2014 National Nuclear Physics Summer School

Lectures on Effective Field Theory

I. Removing heavy particles
II. Removing large scales
III. Describing Goldstone bosons
IV. Interacting with Goldstone bosons

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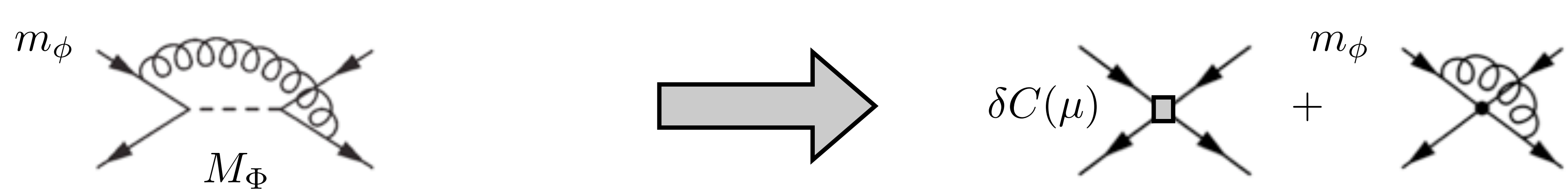
Effective Recap

- Heavy particles can be systematically integrated out resulting in EFTs

Coordinate Space

$$S_{\text{eff}}[\phi] = -\frac{1}{2}\int d^4x\, d^4y\, J(x)G(x-y)J(y)$$

$$S_{\text{eff}}[\phi] = \frac{1}{2M_\Phi^2}\int d^4x\, C(\mu)\, \mathcal{O}(x,\mu) + \ldots$$



Full Theory

Effective Theory

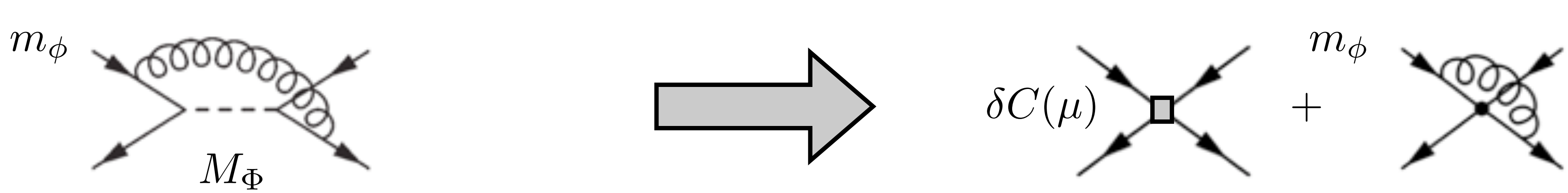
- OPE: product of two currents represented as a tower of local operators

Effective Recap

- Heavy particles can be systematically integrated out resulting in EFTs

Momentum Space

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-M_\Phi^2}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots \qquad\qquad \delta C(\mu)-\frac{1}{M_\Phi^2}\int_\mu \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots$$



Full Theory

Effective Theory

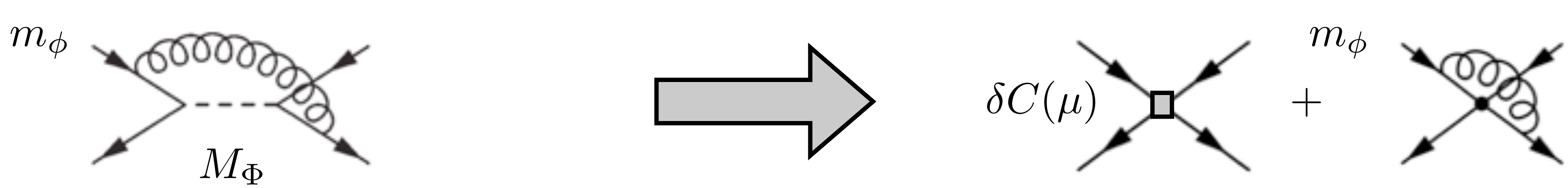
Effective Recap

- Heavy particles can be systematically integrated out resulting in EFTs

IR non-analyticities reproduced

Momentum Space

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-M_\Phi^2}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots \qquad\qquad \delta C(\mu)-\frac{1}{M_\Phi^2}\int_\mu \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots$$



Full Theory

Effective Theory

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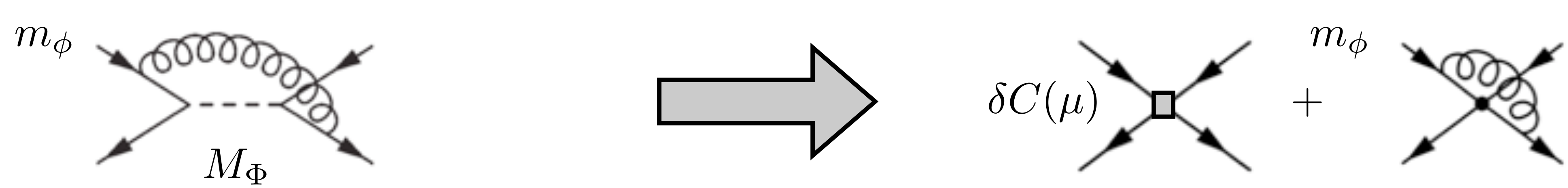
Momentum Space

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M_\Phi^2} \frac{1}{k^2 - m_\phi^2} \frac{1}{k^2} \cdots \qquad\qquad \delta C(\mu) - \frac{1}{M_\Phi^2} \int_\mu \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\phi^2} \frac{1}{k^2} \cdots$$

UV finite

UV divergent



UV singularity

$$\frac{1}{k^2 - M_\Phi^2} = -\frac{1}{M_\Phi^2}\left(1 + \frac{k^2}{M_\Phi^2} + \cdots\right)$$

analytic expansion

Full Theory

Effective Theory

Effective Recap

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IR non-analyticities reproduced

Momentum Space

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-M_\Phi^2}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots \qquad\qquad \delta C(\mu) - \frac{1}{M_\Phi^2}\int_\mu \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_\phi^2}\frac{1}{k^2}\cdots$$

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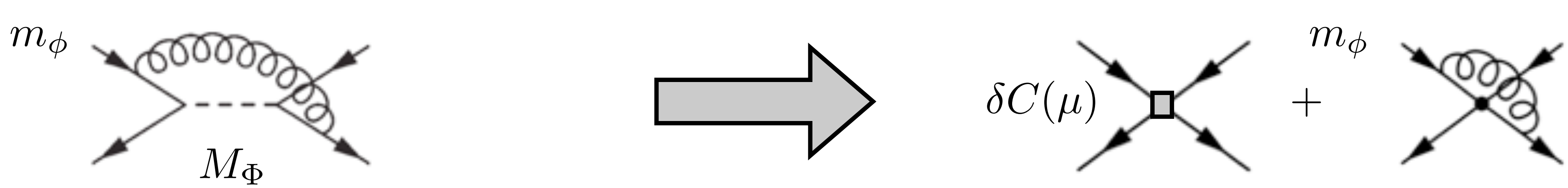

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$$\frac{1}{k^2-M_\Phi^2} = -\frac{1}{M_\Phi^2}\left(1+\frac{k^2}{M_\Phi^2}+\cdots\right)$$

analytic expansion

Full Theory

Effective Theory

- Identifying effective d.o.f = retaining their non-analyticities, all else $\approx$ analytic

Effective Field Theory

II. Removing large scales

Large Scales

- Large mass but now retain the heavy particle in the EFT

Heavy Quark Effective Theory (HQET)

*Heavy Baryon Chiral EFT

- Interactions of multiple heavy particles

Non-Relativistic QED (NRQED)
Non-Relativistic QCD (NRQCD)

*Nucleon-Nucleon EFT$(\not\pi)$

- Particles with large energies, jets, ...

Soft-Collinear Effective Theory (SCET)

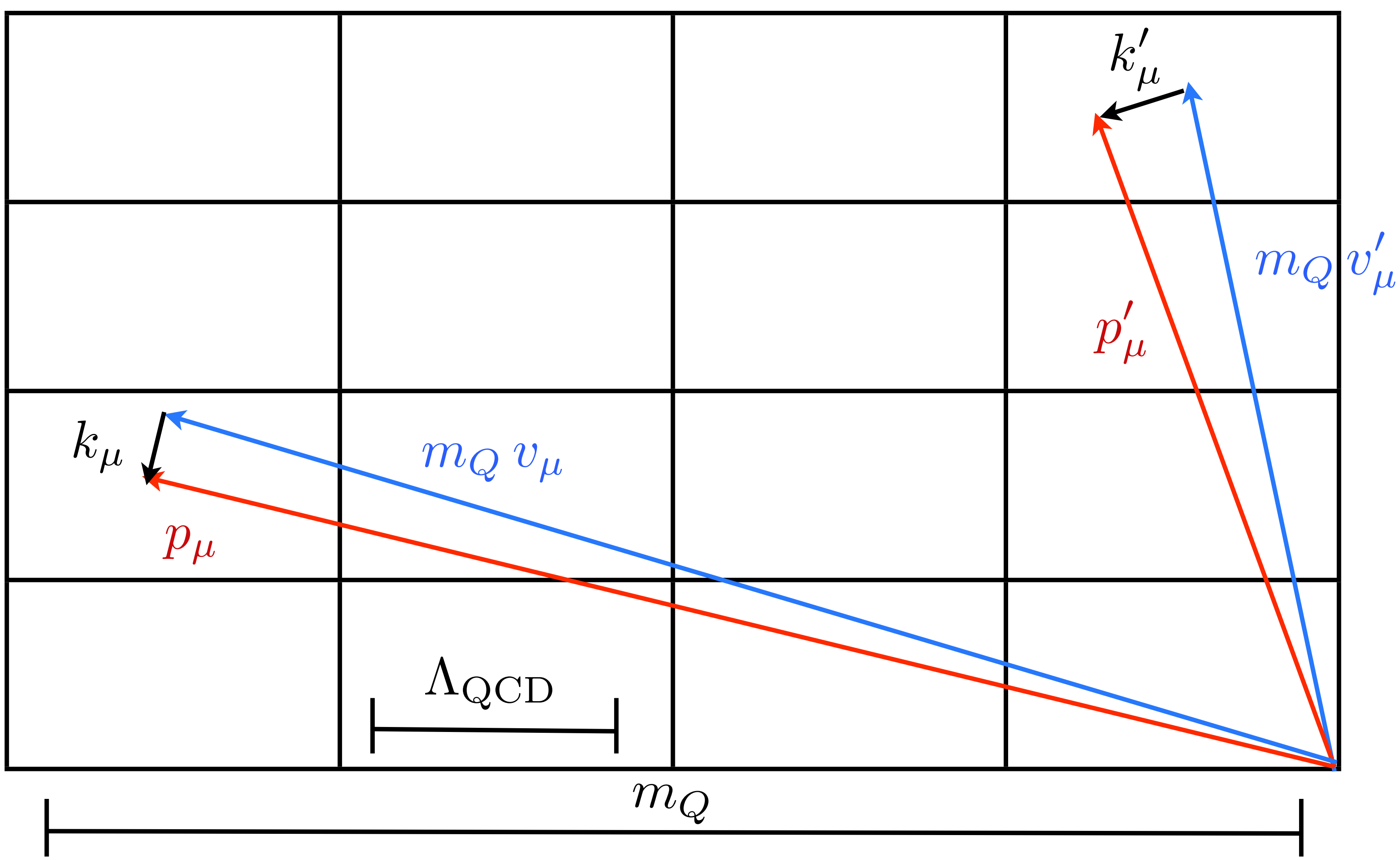
* "bottom up" EFTs

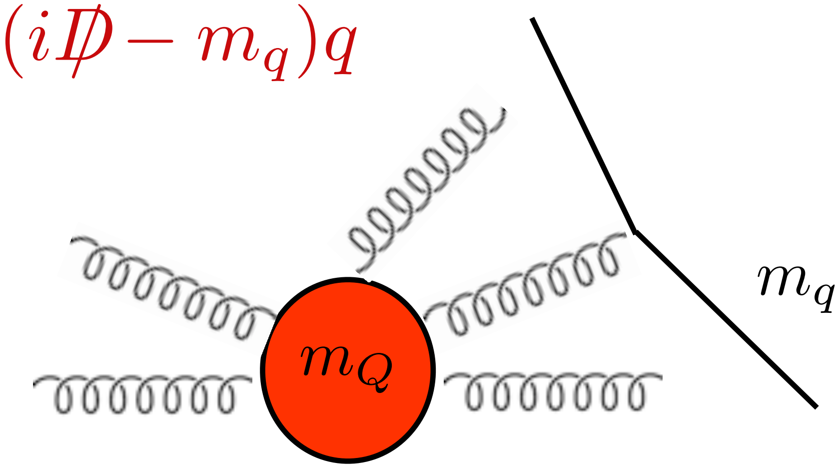
Heavy Quark Effective Theory

- Treat interactions of large mass particles with small momentum exchanged

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \overline{Q}(i\not{D} - m_Q)Q + \overline{q}(i\not{D} - m_q)q$$

- Static limit $p_\mu = m_Q\, v_\mu + k_\mu$, with $k \ll m_Q$





$v_\mu v^\mu = 1$

Georgi's velocity super-selection rule

- Integrate out large momentum modes from heavy quark field --> **HQET**

Heavy Quark Effective Theory

$v_\mu v^\mu = 1$

- Momentum space: heavy quark propagator $p_\mu = m_Q\, v_\mu + k_\mu$, with $k \ll m_Q$

$$\frac{i}{\not p - m_Q + i\epsilon} = i\frac{\not p + m_Q}{p^2 - m_Q^2 + i\epsilon} = i\frac{m_Q(1+\not v) + \not k}{m_Q^2 v^2 + 2m_Q\, v\cdot k + k^2 - m_Q^2 + i\epsilon}$$

Full non-analytic structure

(single particle pole)

Heavy Quark Effective Theory

$v_\mu v^\mu = 1$

- Momentum space: heavy quark propagator $p_\mu = m_Q\, v_\mu + k_\mu$, with $k \ll m_Q$

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Full non-analytic structure

(single particle pole)

$$= i\frac{\frac{1}{2}(1+\not{v}) + \frac{\not{k}}{2m_Q}}{v\cdot k + \frac{k^2}{2m_Q} + i\epsilon} = \frac{i\mathcal{P}_+}{v\cdot k + i\epsilon} + \mathcal{O}\left(\frac{k}{2m_Q}\right)$$

Static heavy quark pole

$\theta(x_0)\,\delta^{(3)}(\vec{x}\,)$

[Quick Exercise]

- Rest frame $v_\mu = (1,0,0,0)$

Heavy Quark Effective Theory

$v_\mu v^\mu = 1$

- Momentum space: heavy quark propagator $p_\mu = m_Q \, v_\mu + k_\mu$, with $k \ll m_Q$

$$\frac{i}{\not{p} - m_Q + i\epsilon} = i \frac{\not{p} + m_Q}{p^2 - m_Q^2 + i\epsilon} = i \frac{m_Q(1 + \not{v}) + \not{k}}{m_Q^2 v^2 + 2m_Q \, v \cdot k + k^2 - m_Q^2 + i\epsilon}$$

Full non-analytic structure

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$$= i \frac{\frac{1}{2}(1 + \not{v}) + \frac{\not{k}}{2m_Q}}{v \cdot k + \frac{k^2}{2m_Q} + i\epsilon} = \frac{i\mathcal{P}_+}{v \cdot k + i\epsilon} + \mathcal{O}\left(\frac{k}{2m_Q}\right)$$

Static heavy quark pole

$\theta(x_0) \, \delta^{(3)}(\vec{x}\,)$

[Quick Exercise]

- Rest frame $v_\mu = (1, 0, 0, 0)$

- Spinor projectors $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \not{v})$

$$\mathcal{P}_+ + \mathcal{P}_- = 1$$

$$\mathcal{P}_\pm^2 = \mathcal{P}_\pm$$

$$\mathcal{P}_\pm \mathcal{P}_\mp = 0$$

Rest frame

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm \gamma_0) \Rightarrow$$

Dirac's basis

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow$$

"Upper components"

$$\mathcal{P}_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Heavy Quark Effective Theory

- Coordinate space: heavy quark field decomposition

$$Q(x) = e^{-im_Q\, v\cdot x}\,[\mathcal{P}_+ Q_v(x) + \mathcal{P}_- \mathfrak{Q}_v(x)]$$

Annihilates quark with v

Creates antiquark with v

Annihilates quark, Creates antiquark

- Free heavy quark action $\mathcal{L} = \overline{Q}\,(i\partial\!\!\!/ - m_Q)\,Q$

$A_\perp^\mu = A^\mu - v\cdot A\, v^\mu$

$$= \overline{Q}_v iv\cdot\partial\, Q_v + \overline{\mathfrak{Q}}_v\,(-iv\cdot\partial - 2m_Q)\,\mathfrak{Q}_v$$
$$+\overline{\mathfrak{Q}}_v\, i\partial\!\!\!/_\perp Q_v + \overline{Q}_v\, i\partial\!\!\!/_\perp \mathfrak{Q}_v$$

Antiquark components energetically separated from quark components

- “Top down” $$e^{iS_{\mathrm{HQET}}[\overline{Q}_v, Q_v]} = \frac{\int \mathcal{D}\overline{\mathfrak{Q}}_v \mathcal{D}\mathfrak{Q}_v\, e^{i\int d^4x \mathcal{L}(\overline{\mathfrak{Q}}_v, \mathfrak{Q}_v, \overline{Q}_v, Q_v)}}{\int \mathcal{D}\overline{\mathfrak{Q}}_v \mathcal{D}\mathfrak{Q}_v\, e^{i\int d^4x \mathcal{L}(\overline{\mathfrak{Q}}_v, \mathfrak{Q}_v, 0, 0)}}$$

Exercise

- Perform the Gaussian path integral over the antiquark field to arrive at the HQET effective action. Repeat for gauge covariant derivative...

$$\mathcal{L} = \overline{Q}\,(i\partial\!\!\!/ - m_Q)\,Q \qquad \Longrightarrow \qquad \mathcal{L}_v = \overline{Q}_v \left(iv\cdot\partial - \frac{\partial_\perp^2}{2m_Q} + \cdots \right) Q_v$$

$$A_\perp^\mu = A^\mu - v\cdot A\, v^\mu$$

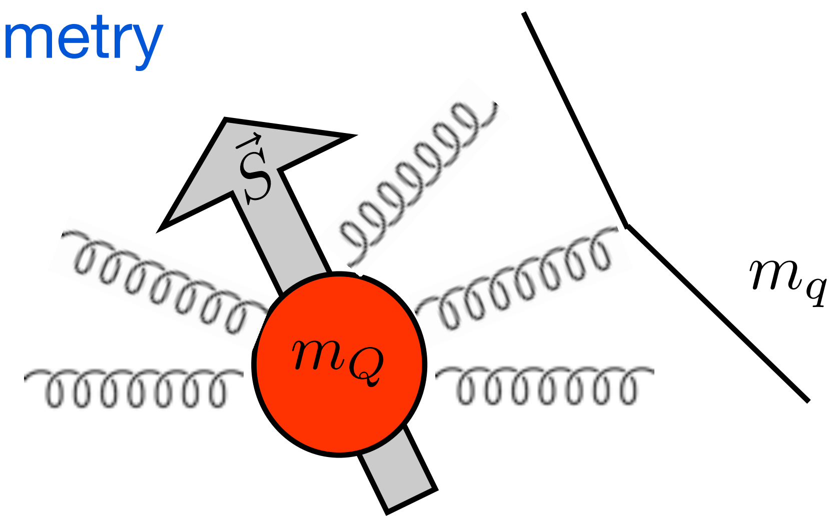
HQET Symmetries

$$D^\mu = \partial^\mu + igA^{\mu a}T^a$$

- New symmetries emerge in the static limit

$$\mathcal{L} = \overline{Q}\,(i\not{D} - m_Q)\,Q \quad \Longrightarrow \quad \mathcal{L}_v = \overline{Q}_v iv\cdot D\,Q_v = \overline{Q}{}^{\uparrow}_v iv\cdot D\,Q^{\uparrow}_v + \overline{Q}{}^{\downarrow}_v iv\cdot D\,Q^{\downarrow}_v$$

Heavy quark spin symmetry



Consequence

Splitting of vector and scalar $\overline{Q}q$ mesons $\propto \dfrac{1}{m_Q}$

$$\mathcal{L} = \bar{b}\,(i\not{D} - m_b)\,b + \bar{c}\,(i\not{D} - m_c)\,c \quad \Longrightarrow \quad \mathcal{L}_v = \bar{b}_v iv\cdot D\,b_v + \bar{c}_v iv\cdot D\,c_v = \overline{\mathbf{Q}}{}^T_v iv\cdot D\,\mathbf{Q}_v$$

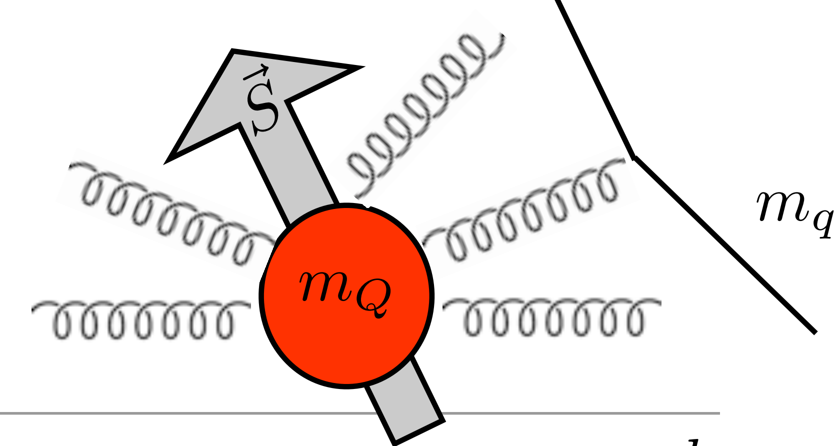
Consequence

Heavy quark flavor symmetry $\quad \mathbf{Q}_v = \begin{pmatrix} b_v \\ c_v \end{pmatrix}$

Splittings of $\bar{b}q$ - $\bar{b}s$ and $\bar{c}q$ - $\bar{c}s$ mesons $\propto \dfrac{1}{m_b} - \dfrac{1}{m_c}$

- HQET has manifest U(4) spin-flavor symmetry for $m_b, m_c = \infty$

Power Corrections



- HQET organized as an expansion in the heavy quark Compton wavelength $\frac{\hbar}{m_Q c}$

$$\mathcal{L}_{\mathrm{HQET}} = \overline{Q}_v \left[iD_0 + c_2 \frac{\vec{D}^2}{2m_Q} + c_F g \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q} + c_D g \frac{\vec{\nabla}\cdot\vec{E}}{8m_Q^2} + ic_S g \frac{\vec{\sigma}\cdot(\vec{D}\times\vec{E}-\vec{E}\times\vec{D})}{8m_Q^2} \right] Q_v \ + \mathcal{O}(\lambda_C^3)$$

Chromomagnetic moment explicitly breaks $U(4) \to U(1)_b \otimes U(1)_c$

- Contains all operators allowed by symmetries:
  parity, time-reversal, gauge invariance, and Galilean invariance

- Underlying Lorentz invariance of QCD implies non-perturbative relations between coefficients at different orders in HQET expansion, **e.g.** $c_2 = 1$

  $c_S = 2c_F - 1$

Exercise

- Demand invariance of the HQET action under an infinitesimal boost to deduce the non-perturbative constraints: $c_2 = 1$, $c_S = 2c_F - 1$.

$$\mathcal{L}_{\rm HQET} = \overline{Q}_v \left[iD_0 + c_2 \frac{\vec{D}^2}{2m_Q} + c_F g \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q} + c_D g \frac{\vec{\nabla}\cdot\vec{E}}{8m_Q^2} + ic_S g \frac{\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})}{8m_Q^2} \right] Q_v$$

Use the variations under the infinitesimal boost $\delta\vec{v} = -\frac{1}{M}\vec{q}$

$$\delta D_0 = \frac{1}{M}\vec{q}\cdot\vec{D} \qquad \delta\vec{D} = \frac{1}{M}\vec{q}D_0$$

$$\delta\vec{E} = \frac{1}{M}\vec{q}\times\vec{B} \qquad \delta\vec{B} = -\frac{1}{M}\vec{q}\times\vec{E}$$

Hint: you will need to deduce the transformation of the heavy quark field...

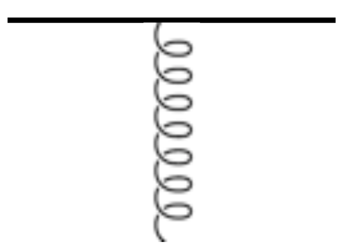
Tree-Level Matching

- To second order in HQET expansion, only two parameters to determine: c_F, c_D

$$\mathcal{L}_{\mathrm{HQET}} = \overline{Q}_v \left[iD_0 + \frac{\vec{D}^2}{2m_Q} + c_F g \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q} + c_D g \frac{\vec{\nabla}\cdot\vec{E}}{8m_Q^2} + i(2c_F - 1) g \frac{\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})}{8m_Q^2} \right] Q_v$$

General quark-gluon vertex

$$\langle p'|J^{a\mu}|p\rangle = g\,\overline{u}(p')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_Q}\right] T^a\, u(p)$$



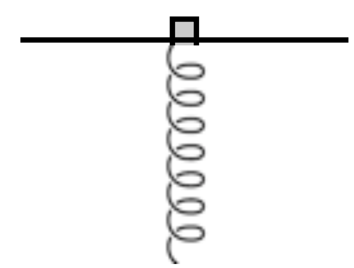
$$u(p) = \sqrt{\frac{m_Q}{E_{\vec{p}}}}\frac{\not{p} + m_Q}{\sqrt{m_Q + E_{\vec{p}}}}\mathcal{P}_+\xi$$

Tree level $\quad g\,\overline{u}(p')\gamma^\mu T^a\, u(p)$

$$= g\,\xi^\dagger\left[v^\mu + \frac{1}{2m_Q}(\vec{p}+\vec{p}\,')^\mu + \frac{i}{2m_Q}(\vec{\sigma}\times\vec{q})^\mu\right]\xi + \mathcal{O}(m_Q^{-2})$$

Full Theory (QCD)

c_F



$$= g\,\xi^\dagger \frac{ic_F}{2m_Q}(\vec{\sigma}\times\vec{q})^\mu \xi$$

$c_F = 1$ Dirac chromomagnetic moment

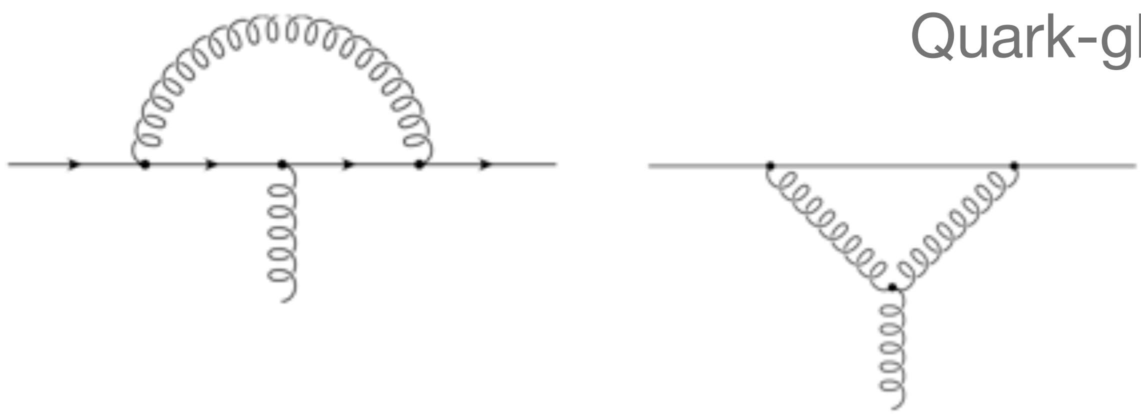
$\ldots c_D = 1$ Darwin term / charge radius

Effective Theory

One-Loop Matching

- pQCD corrections to the coefficients are needed in practice $\alpha_s(m_Q)$
- HQET has different short-distance behavior than QCD

Quark-gluon vertex



+ wavefunction renormalization

$$\frac{A}{\epsilon_{\mathrm{UV}}} + \frac{B}{\epsilon_{\mathrm{IR}}} + (A+B)\log\frac{\mu}{m_Q} + C$$

Full Theory (QCD)

UV and IR poles

$$\int \frac{d^dk}{k^4} \stackrel{\mathrm{dim\ reg}}{=} 0$$

No scale!

One-Loop Matching

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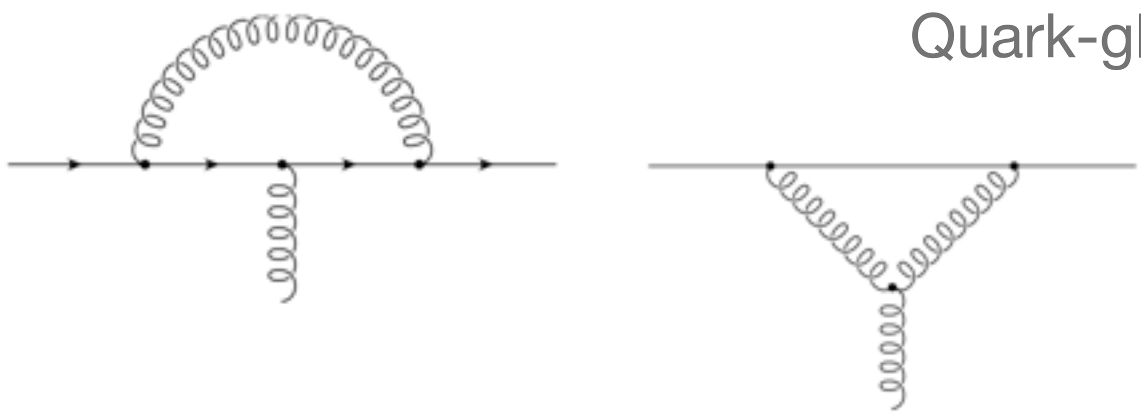
Quark-gluon vertex



+ wavefunction renormalization

$$\frac{A}{\epsilon_{\rm UV}} + \frac{B}{\epsilon_{\rm IR}} + (A+B)\log\frac{\mu}{m_Q} + C$$

Cancel with QCD counterterms

Full Theory (QCD)

UV and IR poles

$$\int \frac{d^dk}{k^4} \overset{\text{dim reg}}{=} 0$$

No scale!

$$\int \frac{d^dk}{k^4} = \int d^dk \left[\frac{1}{k^2(k^2-\Lambda^2)} - \frac{\Lambda^2}{k^4(k^2-\Lambda^2)}\right]$$

$$= i\Omega_3 \left[\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right] = 0$$

One-Loop Matching

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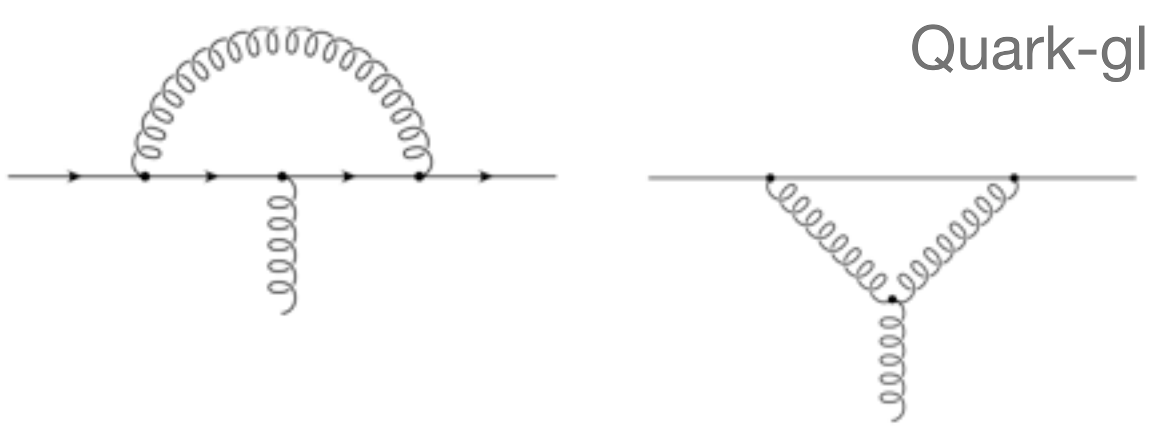
Quark-gluon vertex



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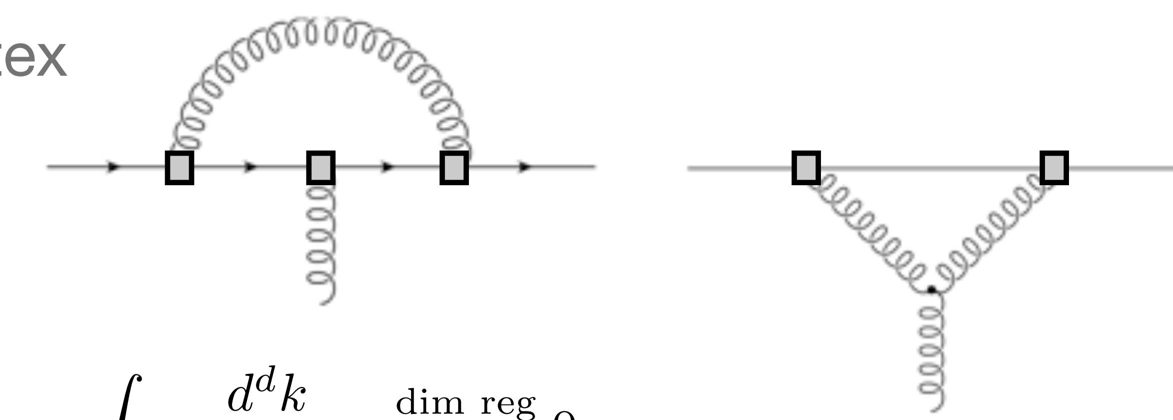
$$\frac{B}{\epsilon_{\rm IR}} + (A+B)\log\frac{\mu}{m_Q} + C$$

Cancelled by QCD counterterms

Full Theory (QCD)



$$\sim \int \frac{d^d k}{(v\cdot k)^2 k^2} \stackrel{\text{dim reg}}{=} 0$$

$$\frac{A_{\rm eff}}{\epsilon_{\rm UV}} + \frac{B_{\rm eff}}{\epsilon_{\rm IR}} + (A_{\rm eff}+B_{\rm eff})\log\frac{\mu}{\Lambda} + C_{\rm eff}$$

Effective Theory

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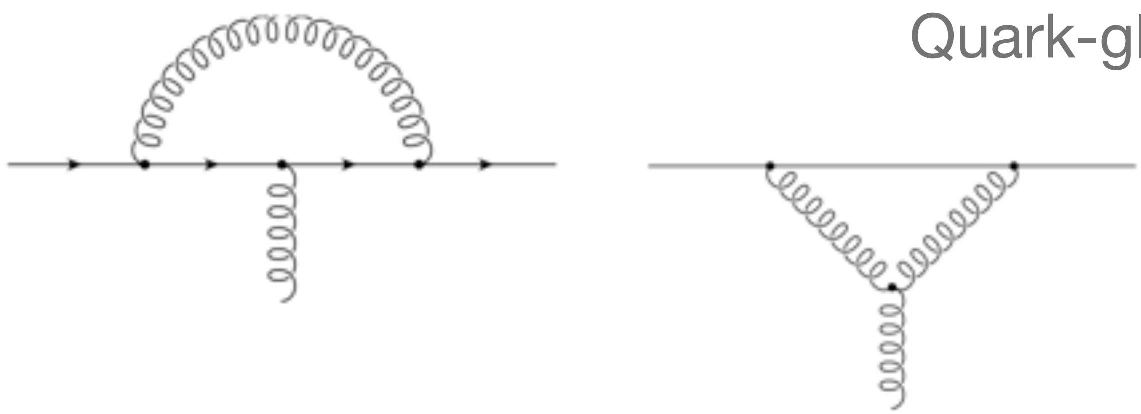
Quark-gluon vertex



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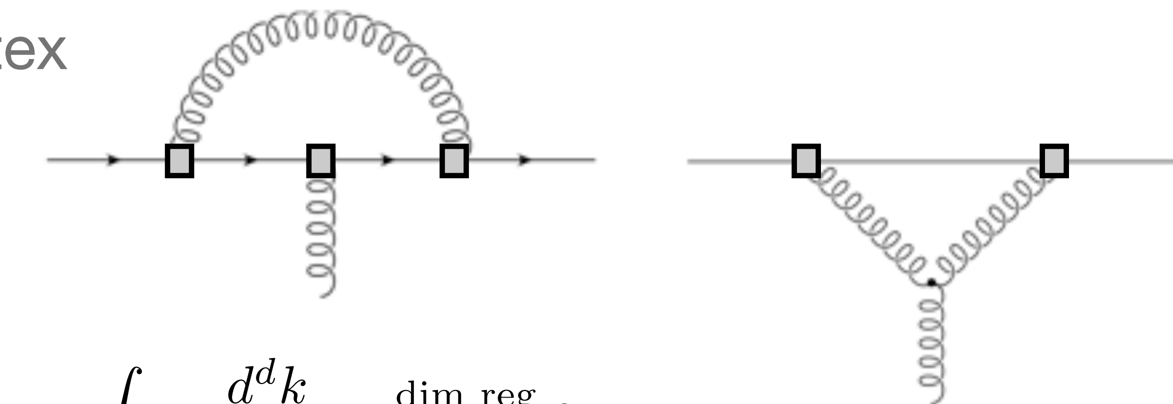
Cancelled by QCD counterterms

Full Theory (QCD)



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$$\frac{A_{\rm eff}}{\epsilon_{\rm UV}} + \frac{B_{\rm eff}}{\epsilon_{\rm IR}} + (A_{\rm eff}+B_{\rm eff})\log\frac{\mu}{\Lambda} + C_{\rm eff}$$

Cancel with EFT counterterms (different than QCD)

Effective Theory

One-Loop Matching

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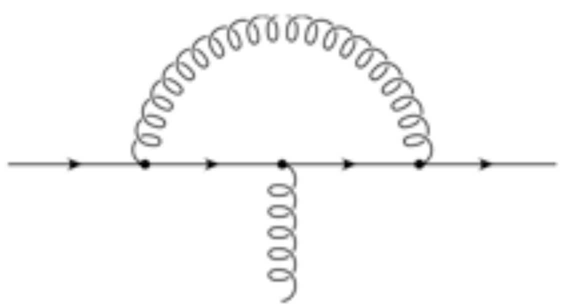
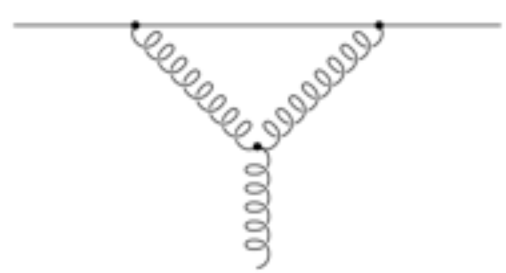
Quark-gluon vertex





+ wavefunction renormalization

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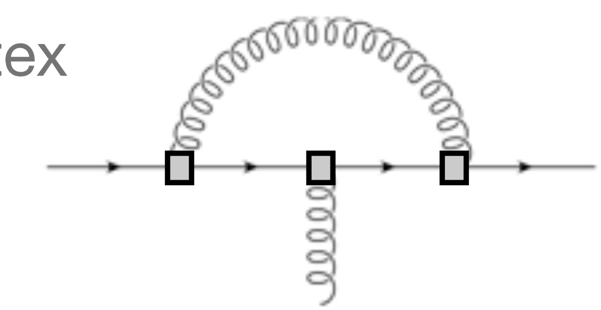
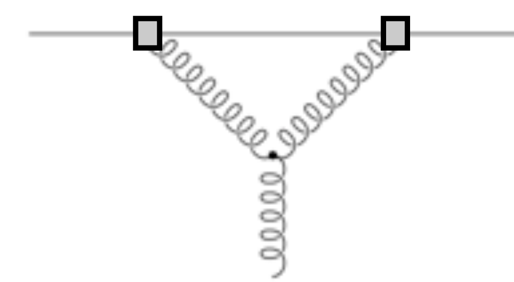
Cancelled by QCD counterterms

Full Theory (QCD)





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$$\frac{B_{\rm eff}}{\epsilon_{\rm IR}} + (A_{\rm eff}+B_{\rm eff})\log\frac{\mu}{\Lambda} + C_{\rm eff}$$

Cancelled by EFT counterterms

Effective Theory

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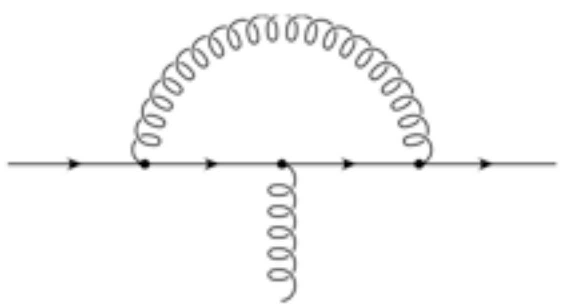
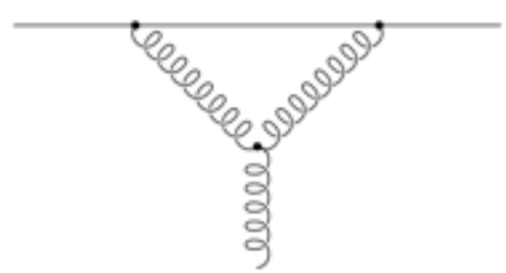
Quark-gluon vertex





+ wavefunction renormalization

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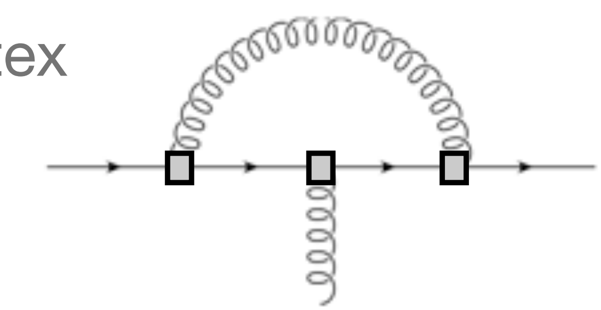
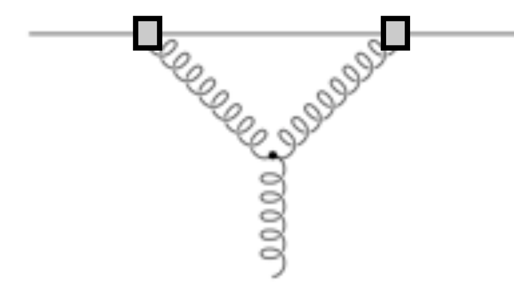
Cancelled by QCD counterterms

Full Theory (QCD)





$$\sim \int \frac{d^d k}{(v\cdot k)^2 k^2} \overset{\text{dim reg}}{=} 0$$

$$\frac{B_{\mathrm{eff}}}{\epsilon_{\mathrm{IR}}} + (A_{\mathrm{eff}}+B_{\mathrm{eff}})\log\frac{\mu}{\Lambda} + C_{\mathrm{eff}}$$

No scale!

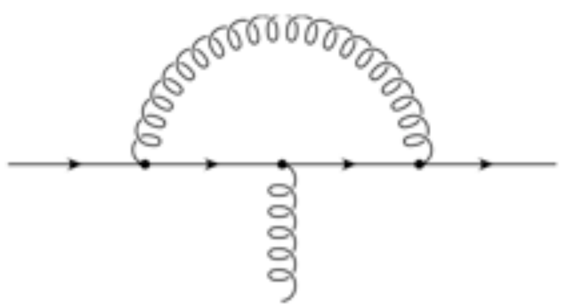
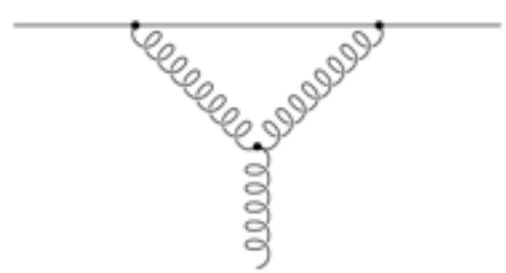
Cancelled by EFT counterterms

Effective Theory

One-Loop Matching

- pQCD corrections to the coefficients are needed in practice $\alpha_s(m_Q)$
- HQET has different short-distance behavior than QCD

Quark-gluon vertex

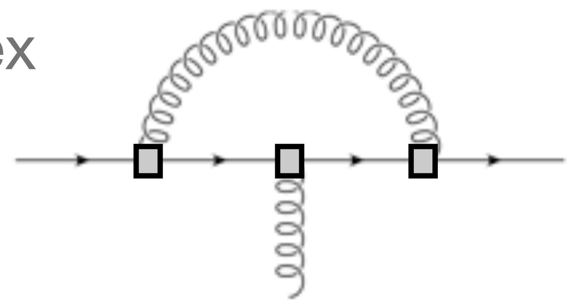
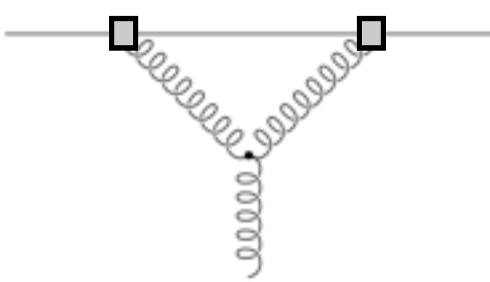



+ wavefunction renormalization

$$\frac{B}{\epsilon_{\mathrm{IR}}} + (A+B)\log\frac{\mu}{m_Q} + C$$

Cancelled by QCD counterterms

Full Theory (QCD)

$$\sim \int \frac{d^d k}{(v\cdot k)^2 k^2} \overset{\text{dim reg}}{=} 0$$

$$\frac{B_{\mathrm{eff}}}{\epsilon_{\mathrm{IR}}} \qquad + C_{\mathrm{eff}}$$

No scale!

Cancelled by EFT counterterms

Net = zero

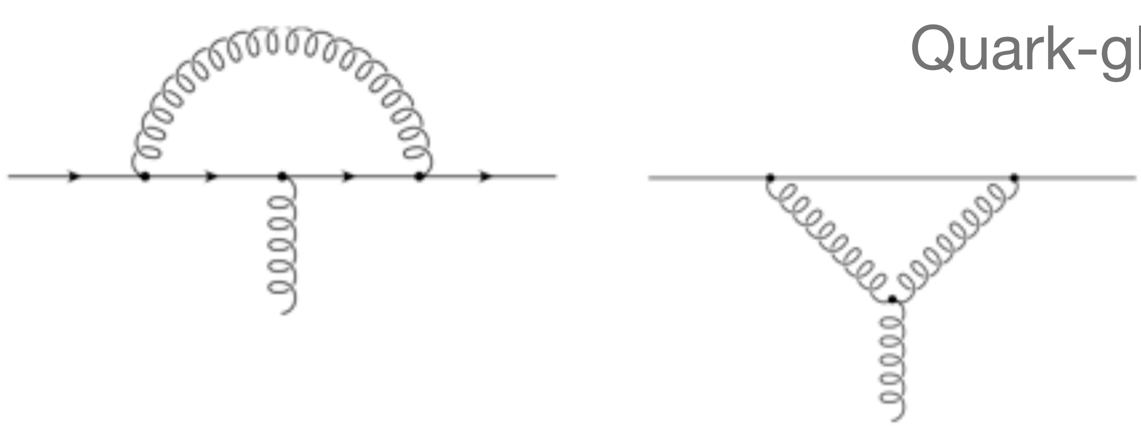
Effective Theory

One-Loop Matching

$$\mathrm{QCD} = \mathrm{HQET} + \Delta C$$

- HQET has same long-distance behavior as QCD $\quad B = B_{\mathrm{eff}}$
- HQET has different short-distance behavior than QCD

Quark-gluon vertex



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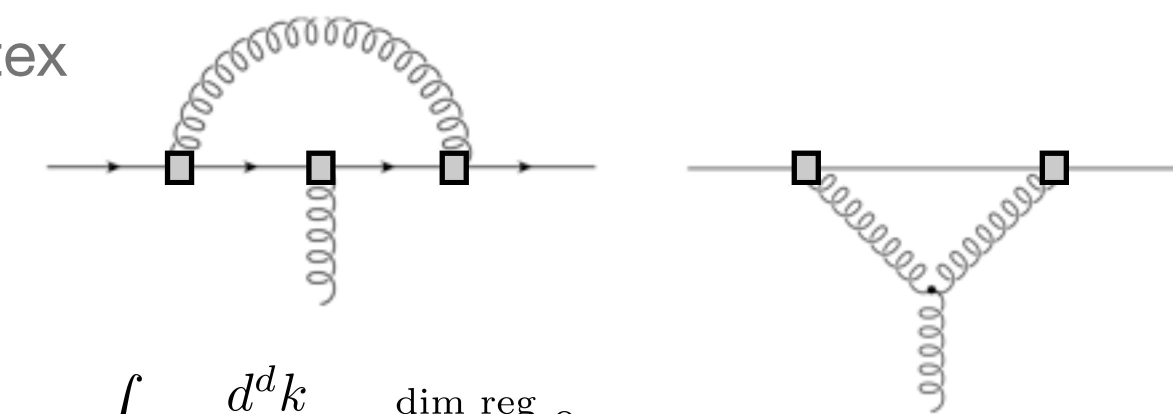

$$\sim \int \frac{d^d k}{(v\cdot k)^2 k^2} \overset{\text{dim reg}}{=} 0$$

$$\frac{B_{\mathrm{eff}}}{\epsilon_{\mathrm{IR}}}$$

No scale!

Cancelled by EFT counterterms

Net = zero

Effective Theory

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$$\mathrm{QCD} = \mathrm{HQET} + \Delta C$$

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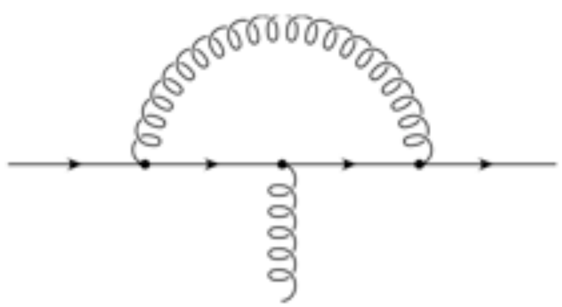
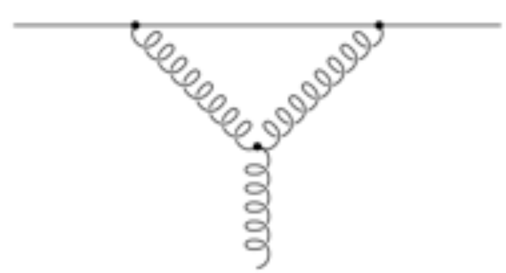
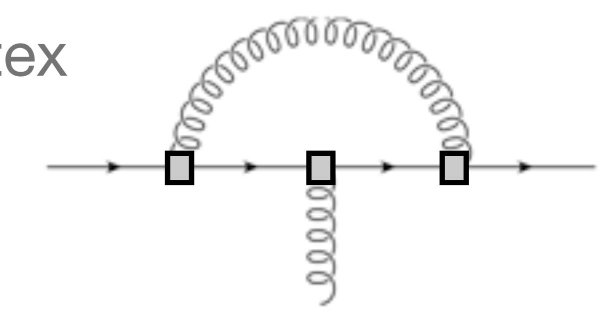
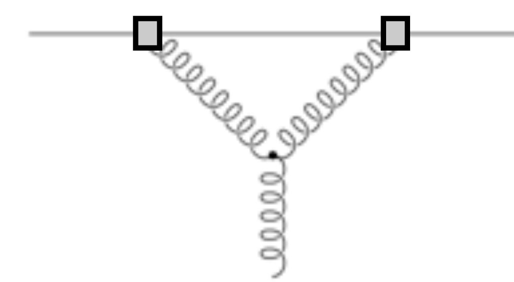








+ wavefunction renormalization

$$\frac{B}{\epsilon_{\mathrm{IR}}} + (A+B)\log\frac{\mu}{m_Q} + C \quad = \quad \frac{B_{\text{eff}}}{\epsilon_{\mathrm{IR}}} \quad + \Delta C$$

Matching condition $\quad \Delta C = (A+B)\log\frac{\mu}{m_Q} + C$

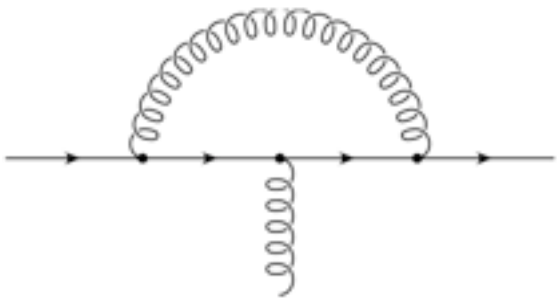
Full Theory (QCD) $\qquad$ Effective Theory

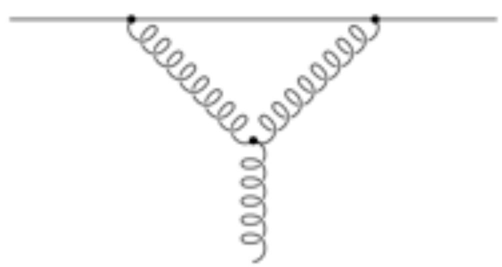
Exercise

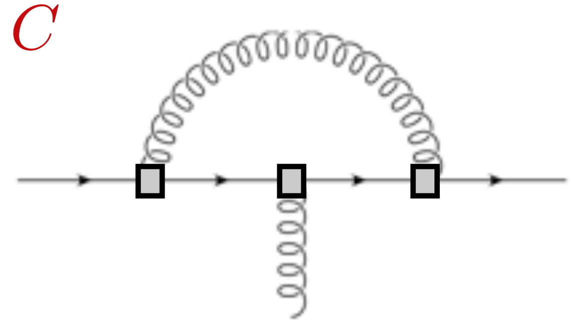
$$\mathrm{QCD} = \mathrm{HQET} + \Delta C$$

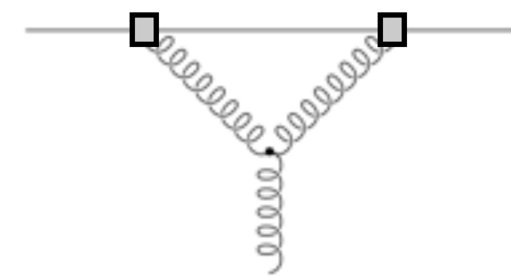
- Verify the matching condition explicitly by isolating the UV and IR divergences of the chromomagnetic moment in QCD and HQET.

$$\Delta C = (A + B)\log\frac{\mu}{m_Q} + C$$









H$\mathcal{Q}$ET for Proton

- Although I don't know what the "$\mathcal{Q}$" stands for, similar analysis can be applied to **E&M** interactions with a composite heavy particle, *e.g.* proton

$$\mathcal{L}_{\mathrm{p}} = \psi_p^\dagger \left[iD_0 + \frac{\vec{D}^2}{2M_p} + c_F e \frac{\vec{\sigma}\cdot\vec{B}}{2M_p} + c_D e \frac{\vec{\nabla}\cdot\vec{E}}{8M_p^2} + i(2c_F - Z)e \frac{\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})}{8M_p^2} \right] \psi_p$$

- Matching cannot be performed in pQCD, need experimental data (or lattice QCD)

$$c_F = Z + \kappa$$

$$c_D = Z + \frac{4}{3} M_p^2 < r_E^2 >$$

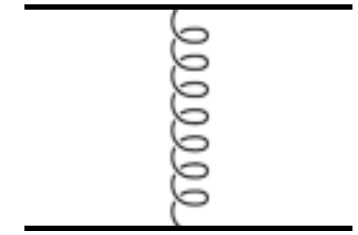
- Coefficients need not be $\mathcal{O}(1)$ in Compton wavelength expansion

$$c_F = 2.8 \qquad 35\%$$

$$c_D = 21 \qquad 5\%$$

Point-like Dirac results

NRQCD/NRQED

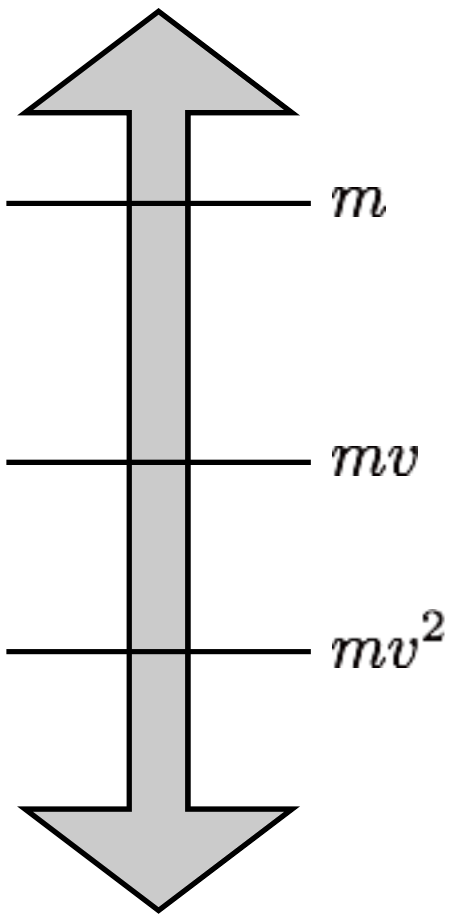


Long-range interactions

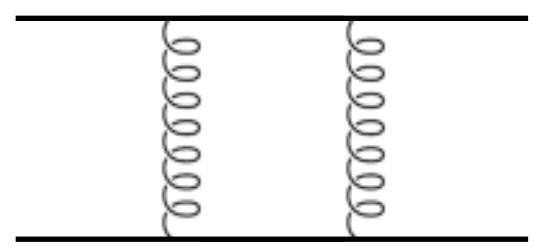
- Non-relativistic bound states of two heavy particles, **e.g.** electron + positron, NRQED

$$\mathcal{L}_e + \mathcal{L}_{\bar{e}} + \mathcal{L}_\gamma + \mathcal{L}_{e\bar{e}}$$

$$\mathcal{L}_e = \psi_e^\dagger \left[iD_0 + \frac{\vec{D}^2}{2m_e} - e\frac{\vec{\sigma}\cdot\vec{B}}{2m_e} - e\frac{\vec{\nabla}\cdot\vec{E}}{8m_e^2} - ie\frac{\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})}{8m_e^2} \right] \psi_e$$



- HQET counting breaks down for two particles interacting



$$\int dk_0 \frac{1}{(k_0 + i\epsilon)(-k_0 + i\epsilon)} \cdots = \infty$$

- Must retain kinetic terms...

$$\int dk_0 \frac{1}{(k_0 - \frac{\vec{k}^2}{2m_e} + i\epsilon)(-k_0 - \frac{\vec{k}^2}{2m_e} + i\epsilon)} = \frac{1}{\frac{\vec{k}^2}{m_e}}$$

- Velocity power counting: *two* scales $k_0 \sim v^2$ $|\vec{k}| \sim v$ space-time asymmetric

$$k_\mu = P_\mu + q_\mu$$

label momentum $\mathcal{O}(v)$

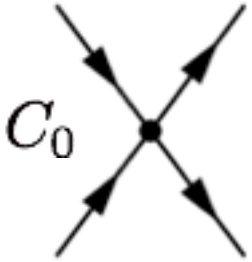
residual momentum $\mathcal{O}(v^2)$

Coulomb photons: label changing $\mathcal{O}(v)$

NNEFT

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

C_0



Short-range interactions

- Low-energy scattering described by effective range expansion... which is an EFT

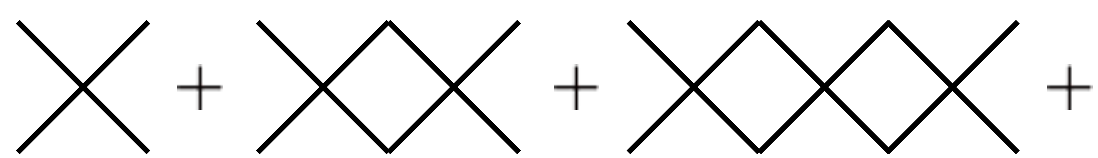
$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right) N - C_0 \left(N^T \mathbf{\Pi} N \right) \cdot \left(N^\dagger \mathbf{\Pi}^\dagger N^{\dagger^T} \right)$$

Propagator $\dfrac{i}{k_0 - \frac{\vec{k}^2}{2M} + i\epsilon}$

Spin x Isospin $\Pi^a = \dfrac{1}{\sqrt{8}} \begin{cases} \sigma_2 \tau_2 \tau^a \\ \sigma_2 \sigma^a \tau_2 \end{cases}$

- Power counting $Q \ll \Lambda \sim m_\pi$ with $k_0 \sim \dfrac{\vec{k}^2}{2M} \sim \dfrac{Q^2}{M}$, i.e. $\vec{k} \sim Q$

$$\int dk_0 d\vec{k} \sim \frac{Q^2}{M} Q^3 = \frac{Q^5}{M}$$



$C_0 \qquad C_0 \dfrac{Q^5}{M} \left(\dfrac{M}{Q^2} \right)^2 C_0$

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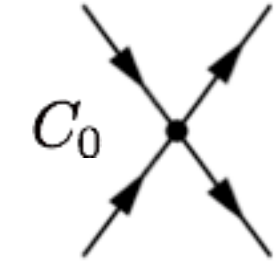

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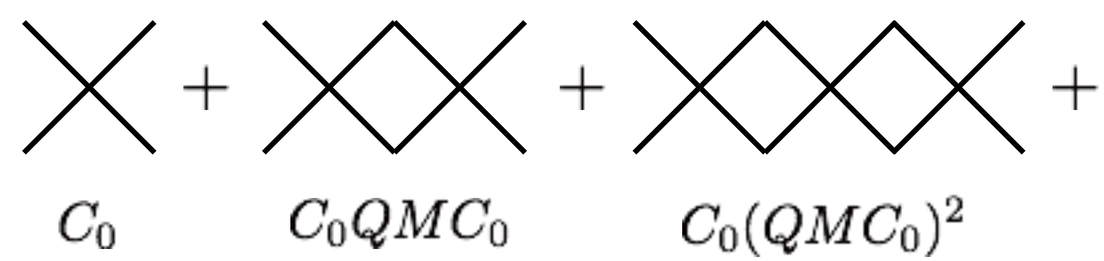

- Perturbative short-range interaction $Q M C_0 \ll 1$

NNEFT

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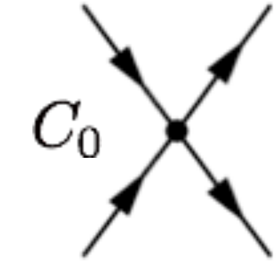

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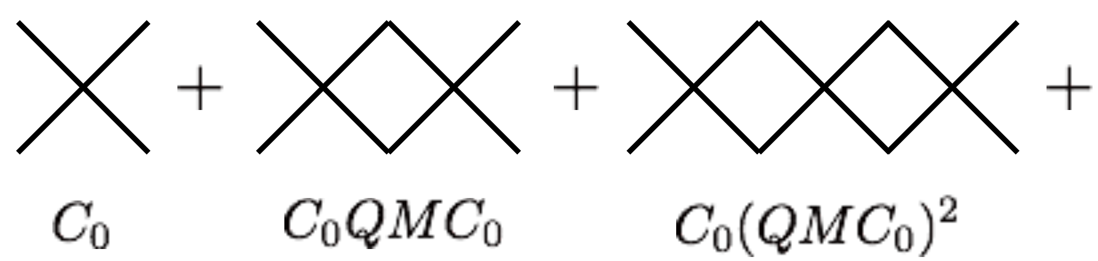

- Strong short-range interaction $QMC_0 \sim 1$, must sum the series

$$T = C_0 + C_0 I(p) C_0 + C_0 [I(p) C_0]^2 + \ldots$$

NNEFT

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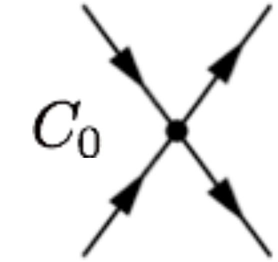

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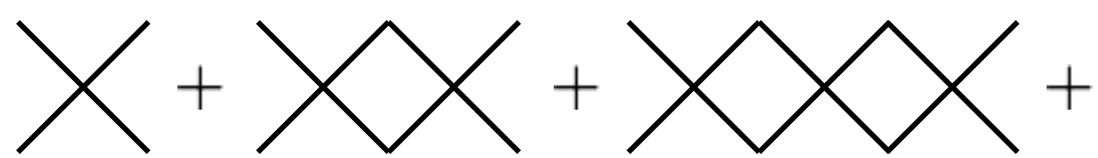

C_0 $\quad$ $C_0 Q M C_0$ $\quad$ $C_0 (Q M C_0)^2$

- Strong short-range interaction $QMC_0 \sim 1$, must sum the series

$$T = \frac{C_0}{1 - I(p) C_0} = \frac{1}{\frac{1}{C_0} - I(p)}$$

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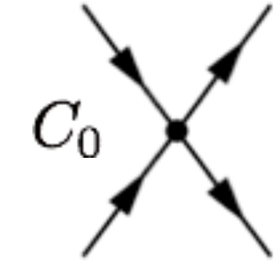

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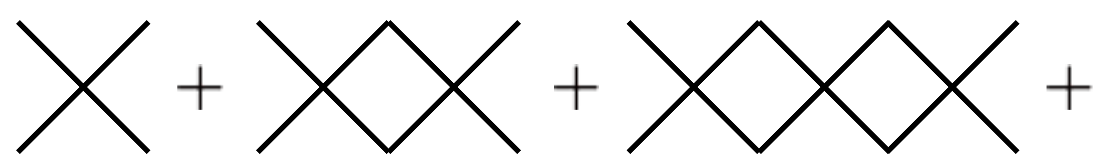


UV divergent

$$I(p) = \int \frac{d\vec{k}}{(2\pi)^3} \frac{M}{p^2 - \vec{k}^2 + i\epsilon}$$

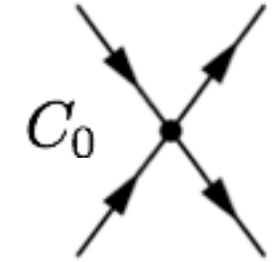
$$T = \frac{C_0}{1 - I(p) C_0} = \frac{1}{\frac{1}{C_0} - I(p)}$$

PDS scheme

$$I(p)_{\text{reg}} \equiv \left[I(p)^d - I(p)^{d \approx 2} \right]_{d \to 3} = -\frac{M}{4\pi} (\mu + ip)$$

NNEFT

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$



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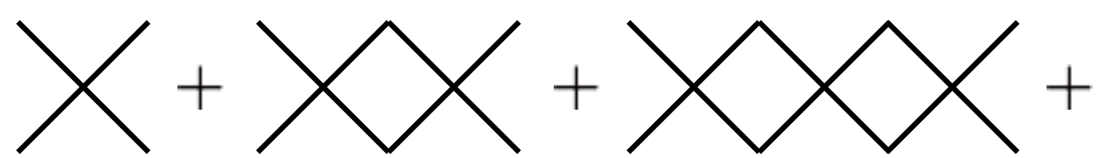

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Compare with $p \cot\delta = -\dfrac{1}{a} + \dfrac{1}{2} r_0 p^2 + \cdots$

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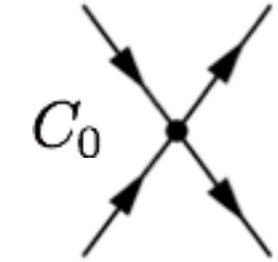

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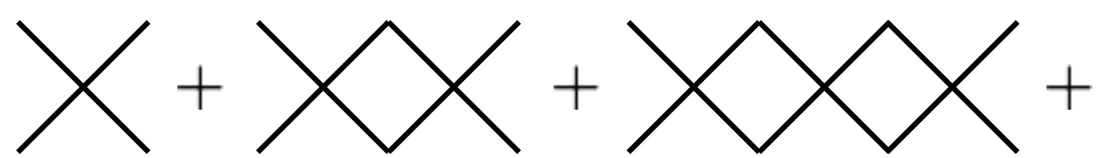

$$T = \frac{-\frac{4\pi}{M}}{-\frac{4\pi}{M}\frac{1}{C_0} - \mu - ip}$$

$$C_0(\mu) = \frac{4\pi}{M}\left(\frac{1}{a} - \mu\right)$$

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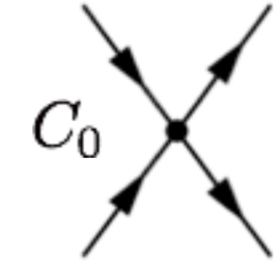

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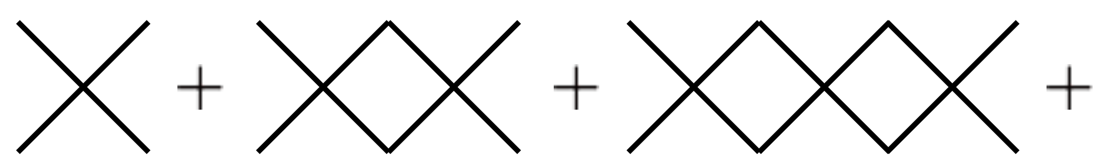


- Strong short-range interaction

$$T = \frac{-\frac{4\pi}{M}}{-\frac{4\pi}{M}\frac{1}{C_0} - \mu - ip}$$

$$C_0(\mu) = \frac{4\pi}{M}\left(\frac{1}{a} - \mu\right)$$

$QMC_0 \sim 1 \Rightarrow a \sim \frac{1}{QM} \gg \frac{1}{\Lambda}$

$a_{np}^{s=0} = -23.7\,\mathrm{fm}$

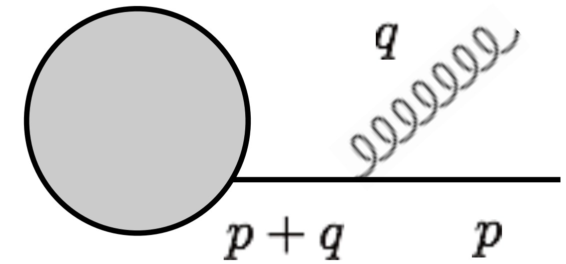
$a_{nn} = -18.4\,\mathrm{fm}$

$a_{np}^{s=1} = +5.4\,\mathrm{fm}$ bound

Compare with $p\cot\delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \cdots$

$$T(p) = -\frac{4\pi}{M} \frac{1}{p\cot\delta - ip}$$

SCET



- IR singularities of intermediate propagator

$$\frac{1}{(p+q)^2 - m^2 + i\epsilon} = \frac{1}{p^2 - m^2 + 2p\cdot q + q^2 + i\epsilon} = \frac{1}{2p\cdot q + i\epsilon} \overset{E \gg m}{\approx} \frac{1}{2E\omega(1-\cos\theta) + i\epsilon}$$

$\omega \to 0$ Soft

$\theta \to 0$ Collinear

- Integrate out large momentum scales of high-energy particle *(propagate along light-cone)*

$$p^2 = p_0^2 - p_3^2 - \vec{p}_\perp^{\,2} = (p_0 + p_3)(p_0 - p_3) - \vec{p}_\perp^{\,2} \equiv p_+ p_- - \vec{p}_\perp^{\,2}$$

Energetic motion along the z-direction: $p_+ = E + p_3 \approx 2E$

$$p_- = E - p_3 \approx \frac{m^2}{2E}$$

$$p_\mu = (p_+ \sim E, p_- \sim \lambda^2 E, p_\perp \sim \lambda E)$$

- SCET mode decomposition $P_\mu = (p_+, 0, \vec{p}_\perp) + k_\mu$ ⟵ residual momentum $\mathcal{O}(\lambda^2)$

$$\frac{1}{P^2 + i\epsilon} = \frac{1}{p_+ k_- - \vec{p}_\perp^{\,2} + i\epsilon} \sim \lambda^{-2}$$

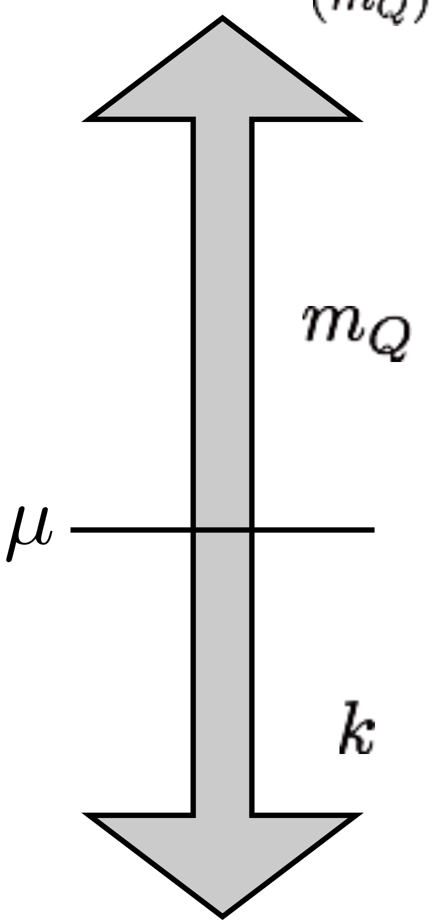
label momentum $\mathcal{O}(1), \mathcal{O}(\lambda)$

- Soft radiation $q_\mu \sim (\lambda^2, \lambda^2, \lambda^2)$
- Collinear radiation $q_\mu \sim (1, \lambda^2, \lambda)$

Summary
II. Removing large scales

- Large scales can be systematically integrated out resulting in EFTs, *e.g.* HQET

$(m_Q)^{-n}$ *power corrections* $\alpha_s(m_Q)$ *perturbative corrections*



- EFT coefficients determined from matching “top-down”

- Theories have different UV behavior

- Only IR behavior is shared and thus cancels in matching

Computations in EFT are simpler
EFT involves only d.o.f. relevant to energy regime

large scales can be interrelated... NRQCD/SCET