Dualities and QCD II

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June 12, 2014

Dualities and QCD

- The meaning of "duality" in physics (Example: The Ising model)
- Quark-Hadron duality (experimental and theoretical evidence)
- The AdS/CFT correspondence (gauge/gravity duality, holographic QCD)

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The AdS/CFT Correspondence

Maldacena - 1997

Witten; Gubser,Klebanov,Polyakov - 1998

Perhaps the most surprising of dualities is the AdS/CFT correspondence, which relates theories in different numbers of spacetime dimensions.

It is a strong-weak coupling duality in certain limits of theory parameters.

It is one of the most active areas of string theory research, and has motivated models of QCD, superconductors, cold atoms, fermi liquids, ...

The AdS/CFT Correspondence

The motivation for the correspondence begins with Black Hole Thermodynamics.

Black holes radiate with a temperature that depends on the black hole mass. The relation between mass and temperature determines an entropy:

$$S = A_H/4G\hbar$$

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Entropy is usually an extensive quantity: it grows with Volume. With gravity, the maximum entropy of a system grows with Area!

The AdS/CFT Correspondence

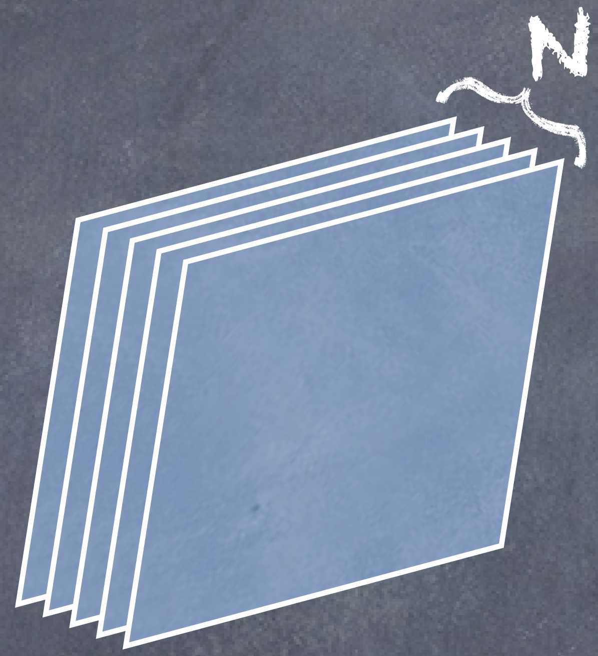
It is as if a nongravitational theory contains the same information as a theory with gravity in one additional dimension. ⟵ "holography" ('t Hooft, Susskind)

Could a weakly-coupled theory with gravity describe strogly-coupled QCD at low energies (in the resonance region)?

To explore this possibility we will first explore what the AdS/CFT correspondence really means.

The AdS/CFT Correspondence

Maldacena, hep-th/9711200



D3 branes

3 spatial dimensions

Dirichlet
(strings end here)

The AdS/CFT Correspondence

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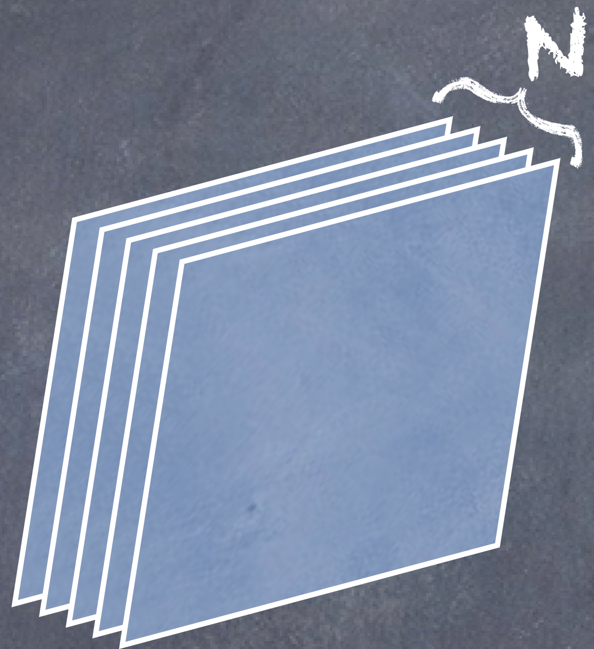

Massless spectrum of open strings attached to D3-branes in Type IIB String Theory is described by N=4 SUSY SU(N) Yang-Mills theory

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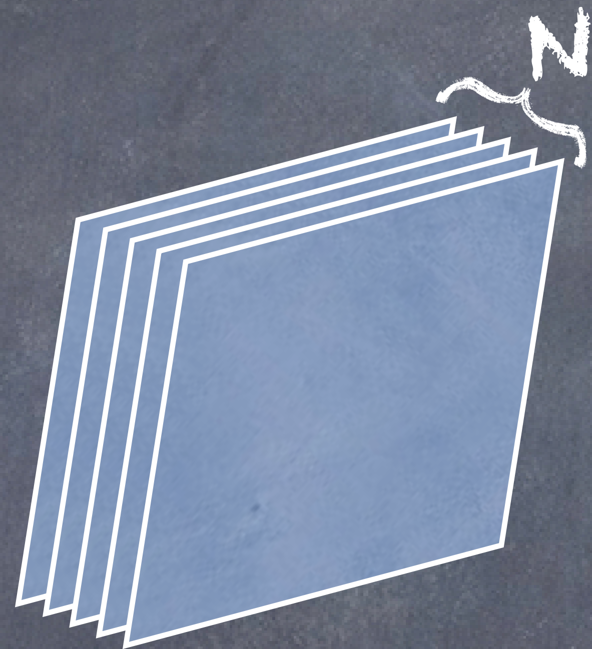

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Massless spectrum of open strings attached to D3-branes in Type IIB String Theory is described by N=4 SUSY SU(N) Yang-Mills theory

@ $N \rightarrow \infty$ with fixed large $g_s N$:
Closed strings describe Type IIB SUGRA in a background with near-horizon geometry $AdS_5 \times S_5$

The AdS/CFT Correspondence

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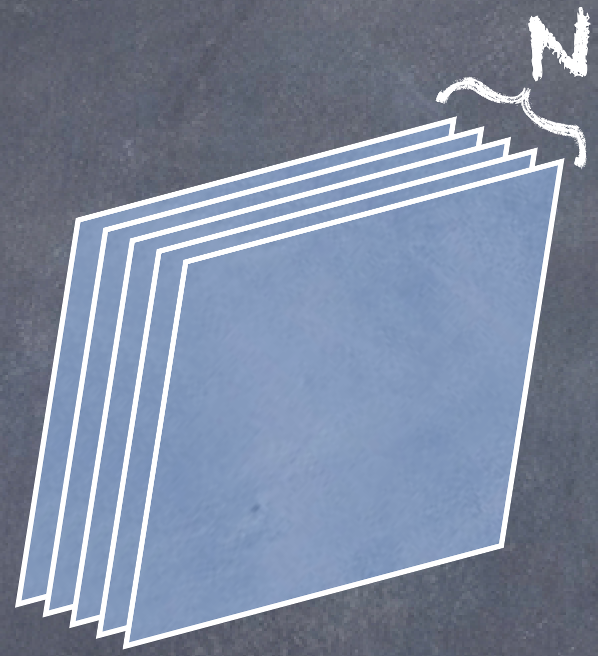

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$\updownarrow$

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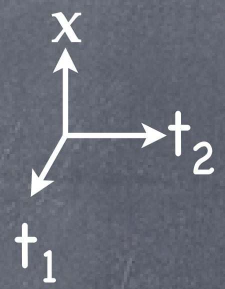
Closed strings describe Type IIB SUGRA in a background with near-horizon geometry $AdS_5 \times S_5$

Anti-de Sitter Space (AdS_5)



$$t_1^2 + t_2^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = R^2$$

$$ds^2 = dt_1^2 + dt_2^2 - dX_1^2 - dX_2^2 - dX_3^2 - dX_4^2$$

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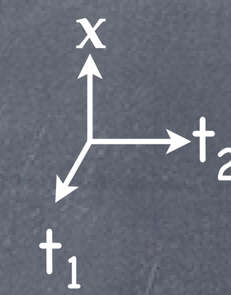

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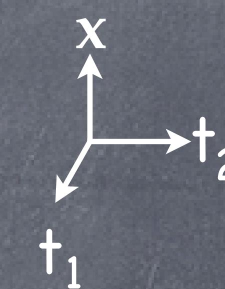

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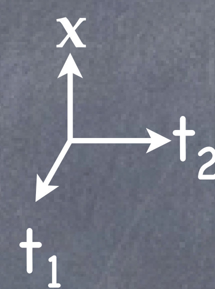

$$t_1^2 + t_2^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 = R^2$$

$$ds^2 = dt_1^2 + dt_2^2 - dX_1^2 - dX_2^2 - dX_3^2 - dX_4^2$$

Poincare coordinates:

$$z = \frac{R^2}{t_2 - X_4} \qquad t = \frac{t_1 z}{R} \qquad x_i = \frac{X_i z}{R} \qquad X_4 = \frac{1}{2z}\left(z^2 - R^2 + \mathbf{x}^2 - t^2\right)$$

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2\right)$$

Anti-de Sitter Space (AdS_5)

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$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2\right)$$

z>0 covers half of the spacetime

Hints of a Conformal Theory

$$t_1^2 + t_2^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = R^2$$

$$ds^2 = dt_1^2 + dt_2^2 - dX_1^2 - dX_2^2 - dX_3^2 - dX_4^2$$

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Isometries of AdS_5: SO(2,4)

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Isometries of AdS_5: SO(2,4)

SO(2,4) is also the group of conformal symmetry transformations in 4D

(Poincare symmetry, dilations, inversions)

The AdS/CFT Dictionary

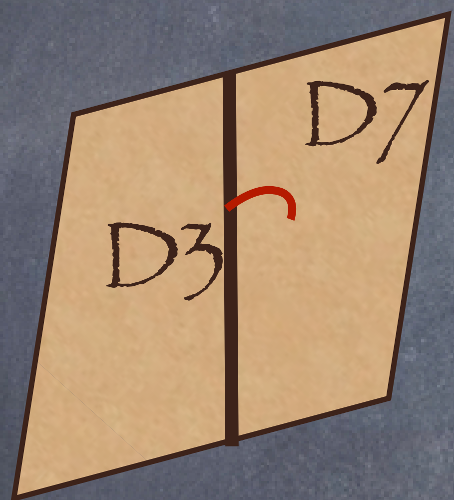
$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2\right)$$

z=0: boundary of the spacetime

Gauge	Gravity
Operator	Field
Scaling dimension of operator	Mass of field
Source for operator	non-normalizable background profile for field near AdS boundary
Generating functional for connected correlation functions	Action with background profiles for fields near AdS boundary

Adding Flavor to AdS/CFT

(Karch, Katz)



Strings from N D3-branes to D7-branes are fundamentals under SU(N)

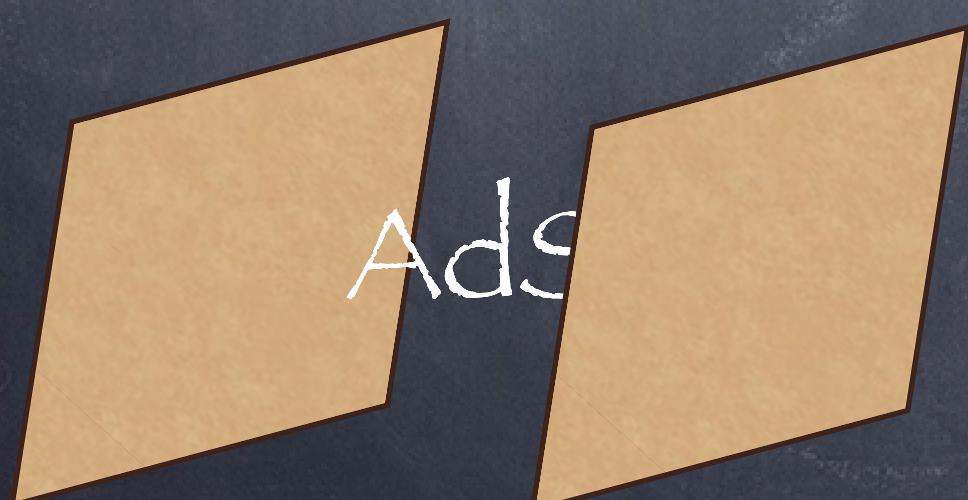
With N_f D7-branes, SU(N_f) gauge fields couple to the flavor current.

The spectrum of those gauge fields corresponds to the spectrum of vector mesons.

Confinement and AdS/CFT

There is no mass gap in a conformal theory. To mimic QCD we need to break the conformal invariance and generate a mass gap.

One way to do this is to introduce a hard wall into the geometry (Polchinski-Strassler).



$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2\right)$$

$$z \in (\epsilon, z_{IR})$$

Particles in Extra Dimensions

Suppose every proton had the same momentum p_5 in a flat extra dimension - like a certain mode of a particle in a box

$$E^2 = \mathbf{p}^2 + p_5^2$$

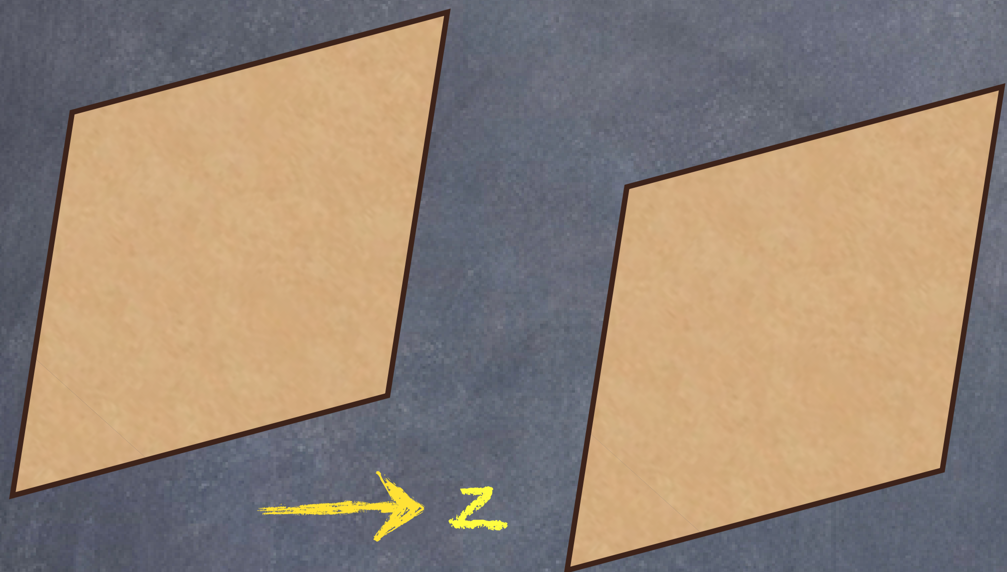
Particles in Extra Dimensions

Suppose every proton had the same momentum p_5 in a flat extra dimension - like a certain mode of a particle in a box

$$E^2 = \mathbf{p}^2 + p_5^2$$

looks like a mass from the 3+1 dim'l perspective

Kaluza-Klein Modes



Example

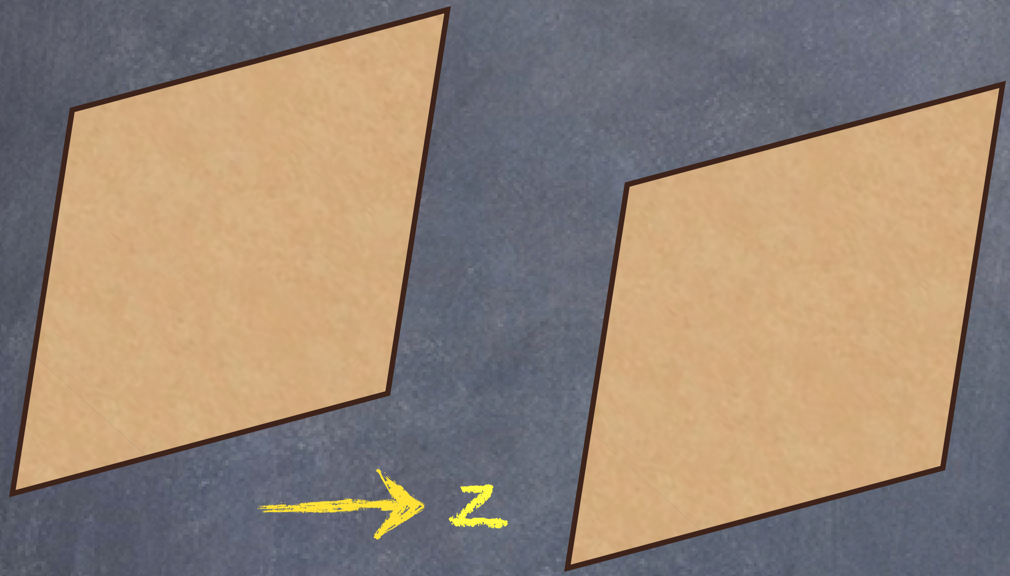
Vector field in slice of 5D Minkowski space $z \in \{0, L\}$

$$S = -\frac{1}{4} \int d^4x \, dz \; \left(F_{\mu\nu} F^{\mu\nu} + F_{\mu z} F^{\mu z} \right)$$

$$\mu \in \{0, 1, 2, 3\}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$F_{\mu z} = \partial_\mu A_z - \partial_z A_\mu$$

Kaluza-Klein Modes



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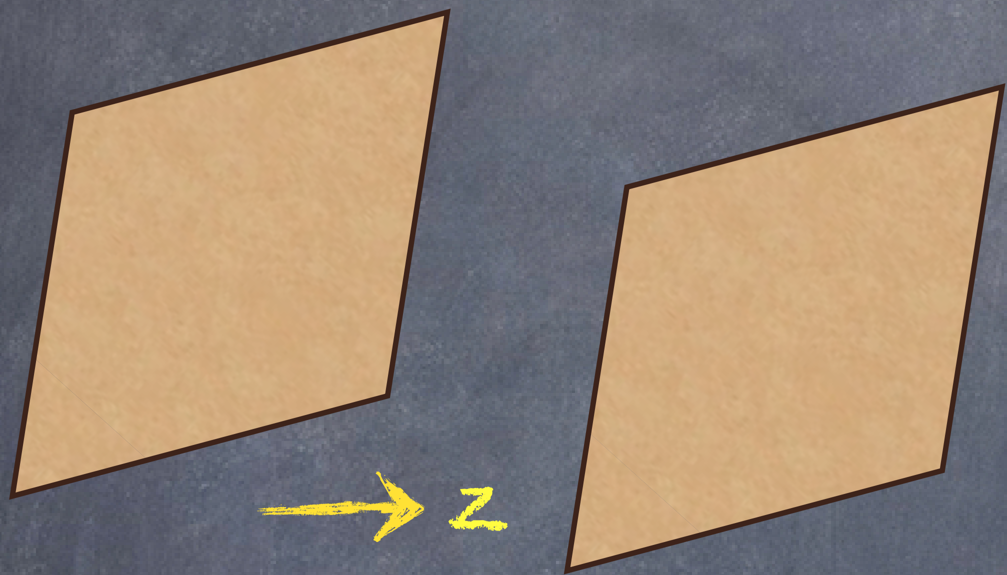
Vector field in slice of 5D Minkowski space $z \in \{0, L\}$

A_ν EOM: $$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + \partial_z(\partial^z A^\nu - \partial^\nu A^z) = 0$$

A_z EOM: $$\partial_\mu(\partial^\mu A^z - \partial^z A^\mu) = 0$$

Kaluza-Klein Modes



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$A_z = 0$ gauge

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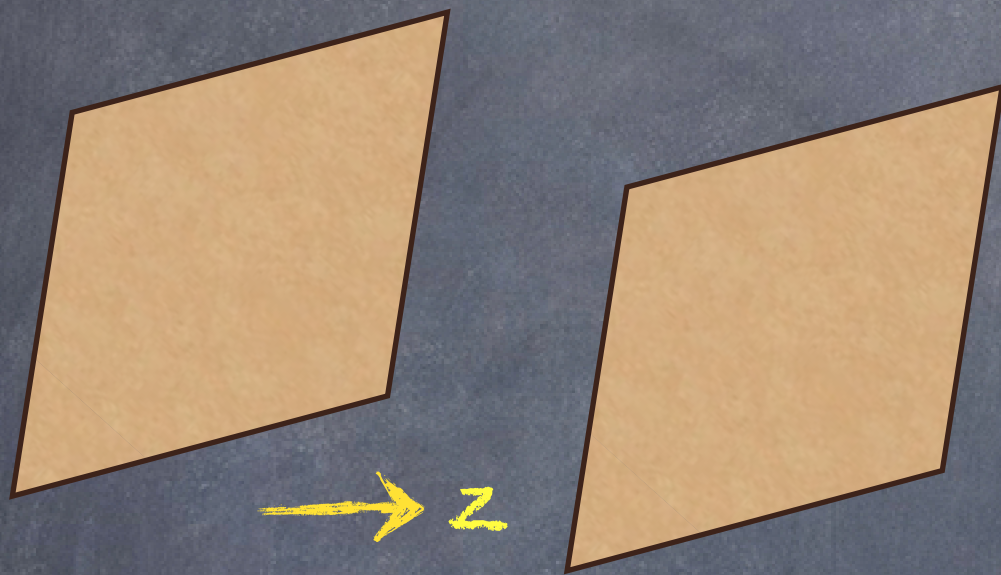

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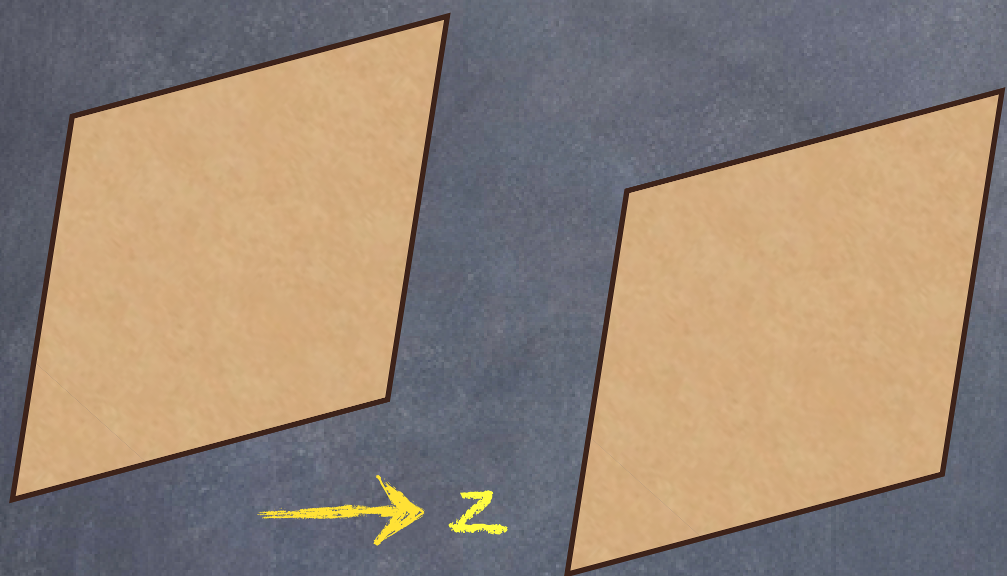

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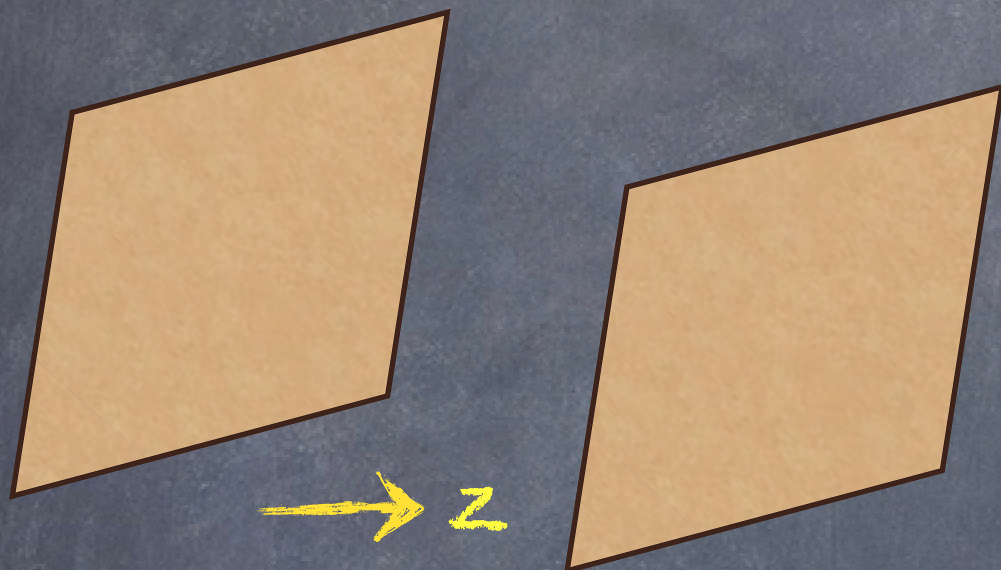
A_z=0 gauge, choose $\partial_\mu A^\mu = 0$

Kaluza-Klein Modes



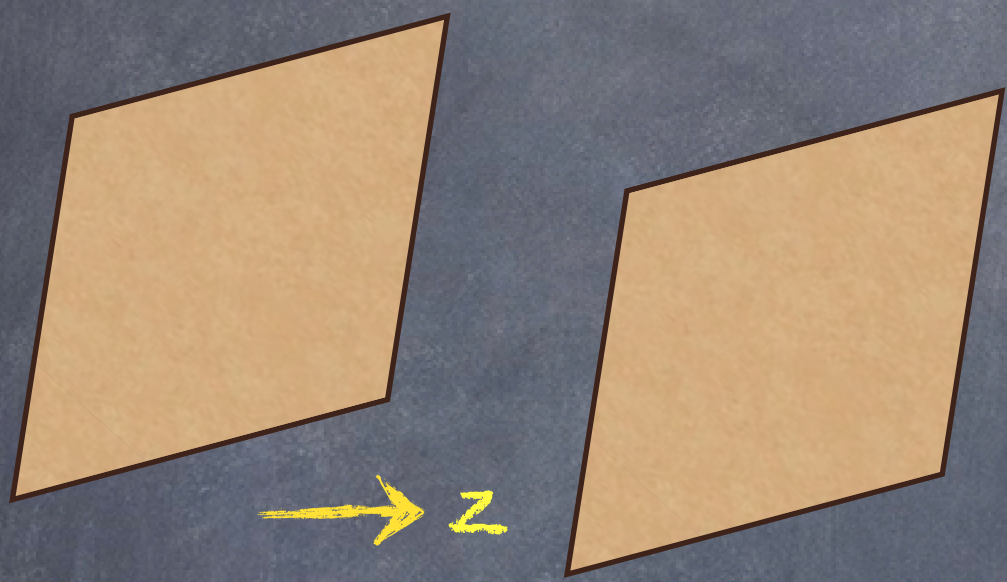
Example

Vector field in slice of 5D Minkowski space $z \in \{0, L\}$

A_ν EOM:

$$\partial_\mu \partial^\mu A^\nu - \frac{\partial^2 A^\nu}{\partial z^2} = 0$$

Kaluza-Klein Modes



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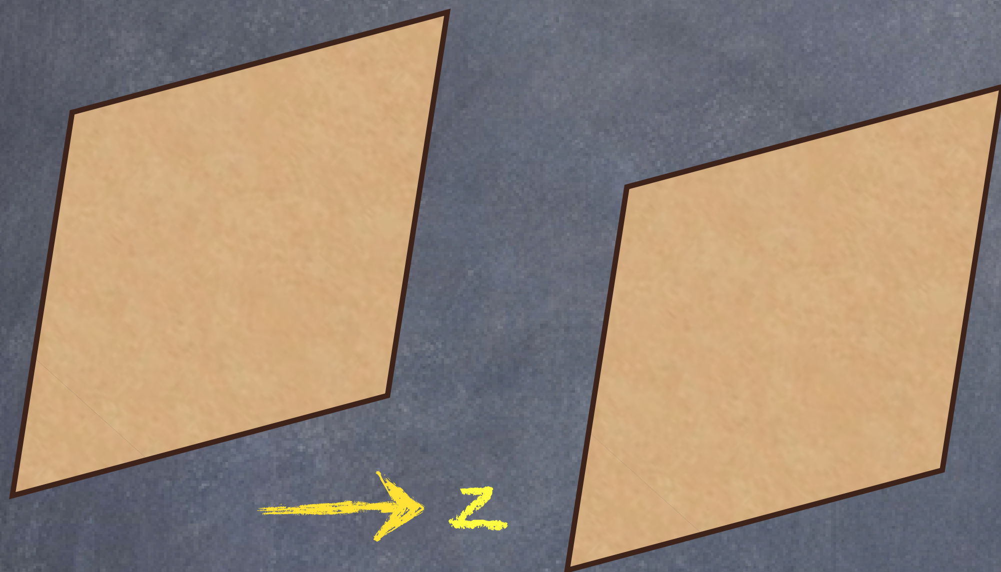
$$\partial_\mu \partial^\mu A^\nu - \frac{\partial^2 A^\nu}{\partial z^2} = 0$$

Separation of Variables: $A^\nu(x, z) = \tilde{A}^\nu(x)\psi(z)$

Kaluza-Klein Modes



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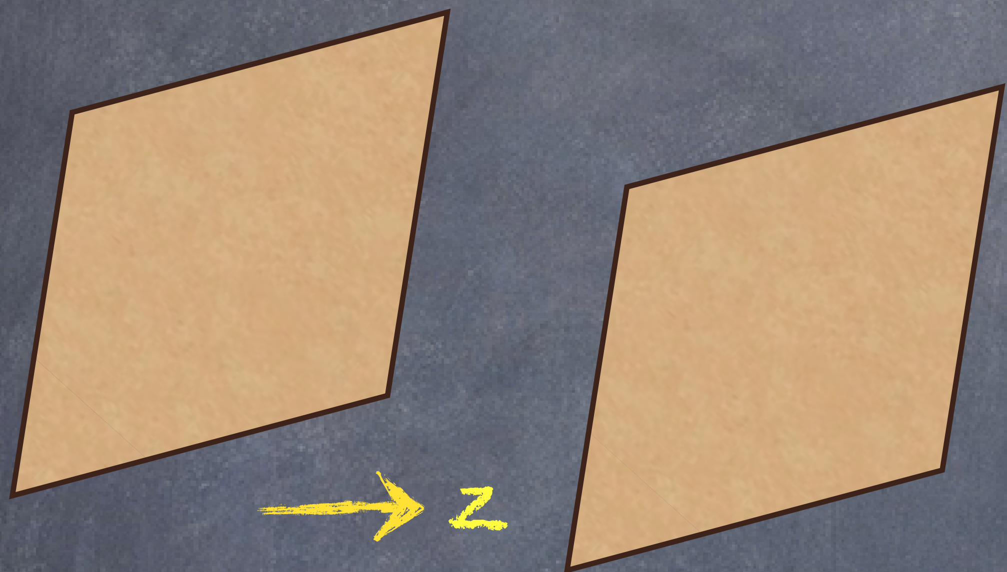
$$\partial_\mu \partial^\mu \tilde{A}^\nu(x) = -q^2\, \tilde{A}^\nu(x)$$

$$\psi''(z) = -q^2\, \psi(z)$$

Kaluza-Klein Modes



Example

Vector field in slice of 5D Minkowski space $z \in \{0, L\}$

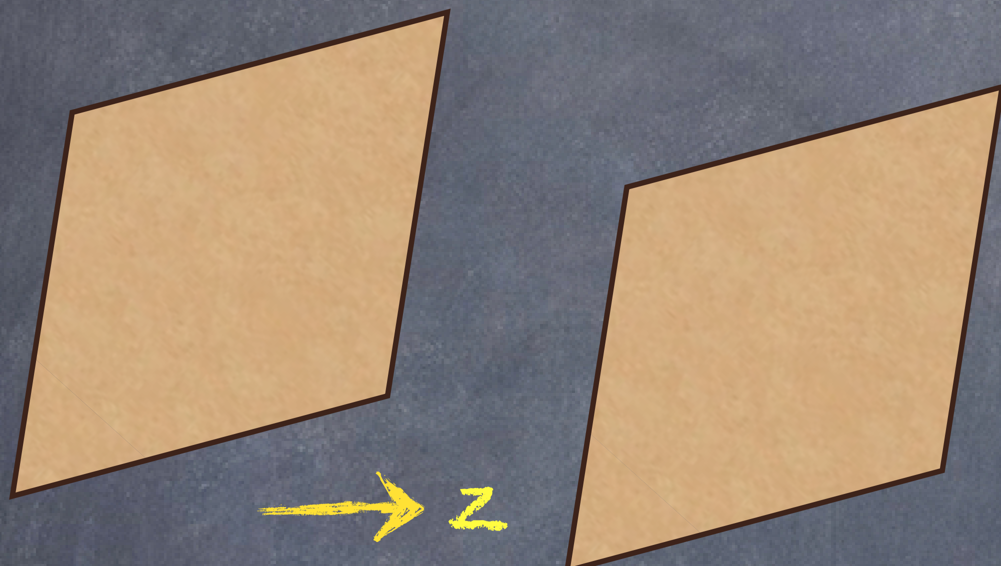
mass² of 4D field $\tilde{A}^\nu(x)$

$$\partial_\mu \partial^\mu \tilde{A}^\nu(x) = -q^2 \tilde{A}^\nu(x)$$

$$\psi''(z) = -q^2 \psi(z)$$

Boundary conditions on $\psi(z)$ determine eigenvalues of q^2

Kaluza-Klein Modes



Example

Vector field in slice of 5D Minkowski space $z \in \{0, L\}$

$$\partial_\mu \partial^\mu \tilde{A}^\nu(x) = -q^2 \, \tilde{A}^\nu(x) \qquad F_{\mu z}(0) = F_{\mu z}(L) = 0$$

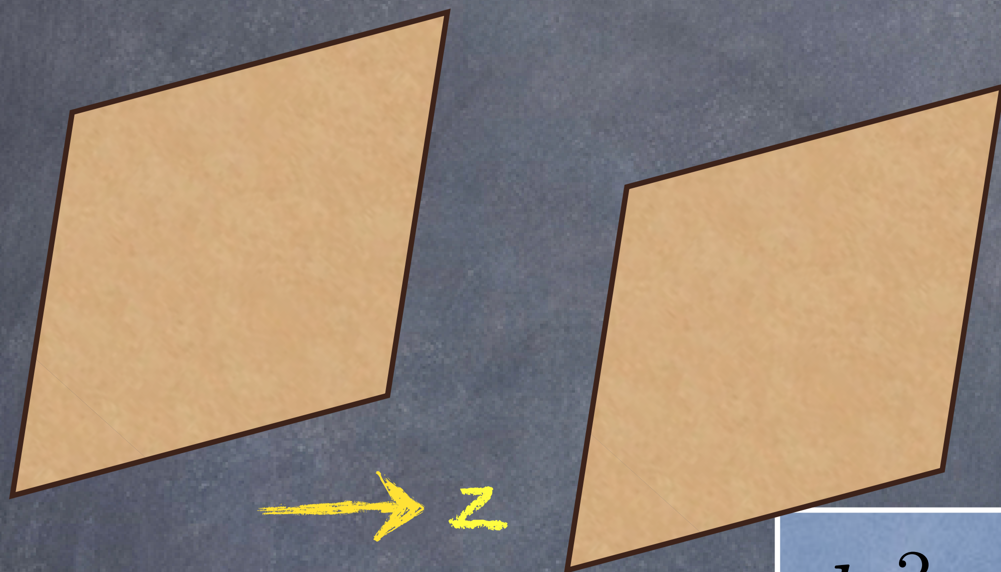
$$\psi''(z) = -q^2 \, \psi(z) \qquad \psi'(0) = \psi'(L) = 0$$

$$\psi_n(z) = \cos(n\pi z/L)$$

$$q_n^2 = \frac{n^2 \pi^2}{L^2}$$

← Kaluza-Klein masses²

Kaluza-Klein Modes in AdS



Example

Vector field in slice of 5D AdS space $z \in \{\epsilon, z_{IR}\}$

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2\right)$$

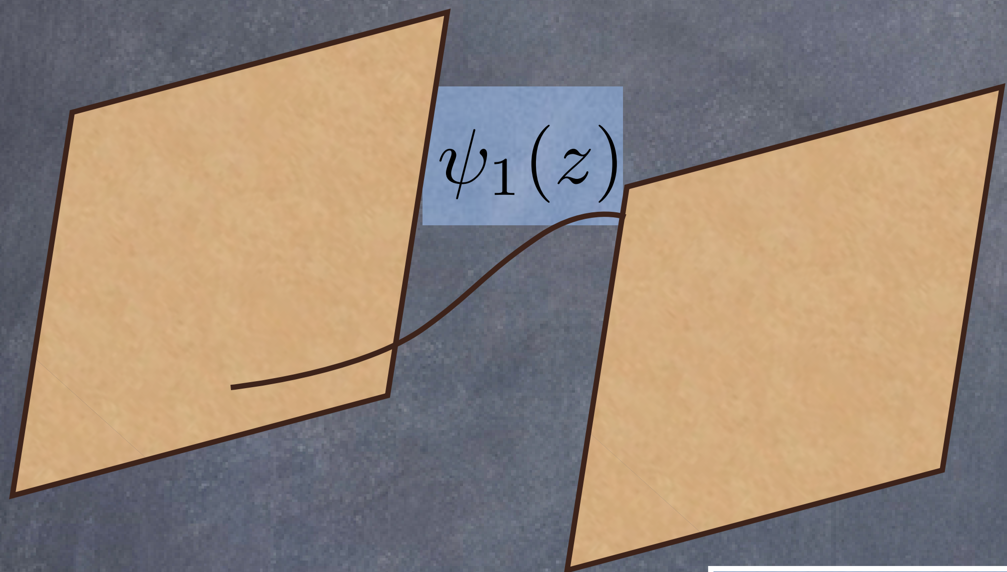
$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g} F_{MN} F_{AB}\, g^{MA} g^{NB}$$

determinant of metric

inverse of metric

$$= -\frac{1}{4}\int d^4x\, dz\, \frac{R}{z} F_{MN} F_{AB}\, \eta^{MA} \eta^{NB}$$

Kaluza-Klein Modes in AdS



Example

Vector field in AdS

EOM:

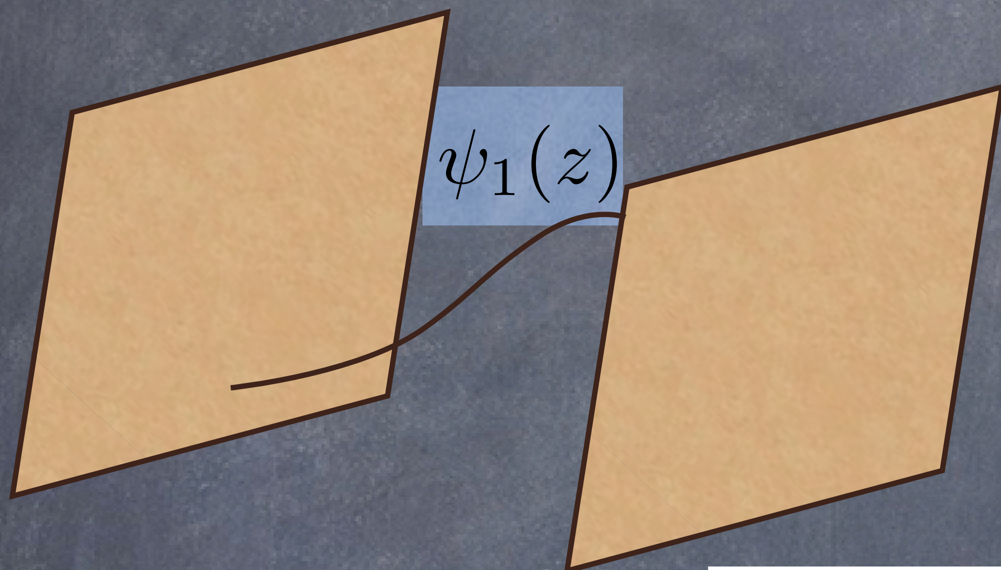
$$\partial_z \left(\frac{1}{z} \partial_z V_\nu(x,z) \right) = \frac{1}{z}\, \partial_\mu \partial^\mu V_\nu(x,z)$$

Kaluza-Klein Modes:

$$V_\nu(x,z) = V_\nu(x) \psi_n(z)$$

$$\partial_z \left(\frac{1}{z} \partial_z \psi_n(z) \right) = -\frac{m_n^2}{z} \psi_n(z)$$

Kaluza-Klein Modes in AdS



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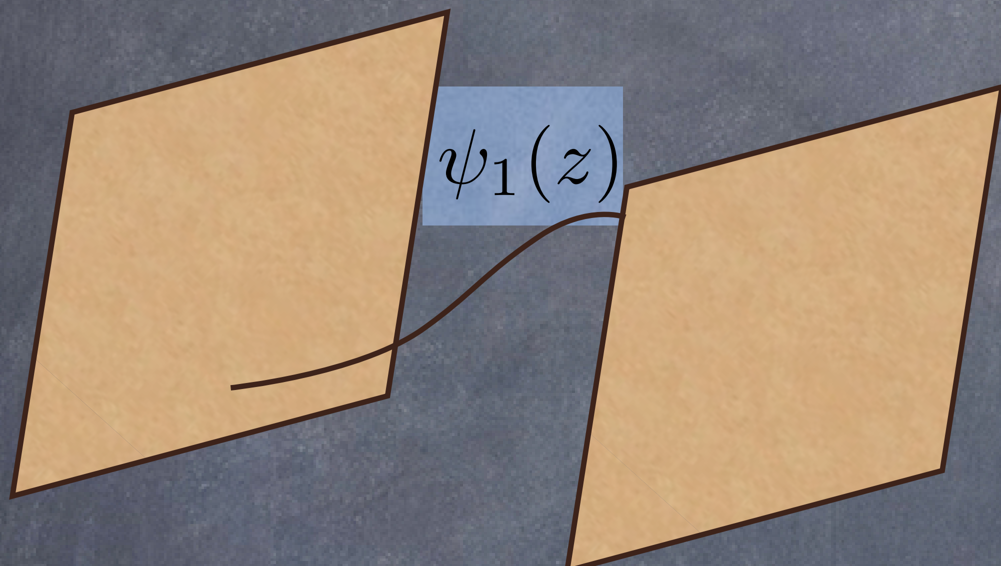
Kaluza-Klein Modes: $$V_\nu(x,z) = V_\nu(x)\psi_n(z)$$
$$\partial_z\left(\frac{1}{z}\partial_z\psi_n(z)\right) = -\frac{m_n^2}{z}\psi_n(z)$$

Boundary Conditions: $\psi_n(\epsilon) = \psi_n'(z_{IR}) = 0$

Kaluza-Klein Modes in AdS



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Vector field in AdS

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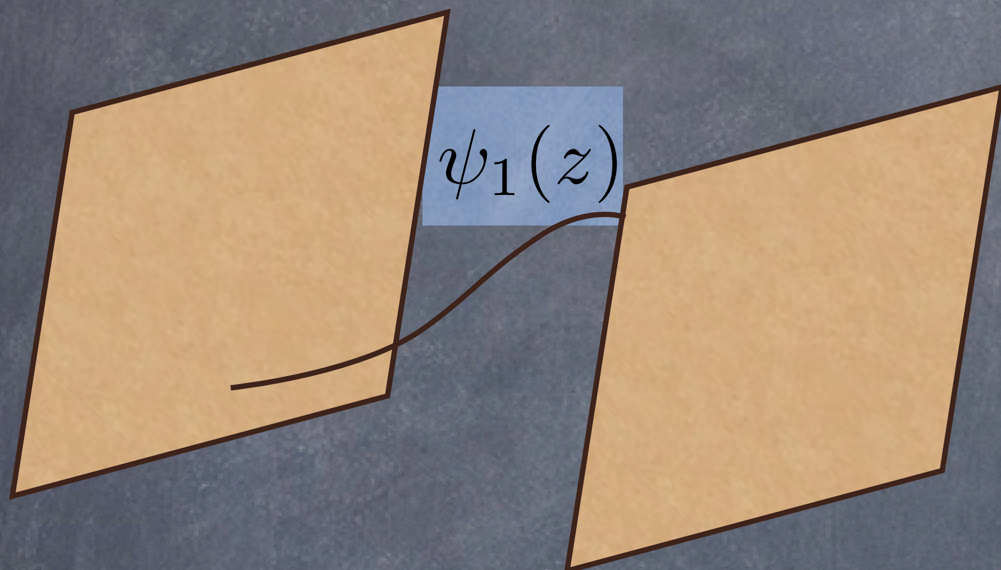
Exercise 1: Write $\psi(z) = z^p\tilde{\psi}(z)$, choose p so that the equation for $\tilde{\psi}(z)$ becomes Bessel's eqn.

Kaluza-Klein Modes in AdS



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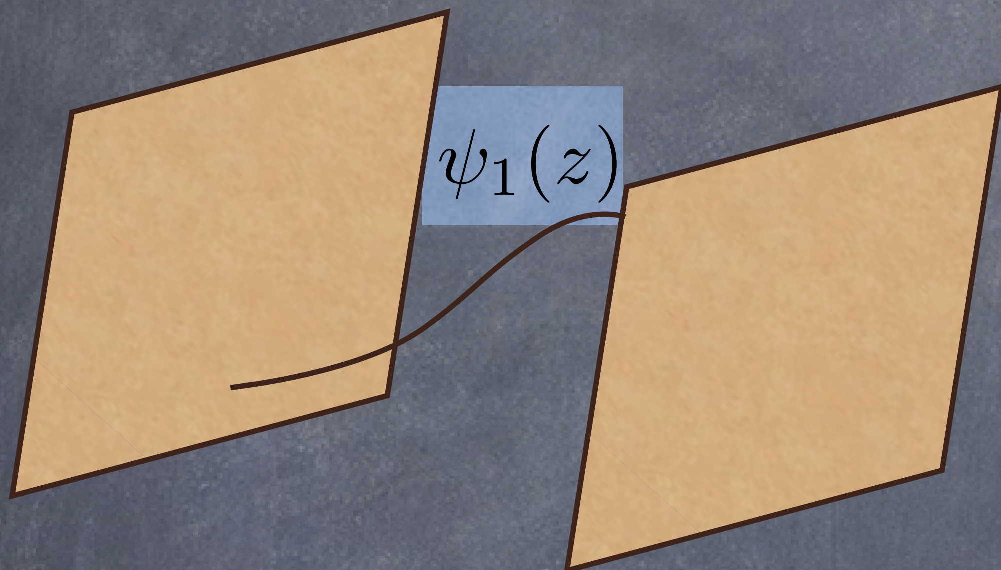

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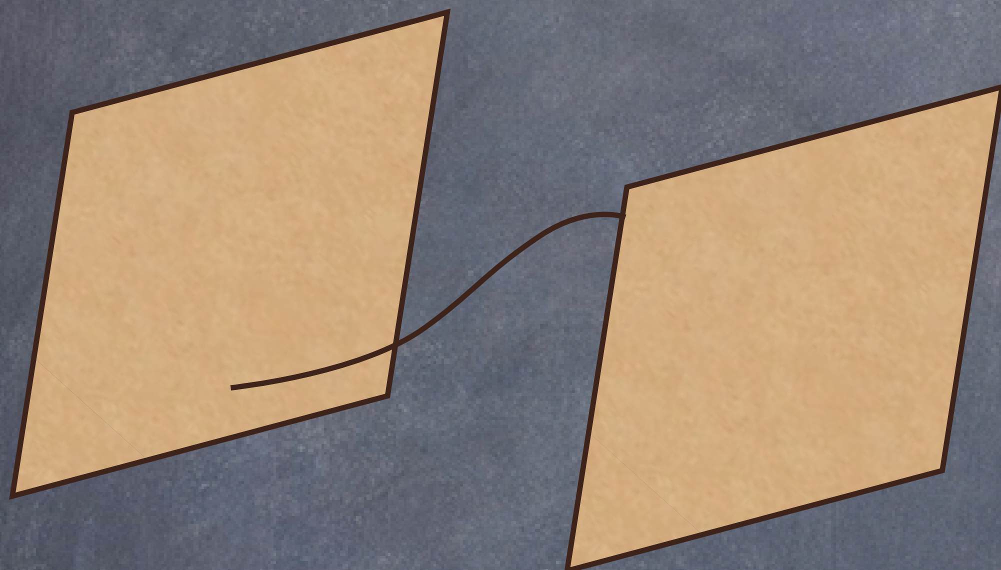
Boundary Conditions: $\psi_n(\epsilon) = \psi_n'(z_{IR}) = 0$

Exercise 2: Show that as $\epsilon \to 0$ the eigenvalues satisfy $J_0(m_n z_{IR}) = 0$.

AdS/CFT:

Kaluza-Klein Modes $\longleftrightarrow$ Bound States



AdS/CFT:

Current $\longleftrightarrow$ Vector field

Kaluza-Klein modes of vector field are rho mesons, and eigenvalues of q^2 determine rho meson masses!

Statement of the AdS/CFT Correspondence

$$\langle e^{i \int d^4x \, \rho(x) \mathcal{O}(x)} \rangle \equiv e^{iW[\rho(x)]}$$

source for operator $\mathcal{O}$

generating functional for connected correlators

AdS/CFT: $$S_{5D}\left[\phi(x,z)\right]_{\phi(x,\epsilon)\sim\rho(x)} = W[\rho(x)]$$

Statement of the AdS/CFT Correspondence

$$\langle e^{i\int d^4x\, A_\mu(x) J^\mu(x)} \rangle \equiv e^{iW[A_\mu(x)]}$$

source for current J

generating functional for connected correlators

AdS/CFT: $S_{5D}\left[A_\mu(x,z)\right]_{A_\mu(x,\epsilon)\sim A_\mu(x)} = W[A_\mu(x)]$

Vector Current Correlators

We need the 5D action on a solution to the EOM that approaches the (transverse) source V(x) at the AdS boundary.

$$\partial_z \left(\tfrac{1}{z}\partial_z V_\nu(x,z)\right) = \tfrac{1}{z}\,\partial_\mu \partial^\mu V_\nu(x,z)$$

5D action vanishes on solution to EOM except for a boundary term:

$$S = -\frac{1}{2g_5^2}\int d^4x \left(\frac{1}{z} V_\mu^a(x,z)\partial_z V^{\mu a}(x,z)\right)_{z=\epsilon}$$

Vector Current Correlators

Fourier transform in 3+1 dim's:

$$S = -\frac{1}{2g_5^2}\int d^4x \left(\frac{1}{z}V_\mu^a(x,z)\partial_z V^{\mu a}(x,z)\right)_{z=\epsilon}$$

$$= -\frac{1}{2g_5^2}\int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{z}V_\mu^a(-q,z)\partial_z V^{\mu a}(q,z)\right)_{z=\epsilon}$$

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"Bulk-to-boundary propagator"

Write $V_\mu^a(q,z) = V(q,z)V_\mu^a(q)$

$$V(q,\epsilon) = 1$$

Vector Current Correlators

$$S = -\frac{1}{2g_5^2}\int \frac{d^4q}{(2\pi)^4} V^{\mu a}(-q) V_\mu^a(q) \left(\frac{1}{z}\partial_z V(q,z)\right)_{z=\epsilon}$$

Vector current-current correlator:

$$\int d^4x \, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0)\rangle \equiv \delta^{ab}\left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi_V(-q^2)$$

AdS/CFT $\longrightarrow$

$$= \frac{\delta}{\delta A_\mu^a(-q)} \frac{\delta}{\delta A_\nu^b(q)} S$$

$$\Pi_V(-q^2) = \frac{1}{g_5^2 q^2}\left(\frac{1}{z}\partial_z V(q,z)\right)_{z=\epsilon}$$

Vector Current Correlators

$$\Pi_V(-q^2) = \frac{1}{g_5^2 q^2}\left(\frac{1}{z}\partial_z V(q,z)\right)_{z=\epsilon}$$

where

$$\partial_z\left(\frac{1}{z}\partial_z V(q,z)\right) + \frac{q^2}{z}V(q,z) = 0$$

$$V(q,\epsilon) = 1$$

$$\partial_z V(q,z)|_{z=z_{IR}} = 0$$

Vector Current Correlators

Can expand solutions at large $-q^2$:

$$\Pi_V(-q^2) = -\frac{1}{2g_5^2}\ln(-q^2) + \ldots$$

One-loop perturbative QCD calculation:

$$\Pi_V(-q^2) \approx -\frac{N_c}{24\pi^2}\ln(-q^2)$$

$\longrightarrow$ Relates g_5 and N_c

Quark-Hadron Duality

Can expand in resonances (Kaluza-Klein modes)

Bulk-to-Boundary Propagator

$$\partial_z \left(\frac{1}{z} \partial_z V(q,z) \right) + \frac{q^2}{z} V(q,z) = 0$$

$$V(q,\epsilon) = 1 \qquad \partial_z V(q,z)|_{z=z_{IR}} = 0$$

Dirichlet Green function:

$$\partial_z \left(\frac{1}{z} \partial_z G(q,z,z') \right) + \frac{q^2}{z} G(q,z,z') = \delta(z-z')$$

$$G(q,\epsilon,z) = 0 \qquad \partial_z G(q,z,z')|_{z=z_{IR}} = 0$$

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$$G(q,\epsilon,z) = 0 \qquad \partial_z G(q,z,z')|_{z=z_{IR}} = 0$$

Quark-Hadron Duality

Consider the integral

$$I \equiv \int_{\epsilon}^{z_{IR}} dz\, V(q,z) \left[\partial_z \frac{1}{z}\partial_z + \frac{q^2}{z}\right] G(q,z,z')$$

$$= V(q,z')$$

Integrate by parts twice:

$$I = \int_{\epsilon}^{z_{IR}} dz\, G(q,z,z') \left[\partial_z \frac{1}{z}\partial_z + \frac{q^2}{z}\right] V(q,z)$$

$$+V(q,z)\, \frac{1}{z}\partial_z G(q,z,z')\Big|_{\epsilon}^{z_{IR}} - G(q,z,z')\, \frac{1}{z}\partial_z V(q,z)\Big|_{\epsilon}^{z_{IR}}$$

Quark-Hadron Duality

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Integrate by parts twice:

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1

Quark-Hadron Duality

We have derived a relation between the bulk-to-boundary propagator and the Dirichlet Green function:

$$V(q,z') = -\frac{1}{z}\partial_z G(q,z,z')\Big|^{z=\epsilon}$$

The Green function can then be expanded in the Kaluza-Klein modes discussed earlier:

$$G(q,z,z') = \sum_n \frac{\psi_n(z)\psi_n(z')}{q^2 - m_n^2}$$

Quark-Hadron Duality

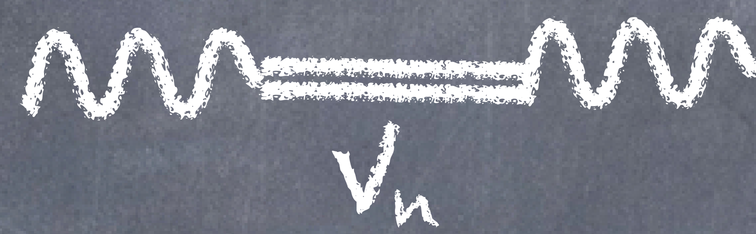
We can now evaluate the expression for the current-current correlator derived earlier as a sum over "rho mesons":

$$\Pi_V(-q^2) = \frac{1}{g_5^2 q^2}\left(\frac{1}{z}\partial_z V(q,z)\right)_{z=\epsilon}$$

$$= -\frac{1}{g_5^2}\sum_n \frac{\left(\psi_n'(\epsilon)/\epsilon\right)^2}{(q^2 - m_n^2)m_n^2}$$

+ contact term

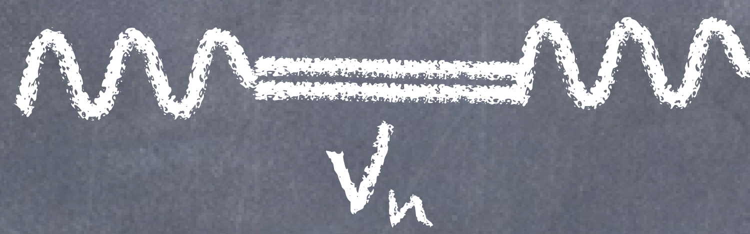
Quark-Hadron Duality



Rho decay constants

$$\Pi_V(-q^2) = -\sum_n \frac{F_n^2}{(q^2 - m_n^2)m_n^2}$$

Quark-Hadron Duality



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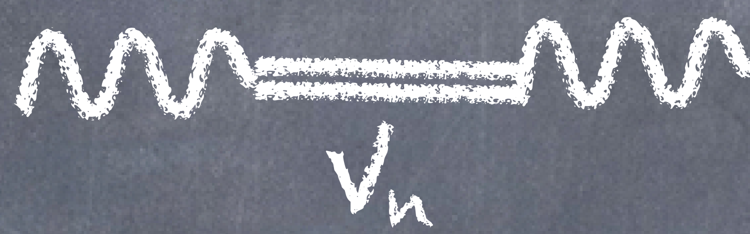
Quark-Hadron Duality



Rho decay constants

$$\Pi_V(-q^2) = -\sum_n \frac{F_n^2}{(q^2 - m_n^2)m_n^2}$$

$$\Pi_V(-q^2) = -\frac{1}{g_5^2}\sum_n \frac{(\psi_n'(\epsilon)/\epsilon)^2}{(q^2 - m_n^2)m_n^2}$$

AdS/QCD →

$$F_n^2 = \frac{1}{g_5^2}\left(\frac{\psi_n'(\epsilon)}{\epsilon}\right)^2$$

Sum Rules

In the deep Euclidean regime $-q^2 \gg m_\rho^2$, perturbative QCD gives

$$i \int d^4x\, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \delta^{ab} \frac{N}{24\pi^2} \log(q^2)$$

We can express the correlator as a sum over resonances:

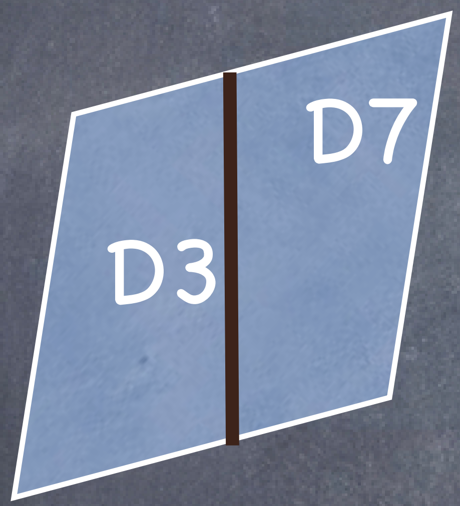
$$i \int d^4x\, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \sum \frac{F_n^2}{q^2 - m_n^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_n^2}\right) \delta^{ab}$$

Agreement of these expressions in the deep Euclidean regime is a Weinberg sum rule.

$m_n = n^{th}$ Kaluza-Klein mass
$F_n =$ Decay constant of n^{th} resonance

Vector Mesons in AdS/CFT

(Kruczenski,Mateos,Myers,Winters)



The large number of D3-branes warp the spacetime.

The D7-branes minimize their volume in that spacetime.

Gauge fields propagate in the induced geometry on the D7-branes, and KK modes are mesons.

What about chiral symmetry?

QCD with massless quarks has an enhanced symmetry

$$\mathcal{L}_{QCD} = \sum_{i=u,d,\ldots} \left[\bar{q}_{iL} \gamma^{\mu} \left(i\partial_{\mu} - gA_{\mu} \right) q_{iL} + \bar{q}_{iR} \gamma^{\mu} \left(i\partial_{\mu} - gA_{\mu} \right) q_{iR} \right]$$

$$q_L = \left(\frac{1-\gamma^5}{2} \right) q \qquad q_R = \left(\frac{1+\gamma^5}{2} \right) q$$

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$$q_L = \left(\frac{1-\gamma^5}{2}\right) q \qquad q_R = \left(\frac{1+\gamma^5}{2}\right) q$$

Chiral symmetry:

SU(N_f) generators

$$q_L \rightarrow e^{i\theta_L^a T^a} q_L \qquad q_R \rightarrow e^{i\theta_R^a T^a} q_R$$

What about chiral symmetry?

Quark masses explicitly break the chiral symmetry. For now pretend quark masses were equal.

$$\mathcal{L}_m = m\left(\bar{q}_L q_R + \bar{q}_R q_L\right)$$

Under chiral symmetry:

$$\mathcal{L}_m \to m\left(\bar{q}_L e^{-i\theta_L^a T^a} e^{i\theta_R^a T^a} q_R + \text{h.c.}\right)$$

Isospin is still preserved: $\theta_L^a = \theta_R^a$

What about chiral symmetry?

The up and down quark masses (few MeV) are small compared to the confining scale (few hundred MeV).

SU(2) chiral symmetry is a pretty good symmetry for the up and down quarks.

What about chiral symmetry?

The up and down quark masses (few MeV) are small compared to the confining scale (few hundred MeV).

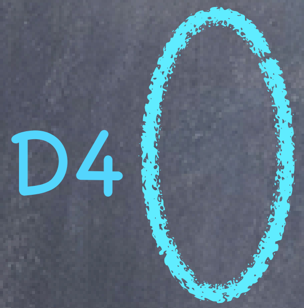
SU(2) chiral symmetry is a pretty good symmetry for the up and down quarks.

However, the chiral symmetry is spontaneously broken by chiral condensates.

$$\langle \overline{q}_L q_R \rangle \neq 0$$

Chiral Fermions in AdS/CFT

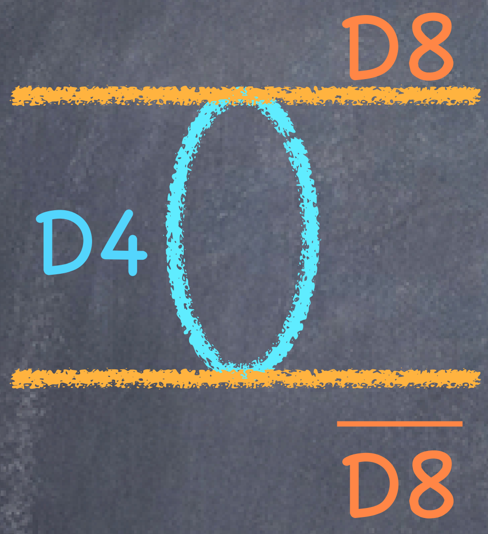
(Sakai,Sugimoto)



D4-branes wrapped on a circle with antiperiodic boundary conditions for fermions ⟶ breaks SUSY (Witten)

Chiral Fermions in AdS/CFT

(Sakai, Sugimoto)



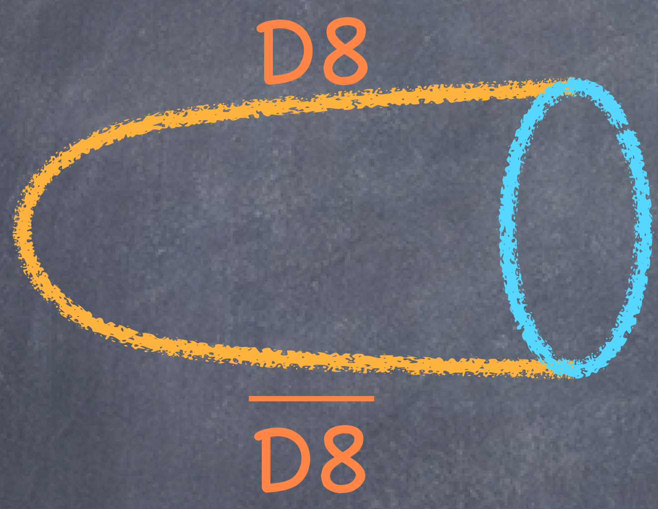
D4-branes wrapped on a circle with antiperiodic boundary conditions for fermions ⟶ breaks SUSY (Witten)

N_f D8-branes and $\overline{D8}$-branes intersect D4-branes

D4-D8 strings contain massless chiral fermions

Chiral Symmetry Breaking

(Sakai,Sugimoto)



D4-branes warp the geometry. D8-branes minimize their volume, connect with $\overline{D8}$-branes.

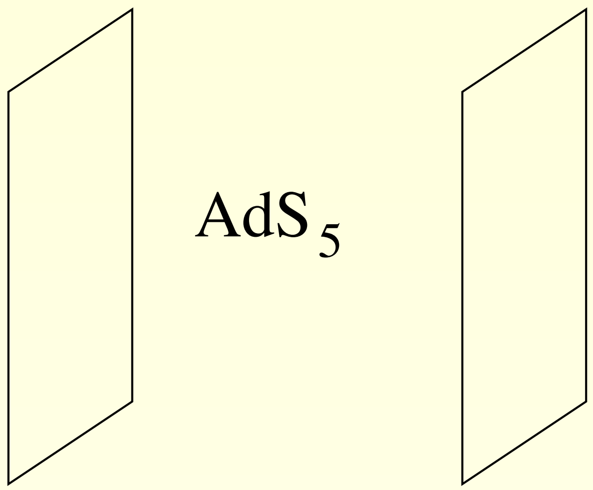
The $SU(N_f) \times SU(N_f)$ chiral symmetry is broken to the diagonal $SU(N_f)$

The spectrum of vector fields on the D8-branes describes vector and axial-vector mesons, and pions.

Bottom-Up AdS/QCD

- Model tower of resonances as Kaluza-Klein modes in an extra dimension (Son,Stephanov'04)
- Model pattern of chiral symmetry breaking by analogy with AdS/CFT correspondence

- *Optional*: Specify details of model (geometry of extra dimension, couplings) by matching to UV as best possible (*e.g.* Brodsky,De Teramond; JE *et al.*; Da Rold,Pomarol)



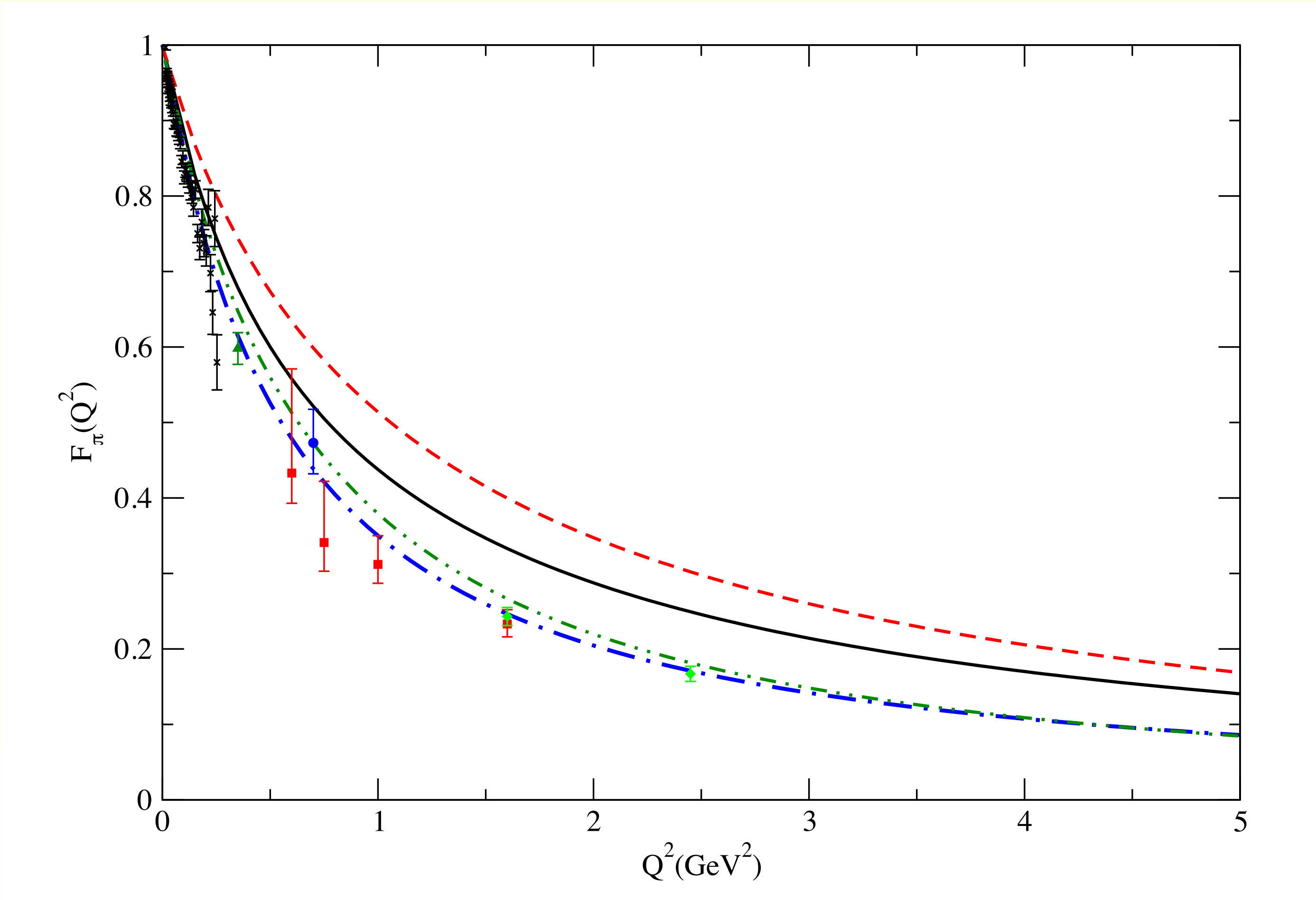
Predictions of Various AdS/QCD Models

Observable	Model A ($\sigma_s = \sigma_q$) (MeV)	Model B ($\sigma_s \neq \sigma_q$) (MeV)	Measured (MeV)
m_π	(fit)	134.3	139.6
f_π	(fit)	86.6	92.4
m_K	(fit)	513.8	495.7
f_K	104	101	113 ± 1.4
$m_{K_0^*}$	791	697	672
$f_{K_0^*}$	28.	36	
m_ρ	(fit)	788.8	775.5
$F_\rho^{1/2}$	329	335	345 ± 8
m_{K^*}	791	821	893.8
$F_{K^*}^{1/2}$	329	337	
m_{a_1}	1366	1267	1230 ± 40
$F_{a_1}^{1/2}$	489	453	433 ± 13
m_{K_1}	1458	1402	1272 ± 7
$F_{K_1}^{1/2}$	511	488	

Abdidin and Carlson '09

Predictions of Various AdS/QCD Models

Pion Form Factor



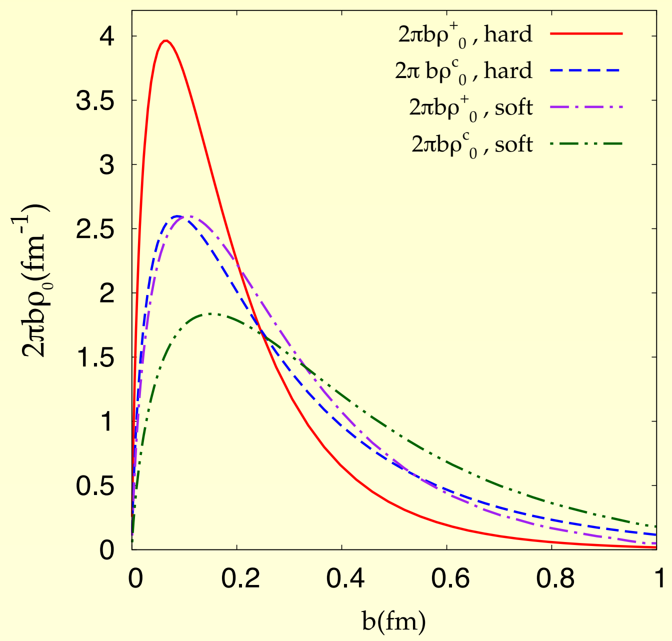
from Kwee and Lebed, arXiv:0807.4565

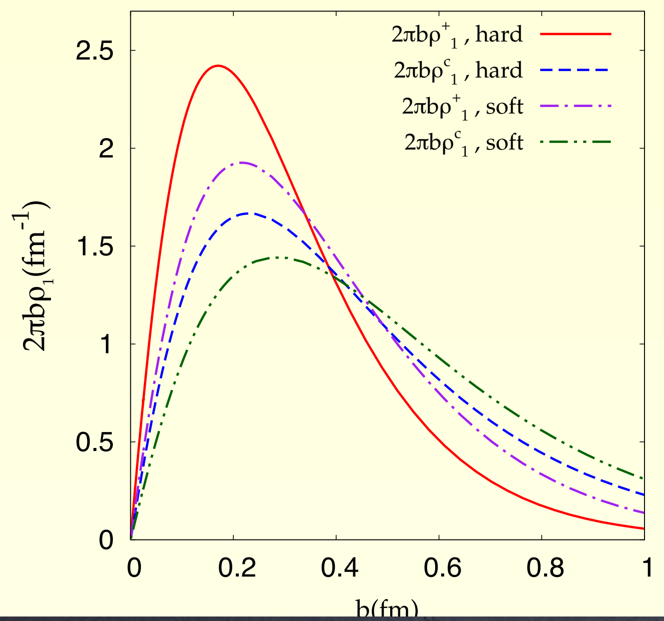
Solid black and blue curves: Hard wall model

Dotted red and green curves: Soft wall model

See also Grigoryan,Radyushkin '08

Predictions of Various AdS/QCD Models





Gravitational Form Factors and Generalized Parton Distributions

From Abidin and Carlson, arXiv:0801.3839

Top: p^+ and charge densities of Helicity-0 rho mesons in hard and soft wall models

Bottom: Same for Helicity-1 rho mesons

See also Lyubovitskij, Vega, ...

Predictions of Various AdS/QCD Models

Can determine meson radii from behavior of form factors near $q^2 = 0$.

Hard wall model:

$$\langle r_\pi^2 \rangle_{charge} = 0.33 \text{ fm}^2$$

$$\langle r_\pi^2 \rangle_{grav} = 0.13 \text{ fm}^2$$

$$\langle r_\rho^2 \rangle_{charge} = 0.53 \text{ fm}^2$$

$$\langle r_\rho^2 \rangle_{grav} = 0.21 \text{ fm}^2$$

$$\langle r_{a_1}^2 \rangle_{charge} = 0.39 \text{ fm}^2$$

$$\langle r_{a_1}^2 \rangle_{grav} = 0.15 \text{ fm}^2$$

H. Grigoryan and A. Radyushkin; Z. Abidin and C. Carlson '07,'08

Dualities Lecture 2 Summary

The AdS/CFT correspondence relates theories in different numbers of spatial dimensions.

Higher-dimensional models which confine with chiral symmetry breaking allow for calculation of hadronic observables, often with surprising quantitative success.