2014 National Nuclear Physics Summer School

WILLIAM & MARY

Lectures on Effective Field Theory

I. Removing heavy particles
II. Removing large scales
III. Describing Goldstone bosons
IV. Interacting with Goldstone bosons

Brian Tiburzi

The City College of New York

RIKEN BNL Research Center

CUNY

Effective Field Theory

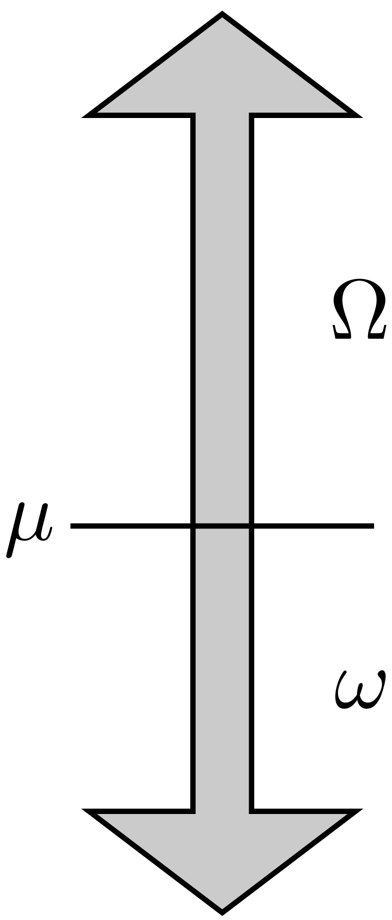
IV. Interacting with Goldstone bosons

“Bottom Up” EFTs

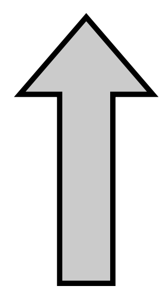
- Effective d.o.f. arise non-perturbatively

CMT: quasi-particles

QCD: low-lying hadrons



- End point: breakdown of EFT at higher-scale



Power corrections $(\omega/\Omega)^n$

Non-analytic corrections $\log \frac{\omega}{\Omega}$

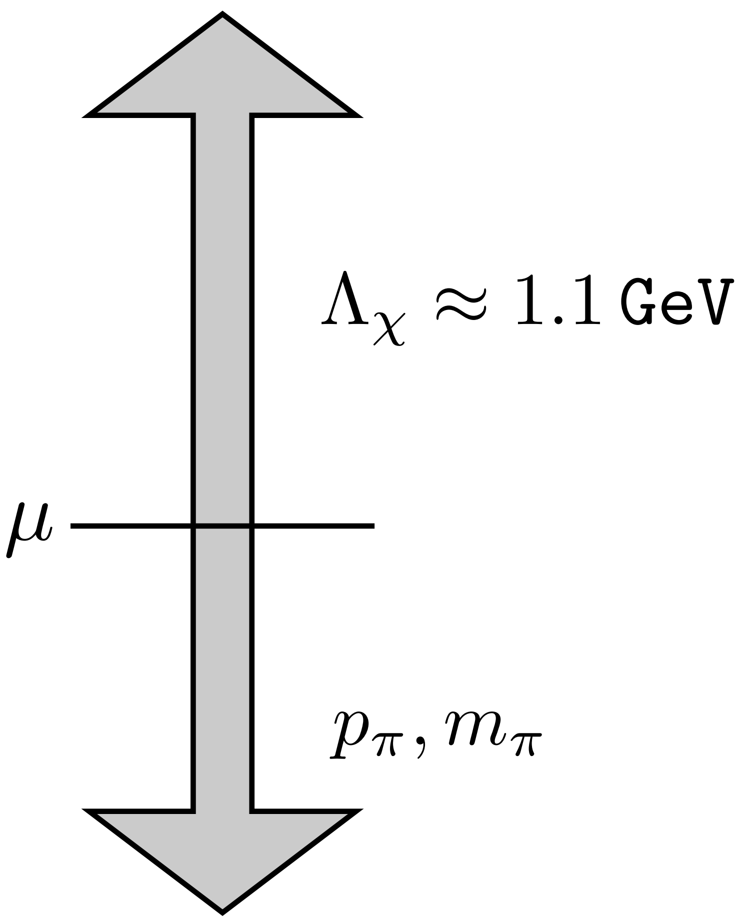
- Starting point: low-scale EFT

Captures non-analyticities of IR d.o.f.

Matching with experiment / non-perturbative calculation

- Focus: spontaneous symmetry breaking for which EFT is constructed to account for Goldstone modes. **QCD** *chiral symmetry & pions... now add* ***nucleon***

Nucleon in Chiral Perturbation Theory



$$\mathcal{L}_N = \overline{N}\,(i\not{D} - M_N)\,N$$

$$M_N = 0.94\,\texttt{GeV}$$

- ChPT is a low-energy effective theory

$$p_\mu = M_N\,v_\mu + k_\mu$$

- Include nucleon as an external flavor source, and describe small energy fluctuations about the nucleon mass

$$k \ll M_N \sim \Lambda_\chi$$

- Account for quark mass dependence, need chiral limit nucleon mass M

Nucleon in Chiral Perturbation Theory

- Digression: chiral limit mass out of nothing! $\langle N(\vec{k}\,)|T_{\mu\nu}|N(\vec{k}\,)\rangle = \dfrac{k_\mu k_\nu}{M}$

  QCD energy-momentum tensor $T_{\mu\nu}$ trace

  $$T^\mu{}_\mu = m_q \overline{\psi}\psi \qquad \langle N(\vec{k}\,)|T^\mu{}_\mu|N(\vec{k}\,)\rangle = M$$

  Classically $M = 0$

- Trace anomaly (QCD cannot be defined without a scale)

$$T^\mu{}_\mu = \frac{\beta}{2g} G^{\mu\nu} G_{\mu\nu} + m_q \overline{\psi}\psi$$

$$M_N = \left\langle N(\vec{k}\,) \left| \frac{\beta}{2g} G^{\mu\nu} G_{\mu\nu} + m_q \overline{\psi}\psi \right| N(\vec{k}\,) \right\rangle \stackrel{?}{=} M + A m_q + B m_q^2 + \ldots$$
$$= M + \mathcal{A} m_\pi^2 + \mathcal{B} m_\pi^4 + \ldots$$

- Higgs doesn't have a monopoly over all masses in the universe

Investigate:

Is the trace of the energy-momentum tensor the divergence of a current?

Heavy Nucleon ChPT

$$p_\mu = M\, v_\mu + k_\mu$$

- Large chiral limit mass phased away: derivative expansion is now valid

$$\mathcal{L} = \overline{N}_v\, iv\cdot\partial\,\mathcal{P}_+ N_v \qquad \partial_\mu N_v(x) \sim k_\mu, \qquad k_\mu \ll M \sim \Lambda_\chi$$

- Combine heavy nucleon limit with chiral perturbation theory:
  quark mass dependence of nucleon properties, pion-nucleon interactions, ...

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N_i \overset{SU(2)_V}{\longrightarrow} V_{ij} N_j$$

$$\Sigma = e^{2i\phi/f} \qquad \Sigma \overset{SU(2)_L \otimes SU(2)_R}{\longrightarrow} L \Sigma R^\dagger$$

- Actually it’s unknown to which chiral multiplet(s) the nucleon belongs

Assume simple $\quad N_R \to R N_R \quad N_L \to L N_L \quad$ N

Dressed nucleon $\quad \tilde{N}_R = \Sigma^\dagger N_L \quad \tilde{N}_L = \Sigma N_R \quad$ πN

- Exploit arbitrary nature for simplicity $\quad \xi = \sqrt{\Sigma} = e^{i\phi/f}$

Heavy Nucleon ChPT

Seems complicated: $\xi \to \sqrt{L\xi^2 R^\dagger} \equiv L\xi U^\dagger$ $\quad U = U\left(L, R, \xi(x)\right)$

Vector subgroup $\quad U(L = R = V, \xi) = V \quad \xi \to V\xi V^\dagger$

$$\Sigma \to L\xi^2 R^\dagger$$
$$\xi^2 \to L\xi U^\dagger L\xi U^\dagger$$

$$\Longrightarrow \quad \xi R^\dagger = U^\dagger L\xi U^\dagger \quad \Longrightarrow \quad U\xi R^\dagger = L\xi U^\dagger$$

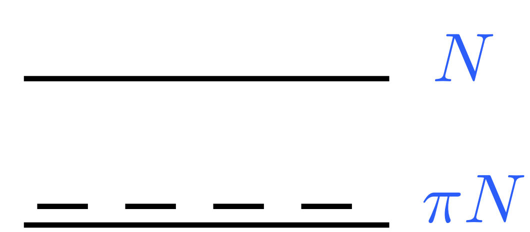
Differently dressed nucleon $\quad \breve{N}_R = \xi^\dagger N_L \quad \breve{N}_L = \xi N_R$

$\breve{N}_R \to U\breve{N}_R \quad \breve{N}_L \to U\breve{N}_L$

Dressed differently, chiral components transform the same way

Assume simple $\quad N_R \to RN_R \quad N_L \to LN_L$

Dressed nucleon $\quad \tilde{N}_R = \Sigma^\dagger N_L \quad \tilde{N}_L = \Sigma N_R$



- Exploit arbitrary nature for simplicity $\quad \xi = \sqrt{\Sigma} = e^{i\phi/f}$

Heavy Nucleon ChPT

- Actually it's unknown to which chiral multiplet(s) the nucleon belongs

1). Within a given chiral multiplet, nucleon field is not unique

2). Invent nucleon field with chiral components transforming the same

$$\xi \to L\xi U^\dagger = U\xi R^\dagger$$

3). Need not know unknown $N_i \to U_{ij} N_j$ (meets known $N_i \to V_{ij} N_j$)

4). Construct heavy nucleon chiral Lagrangian based on symmetry

$$\xi^\dagger \partial_\mu \xi \to U\xi^\dagger L^\dagger \partial_\mu \left(L\xi U^\dagger\right) = U\xi^\dagger \partial_\mu \xi U^\dagger + U\partial_\mu U^\dagger$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right) \Longrightarrow \mathcal{A}_\mu \to U\mathcal{A}_\mu U^\dagger$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right) \Longrightarrow \mathcal{V}_\mu \to U\mathcal{V}_\mu U^\dagger + U\partial_\mu U^\dagger$$

$$D_\mu N \equiv \partial_\mu N + \mathcal{V}_\mu N \qquad D_\mu N \to U(D_\mu N)$$

Heavy Nucleon Chiral Lagrangian

$$S^\mu = \left(0, \frac{\vec{\sigma}}{2}\right)$$

$$\mathcal{L} = N^\dagger i v \cdot D\, N + 2 g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$

$$D_\mu N = \partial_\mu N + \mathcal{V}_\mu N$$

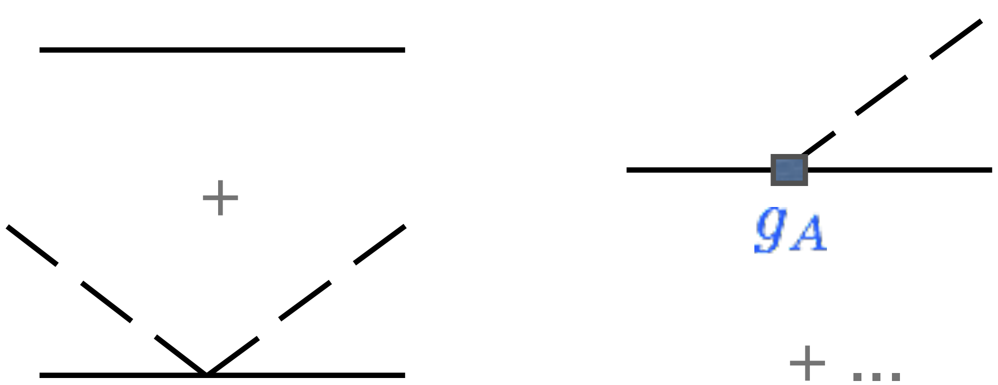
$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right)$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)$$

Two more invariants:

$$(N^\dagger v^\mu N)\, \mathrm{Tr}(\mathcal{V}_\mu)$$

$$(N^\dagger S^\mu N)\, \mathrm{Tr}(\mathcal{A}_\mu)$$



Axial coupling free parameter in chiral limit (depends upon chiral multiplet)

Vector coupling exactly fixed by pattern of chiral symmetry breaking

Heavy Nucleon Chiral Lagrangian

$$\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right) \qquad \mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)$$

- Include **external** vector and axial-vector fields: local chiral transformation

$$\xi^\dagger \partial_\mu \xi \to U \xi^\dagger L^\dagger \partial_\mu \left(L \xi U^\dagger\right) = U\xi^\dagger (L^\dagger \partial_\mu L)\xi U^\dagger + U\xi^\dagger \partial_\mu \xi U^\dagger + U \partial_\mu U^\dagger$$

$$L_\mu \to L L_\mu L^\dagger + i(\partial_\mu L) L^\dagger \qquad \xi^\dagger D_{L\mu} \xi = \xi^\dagger (\partial_\mu + i L_\mu)\xi$$

- Vector and axial-vector pion fields become gauged

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger D_{L\mu} \xi + \xi D_{R\mu} \xi^\dagger\right) \qquad \mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger D_{L\mu} \xi - \xi D_{R\mu} \xi^\dagger\right)$$

- Singlet couplings can be turned on externally

$$\mathrm{Tr}\left(\mathcal{V}_\mu\right) = \frac{i}{2}\mathrm{Tr}\left(L_\mu + R_\mu\right) = i\mathrm{Tr}\left(V_\mu\right) \qquad \mathrm{Tr}\left(\mathcal{A}_\mu\right) = -\frac{1}{2}\mathrm{Tr}\left(L_\mu - R_\mu\right) = \mathrm{Tr}\left(A_\mu\right)$$

Singlet vector coupling exactly fixed by electric charge assignments

$$D_\mu N = \left[\partial_\mu + \mathcal{V}_\mu + \mathrm{Tr}(\mathcal{V}_\mu)\right] N$$

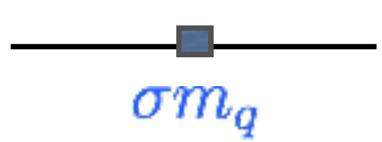
Quark Mass Dependence of the Nucleon

- Now turn on explicit chiral symmetry breaking due to the quark mass

$$\Delta\mathcal{L} = -\overline{\psi}_L s\, \psi_R - \overline{\psi}_R s^\dagger \psi_L \qquad s \to L\, s\, R^\dagger \qquad s = m_q + \cdots$$

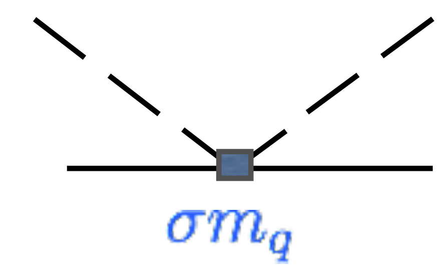
- Dress the scalar source with pions $\quad \xi \to L\xi U^\dagger = U\xi R^\dagger \qquad N \to UN$

$$\mathcal{M}_\pm = \frac{1}{2}\left(\xi s^\dagger \xi \pm \xi^\dagger s\, \xi^\dagger\right) \to U\mathcal{M}_\pm U^\dagger \qquad \mathcal{M}_\pm = m_q\left(\Sigma \pm \Sigma^\dagger\right) + \cdots$$



Leading quark mass dependence

$$\mathcal{L}_{m_q} = -\sigma N^\dagger \mathcal{M}_+ N + \mathcal{O}(m_q^2)$$



$$M_N = M + \sigma m_q + \ldots$$

Recall $\quad M_N = \left\langle N(\vec{k}\,)\left|\frac{\beta}{2g}G^{\mu\nu}G_{\mu\nu} + m_q\overline{\psi}\psi\right|N(\vec{k}\,)\right\rangle$

Aside: The Pion-Nucleon Sigma Term

$$\sigma_N \equiv \frac{1}{2M_N} \langle N(\vec{k})|\, m_q \overline{\psi}\psi \,|N(\vec{k})\,\rangle$$

$$m_q = \frac{1}{2}(m_u + m_d) \quad \overline{\psi}\psi = \overline{u}u + \overline{d}d$$

- Leading-order result

$$\sigma_N = \frac{\sigma m_q}{2M_N} + \ldots$$

- Sigma term relevant for: mass spectrum, strangeness content, quark mass ratios, pion-nucleon scattering, new physics searches, ...

$$\sigma_N = \frac{m_q}{2M_N}\frac{\partial M_N}{\partial m_q}$$

mass spectrum

$$y = \frac{\langle N(\vec{k})\,|\,\overline{s}s\,|\,N(\vec{k})\,\rangle}{\frac{1}{2}\langle N(\vec{k})\,|\,\overline{u}u + \overline{d}d\,|\,N(\vec{k})\,\rangle}$$

strangeness content

quark mass ratio

$$\left(\frac{m_s}{m_q} - 1\right)(1-y)\sigma_N = \frac{m_s - m_q}{2M_N}\langle N(\vec{k})|\overline{u}u + \overline{d}d - 2\overline{s}s|N(\vec{k})\rangle$$

End of lecture will be strange

Aside: The Pion-Nucleon Sigma Term

$$\sigma_N \equiv \frac{1}{2M_N}\langle N(\vec{k})|\, m_q\overline{\psi}\psi \,|N(\vec{k})\,\rangle$$

$$m_q = \frac{1}{2}(m_u + m_d) \quad \overline{\psi}\psi = \overline{u}u + \overline{d}d$$

- Leading-order result

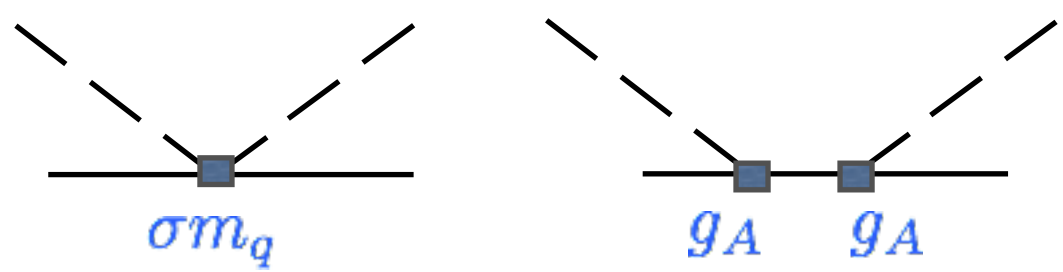
$$\sigma_N = \frac{\sigma m_q}{2M_N} + \ldots$$

- Sigma term relevant for: mass spectrum, strangeness content, quark mass ratios, pion-nucleon scattering, new physics searches, ...

Low-Energy Theorem (Cheng-Dashen)

pion-nucleon scattering

$$t = (k' - k)^2$$



$$D^{I=0}(\nu = 0, t = 2m_\pi^2) - \text{Born} = \frac{2\sigma_N}{f^2} + \ldots$$ (large corrections)

$$2M_N\sigma_N =$$

$$= \frac{2}{f^2}\left[\sigma_N(t = 2m_\pi^2) + \Delta_R\right]$$

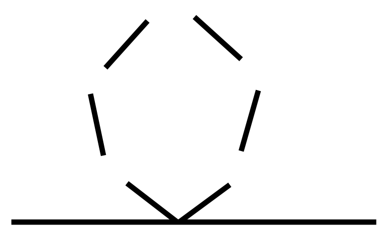
$45(8)\,\mathtt{MeV}\,[1990\text{'s}]$ $64(7)\,\mathtt{MeV}\,[2000\text{'s}]$ $39(4)\,\mathtt{MeV}\,[2010\text{'s}]$

(experiment coupled with ChPT analysis) (BMW nucleon spectrum from lattice QCD)

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $$\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$

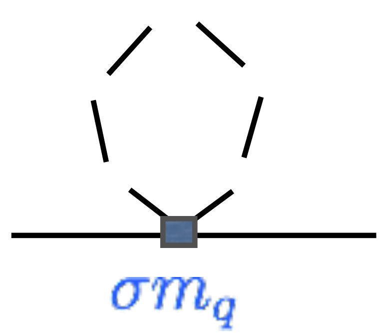


$$\sim \int d^4p\, \frac{1}{p^2}\, p = p^3$$

Odd powers!

$\mathcal{O}(p^2)$ $$\mathcal{L}_{m_q} = -\sigma N^\dagger \mathcal{M}_+ N$$

Form all one-loop diagrams from leading-order vertices

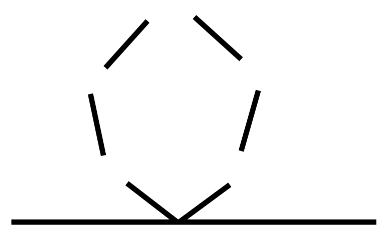


$$\sim p^2 \int d^4p\, \frac{1}{p^2} = p^4$$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots \quad \mathcal{O}(p^2)$
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $$\mathcal{L} = N^\dagger iv\cdot D\,N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$

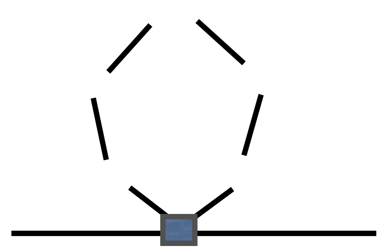


$$\sim \int d^4p\,\frac{v\cdot p}{p^2 - m_\pi^2} = 0$$

Odd powers!

$\mathcal{O}(p^2)$ $$\mathcal{L}_{m_q} = -\sigma N^\dagger \mathcal{M}_+ N$$

Form all one-loop diagrams from leading-order vertices



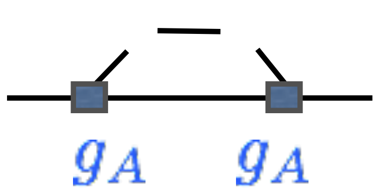
$$\sim p^2 \int d^4p\,\frac{1}{p^2} = p^4$$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$

- Power counting one-loop diagrams

$\mathcal{O}(p)$ $$\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$



$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{\vec{p}\cdot\vec{\sigma}\;\vec{p}\cdot\vec{\sigma}}{p_0\,(p^2 - m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

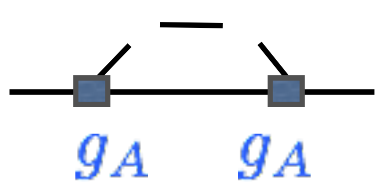
$$= 0$$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$

- Power counting one-loop diagrams

$\mathcal{O}(p)$ $$\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$



$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{\vec{p}\cdot\vec{\sigma}\;\vec{p}\cdot\vec{\sigma}}{p_0\,(p^2 - m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$\frac{i}{p\cdot v + i\epsilon} = \frac{i}{p_0 + i\epsilon} = i\,\mathrm{PV} + \pi\delta(p_0)$$

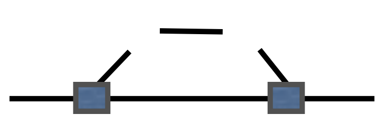
forward propagating heavy nucleon

Rest frame $v_\mu = (1,0,0,0)$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $$\mathcal{L} = N^\dagger i v \cdot D\, N + 2 g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$$



$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{\vec{p}\cdot\vec{\sigma}\; \vec{p}\cdot\vec{\sigma}}{p_0\,(p^2 - m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$\sim \int d\vec{p}\, \frac{\vec{p}\cdot\vec{\sigma}\;\; \vec{p}\cdot\vec{\sigma}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\, \frac{\vec{p}^{\,2}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\; - m_\pi^2 \int d\vec{p}\, \frac{1}{\vec{p}^{\,2} + m_\pi^2}$$

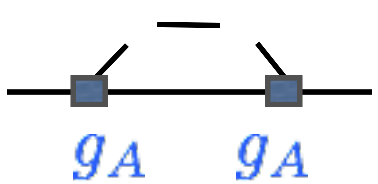
Λ^3 $\quad$ $m_\pi^2(\Lambda + \text{finite})$

Rest frame $v_\mu = (1,0,0,0)$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots \quad \mathcal{O}(p^2)$
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $\quad \mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger S^\mu \mathcal{A}_\mu\, N$



$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{\vec{p}\cdot\vec{\sigma}\;\vec{p}\cdot\vec{\sigma}}{p_0\,(p^2 - m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

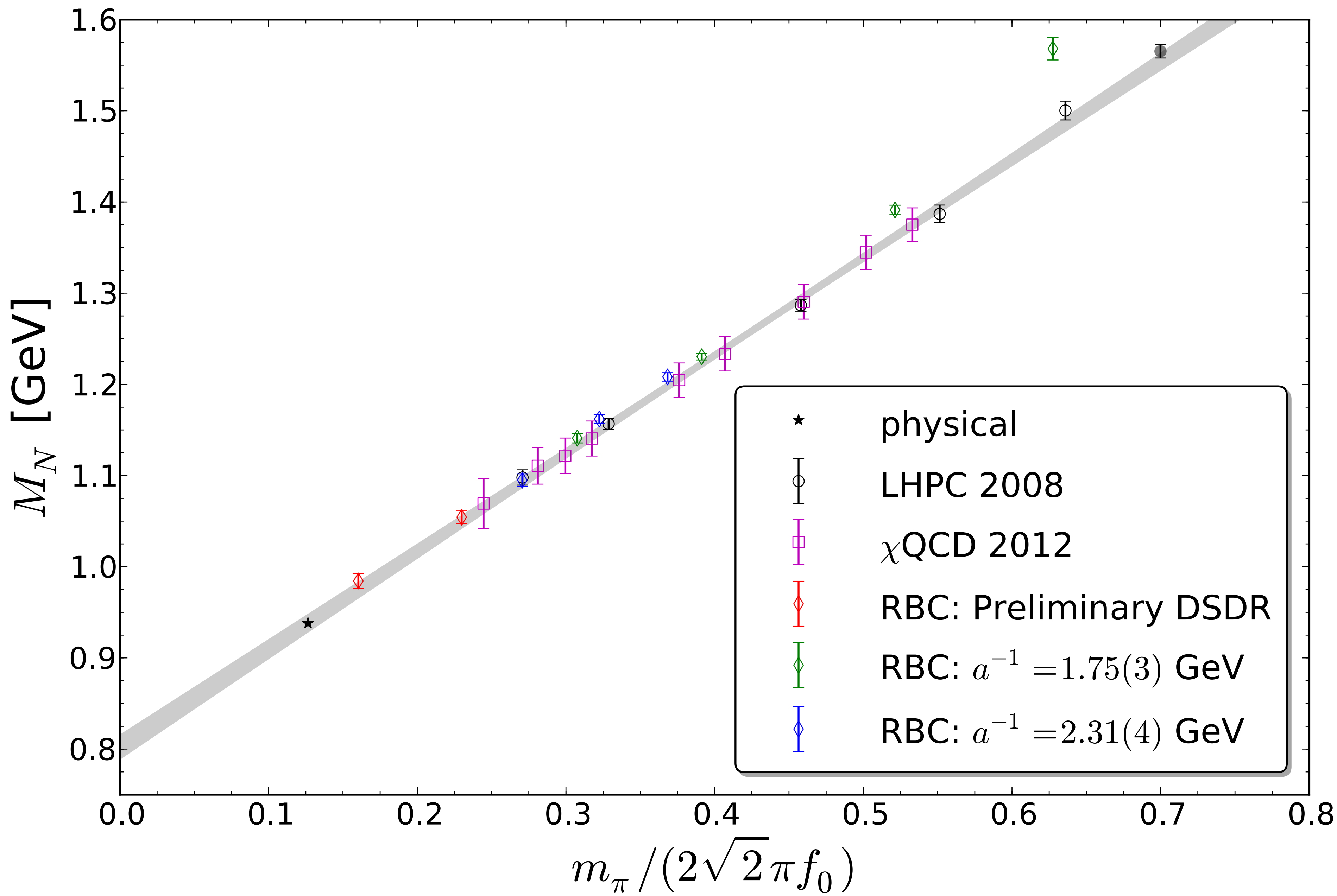
$$\sim \int d\vec{p}\, \frac{\vec{p}\cdot\vec{\sigma}\;\;\vec{p}\cdot\vec{\sigma}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\, \frac{\vec{p}^{\,2}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\; - m_\pi^2 \int d\vec{p}\, \frac{1}{\vec{p}^{\,2} + m_\pi^2}$$

$\Lambda^3 \qquad m_\pi^2(\Lambda + \text{finite})$

Or just use dimensional regularization $= m_\pi^3$

Chiral Expansion of Nucleon Mass

$$M_N = M + \mathcal{A}\, m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f)^2} m_\pi^3 + \mathcal{B}\, m_\pi^4 \left(\log \frac{\mu^2}{m_\pi^2} + C \right) + \dots$$



Compilation courtesy of A. Walker-Loud (Chiral Dynamics 2012)

Chiral Expansion of Nucleon Properties

- Can compute chiral corrections to matrix elements of various currents. For example: quark bilinears $\overline{\psi}\,\Gamma\,\psi$, four quark operators $\left(\overline{\psi}\,\Gamma_1\,\psi\right)\left(\overline{\psi}\,\Gamma_2\,\psi\right)$.

Isovector axial current $\langle N(\vec{p}\,')|J^{+}_{5\mu}|N(\vec{p})\rangle = u'^{\dagger}\left[2S_\mu G_A(q^2) + q_\mu S\cdot q\, G_P(q^2)\right] u$

$$G_A = g_A + Am_\pi^2\left(\log m_\pi^2 + B\right) + \ldots$$

$$< r_A^2 >= r^2 + A\, m_\pi^2\left(\log m_\pi^2 + B\right) + \ldots$$

$$G_P(q^2) = \frac{g_A}{q^2 - m_\pi^2} + \frac{< r_A^2 >}{3} + \mathcal{O}(m_\pi^2)$$

Form factors and radii

$$\mathcal{G}(q^2) = \mathcal{G}(0) - \frac{1}{6}q^2 < r^2 > + \ldots$$

Isovector EM current $\langle N(\vec{p}\,')|J^{+}_{\mu}|N(\vec{p})\rangle = u'^{\dagger}\left[v_\mu G_E(q^2) + \frac{\varepsilon_{ijk}q_j\sigma_k}{2M_N}G_M(q^2)\right] u$

$$< r_E^2 >= A\left(\log m_\pi^2 + B\right) + \mathcal{O}(m_\pi)$$

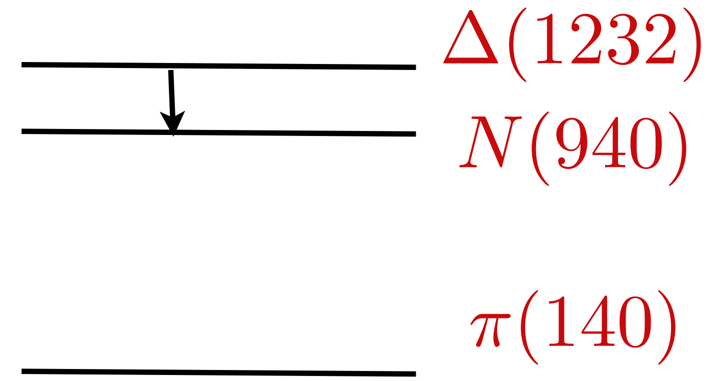
$$\mu = \mu_0 + Am_\pi + B\,m_\pi^2(\log m_\pi^2 + C)$$

$$< r_M^2 >= A\frac{1}{m_\pi} + B\,(\log m_\pi + C)$$

Experimental numbers

$$\sqrt{< r_A^2 >} = 0.65\,\mathtt{fm}$$

$$\sqrt{< r_E^2 >^{p-n}} = 0.94\,\mathtt{fm}$$



Delta Resonances $I = \frac{3}{2} \quad J^P = \frac{3}{2}^+$

- Low-energy expansion limited by nearest-lying excluded states

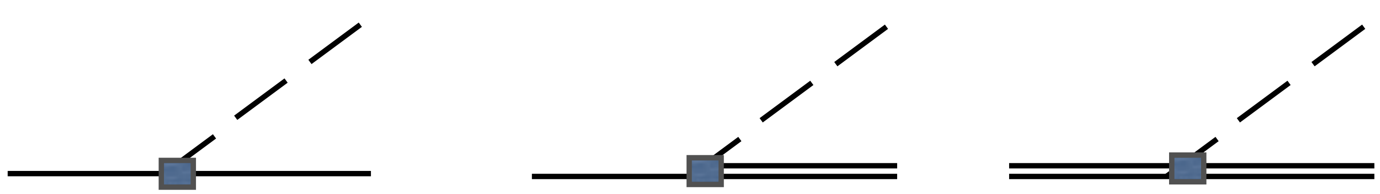
  $\Delta \equiv M_\Delta - M_N = 290\,\mathtt{MeV}$ strong decays $\Delta \to N\pi$

Higher resonances:
Too much momentum available for decays.

- New dimensionless parameter $\frac{m_\pi}{\Delta}$

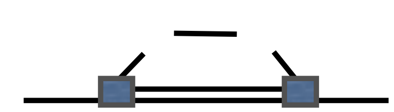
  $\frac{m_\pi}{\Delta} \ll 1$ chiral limit, nucleon only

  $\frac{m_\pi}{\Delta} \sim 1$ nucleon with explicit deltas



g_A	$g_{\Delta N}$	$g_{\Delta\Delta}$
1.27	~ 1.5	~ 2.2

δM_N



$\mathcal{O}(p^3)$

$$\frac{g_{\Delta N}^2}{(4\pi f)^2}\mathcal{F}\left(\frac{m_\pi}{\Delta}\right)$$

Exercises:

Write down all strong *isospin breaking* mass operators up to second order in the quark mass. What effect does isospin breaking in the pion mass have on the nucleon mass? Deduce the behavior of the nucleon mass splitting as a function of the quark masses.

In the chiral limit, the isovector axial current is a conserved current. Is there a constraint on the quark isovector axial charge due to the non-renormalization of this current? What about on the nucleon axial charge?

Reminder: Asymptotic Expansions

- Non-analytic quark mass dependence implies asymptotic expansion (but obviously so: zero radius of convergence)

Chiral expansion

$$m_\pi^2/\Lambda_\chi^2 \sim 0.02$$
$$m_\pi^2/m_\rho^2 \sim 0.03$$
$$m_\pi^2/m_\sigma^2 \sim 0.08$$

(may even be OK for larger-than-physical pion masses)

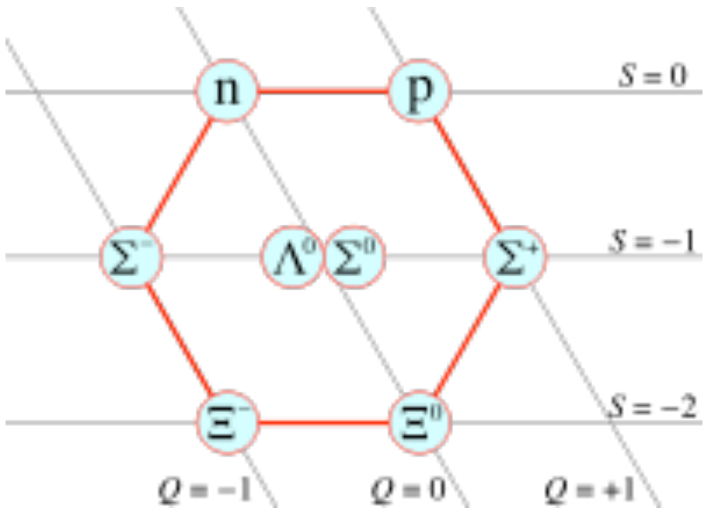
Heavy nucleon expansion $M \sim 800\,\mathtt{MeV}$ $\quad m_\pi/M \sim 0.2$

Delta resonance contributions

$$m_\pi/\Delta \sim 0.5$$
$$\Delta/\Lambda_\chi \sim 0.3$$

- Higher orders introduce more parameters (low-energy constants)
- Makes addressing convergence difficult without knowing the chiral limit values of these parameters

Heavy Baryon Chiral Perturbation Theory



Lowest lying spin-half baryons form an octet of $SU(3)_V$

$$B \to V B V^\dagger$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Couple to the Goldstone modes via $\xi = \sqrt{\Sigma} = e^{i\phi/f}$

$$\xi \to L \xi U^\dagger = U \xi R^\dagger$$

Free to choose the chiral transformation of baryon octet of the form $B \to U B U^\dagger$

$$\mathcal{A}_\mu \to U \mathcal{A}_\mu U^\dagger$$

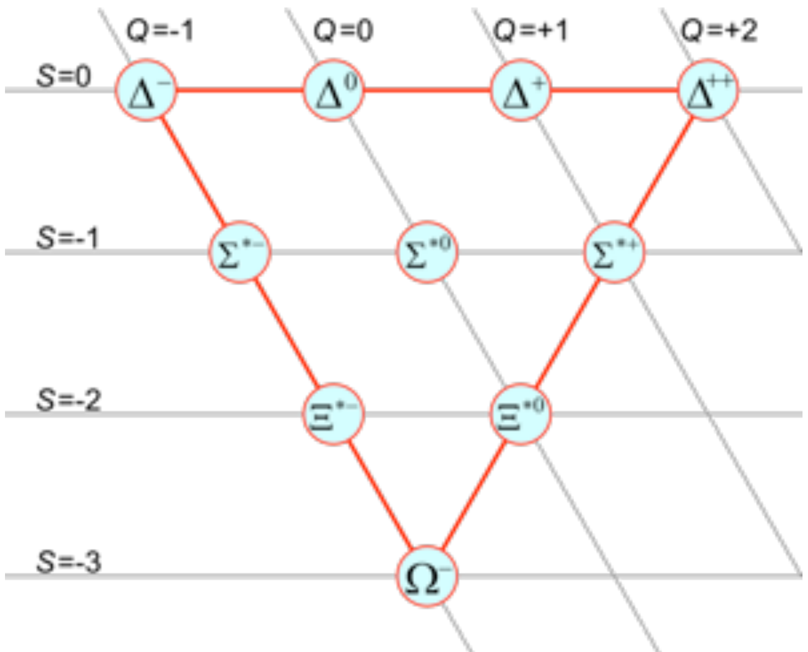
$$\mathcal{V}_\mu \to U \mathcal{V}_\mu U^\dagger + U \partial_\mu U^\dagger$$

$$D_\mu B = \partial_\mu B + [\mathcal{V}_\mu, B]$$

$\mathcal{O}(p)$ $$\mathcal{L} = \mathrm{Tr}\left(\overline{B} i v \cdot D\, B\right) + 2D\, \mathrm{Tr}\left(\overline{B} S^\mu \{A_\mu, B\}\right) + 2F\, \mathrm{Tr}\left(\overline{B} S^\mu [A_\mu, B]\right)$$

Phased away SU(3) chiral limit mass $M_B(m_q = m_s = 0)$

Heavy Baryon Chiral Perturbation Theory

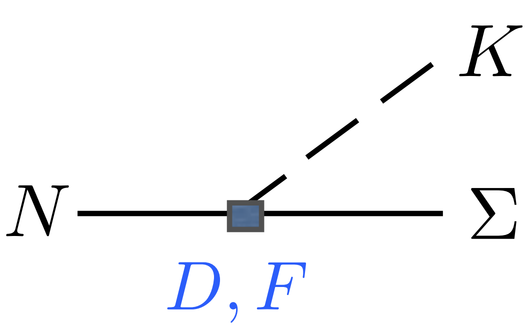


Lowest lying spin three-half baryons form a decuplet of $SU(3)_V$

$$T_{ijk} \to V_i{}^{i'} V_j{}^{j'} V_k{}^{k'} T_{i'j'k'}$$

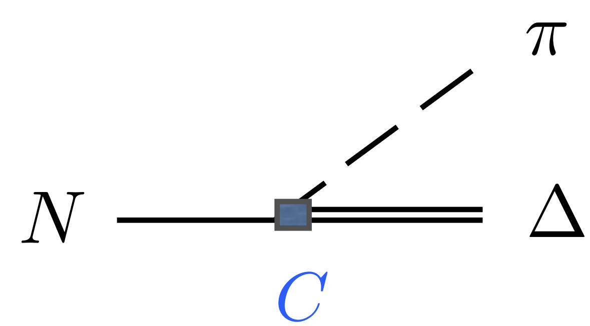
$$\Delta \equiv M_T - M_B = 270\,\texttt{MeV}$$

No question about inclusion for three flavors



$m_K \sim 500\,\texttt{MeV}$

$M_\Sigma \sim 1200\,\texttt{MeV}$



$m_\pi \sim 140\,\texttt{MeV}$

$M_\Delta \sim 1230\,\texttt{MeV}$

$\mathcal{O}(p)$
$$\mathcal{L} = -\overline{T}^\mu \left(iv \cdot D - \Delta\right) T_\mu + 2H\, \overline{T}^\mu S \cdot \mathcal{A}\, T_\mu + 2C \left(\overline{T}^\mu A_\mu B + \overline{B} A^\mu T_\mu\right)$$

Quark Mass Dependence of the Octet Baryons

- Now turn on explicit chiral symmetry breaking due to the quark masses

$$\Delta\mathcal{L} = -\overline{\psi}_L s\,\psi_R - \overline{\psi}_R s^\dagger \psi_L \qquad s \to L\,s\,R^\dagger \qquad s = m + \ldots$$

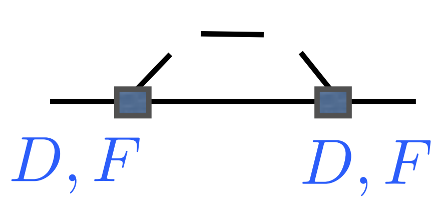
- Dress the scalar source with pions $\mathcal{M}_\pm = \frac{1}{2}\left(\xi s^\dagger \xi \pm \xi^\dagger s\,\xi^\dagger\right) \to U\mathcal{M}_\pm U^\dagger$

$$\mathcal{O}(p^2) \quad -\mathcal{L}_m = b_D \mathrm{Tr}\left(\overline{B}\{\mathcal{M}_+, B\}\right) + b_F \mathrm{Tr}\left(\overline{B}[\mathcal{M}_+, B]\right) + \sigma\,\mathrm{Tr}\left(\overline{B}B\right)\mathrm{Tr}\left(\mathcal{M}_+\right)$$

- Similar Gell-Mann Okubo constraint on octet baryon masses from tree-level

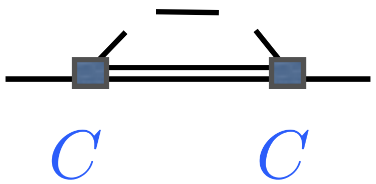
$$M_{\mathrm{GMO}} = M_\Lambda + \frac{1}{3}M_\Sigma - \frac{2}{3}M_N - \frac{2}{3}M_\Xi \stackrel{!}{=} 0 \qquad M_{\mathrm{GMO}}/\overline{M}_B \sim 1\%$$

- One-loop correction predicted in terms of axial couplings C, D, F



$$M_{\mathrm{GMO}} = \frac{4\pi}{3(4\pi f)^2}(D^2 - 3F^2)\left(\frac{4}{3}m_K^3 - m_\eta^3 - \frac{1}{3}m_\pi^3\right) + \mathrm{Decuplet}$$

Anatomy of Decuplet Contribution



- Intermediate-state angular momentum, pion p-wave $\quad p^{2\ell}$

$$\sim \frac{C^2}{f^2} \int d^4p \, \frac{\vec{p}^{\,2}}{[p_0 - \Delta + i\epsilon]\,[(p_0)^2 - E_\pi^2 + i\epsilon]} \qquad p_0 = \begin{cases} \Delta - i\epsilon \\ \pm E_\pi \mp i\epsilon \end{cases} \qquad E_\pi = \sqrt{\vec{p}^{\,2} + m_\pi^2}$$

poles

- Contour integration $\equiv$ anti-pion on shell

$$\sim \frac{C^2}{f^2} \int d\vec{p} \, \frac{\vec{p}^{\,2}}{E_\pi(E_\pi + \Delta)} \sim \frac{C^2}{f^2} \int_{m_\pi}^{\infty} dE_\pi \, \frac{(E_\pi^2 - m_\pi^2)^{3/2}}{E_\pi + \Delta}$$

two-body phase space near threshold $\sqrt{s - m_\pi^2}$

- Divergences

$$\int^{\Lambda} dE\, E^2 \left(1 - \frac{\Delta}{E} + \frac{\Delta^2}{E^2} - \frac{\Delta^3}{E^3} + \dots\right)\left(1 - \frac{3}{2}\frac{m_\pi^2}{E^2} + \dots\right)$$

$$\Lambda^3 + \Delta\Lambda^2 + \Delta^2\Lambda + \Delta^3 \log\Lambda$$
$$+ m_\pi^2\Lambda + \Delta m_\pi^2 \log\Lambda + \text{finite}$$

Renormalization condition

$$M_{N,\Sigma,\Lambda,\Xi}\big|_{m_q=0} = M_B$$

Gell-Mann Okubo Relation to One-Loop

$$M_{\mathrm{GMO}} = \frac{4\pi}{3(4\pi f)^2}\left[\pi(D^2 - 3F^2)\Delta_{\mathrm{GMO}}(m_\phi^3) - \frac{1}{6}C^2\Delta_{\mathrm{GMO}}\big(\mathcal{F}(m_\phi, \Delta, \mu)\big)\right]$$

$$\mathcal{F}(m,\delta,\mu) = (m^2 - \delta^2)\left[\sqrt{\delta^2 - m^2}\log\left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}}\right) - \delta\log\frac{m^2}{\mu^2}\right] - \frac{1}{2}\delta m^2\log\frac{m^2}{\mu^2}$$

Scale dependence?

Chiral limit?

M_{GMO}/M_B	Source	D	F	C
0.79%	ChPT	0.61	0.40	1.2
1.12%	Lattice QCD	0.72	0.45	1.6
1.29%	SU(6)	3/4	1/2	3/2

Experiment $M_{\mathrm{GMO}}/\overline{M}_B \sim 1\%$

- BUT: One-loop chiral corrections to the individual masses are LARGE

$$\delta M_N(\mu = \Lambda_\chi)/M_N = -39\%$$
$$\delta M_\Lambda(\mu = \Lambda_\chi)/M_\Lambda = -67\%$$
$$\delta M_\Sigma(\mu = \Lambda_\chi)/M_\Sigma = -89\%$$
$$\delta M_\Xi(\mu = \Lambda_\chi)/M_\Xi = -98\%$$

Expansion stranger with increasing strangeness

Heavy baryons

$$m_\pi/M_B \sim 0.1$$
$$m_K/M_B \sim 0.5$$
$$m_\eta/M_B \sim 0.5$$

Confronting SU(3): the strange quark mass

$SU(3)_L \otimes SU(3)_R$ $\qquad m_u, m_d \sim m_s \ll \Lambda_{\rm QCD}$

- Unless you're exceptionally lucky, the strange quark mass is probably too large for the success of SU(3) baryon chiral expansion...

- One approach: integrate out the heavy strange quark mass to use an SU(2) theory

$SU(2)_L \otimes SU(2)_R$ $\qquad m_u, m_d \ll m_s \sim \Lambda_{\rm QCD}$

- For the nucleon (and pion) this is just SU(2) chiral perturbation theory. **Done**

- For the nucleon, we treated it as a heavy external flavor source.
Nothing stops us from having strangeness in such a source.

... SU(2) chiral perturbation theory for strange hadrons

Limited predictive power, but ideal for lattice applications

Integrating out the strange quark

- Use the kaon mass to exemplify

$$m_K^2 = \frac{4\lambda}{f^2}(m_q + m_s) + \ldots = \frac{1}{2}m_\pi^2 + M_K^2 + \ldots = M_K^2\left(1 + \frac{m_\pi^2}{2M_K^2}\right) + \ldots$$

$$M_K \equiv m_K\Big|_{m_q=0}$$

Estimate using SU(3) and pion, kaon masses

$$M_K = 0.486(5)\,\texttt{GeV} \qquad m_{K^0} = 0.497\,\texttt{GeV}$$

- Consider the SU(3) expansion of the Sigma baryon mass, schematically

$$M_\Sigma = M_B + a m_K^2 + b m_K^3 + \ldots$$

Expand out the strange quark contribution

$$M_\Sigma = M_B + a' M_K^2 + a'' m_\pi^2 + b' M_K^3 + b'' M_K m_\pi^2 + b''' \frac{1}{M_K} m_\pi^4 + \ldots$$

Reorganize into SU(2) chiral limit expansion

$$M_\Sigma = M_\Sigma^{(2)} + \sigma_{\pi\Sigma}\, m_\pi^2 + A m_\pi^3 + B m_\pi^4 \left(\log m_\pi^2 + C\right) + \ldots$$

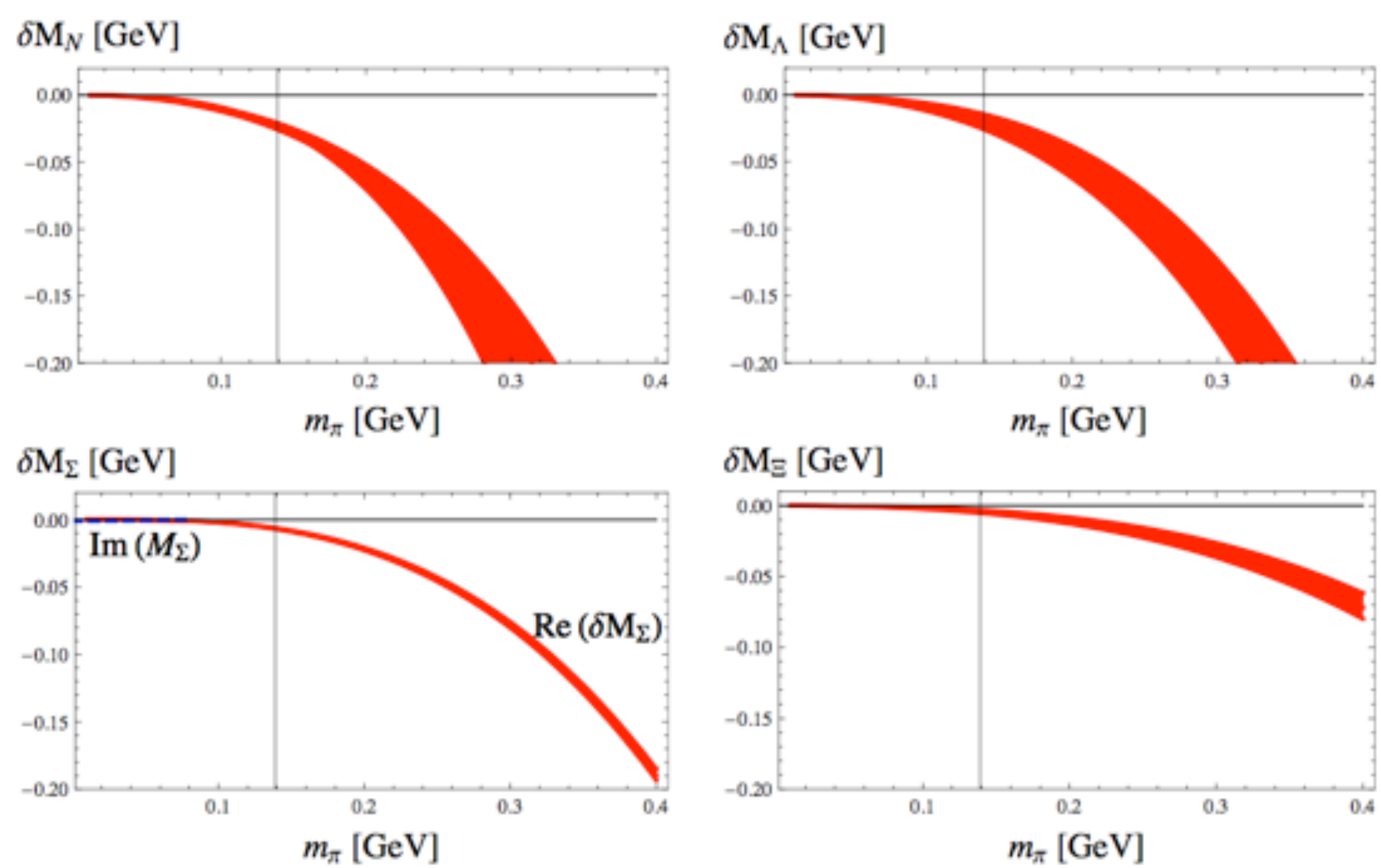
E.g. SU(2) Chiral Perturbation Theory for Hyperons

	$SU(3)$	$SU(2)_{S=0}$	$SU(2)_{S=1}$	$SU(2)_{S=2}$	$SU(2)_{S=3}$
Expansion Parameters	p m_π, m_K, m_η Δ	p m_π $\Delta_{\Delta N}$	p m_π $\Delta_{\Sigma\Lambda}, \Delta_{\Sigma^*\Sigma}$	p m_π $\Delta_{\Xi^*\Xi}$	p m_π
Baryon Multiplets	$\mathbf{8}\,B$ $\mathbf{10}\,T$	$\mathbf{2}\,N$ $\mathbf{4}\,\Delta$	$\mathbf{1}\,\Lambda, \mathbf{3}\,\Sigma$ $\mathbf{3}\,\Sigma^*$	$\mathbf{2}\,\Xi$ $\mathbf{2}\,\Xi^*$	$\mathbf{1}\,\Omega$
Axial Couplings	D, F C H	g_A $g_{\Delta N}$ $g_{\Delta\Delta}$	$g_{\Lambda\Sigma}, g_{\Sigma\Sigma}$ $g_{\Lambda\Sigma^*}, g_{\Sigma\Sigma^*}$ $g_{\Sigma^*\Sigma^*}$	$g_{\Xi\Xi}$ $g_{\Xi\Xi^*}$ $g_{\Xi^*\Xi^*}$	

$\mathcal{O}(p^3)$

cf. SU(3) expansion at physical pion mass

$$\delta M_N(\mu = \Lambda_\chi)/M_N = -39\%$$
$$\delta M_\Lambda(\mu = \Lambda_\chi)/M_\Lambda = -67\%$$
$$\delta M_\Sigma(\mu = \Lambda_\chi)/M_\Sigma = -89\%$$
$$\delta M_\Xi(\mu = \Lambda_\chi)/M_\Xi = -98\%$$

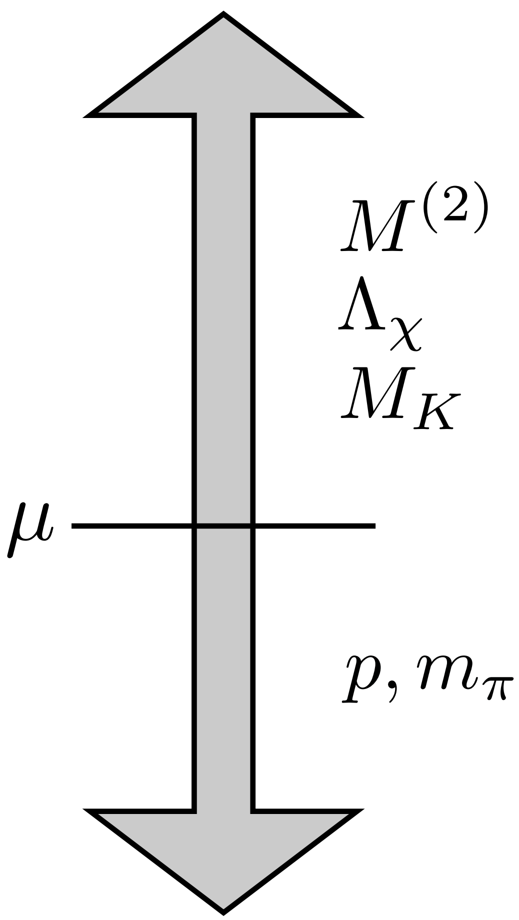


SU(2) chiral expansion $\frac{m_\pi^2}{\Lambda_\chi^2}, \quad \frac{m_\pi^2}{2M_K^2}$

Heavy baryon expansion $\frac{m_\pi}{M^{(2)}}$

Summary
IV. Interacting with Goldstone bosons

- Spontaneous symmetry breaking can be systematically addressed in EFTs



- EFT construction is “bottom down”

  In essence effective d.o.f. arise non-perturbatively

- Goldstone boson dynamics consequence of pattern of spontaneous and explicit symmetry breaking

  ... also true of their interactions with low-lying hadrons

- **E.g.** Chiral perturbation theory provides the tool to account for light quark mass dependence of low-energy QCD observables.

  Size of light quark mass controls efficacy compared with chiral limit mass of included low-lying hadrons