Relativistic Heavy Ion Collisions and the Quark Gluon Plasma

I. Idealized Partonic Matter (1st lecture)

II. Modeling Heavy Ion Collisions and connecting QGP properties to experiment (2nd and 3rd lectures)

III. Quantifying our knowledge of the QGP (3rd lecture)

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Hadron Gas

Density depends on temperature *T*..

$$n_{\text{hadrons}} = \sum_{\alpha} (2S_{\alpha} + 1) \int \frac{d^3p}{(2\pi)^3} e^{-E_p/T} \quad ,$$

$$E_p = \sqrt{m_{\alpha}^2 + p^2}$$

Masses (MeV):

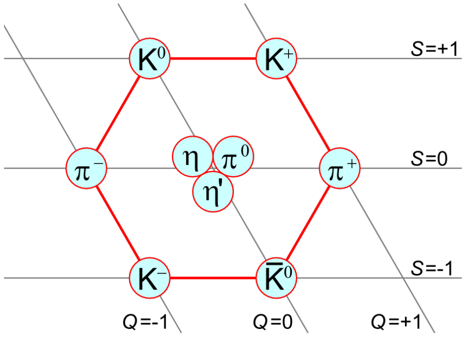
Mesons: $\pi^{+/-/0}$(138), $K^{+/-/0}$(495),η(549), η ´(980),

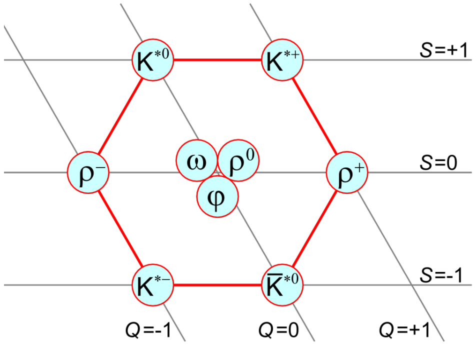
$\rho^{+/-/0}$(770), ω(783), $K^{+/-/0}$(850),ϕ(1020)

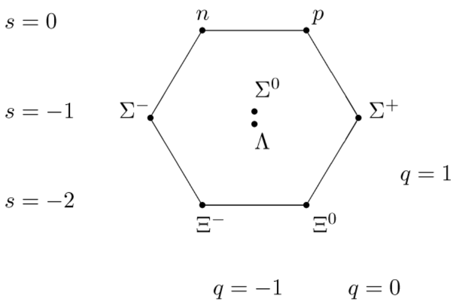
Baryons: p(938), n(940), $\Delta^{++/+/-/0}$(1232), Λ(1116),

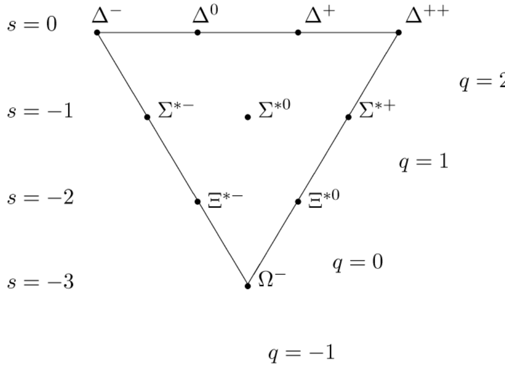
$\Sigma^{+/-/0}$(1195), $\Sigma^{*+/-/0}$(1195), $\Xi^{-/0}$(1314), $\Xi^{*-/0}$(1530), Ω^{-}(1672)

Hundreds of states with M_α < 2 GeV



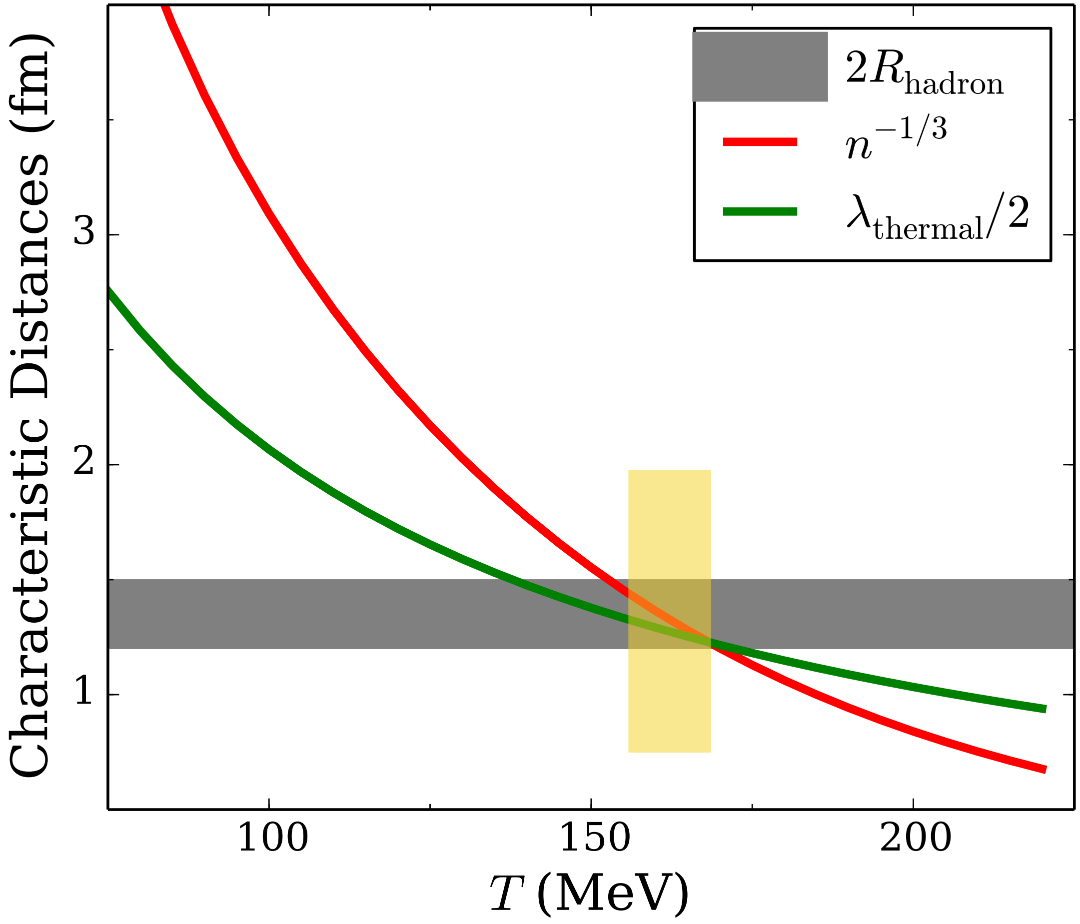




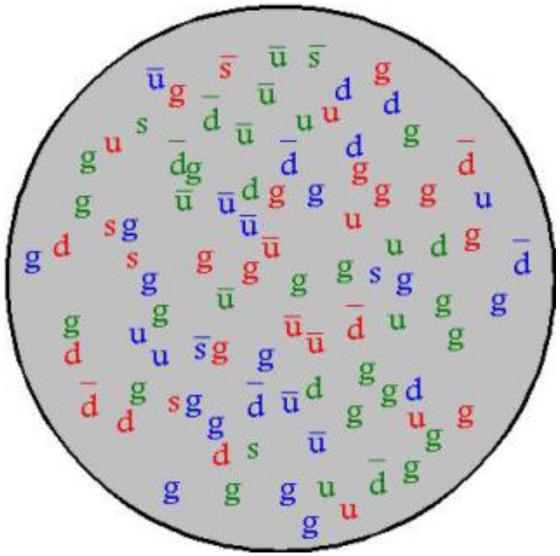


Hadron Gas

- **Hadrons overlap for *T*> ~160 MeV (*T* of universe ~20 μsec after big bang)**
- **Approximately 1 particle per λ_{th}^3**



Parton Gas



52 light degrees of freedom

- **36 quarks**
  (3 colors, 2 spins, part/antipart, uds)
- **16 gluons**
  (8 colors, 2 spins)
- **~ignore leptons, photons or heavy quarks**

$$n \sim T^3$$

$$\epsilon \sim T^4$$

$$P = \epsilon / 3 \sim T^4$$

$$s = \frac{P + \epsilon}{T} \sim T^3$$

Inside ~ 1 λ_{th}^3,

- **Bose condensed ^{4}He: one particle**
- **Photon gas: 2 particles**
- **Parton gas: 52 particles**

With Interactions

Properties to discuss:

1. Eq. of State (μ_B=0, μ_B≠0)
2. Chemistry
3. Chiral Symmetry
4. Color screening
5. Viscosity
6. Diffusion Constant*
7. Jet damping*
8. Stopping and Thermalization

*will skip

- No.s 1 - 4 require lattice gauge theory
- all can be connected to measurement (next lecture)

Lattice Gauge Theory

First, outline derivation of path-integral form of partition function

$$Z(\beta) = \frac{1}{(2\pi)^N} \prod_{i_1 i_2 \cdots i_N} \int dp_1 dq_1 dp_2 dq_2 \cdots dp_N dq_N \exp\left\{ i \int_0^{i\beta} d\tau \;\; L(p(\tau), q(\tau)) \right\}$$

$$\phi = (p + iq) / \sqrt{2}$$

Lattice Gauge Theory

“coherent” state is eigenstate of destruction operator

$$\left|\phi\right\rangle = \exp\left\{-\phi^{*}a + \phi a^{\dagger})\right\}\left|0\right\rangle$$

$$= e^{-|\phi|^{2}/2}e^{\phi a^{\dagger}}\left|0\right\rangle$$

$$a\left|\phi\right\rangle = \phi\left|\phi\right\rangle$$

Lattice Gauge Theory

Exercise 1.

Show that:

$a: |\eta\rangle \equiv e^{(\eta a^{\dagger} - \eta^{*} a)} |0\rangle = e^{-\eta^{*}\eta/2} e^{\eta a^{\dagger}} |0\rangle$ **Use Baker-Campbell-Hausdorff**

$b: a^{\dagger} e^{-i\eta a^{\dagger}} |0\rangle = \eta a^{\dagger} e^{\eta a^{\dagger}}$ **Expand exponential**

$c: \langle \eta | \eta + \delta\eta \rangle = e^{(\eta^{*}\delta\eta - \delta\eta^{*}\eta)/2}$ **Use (a) and (b)**

Lattice Gauge Theory

completeness proof

$$
\begin{aligned}
\langle m|\phi\rangle\langle\phi|n\rangle &= \frac{1}{\sqrt{m!n!}}\langle 0|a^m|\phi\rangle\langle\phi|(a^\dagger)^n|0\rangle \\
&= \frac{1}{\sqrt{m!n!}}(-i\phi^*)^m(i\phi)^n\langle 0|\phi\rangle\langle\phi|0\rangle \\
&= \frac{(-i\phi^*)^m(i\phi)^n}{\sqrt{m!n!}}e^{-|\phi|^2}
\end{aligned}
$$

$$
\begin{aligned}
\int d\phi_r d\phi_i\ \langle m|\phi\rangle\langle\phi|n\rangle &= 2\pi\delta_{mn}\int|\phi|\,d|\phi|\frac{|\phi|^{2n}}{n!}e^{-|\phi|^2} \\
&= \pi\delta_{mn}
\end{aligned}
$$

$$
\int\frac{d\phi_r d\phi_i}{\pi}\ |\phi\rangle\langle\phi| = I
$$

Lattice Gauge Theory

Take trace of Lagrangian

$$Z(\beta)=\frac{1}{(2\pi)^N}\prod_{i_1 i_2\cdots i_N}\int dp_1 dq_1 dp_2 dq_2\cdots dp_N dq_N$$

$$\cdot\left\langle\phi_1\right|e^{-\delta\beta H(p,q)}\left|\phi_2\right\rangle\left\langle\phi_2\right|e^{-\delta\beta H(p,q)}\cdots\left|\phi_n\right\rangle\left\langle\phi_n\right|e^{-\delta\beta H(p,q)}\cdots\left|\phi_1\right\rangle\ ,$$

$$\delta\beta=\beta/N,\ \beta=1/T$$

$$=\frac{1}{(2\pi)^N}\prod_{i_1 i_2\cdots i_N}\int dp_1 dq_1 dp_2 dq_2\cdots dp_N dq_N\exp\left\{i\int_0^{i\beta}d\tau\ L(p(\tau),q(\tau))\right\}$$

$$\left\langle\phi_1\right|e^{-\delta\beta H(p,q)}\left|\phi_2\right\rangle\approx(1-\delta\beta H(p_1,q_1))\langle\phi_1|\phi_1+\delta\phi\rangle=(1-\delta\beta H(p_1,q_1)+p\delta q/2-q\delta p/2)$$

$$=1+\delta\beta\left(p\dot{q}/2-q\dot{p}/2-H(p,q)\right)$$

$$Z(\beta)=\frac{1}{(2\pi)^N}\prod_{i_1 i_2\cdots i_N}\int dp_1 dq_1 dp_2 dq_2\cdots dp_N dq_N\exp\left\{\int_0^{\beta}\mathcal{L}(p,q)\right\}$$

$$\phi=(p+iq)/\sqrt{2}$$

Path integral for evolution operator,
but in imaginary time

Lattice Gauge Theory (Review)

Integrate over field configurations —> Partition function

$$Z(\beta = 1/T) = \sum_i \langle i | e^{-\beta H} | i \rangle$$

$$= \sum_{i_1 \cdots i_N} \langle i_1 | e^{-\delta\beta H} | i_2 \rangle \langle i_2 | e^{-\delta\beta H} \cdots | i_N \rangle \langle i_N | e^{-\delta\beta H} | i_1 \rangle, \quad \delta\beta = \beta / N$$

Change basis to “fields”

$$|\phi\rangle = \exp\left\{ i\phi a - i\phi^* a^\dagger) \right\} |0\rangle, \ \ \phi = (p + iq)/\sqrt{2}$$

$$\sum_i |i\rangle\langle i| \to \frac{1}{2\pi} \int dp\, dq\, |\phi\rangle\langle\phi|$$

$$\langle \phi(t) | \phi(t + \delta t) \rangle = \exp\left\{ (ip\dot{q} - iq\dot{p})\delta t / 2 \right\}, \sim\sim\sim \langle \phi(t) | e^{-iH\delta t} | \phi(t + \delta t) \rangle = \exp\left\{ iL(p,q)\delta t \right\}$$

Problem reduced to high-dimensional integral

$$Z(\beta) = \frac{1}{(2\pi)^N} \prod_{i_1 i_2 \cdots i_N} \int dp_1 dq_1 dp_2 dq_2 \cdots dp_N dq_N \exp\left\{ i \int_0^{i\beta} d\tau \ L(p(\tau), q(\tau)) \right\}$$

Lattice Gauge Theory

Advantages

- Can handle configurations where particle number is not well conserved (gluons)
- Relativistic

Disadvantages

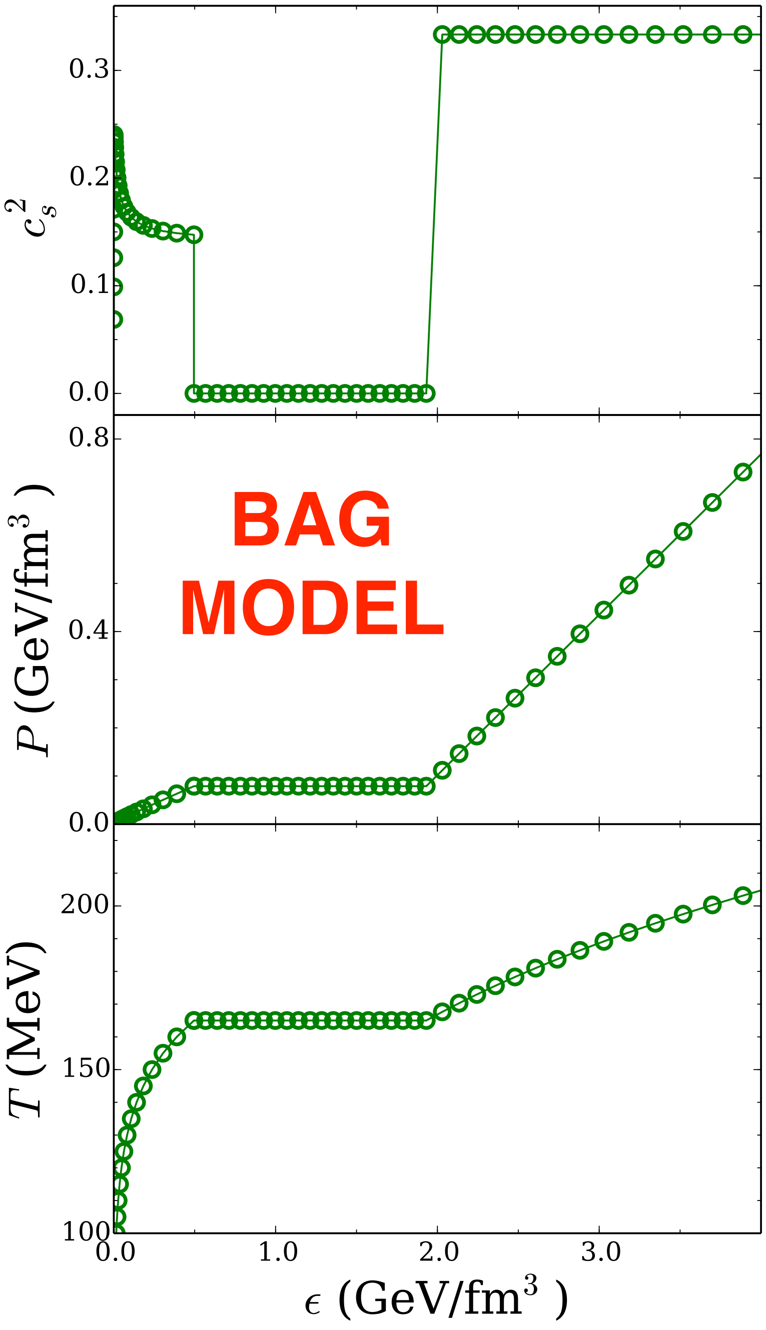
- Poor choice for systems with fixed number of well-defined quasiparticles (nuclei)
- Has trouble with correlators in real time

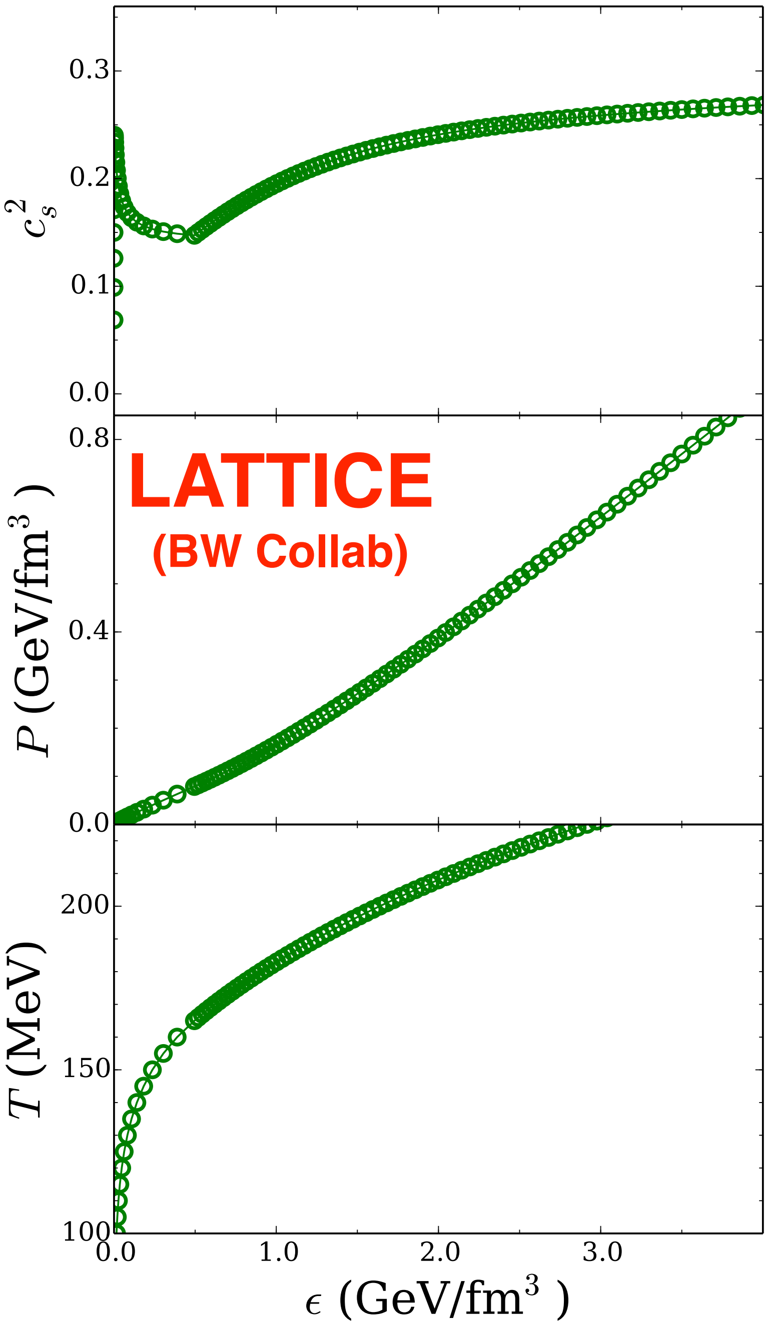
  $\langle A(0)A(t)\rangle$

  Examples: viscosity, conductivity, diffusion constant
- Numerically expensive

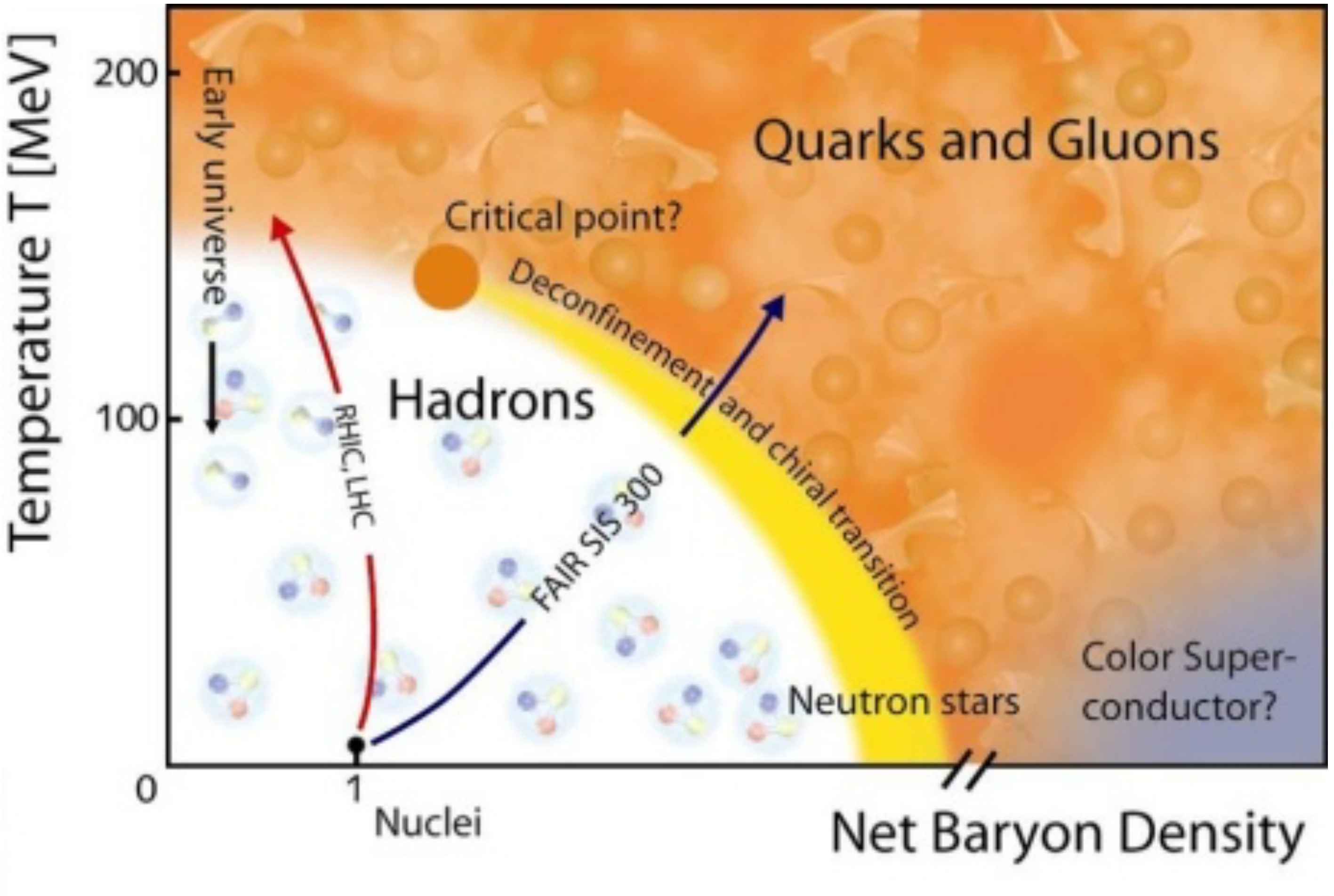
1. Eq. of State

First order?





1. EoS at Finite Baryon Density



First-order phase transition and critical point?

- **If first-order there should be critical point**
- **Lattice has trouble at finite μ**
- **NJL Models can lead to 1st-order transition**

2. Chemistry

Parton number undefined in interacting system and $\langle \rho_{u,d,s} \rangle = 0$ so, considers fluctuations:

$$\chi_{ab} \equiv \frac{\langle Q_a Q_b \rangle}{V}$$

For parton gas (non-interacting)

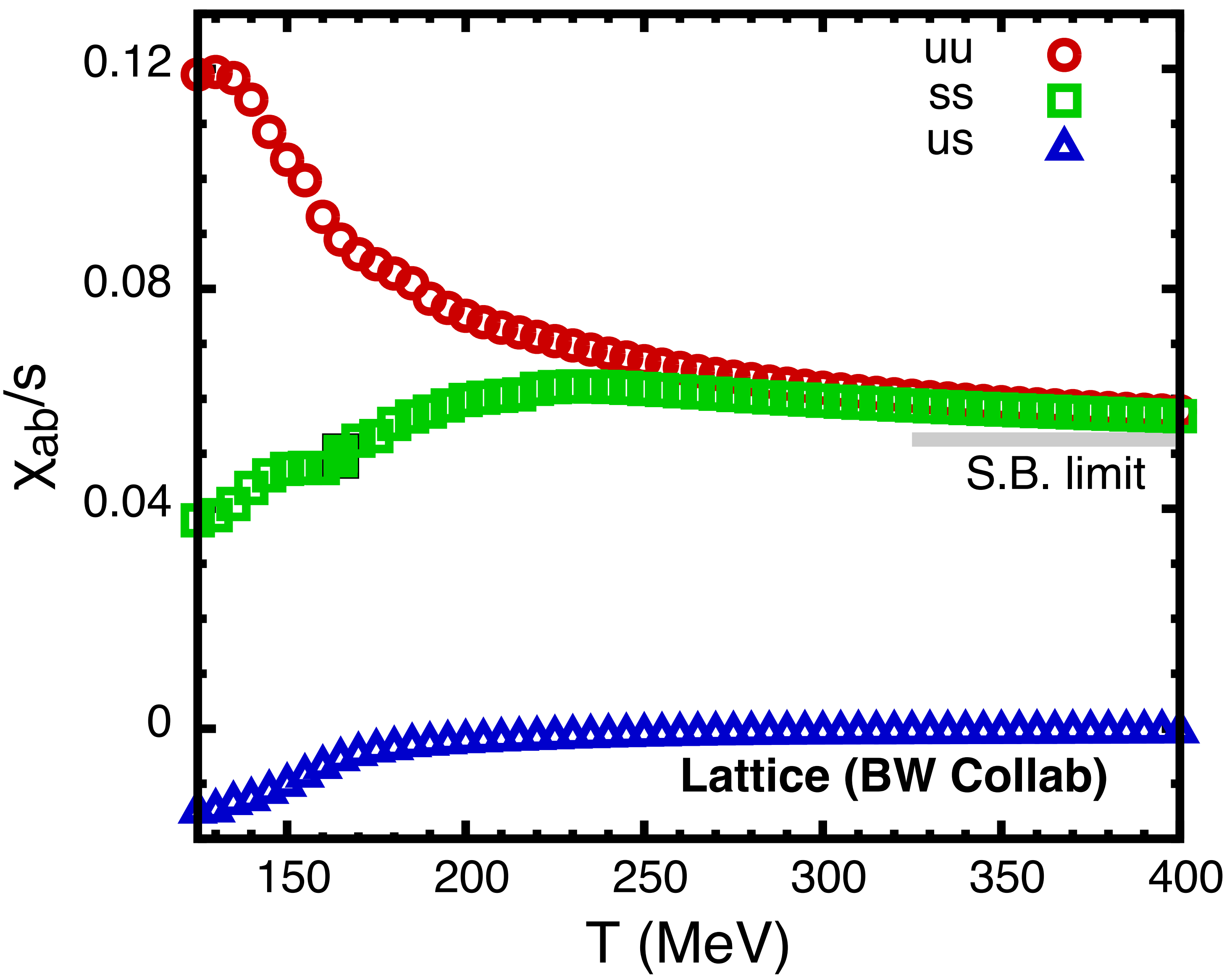
$$\chi_{ab} = (n_a + n_{\bar{a}})\delta_{ab} \ , \chi / s = \text{constant for } m = 0$$

For hadron gas (non-interacting)

$$\chi_{ab} = \sum_{\alpha} n_{\alpha} q_{\alpha a} q_{\alpha b}$$

2. Chemistry

behavior approaches parton gas at high *T*



3. Chiral Symmetry

$$\mathcal{L} = \bar{\Psi}_a (i\partial_\mu - eA_\mu)\gamma^\mu \Psi_a + \cdots$$

$$\Psi \rightarrow e^{i\gamma_5 \phi}\Psi$$

$$\Psi \rightarrow e^{i\gamma_5 \vec{\tau}\cdot\vec{\phi}}\Psi$$

Invariant to axial and iso-axial rotations

Noether's theorem leads to conserved currents

$$\xcancel{j_5^\mu = \bar{\Psi}\gamma^\mu \gamma_5 \Psi}$$

$$j_{5a}^\mu = \bar{\Psi}\gamma^\mu \gamma_5 \tau_a \Psi$$

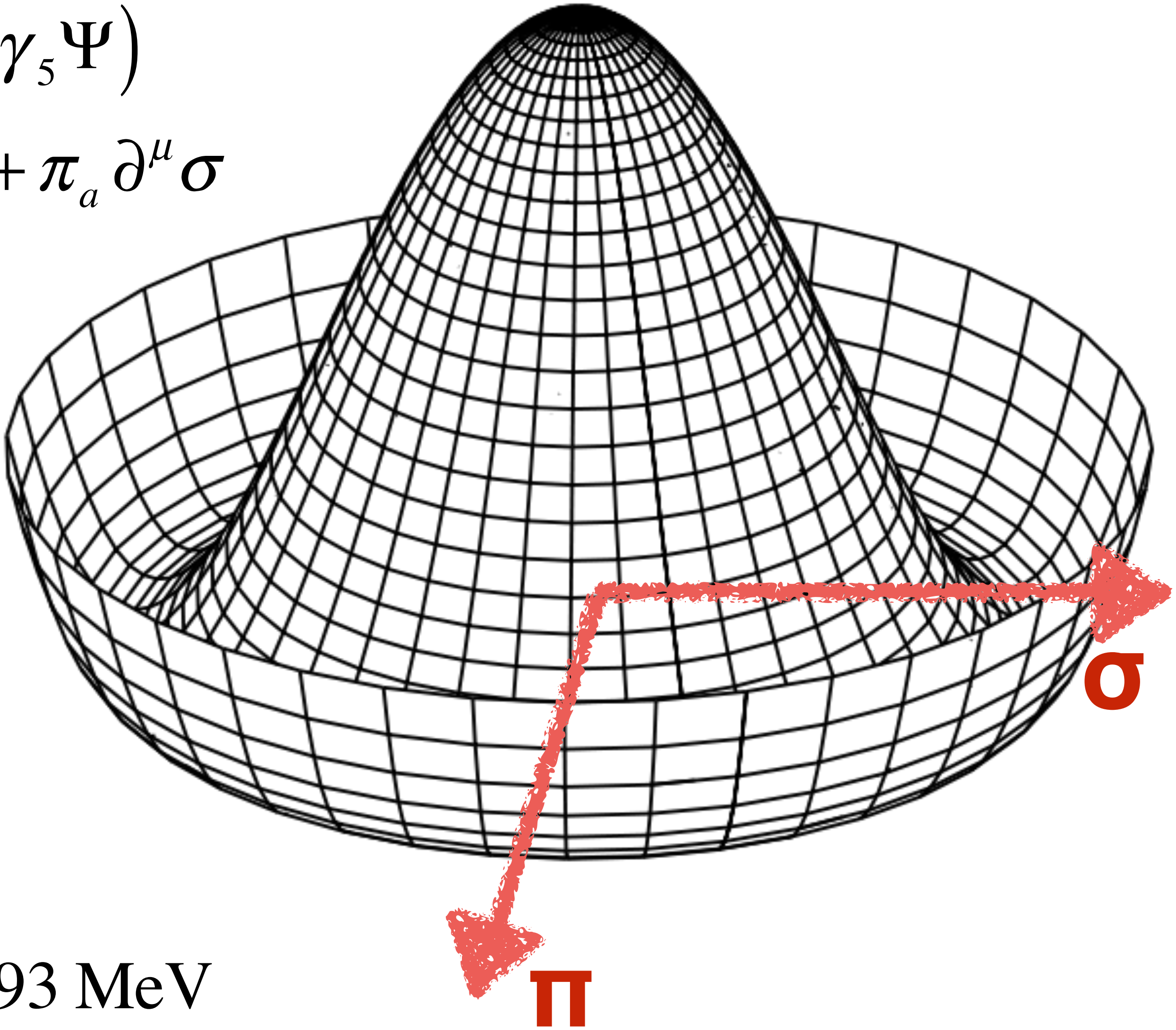
ruined by chiral anomaly

3. Chiral Symmetry (hadronic perspective)

$$\mathcal{L} = \frac{-1}{2}\left\{\sigma\partial^2\sigma + \vec{\pi}\partial^2\cdot\vec{\pi}\right\} + \frac{1}{2}M_0^2\left\{\sigma^2 + |\vec{\pi}|^2\right\} - \frac{\lambda}{4}\left\{\sigma^2 + |\vec{\pi}|^2\right\}^2$$

$$+ g_{\pi N}\left(\sigma\bar{\Psi}\Psi + i\vec{\pi}\cdot\bar{\Psi}\vec{\tau}\gamma_5\Psi\right)$$

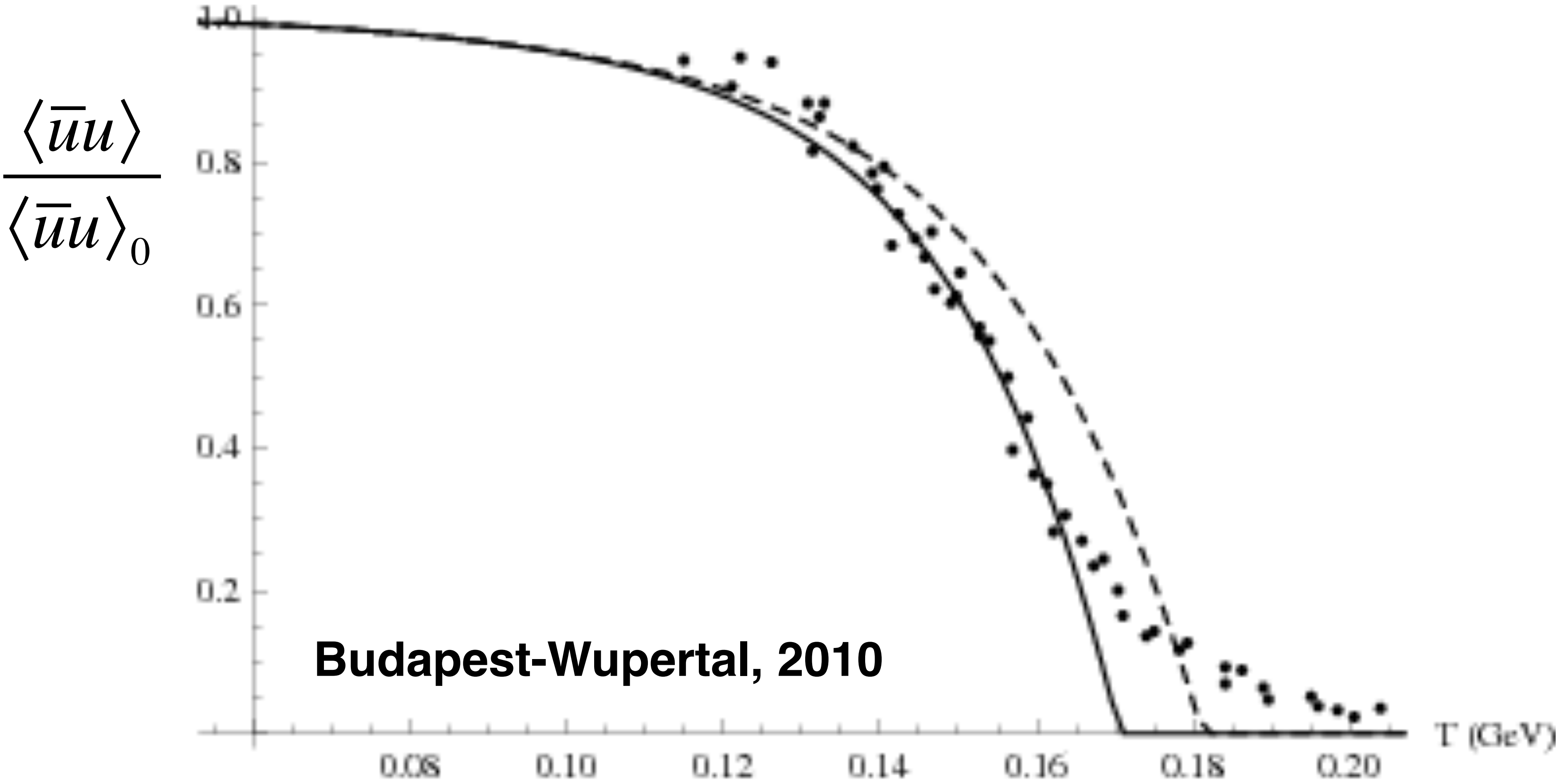
$$j_a^\mu = \bar{\Psi}\gamma_5\gamma^\mu\tau_a\Psi + \sigma\partial^\mu\pi_a + \pi_a\partial^\mu\sigma$$



$$M_N \approx g_{\pi N}\langle\sigma\rangle, \langle\sigma\rangle = f_\pi = 93\ \text{MeV}$$

3. Chiral Symmetry (lattice)

3. q-qbar condensate, is related to sigma condensate



Budapest-Wupertal, 2010

Condensate leads to constituent quark mass

4. Color Screening

Debye Screening: Charge +Q_0 in plasma, will attract negative charges

$$\Delta n_e(r) = n_e(e^{-V(r)/T} - 1),$$

$$\approx -n_0 V(r) / T$$

$$V(r) = \frac{-eQ_0}{4\pi\epsilon_0 r} e^{-r/\lambda},$$

Includes contribution from screening charges

$$\lambda_{\mathrm{Debye}} = \sqrt{\frac{\epsilon_0 T}{n_0 e^2}}$$

Screens confining potential ➜ “free color charges”

4. Color Screening

Exercise 2. Show form

$$\Delta n_e(r) = n_e(e^{-V(r)/T} - 1),$$

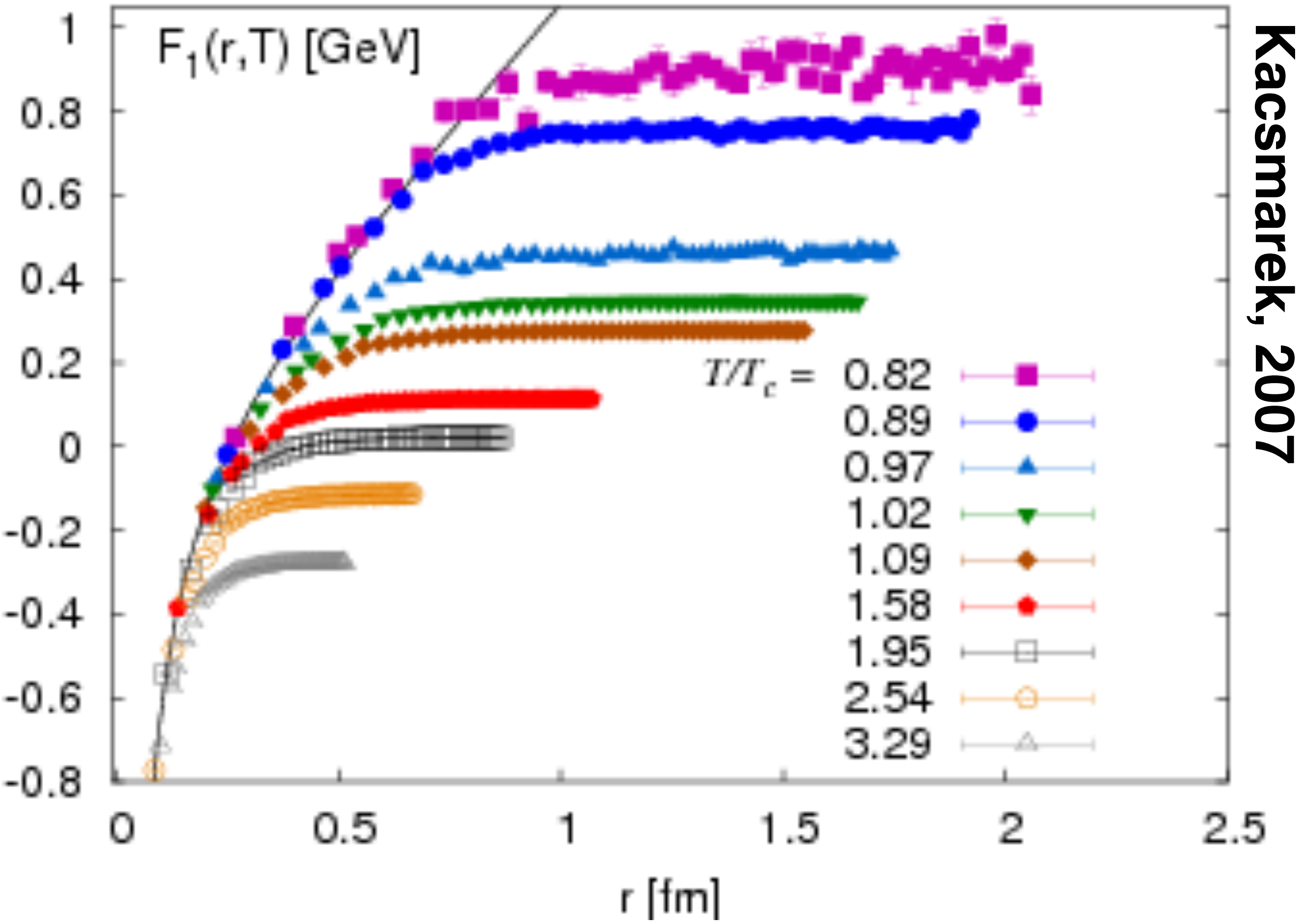
$$\approx -n_0 V(r)/T$$

$$V(r) = \frac{-eQ_0}{4\pi\epsilon_0 r} e^{-r/\lambda}, \lambda_{\text{Debye}} = \sqrt{\frac{\epsilon_0 T}{n_0 e^2}}$$

is consistent with Gauss's law. I.e. calculate *E(r)* and *Q(r)*=charge inside *r*.

4. Color Screening

Free energy vs. separation



Kacsmarek, 2007

For T> 200 MeV, charges can separate

5. Viscosity

$$\partial_t T_{00} = -\partial_x T_{0x} - \partial_y T_{0y} + \partial_z T_{0z}$$

$$\partial_t T_{0x} = \partial_x T_{xx} + \partial_y T_{yx} + \partial_z T_{zx}$$

Local conservation of *E* and *P*

$$T_{i\neq j} = 0$$

Ideal hydro

$$T_{ij} = P\delta_{ij} - \eta(\partial_i v_j + \partial_j v_i) - \zeta \nabla \cdot \vec{v}$$

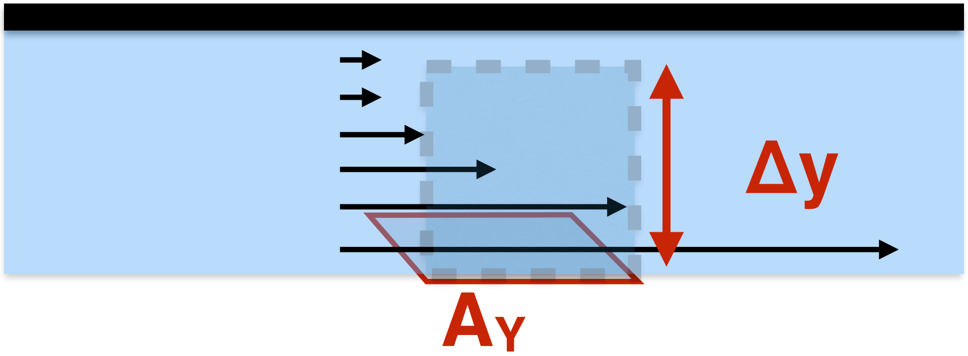
Navier-Stokes

η = shear viscosity

ζ = bulk viscosity

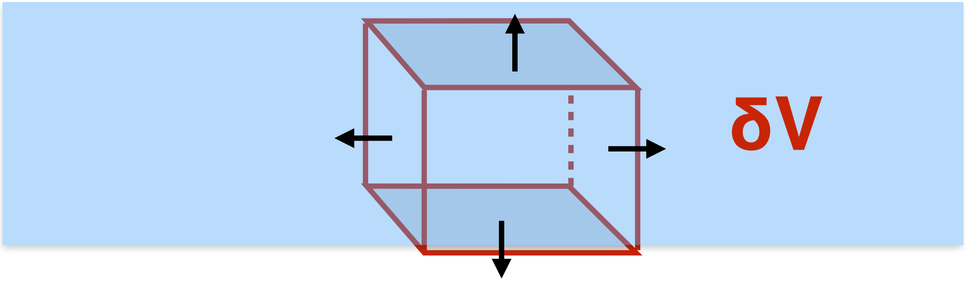
5. Viscosity

shear represents friction between layers of fluid



$$\frac{d}{dt}P_x = A_y \eta \partial_y v_x$$

bulk describes dissipation of diverging flow



$$\delta E = -P\delta V + \zeta \nabla \cdot \vec{v}\delta V$$

5. Viscosity (Kubo relations)

Linear response theory — example conductivity

$$\delta\langle j(x=0,t=0)\rangle = \langle \Psi_0 | \left(1+i\int_{-\infty}^{0} dt\, V(t)\right) j(0,0) \left(1-i\int_{-\infty}^{0} dt\, V(t)\right) | \Psi_0\rangle$$

$$V(t) = \int dx\, x E_x \rho(x,t) = E_x \int dx\, x\, t\, \partial_t \rho(x,t),$$

$$\int_{-\infty}^{0} dt\;\; V(t) = E_x \int_{-\infty}^{0} dt\, t\, dx\, j(x,t)$$

$$\sigma = -i\int_{0}^{\infty} dt\;\; t\;\; dx\;\; \langle [j(0,0), j(x,t)]\rangle$$

$$= -i\int_{-\infty}^{\infty} dt\;\; t\;\; dx\;\; \langle j(0,0) j(x,t)\rangle$$

$$= \frac{1}{2T}\int_{-\infty}^{\infty} dt\;\; \langle j(0,0) j(x,t)\rangle$$

analyticity+trace properties of *Z*

Transport coefficients derived from correlations integrated over relative time

5. Viscosity (Kubo relations)

Exercise 3: Derive Kubo relation for viscosity

$$\eta = -i\int d^3r\,dt\;t\langle[T_{xy}(0,0),T_{xy}(\vec{r},t=0)]\rangle$$

First assume velocity gradient $\partial_x v_y$ as external field:

$$\langle\Delta T_{xy}(r=0)\rangle = \eta\,\partial_x v_y$$

Then use interaction:

$$V = \int d^3r\,T_{0y}\,x\,\partial_x v_y$$

Folllow steps for conductivity, but with

$$E \to \partial_x v_y,\quad x\rho \to xT_{0y}$$

5. Viscosity (Kubo relations)

Exercise 3*: Derive Kubo relation for viscosity

Show:

$$\eta = -i\int d^3r\,dt\;t\,\langle T_{xy}(0,0)T_{xy}(\vec{r},t=0)\rangle$$

leads to:

$$\eta = \frac{\beta}{2}\int d^3r\,dt\;\langle T_{xy}(0,0)T_{xy}(\vec{r},t=0)\rangle$$

Hint:

$$g(t) = \langle A(0)A(t)\rangle = \mathrm{Tr}\,e^{-\beta H}A(0)A(t)$$

First show:

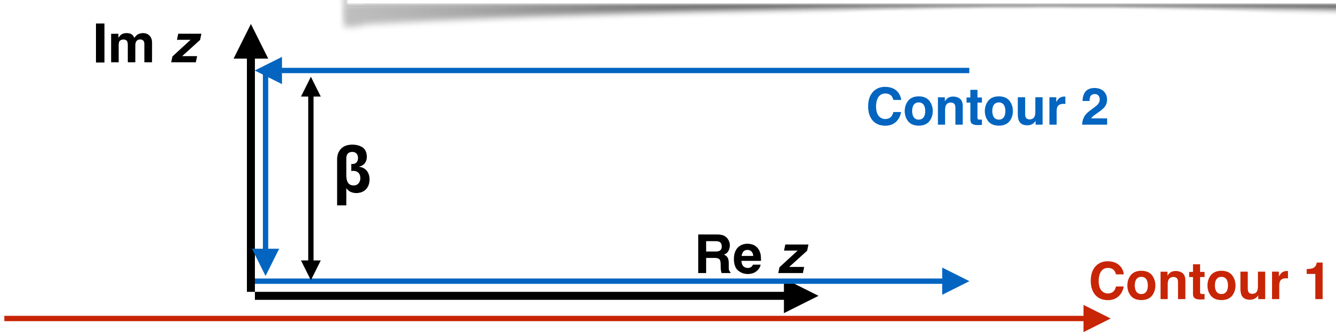
$$g(i\beta/2+z) = g(i\beta/2-z)$$

cyclic prop. of trace

Then show:

$$\oint_{\text{contour}\,1} dz\,g(z)(z-i\beta/2) = \oint_{\text{contour}\,2} dz\,g(z)(z-i\beta/2) = 0$$

analyticity

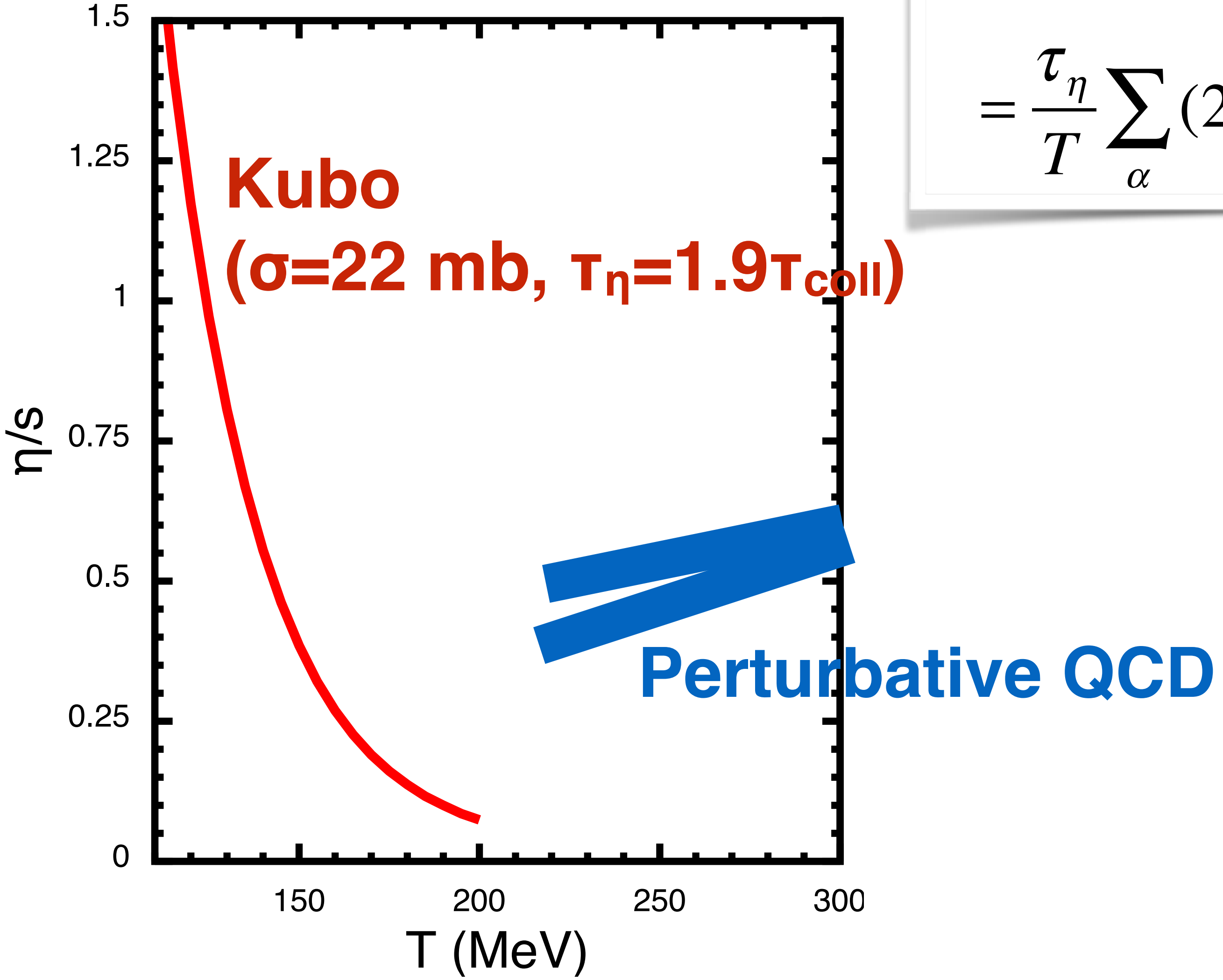


5. Viscosity (Kubo relations)

For gas, correlation of particles with themselves multiplied by relaxation time:

$$\eta = \frac{\tau_\eta}{T}\int d^3r\,\langle T_{xy}(0,0)T_{xy}(\vec{r},t=0)\rangle$$

$$= \frac{\tau_\eta}{T}\sum_\alpha (2S_\alpha+1)\int \frac{d^3p}{(2\pi)^3}\,e^{-E/T}\,\frac{p_x^2 p_y^2}{E^2}$$



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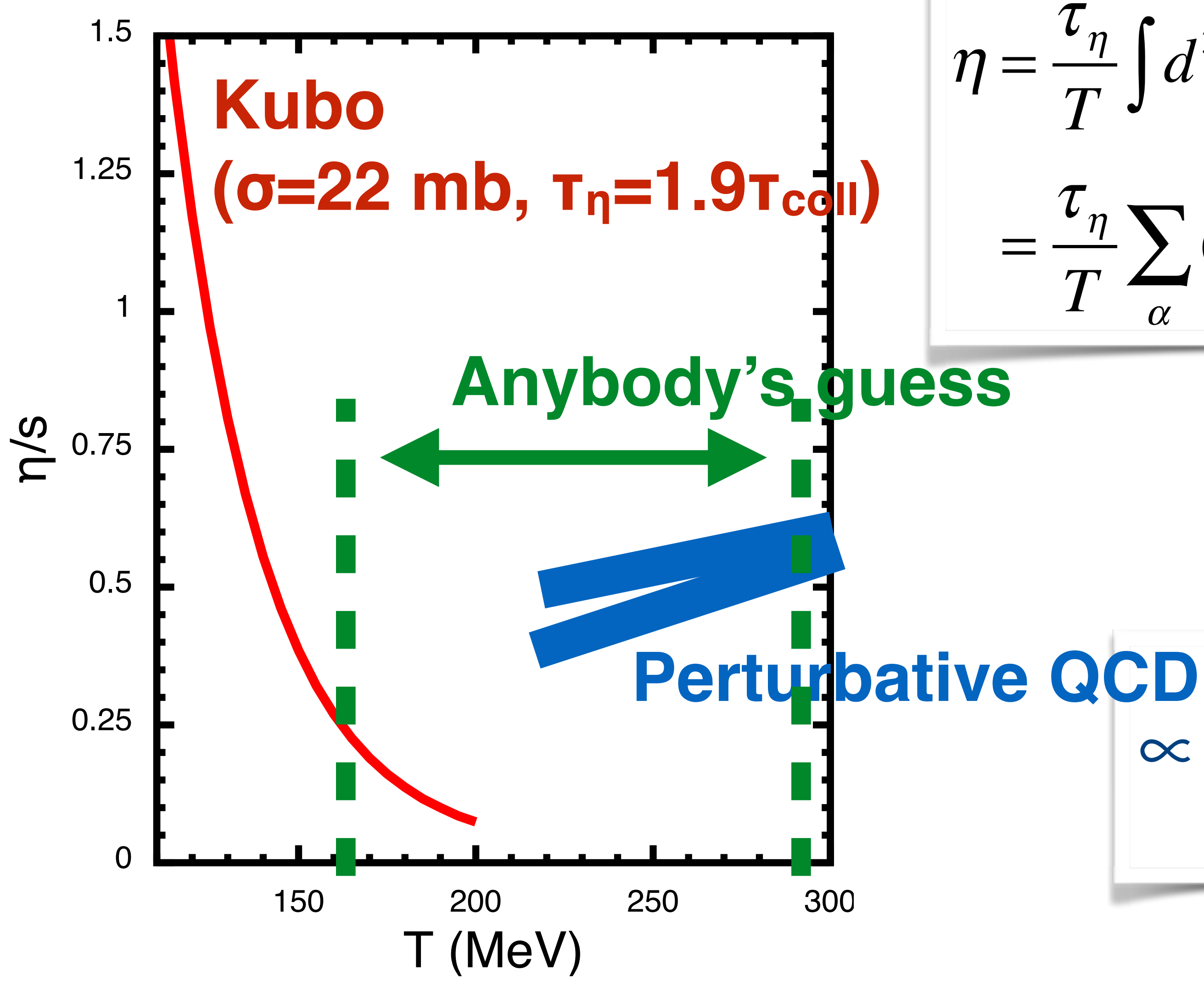
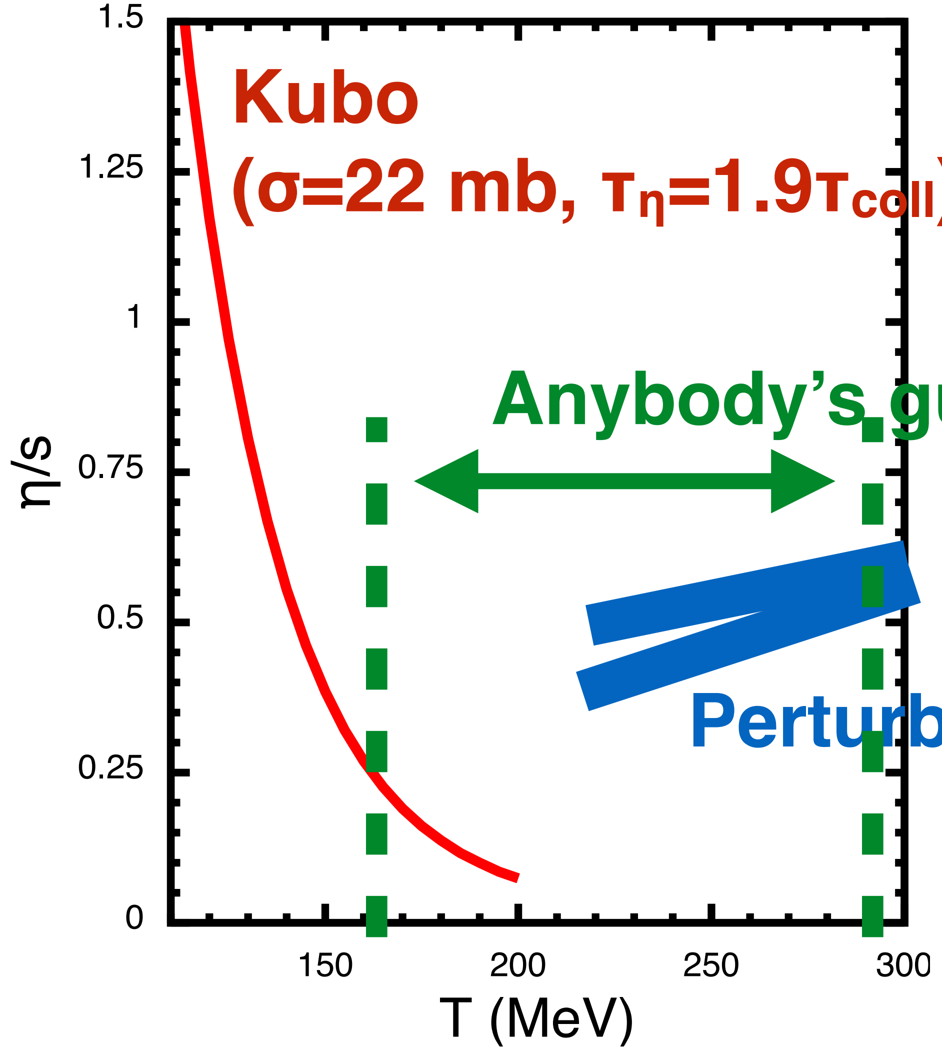


$$\propto \frac{-1}{\alpha^2 \ln \alpha_s}$$

5. Viscosity

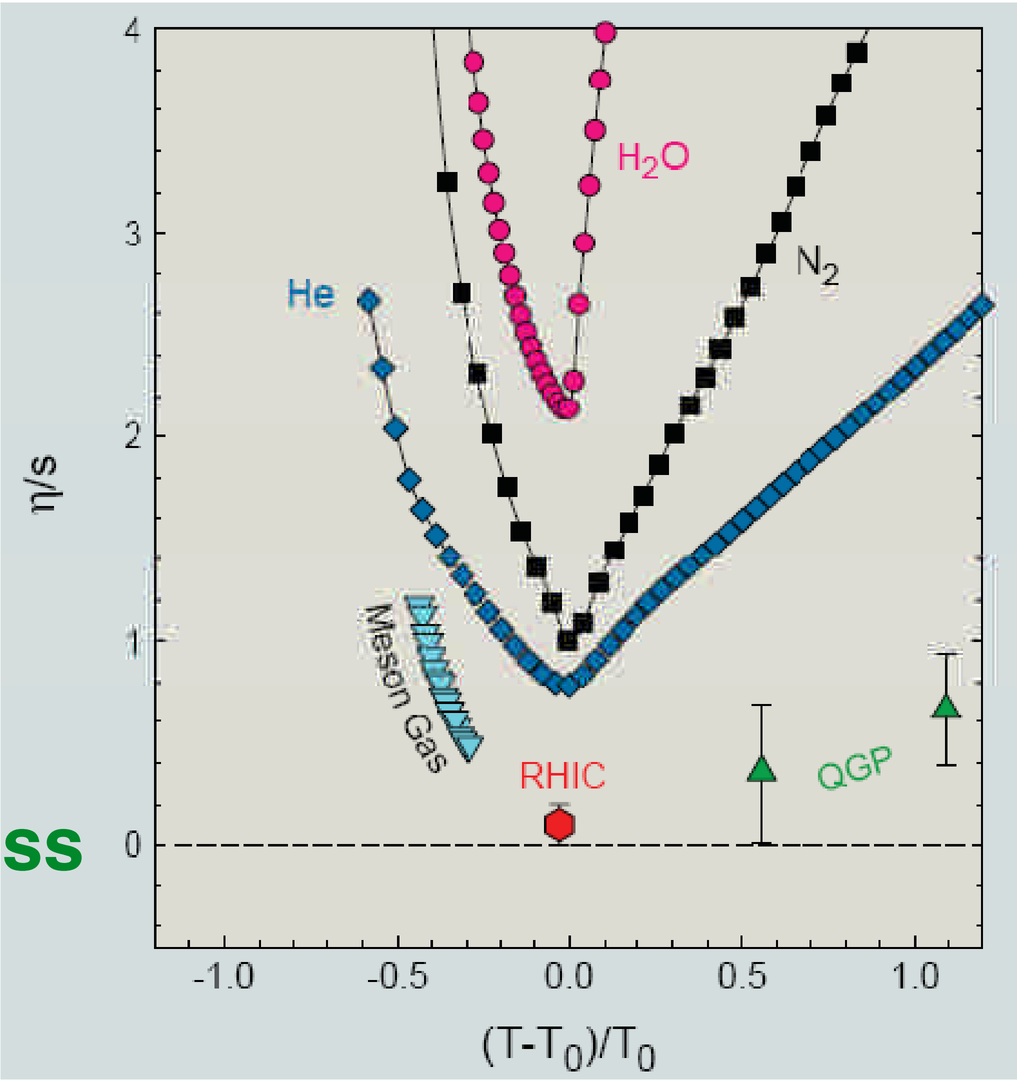
similar behavior to other fluids near T_c



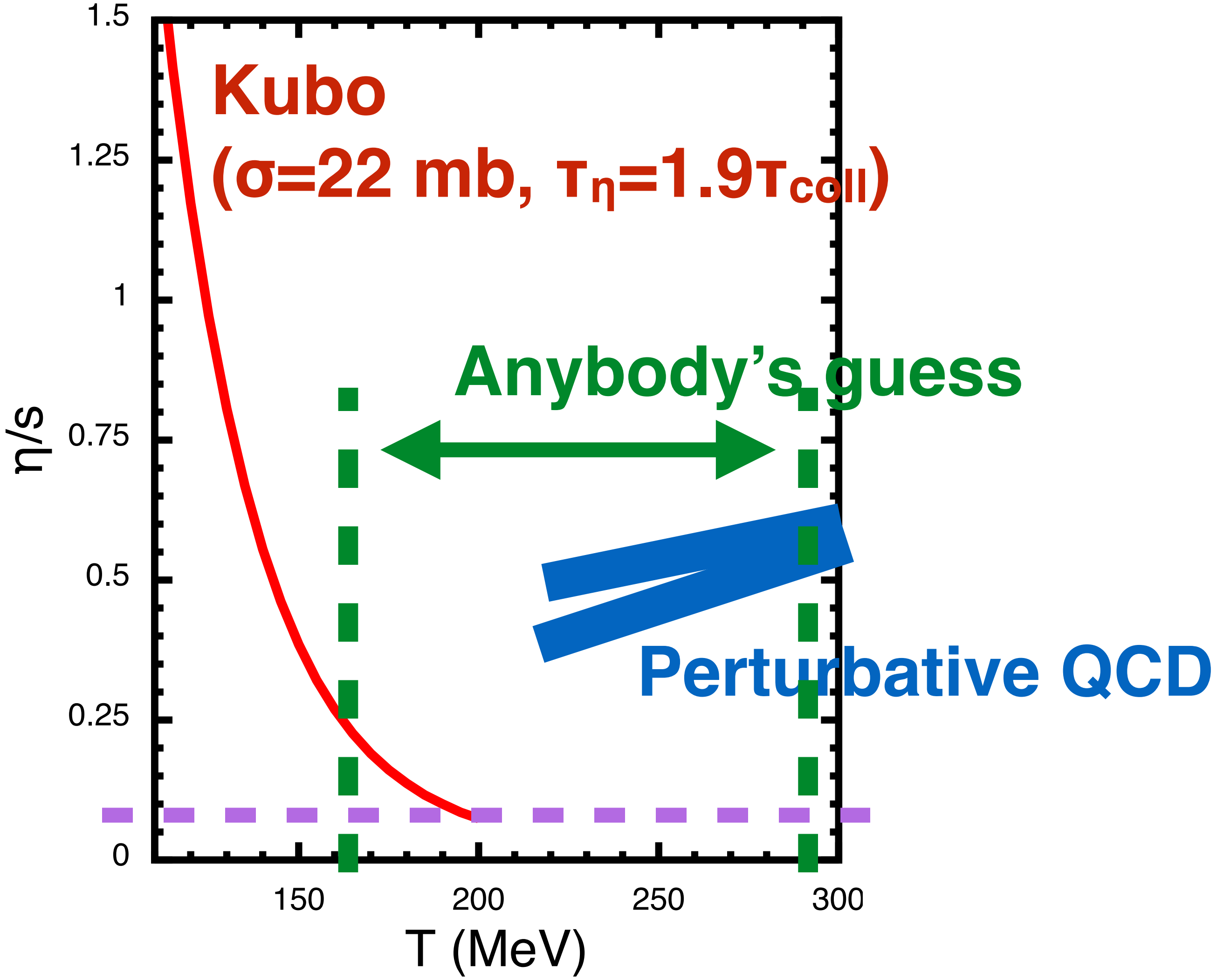
Kubo
(σ=22 mb, τ_η=1.9τ_{coll})

Anybody's guess

Perturbative QCD



5. Viscosity



Some values:

0.08 : $\lambda_{therm} \sim \lambda_{mfp}$ (Danielewicz and Gyulassy)

1/4π : AdS/CFT (Kovton, Starinets, Son)

in principle can approach zero