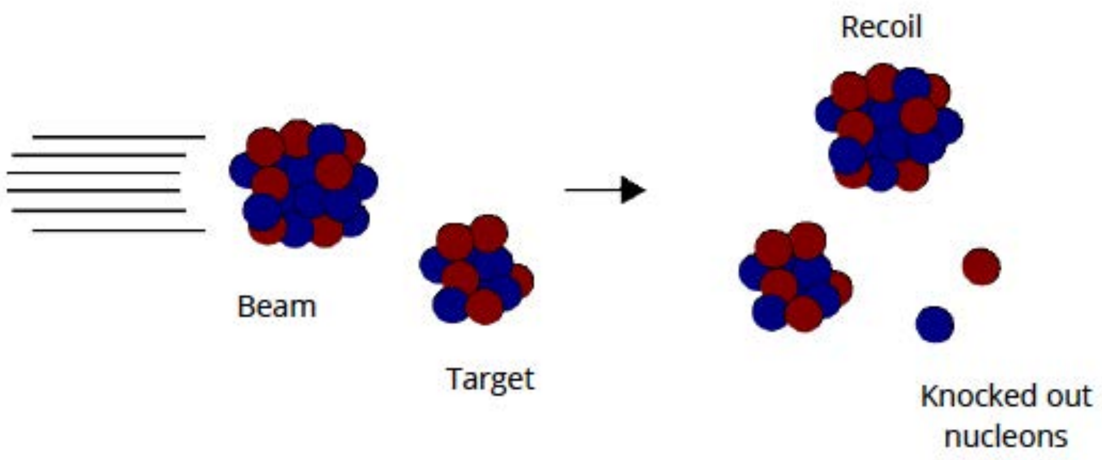


- A + b = C -> D + X
  - $^{12}C(^{18}O,3n)^{27}Si^*$
- Compound system has NO memory of its formation
- Evaporated particle energies give excitation energies of final states

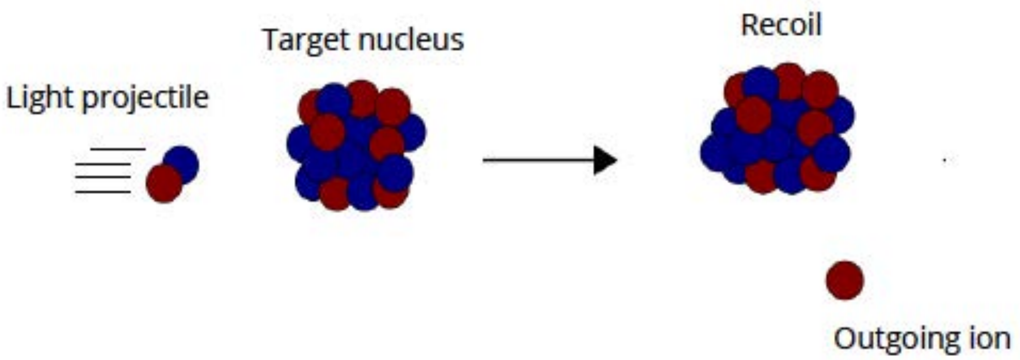


- Two-body A(b,c)D
  - $^{16}O(d,p)^{17}O^*$
- Outgoing particle DO retain knowledge of transferred particles

Knockout reaction vs. direct transfer



- A + b = c – Xn - Xp
  - $^{9}\mathrm{Be}(^{44}\mathrm{S},-1p1n)^{42}\mathrm{P}^*$
- Momentum distribution of recoil reflects orbital momentum transfer



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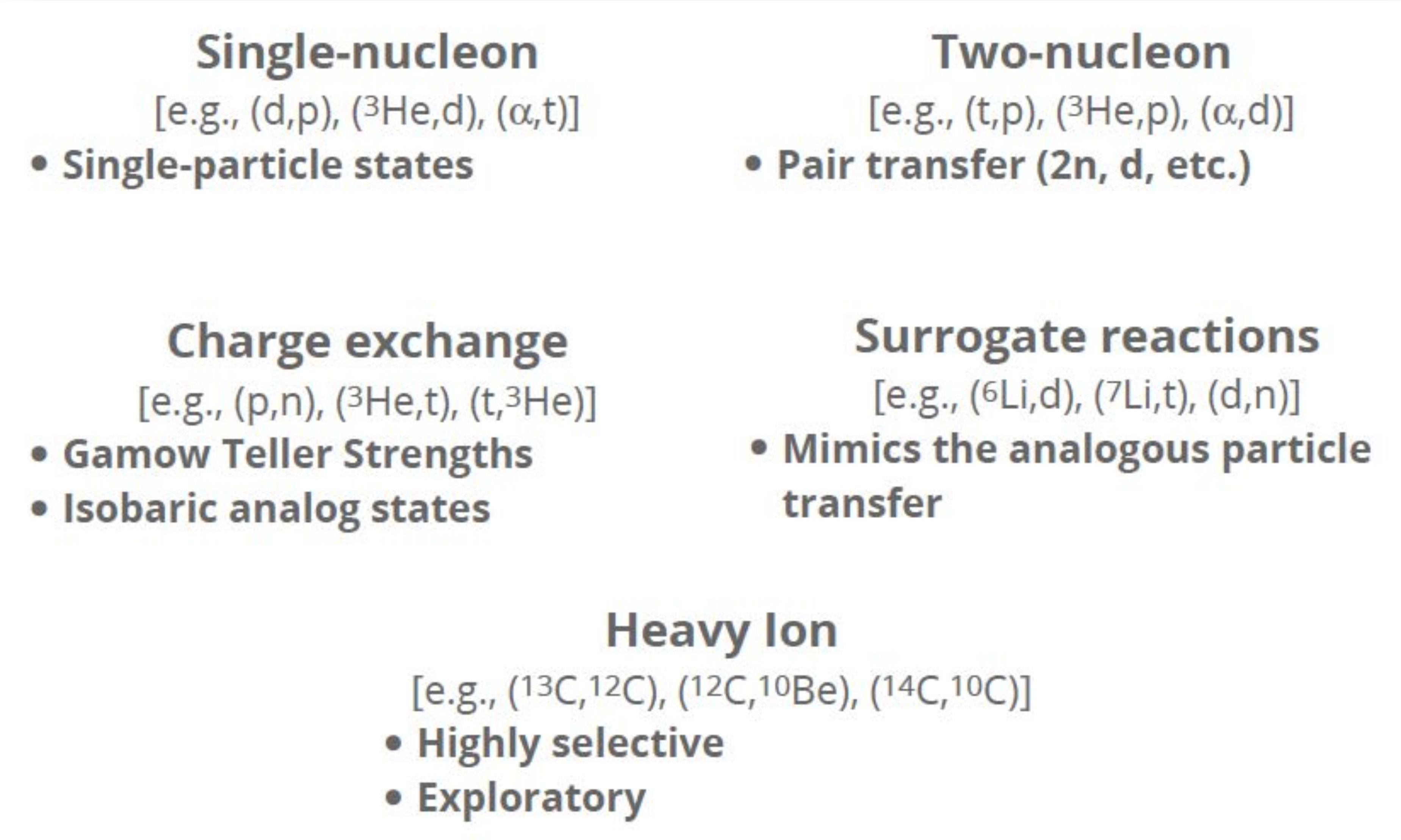
Transfer reactions



Single-nucleon

[e.g., (d,p), (^{3}He,d), (α,t)]

- Single-particle states

Two-nucleon

[e.g., (t,p), (^{3}He,p), (α,d)]

- Pair transfer (2n, d, etc.)

Charge exchange

[e.g., (p,n), (^{3}He,t), (t,^{3}He)]

- Gamow Teller Strengths
- Isobaric analog states

Surrogate reactions

[e.g., (^{6}Li,d), (^{7}Li,t), (d,n)]

- Mimics the analogous particle transfer

Heavy Ion

[e.g., (^{13}C,^{12}C), (^{12}C,^{10}Be), (^{14}C,^{10}C)]

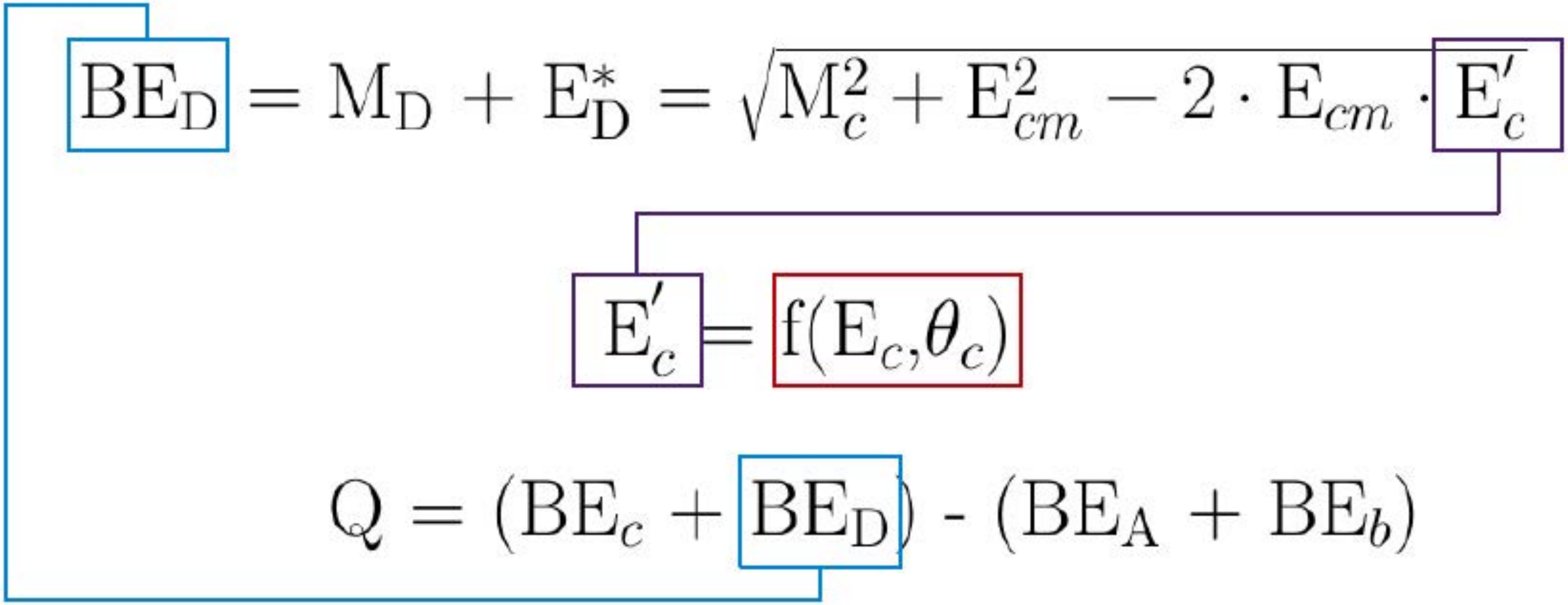
- Highly selective
- Exploratory

Transfer reactions: measured quantities

- Momenta and angles of outgoing light particles [or heavy-ion recoils]

Reaction: A(b,c)D

[e.g., ^{208}Pb(^{3}He,d)^{209}Bi]



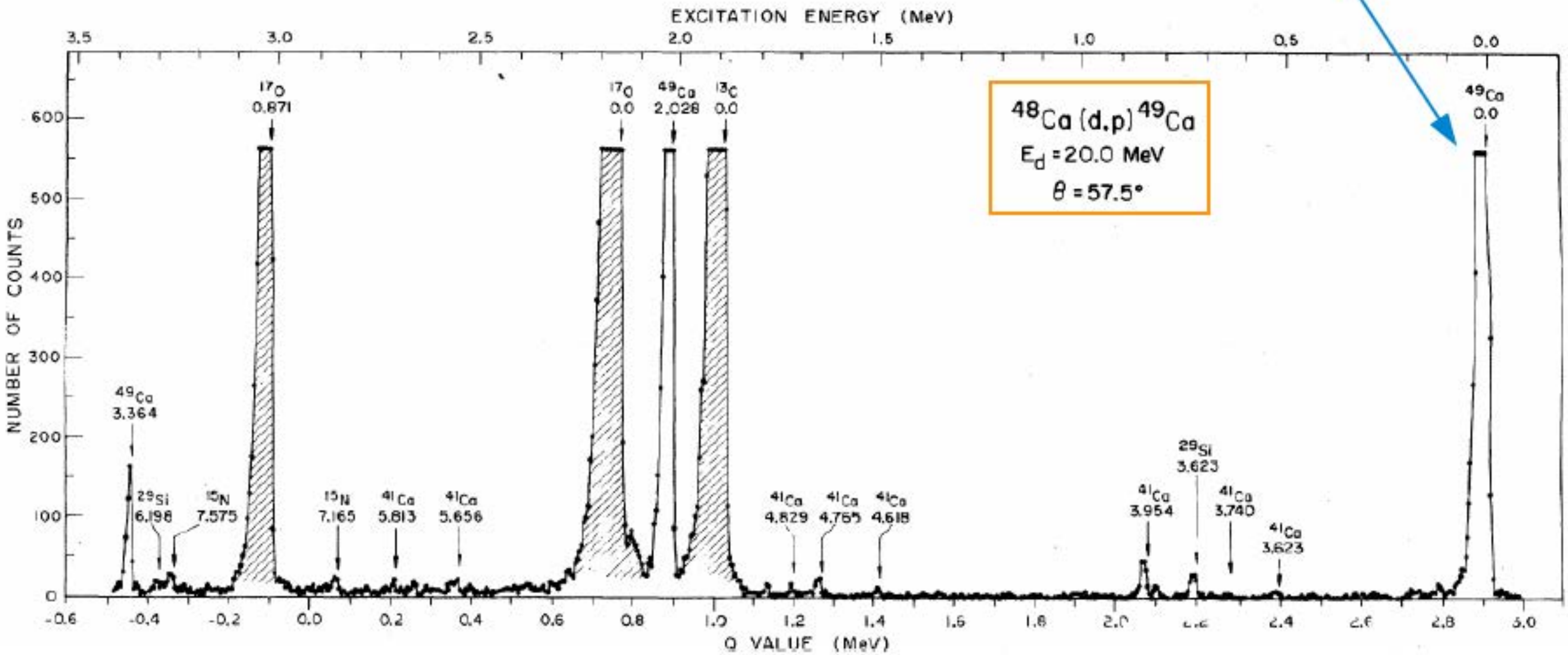
$$\boxed{\mathrm{BE_D}} = \mathrm{M_D} + \mathrm{E^*_D} = \sqrt{\mathrm{M}_c^2 + \mathrm{E}_{cm}^2 - 2\cdot \mathrm{E}_{cm}\cdot \boxed{\mathrm{E}'_c}}$$

$$\boxed{\mathrm{E}'_c} = \boxed{\mathrm{f}(\mathrm{E}_c,\theta_c)}$$

$$\mathrm{Q} = (\mathrm{BE}_c + \boxed{\mathrm{BE_D}}) - (\mathrm{BE_A} + \mathrm{BE}_b)$$

Transfer reactions: measured quantities

[Q(g.s) = +2.92 MeV]

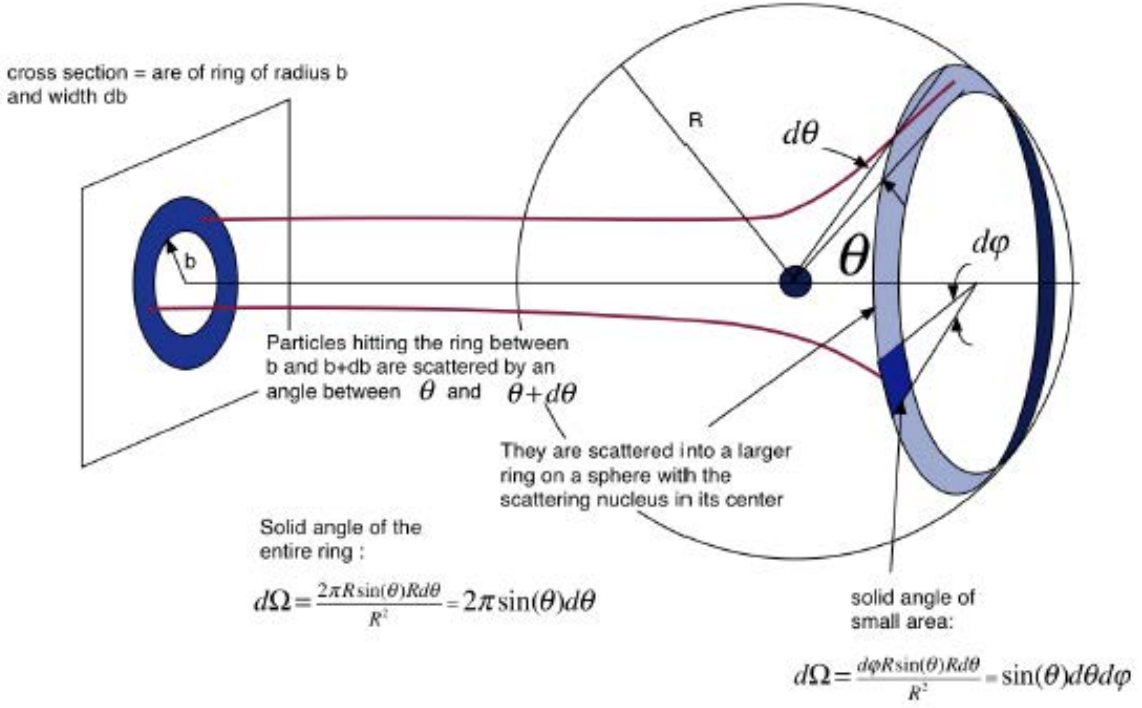


$$Q = (BE_c + \boxed{BE_D}) - (BE_A + BE_b)$$

Transfer reactions: measured quantities

Cross sections – Yields as a function of angle

[differential cross section: millibarns per ster radians (mb/sr)]



cross section = are of ring of radius b and width db

Particles hitting the ring between b and b+db are scattered by an angle between θ and $\theta + d\theta$

They are scattered into a larger ring on a sphere with the scattering nucleus in its center

Solid angle of the entire ring :

$$d\Omega = \frac{2\pi R\sin(\theta)Rd\theta}{R^2} = 2\pi\sin(\theta)d\theta$$

solid angle of small area:

$$d\Omega = \frac{d\varphi R\sin(\theta)Rd\theta}{R^2} = \sin(\theta)d\theta d\varphi$$

Rutherford Scattering

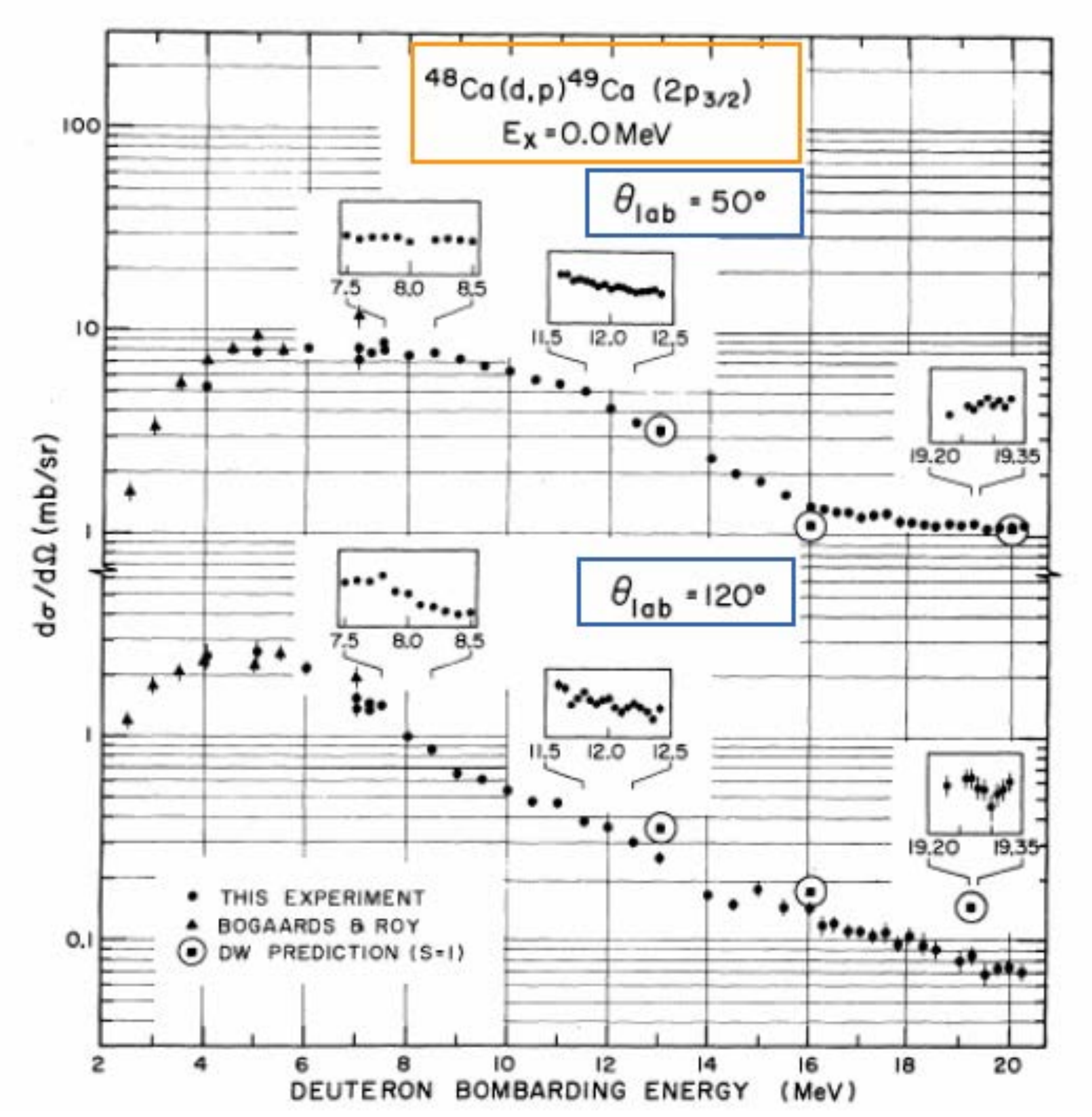
[V = Coulomb]

$$\frac{d\sigma}{d\Omega} = \frac{(zZe^2)^2}{(4\pi\varepsilon_0)^2(4E_{kin})^2}\frac{1}{\sin^4(\theta/2)}$$

Transfer Reaction

[V = Nuclear + Coulomb]

Cross section vs. incident beam energy



Transfer reactions: extracted quantities

Sensitivity of the differential cross sections to orbital angular momenta (*l*) of transferred nucleon(s)

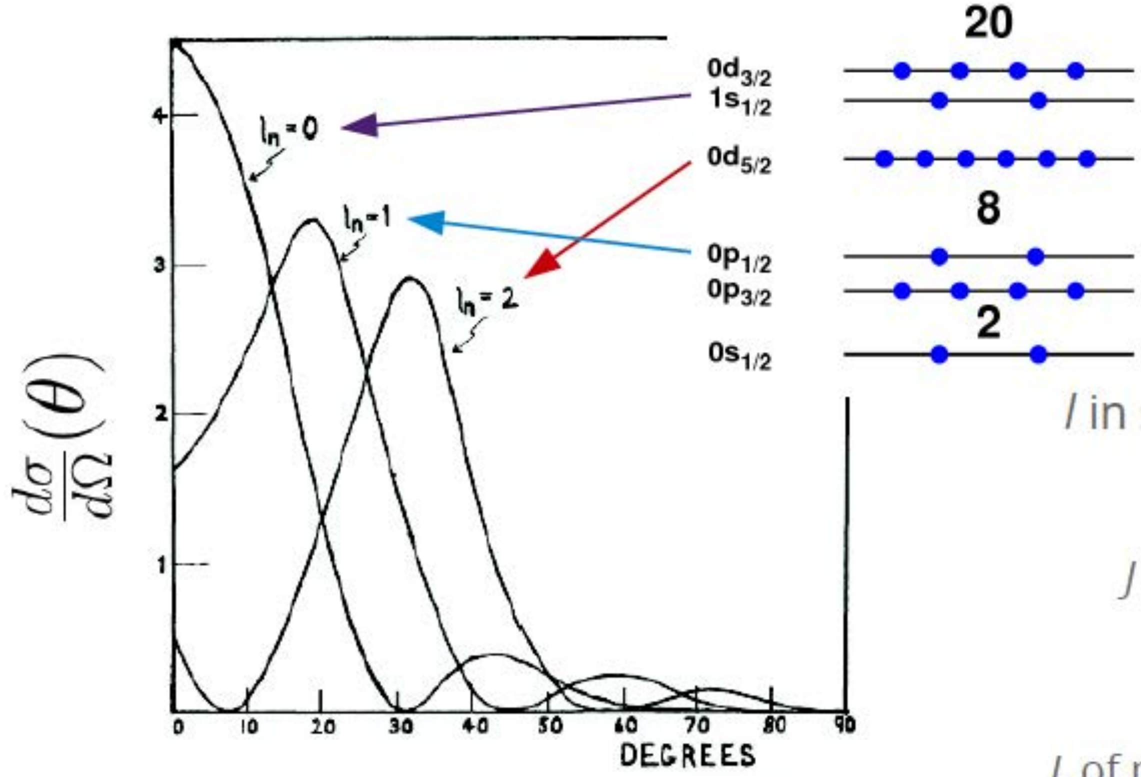


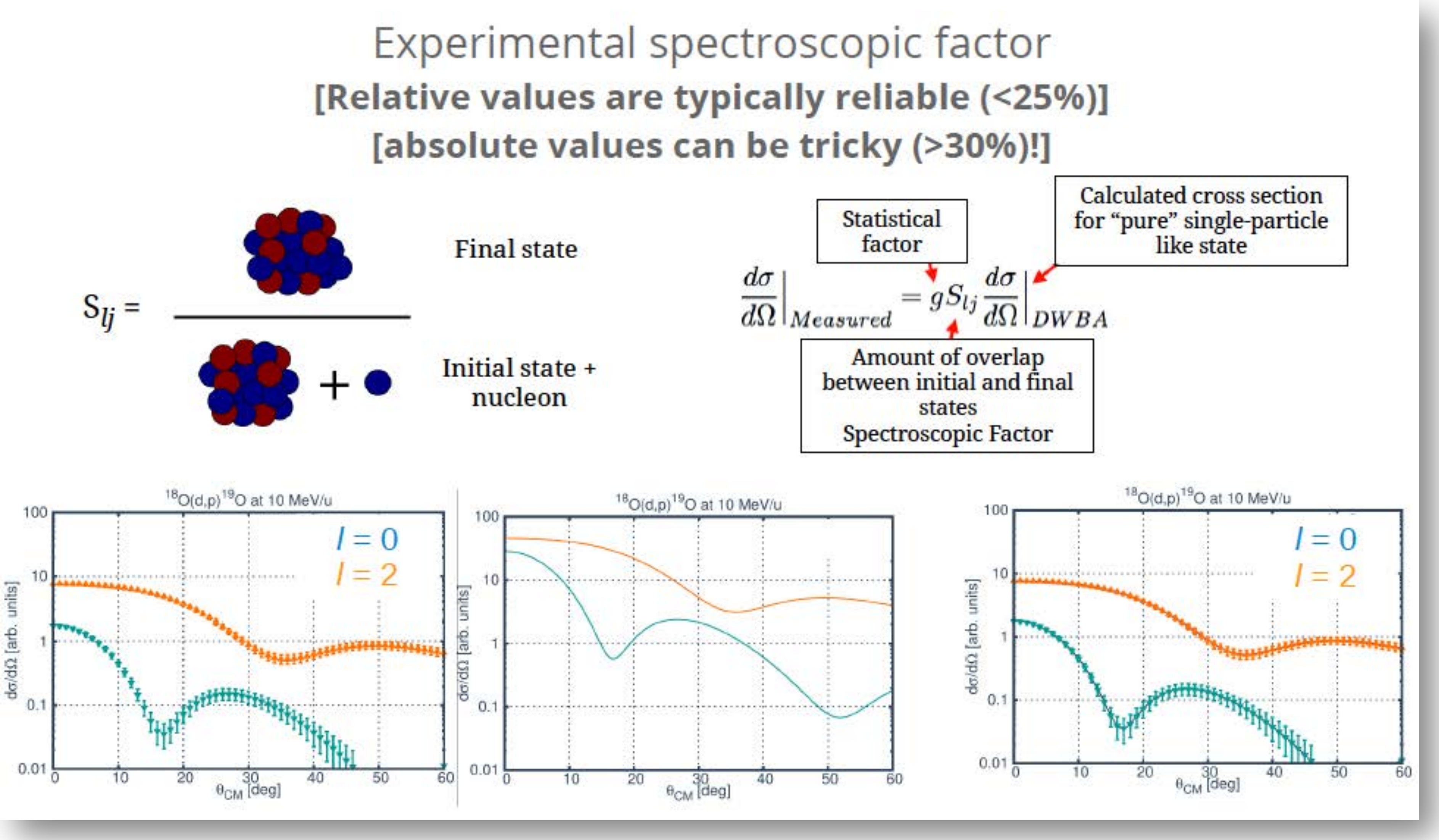
FIG. 1. Theoretical angular distributions for (*d*, *p*) and (*d*, *n*) reactions for different angular momentum transfers to the initial nucleus.

l in single-particle transfer
[e.g., (d,p)]

J if incoming particle is polarized
[analyzing power]

L of pair in two-particle transfer
[e.g., (t,p)]

Transfer reaction: extracted quantities



Experimental spectroscopic factor

[Relative values are typically reliable (<25%)]

[absolute values can be tricky (>30%)!]

S_{lj} =

Final state

Initial state + nucleon

Statistical factor

Calculated cross section for "pure" single-particle like state

$$\left.\frac{d\sigma}{d\Omega}\right|_{Measured} = g S_{lj} \left.\frac{d\sigma}{d\Omega}\right|_{DWBA}$$

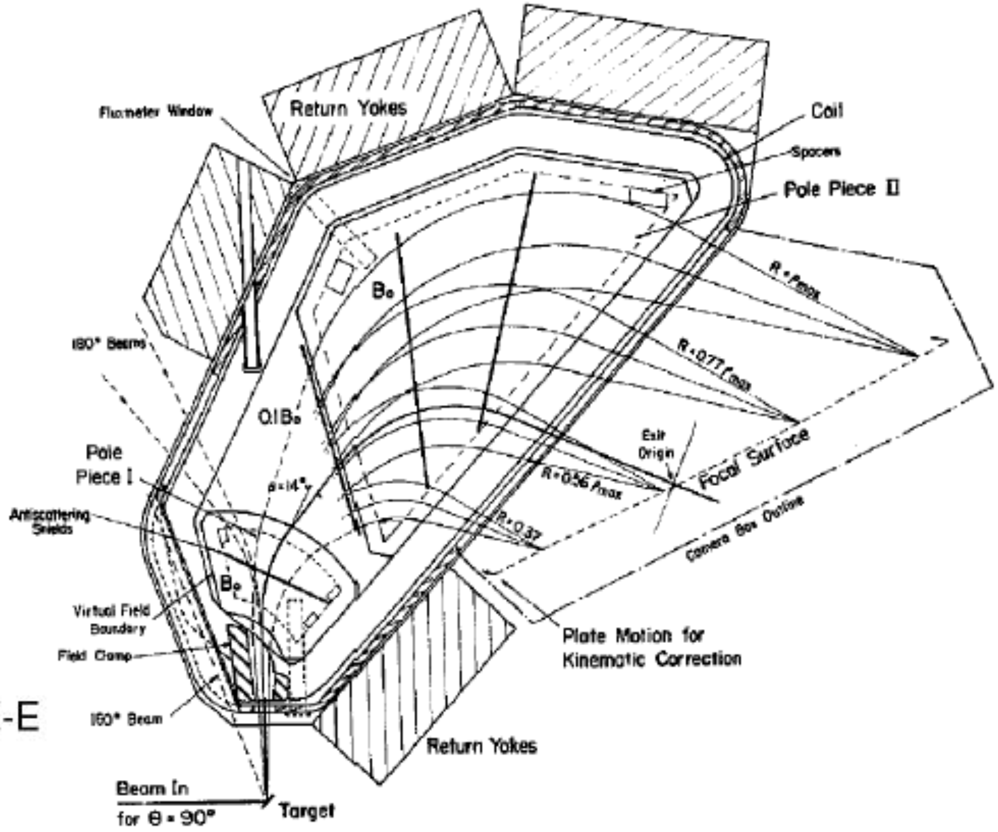
Amount of overlap between initial and final states
Spectroscopic Factor

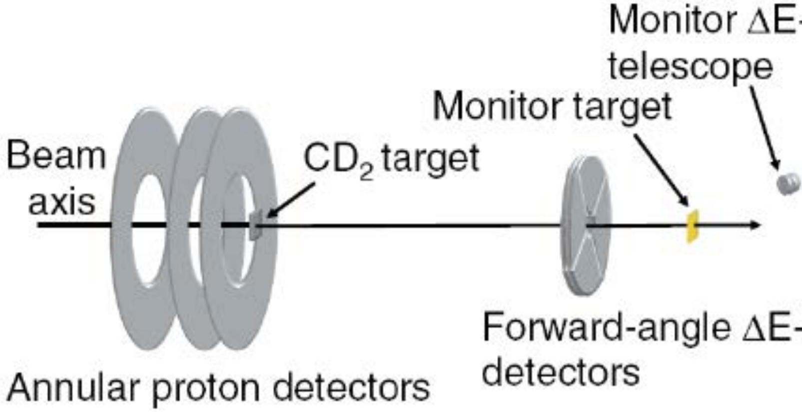
Low-energy transfer experiments

Detection systems depend on kinematics of the reaction

--> ‘normal kinematics’ with a light beam on a heavy target – spectrographs can analyze the light outgoing particle

--> ‘inverse kinematics’ with a heavy beam on a light target – detect the light outgoing particle, or analyze the beam-like particle





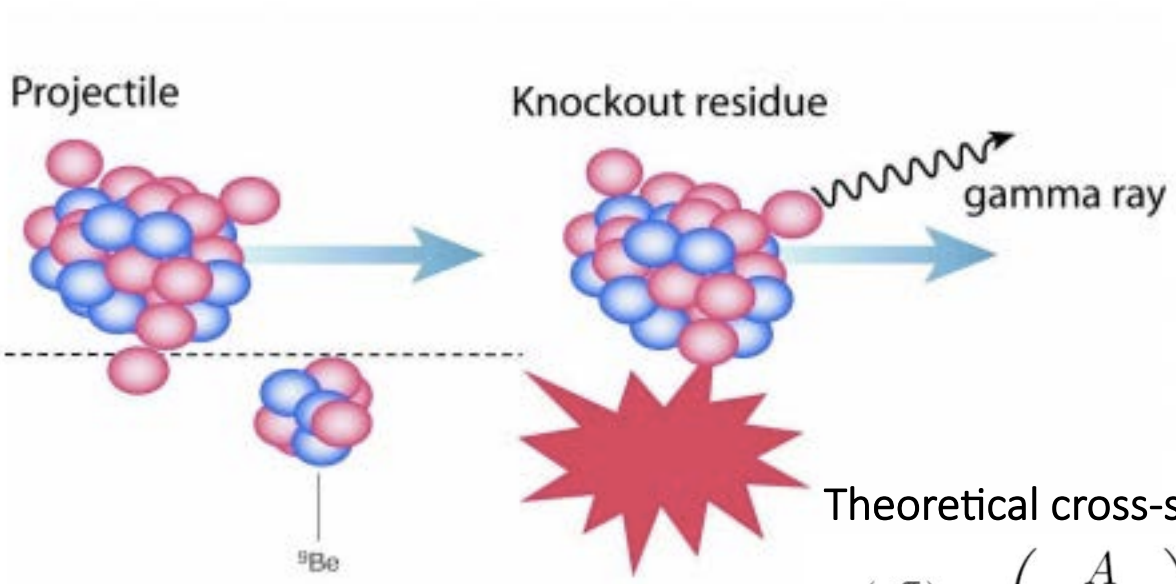
Nucleon knockout reactions

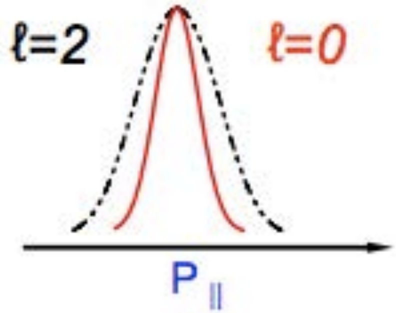
Intermediate energy beams (> 50 MeV/nucleon)

- Sudden approximation + eikonal approach for reaction theory

Spectroscopic strengths --> exclusive cross-sections

- Populated states in A-1 residue provide detailed measure of beam structure





residue moment distribution
→ ℓ-value of knocked-out n

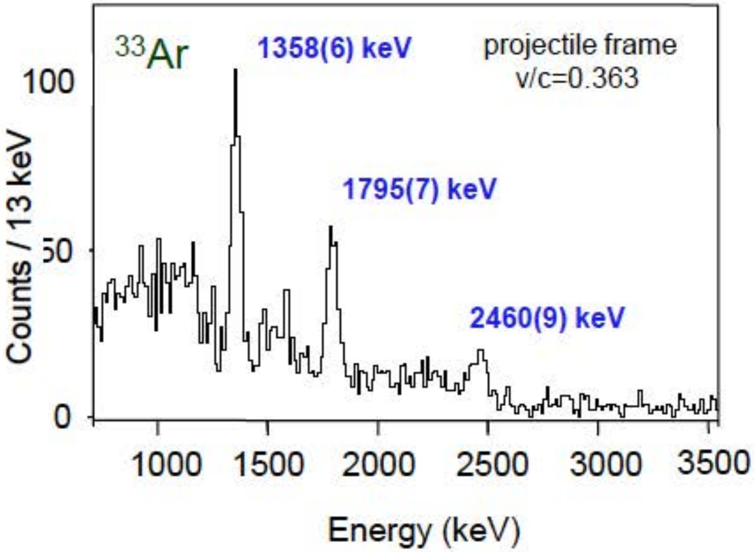
Theoretical cross-section

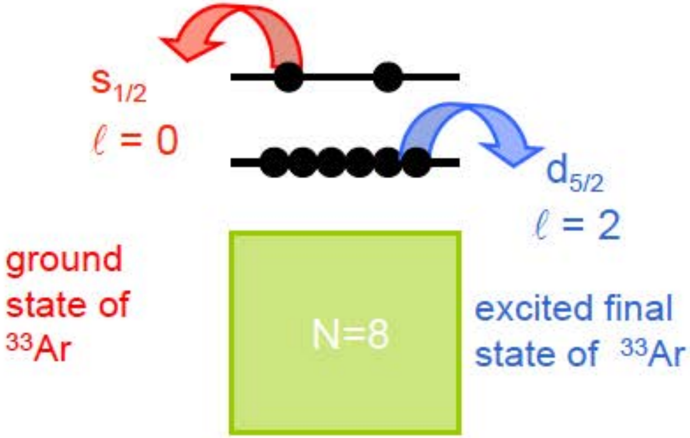
Reaction theory

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2S(j^\pi)\sigma_{sp}(j, S_N + E_x[j^\pi])$$

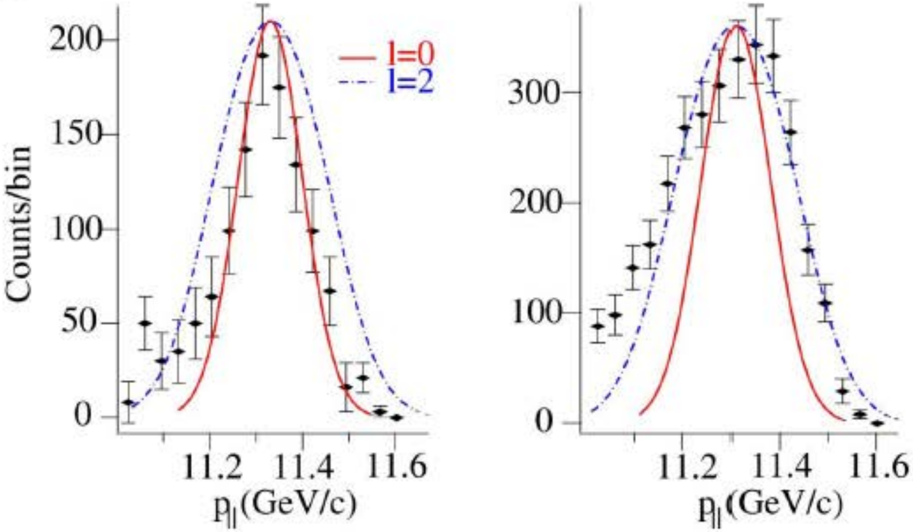
Structure theory

Neutron knockout – ^{9}Be(^{34}Ar, ^{33}Ar)X





	BR (%)	σ_{exp} (mb)	C^2S_{exp}
1/2+	30.2(46)	4.7(9)	0.38(6)
3/2+	20.2(44)	3.2(8)	0.36(9)
5/2+	31.7(31)	4.9(7)	0.56(8)
(5/2+)	17.9(30)	2.8(6)	>0.34(7)



A. Gade et al., PRC 69, 034311 (2004).

Excited state lifetimes

Lifetimes and transition probabilities

Transition probability for gamma-decay relates strongly to specific nuclear matrix elements --> provide a stringent test of theoretical wavefunctions

Consider the case of the first 2+ states in even-even nuclei

$$\tau_\gamma = 40.81 \times 10^{13} E^{-5} [B(E2)\uparrow/\mathrm{e}^2\mathrm{b}^2]^{-1}$$

$$B(E2 : J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \langle \psi_f || E2 || \psi_i \rangle^2$$

Lifetimes are of order ps --> how do we measure these lifetimes?

Recoil-distance (plunger) method

The lifetime of excited states in the range of 10-100s of ps can be measured by populating the state via Coulomb excitation or knock-out reactions, and observing the Doppler-shift of the decay gamma-ray.

c=300 µm/ps β ~ 0.3c 10 ps ~ 1mm

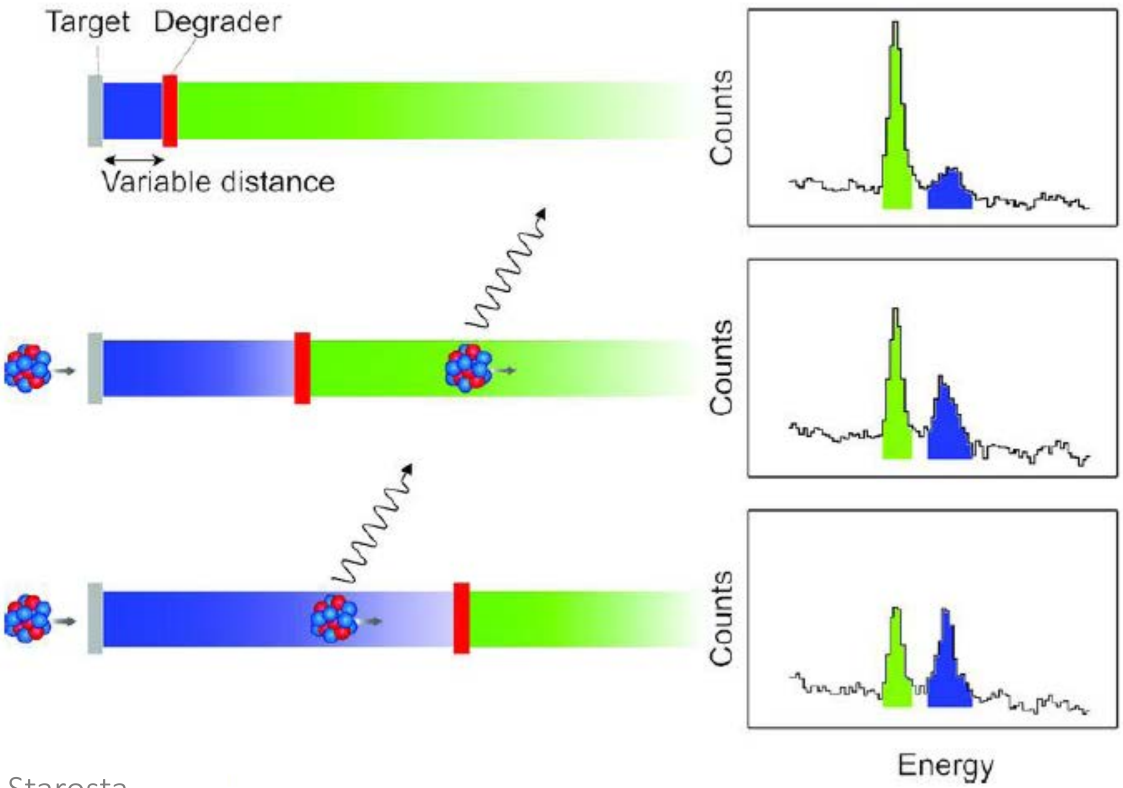
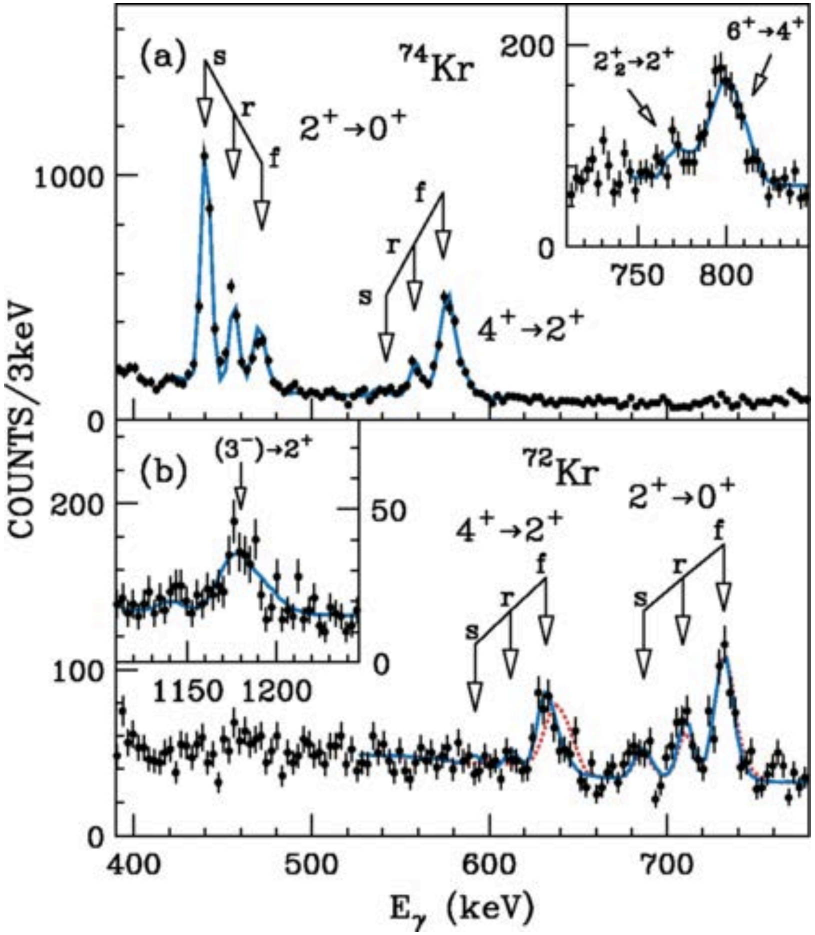
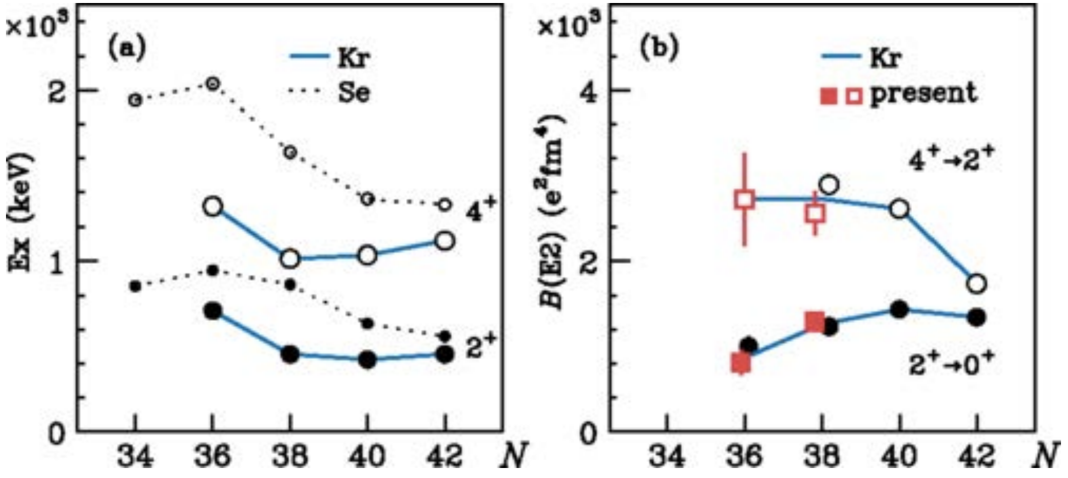


Figure: Adapted from K. Starosta

Lifetime in ^{72,74}Kr





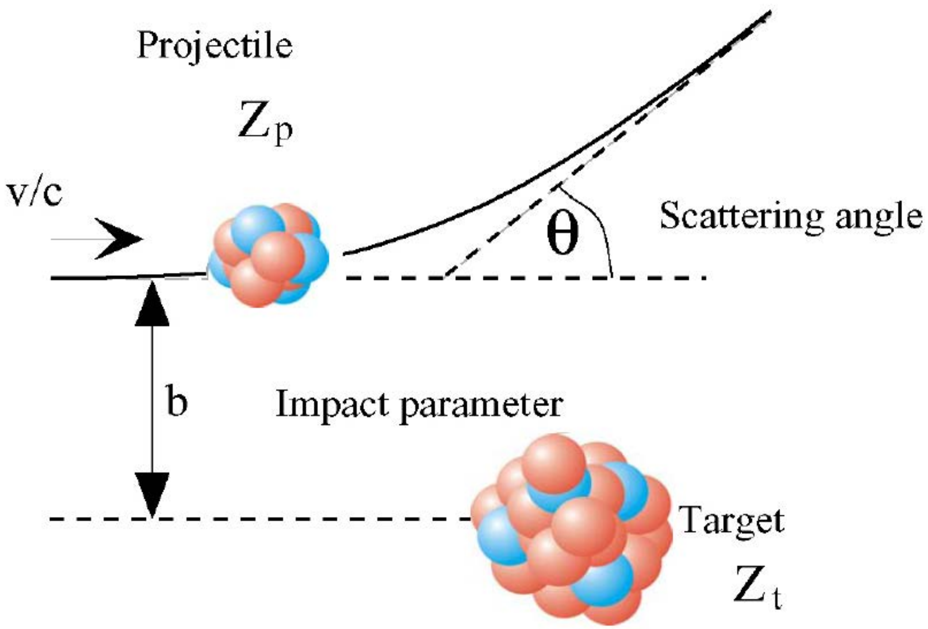
Lifetimes are related to the reduced transition probabilities B(E2), which are an indicator for collectivity in the nuclear structure.

Here, the irregular behaviour for the 4+ and 2+ states suggest a rapid shape evolution in ^{72}Kr

H. Iwasaki *et al.*, Phys. Rev. Lett. **112**, 142502 (2014).

Coulomb excitation

Collectivity: B(E2) from excitation probability

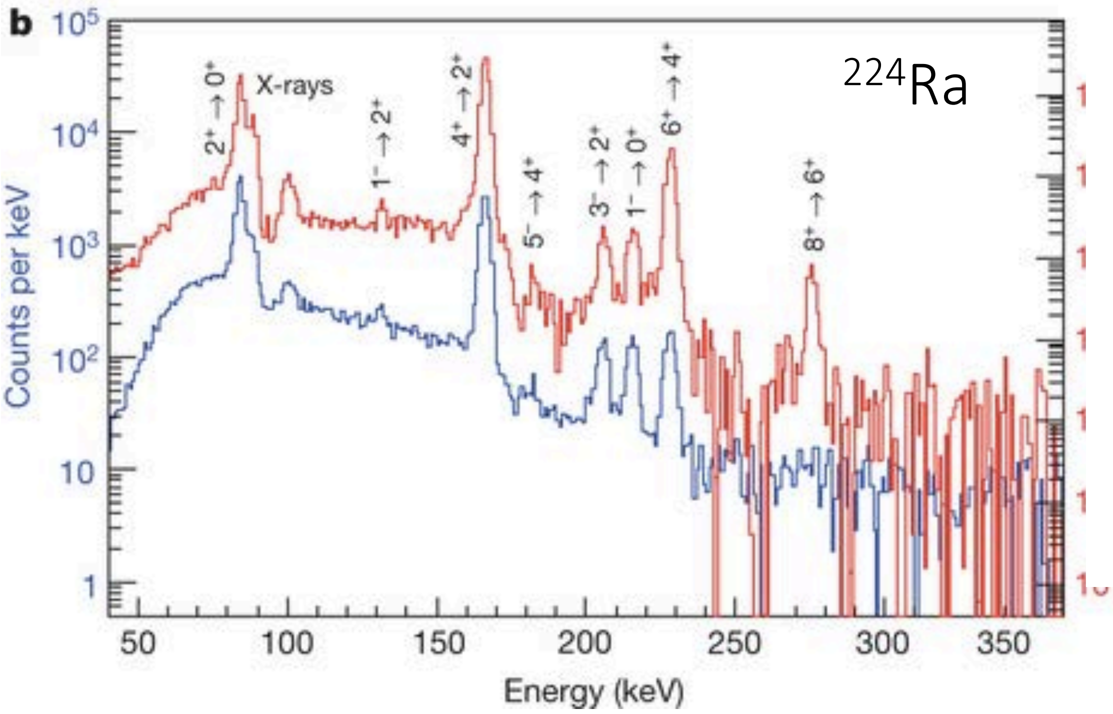


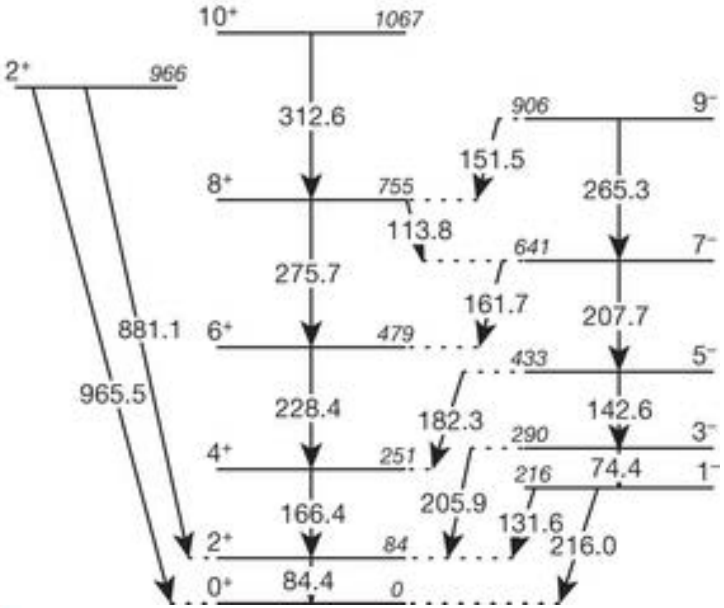
Coulomb excitation:

- purely Coulomb interaction causes excitation of the nucleus of interest
- well described interaction, and cross-section relates to transition matrix element, i.e. B(E2) for 0+ --> 2+ in even-even nuclei.

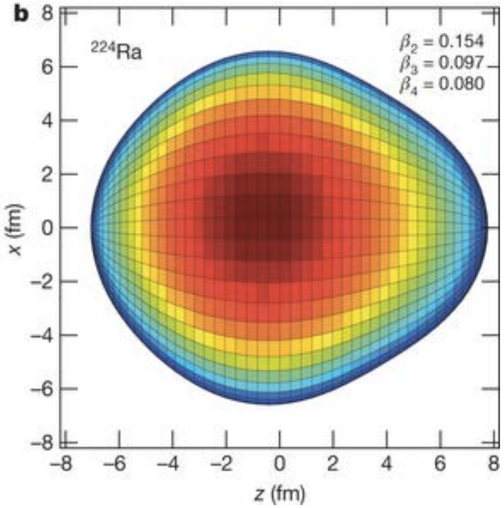
$$\sigma_{\pi\lambda} \approx \left(\frac{Z_{\mathrm{pro}}e^2}{\hbar c}\right)^2 \frac{\pi}{e^2 b_{\mathrm{min}}^{2\lambda-2}} B(\pi\lambda, 0 \to \lambda) \begin{cases} 1/(\lambda-1) & \text{for } \lambda \geq 2 \\ 2\ln(b_a/b_{\mathrm{min}}) & \text{for } \lambda = 1 \end{cases}$$

Pear shaped nuclei and atomic EDM





$$\langle I'||\mathrm{E}\lambda||I\rangle = \sqrt{(2I'+1)(2\lambda+1)/16\pi}(I'0\lambda0|I0)Q_\lambda$$

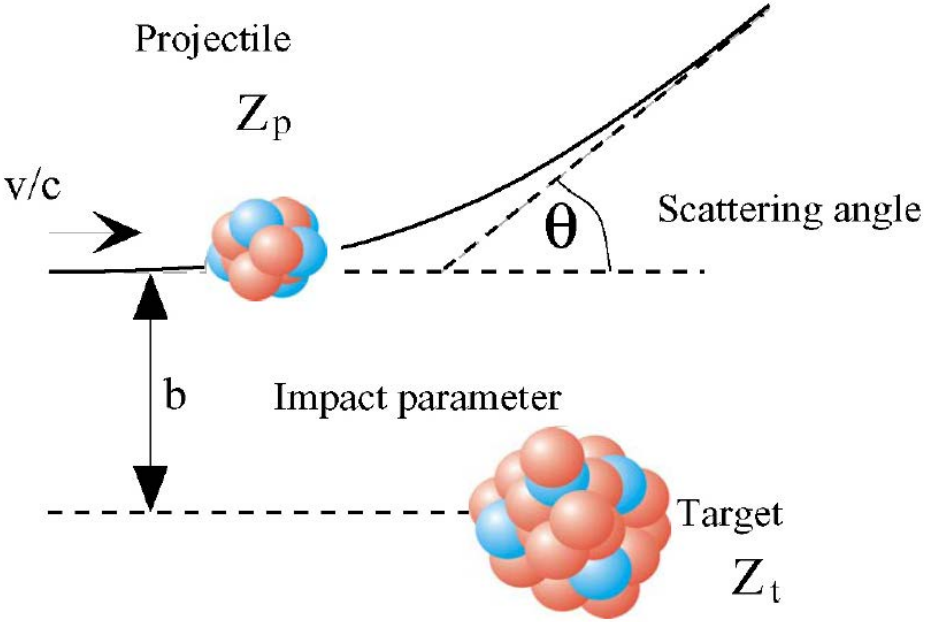


L. P. Gaffney *et al.*, Nature **497**, 199 (2013).

Intermediate-energy Coulex

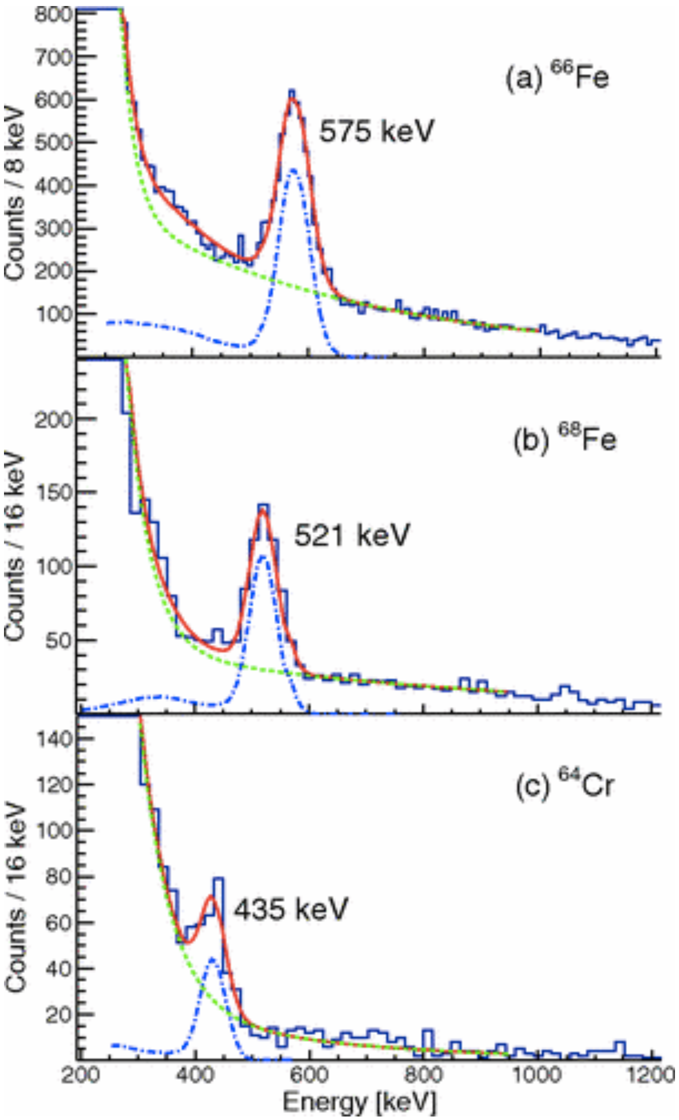
- In conventional (low-energy) Coulomb excitation, bombarding energies are well below the Coulomb barrier

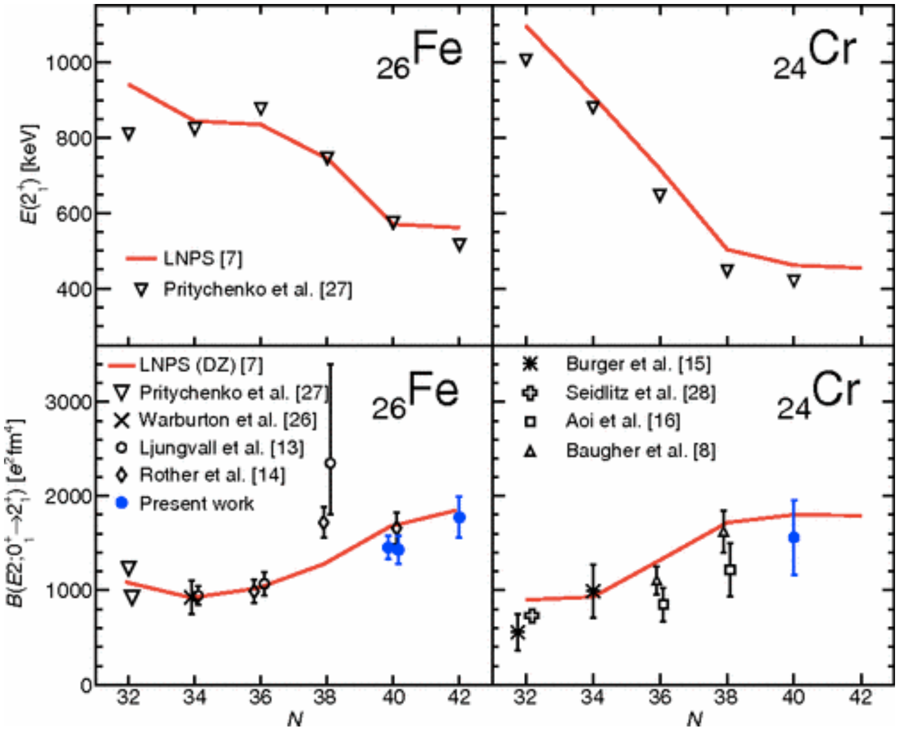
- At high energies (~100 MeV/A), nuclear contribution can be significant for small impact parameters, but for b > R_{int} Coulomb dominates



- At a given beam velocity, b relates to the scattering angle θ, so restricting analysis to forward scattering angles ensures 'safe' Coulex

Neutron-rich Fe and Cr





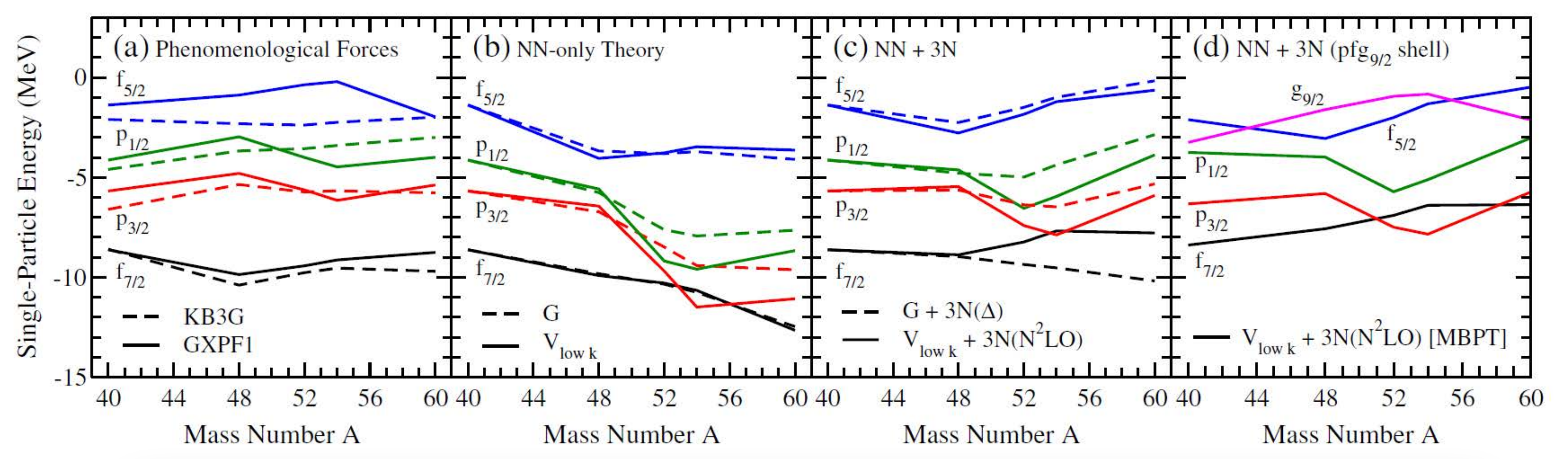
HLC *et al.*, Phys. Rev. Lett. **110**, 242701 (2013).

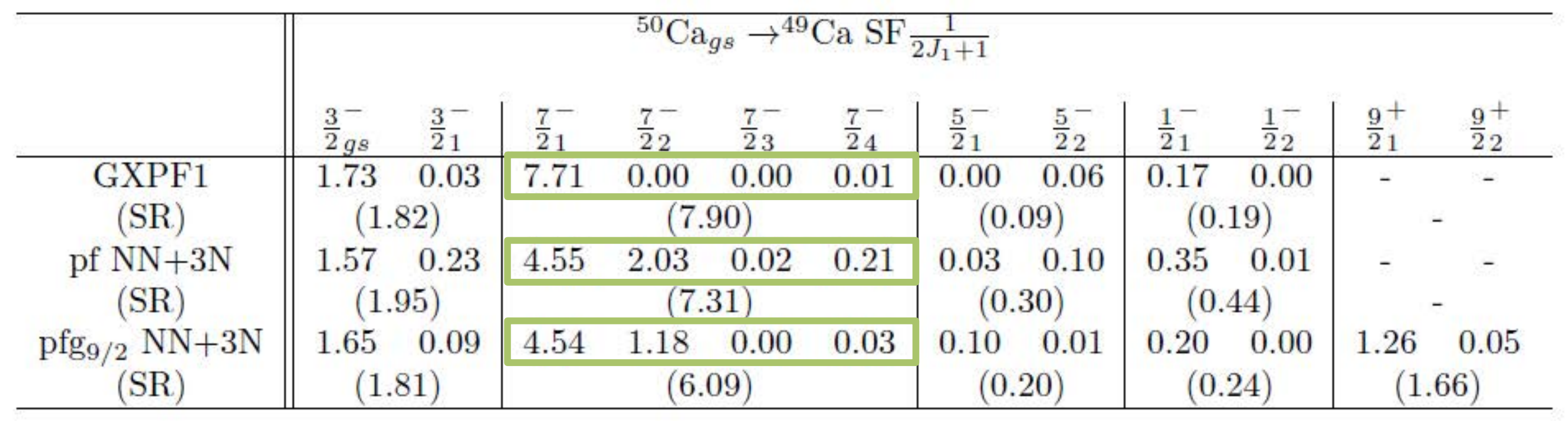
And what have I skipped?

- ‘Exotic’ decay modes
  - 1p and 2p decay at the proton dripline
  - Neutron decay --> recent sequential 2n decay at NSCL
- Resonance spectroscopy – properties of unbound states (beyond the proton and neutron driplines)
- Reactions for spectroscopy and more --> deep inelastic reactions, multi-nucleon transfer, charge-exchange, etc.
- And much, much more…

Example: Designing an experiment to access the physics

We read this theory paper…





	$^{50}\mathrm{Ca}_{gs} \rightarrow {}^{49}\mathrm{Ca}\ \mathrm{SF}\frac{1}{2J_1+1}$											
	$\frac{3}{2}^{-}_{gs}$	$\frac{3}{2}^{-}_{1}$	$\frac{7}{2}^{-}_{1}$	$\frac{7}{2}^{-}_{2}$	$\frac{7}{2}^{-}_{3}$	$\frac{7}{2}^{-}_{4}$	$\frac{5}{2}^{-}_{1}$	$\frac{5}{2}^{-}_{2}$	$\frac{1}{2}^{-}_{1}$	$\frac{1}{2}^{-}_{2}$	$\frac{9}{2}^{+}_{1}$	$\frac{9}{2}^{+}_{2}$
GXPF1	1.73	0.03	7.71	0.00	0.00	0.01	0.00	0.06	0.17	0.00	-	-
(SR)	(1.82)		(7.90)				(0.09)		(0.19)		-	
pf NN+3N	1.57	0.23	4.55	2.03	0.02	0.21	0.03	0.10	0.35	0.01	-	-
(SR)	(1.95)		(7.31)				(0.30)		(0.44)		-	
pfg$_{9/2}$ NN+3N	1.65	0.09	4.54	1.18	0.00	0.03	0.10	0.01	0.20	0.00	1.26	0.05
(SR)	(1.81)		(6.09)				(0.20)		(0.24)		(1.66)	

J.D. Holt et al., J. Phys. G: Nucl. Part. Phys. **39**, 085111 (2012).

J.D. Holt, J. Menendez, A. Schwenk, private communication.

Can we inform this physics question?

- Theory tells us there is a difference in spectroscopic factor for removal of neutrons in ^{50}Ca to states in ^{49}Ca
  - Is this observable? Can we design a measurement to test the different predictions? What could we do? What would our experiment observables be?
  - Where could we do this type of experiment? What facility could we use? What type of equipment?
  - What exactly would we **measure**? How would we have to interpret the data? Do we need theory to interpret the data?