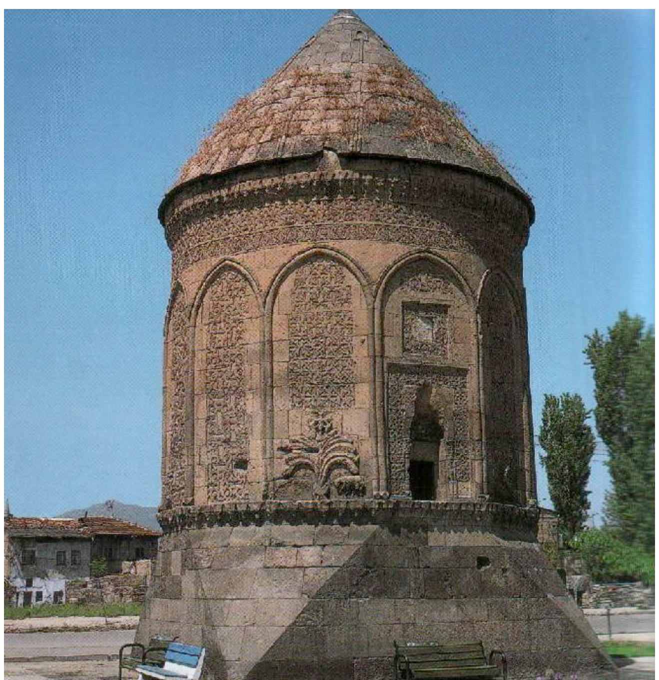
Fundamental Symmetries and Neutrinos - Theory

A.B. Balantekin
University of Wisconsin - Madison

Translational symmetry

Mirror symmetry



Rotational symmetry

Symmetries of space-time: Lorentz invariance, Poincare invariance

Exact Symmetries of interactions:

- Invariance under space translations: momentum conservation
- Rotational invariance: angular momentum conservation.
- Invariance under time translations: energy conservation.

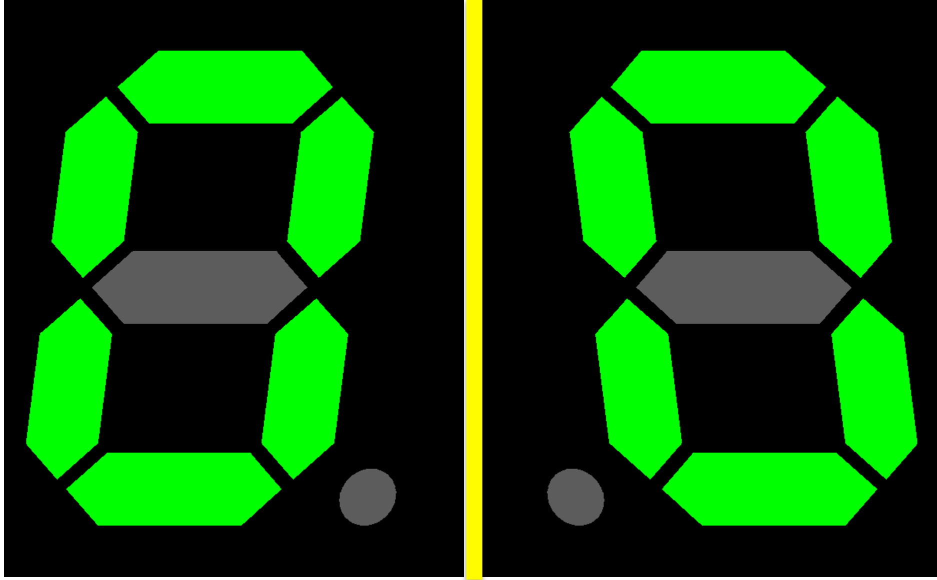
All others are approximate!

"Fundamental" Symmetries: C, P, and T

PARITY (P)

If a process is permitted by the laws of physics, its mirror image is also permitted.

Not always true



$$\mathbf{r} \xrightarrow{P} -\mathbf{r}$$

$$\mathbf{p} = -i\hbar\nabla \xrightarrow{P} -\mathbf{p}$$

$$\mathbf{A} \xrightarrow{P} -\mathbf{A}$$

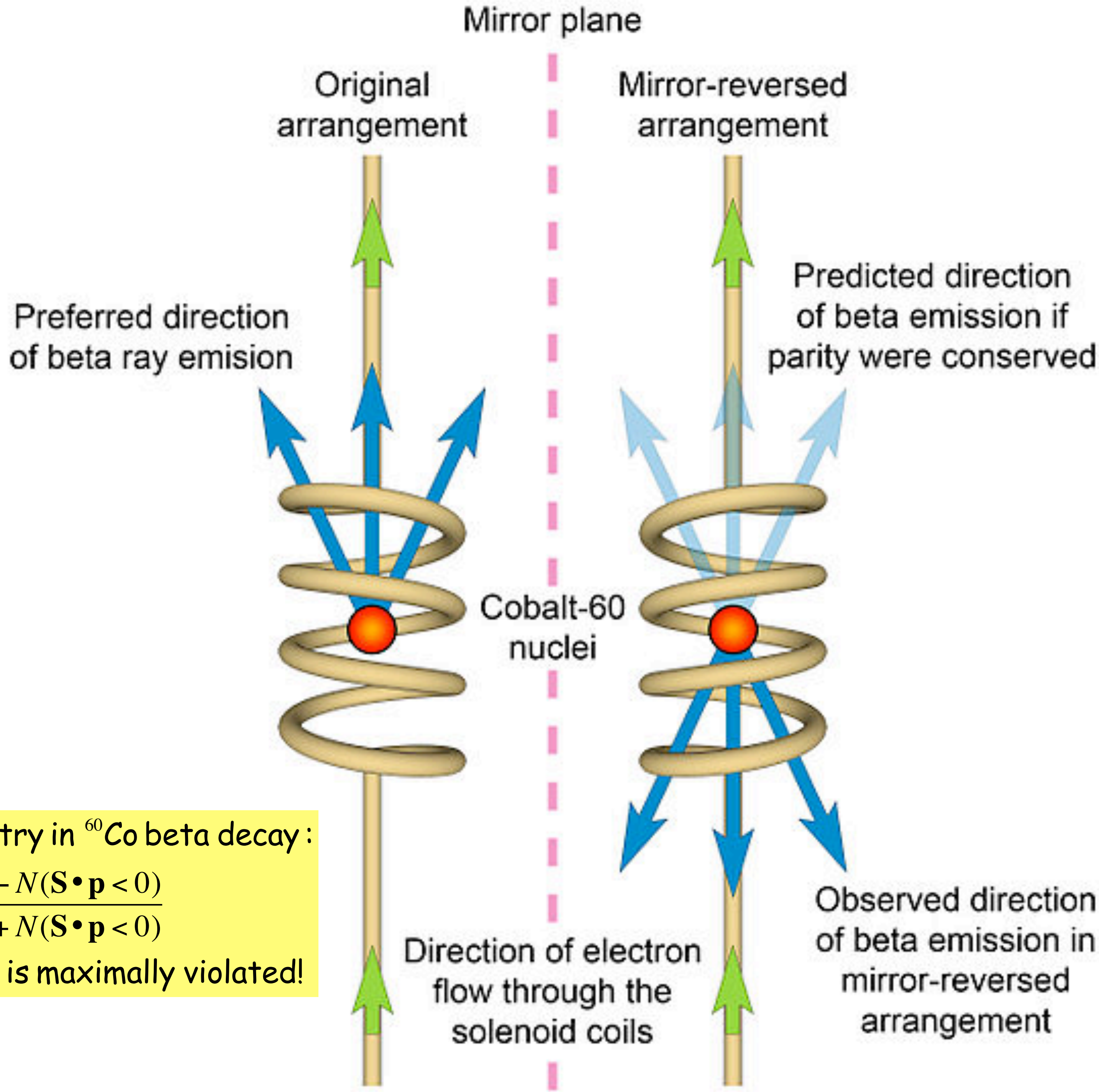
$$\underbrace{\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \xrightarrow{P} -\mathbf{E}}_{\text{vectors}}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \xrightarrow{P} \mathbf{L}$$

$$\mathbf{S} \xrightarrow{P} \mathbf{S}$$

$$\underbrace{\mathbf{B} = \nabla \times \mathbf{A} \xrightarrow{P} \mathbf{B}}_{\text{pseudo-vectors}}$$

Wu's beta decay expt.



Wu searched for asymmetry in ^{60}Co beta decay:

$$A = \frac{N(\mathbf{S}\bullet\mathbf{p}>0) - N(\mathbf{S}\bullet\mathbf{p}<0)}{N(\mathbf{S}\bullet\mathbf{p}>0) + N(\mathbf{S}\bullet\mathbf{p}<0)}$$

Large asymmetry: parity is maximally violated!

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \xrightarrow{P} i\gamma_\mu \partial'^\mu \psi' - m\psi' = 0$$

$$\text{Assume} \quad \psi'(-\mathbf{r}, t) = S\psi(\mathbf{r}, t) \Rightarrow \begin{cases} i\gamma_\mu \partial^\mu S\psi - mS\psi = 0 & \mu = 0 \\ -i\gamma_\mu \partial^\mu S\psi - mS\psi = 0 & \mu = 1,2,3 \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{ll} \gamma_\mu S = S\gamma_\mu & \mu = 0 \\ \gamma_\mu S = -S\gamma_\mu & \mu = 1,2,3 \end{array} \right\} \Rightarrow S = \eta_P \gamma_0 \quad \text{Note that } |S|^2 = 1!$$

$$V_i = \bar{\psi}\gamma_i\psi \xrightarrow{P} \psi^{\dagger'}\gamma_0\gamma_i\psi' = \psi^\dagger S^\dagger \gamma_0\gamma_i S\psi = \psi^\dagger\gamma_i\gamma_0\psi = -V_i$$

$$A_i = \bar{\psi}\gamma_i\gamma_5\psi \xrightarrow{P} \psi^{\dagger'}\gamma_0\gamma_i\gamma_5\psi' = \psi^\dagger S^\dagger \gamma_0\gamma_i\gamma_5 S\psi = -\psi^\dagger\gamma_i\gamma_0\gamma_5\psi = A_i$$

Chiral representation of Dirac matrices

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\psi = \begin{pmatrix} R \\ L \end{pmatrix} \Rightarrow \begin{cases} \psi_L = \frac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 \\ L \end{pmatrix} \\ \psi_R = \frac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} R \\ 0 \end{pmatrix} \end{cases}$$

Right-eyed flounder (both eyes are on the right side)

Left-eyed flounder (both eyes are on the left side)

Different taste!

CHARGE CONJUGATION (C)

The laws of physics do not change if particles are replaced by antiparticles.

Not always true

$$\psi \xrightarrow{C} \psi^C = \eta_C C \overline{\psi}^T = \eta_C C \gamma_0^T \psi^*$$

$$-C = C^{-1} = C^T = C^\dagger$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$C^2 = 1$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \xrightarrow{\text{Hermitian conjugate}} i\partial^\mu \overline{\psi}\gamma_\mu + m\overline{\psi} = 0$$

$$\xrightarrow{\text{transpose}} i\gamma_\mu^T \partial^\mu \overline{\psi}^T + m\overline{\psi}^T = 0 \Rightarrow -iC^{-1}\gamma_\mu C \partial^\mu \overline{\psi}^T + m\overline{\psi}^T = 0$$

$$\Rightarrow -i\gamma_\mu \partial^\mu C\overline{\psi}^T + mC\overline{\psi}^T = 0 \quad \text{or} \quad i\gamma_\mu \partial^\mu \psi^C - m\psi^C = 0$$

$$\psi^C = \eta_C C\overline{\psi}^T = \eta_C C\gamma_0^T \psi^*$$

Chiral representation of Dirac matrices

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}$$

$$\psi = \begin{pmatrix} R \\ L \end{pmatrix} \Rightarrow \begin{cases} \psi_L = \frac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 \\ L \end{pmatrix} \\ \psi_R = \frac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} R \\ 0 \end{pmatrix} \end{cases}$$

$$\psi^C = \eta_C C \gamma_0^T \psi^* = \begin{pmatrix} i\sigma_2 L^* \\ -i\sigma_2 R^* \end{pmatrix} \Rightarrow \begin{cases} (\psi_L)^C \text{ is right-handed} \\ (\psi_R)^C \text{ is left-handed} \end{cases}$$

TIME-REVERSAL (T)

The laws of physics are the same whether time is running forward or backward

Not always true

$$\mathbf{r} \xrightarrow{T} \mathbf{r}$$

$$\mathbf{p} \xrightarrow{T} -\mathbf{p}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \xrightarrow{T} -\mathbf{L}$$

$$\mathbf{S} \xrightarrow{T} -\mathbf{S}$$

$$\mathbf{E} \xrightarrow{T} \mathbf{E}$$

$$\mathbf{B} \xrightarrow{T} -\mathbf{B}$$

$$\left.\begin{array}{l} \mathbf{E}^2 - \mathbf{B}^2 \xrightarrow{T} \mathbf{E}^2 - \mathbf{B}^2 \\ \mathbf{E} \bullet \mathbf{B} \xrightarrow{T} -\mathbf{E} \bullet \mathbf{B} \end{array}\right\} \text{Lorentz invariants}$$

Dipole moments: $H_{md} = -\vec{\mu} \bullet \mathbf{B} \quad \vec{\mu} = \mu \mathbf{S} \qquad H_{ed} = -\mathbf{d} \bullet \mathbf{E} \quad \mathbf{d} = d\mathbf{S}$

$$-\vec{\mu} \bullet \mathbf{B} \xrightarrow{T} -\vec{\mu} \bullet \mathbf{B} \qquad -\mathbf{d} \bullet \mathbf{E} \xrightarrow{T} \mathbf{d} \bullet \mathbf{E}$$

Transformation of angular momentum states under time reversal

$$i\frac{\partial}{\partial t}\Psi = H\Psi \xrightarrow{\text{complex conjugate}} -i\frac{\partial}{\partial t}\Psi^* = H^*\Psi^*$$

Most of the time is H real (except when it contains σ_2).

Hence $\hat{T} = \hat{U}\hat{K}$ $\quad \hat{K}$: complex conjugation operator

$$\hat{T}\mathbf{J}\hat{T}^{-1} = -\mathbf{J} \Rightarrow \begin{cases} \hat{T}\mathbf{J}^2\hat{T}^{-1} = \mathbf{J}^2 \Rightarrow \hat{T}\left|j,m\right\rangle = \eta_{jm}\left|j,m'\right\rangle \\ \hat{T}J_z\hat{T}^{-1} = -J_z \Rightarrow m' = -m \\ \hat{T}\left(J_x \pm iJ_y\right)\hat{T}^{-1} = (-J_x) \pm (-i)(-J_y) = -J_\mp \end{cases}$$

$$\left.\begin{array}{c} \hat{T}J_\pm\hat{T}^{-1} = -J_\mp \\ J_\pm\left|j,m\right\rangle = \sqrt{j(j+1)-m(m\pm 1)}\left|j,m\pm 1\right\rangle \end{array}\right\} \Rightarrow \eta_{jm} \propto (-1)^m \quad \text{Choice: } \eta_{jm} = (-1)^{j+m}$$

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \xrightarrow{\text{complex conjugate}} -i\gamma_\mu^* \partial^\mu \psi^* - m\psi^* = 0$$

$$\left.\begin{array}{l} -iC\gamma_\mu^* \partial^\mu \psi^* - mC\psi^* = 0 \\ C\gamma_\mu^* = -\gamma_\mu^\dagger C = \begin{cases} -\gamma_\mu^\dagger C \text{ for } \mu=0 \\ \gamma_\mu^\dagger C \text{ for } \mu=1,2,3 \end{cases} \end{array}\right\} \Rightarrow i\gamma_0 \partial^0 C\psi^* + i\gamma_i \partial^i C\psi^* - mC\psi^* = 0$$

$$(i\gamma_0\partial^0 + i\gamma_i\partial^i - m)C\psi^*(\mathbf{r},t) \xrightarrow{t\to -t} (-i\gamma_\mu\partial^\mu - m)C\psi^*(\mathbf{r},-t) = 0$$

$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \Rightarrow (i\gamma_\mu\partial^\mu - m)\gamma_5 C\psi^*(\mathbf{r},-t) = 0$$

$$\Rightarrow \psi(\mathbf{r},t) \xrightarrow{t\to -t} \eta_T \gamma_5 C\psi^*(\mathbf{r},-t)$$

CPT Theorem

$$\psi(\mathbf{r},t) \xrightarrow{P} \gamma_0\psi(-\mathbf{r},t)$$

$$\xrightarrow{C} \gamma_0 C\gamma_0^T\psi^*(-\mathbf{r},t)$$

$$\xrightarrow{T} \gamma_5 \underbrace{C\gamma_0 C}_{\gamma_0^T}\gamma_0^T\psi(-\mathbf{r},-t) = \gamma_5\psi(-\mathbf{r},-t)$$

$$(i\gamma_\mu\partial^\mu - m)\gamma_5\psi(-\mathbf{r},-t) = 0 \Rightarrow (-i\gamma_\mu\partial^\mu - m)\gamma_5\psi(\mathbf{r},t) = 0$$

$$\Rightarrow \gamma_5(i\gamma_\mu\partial^\mu - m)\psi(\mathbf{r},t) = 0$$

Under rather general assumptions (locality, Lorentz invariance, etc.) physical laws remain the same under combined P, C, and T operations. The derivation above is an illustration of this theorem

The World according to Escher

P C T

start

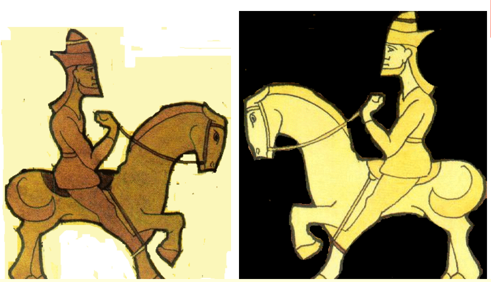
matter

anti-matter

identical to start

mirror image

time → ← time

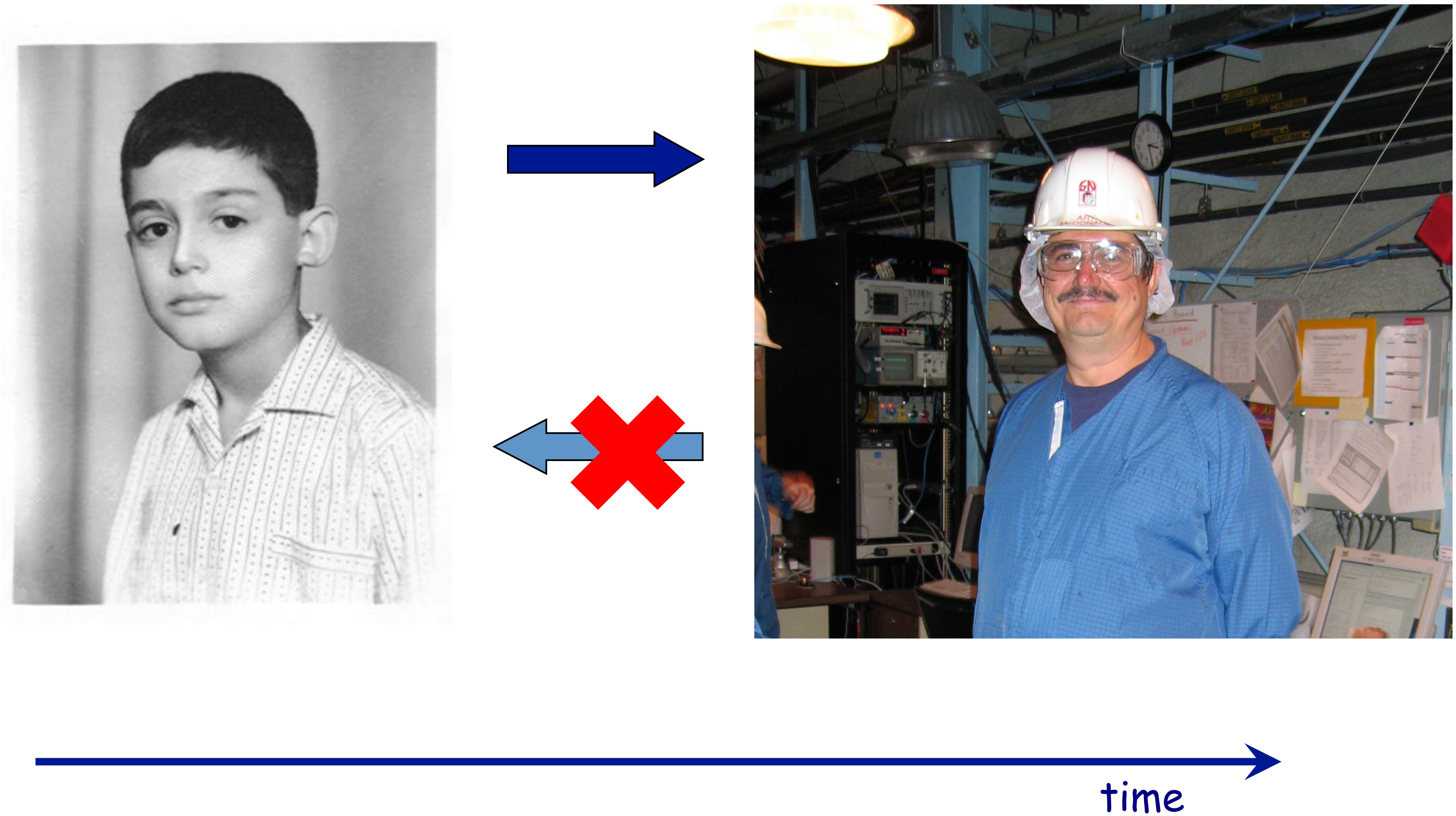


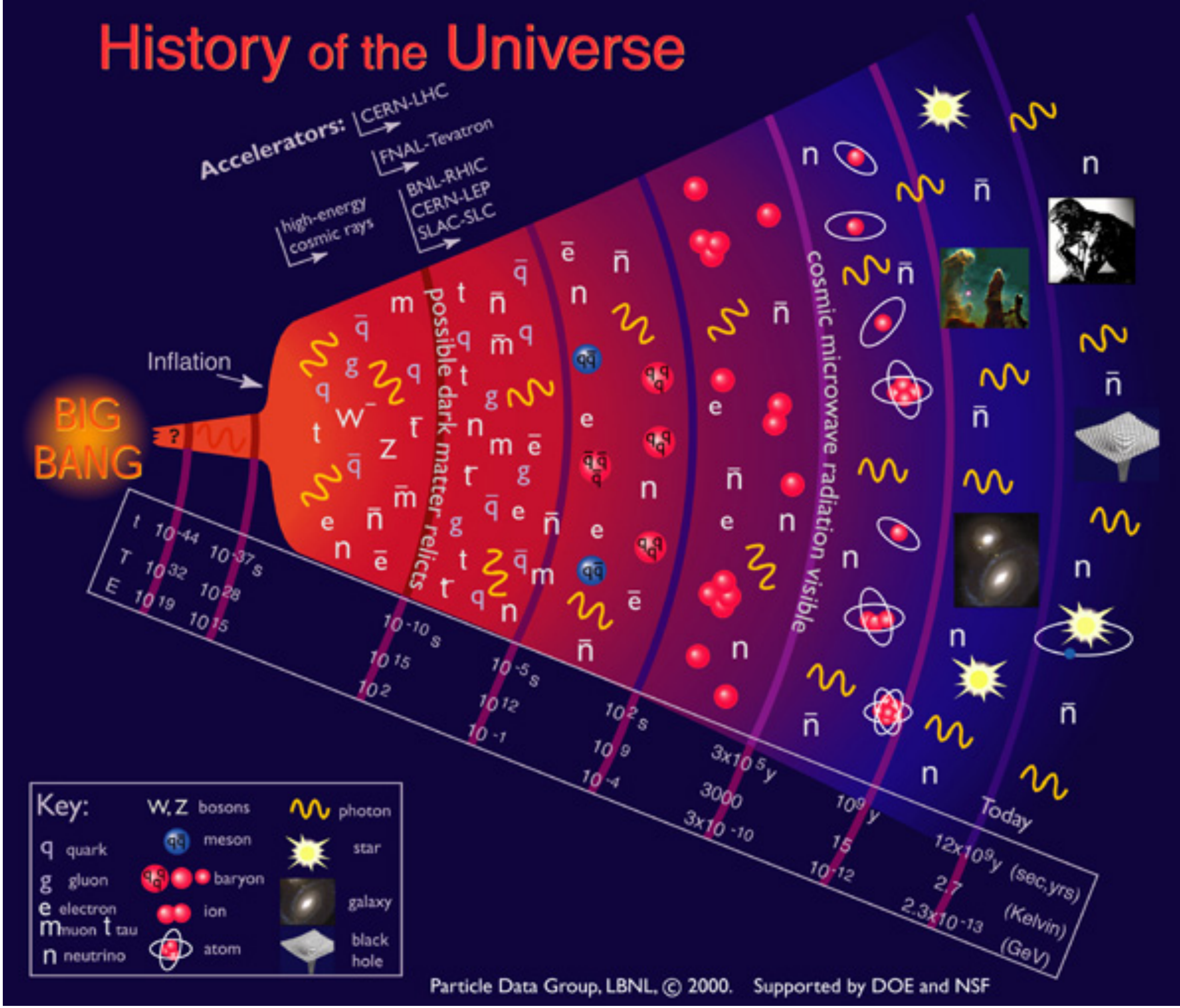
anti-particle e^+

particle e^-

Time Reversal Violation can be measured at low energies

From H.W. Wilschut via K. Jungmann


time



History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 10 32 10 28
E 10 19 10 15
10-10 s
10 15
10 2
10-5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

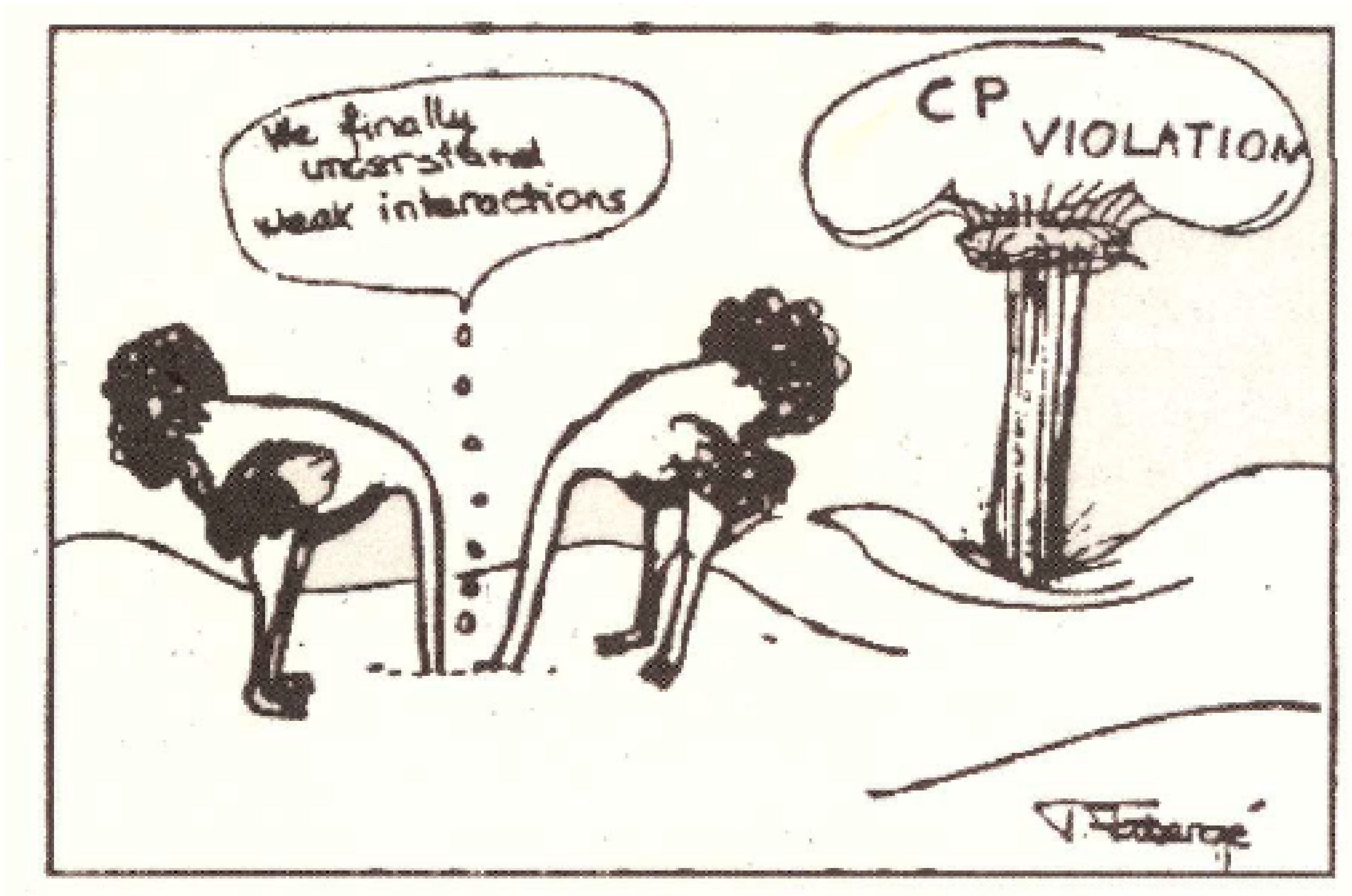
CP asymmetry at high temperatures was proposed by Okubo in 1958.

In 1967, Sakharov wrote:
"From S. Okubo's effect at high temperature a coat is tailored for the Universe to fit its skewed shape".

He proposed the conditions under which one can achieve matter-antimatter asymmetry via baryogenesis:

- C and CP-violation
- B violation
- Deviation from thermal equilibrium

In 1986 Fukugita & Yanagida explored conditions for leptogenesis. For example heavy partners of neutrinos in the see-saw mechanism must have been produced in the Early Universe. If their decays are CP-violating they may have given rise to particle-antiparticle asymmetry.



CP-violation is observed in neutral kaon decays, but what is seen there is not big enough to provide baryon-antibaryon asymmetry!

A direct search for CP-violation by neutrinos would search for
$P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$

What is a particle?

Wigner: A particle is an irreducible representation of the Poincare group.

Recall the quantities Lorentz transformations leave invariant:

$$t^2 - \mathbf{r}^2 = \tau^2$$

$$E^2 - \mathbf{p}^2 = m^2$$

$$\mathbf{E}^2 - \mathbf{B}^2$$

$$\mathbf{E} \cdot \mathbf{B}$$

What is a particle?

Wigner: A particle is an irreducible representation of the Poincare group.

Recall the quantities Lorentz transformations leave invariant:

$$t^2 - \mathbf{r}^2 = \tau^2$$

$$E^2 - \mathbf{p}^2 = m^2$$

$$\mathbf{E}^2 - \mathbf{B}^2$$

$$\mathbf{E} \cdot \mathbf{B}$$

What is a particle?

Wigner: A particle is an irreducible representation of the Poincare group.

Recall the quantities Lorentz transformations leave invariant:

$$t^2 - \mathbf{r}^2 = \tau^2$$

$$E^2 - \mathbf{p}^2 = m^2$$

$$\mathbf{E}^2 - \mathbf{B}^2$$

$$\mathbf{E} \cdot \mathbf{B}$$

Poincare group is the group including Lorentz boosts, translations and rotations.

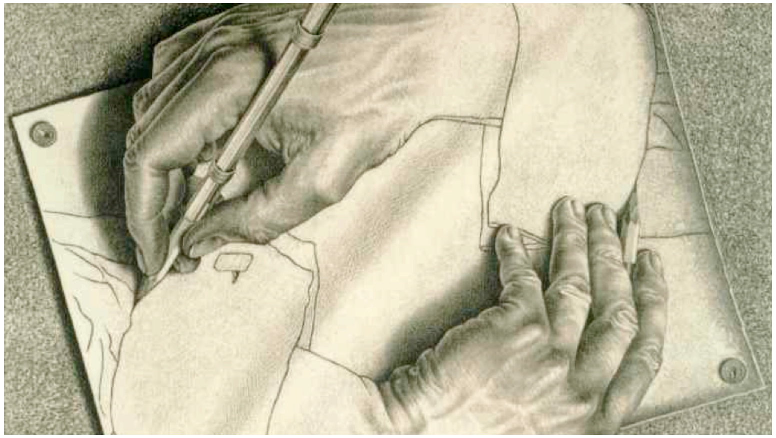
Next let us explore the concept of mass

What is mass?

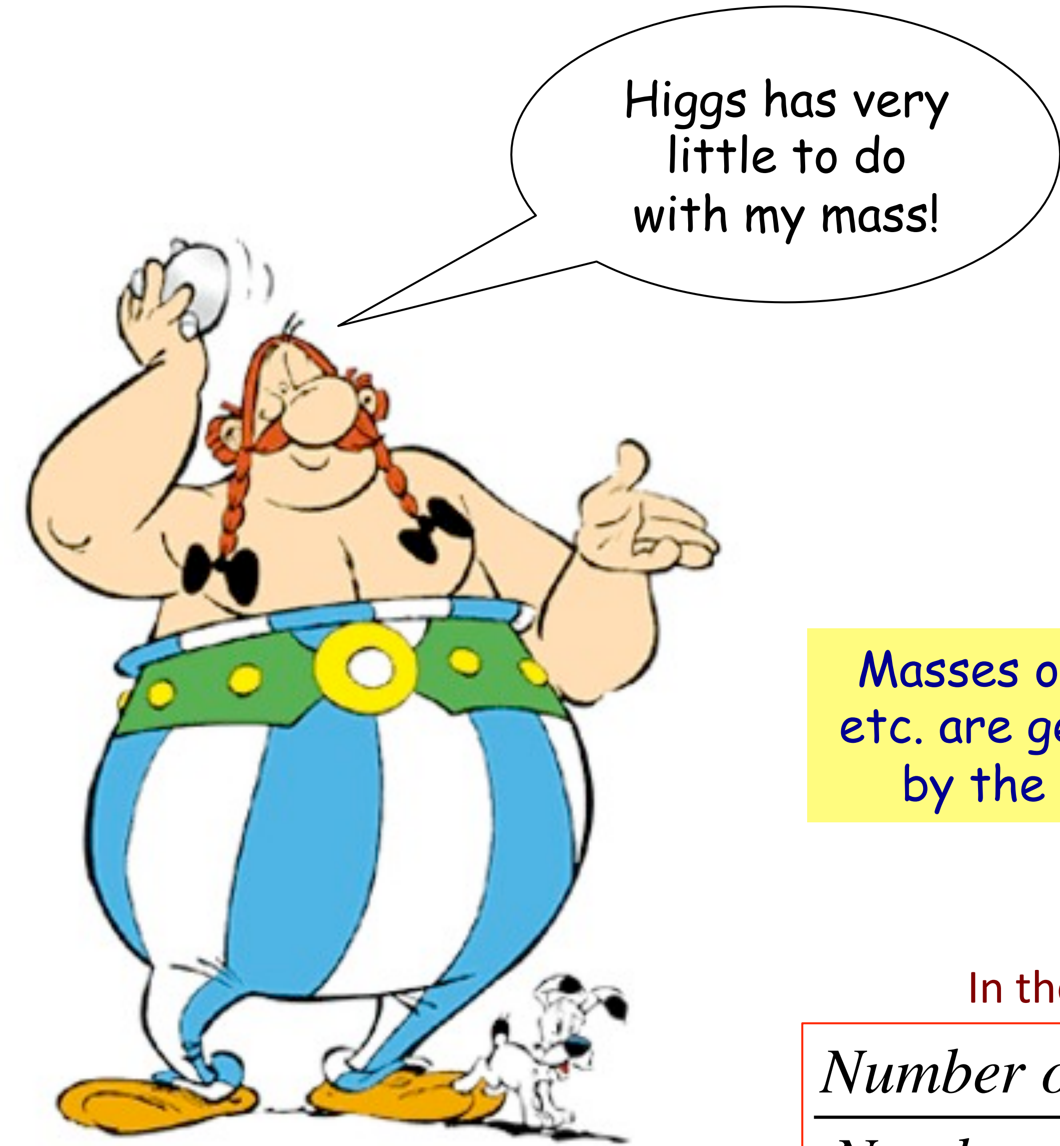
$$\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi$$

$$\Psi_R = \frac{1}{2}(1+\gamma_5)\Psi$$

$$L = m\bar{\Psi}\Psi = m\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_l\right)$$



In the Standard Model all elementary masses possibly except those for neutrinos are generated by the Yukawa couplings of the Higgs.



Masses of protons, neutrons, etc. are generated dynamically by the QCD interactions!

In the Early Universe

$$\frac{Number\ of\ neutrons}{Number\ of\ protons} = \frac{e^{-m_n/T}}{e^{-m_p/T}}$$

Chiral representation of Dirac matrices

$$\gamma^0 = \beta = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Helicity and chirality

Chirality: γ_5, Helicity: $\dfrac{\vec{\Sigma}\cdot\vec{k}}{\left|\vec{k}\right|}$, $\left[\gamma_5, \dfrac{\vec{\Sigma}\cdot\vec{k}}{\left|\vec{k}\right|}\right] = 0$

$$\gamma_5 \left|\lambda,\chi\right\rangle = \lambda\left|\lambda,\chi\right\rangle$$

$$\frac{\vec{\Sigma}\cdot\vec{k}}{\left|\vec{k}\right|}\left|\lambda,\chi\right\rangle = \chi\left|\lambda,\chi\right\rangle$$

$$\lambda = \pm 1, \quad \chi = \pm 1$$

These operators act on the fermion fields:

$$\Psi_s(\vec{r}) = \sum_{\vec{k}} \left\langle \vec{r}\,\middle|\,\vec{k}\right\rangle a_s(\vec{k}), \quad s = 1,2,3,4$$

$$b^\dagger\left(\vec{k},\chi\right)=\sum_s a_s^\dagger\left(\vec{k}\right)\left\langle s\middle|\chi,\chi\right\rangle$$

$$d\left(-\vec{k},\chi\right)=\sum_s a_s^\dagger\left(\vec{k}\right)\left\langle s\middle|-\chi,\chi\right\rangle$$

Free, massless particle Hamiltonian:

$$H=\int d^3\vec{r}\ \Psi^\dagger(\vec{r})\vec{\alpha}\cdot\hat{p}\Psi(\vec{r})$$

$$=\sum_{\vec{k},\chi}\left|\vec{k}\right|\left[b^\dagger\left(\vec{k},\chi\right)b\left(\vec{k},\chi\right)-d\left(-\vec{k},\chi\right)d^\dagger\left(-\vec{k},\chi\right)\right]$$

"Dirac" mass term:

$$m_D\int d^3\vec{r}\ \Psi^\dagger(\vec{r})\beta\Psi(\vec{r})$$

$$=-\sum_{\vec{k},\chi}m_D\left[b^\dagger\left(\vec{k},\chi\right)d^\dagger\left(-\vec{k},-\chi\right)-d\left(-\vec{k},-\chi\right)b\left(\vec{k},\chi\right)\right]$$

Hence the total Hamiltonian is

$$H = \sum_{\vec{k},\chi} \left\{ \left|\vec{k}\right| \left[b^{\dagger}_{k\chi} b_{k\chi} - d_{-k\chi} d^{\dagger}_{-k\chi} \right] - m_D \left[b^{\dagger}_{k\chi} d^{\dagger}_{-k\chi} - d_{-k\chi} b_{k\chi} \right] \right\}$$

This Hamiltonian can be diagonalized by the transformation

$$\begin{pmatrix} B_{k\chi} \\ D^{\dagger}_{-k\chi} \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} b_{k\chi} \\ d^{\dagger}_{-k\chi} \end{pmatrix}$$

$$H = \sum_{k\chi} \sqrt{\vec{k}^{\,2} + m_D^2} \left(B^{\dagger}_{k\chi} B_{k\chi} - D_{-k\chi} D^{\dagger}_{-k\chi} \right)$$

$$\cos 2\vartheta = \frac{\left|\vec{k}\right|}{\sqrt{\vec{k}^{\,2} + m_D^2}} \qquad \sin 2\vartheta = \frac{m_D}{\sqrt{\vec{k}^{\,2} + m_D^2}}$$

Majorana mass term:

$$m_M\left(\left(\Psi_L\right)^C\right)^{\dagger}\beta\,\Psi_L$$

• Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.

- Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W \times U(1)$.

- In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet (I_W =1/2) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet (I_W=0).

SU(2) × U(1) Standard Model

Weak isospin

$$SU(2)_W: \quad I^{(W)} = \frac{1}{2} \Rightarrow \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{matrix} I_3^{(W)} = +\frac{1}{2} \\ I_3^{(W)} = -\frac{1}{2} \end{matrix}$$

Weak singlets

$$I^{(W)} = 0: \quad \nu_R, \, e_R$$

Higgs Field sits in a weak doublet with $I_3^{(W)} = -\frac{1}{2}$.

• Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W \times U(1)$.

• In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet (I_W =1/2) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet (I_W=0).

• A mass term connects left- and right-handed components. The usual Dirac mass term is $L = m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$. But such a neutrino mass term requires a right-handed neutrino, hence it is not in the Standard Model.

• The right-handed component of the neutrino carries no weak isospin quantum numbers.

A very brief introduction to the effective field theories

A note on dimensional counting

- Lagrangian, L, has dimensions of energy (or mass).
- $L = \int d^3x\, \mathcal{L} \Rightarrow$ Lagrangian density, $\mathcal{L}$, has dimensions of energy/volume or M^4.
- Define the scaling dimension of x, [x] to be -1 $\Rightarrow$ scaling dimension of momentum (or mass) is [m] = +1 (recall that (p.x/h) is dimensionless and we take [h]=0).
- Clearly $[\mathcal{L}] = 4$. This should be true for any Lagrangian density of any theory.
- Consider the mass term for fermions, $\mathcal{L}_m = m\,\bar{\Psi}\Psi$. Then $[\bar{\Psi}\Psi] = 3$ or $[\Psi] = 3/2$.
- In the Standard Model the Higgs field vacuum expectation value gives the particle mass: $\mathcal{L} = H\,\bar{\Psi}\Psi$. Hence [H] = 1.

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \underbrace{\text{terms which are higher order in fields}}_{\text{Consistent with the symmetries of the system}}$$

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E}\bullet\mathbf{B}\right)^2}_{\text{mass dimension 8}} + c_3 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)\left(\mathbf{E}\bullet\mathbf{B}\right)}_{\text{mass dimension 8}} \right]$$

These operators have mass dimension 8, so there should be a power of 4 here

Symmetries of the Electromagnetism

Lorentz Invariants: $\mathbf{E}^2 - \mathbf{B}^2$ and $\mathbf{E} \cdot \mathbf{B}$

$$\underbrace{\begin{aligned} \mathbf{E} &\to \mathbf{E} \\ \mathbf{B} &\to -\mathbf{B} \\ \mathbf{E}^2 - \mathbf{B}^2 &\to \mathbf{E}^2 - \mathbf{B}^2 \\ \mathbf{E} \cdot \mathbf{B} &\to -\mathbf{E} \cdot \mathbf{B} \end{aligned}}_{Under\ time-reversal}$$

Since we want the Lagrangian density to be *invariant* under *both* Lorentz *and* time-reversal transformations we pick $\mathbf{E}^2 - \mathbf{B}^2$.

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E}\bullet\mathbf{B}\right)^2}_{\text{mass dimension 8}} + c_3 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)\left(\mathbf{E}\bullet\mathbf{B}\right)}_{\text{mass dimension 8}} \right]$$

Not good
(not T-
invariant)

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{g^4}{\Lambda^4}\left[c_1 \underbrace{\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2}_{\text{mass dimension 8}} + c_2 \underbrace{\left(\mathbf{E} \bullet \mathbf{B}\right)^2}_{\text{mass dimension 8}} \right]$$

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}\,(\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45 m_e^4}\left[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7\,(\mathbf{E}\cdot\mathbf{B})^2\right]$$

Another example: Low-energy limit of weak interactions

$$\mathcal{L} = \Bigg(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \Bigg)$$

Another example: Low-energy limit of weak interactions

$$L = \left(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \right)$$

$$L = \frac{g^2}{\Lambda^2} \left(\underbrace{\bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L}_{\text{mass dimension 6}} \right)$$

$$G_F = \frac{\sqrt{2} g^2}{8 M_W^2}$$

Using the Standard Model degrees of freedom one can parameterize the neutrino mass by a dimension 5 operator. (Recall that $I_3^W = 1/2$ for the ν_L and -1/2 for H_{SM}).

$$L = X_{\alpha\beta} H_{SM} H_{SM} \overline{\nu_{L\alpha}^{C}} \nu_{L\beta} / \Lambda$$

$$v^2 X_{\alpha\beta} / \Lambda = U m_\nu^{diagonal} U^T$$

This term is not renormalizable! It is the only dimension-five operator one can write using the Standard Model degrees of freedom. Hence the neutrino mass is the most accessible new physics beyond the Standard Model!

There are other ways to obtain neutrino mass:

$$L = H_{I=1}\,\overline{\nu_{L\alpha}^{\;c}}\,\nu_{L\beta}$$

Note: This Higgs is not in the Standard Model!

At lower energies, Beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

Majorana neutrino mass (unique)

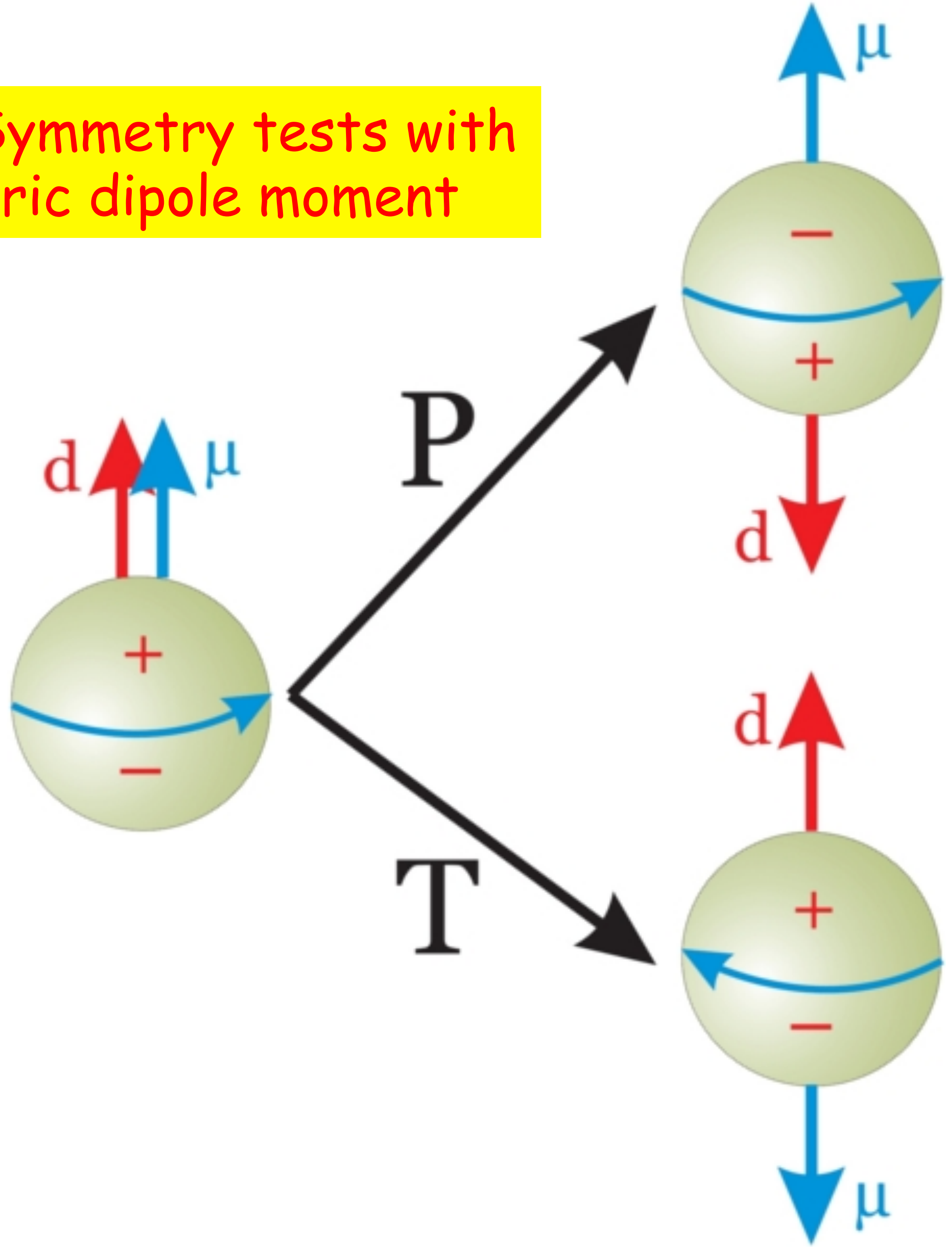
Includes Majorana neutrino magnetic moment

Majorana mass term:

$$m_M\left(\left(\Psi_L\right)^C\right)^{\dagger}\beta\,\Psi_L$$

- Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.
- It is permitted by the weak-isospin invariance of the Standard Model.
- Neutrino mass terms are not included in the fundamental Lagrangian of the Standard Model. They arise from new physics. Of course it is possible to write down an *effective* Lagrangian for the neutrino mass in terms of only the Standard Model fields if you give up renormalizability.

Fundamental Symmetry tests with nuclei: Electric dipole moment



Electric dipole operator: $\mathbf{d} = \sum_i q_i \mathbf{r}_i$ (vector operator)

$$\left.\begin{aligned} e^{i\pi J_y} J_z e^{-i\pi J_y} &= -J_z \\ e^{i\pi J_y} d_z e^{-i\pi J_y} &= -d_z \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} \langle j,m|d_z|j,m\rangle &= \langle j,m|e^{i\pi J_y} e^{-i\pi J_y} d_z e^{i\pi J_y} e^{-i\pi J_y}|j,m\rangle = \langle j,-m|(-d_z)|j,-m\rangle \\ \langle j,m|d_z|j,m\rangle &= \langle j,m|\hat{T}^{-1}\hat{T} d_z \hat{T}^{-1}\hat{T}|j,m\rangle = \langle j,-m|\left(\hat{T} d_z \hat{T}^{-1}\right)|j,-m\rangle \end{aligned}\right\}$$

If $\left(\hat{T} d_z \hat{T}^{-1}\right) = d_z$ then $\langle j,m|d_z|j,m\rangle = 0$

Observation of a non-zero value of the dipole operator requires CP violation!

In effective field theories at lower energies, beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

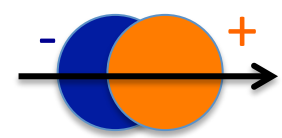
Majorana neutrino mass (unique)

$$+$$

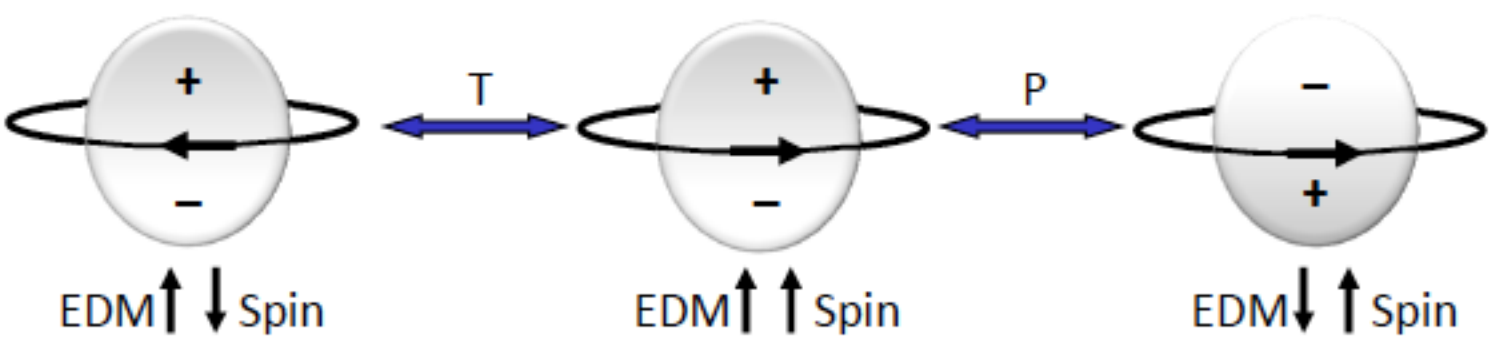
$$-\theta \frac{g_s^2}{32\pi^2} \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Electric dipole moment

$$H \approx -d\, \mathbf{J} \cdot \mathbf{E}$$



$$d_i \propto \frac{m_i}{\Lambda^2} \sin \phi_{CP}$$



Schiff's theorem

Consider a system of non-relativistic particles which possess electric dipole moments and held together by Coulomb forces:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + \sum_i V_{\text{Coulomb}}(\mathbf{r}_i) - \sum_i \mathbf{d}_i \bullet \mathbf{E}_{\text{Coulomb}} = H_0 - \sum_i \mathbf{d}_i \bullet \mathbf{E}_{\text{Coulomb}}$$

$$\mathbf{d}_i \bullet \mathbf{E}_{\text{Coulomb}} = -\frac{1}{q_i}\mathbf{d}_i \bullet \nabla V_{\text{Coulomb}}(\mathbf{r}_i) = -\frac{i}{q_i}\left[\mathbf{d}_i \bullet \mathbf{p}_i, H_0\right]$$

$$\Rightarrow H = H_0 + i\sum_i \frac{1}{q_i}\left[\mathbf{d}_i \bullet \mathbf{p}_i, H_0\right]$$

Assuming the dipole moment is small treat the second term in perturbation theory. The shift in the ground state of H_0 is

$$|0\rangle \rightarrow |0\rangle + \sum_n |n\rangle \frac{\langle n| i\sum_i \frac{1}{q_i}\left[\mathbf{d}_i \bullet \mathbf{p}_i, H_0\right]|0\rangle}{E_0 - E_n} = \left(1 + i\sum_i \frac{1}{q_i}\mathbf{d}_i \bullet \mathbf{p}_i\right)|0\rangle = |0\rangle_S$$

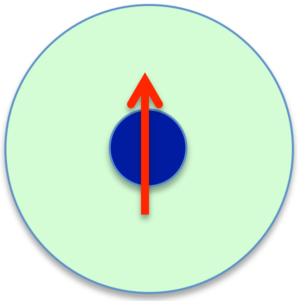
Schiff's theorem

Perturbed ground state : $|0\rangle_S = \left(1 + i\sum_i \frac{1}{q_i}\mathbf{d}_i \bullet \mathbf{p}_i\right)|0\rangle$

Induced electric moment : ${}_S\langle 0|\sum_i q_i\mathbf{r}_i|0\rangle_s$

$${}_S\langle 0|\sum_i q_i\mathbf{r}_i|0\rangle_s = i\langle 0|\left[\sum_q q_i\mathbf{r}_i, \sum_j \frac{1}{q_j}\mathbf{d}_j \bullet \mathbf{p}_j\right]|0\rangle = -\langle 0|\sum_i \mathbf{d}_i|0\rangle$$

Nuclear EDM's and the Schiff moment



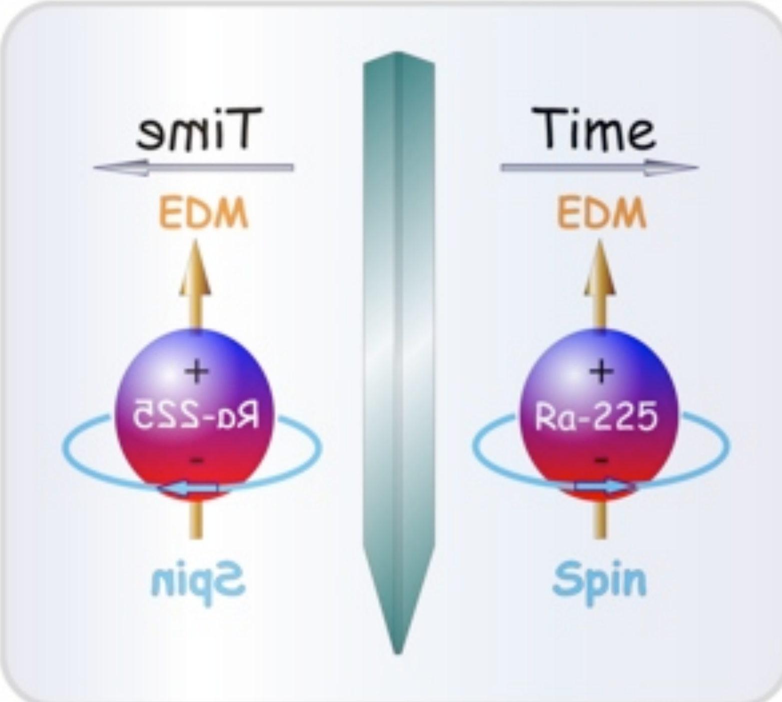
Schiff shielding: $d_{\text{atom}} = d_{\text{electronic}} + d_{\text{nuclei}} \cong 0$

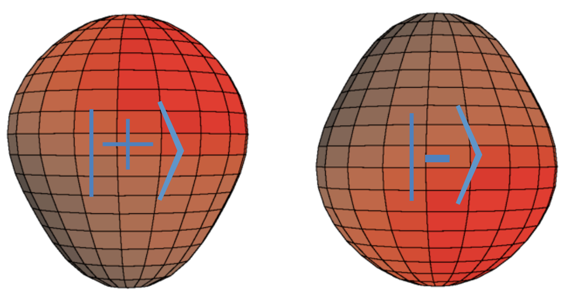
However, since the nuclear charge distribution is not isotropic, one obtains the **Schiff moment**:

$$\mathbf{S} = \frac{1}{10}\sum_i e_i\left(r_i^2 - \frac{5}{3}\langle r^2\rangle_{\text{ch}}\right)\mathbf{r}_i \times \begin{cases} 1 & \text{isoscalar} \\ \tau_{zi} & \text{isovector} \end{cases}$$

Note that, since this is a difference of two large quantities, it involves many subtle nuclear physics issues, such as identifying contributions of single-particle states, low-lying dipole resonances…

Auerbach, Dobaczewski, Engel, Flambaum, Haxton, Kriplovich, Ramsey-Musolf, Shlomo, Zelevinsky……





$$\Psi^- = (|+\rangle - |-\rangle)/\sqrt{2}$$

ΔE

$$\Psi^+ = (|+\rangle + |-\rangle)/\sqrt{2}$$

Enhancement of EDM from octupole deformation

No spin-correlation suggests

$$\langle \Psi^+ | \mathbf{d}_{internal} | \Psi^+ \rangle = 0$$

A interaction which is T- and P- odd would mix the states

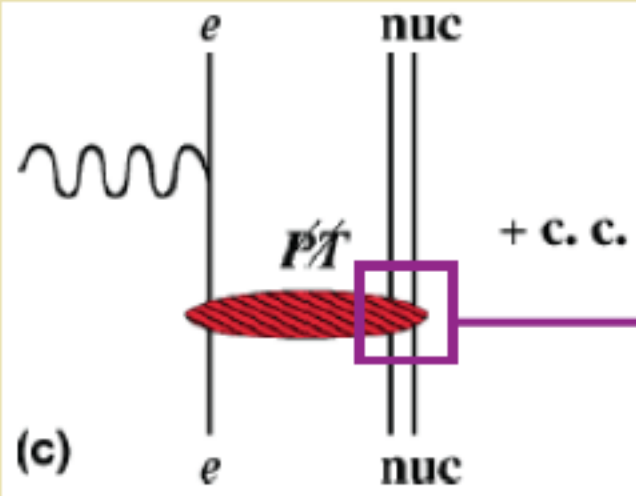
$$\Psi = \Psi^+ + \alpha \Psi^-$$

$$\alpha = \frac{\langle \Psi^+ | \mathbf{d}_{internal} | \Psi^- \rangle}{\Delta E}$$

$$\langle d_z \rangle_{lab} = 2\alpha d_{internal} \frac{I}{I+1}$$

Haxton & Henley; Auerbach, Flambaum & Spevak; Dobaczewski & Engel

Ramsey-Musolf



Schiff moment, MQM,…

Nuclear Schiff Moment

$$S \sim \int d^3x\, x^2\, \vec{x}\, \rho(\vec{x})^{\mathrm{CPV}}$$

$(R_N / R_A)^2$ suppression

Parameterize Schiff moment as

$$S = \frac{2 m_N g_A}{F_\pi}\left(a_0 \bar{g}_\pi^{(0)} + a_1 \bar{g}_\pi^{(1)} + a_2 \bar{g}_\pi^{(2)}\right)$$

$\bar{g}_\pi^{(0)}$: isoscalar

$\bar{g}_\pi^{(1)}$: isovector

$\bar{g}_\pi^{(2)}$: isotensor

Nuclear Matrix Elements

$$S = a_0\, g\, \bar{g}_\pi^{(0)} + a_1\, g\, \bar{g}_\pi^{(1)} + a_2\, g\, \bar{g}_\pi^{(2)}$$

Nucl.	Best value		
	a_0	a_1	a_2
^{199}Hg	0.01	±0.02	0.02
^{129}Xe	−0.008	−0.006	−0.009
^{225}Ra	−1.5	6.0	−4.0

Range		
a_0	a_1	a_2
0.005–0.05	−0.03–(+0.09)	0.01–0.06
−0.005–(−0.05)	−0.003–(−0.05)	−0.005–(−0.1)
−1–(−6)	4–24	−3–(−15)

Engel, R-M, van Kolck '13

Neutrino mixing

$$\left|\nu_{flavor}\right\rangle = T\left|\nu_{mass}\right\rangle$$

$$T = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\textit{atmospheric neutrinos}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\textit{reactor neutrinos}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\textit{solar neutrinos}}$$

$$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2(\Delta_{31}L) + \sin^2\theta_{12}\sin^2(\Delta_{32}L)\right] - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2(\Delta_{21}L)$$

$$\Delta_{ij} = \frac{\delta m_{ij}^2}{4E_\nu} = \frac{m_i^2 - m_j^2}{4E_\nu}, \qquad \Delta_{32} = \Delta_{31} - \Delta_{21}$$

The MSW Effect

In vacuum: $E^2 = \mathbf{p}^2 + m^2$

In matter:

$(E - V)^2 = (\mathbf{p} - \mathbf{A})^2 + m^2$

$\Rightarrow E^2 = \mathbf{p}^2 + m^2_{\text{eff}}$

V $\propto$ background density

$\mathbf{A} \propto \mathbf{J}_{\text{background}}$ (currents) or

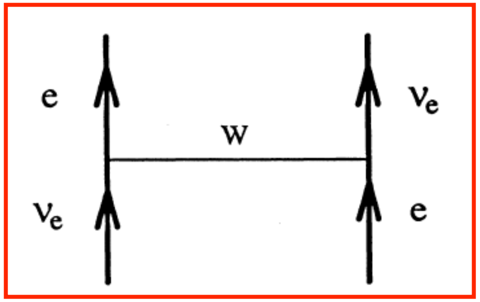
$\mathbf{A} \propto \mathbf{S}_{\text{background}}$ (spin)

In the limit of static, charge-neutral, and unpolarized background

V$\propto N_e$ and $\mathbf{A} = 0$

$\Rightarrow m^2_{\text{eff}} = m^2 + 2EV + \mathcal{O}(V^2)$

The potential is provided by the coherent forward scattering of ν_e's off the electrons in dense matter



There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute to phase differences *unless* we invoke a sterile neutrino.

Note that matter effects induce an effective CP-violation since the matter in the Earth and the stars is not CP-symmetric!

Matter effects

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix} = \left[T\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}T^\dagger + \begin{pmatrix}V_c+V_n & 0 & 0\\ 0 & V_n & 0\\ 0 & 0 & V_n\end{pmatrix}\right]\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}$$

$$V_c = \sqrt{2}G_F N_e$$

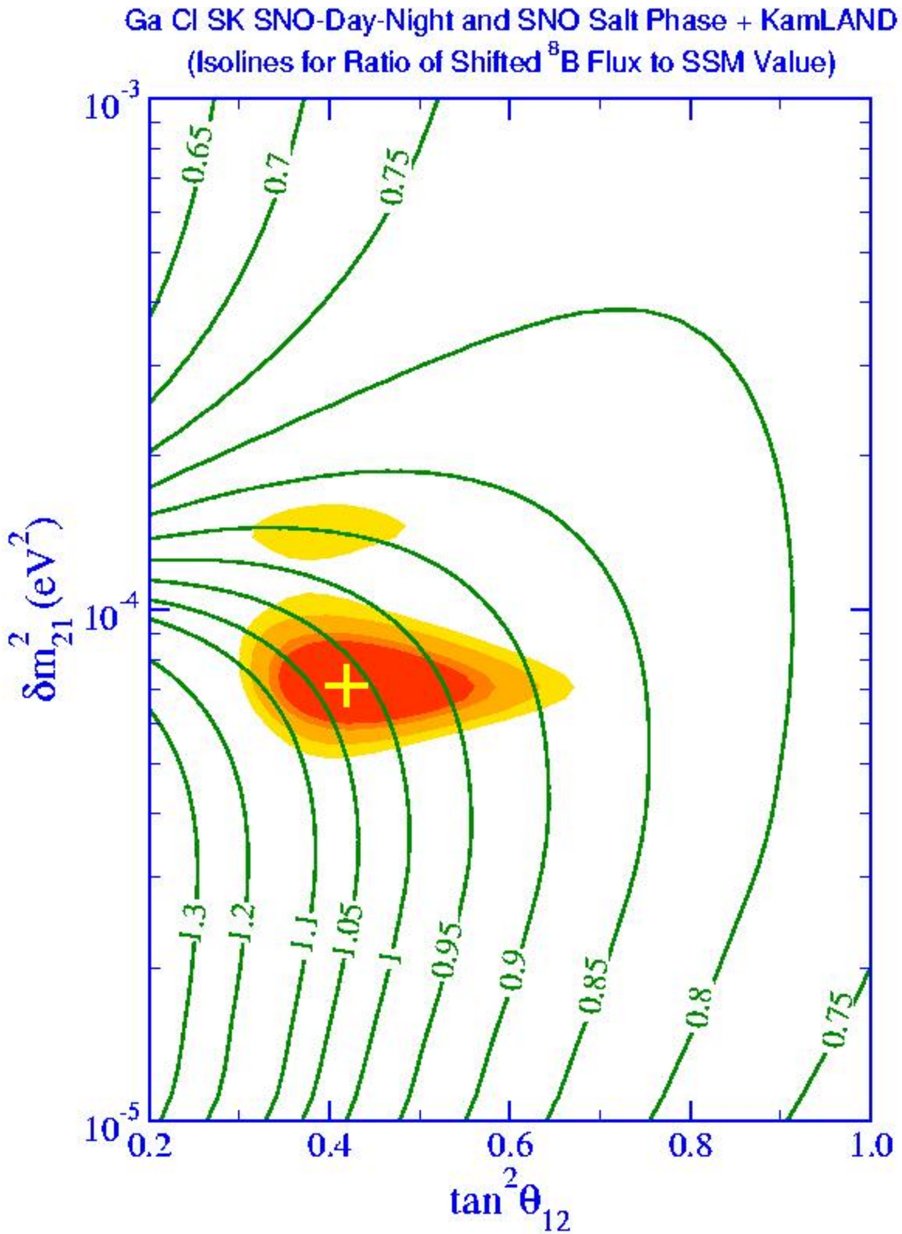
$$V_n = -\frac{1}{\sqrt{2}}G_F N_n$$

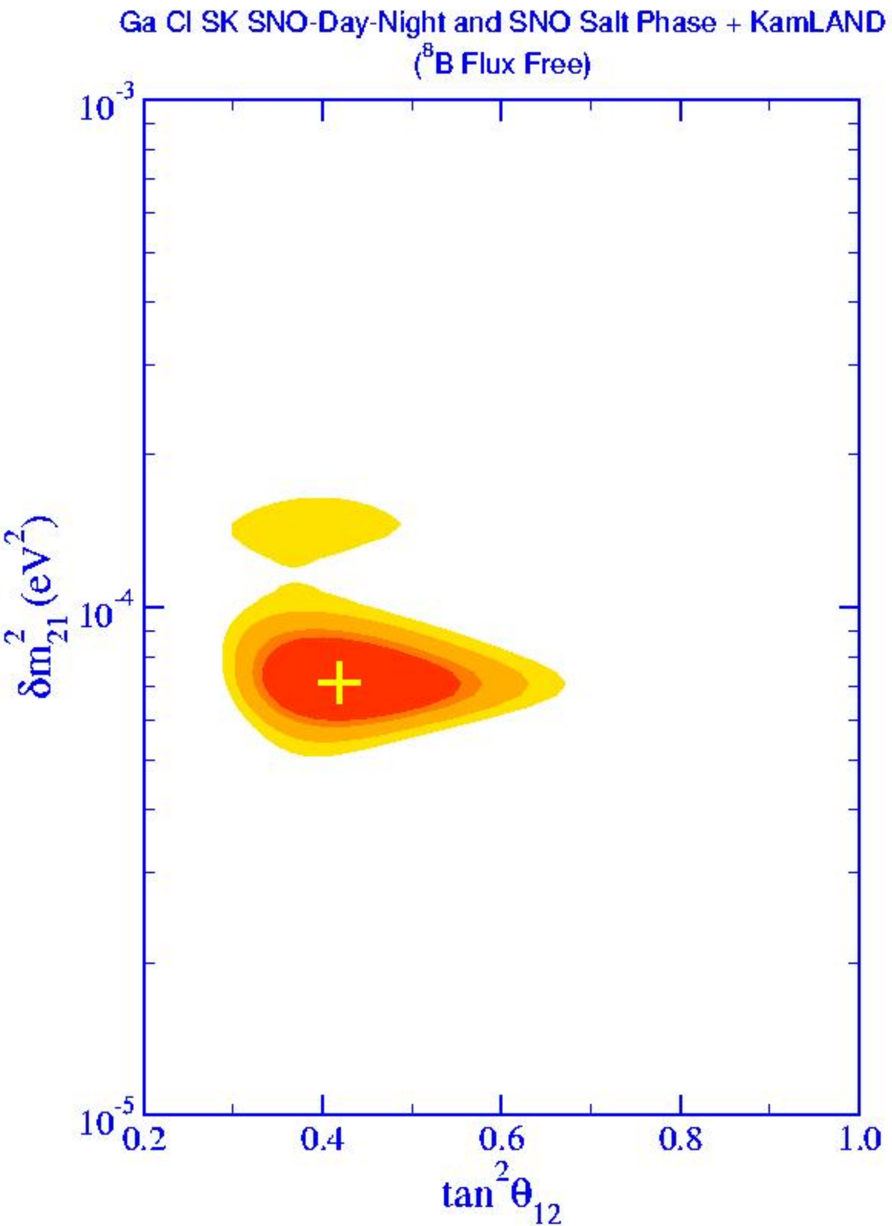
Two-flavor limit

$$i\frac{\partial}{\partial t}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix} = \begin{pmatrix}\varphi & \frac{\delta m^2}{4E}\sin 2\theta\\ \frac{\delta m^2}{4E}\sin 2\theta & -\varphi\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix}$$

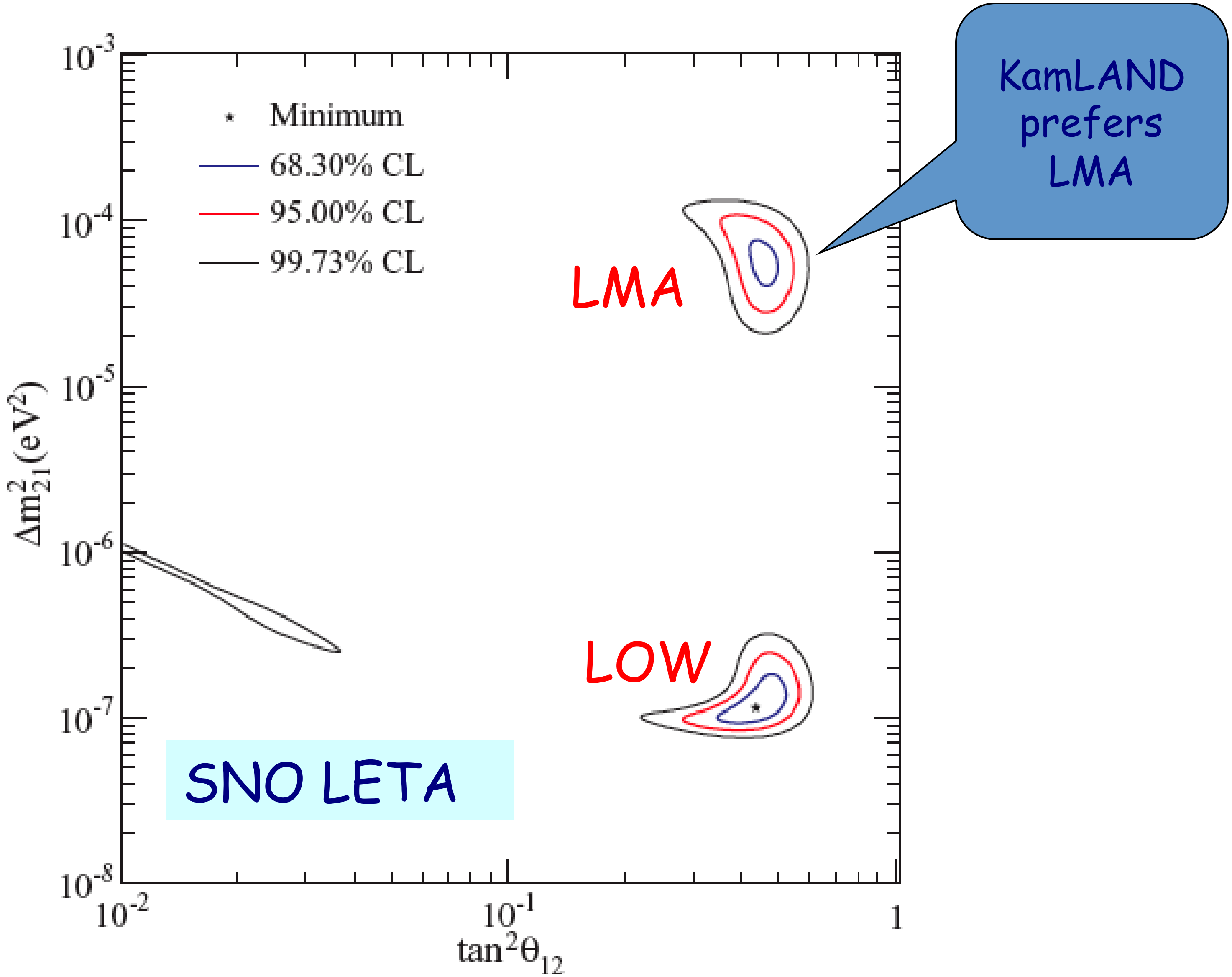
$$\varphi = -\frac{\delta m^2}{4E}\cos 2\theta + \frac{1}{\sqrt{2}}G_F N_e$$

Already first SNO neutral current (salt) results could be analyzed without referring to the Standard Solar Model, A.B.B. & Yuksel, PRD 68, 113002 (2003)

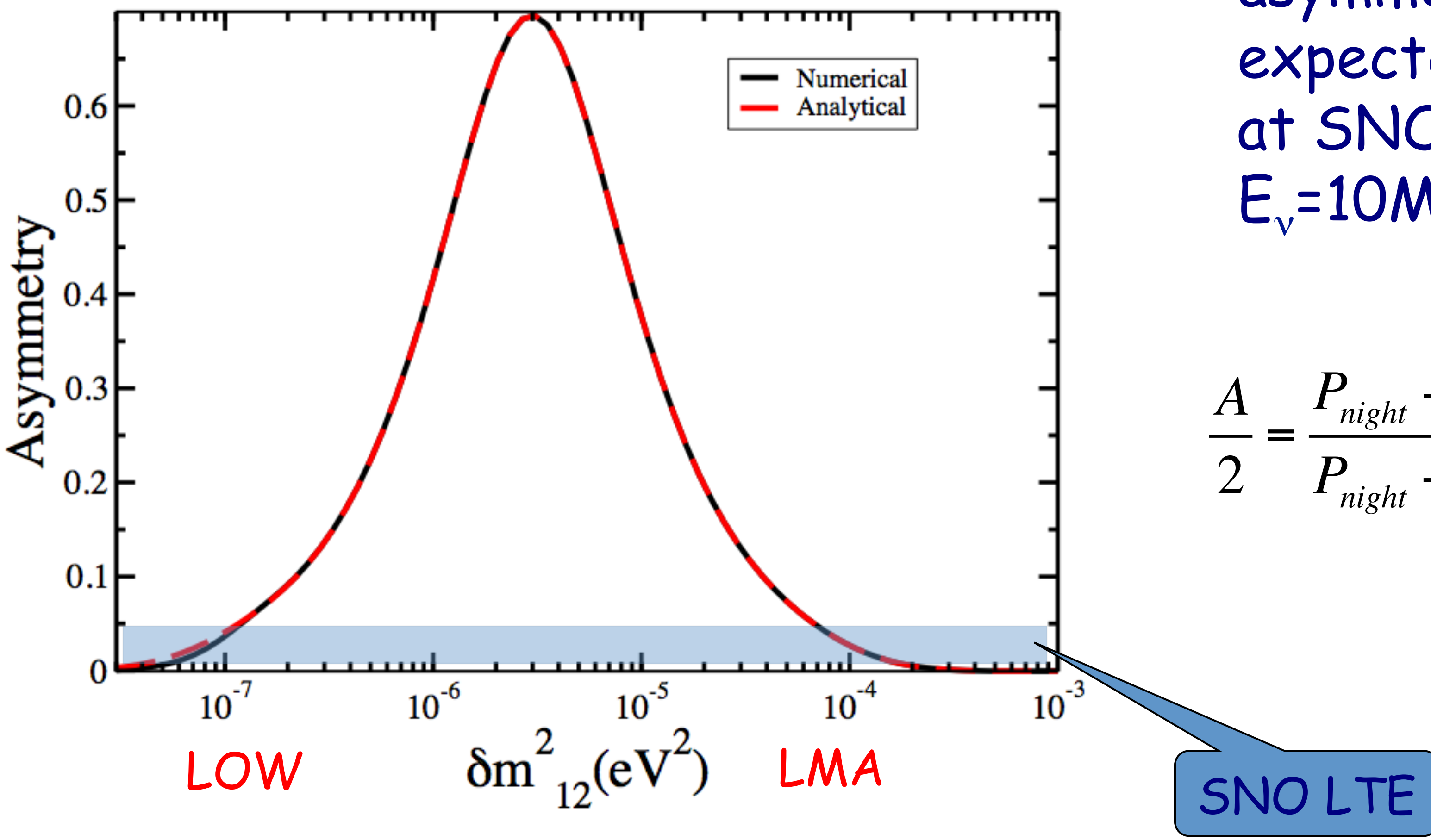




Do antineutrinos mix the same way neutrinos do?

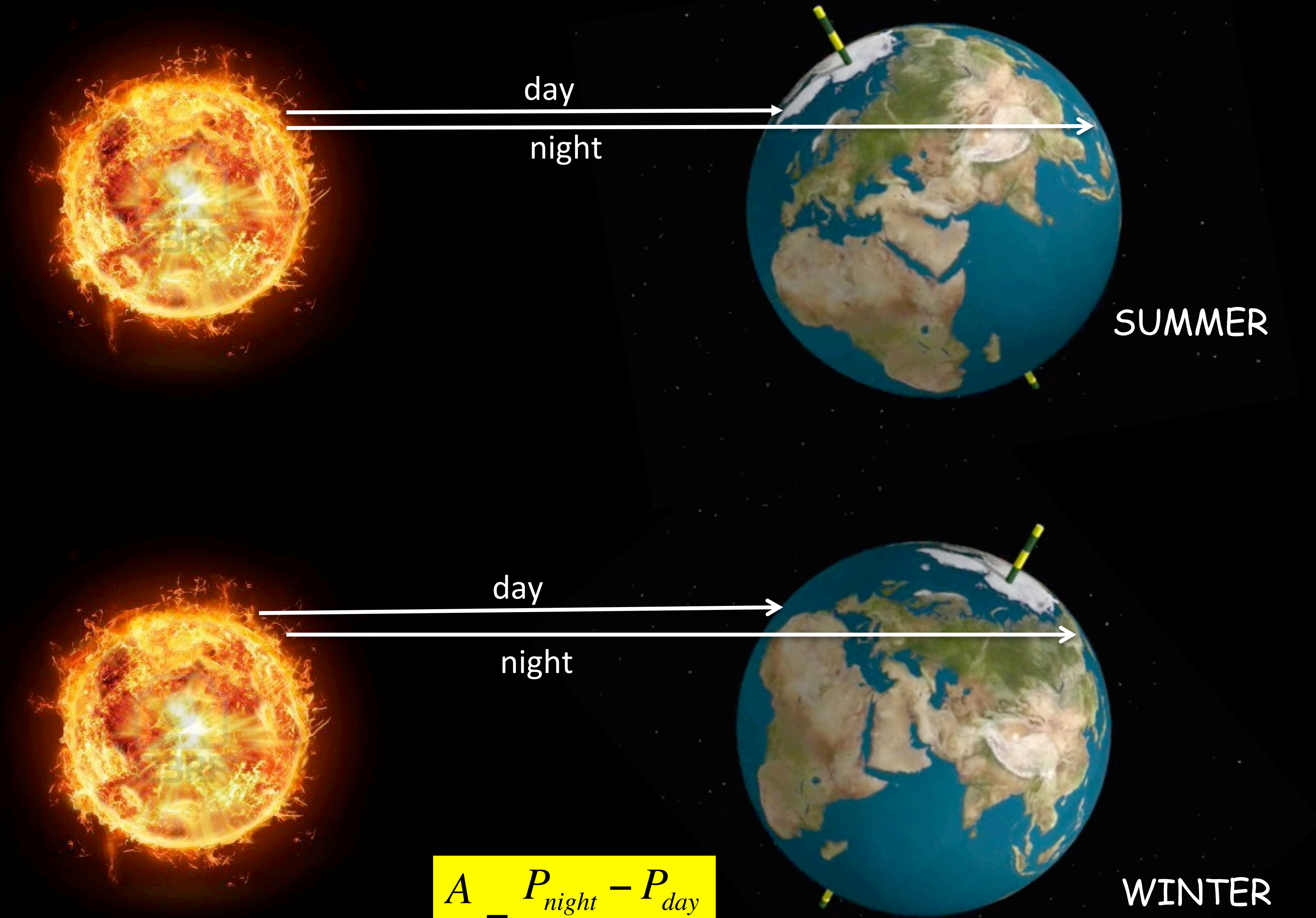


Day-night asymmetry expected at SNO for E_ν=10MeV



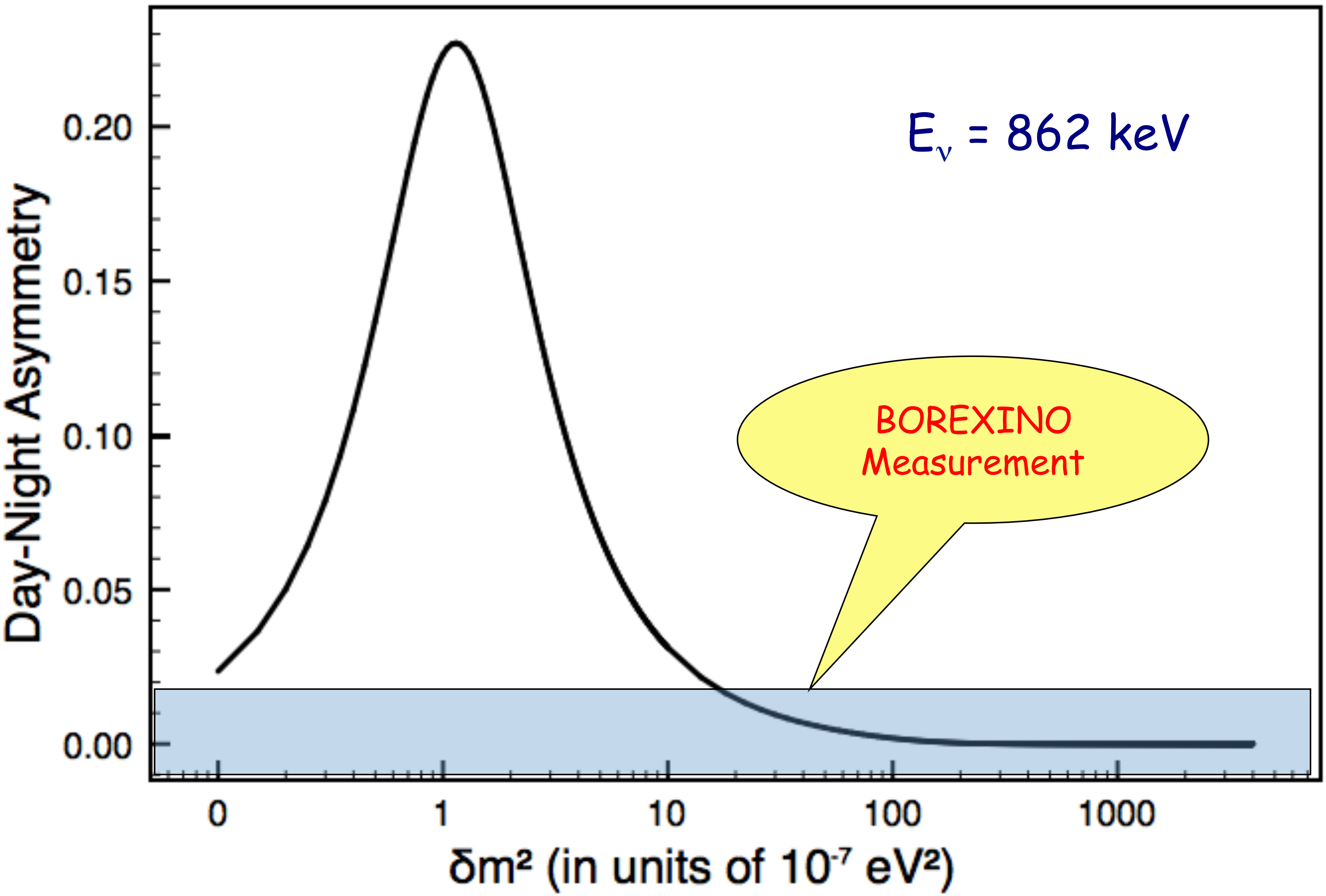
$$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$$

Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.



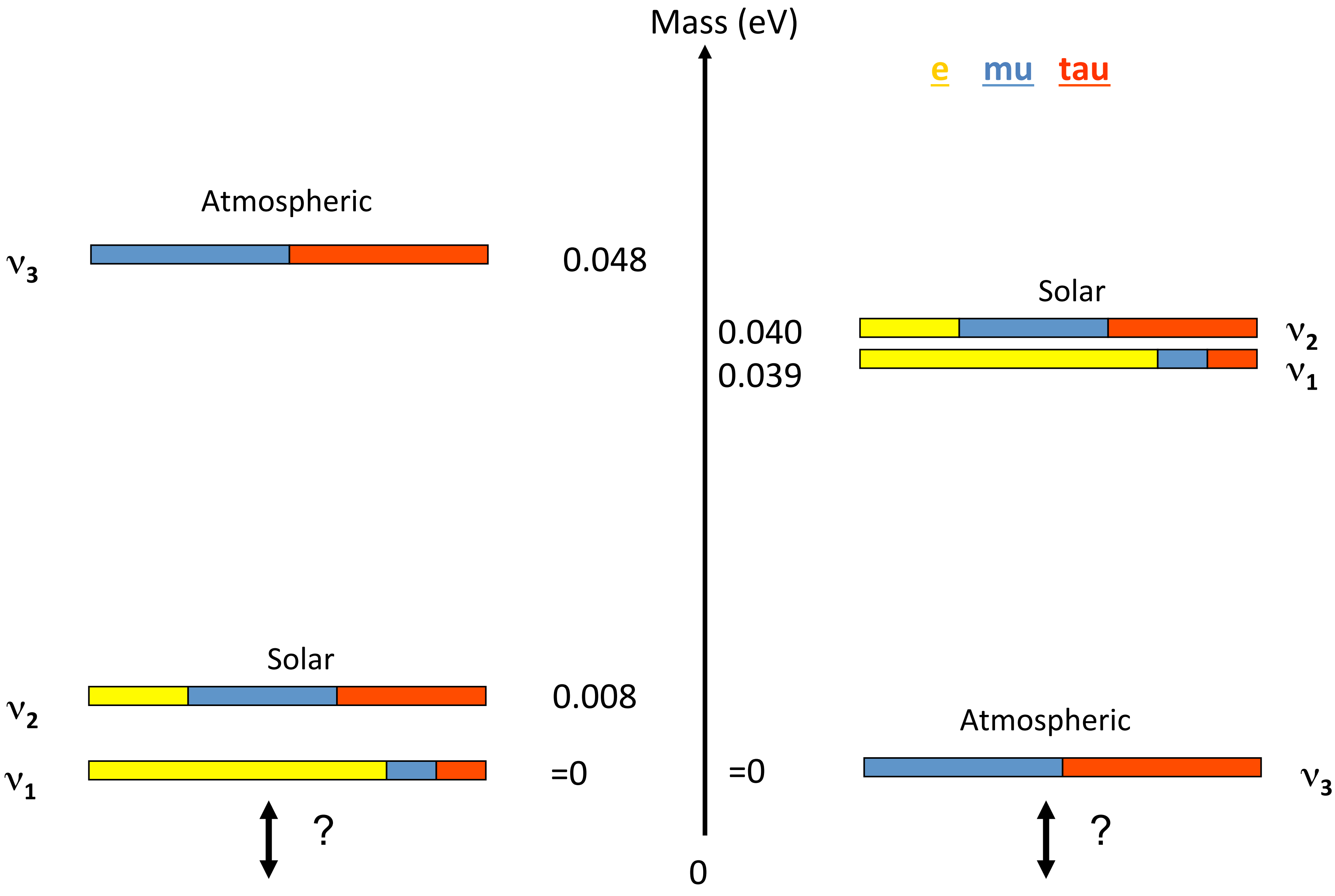
Day-night asymmetry

$$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$$



Day-Night Asymmetry
0.20
0.15
0.10
0.05
0.00
0
1
10
100
1000
δm² (in units of 10⁻⁷ eV²)
E_ν = 862 keV
BOREXINO
Measurement

Neutrino Masses and Flavor Content

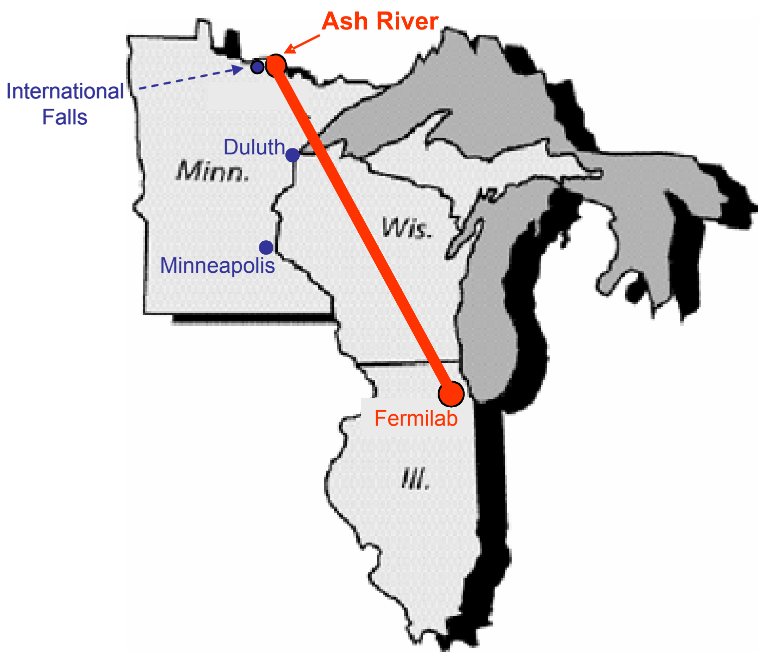


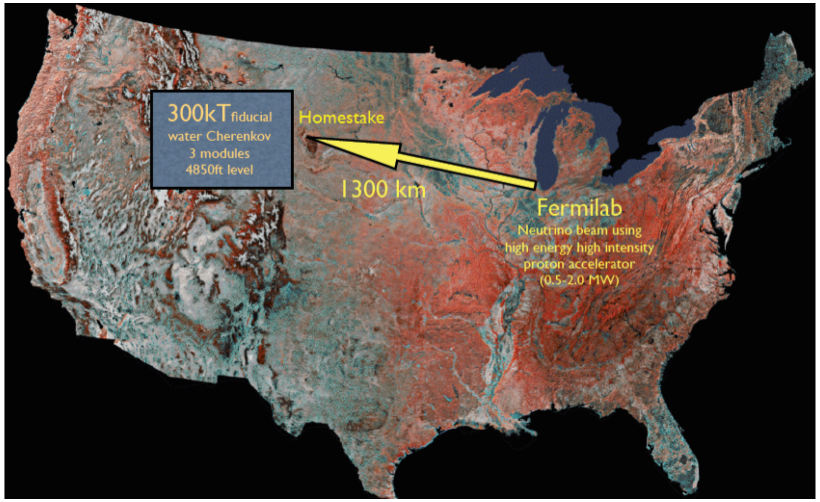
Long-baseline oscillations at GeV energies

Osc. max. L ~ E **+** Flux at source ~ E^2; Flux (L) = Flux (L=0)/L^2

Flux (L) ~ 1 **+** σ ~ E (DIS)

Event rate ~ E





Matter effects in long-baseline oscillations

Example: two flavors and normal hierarchy

$$P(\nu_e \to \nu_e) = \sin^2 2\theta \left[1 + \frac{4\sqrt{2}G_F N_e}{\delta m^2}\cos 2\theta\right] \sin^2\left[\left(\frac{\delta m^2}{4E} + \ldots\right)L\right]$$

$$P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^2 2\theta \left[1 - \frac{4\sqrt{2}G_F N_e}{\delta m^2}\cos 2\theta\right] \sin^2\left[\left(\frac{\delta m^2}{4E} + \ldots\right)L\right]$$

- This can be used to distinguish normal from inverted hierarchy
- Matter effects mimic CP-violation!
- Matter effects increase with energy, E_{MSW} ~ 10 GeV for Earth's mantle

Typical Appearance Experiment

$$P_{\nu_\mu \to \nu_e} \sim \frac{\sin^2 2\theta_{13} \sin^2 \theta_{23}}{(1 - 2\sqrt{2} G_F N_e E / \delta m^2)^2} \sin^2 \left[\left(\frac{\delta m^2_{31}}{4E} - \frac{G_F N_e}{\sqrt{2}} \right) L \right]$$

$$+\mathcal{O}(g)$$

$$g = \frac{\delta m^2_{21}}{\delta m^2_{31}} \sim 0.03$$

Typical Appearance Experiment

$$
\begin{aligned}
P_{\nu_\mu \to \nu_e} \sim\ & \frac{\sin^2 2\theta_{13} \sin^2 \theta_{23}}{(1 - 2\sqrt{2} G_F N_e E / \delta m^2)^2} \sin^2 \left[\left(\frac{\delta m^2_{31}}{4E} - \frac{G_F N_e}{\sqrt{2}} \right) L \right] \\
- \quad & g \frac{\sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}}{(1/2 - 2\sqrt{2} G_F N_e E / \delta m^2_{31}) - 1/4} \cos \left(\delta + \frac{\delta m^2_{31} L}{4E} \right) \\
\times \quad & \cos \left(\frac{\delta m^2_{31} L}{4E} \right) \sin \left(\frac{G_F N_e L}{\sqrt{2}} \right) \sin \left[\left(\frac{\delta m^2_{31}}{4E} - \frac{G_F N_e}{\sqrt{2}} \right) L \right] \\
+ \quad & \mathcal{O}(g^2)
\end{aligned}
$$

$$
g = \frac{\delta m^2_{21}}{\delta m^2_{31}} \sim 0.03
$$

Typical Appearance Experiment

$$
\begin{aligned}
P_{\nu_\mu \to \nu_e} &\sim \frac{\sin^2 2\theta_{13} \sin^2 \theta_{23}}{(1 - 2\sqrt{2} G_F N_e E/\delta m^2)^2} \sin^2 \left[\left(\frac{\delta m^2_{31}}{4E} - \frac{G_F N_e}{\sqrt{2}} \right) L \right] \\
- \quad & g \frac{\sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}}{(1/2 - 2\sqrt{2} G_F N_e E/\delta m^2_{31}) - 1/4} \cos \left(\delta + \frac{\delta m^2_{31} L}{4E} \right) \\
\times \quad & \cos \left(\frac{\delta m^2_{31} L}{4E} \right) \sin \left(\frac{G_F N_e L}{\sqrt{2}} \right) \sin \left[\left(\frac{\delta m^2_{31}}{4E} - \frac{G_F N_e}{\sqrt{2}} \right) L \right] \\
+ \quad & \mathcal{O}(g^2)
\end{aligned}
$$

Is equal to
zero for
the magic baseline

$$g = \frac{\delta m^2_{21}}{\delta m^2_{31}} \sim 0.03$$

CP-violation

$$T_{23}T_{13}T_{12}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$

$$c_{ij}=\cos\theta_{ij}\qquad s_{ij}=\sin\theta_{ij}$$

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\\tilde{\psi}_\mu\\\tilde{\psi}_\tau\end{pmatrix}=\left[T_{13}T_{12}\begin{pmatrix}E_1&0&0\\0&E_2&0\\0&0&E_3\end{pmatrix}T_{12}^\dagger T_{13}^\dagger+\begin{pmatrix}V_{e\mu}&0&0\\0&s_{23}^2V_{\tau\mu}&-c_{23}s_{23}V_{\tau\mu}\\0&-c_{23}s_{23}V_{\tau\mu}&c_{23}^2V_{\tau\mu}\end{pmatrix}\right]\begin{pmatrix}\psi_e\\\tilde{\psi}_\mu\\\tilde{\psi}_\tau\end{pmatrix}$$

$$\tilde{\psi}_\mu=\cos\theta_{23}\psi_\mu-\sin\theta_{23}\psi_\tau$$

$$\tilde{\psi}_\tau=\sin\theta_{23}\psi_\mu+\cos\theta_{23}\psi_\tau$$

$$V_{e\mu}=2\sqrt{2}G_FN_e\left[1+O\left(\alpha\frac{m_\mu}{m_W}\right)^2\right]$$

$$V_{\tau\mu}=-\frac{3\sqrt{2}\alpha G_F}{\pi\sin^2\theta_W}\left(\frac{m_\tau}{m_W}\right)^2\left[\left(N_p+N_n\right)\log\frac{m_\tau}{m_W}+\left(\frac{N_p}{2}+\frac{N_n}{3}\right)\right]$$

We need to solve an evolution equation

$$i\frac{\partial}{\partial t}U = HU$$

If we ignore $V_{\tau\mu}$ it is easy to show that the CP-violating phase factorizes:

$$U(\delta) = SU(\delta = 0)S^{\dagger} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

This factorization still holds when collective oscillations are include, but breaks down if there is spin-flavor precession

This factorization implies that neither

$$P\left(\nu_e \rightarrow \nu_e\right)$$

nor

$$P\left(\nu_\mu \rightarrow \nu_e\right) + P\left(\nu_\tau \rightarrow \nu_e\right)$$

depend on the CP-violating phase δ.

If the ν_μ and ν_τ luminosities are the same at the neutrinosphere of a core-collapse supernova, this factorization implies that ν_e and ν_e fluxes observed at terrestrial detectors will not be sensitive to the CP-violating phase! To see its effects you need to measure ν_μ and ν_τ luminosities separately!

If you see the effects of δ in either charged- or neutral current scattering that may mean any of the following:

- There are new neutrino interactions beyond the standard model operating either within the neutron star or during propagation.
- Standard Model loop corrections (very easy to quantify) are seen.
- There are sterile neutrino states.

Factorization of the CP-violating phase if there are no sterile neutrinos

$$H(\delta) = H_{\nu} + H_{\nu\nu} = \mathbf{S} H(\delta = 0) \mathbf{S}^{\dagger}$$

$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

Holds if neutrino magnetic moment is ignored.

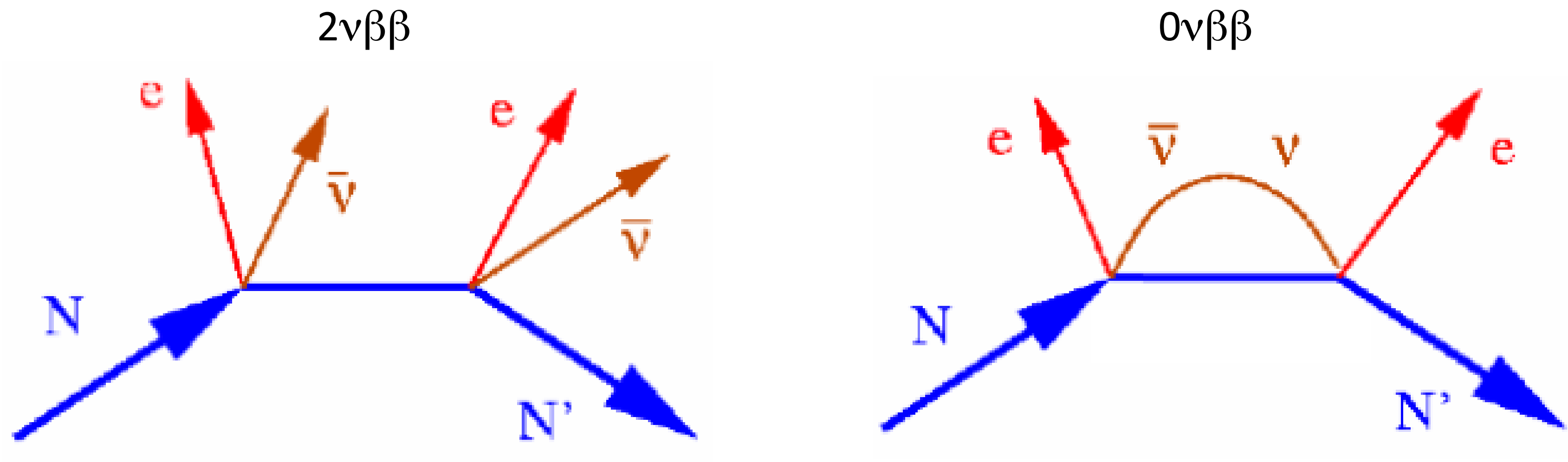
- MSW Hamiltonian: Balantekin, Gava, Volpe, Phys. Lett B662, 396 (2008).
- Collective Hamiltonian in the mean-field approximation: Gava, Volpe, Phys. Rev. D78, 083007 (2008).
- Exact collective Hamiltonian: Pehlivan, Balantekin, Kajino, Phys. Rev. D90, 065011 (2014).

Double Beta Decay

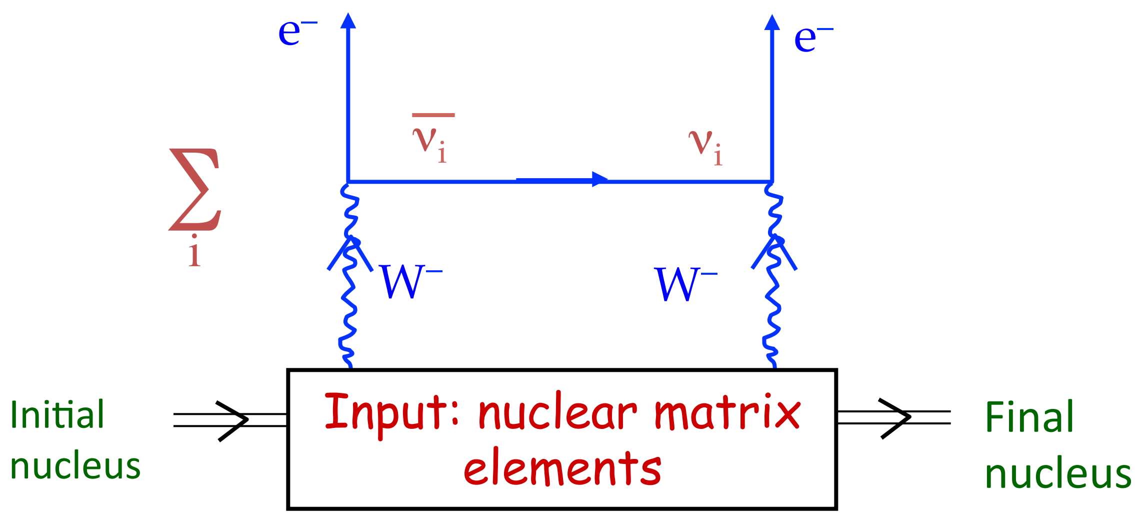
The second order process, where two neutrinos are emitted, is also possible.

Maria Mayer, 1935

Maria Goeppert *Mayer* was *awarded* the 1963 *Nobel* for the nuclear shell model, the San Diego Union headline read "*San Diego Housewife Wins Nobel Prize*".



Majorana nature of the neutrinos permit neutrinoless double beta decay:



Suggestion of neutrinoless double beta decay

Nuovo Cimento, **14**, pp 322-328 (1937)

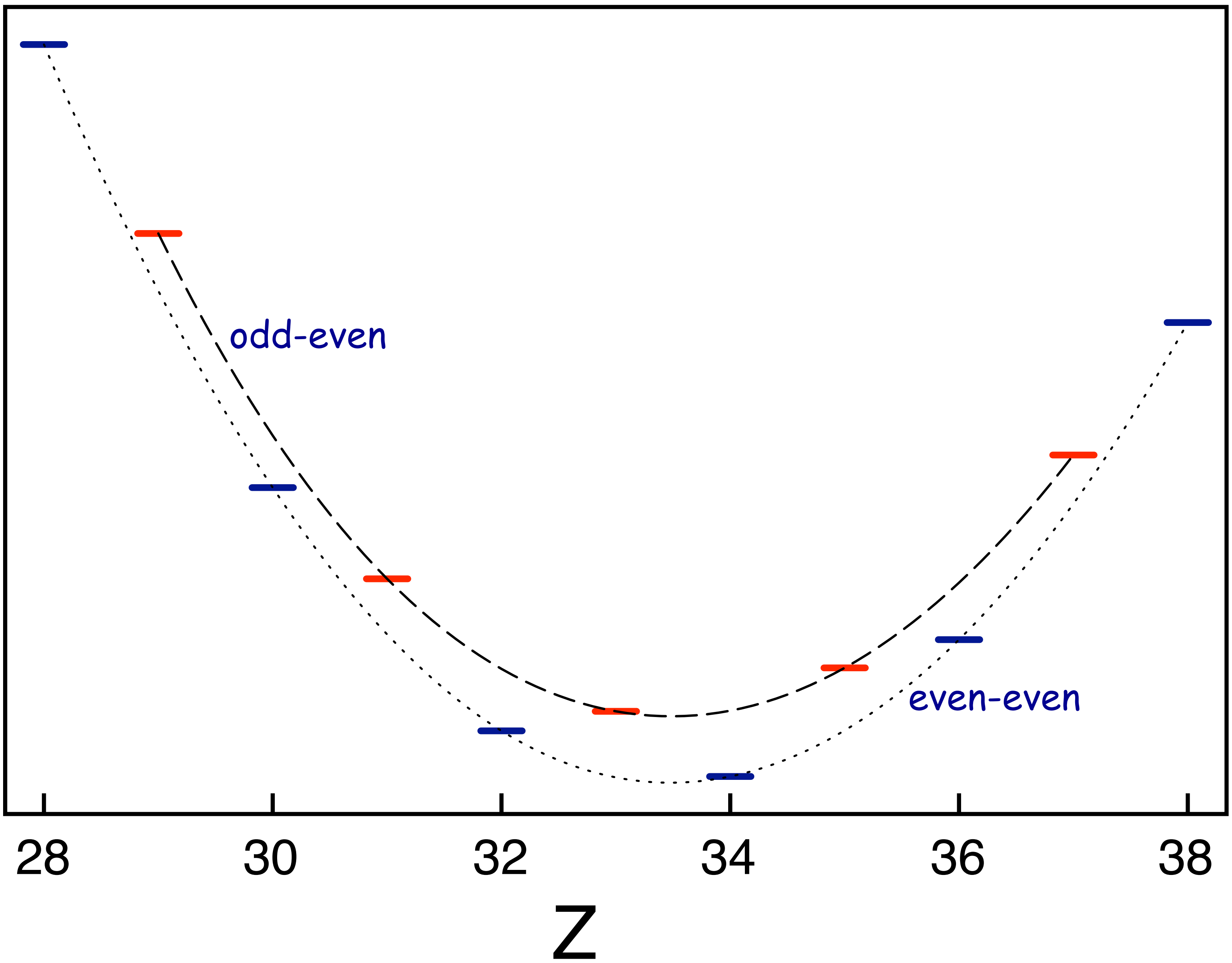
SULLA SIMMETRIA TRA PARTICELLE E ANTIPARTICELLE

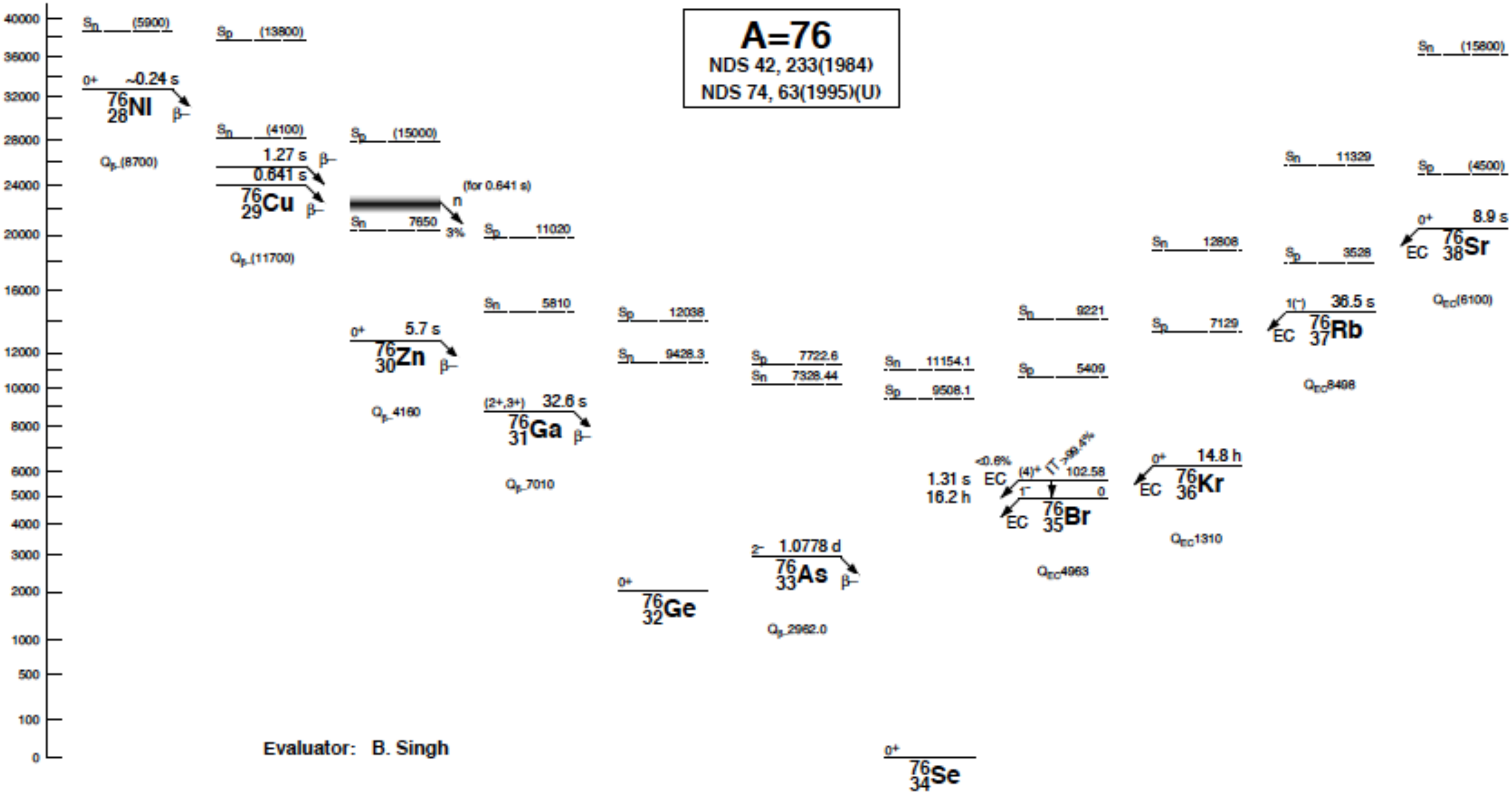
Nota di GIULIO RACAH

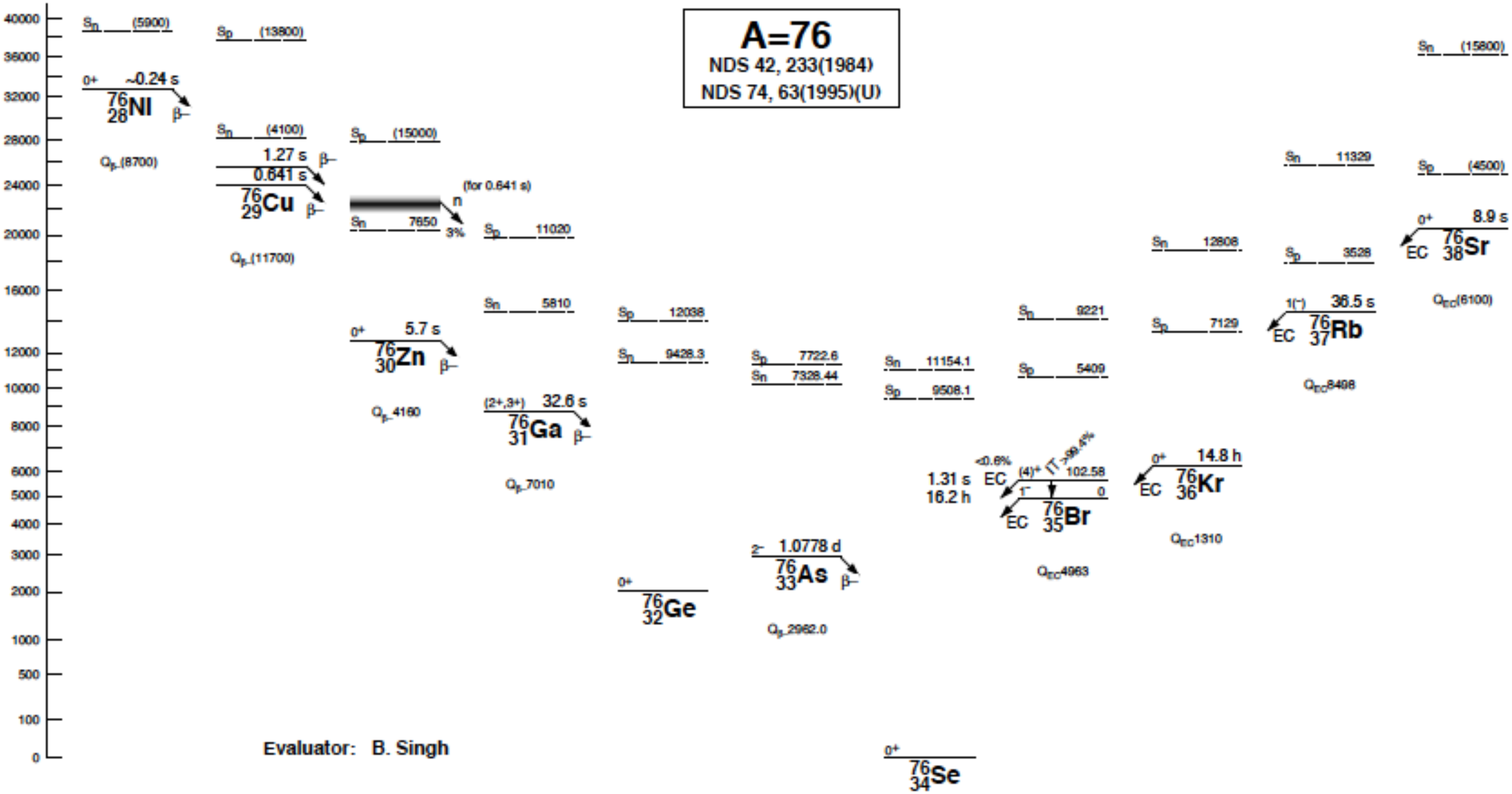
Sunto. - *Si mostra che la simmetria tra particelle e antiparticelle porta alcune modificazioni formali nella teoria di* FERMI *sulla radioattività* β, *e che l'identità fisica tra neutrini ed antineutrini porta direttamente alla teoria di* E. MAJORANA.

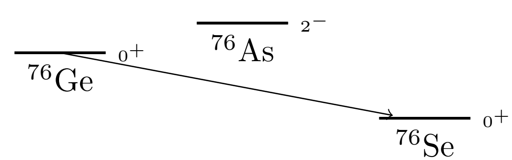
Summary - This article shows that the symmetry between particles and antiparticles leads some formal amendments in the theory of Fermi β radioactivity, and that the physical identity between neutrinos and antineutrinos leads directly to the theory of E. Majorana.

Pairing gives rise to double beta decay:




A=76
NDS 42, 233(1984)
NDS 74, 63(1995)(U)
76 28 Ni
0+ ~0.24 s
β−
Sn (5900)
Qβ- (8700)
76 29 Cu
1.27 s
0.641 s
β−
Sp (13800)
Sn (4100)
Qβ- (11700)
(for 0.641 s)
n
3%
Sp (15000)
Sn 7650
76 30 Zn
0+ 5.7 s
β−
Qβ- 4160
76 31 Ga
(2+,3+) 32.6 s
β−
Sp 11020
Sn 5810
Qβ- 7010
76 32 Ge
0+
Sp 12038
Sn 9428.3
76 33 As
2− 1.0778 d
β−
Sp 7722.6
Sn 7328.44
Qβ- 2962.0
76 34 Se
0+
Sn 11154.1
Sp 9508.1
76 35 Br
(4)+ 102.58
1− 0
IT >99.4%
<0.6%
EC
1.31 s
16.2 h
Sn 9221
Sp 5409
QEC 4963
76 36 Kr
0+ 14.8 h
EC
Sn 12808
Sp 7129
QEC 1310
76 37 Rb
1(−) 36.5 s
EC
Sn 11329
Sp 3528
QEC 8498
76 38 Sr
0+ 8.9 s
EC
Sn (15800)
Sp (4500)
QEC (6100)
Evaluator: B. Singh


A=76
NDS 42, 233(1984)
NDS 74, 63(1995)(U)
Evaluator: B. Singh
0+ ~0.24 s
$^{76}_{28}$Ni β−
$Q_{β-}$(8700)
1.27 s β−
0.641 s
$^{76}_{29}$Cu β−
$Q_{β-}$(11700)
(for 0.641 s)
n
3%
0+ 5.7 s
$^{76}_{30}$Zn β−
$Q_{β-}$4160
(2+,3+) 32.6 s
$^{76}_{31}$Ga β−
$Q_{β-}$7010
0+
$^{76}_{32}$Ge
2− 1.0778 d
$^{76}_{33}$As β−
$Q_{β-}$2962.0
0+
$^{76}_{34}$Se
1.31 s
16.2 h
<0.6%
EC
(4)+ 102.58
1− 0
$^{76}_{35}$Br
Q_{EC}4963
0+ 14.8 h
EC $^{76}_{36}$Kr
Q_{EC}1310
1(−) 36.5 s
EC $^{76}_{37}$Rb
Q_{EC}8498
0+ 8.9 s
EC $^{76}_{38}$Sr
Q_{EC}(6100)

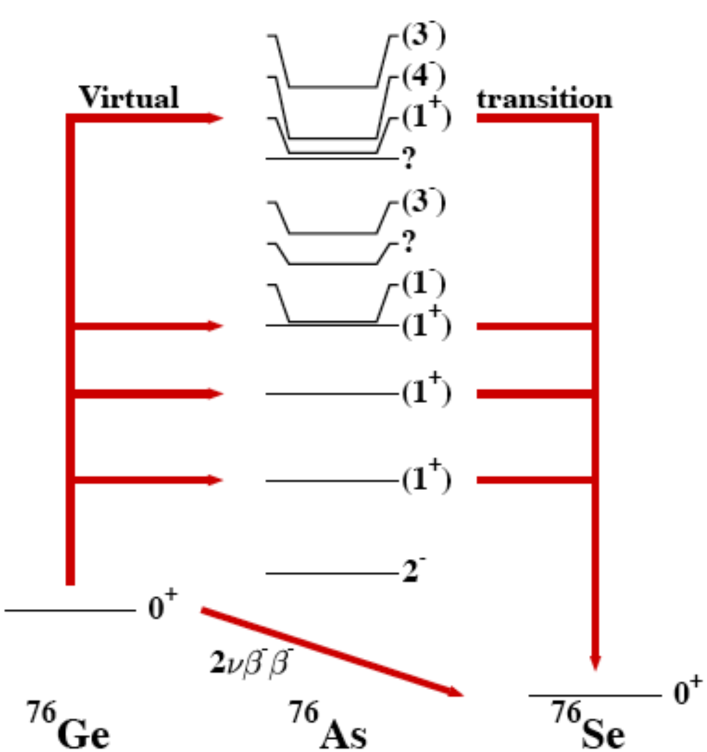

^{76}Ge 0+
^{76}As 2−
^{76}Se 0+

Current limits on 0ββ decay

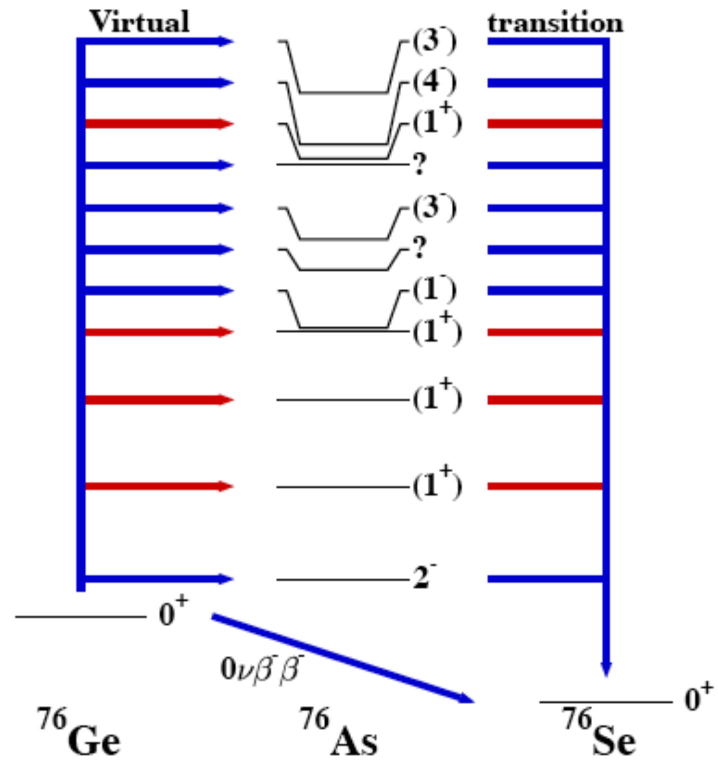
Nucleus	Q-value (MeV)	$T_{1/2}$ (years) limit	$\langle m_\nu \rangle$ (eV) limit
^{48}Ca	4.276	$> 1.14 \times 10^{22}$	< 7.2
^{76}Ge	2.039	$> 1.6 \times 10^{25}$	< 0.33
^{82}Se	2.992	$> 1.9 \times 10^{23}$	< 1.3
^{100}Mo	3.034	$> 5.8 \times 10^{23}$	< 0.8
^{116}Cd	2.804	$> 1.7 \times 10^{23}$	< 1.7
^{128}Te	0.876	$> 7.7 \times 10^{24}$	< 1.1
^{130}Te	2.529	$> 3 \times 10^{23}$	< 0.46
^{136}Xe	2.467	$> 4.4 \times 10^{23}$	< 1.8
^{150}Nd	3.368	$> 1.2 \times 10^{21}$	< 7

$$\langle m_\nu \rangle = \sum_{i=1}^{3} U_{ie}^2 m_i$$

Why are matrix elements of 0νββ and 2νββ different?



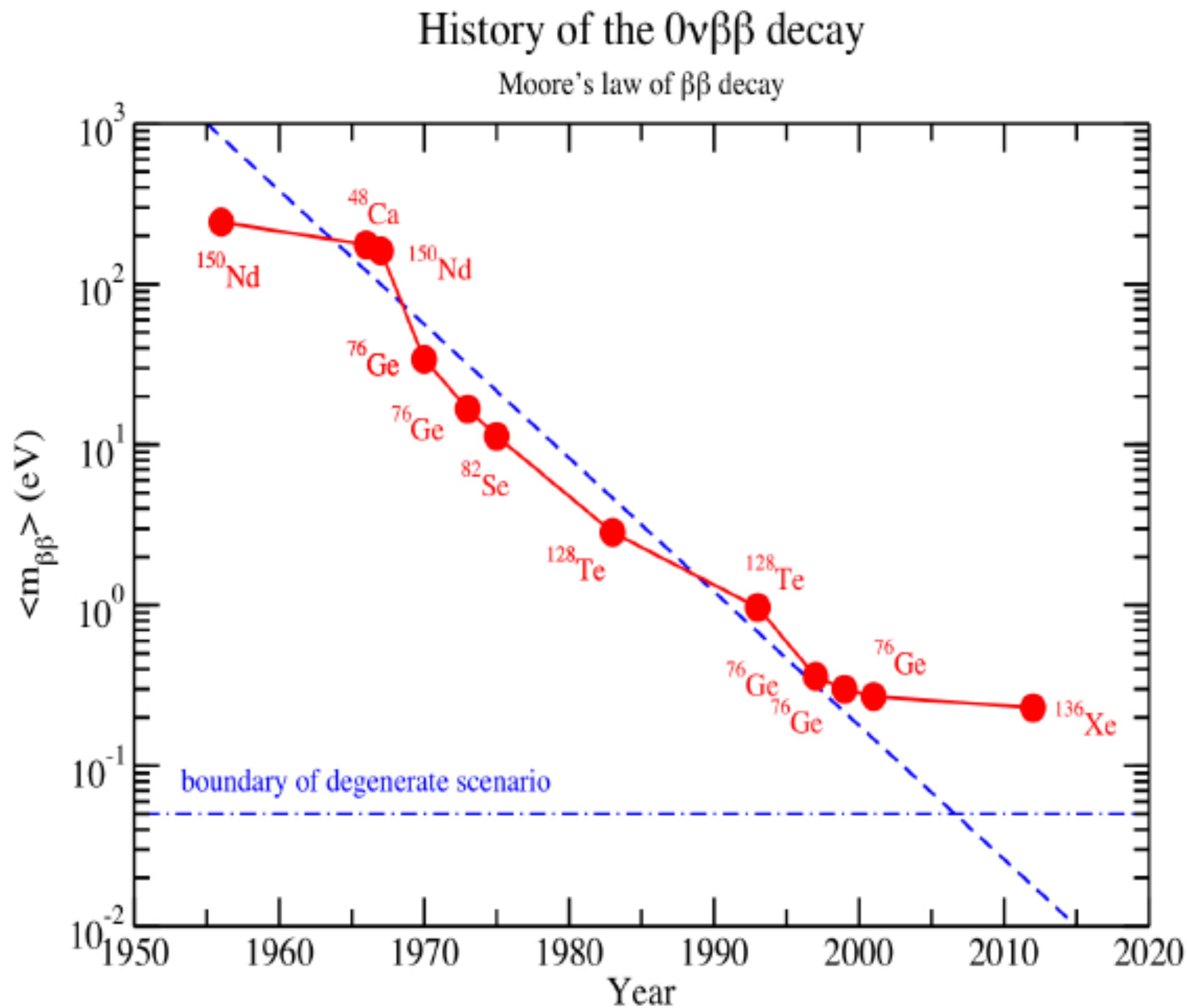
2νββ



0νββ

Only intermediate 1^+ states contribute (single-state dominance approximation?) q< a few MeV: $e^{iqr} \sim 1$

All intermediate states contribute (closure approximation?) q ~ a few 100 MeV: $e^{iqr} = 1 + iqr - (qr)^2 + \ldots$


History of the 0νββ decay
Moore's law of ββ decay
<m_ββ> (eV)
Year
10^3
10^2
10^1
10^0
10^-1
10^-2
1950
1960
1970
1980
1990
2000
2010
2020
150Nd
48Ca
150Nd
76Ge
76Ge
82Se
128Te
128Te
76Ge
76Ge
76Ge
136Xe
boundary of degenerate scenario

Vogel

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \,\|\, \vec{\sigma}\tau_+ \,\|\, n > \cdot < n \,\|\, \vec{\sigma}\tau_+ \,\|\, i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay

$$M^{0\nu} = M_{GT}^{0\nu} - \frac{M_F^{0\nu}}{g_A^2} + M_T^{0\nu}$$

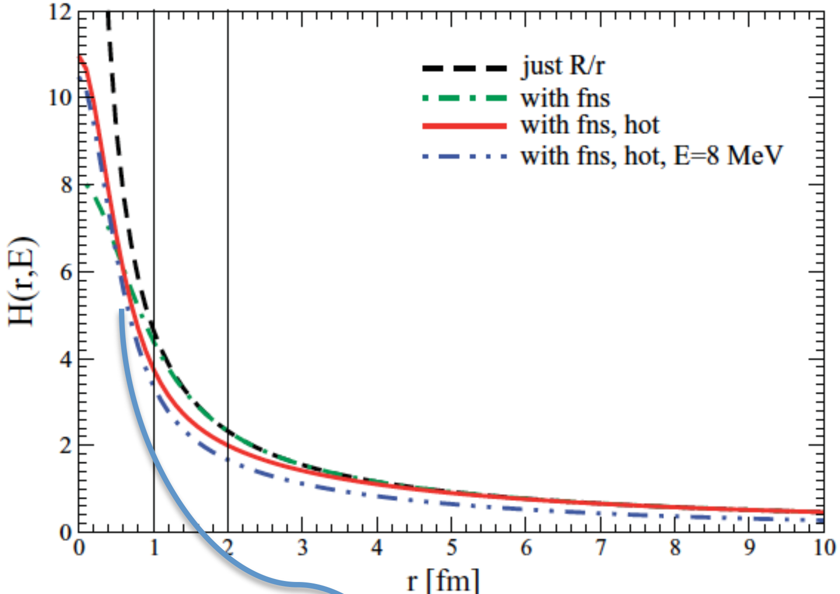
$$M_{GT}^{0\nu} \approx < f \,|\, \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j)\cdot\vec{\sigma}(k)\, \tau_+(j)\tau_+(k) \,|\, f >$$

Neutrinoless ββ decay

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \| \vec{\sigma}\tau_+ \| n > \cdot < n \| \vec{\sigma}\tau_+ \| i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay



$$M^{0\nu} = M_{GT}^{0\nu} - \frac{M_F^{0\nu}}{g_A^2} + M_T^{0\nu}$$

$$M_{GT}^{0\nu} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j)\cdot\vec{\sigma}(k)\, \tau_+(j)\tau_+(k) | f >$$

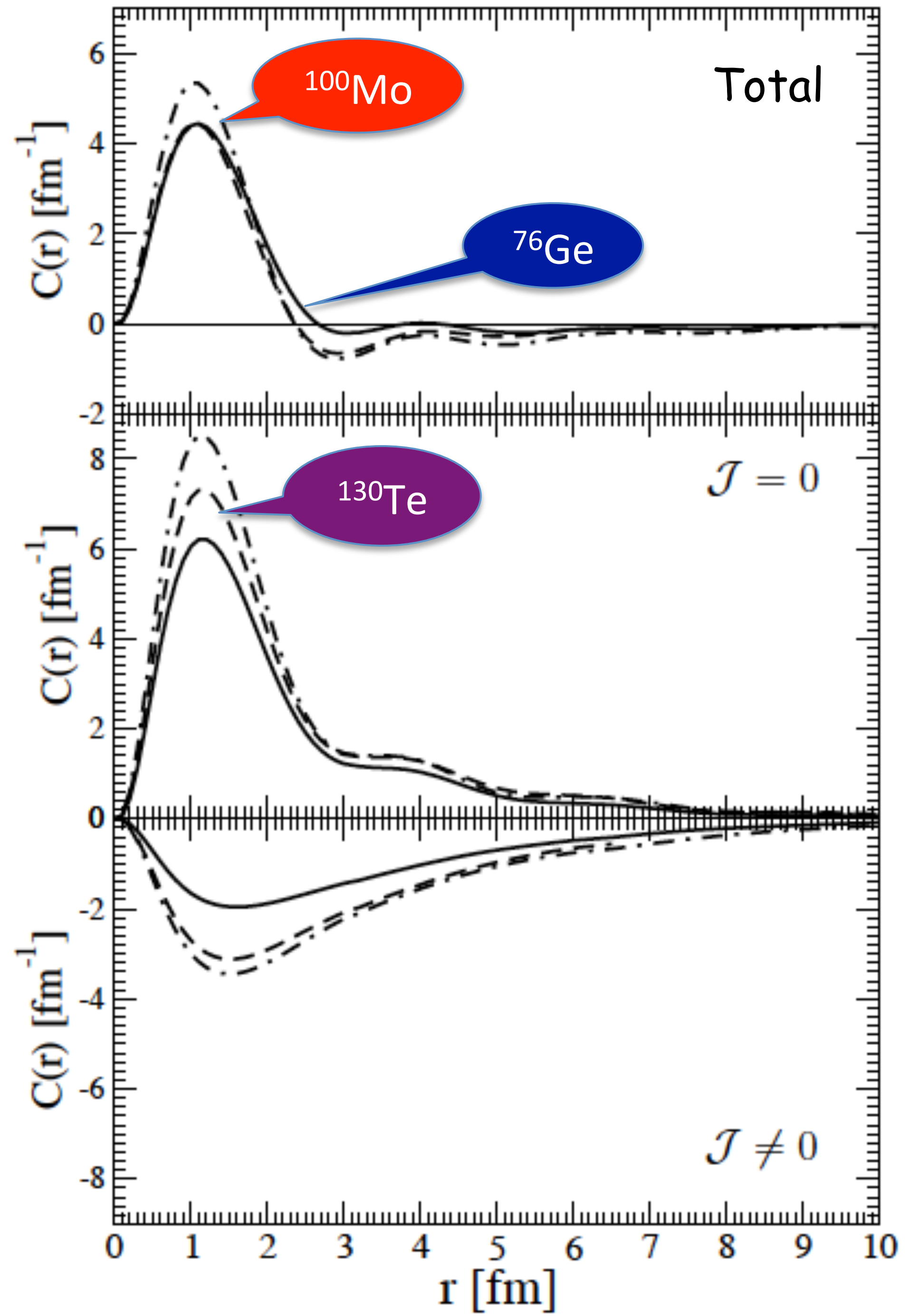
Neutrinoless ββ decay

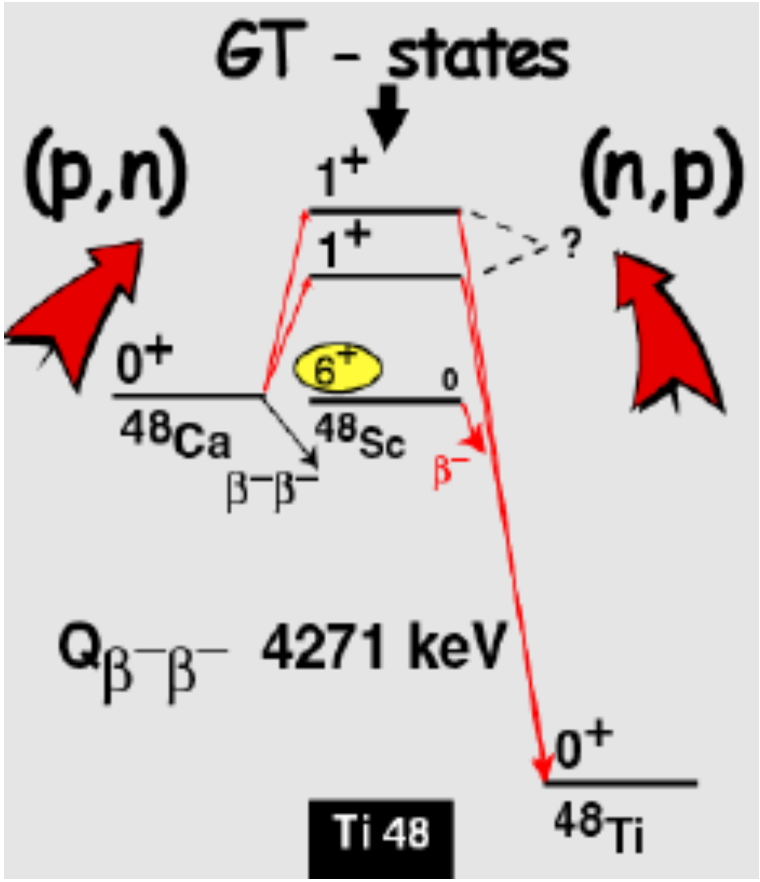
Nuclear matrix elements

$$M_{GT}^{0\nu} = \int_0^\infty C_{GT}^{0\nu}(r)\,dr$$

Momentum of virtual neutrino, q ~ 1/r
r ~ 2 fm
q ~ 100 MeV

P. Vogel, J. Phys G **39**, 124002 (2012)

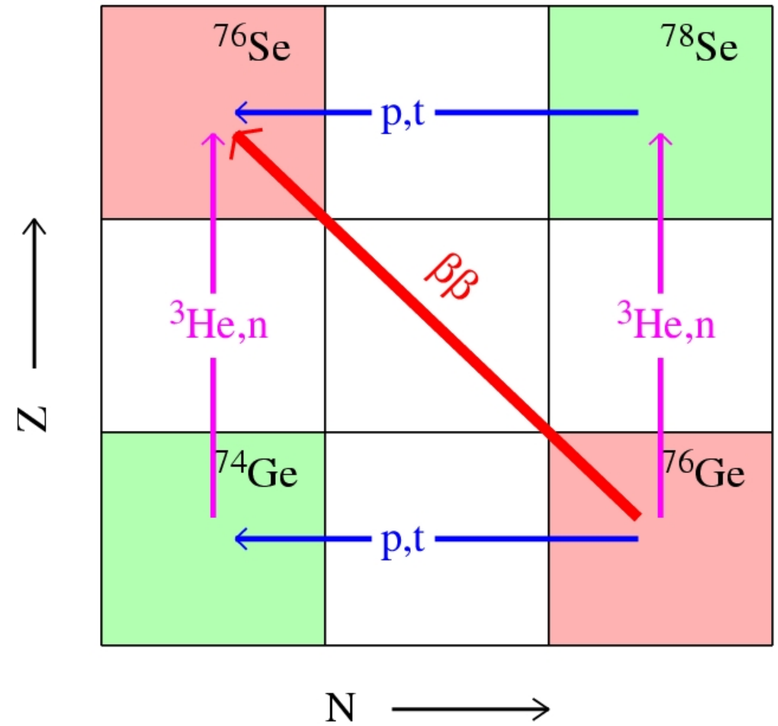




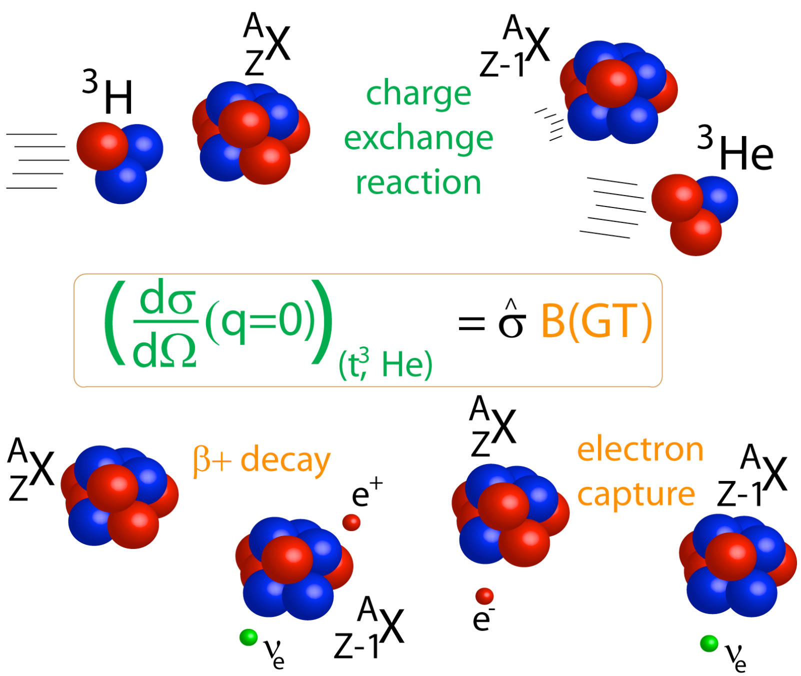
For 2νββ decay only intermediate 1* states contribute. These are, in principle, accessible with charge-exchange reactions (at θ= 0°). For 0νββ we do not have such help.

- 2ν $\beta\beta$-decay: small momentum transfer
- 0ν $\beta\beta$-decay: large momentum transfer (~ 100 MeV)
- 100 MeV covers all giant resonances and details of the structure of the intermediate nucleus cannot be very important.
- Since only initial and final states matter, determine the composition of the Fermi surface: the distribution of the valence nucleons that participate in the decay.

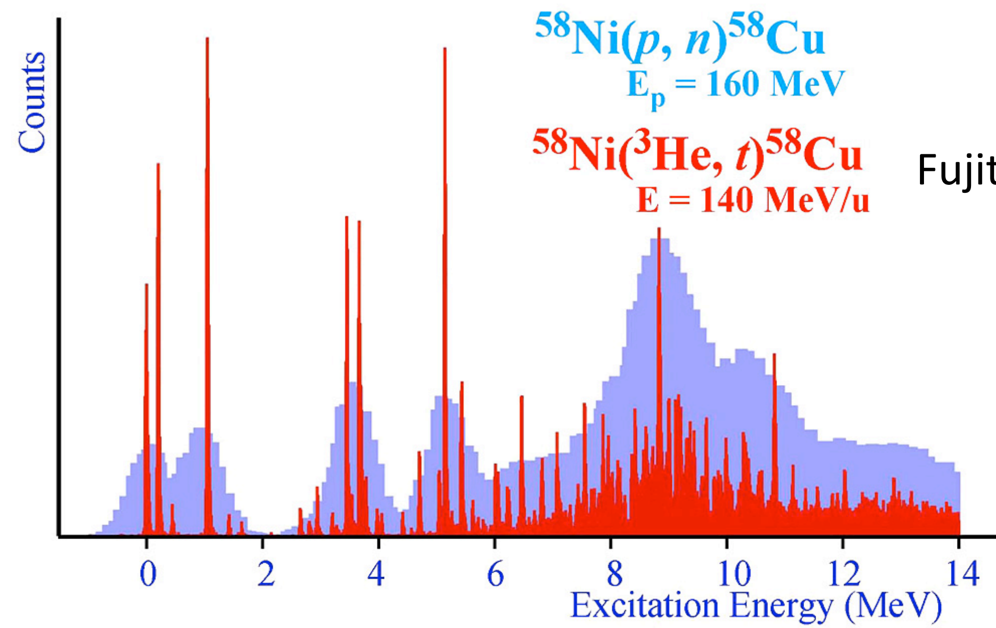
Charge-exchange reaction experiments both with direct and inverse kinematics will help. Recently there have been significant developments in this area.



Schiffer



Zegers



Fujita

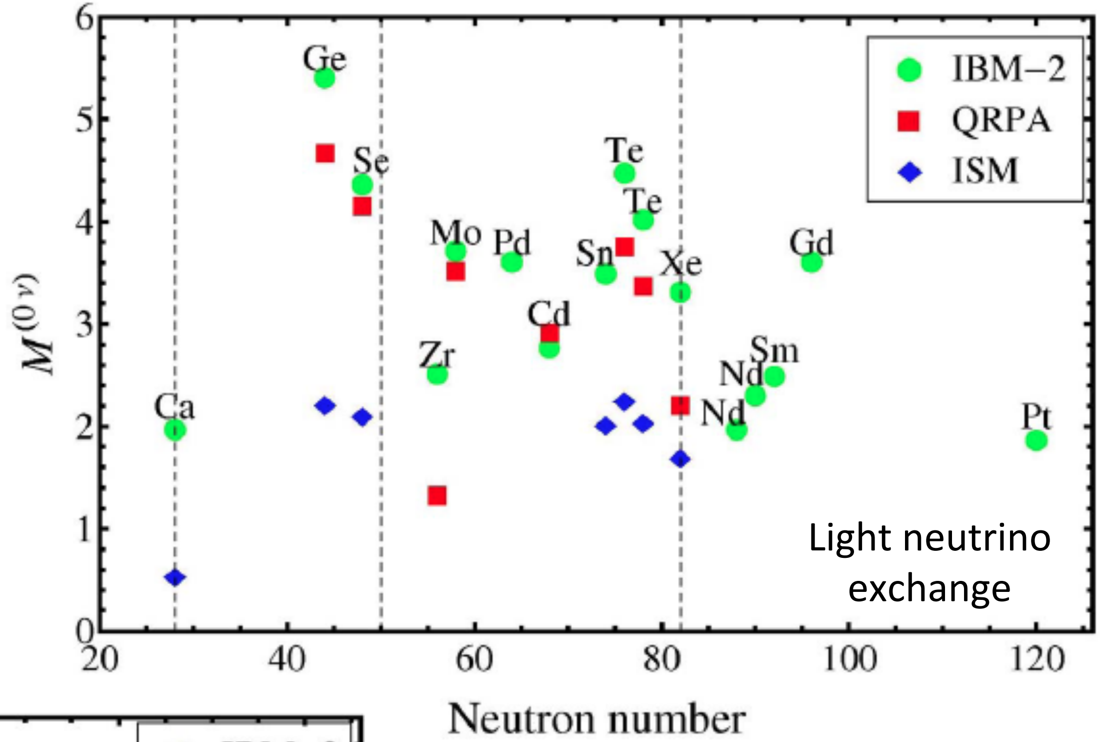
0ν double beta decay

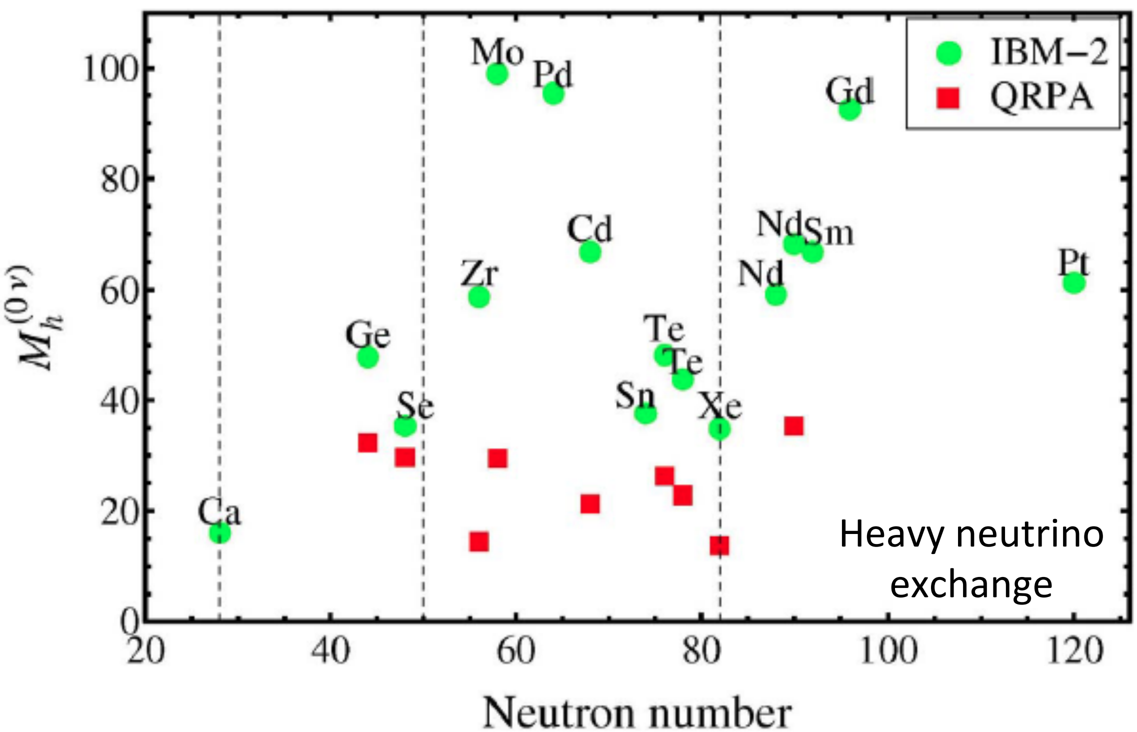
$$(1/T_{1/2}) = G(E,Z)\, M^2\, \langle m_{\beta\beta} \rangle^2$$

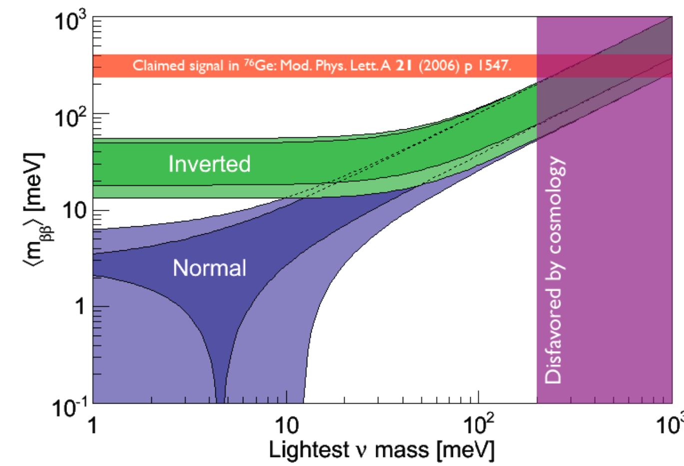
$G(E,Z)$: phase space

M : nuclear matrix element

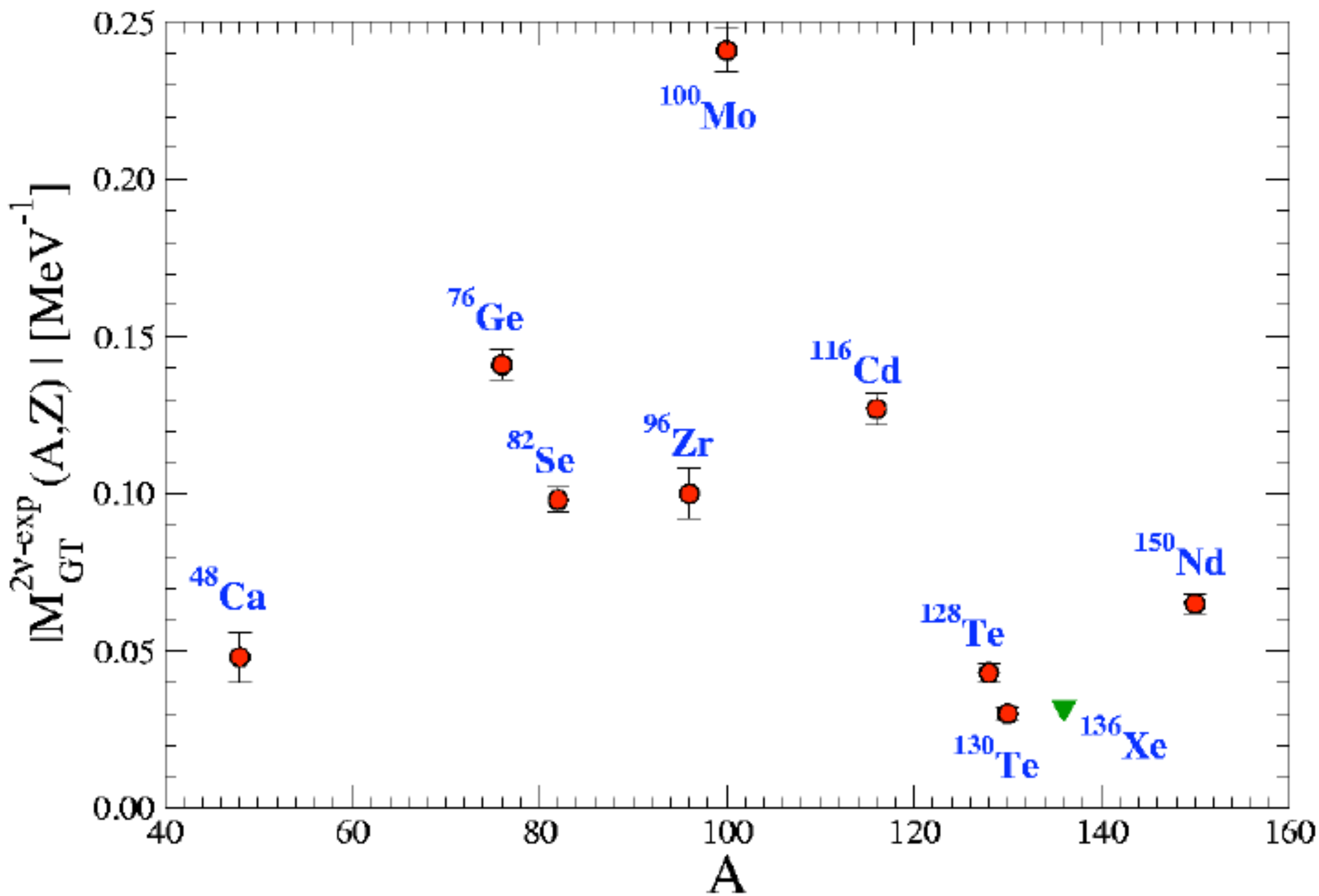
$$\langle m_{\beta\beta} \rangle = |\Sigma_j\, |U_{ej}|^2\, m_j\, e^{i\delta(j)}|$$







For $2\nu\beta\beta$ there is a strong shell-model dependence of the matrix elements



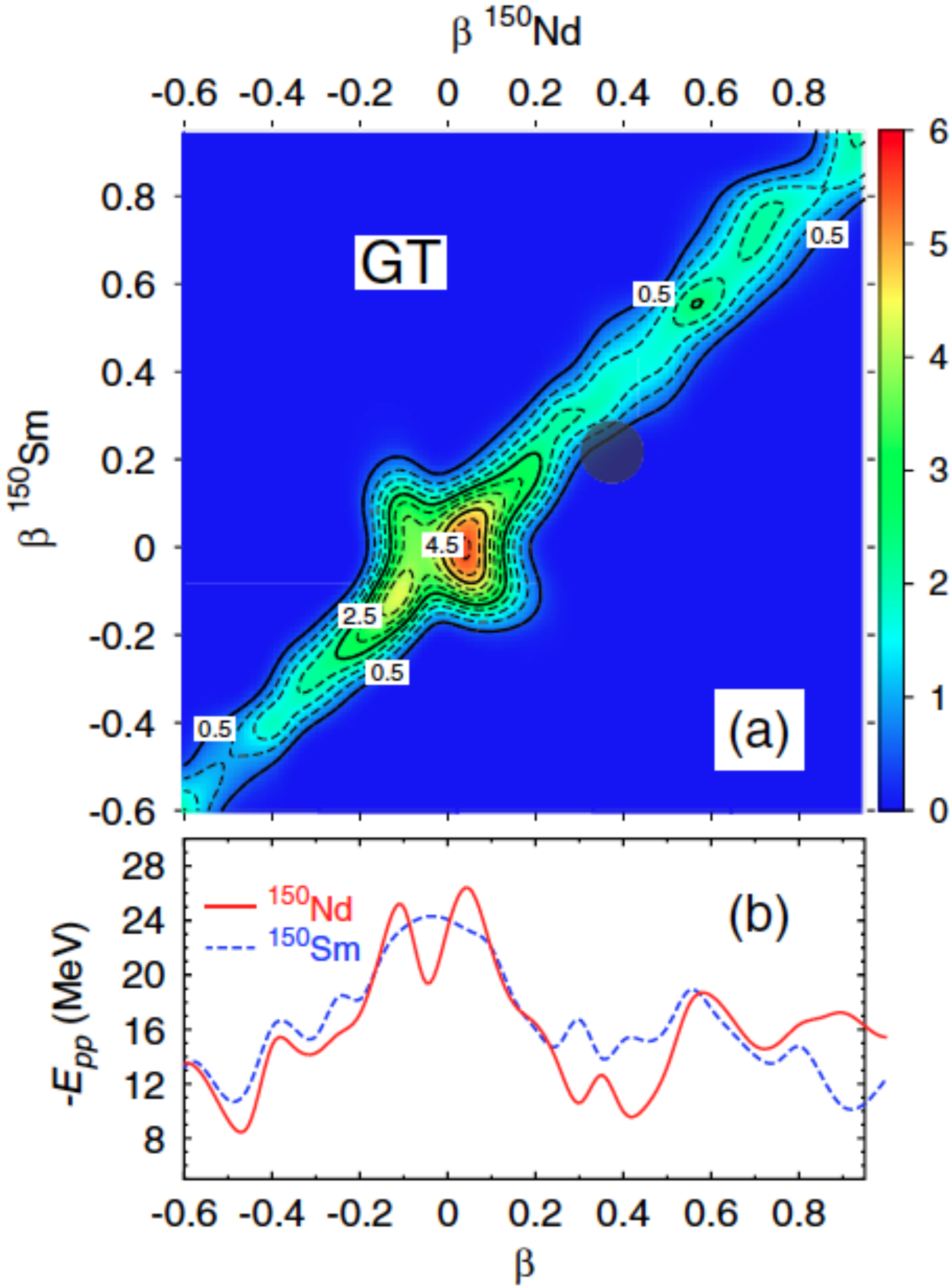
Vogel

In neutrinoless double beta decay, the overlap between initial and final states should be not too small!

Example:

$^{150}Nd \rightarrow ^{150}Sm+ee$

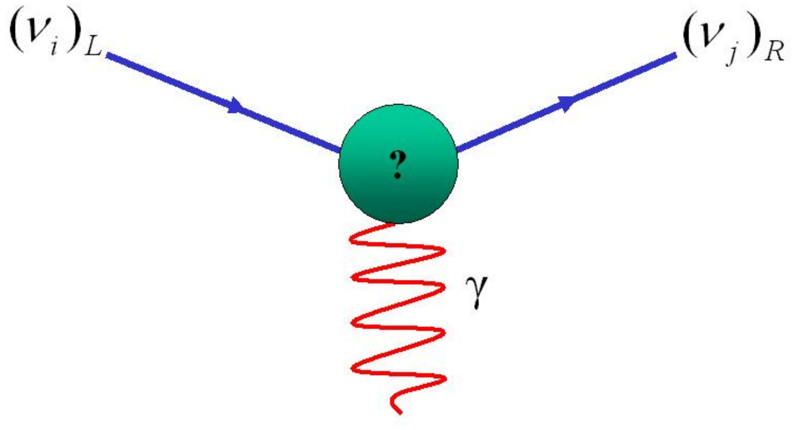
Rodriguez & Martinez-Pinedo, PRL **105,** 252503 (2010)



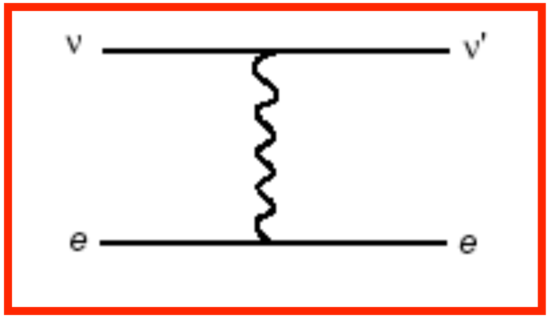
Neutrino Magnetic Moment

$$L_{\text{int}} = \frac{1}{2}\overline{\psi}_j \sigma_{\alpha\beta}(\beta_{ij} + \varepsilon_{ij}\gamma_5)\psi_i F^{\alpha\beta} + h.c.$$

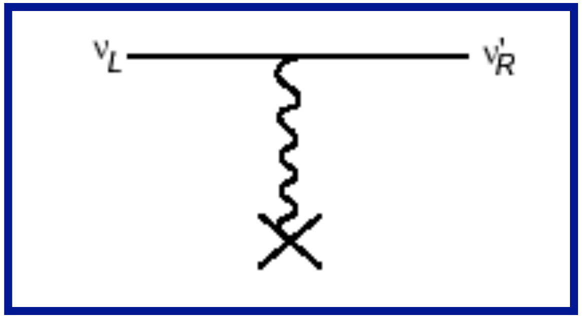
$$\mu_{ij} \equiv \left|\beta_{ij} - \varepsilon_{ij}\right|$$



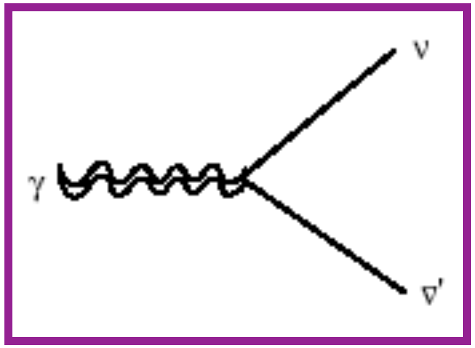
Physical Processes with a Neutrino Magnetic Moment



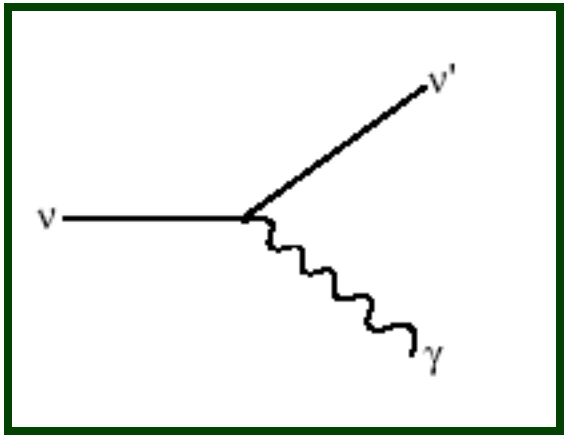
ν-e scattering



Spin-flavor precession



Plasmon decay



Neutrino decay

In effective field theories at lower energies, beyond Standard Model physics is described by local operators

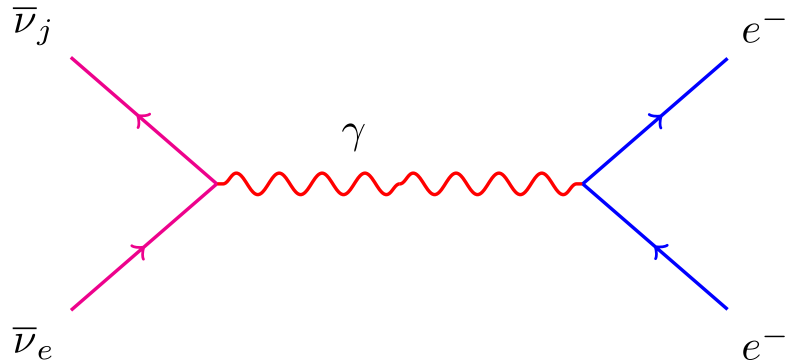
$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

Majorana neutrino mass (unique)

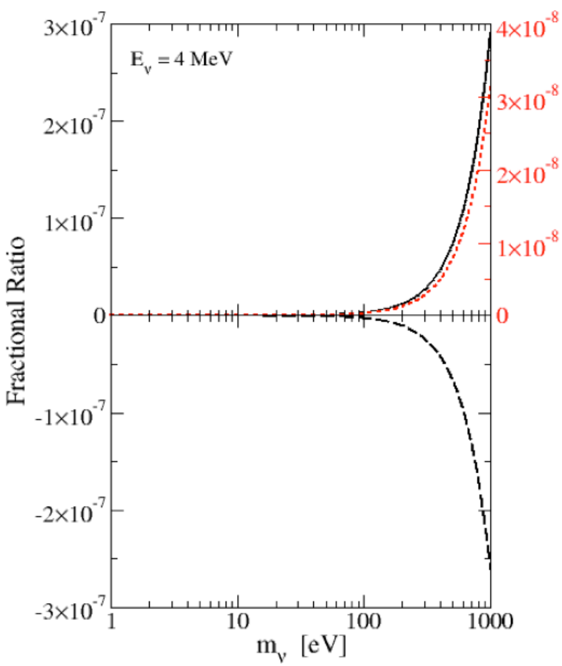
Includes Majorana neutrino magnetic moment

$$\mu_\nu \propto \frac{m_\nu}{\Lambda^2}$$

Neutrino-electron scattering at reactors



$$\frac{d\sigma_{ij}}{dt} = \frac{e^2\mu_{ij}^2}{8\pi\lambda}\left[\frac{1}{t}2\left(2\lambda + 4m_e^2 m_i^2 + 2A\,\delta m^2 + 2m_e^2\,\delta m^2 + \left[\delta m^2\right]^2\right) + \left(2A + \delta m^2\right) + \frac{2m_e^2\left(\delta m^2\right)^2}{t^2}\right]$$



So we can safely ignore the neutrino masses in the above equation. But we have to account for the possibility of neutrinos oscillating before they reach the detector

$$\frac{d\sigma}{dT_e} = \frac{\alpha^2\pi}{m_e^2}\mu_{\text{eff}}^2\left[\frac{1}{T_e} - \frac{1}{E_\nu}\right]$$

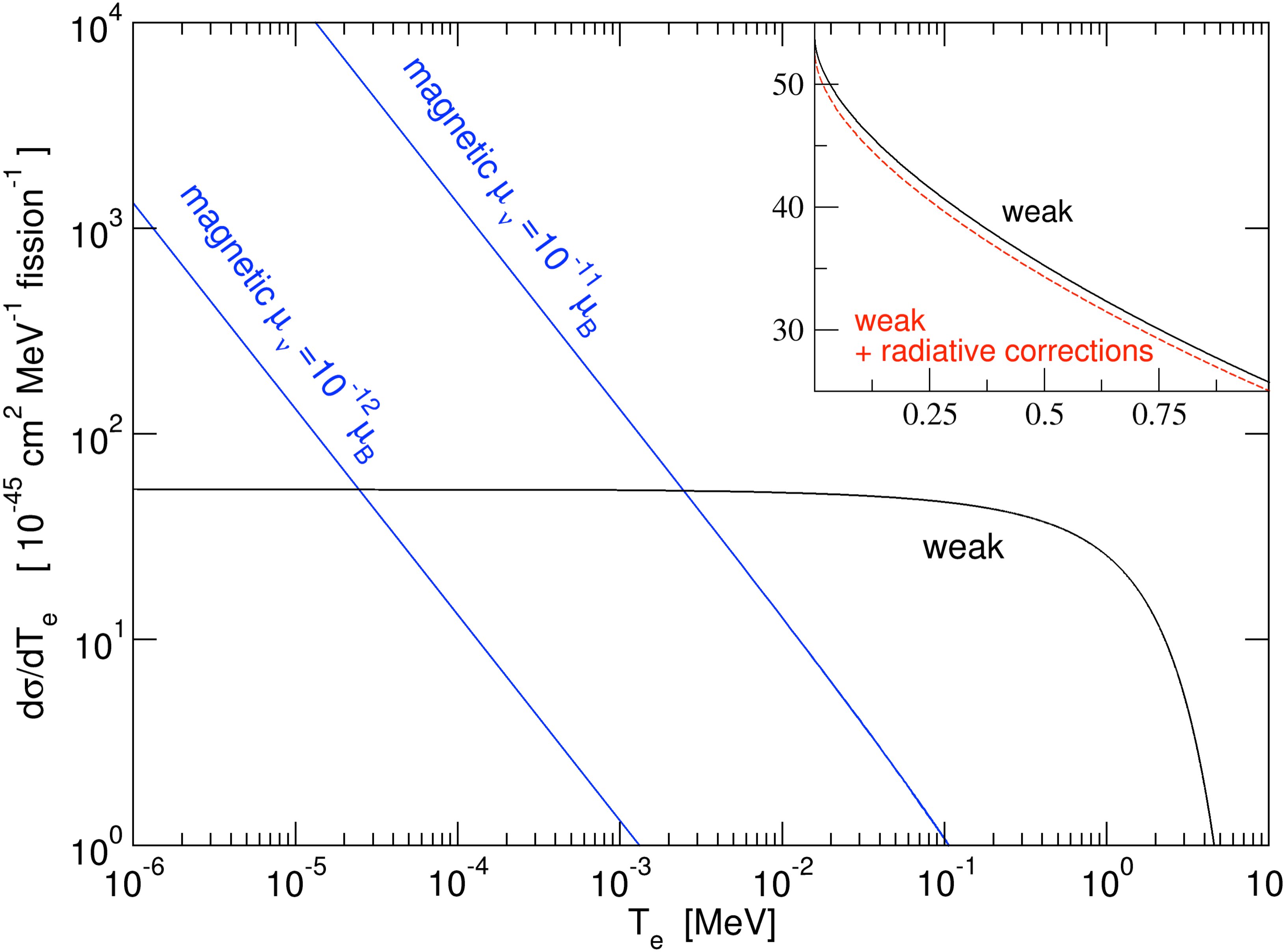
$$\mu_{\text{eff}}^2 = \sum_i\left|\sum_j U_{ej}e^{-iE_jL}\mu_{ji}\right|^2$$

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right]$$ ← weak

$$+\frac{\pi\alpha^2\mu^2}{m_e^2}\left(\frac{1}{T} - \frac{1}{E_\nu}\right)$$ ← magnetic

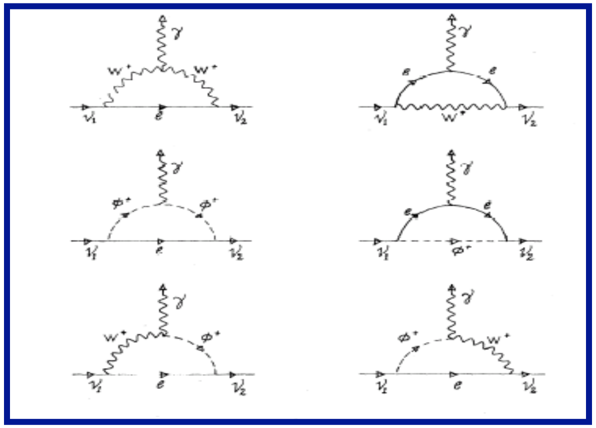
$g_v = 2\sin^2\theta_W + 1/2$

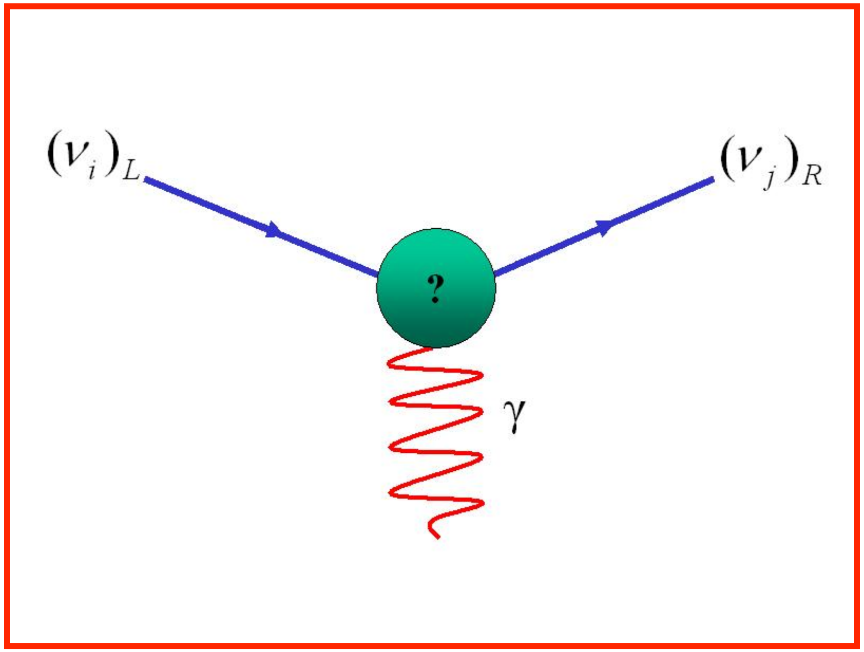
$g_A = \begin{cases} +1/2 \text{ for electron neutrinos} \\ -1/2 \text{ for electron antineutrinos} \end{cases}$



magnetic $\mu_\nu = 10^{-11}\,\mu_B$
magnetic $\mu_\nu = 10^{-12}\,\mu_B$
weak
weak
weak
+ radiative corrections
$d\sigma/dT_e$ [10^{-45} cm^2 MeV^{-1} fission^{-1}]
T_e [MeV]
10^0
10^1
10^2
10^3
10^4
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
30
40
50
0.25
0.5
0.75

Neutrino Magnetic Moment in the Standard Model





$$\mu_{ij} = -\frac{eG_F}{8\sqrt{2}\pi^2}(m_i + m_j)\sum_{\ell} U_{\ell i}U^*_{\ell j} f(r_\ell)$$

$$f(r_\ell) \approx -\frac{3}{2} + \frac{3}{4} r_\ell + \dots, \quad r_\ell = \left(\frac{m_\ell}{M_W}\right)^2$$

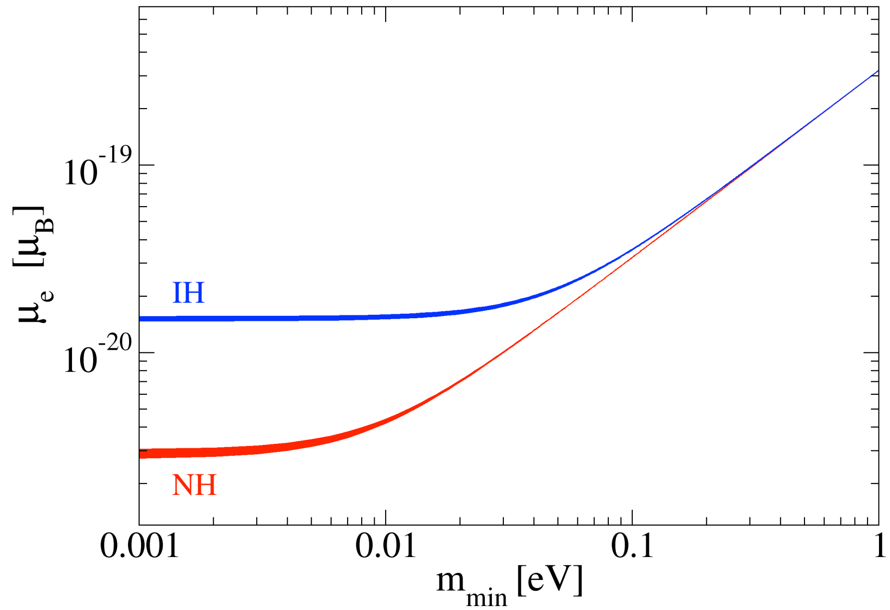
Standard Model (Dirac)

Symmetry Principles:

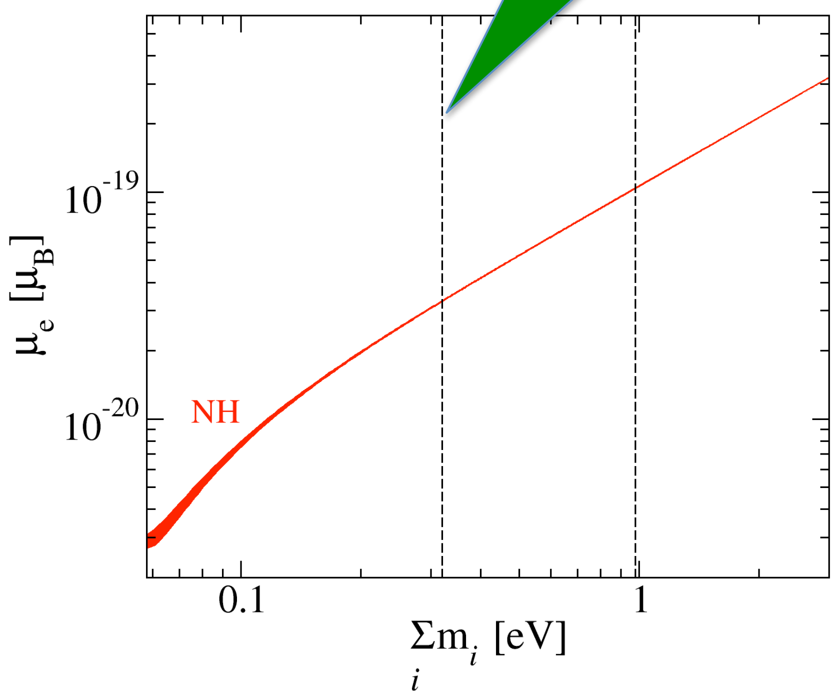
$\mu_\nu \rightarrow 0$ as $m_\nu \rightarrow 0$

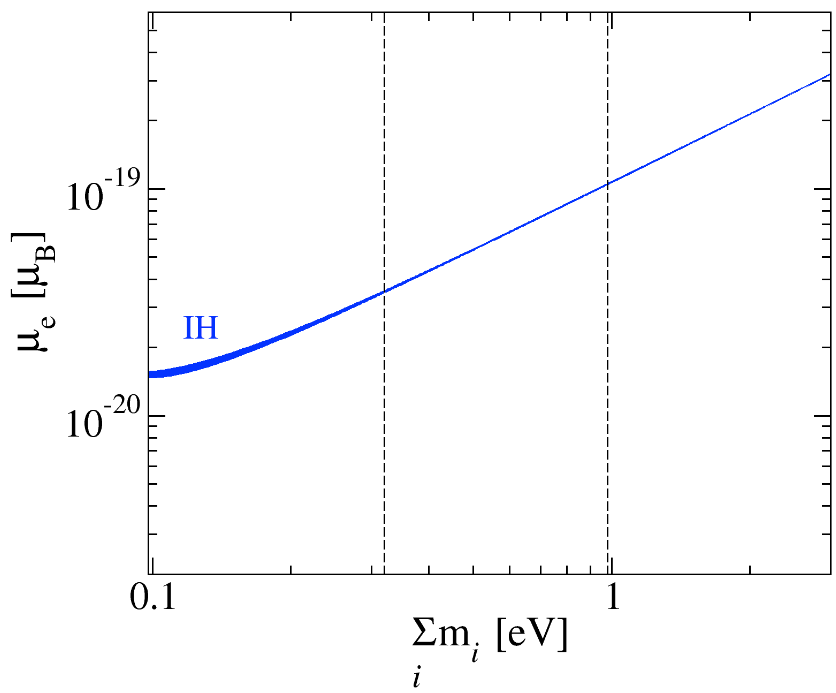
Standard Model (only) contribution to the Dirac neutrino magnetic moment

A.B.B., N. Vassh, arXiv:1312.6858

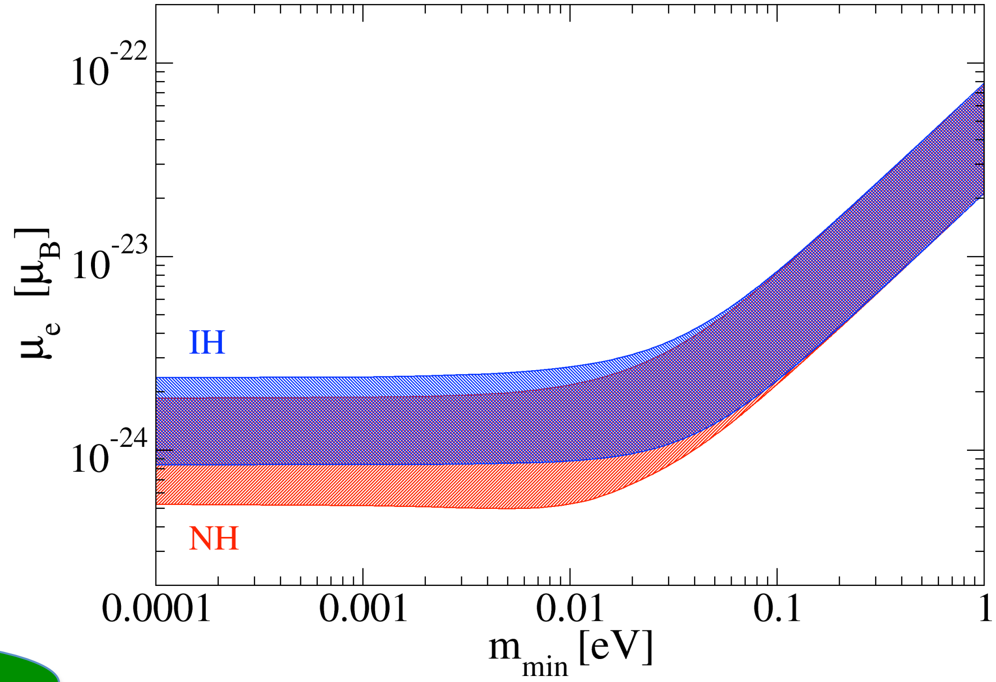


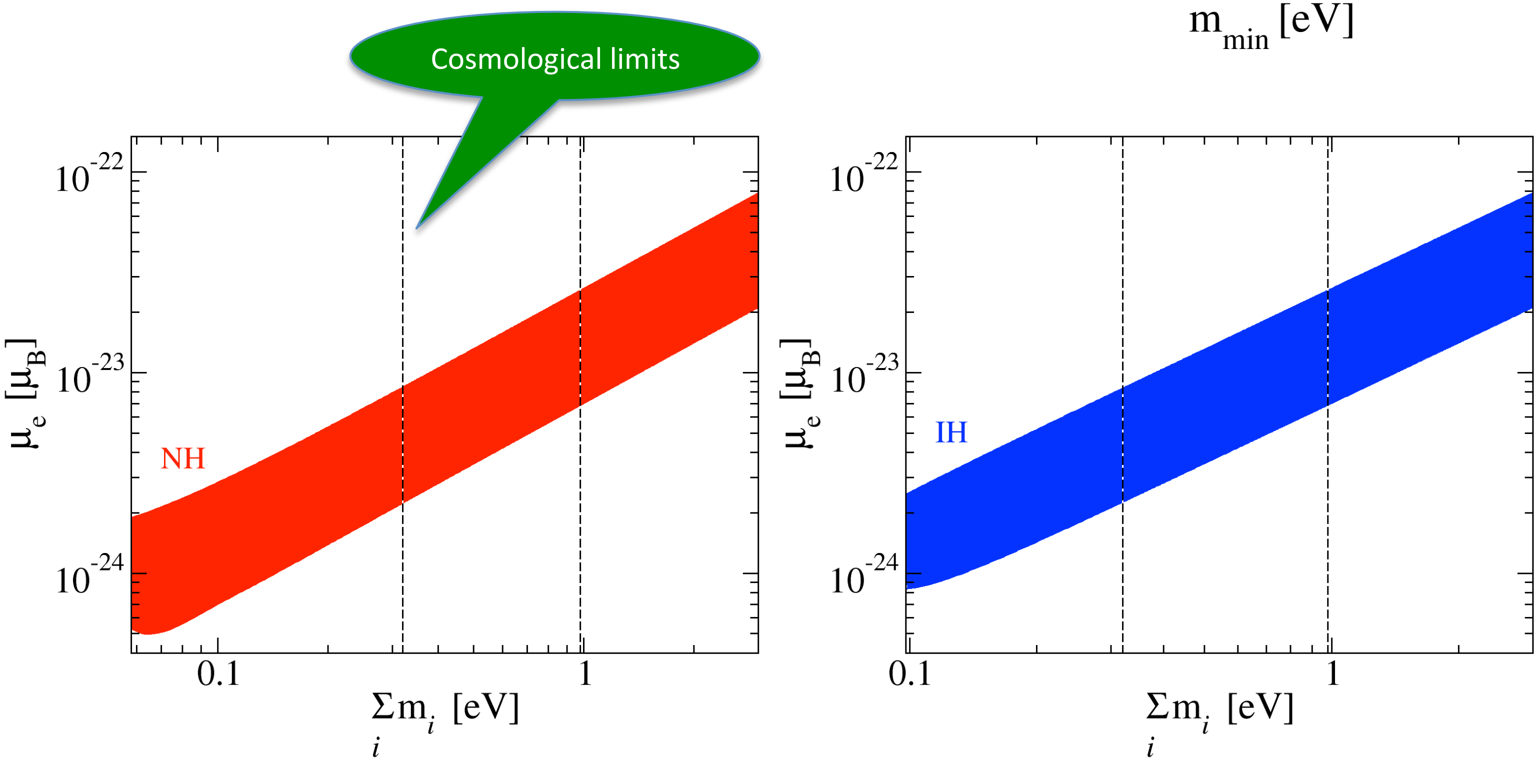
Cosmological limits

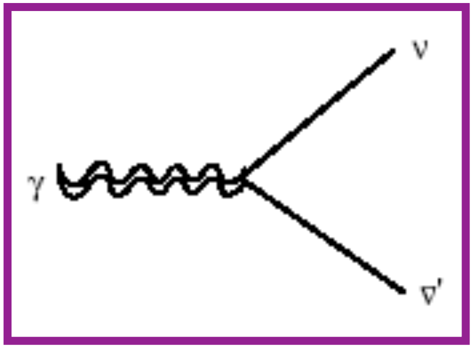




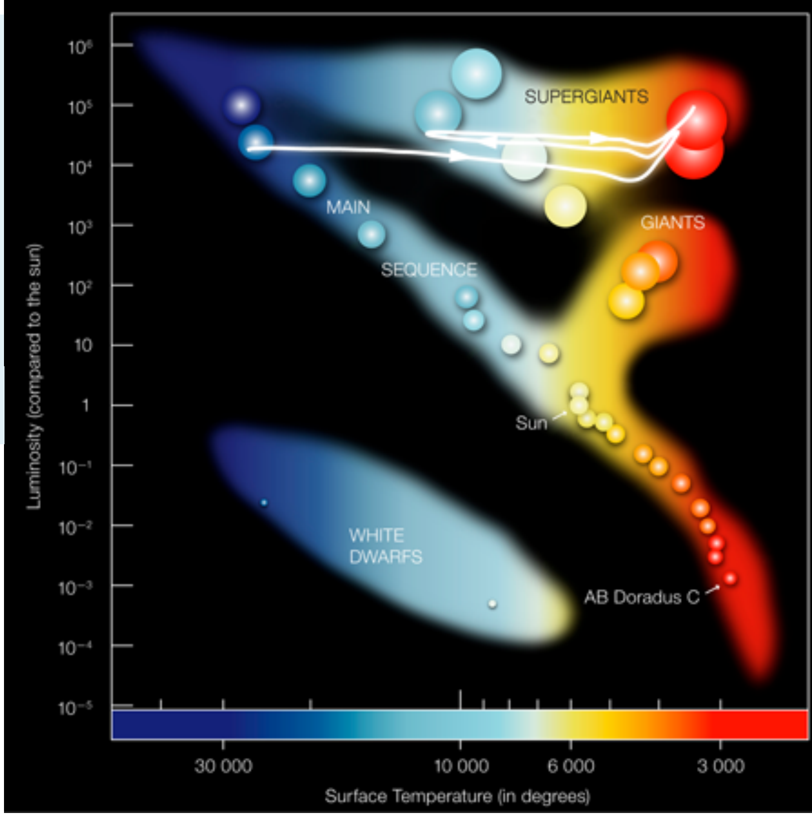
Standard Model (only) contribution to the Majorana neutrino magnetic moment

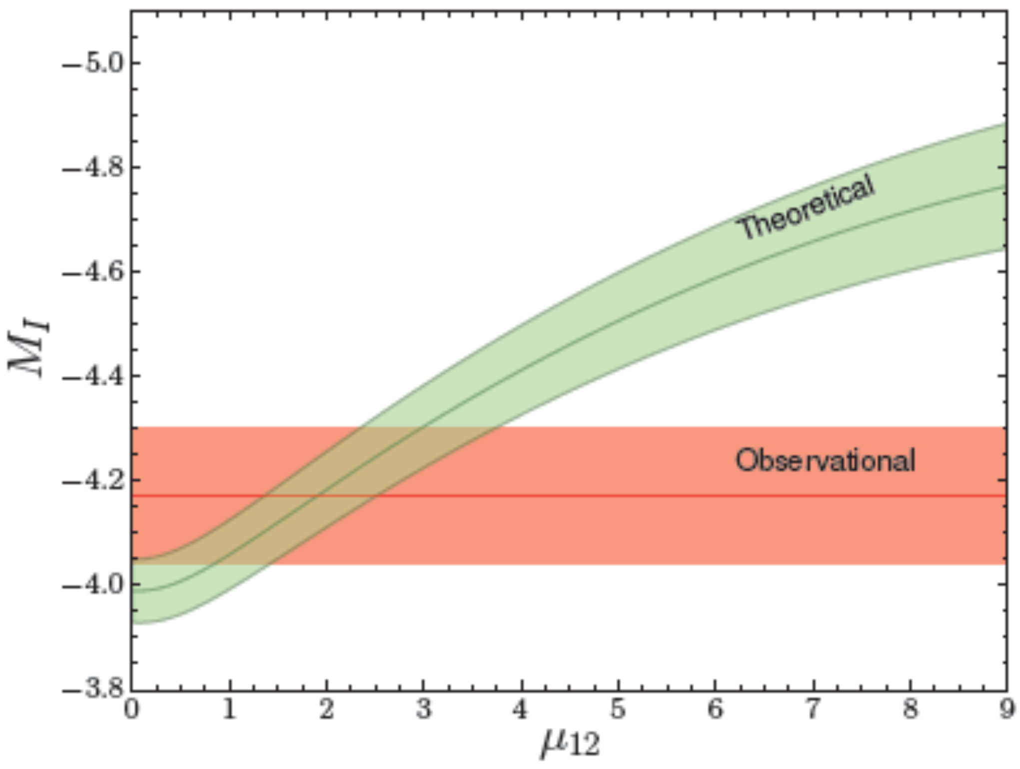






A large enough neutrino magnetic moment implies enhanced plasmon decay rate: $\gamma \to \nu\nu$. Since the neutrinos freely escape the star, this is turn cools a red giant star faster delaying helium ignition.





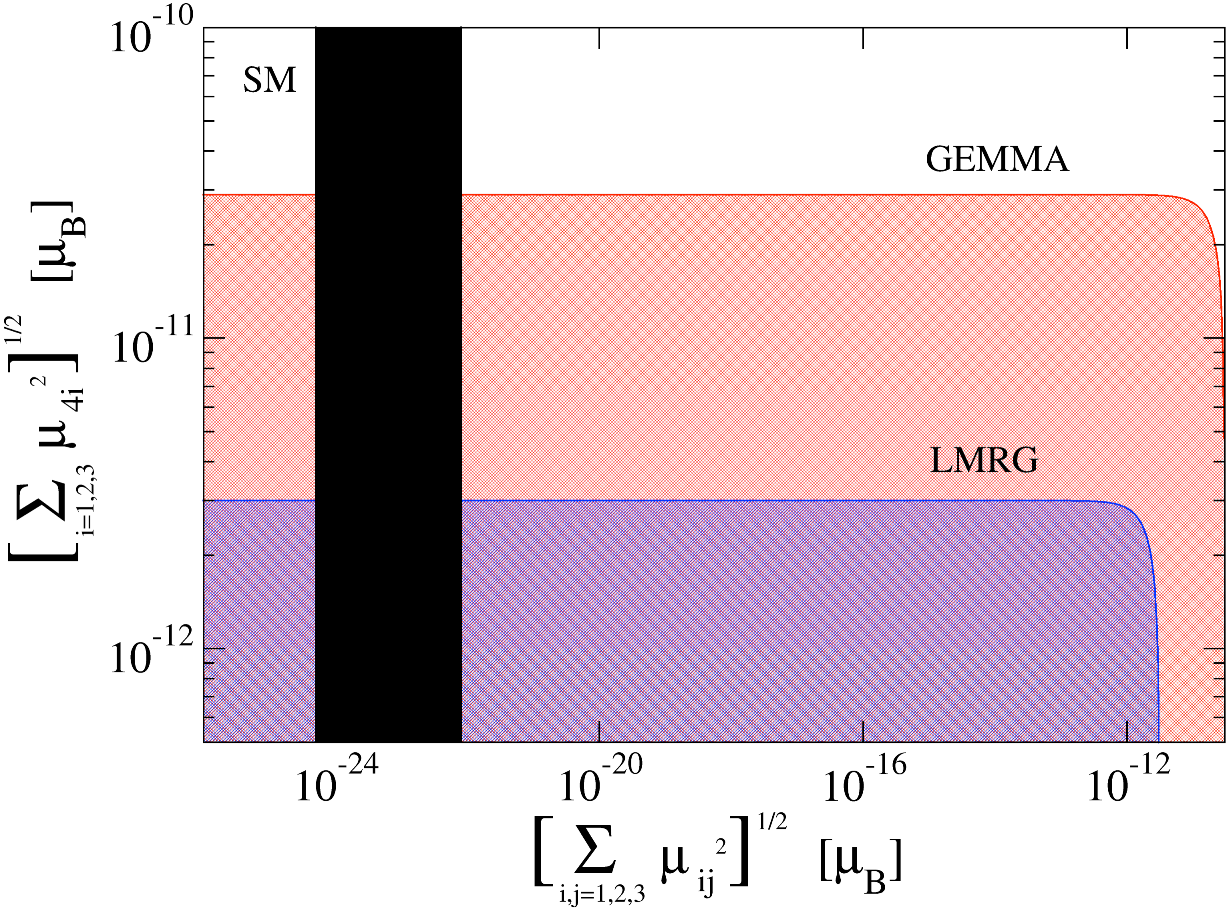
Globular cluster M5
➔ $\mu_\nu < 4.5 \times 10^{-12}\ \mu_B$ (95% C.L.)

arXiv:1308.4627

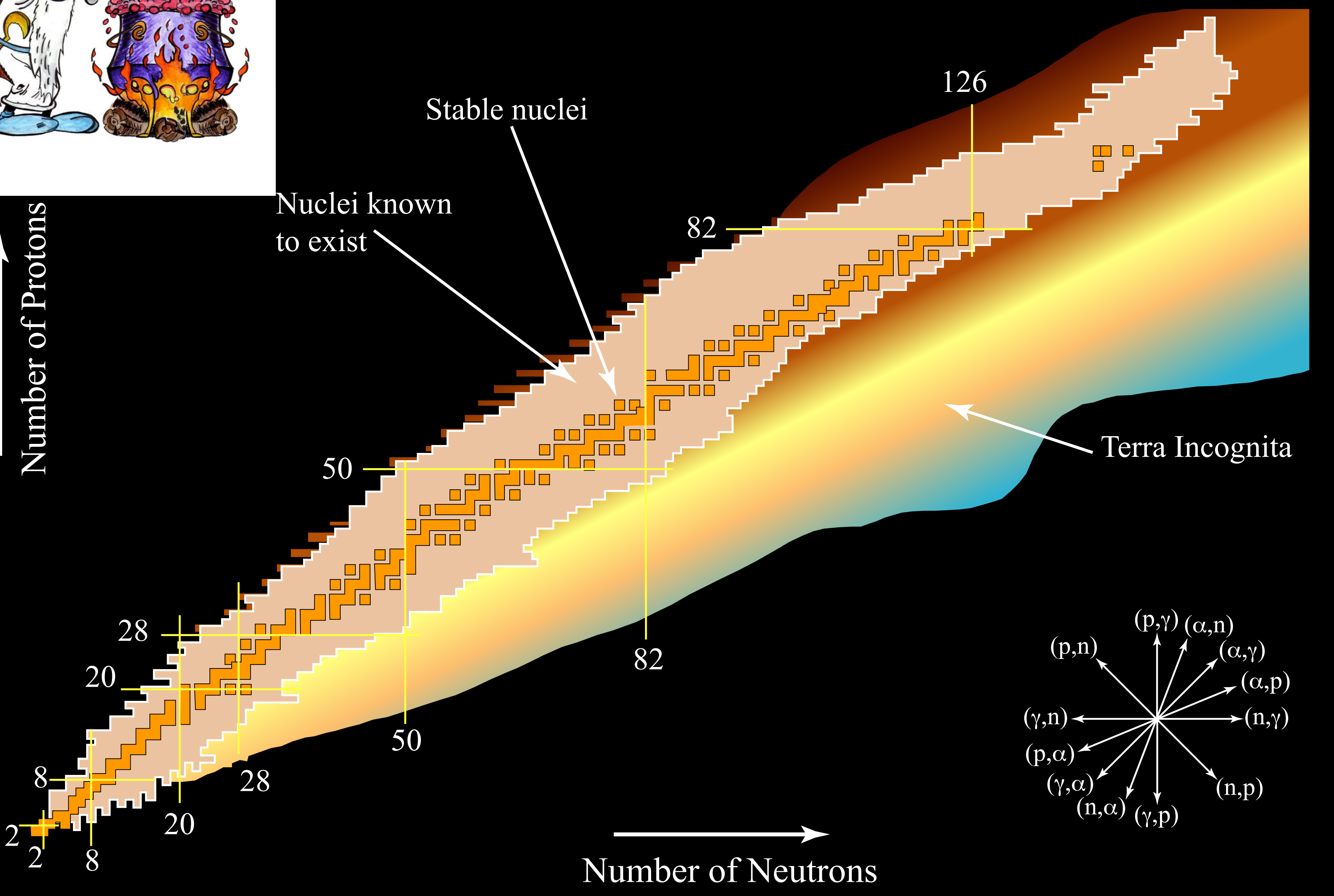
For a sufficiently heavy sterile neutrino the phases with $(E_4 - E_i)L$ average to zero

$$\mu_{eff}^2 = \sum_{i,j=1}^{3}\left[U_{ei}\left(\mu\mu^+\right)_{ij}U_{je}^+\right] + U_{e4}\left(\mu\mu^+\right)_{44}U_{4e}^+$$

$$\Rightarrow \mu_{eff}^2 \leq \sum_{i=1}^{3}\mu_{i4}^2 + \left(1-\left|U_{e4}\right|^2\right)\sum_{i,j=1}^{3}\mu_{ij}^2$$



How do you cook elements around us?



How do you cook elements around us?

How do you cook elements around us?

Pop III stars
(very big and very
metal poor)

How do you cook elements around us?

How do you cook elements around us?

Ti

Mg O

C

Sr

N

Ca

Fe Si

How do you cook elements around us?

How do you cook elements around us?

Some go supernova, producing U, Eu,Th… via the r-process

Pop II stars (metal poor)

AGB stars produce Ba, La, Y,…. via the s-process

Core-collapse supernovae are very sensitive to ν physics

Gravitational collapse yields very large values of the Fermi energy for electrons and ν_e's (~10^{57} units of electron lepton number). ν_μ's and ν_τ's are pair-produced, so they carry no μ or τ lepton number. Any process that changes neutrino flavor could increase electron capture and reduce electron lepton number.

Almost the entire gravitational binding energy of the progenitor star is emitted in neutrinos. Neutrinos transport entropy and the lepton number.

Electron fraction, or equivalently neutron-to-proton ratio (the controlling parameter for nucleosynthesis) is determined by the neutrino capture rates:

$$\nu_e + n \rightleftarrows p + e^-$$

$$\overline{\nu}_e + p \rightleftarrows n + e^+$$

λ_p: proton weak loss rate (rate for $\bar{\nu}_e + p \rightarrow e^+ + n$ and $e^- + p \rightarrow \nu_e + n$ reactions)

λ_n: neutron weak loss rate (rate for $\nu_e + n \rightarrow e^- + p$ and $e^+ + n \rightarrow \bar{\nu}_e + p$ reactions)

$$\frac{dN_p}{dt} = -\lambda_p N_p + \lambda_n N_n$$

Electron fraction: $Y_e \equiv \dfrac{\text{Net number of electrons}}{\text{Number of baryons}}$

Neutral medium, only protons and neutrons: $Y_e = \dfrac{N_p}{N_p + N_n}$

$$\frac{d}{dt} Y_e = \lambda_n - \left(\lambda_p + \lambda_n\right) Y_e$$

λ_p: proton weak loss rate (rate for $\bar{\nu}_e + p \to e^+ + n$ and $e^- + p \to \nu_e + n$ reactions)

λ_n: neutron weak loss rate (rate for $\nu_e + n \to e^- + p$ and $e^+ + n \to \bar{\nu}_e + p$ reactions)

$$\frac{dN_p}{dt} = -\lambda_p N_p + \lambda_n N_n$$

Electron fraction: $Y_e \equiv \dfrac{\text{Net number of electrons}}{\text{Number of baryons}}$

Neutral medium, only protons and neutrons: $Y_e = \dfrac{N_p}{N_p + N_n}$

Neutral medium, with protons, neutrons and alphas: $Y_e = \dfrac{N_p + 2N_\alpha}{N_p + N_n + 4N_\alpha}$

Mass fraction of alphas: $X_\alpha = \dfrac{4N_\alpha}{N_p + N_n + 4N_\alpha}$

$$\frac{d}{dt}\left[Y_e - \frac{1}{2}X_\alpha\right] = \lambda_n - \left(\lambda_p + \lambda_n\right)Y_e + \frac{1}{2}\left(\lambda_p - \lambda_n\right)X_\alpha$$

Vanishes if weak interactions of alphas are ignored

$dY_e/dt = 0$

$$Y_e = \frac{\lambda_n}{\lambda_p + \lambda_n} + \frac{1}{2}\frac{\lambda_p - \lambda_n}{\lambda_p + \lambda_n} X_\alpha$$

If alpha particles are present

$$Y_e^{(0)} = \frac{1}{1 + \lambda_p / \lambda_n}$$

If alpha particles are absent

$$Y_e = Y_e^{(0)} + \left(\frac{1}{2} - Y_e^{(0)}\right) X_\alpha$$

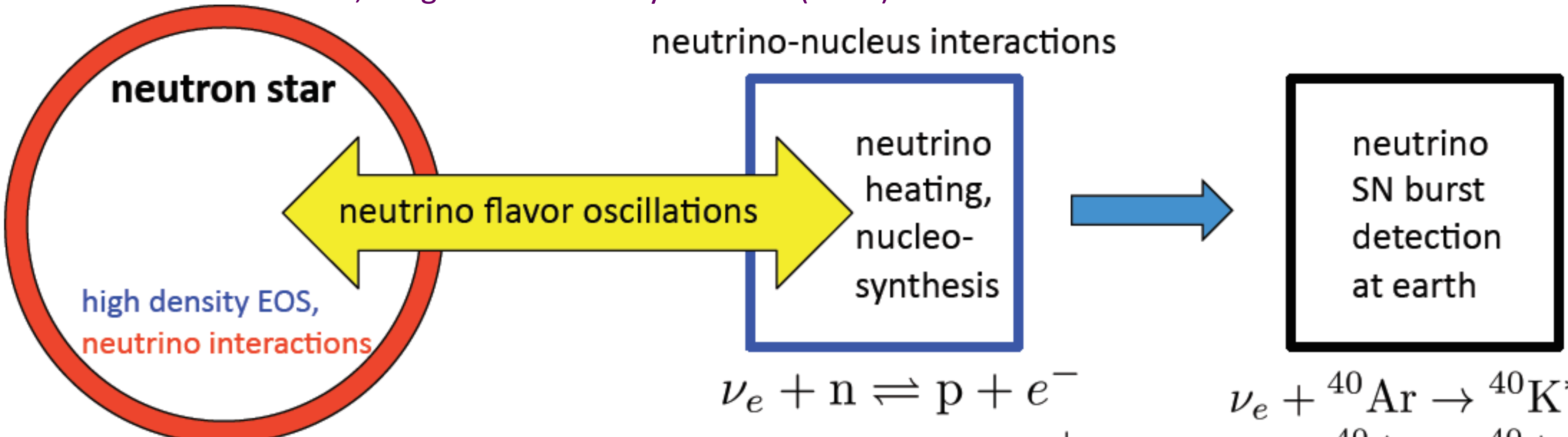
If $Y_e^{(0)} < 1/2$, non-zero X_α increases Y_e. If $Y_e^{(0)} > 1/2$, non-zero X_α decreases Y_e.

Non-zero X_α pushes Y_e to 1/2

Alpha effect

For example understanding a core-collapse supernova requires answers to a variety of questions some of which need to be answered by nuclear physics, both theoretically and experimentally.

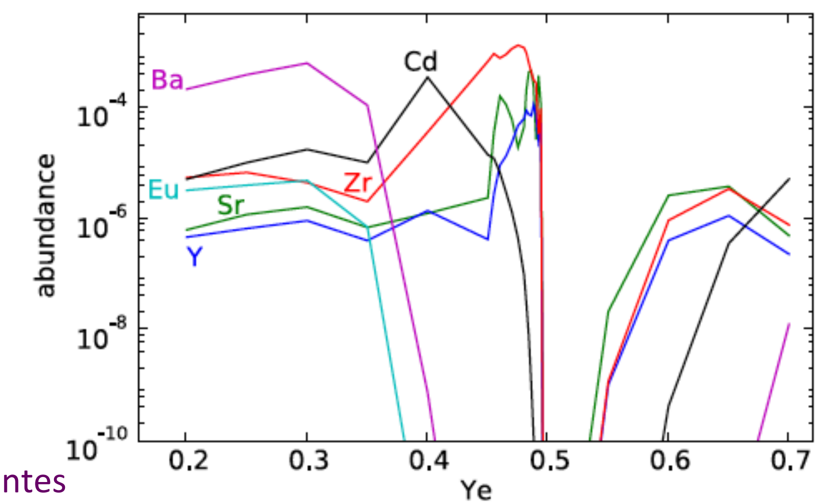
Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)



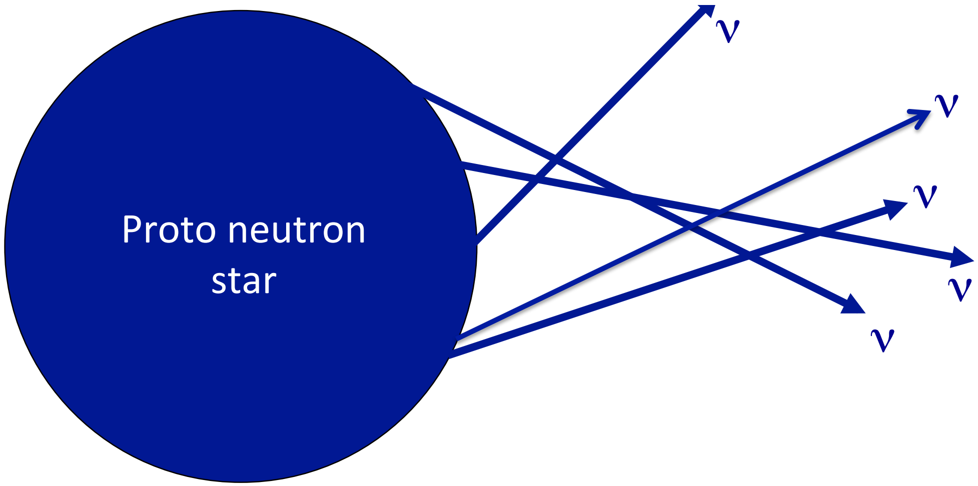
$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{Ar}^* + \nu_x$$

$$Y_e = \frac{N_p}{N_p + N_n} = \frac{1}{1 + \lambda_p / \lambda_n}$$



Arcones and Montes



Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
99% of this energy is carried away by neutrinos and antineutrinos!
~ 10^{58} Neutrinos!
This necessitates including the effects of $\nu\nu$ interactions!

$$H = \underbrace{\sum a^\dagger a}_{\text{describes neutrino oscillations interaction with matter (MSW effect)}} + \underbrace{\sum (1-\cos\theta) a^\dagger a^\dagger a a}_{\text{describes neutrino-neutrino interactions}}$$

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Many neutrino system

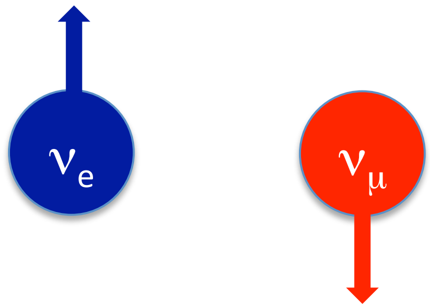
This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most ∼250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$
$$= \frac{\delta m^2}{4E}\cos 2\theta\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'\,\hat{1}$$

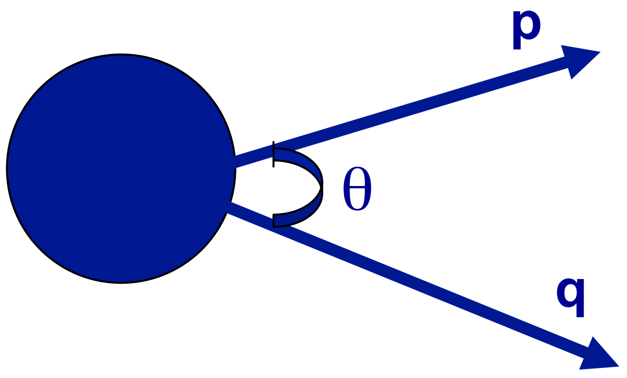
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\,\hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Raffelt, Duan, Balantekin, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp \left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

What is the mean-field approximation?

$$\left[\hat{O}_1, \hat{O}_2\right] \cong 0$$

$$\hat{O}_1\hat{O}_2 \approx \hat{O}_1\left\langle\hat{O}_2\right\rangle + \left\langle\hat{O}_1\right\rangle\hat{O}_2 - \left\langle\hat{O}_1\hat{O}_2\right\rangle$$

Expectation values should be calculated with a state $\left|\Psi\right\rangle$ chosen to satisfy:

$$\left\langle\hat{O}_1\hat{O}_2\right\rangle = \left\langle\hat{O}_1\right\rangle\left\langle\hat{O}_2\right\rangle$$

This reduces the two-body problem to a one-body problem:

$$a^\dagger a^\dagger aa \Rightarrow \left\langle a^\dagger a\right\rangle a^\dagger a + \left\langle a^\dagger a^\dagger\right\rangle aa + \text{h.c.}$$

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q \cong \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\left\langle\vec{\mathbf{J}}_p\right\rangle\cdot\vec{\mathbf{J}}_q$$

Mean field

Neutrino-neutrino interaction

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\nu L}\gamma_{\mu}\psi_{\nu L} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\left\langle \bar{\psi}_{\nu L}\gamma_{\mu}\psi_{\nu L}\right\rangle + \cdots$$

Antineutrino-antineutrino interaction

$$\bar{\psi}_{\bar{\nu} R}\gamma^{\mu}\psi_{\bar{\nu} R}\bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \Rightarrow \bar{\psi}_{\bar{\nu} R}\gamma^{\mu}\psi_{\bar{\nu} R}\left\langle \bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R}\right\rangle + \cdots$$

Neutrino-antineutrino interaction

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\left\langle \bar{\psi}_{\bar{\nu} R}\gamma_{\mu}\psi_{\bar{\nu} R}\right\rangle + \cdots$$

Balantekin and Pehlivan, JPG **34**,1783 (2007)

Neutrino-antineutrino can also have an additional mean field

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\psi_{\bar{\nu}R} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle \psi_{\bar{\nu}R} + \cdots$$

However note that

$$\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle \propto m_{\nu} \quad \text{(negligible if the medium is isotropic)}$$

Fuller *et al.*
Volpe