WILLIAM & MARY

2014 National Nuclear Physics Summer School

Lectures on Effective Field Theory

I. Removing heavy particles
II. Removing large scales
III. Describing Goldstone bosons
IV. Interacting with Goldstone bosons

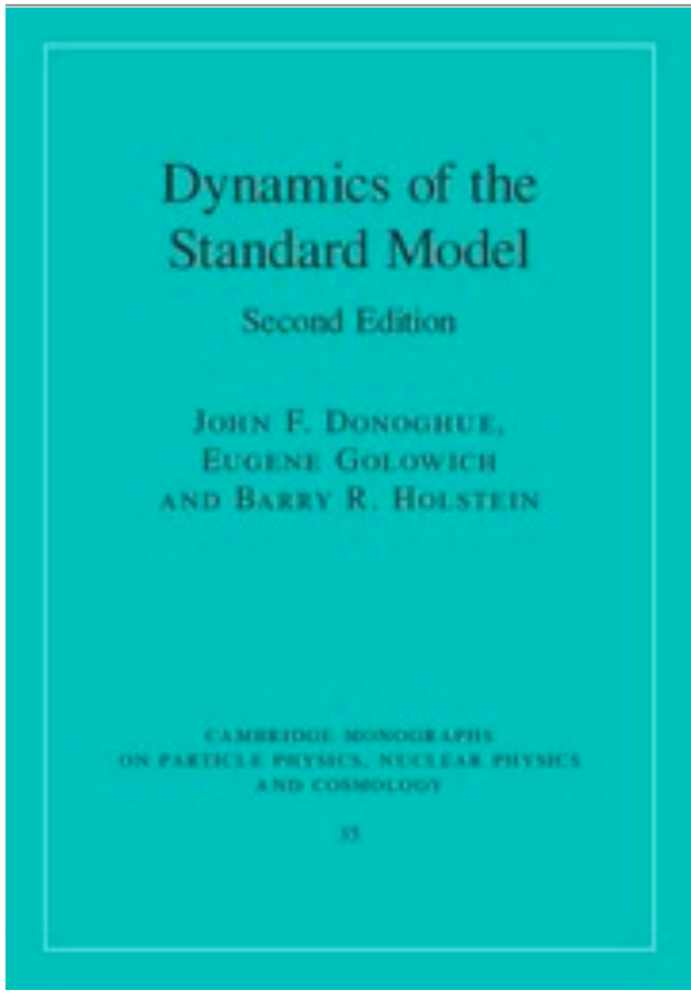
Brian Tiburzi

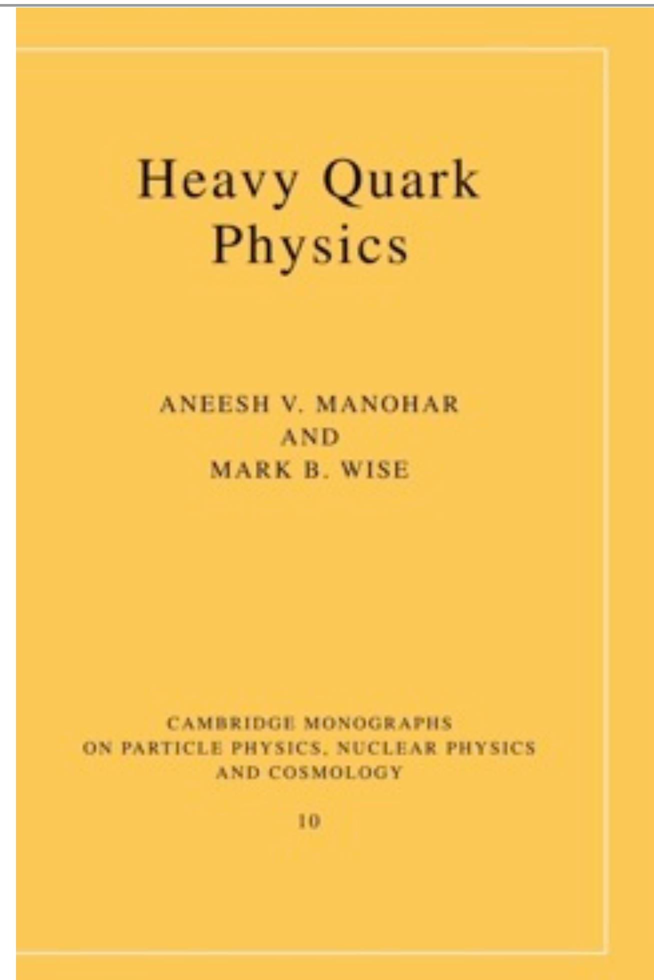
The City College of New York

RIKEN BNL Research Center

CUNY

Effective Reviews





Five lectures on effective field theory
David B. Kaplan. Oct 2005. 79 pp.
e-Print: **nucl-th/0510023** | **PDF**

TASI lectures on effective field theories
Ira Z. Rothstein (Carnegie Mellon U.). Aug 2003. 90 pp.
e-Print: **hep-ph/0308266** | **PDF**

Effective field theory for few nucleon systems
Paulo F. Bedaque (LBL, Berkeley), Ubirajara van Kolck (Arizona U. & RIKEN BNL)
Published in **Ann.Rev.Nucl.Part.Sci. 52 (2002) 339-396**
DOI: 10.1146/annurev.nucl.52.050102.090637
e-Print: **nucl-th/0203055** | **PDF**

On the foundations of chiral perturbation theory
H. Leutwyler (Bern U.). Aug 1993. 52 pp.
Published in **Annals Phys. 235 (1994) 165-203**
BUTP-93-24
DOI: 10.1006/aphy.1994.1094
e-Print: **hep-ph/9311274** | **PDF**

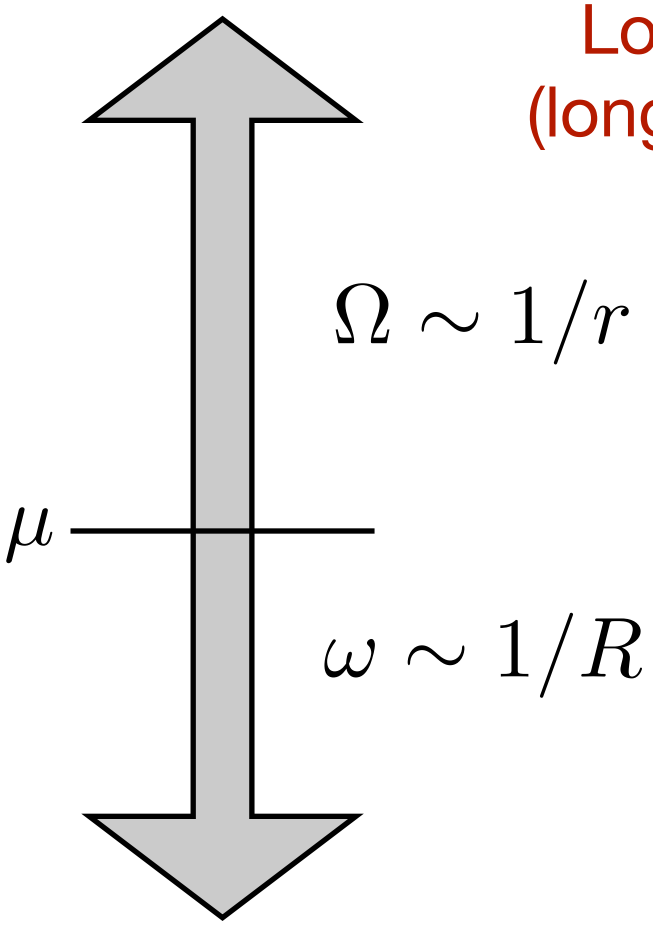
Effective Overview

$(\hbar = c = 1)$

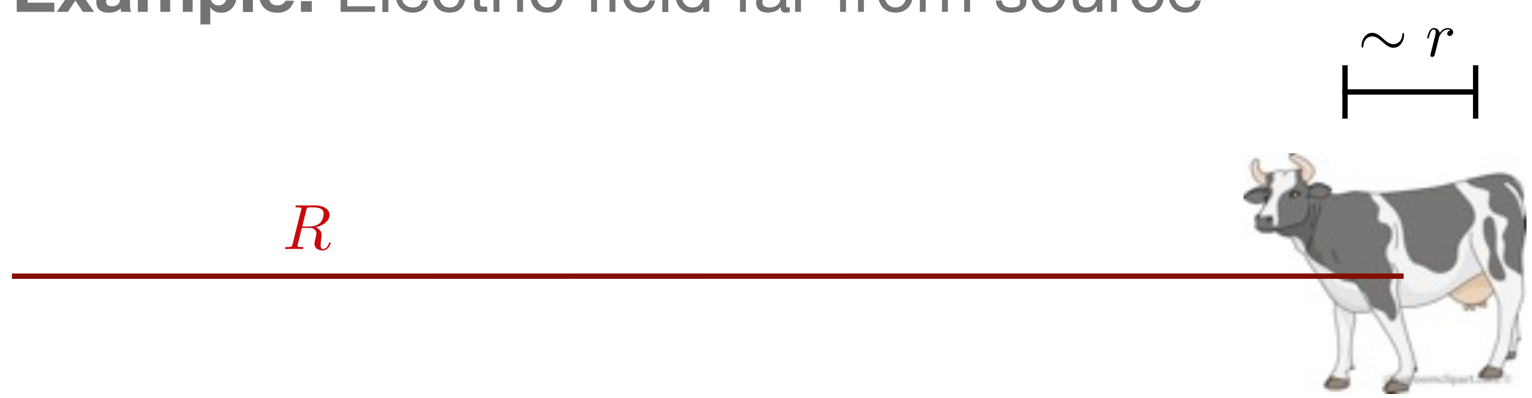
Effective (quantum) field theories exploit a separation of scales $\Omega \gg \omega$

Low-scale physics is largely insensitive to high-scale physics
(long-distance) (short-distance)

$r \ll R$



Example: Electric field far from source



physical distribution of charge

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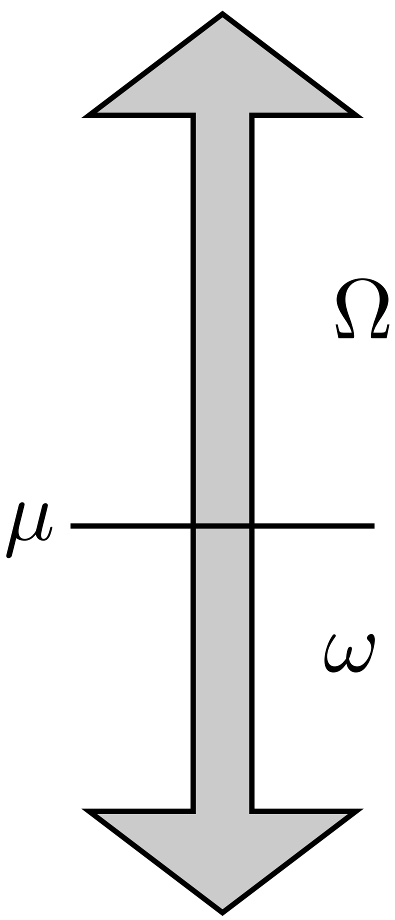


$\Omega \sim 1/r$

μ

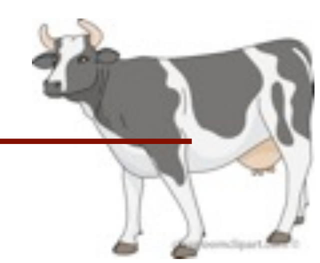
$\omega \sim 1/R$

$r \ll R$

Example: Electric field far from source

$\sim r$

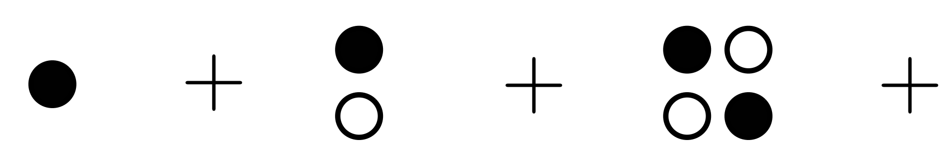
R



physical distribution of charge

multipolarity
(power counting)

$$\sim \left(\frac{r}{R}\right)^{\ell} \sim \left(\frac{\omega}{\Omega}\right)^{\ell}$$



L multipoles to describe $\vec{E}$ to $p\%$

effective multipole theory

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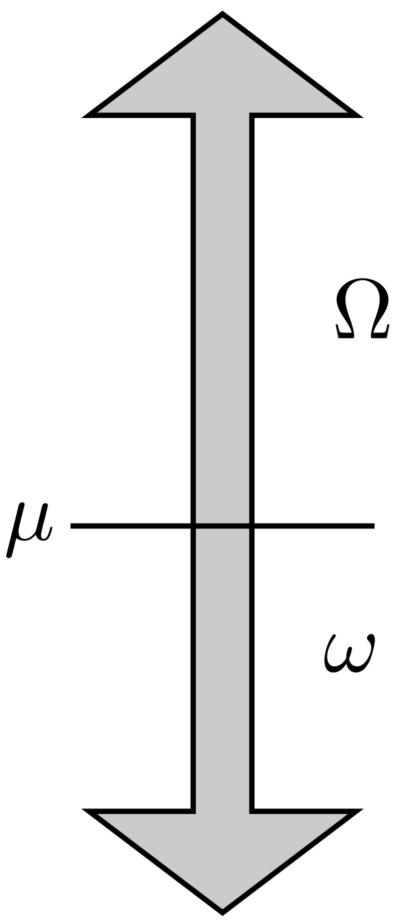

$\Omega \sim 1/r$

$\omega \sim 1/R$

Goal: investigate power counting for various effective QFTs

QFTs require a regulator and renormalization

UV regulator alters high-scale physics to compute quantum effects

Renormalization: absorb effects of regulator into theory parameters

Effective Overview

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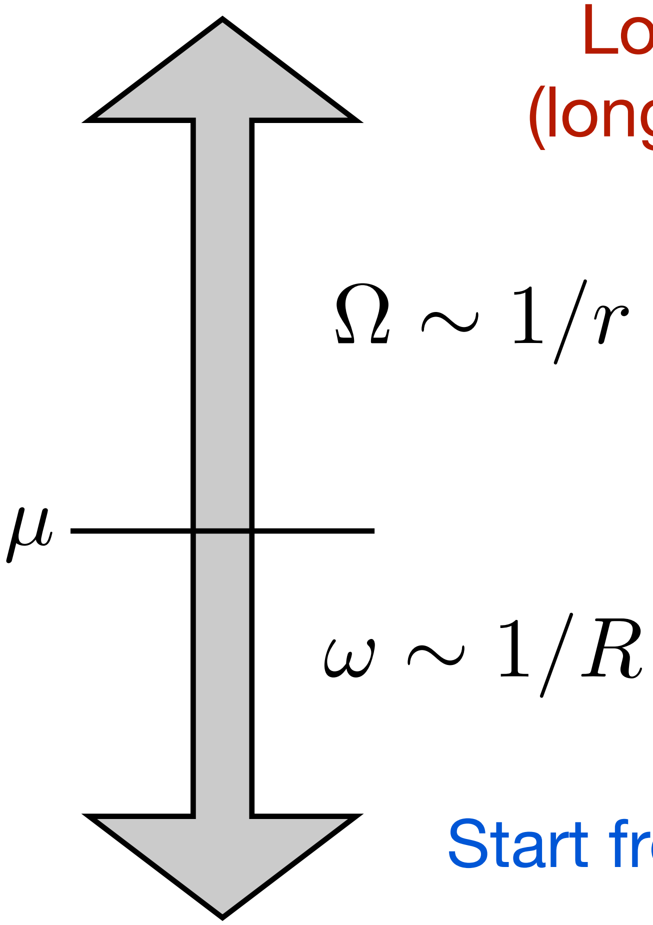

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Goal: investigate power counting for various effective QFTs

"*Top Down*" I. Removing heavy particles
II. Removing large scales

Start from perturbative high-scale theory, arrive at simpler low-scale EFT

"*Bottom Up*" III. Describing Goldstone bosons
IV. Interacting with Goldstone bosons

Construct low-scale EFT using non-perturbative degrees of freedom

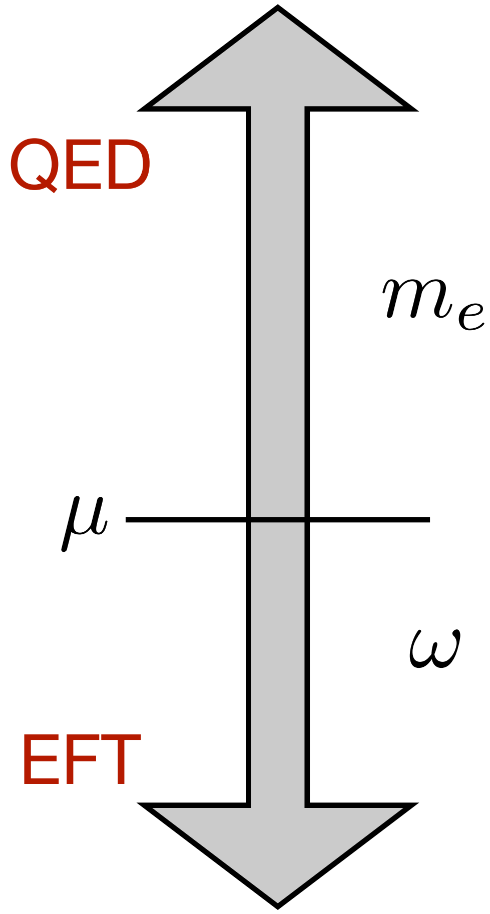
Effective Field Theory

I. Removing heavy particles

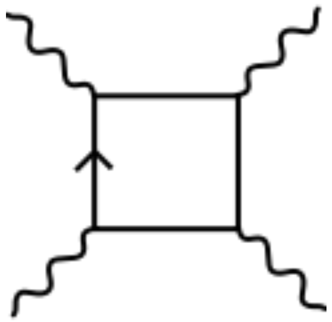
Euler-Heisenberg EFT

$$\mathcal{L} = \dim 4$$
$$F_{\mu\nu} = \dim 2$$
$$\psi = \dim \frac{3}{2}$$

EFT for low-frequency photons $\omega \ll m_e$ obtained by "integrating out" electron



$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{D} - m_e)\psi$$



Photon-photon scattering mediated by virtual electron in QED

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QED

m_e

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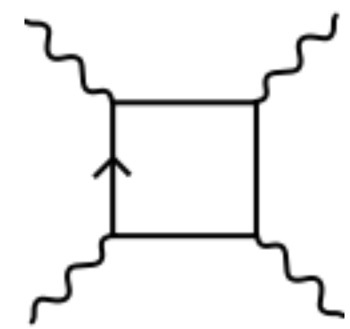


Photon-photon scattering mediated by virtual electron in QED

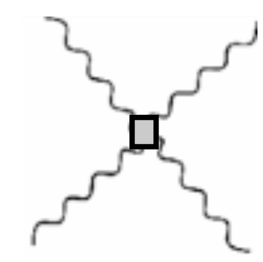
μ

Photon is the only degree of freedom in the EFT

ω

EFT

$$\mathcal{L}_{\text{EFT}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots$$



Low-frequency photon-photon scattering requires 4-photon operators $\sim F^4 = \dim 8$

Lorentz invariance, Gauge invariance, C, P, T

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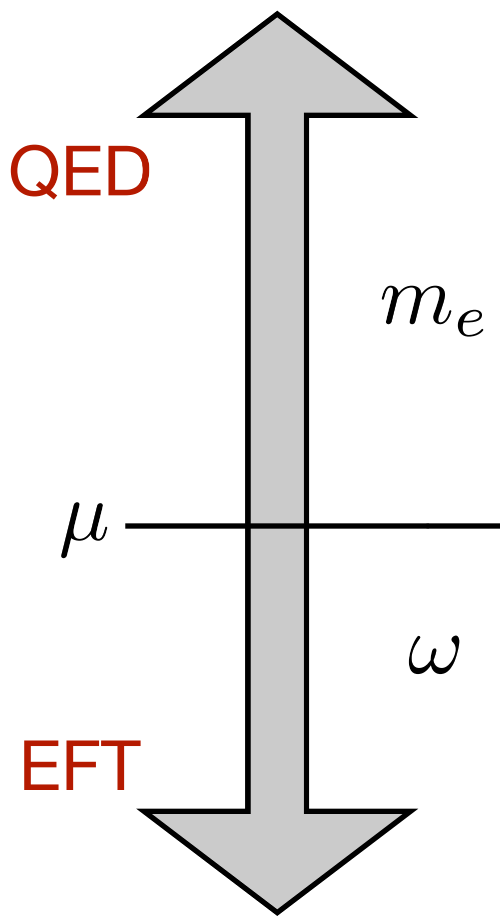

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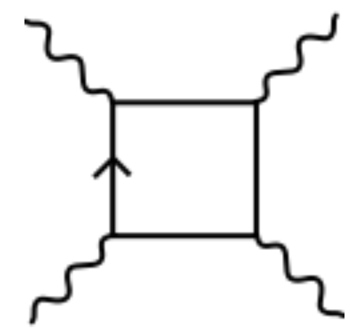
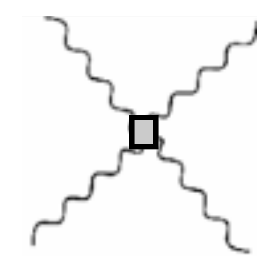


Photon-photon scattering mediated by virtual electron in QED

Photon is the only degree of freedom in the EFT

$$\mathcal{L}_{\rm EFT} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{m_e^4}\left[c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right]$$



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$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

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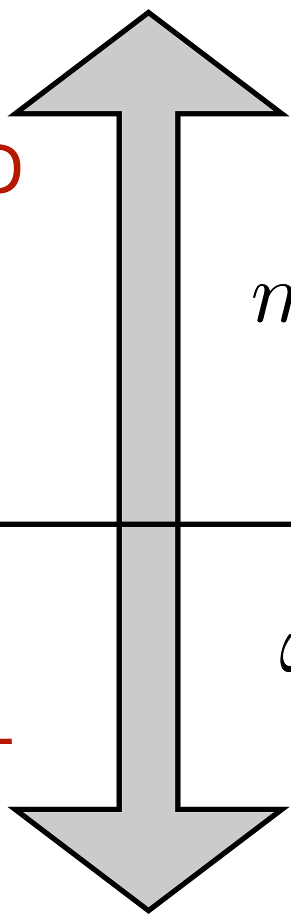

m_e

μ

ω

EFT

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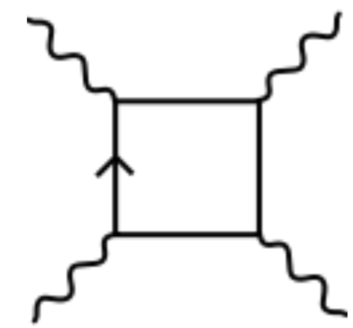
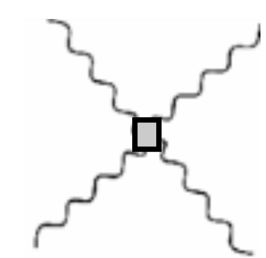


Photon-photon scattering mediated by virtual electron in QED

Photon is the only degree of freedom in the EFT



$$\mathcal{L}_{\text{EFT}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right]$$

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$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

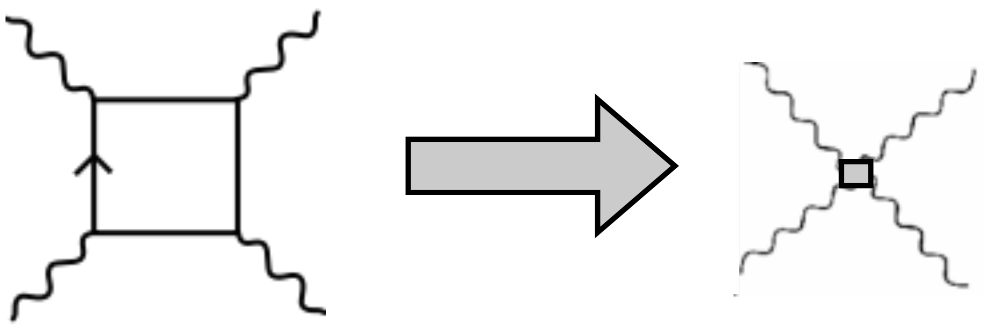
Exercise

- Include low-energy *virtual* photons in the Euler-Heisenberg EFT. What new local operator of lowest dimension is required? Determine the coefficient of this operator from matching to QED at one-loop order.

$$\mathcal{L}_{\rm EFT} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \quad ???$$
$$+ \frac{\alpha^2}{90 m_e^4} \left[(F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right]$$

Effective Lessons

- EFT as low-energy limit of a QFT



- Short-distance physics encoded in coefficients of local operators

$$c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Operators built from effective d.o.f.

Respect symmetries of underlying theory

- Infinite tower of such higher-dimensional operators requires power counting

$$F^2 \sim \omega^4, \quad F^4 \sim \omega^8 \quad \Rightarrow \quad \gamma\gamma \to \gamma\gamma \quad \sim \frac{\omega^4}{m_e^4}$$

- Can make predictions without employing full QFT, and can systematically improve

Finitely many operators to a given order

EFT is itself a QFT... compute radiative corrections (non-analytic)

- Coefficients of higher dimensional operators must be determined

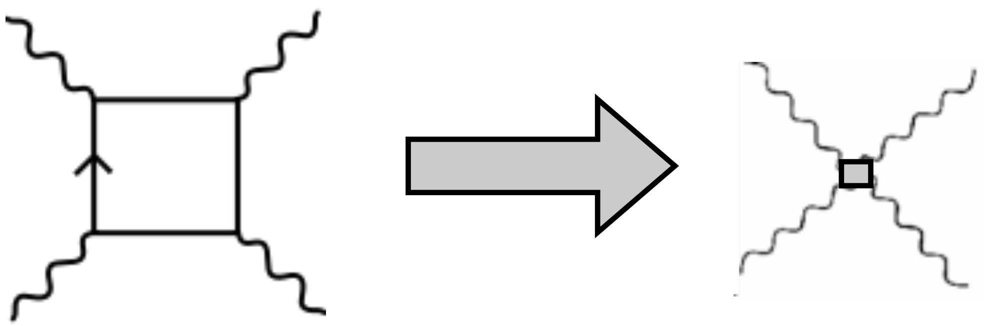
"Top Down" — Perturbation Theory

"Bottom Up" — Experiment

Non-perturbative (lattice QCD)

Effective Lessons

- EFT is low-energy limit of a QFT



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“Top Down”

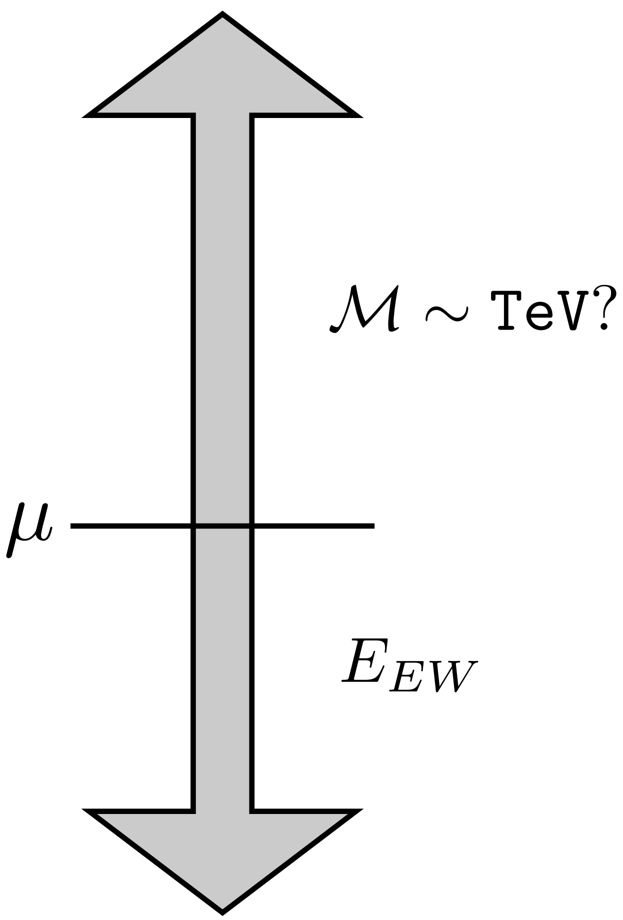
Perturbation Theory

✓ Euler-Heisenberg EFT from finite one-loop diagram in QED perturbation theory

Standard Model as an EFT

$$\mathcal{L} = \dim 4$$
$$F_{\mu\nu} = \dim 2$$
$$\psi = \dim \frac{3}{2}$$

- Renormalizable interactions of SM are the low-energy limit of some high-energy theory
  - Physics beyond SM encoded in a tower of higher-dimension ops.



$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{j=1}^{\infty} \sum_{k} \frac{c_k^{(j)}}{\mathcal{M}^j} \mathcal{O}_k^{(4+j)}$$

$$\dim \mathcal{O}_k^{(4+j)} = 4 + j$$

$$\dim c_k^{(j)} = 0$$ naturalness $$c_k^{(j)} \sim 1$$

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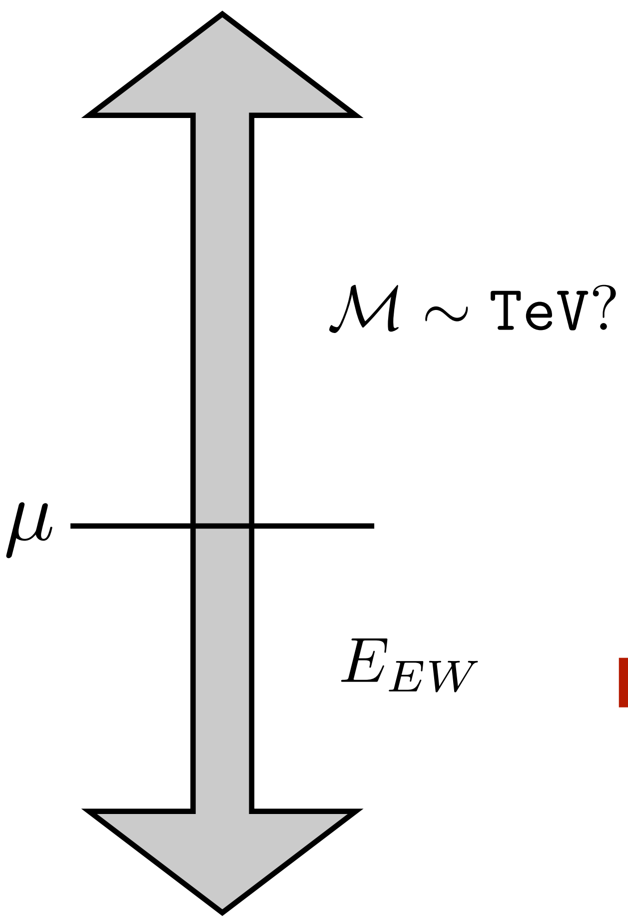

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Example: CP violation beyond SM

$$\mathcal{O}^{(4)} = G_{\mu\nu} \tilde{G}^{\mu\nu}$$

$$\mathcal{O}^{(6)} = \overline{\psi}_L \sigma_{\mu\nu} \tilde{G}^{\mu\nu} \Phi \, \psi_R$$

$$\mathcal{O}^{(6)} = f^{abc} \tilde{G}^{a\,\nu}_{\mu} G^{b\,\alpha}_{\nu} G^{c\,\mu}_{\alpha}$$

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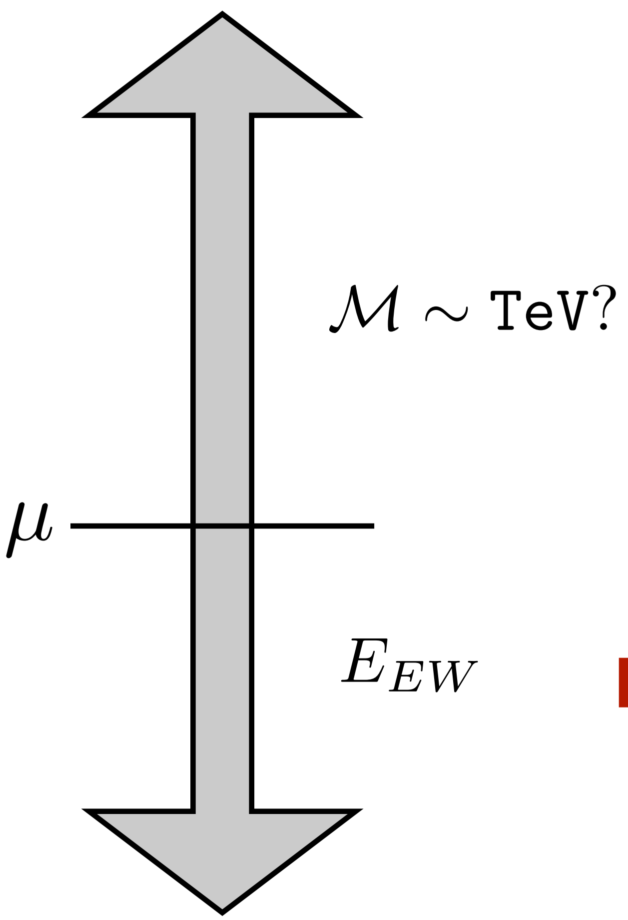

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$\dim c_k^{(j)} = 0$ naturalness $c_k^{(j)} \sim 1$

Example: CP violation beyond SM

$\mathcal{O}^{(4)} = G_{\mu\nu} \tilde{G}^{\mu\nu}$ $c^{(4)} \equiv \theta \ll 1$ unnatural = strong CP problem

$\mathcal{O}^{(6)} = \overline{\psi}_L \sigma_{\mu\nu} \tilde{G}^{\mu\nu} \Phi \, \psi_R$

$\mathcal{O}^{(6)} = f^{abc} \tilde{G}^{a\,\nu}_{\mu} G^{b\,\alpha}_{\nu} G^{c\,\mu}_{\alpha}$

$\left(\frac{E_{EW}}{\mathcal{M}}\right)^2$

Neutron EDM

$d_E \, \vec{S} \cdot \vec{E}$

Generate electric dipole moments

- QFT prejudice: associate energy scale with mass of new heavy particle

Exercise

- Enumerate *all* dimension-6 CP violating operators that respect Standard Model symmetries. How do these operators appear after electroweak symmetry breaking?

$$\mathcal{O}^{(6)} = \overline{\psi}_L \sigma_{\mu\nu} \tilde{G}^{\mu\nu} \Phi \, \psi_R$$

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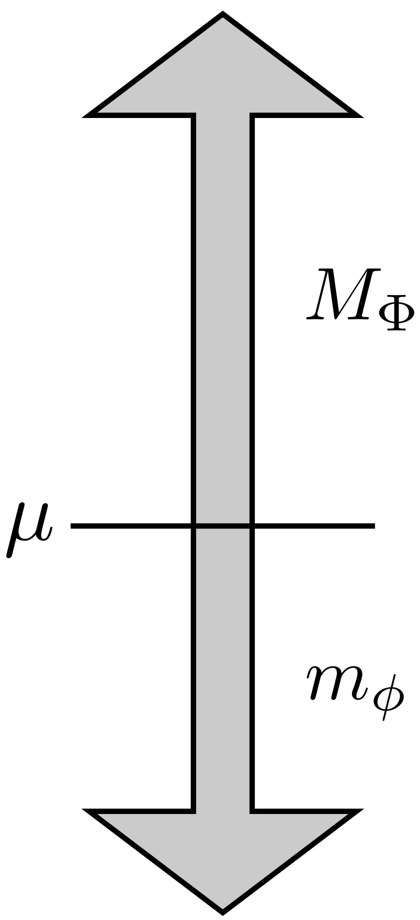
Integrating Out Heavy Particles

$\Phi,\ \phi$

$M_\Phi \gg m_\phi$

- Perform path integral over heavy field
- Expand result in local operators built from light field

$$e^{iS_{\text{eff}}[\phi]} = \frac{\int \mathcal{D}\Phi\, e^{i\int d^4x \mathcal{L}(\Phi,\phi)}}{\int \mathcal{D}\Phi\, e^{i\int d^4x \mathcal{L}(\Phi,0)}}$$



Illustrative toy model (Gaussian path integral)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M_\Phi^2\Phi^2 + \Phi\, J$$

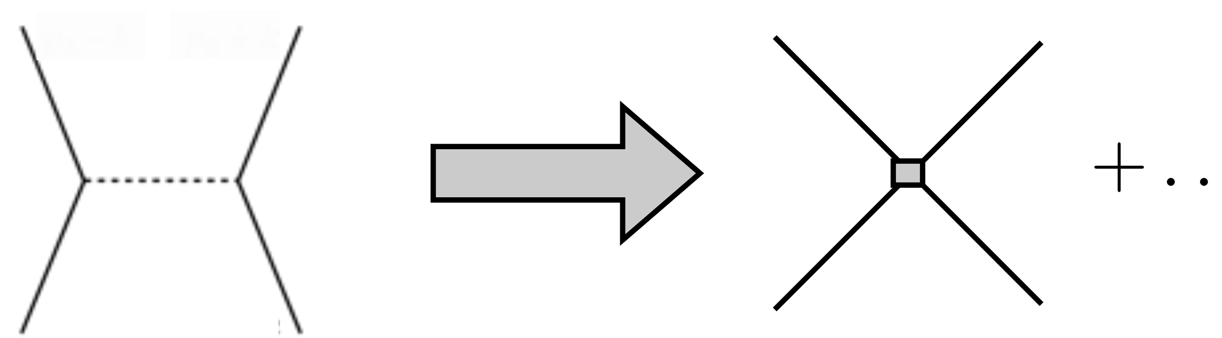
<-- coupling to ϕ

Complete the square to deduce

$$S_{\text{eff}}[\phi] = -\frac{1}{2}\int d^4x\, d^4y\, J(x)G(x-y)J(y)$$

Operator product expansion

$$J(y) = J(x) + (y-x)_\mu\partial^\mu J(x) + \dots$$



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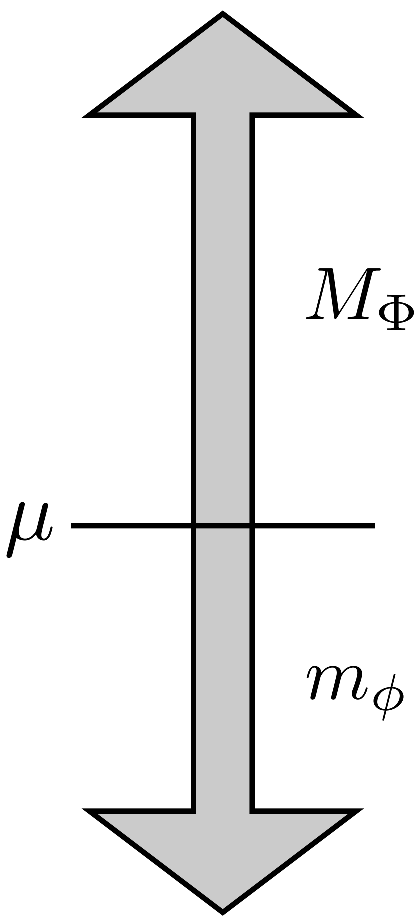

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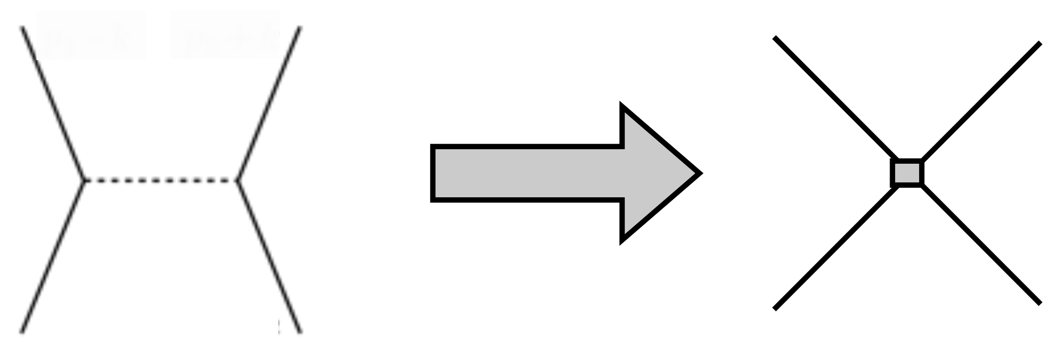

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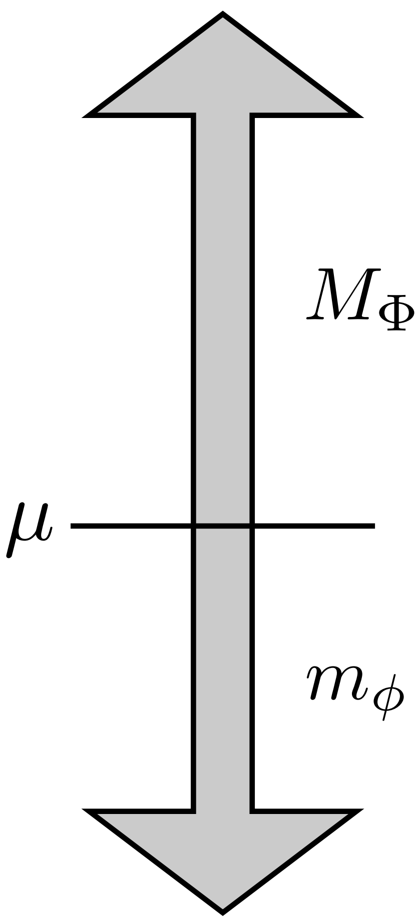

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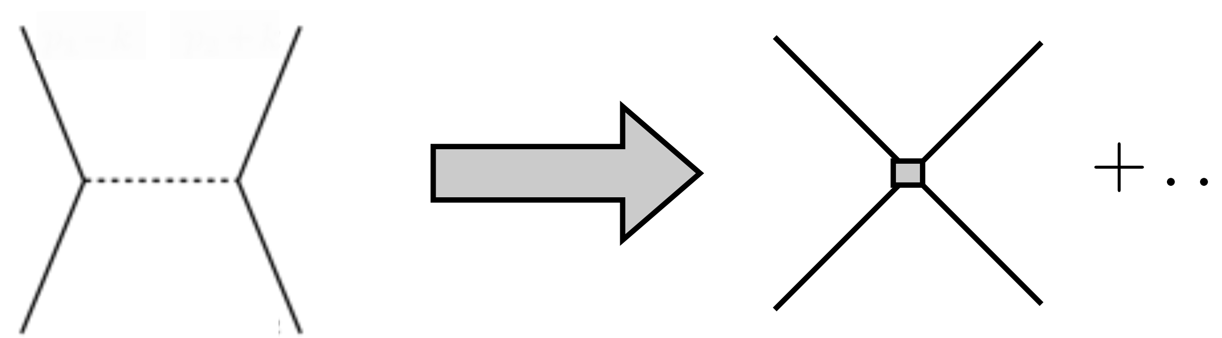

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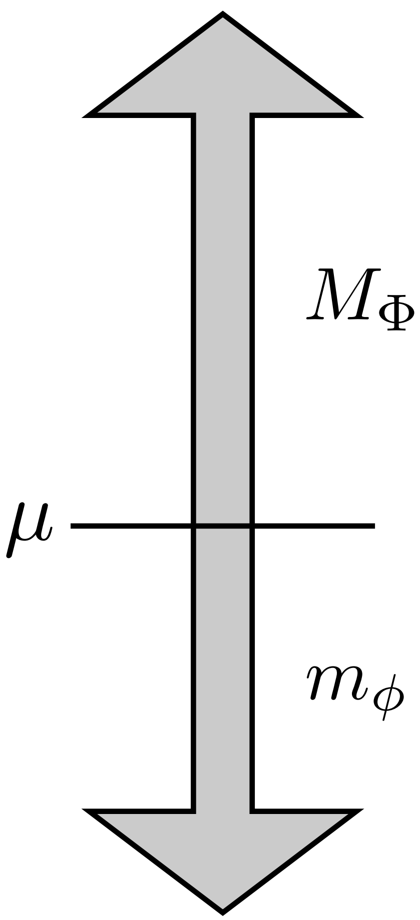

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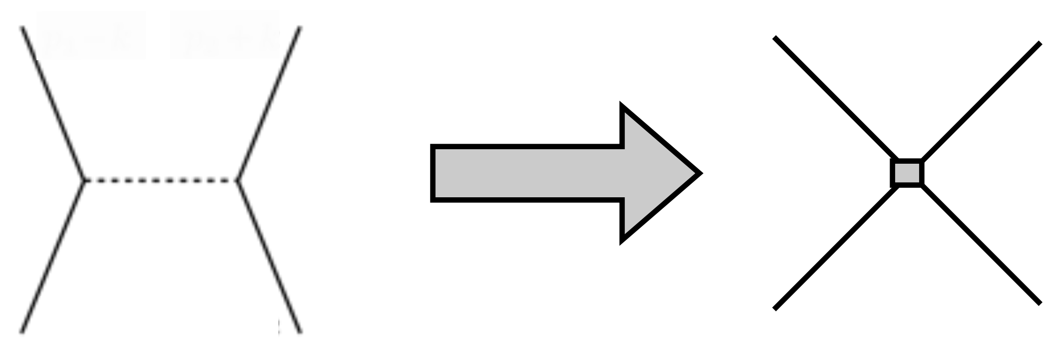
$$\int d^4y\, G(y) = \tilde{G}(k=0) = -\frac{1}{M_\Phi^2}$$



$+ \dots$

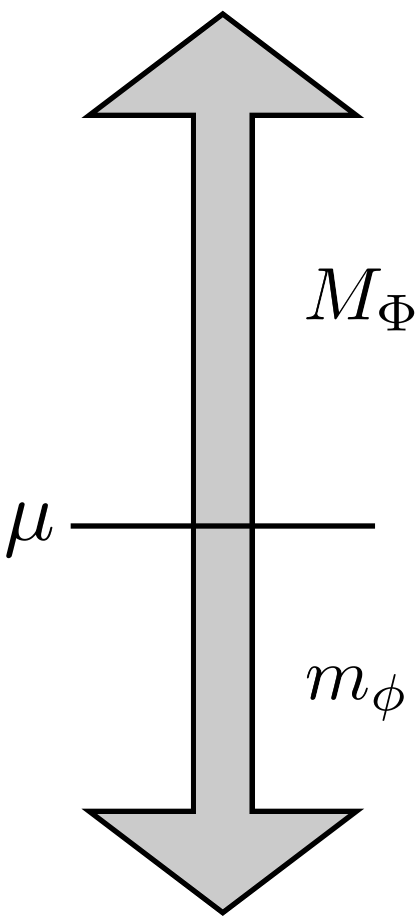
Integrating Out Heavy Particles

$\Phi,\ \phi$

$M_\Phi \gg m_\phi$

- Perform path integral over heavy field
- Expand result in local operators built from light field

$$e^{iS_{\rm eff}[\phi]} = \frac{\int \mathcal{D}\Phi\, e^{i\int d^4x \mathcal{L}(\Phi,\phi)}}{\int \mathcal{D}\Phi\, e^{i\int d^4x \mathcal{L}(\Phi,0)}}$$



Illustrative toy model (Gaussian path integral)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M_\Phi^2\Phi^2 + \Phi\, J$$ <-- coupling to ϕ

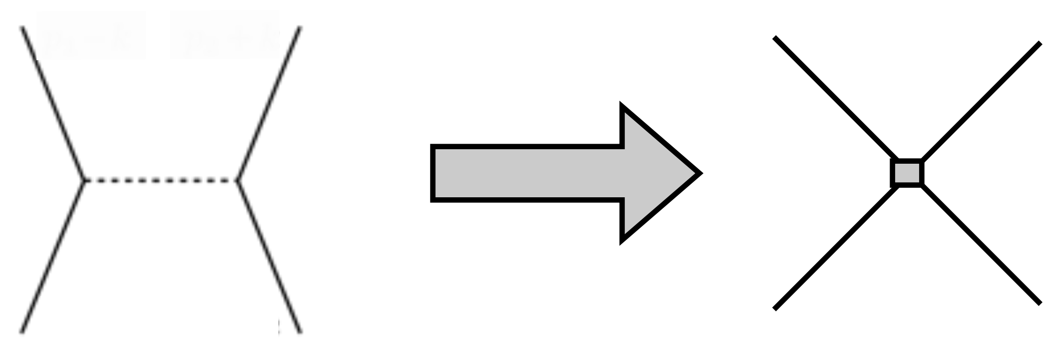
Complete the square to deduce

$$S_{\rm eff}[\phi] = \frac{1}{2M_\Phi^2}\int d^4x\, J(x)J(x) + \dots$$

Operator product expansion

$$J(y) = J(x) + (y-x)_\mu\partial^\mu J(x) + \dots$$

$$\int d^4y\, G(y) = \tilde{G}(k=0) = -\frac{1}{M_\Phi^2}$$

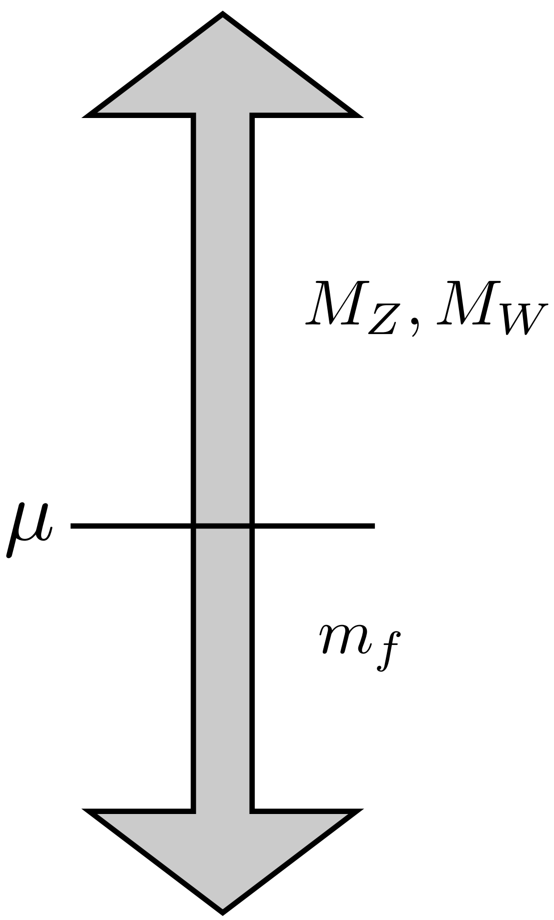


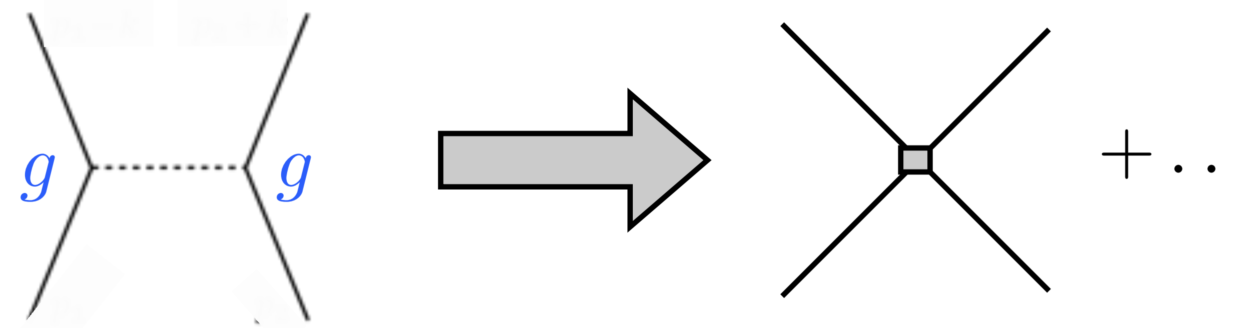
$+\dots$

$$\mathcal{L} = \dim 4$$
$$F_{\mu\nu} = \dim 2$$
$$\psi = \dim \frac{3}{2}$$

Fermi Theory as an EFT

- Weak interactions are so because of large masses of W and Z bosons





$$\sim \frac{1}{q^2 - M_W^2}$$

$$\frac{g^2}{M_W^2}\left[1 + \mathcal{O}\left(\frac{q^2}{M_W^2}\right)\right]$$

- Four-Fermion interaction $\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\overline{\psi}\psi\right)_{V-A} \left(\overline{\psi}\psi\right)_{V-A}$

- Non-renormalizability of Fermi theory implies energy scale

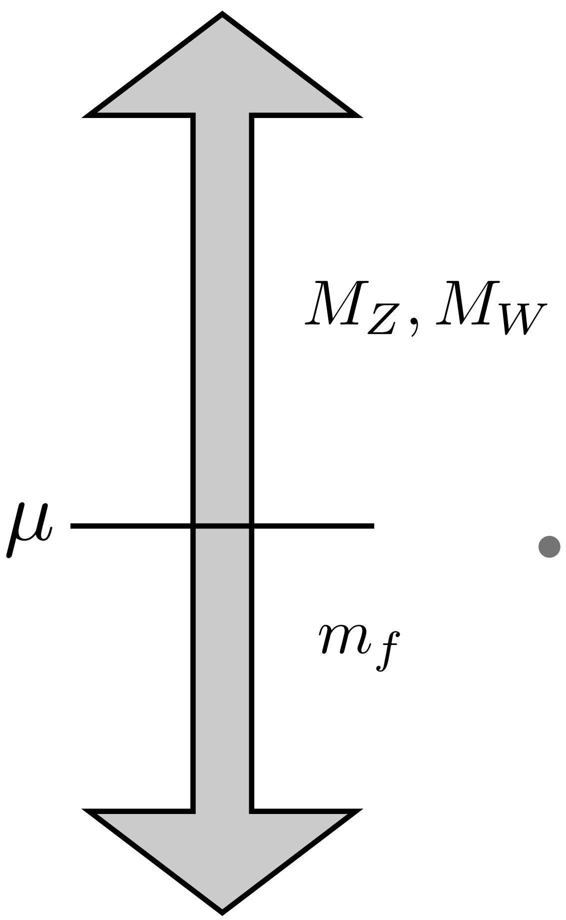
  $G_F = \dim(-2)$ matching to standard model reveals $G_F = \frac{\sqrt{2} g^2}{8 M_W^2}$

- Efficacy of Fermi theory controlled by power counting, **e.g.** ß-decay

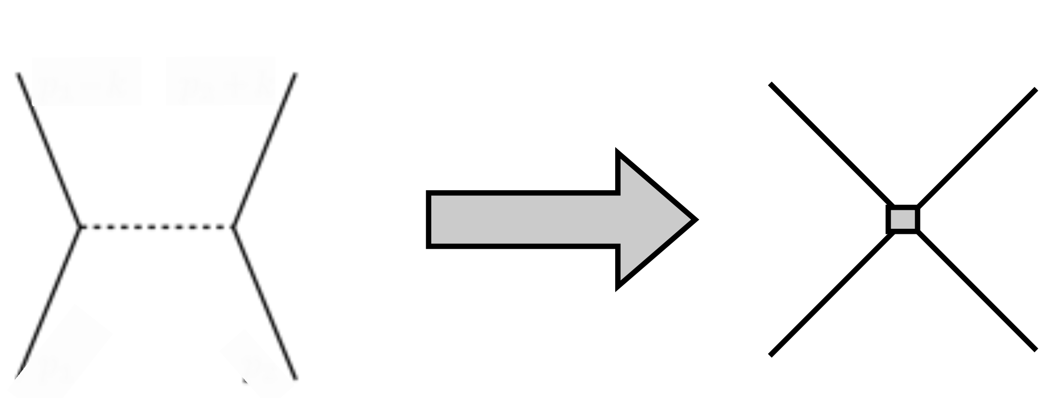
  $(\overline{u}_L \gamma_\mu d_L)(\overline{e}_L \gamma^\mu \nu_L)$ → $n \to p + e + \overline{\nu}_e$ power corrections $\frac{(\delta M_N)^2}{M_W^2} \sim 10^{-10}$

Beyond Tree Level

- "Top-down" uses perturbation theory to integrate out heavy particles



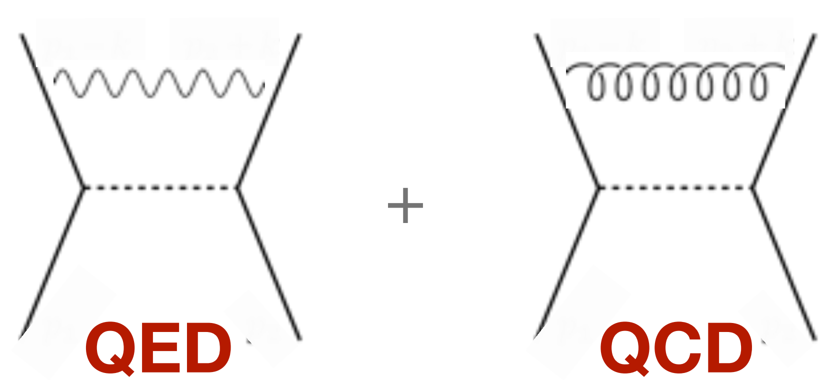
- In addition to power corrections, there are perturbative corrections



+ power corrections

- Fortunately QED and QCD are perturbative at the weak scale

$\alpha(M_Z) \sim \frac{1}{128}$



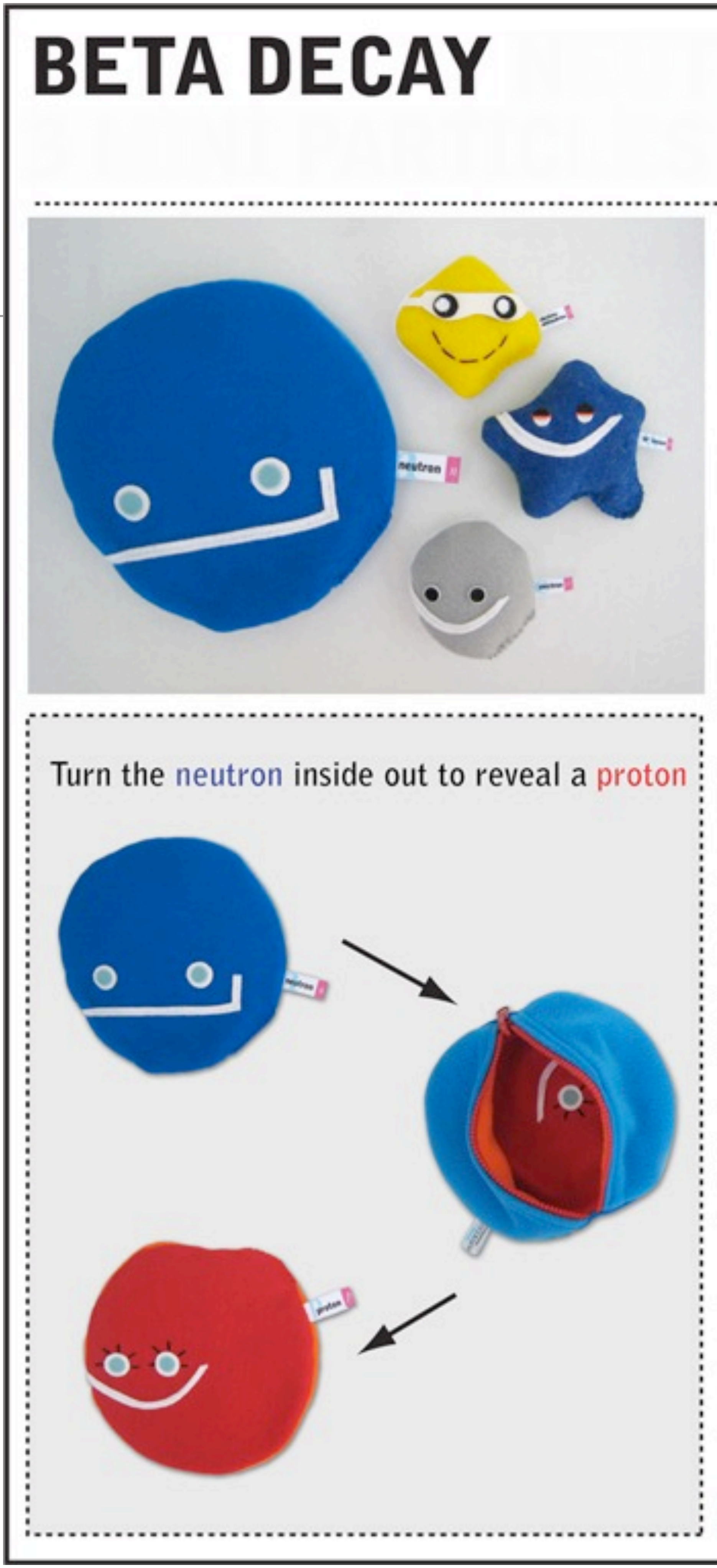
$\alpha_s(M_Z) \sim 0.1$

- Introduces renormalization scale and scheme dependence in EFT $c\,\mathcal{O}(x) \to c(\mu)\mathcal{O}(x,\mu)$

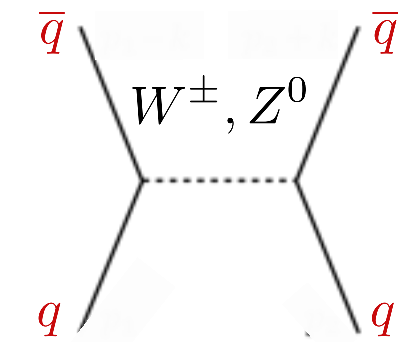
Exercise

- Are there pQCD corrections to the ß-decay operator in Fermi EFT? If so, characterize them. If not, explain why.

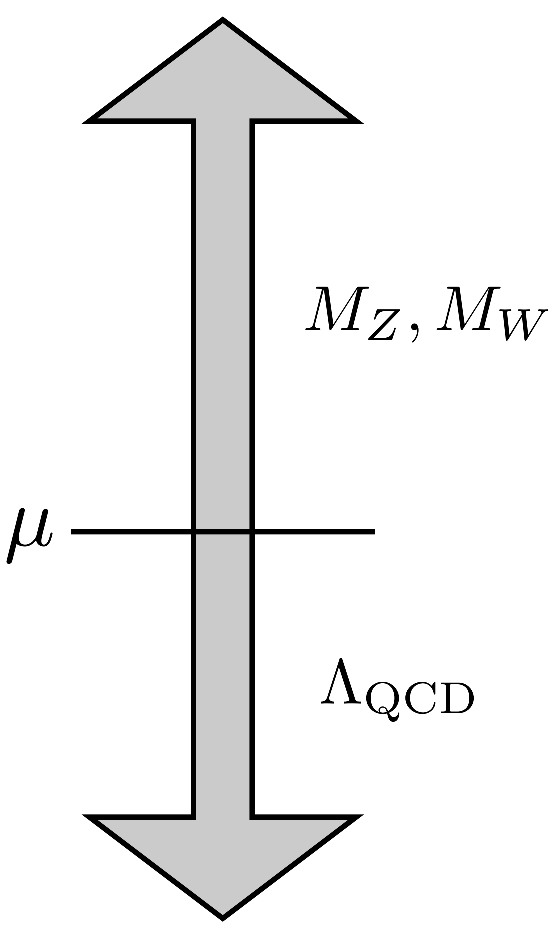
$$(\overline{u}_L \gamma_\mu d_L)(\overline{e}_L \gamma^\mu \nu_L) \Rightarrow n \to p + e + \overline{\nu}_e$$



The Dirtiest Corner of Standard Model



- Hadronic weak interaction observable through processes that violate QCD symmetries



Flavor changing

$\Delta s = 1, \quad K \to \pi\pi$

[Particle Physics]

Flavor conserving but parity violating

$\Delta s = 0, \quad p + p \xrightarrow{\text{PV}} p + p$

$p + {}^4\text{He} \xrightarrow{\text{PV}} p + {}^4\text{He}$

+ PV nuclear reactions

[Nuclear Physics]

Quark weak interactions known at weak scale

Must determine EFT at low scales using renormalization group...

$c(\mu)\ \langle \pi\,\pi\,|\mathcal{O}_{\Delta s=1}(x,\mu)|K\rangle$

$c(\mu)\ \langle p\,p\,|\mathcal{O}_{PV}(x,\mu)|p\,p\,\rangle$

Non-perturbative matrix elements ^will be^ computed at QCD scales with lattice

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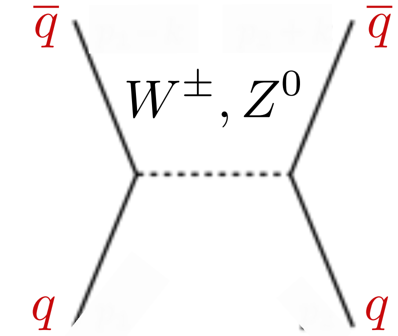

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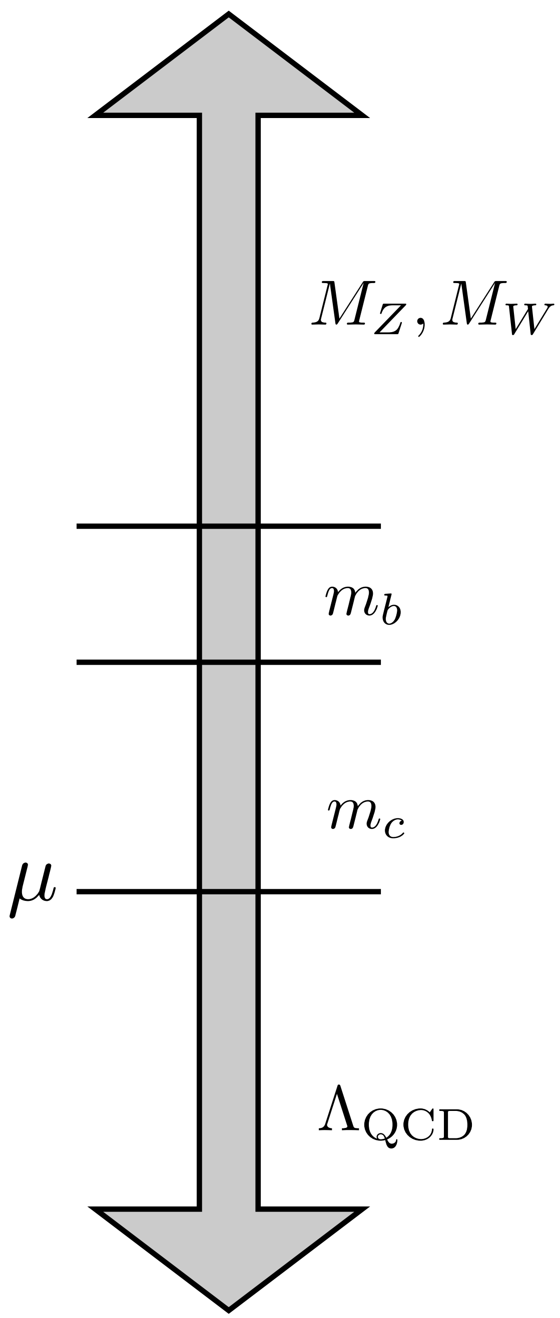

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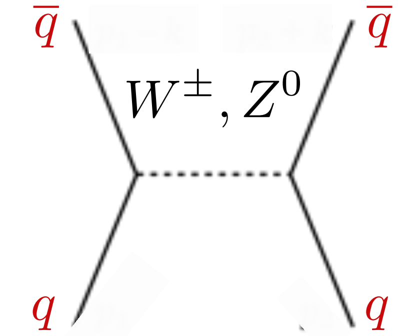
Non-perturbative matrix elements will be computed at QCD scales with lattice

New features:

Multiple scales	(treat one at a time)
Operator mixing	(coeffients --> matrix)
Matching to lattice	(scheme dependence)

The Dirtiest Corner of Standard Model



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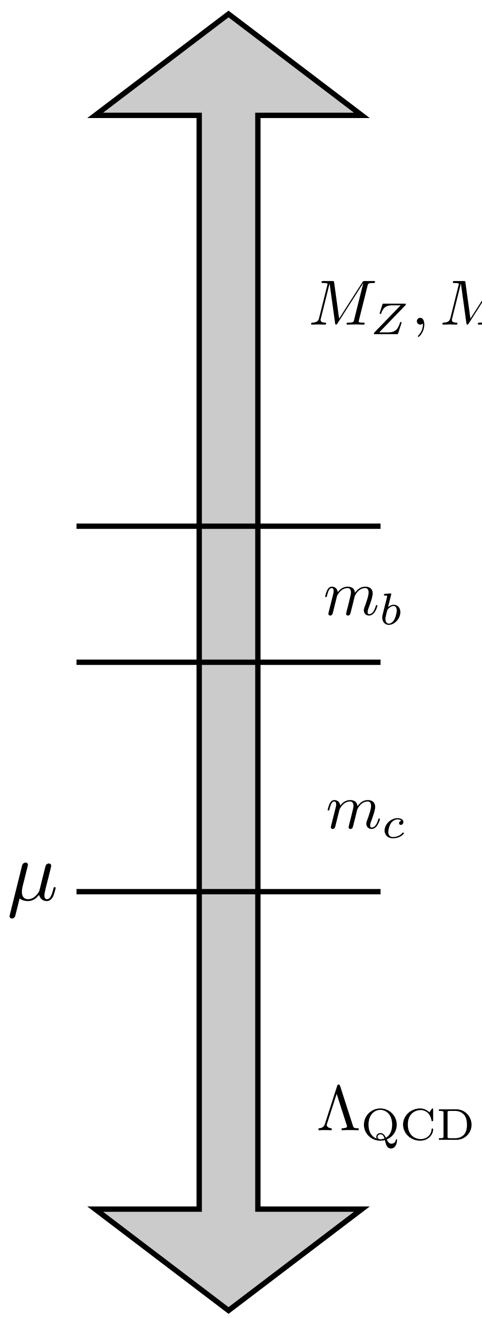

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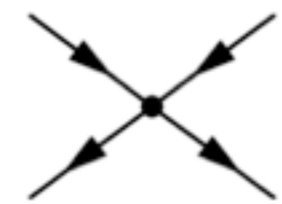
~~Matching to lattice (scheme dependence)~~

Hadronic Parity Violation Beyond Trees

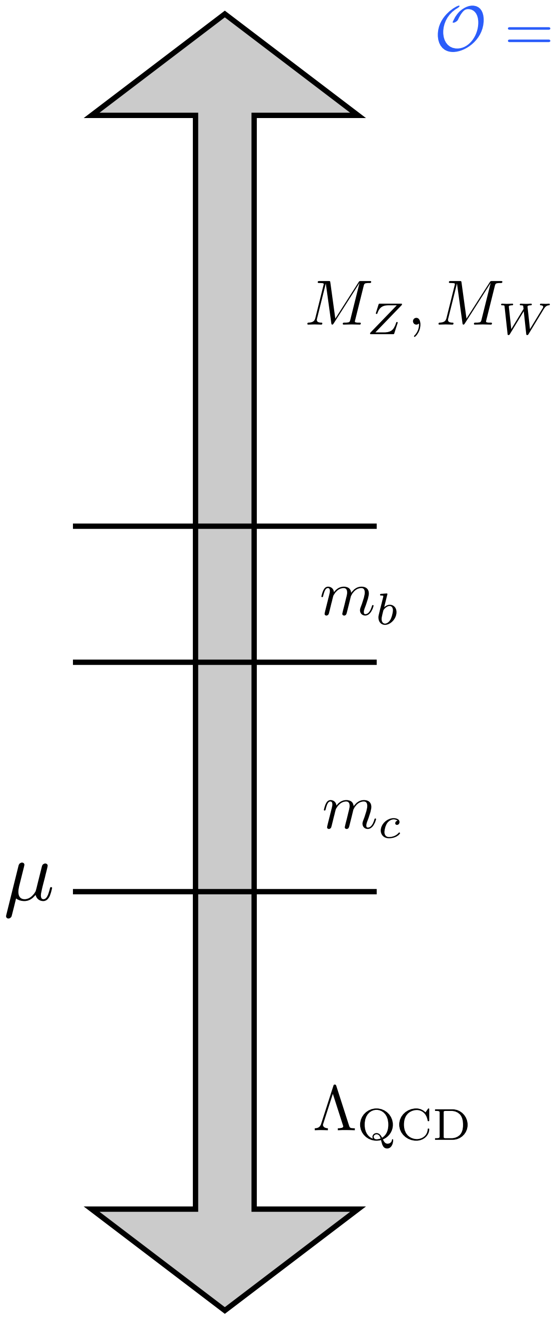
- Standard Model has *isotensor* PV interactions
- Fermi PV EFT described by one operator

$$\mathcal{L}_{PV}^{\Delta I=2} = \frac{G_F}{\sqrt{2}} C(\mu)\mathcal{O}(\mu)$$



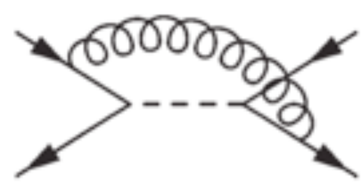
$$\mathcal{O} = (\bar{q}\tau^3\gamma_\mu q)_L(\bar{q}\tau^3\gamma^\mu q)_L - \frac{1}{3}(\bar{q}\vec{\tau}\gamma_\mu q)_L \cdot (\bar{q}\vec{\tau}\gamma^\mu q)_L - \{L \to R\}$$

$$C_{\text{tree}} \approx -\sin^2\theta_W$$

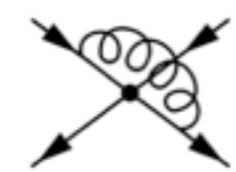


QCD Renormalization at one loop

Full theory



Effective theory



UV finite $\log \frac{M_W^2}{-p^2}$

UV divergent $\log \frac{\mu^2}{-p^2}$

$$\log \frac{M_W^2}{-p^2} = \log \frac{\mu^2}{-p^2} - \log \frac{\mu^2}{M_W^2}$$

EFT will reproduce full theory by altering the low-energy coefficient

$$\Delta C(\mu) = -\gamma_{\mathcal{O}} \frac{\alpha_s(\mu)}{4\pi} \log \frac{\mu^2}{M_W^2}$$

EFT @ M_W has no log, EFT @ m_b has large log --> sum using RG

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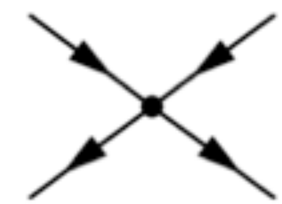

$$\mathcal{O} = (\bar{q}\tau^3\gamma_\mu q)_L(\bar{q}\tau^3\gamma^\mu q)_L - \frac{1}{3}(\bar{q}\vec{\tau}\gamma_\mu q)_L \cdot (\bar{q}\vec{\tau}\gamma^\mu q)_L - \{L \to R\}$$

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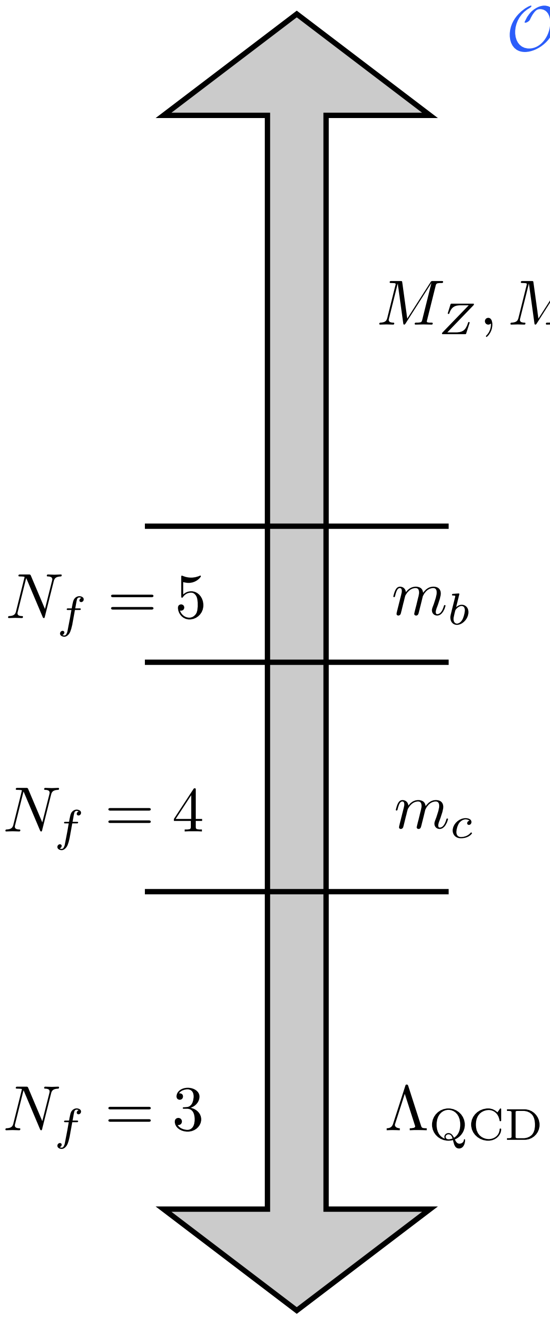
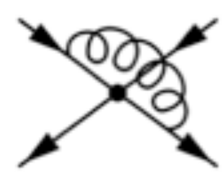


Effective theory



$$4\pi\mu\frac{d}{d\mu}C(\mu) = \gamma_{\mathcal{O}}\,\alpha_s(\mu)\,C(\mu)$$

N_f-dependent

Solution to RG evolution $C(\mu') = U(\mu',\mu)C(\mu)$

$$\beta_0 = 11 - \frac{2}{3}N_f \qquad U(\mu',\mu) = \left[\frac{\alpha_s(\mu')}{\alpha_s(\mu)}\right]^{-\gamma_{\mathcal{O}}/2\beta_0}$$

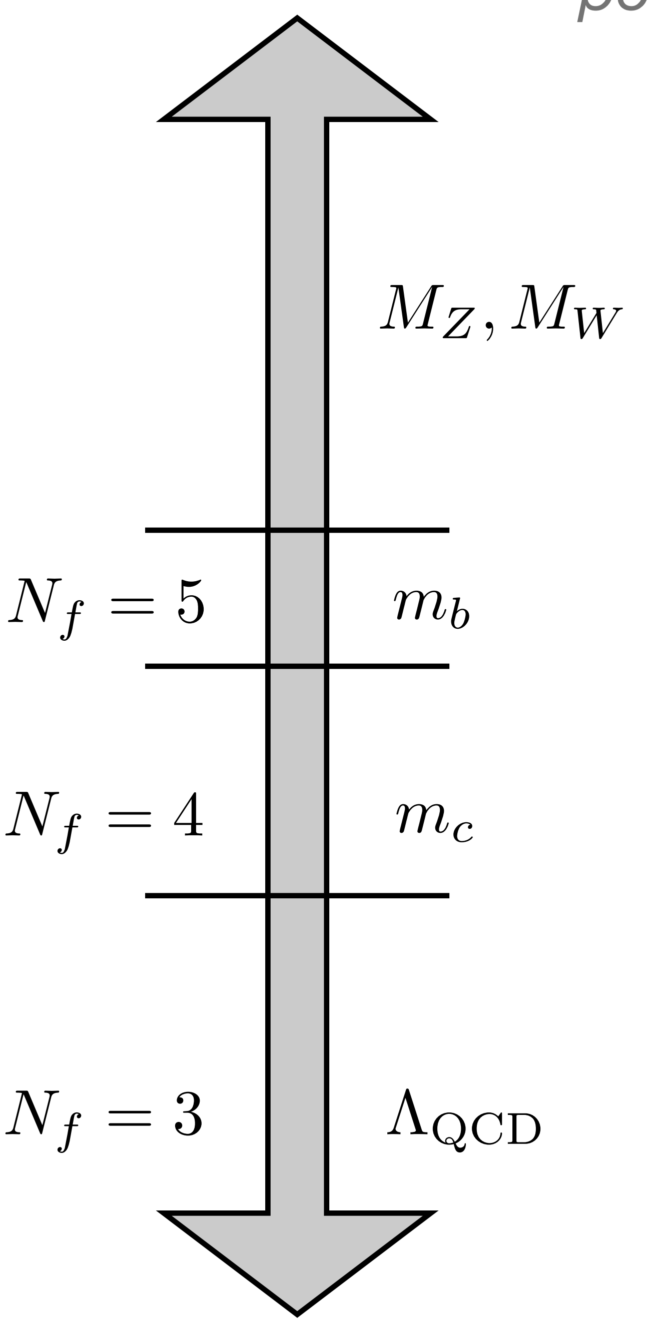
Multiple scales: once below heavy quark threshold integrate out, then match $N_f - 1$ and N_f EFTs at the scale m_Q

$$C(\Lambda_{\text{QCD}}) = \left[\frac{\alpha_s(\Lambda_{\text{QCD}})}{\alpha_s(m_c)}\right]^{-\frac{6}{27}} \left[\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right]^{-\frac{6}{25}} \left[\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right]^{-\frac{6}{23}} C(M_W)$$

Effective Summary
I. Removing heavy particles

- Heavy particles can be systematically integrated out resulting in EFTs

power corrections $\left(\frac{m_\phi}{M_\Phi}\right)^n$ *perturbative corrections* $\left[\frac{\alpha_s(m_\phi)}{\alpha_s(M_\Phi)}\right]^{-\gamma_{\mathcal{O}}/2\beta_0}$



- EFT coefficients determined from matching "top-down"

$$\log \frac{M_W^2}{-p^2} = \log \frac{\mu^2}{-p^2} - \log \frac{\mu^2}{M_W^2}$$

- Theories have different UV behavior

- Only IR behavior is shared and thus cancels in matching

Computations in EFT are simpler (one scale at a time)
EFT involves only d.o.f. relevant to energy regime

Standard Model is an EFT ... arises from integrating out heavy new particles?