Dualities and QCD I

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National Nuclear Physics Summer School
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Dualities and QCD

- The meaning of "duality" in physics (Example: The Ising model)
- Quark-Hadron duality (experimental and theoretical evidence)
- The AdS/CFT correspondence (gauge/gravity duality, holographic QCD)

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image from belowbeltway@Flickr

Challenge:

Analyze this using QCD and QED:

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image from taosecurity.blogspot.com

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The Basic Question:

Starting with the Standard Model, how can we make predictions at different length scales?

What is duality?

Dualities exist where there are multiple descriptions of the same physical situation.

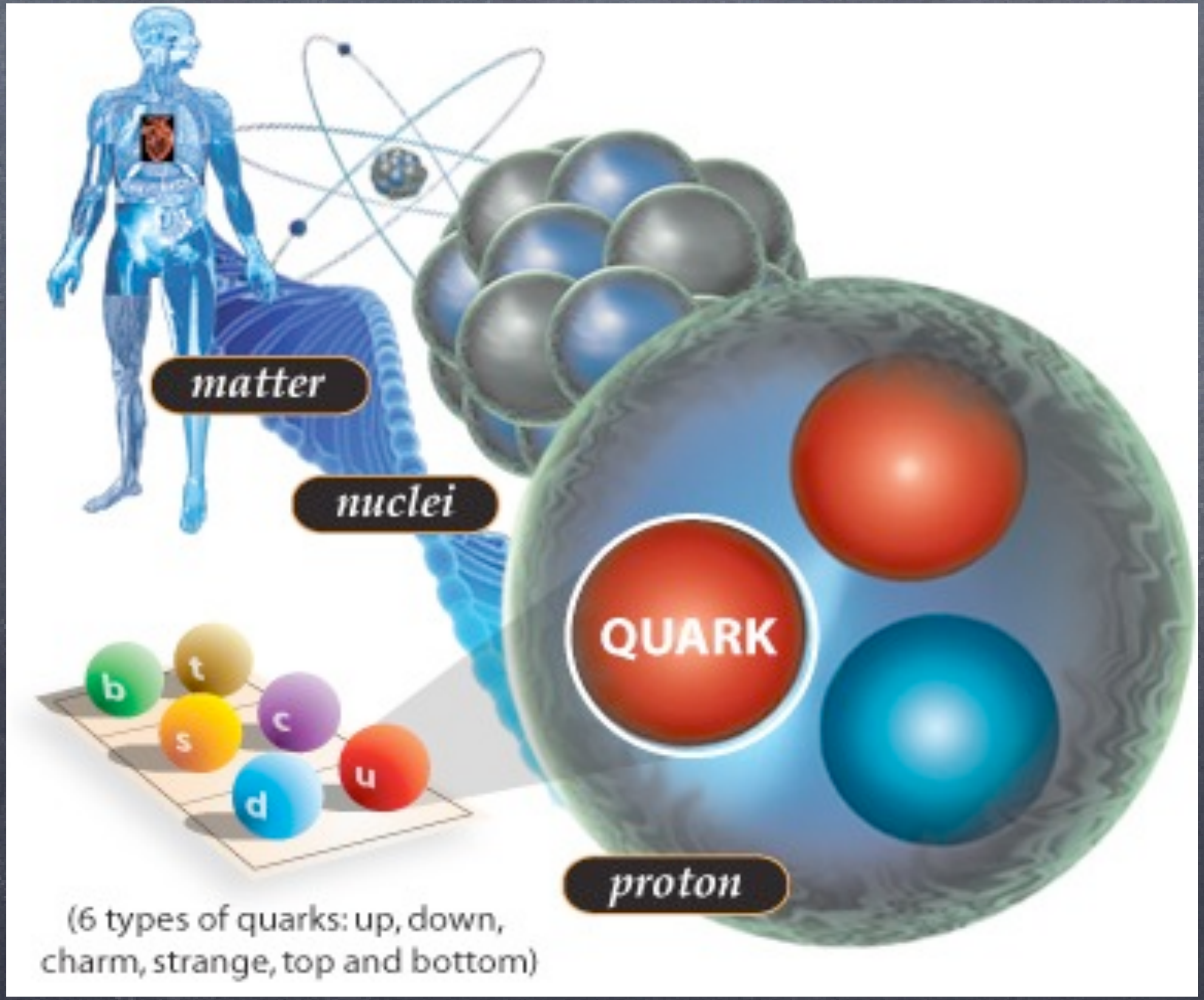

image from JLab website

Dualities Abound in SUSY and String Theory

Seiberg Duality

Montonen-Olive Duality

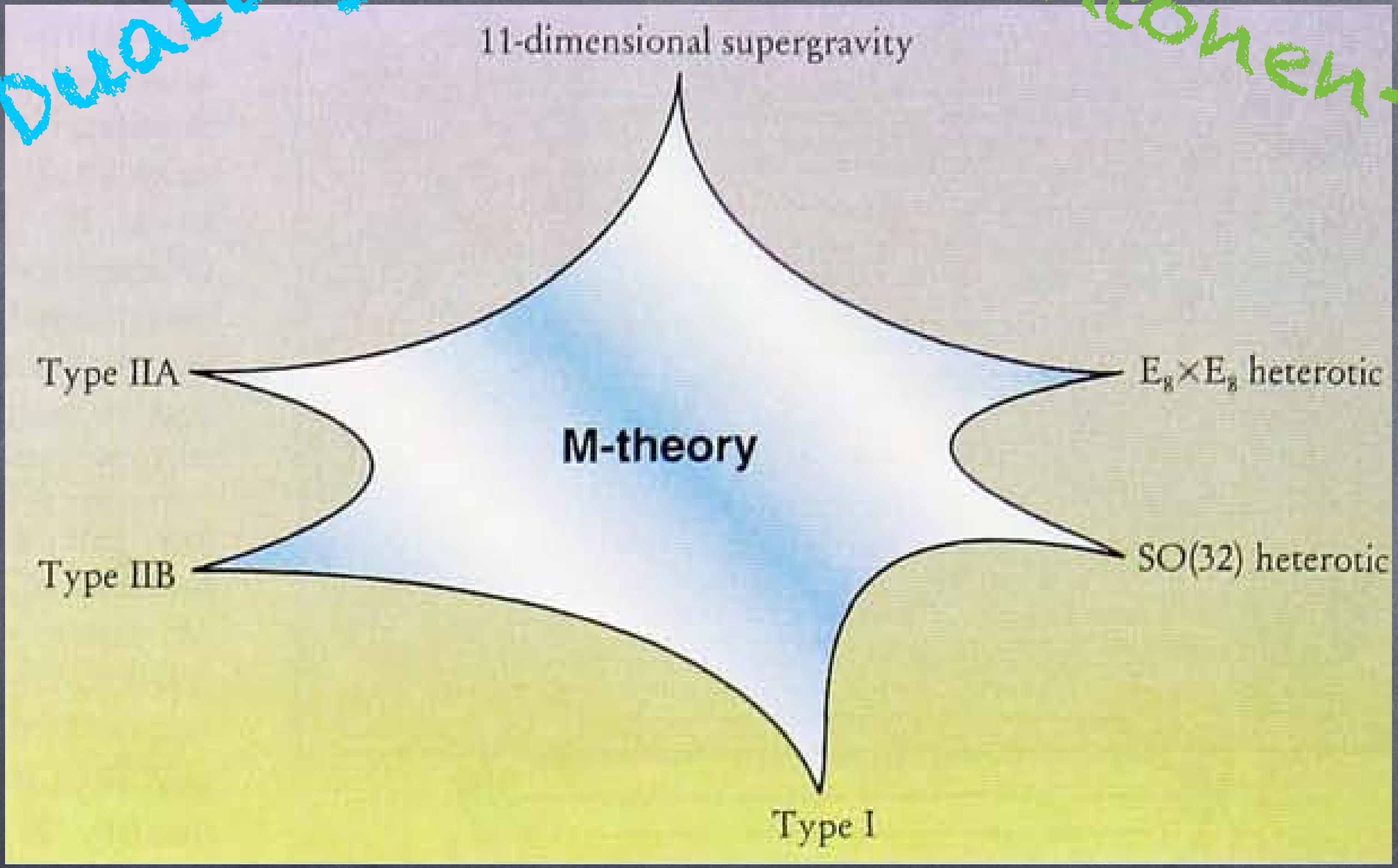

image from Witten, Phys Today, May 97

AdS/CFT

Orbifold Projection

Mirror Symmetry

Seiberg-Witten Theory

An Example of Duality: The 2D Ising Model

(from Joel Moore's Phys 212 course notes at Berkeley)

$s_i = \pm 1$

$$-\beta E = K \sum_{\langle ij \rangle} s_i s_j$$

$$K = \beta J$$

sum over bonds

T=0	Low T
↑↑↑↑↑↑	↑↓↑↑↑↑
↑↑↑↑↑↑	↑↑↑↓↓↑
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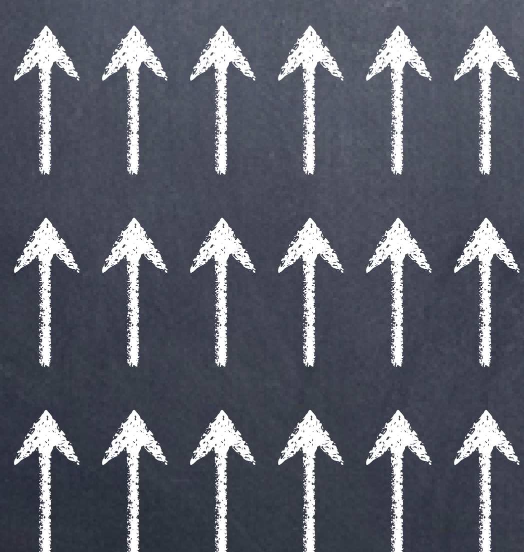
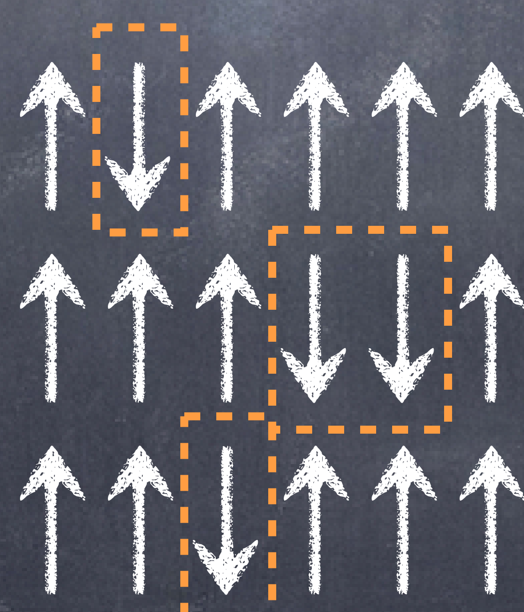


Low T



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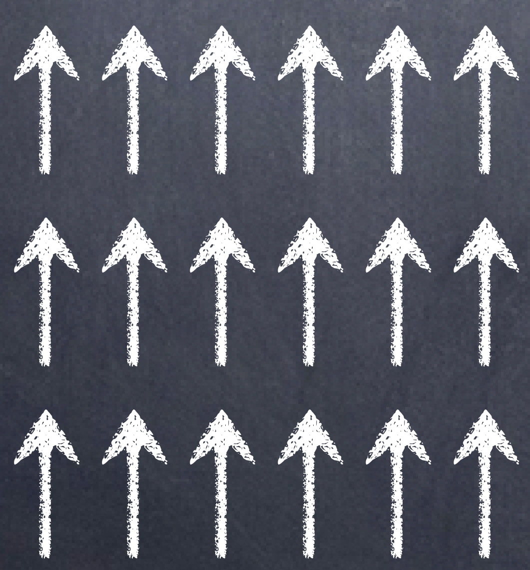

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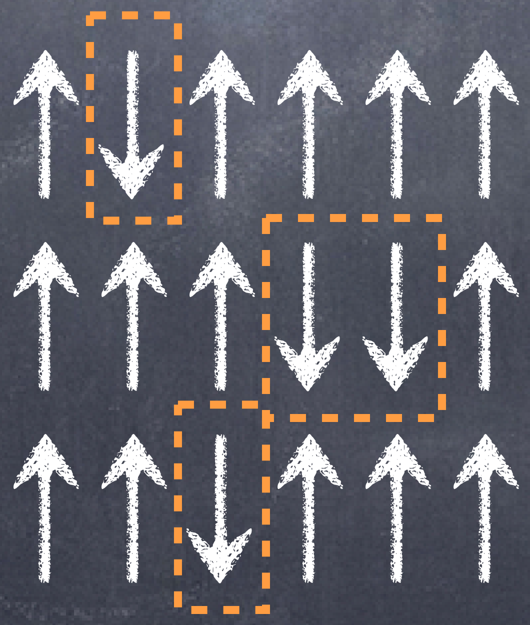

Energy increase=2Jl(P).
l(P)= #broken bonds
along closed path P

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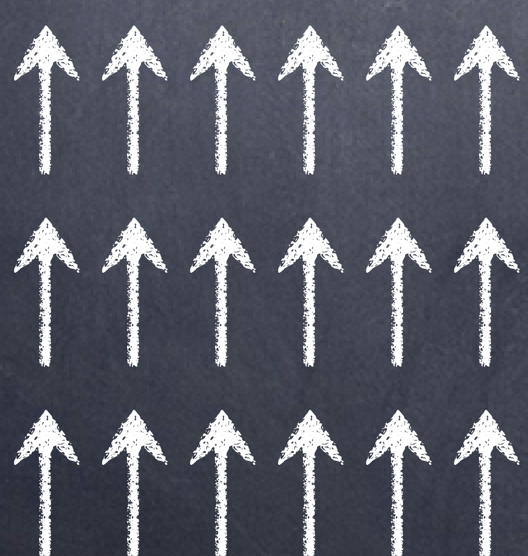
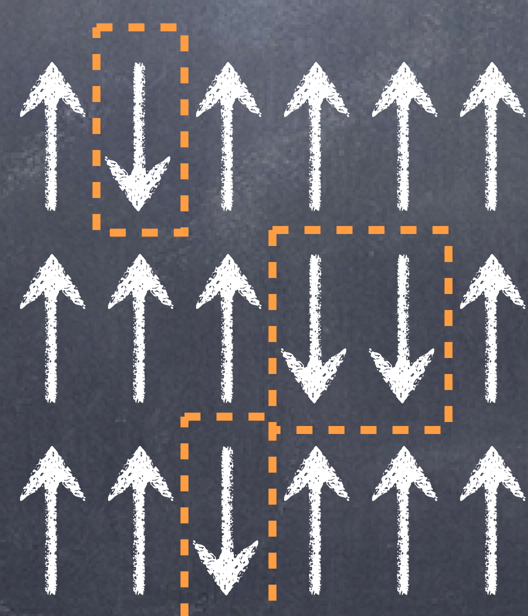


Low T



Energy increase=2Jl(P).
l(P)= #broken bonds
along closed path P

$$Z = 2e^{N_b K} \sum_P e^{-2K\ell(P)}$$

#bonds

small @low T

The 2D Ising Model

$$-\beta E = K \sum_{\langle ij \rangle} s_i s_j$$

$$K = \beta J$$

$$Z = \sum_s e^{\sum_{\langle ij \rangle} K s_i s_j} = \sum_s \prod_{\langle ij \rangle} e^{K s_i s_j} = \sum_s \prod_{\langle ij \rangle} (\cosh K + s_i s_j \sinh K)$$

small @ high T

High T

↑↓↑↓↑↑

↑↓↑↓↓↑

↑↑↓↑↑↓

$$Z = (\cosh K)^{N_b} \sum_s \prod_{\langle ij \rangle} (1 + s_i s_j \tanh K)$$

Expand in tanh K. Only terms where s_i appears an even number of times survive.

The 2D Ising Model

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#bonds

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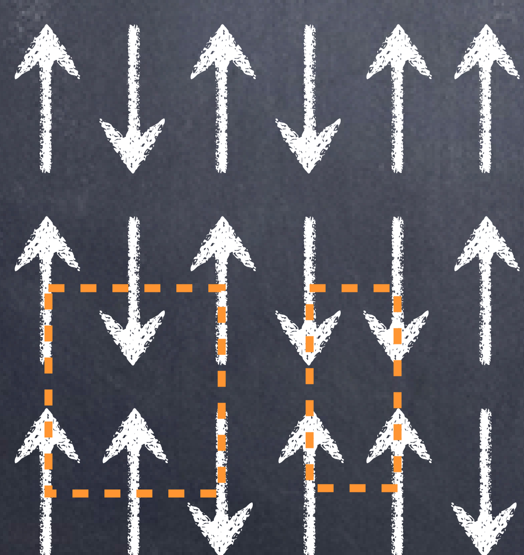
High T



$$Z = 2^{N_s} (\cosh K)^{N_b} \sum_P (\tanh K)^{\ell(P)}$$

#sites

The 2D Ising Model

Simpler at low T (large K):

$$Z = 2e^{N_b K} \sum_P e^{-2K\ell(P)}$$

Simpler at high T (small K):

$$Z = 2^{N_s} (\cosh K)^{N_b} \sum_P (\tanh K)^{\ell(P)}$$

The partition function at low T and high T are the same up to an overall rescaling if we identify

$$e^{-2K^*} = \tanh K \Rightarrow K^* = -\frac{1}{2} \log \tanh K$$

The 2D Ising Model

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This is called Kramers-Wannier duality. It is a strong-weak coupling duality:

When K is large (small), K^* is small (large). One description is simpler at high T, and the other at low T.

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Critical temperature: singularities in K, K^* at same point.

What does this have to do with particle physics?

There's an analogy:
QCD is adequately described at high energies by quarks and gluons.

However, at low energies a hadronic description is "better."

Definition: Better = Simpler/More weakly coupled

The Running Coupling

A theory may be better described by varying the couplings as the scale of interest changes, by integrating out short-distance fluctuations.

Renormalization of couplings can be thought of as a type of duality.

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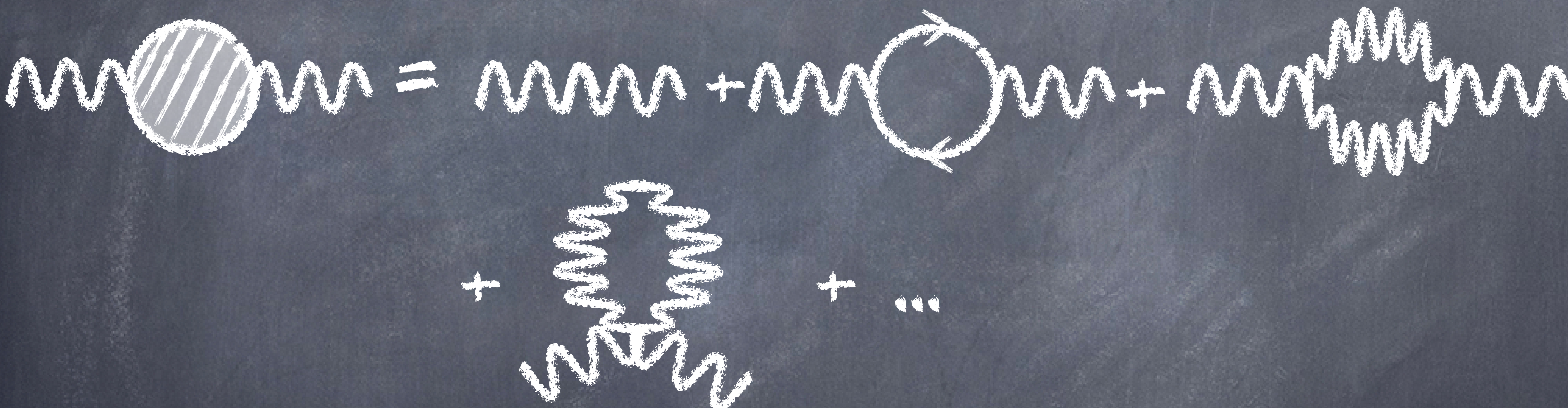
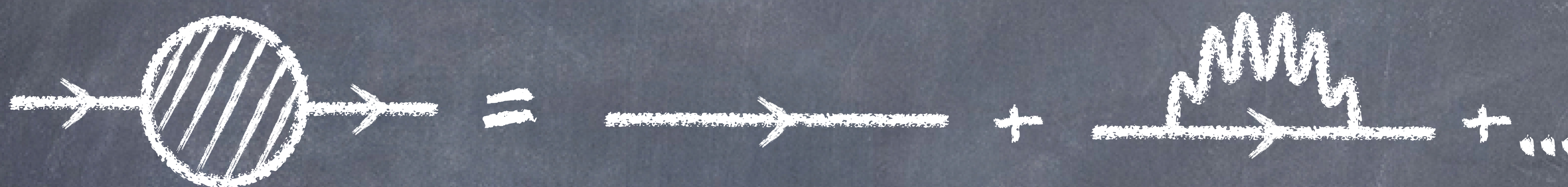
Kenneth G. Wilson

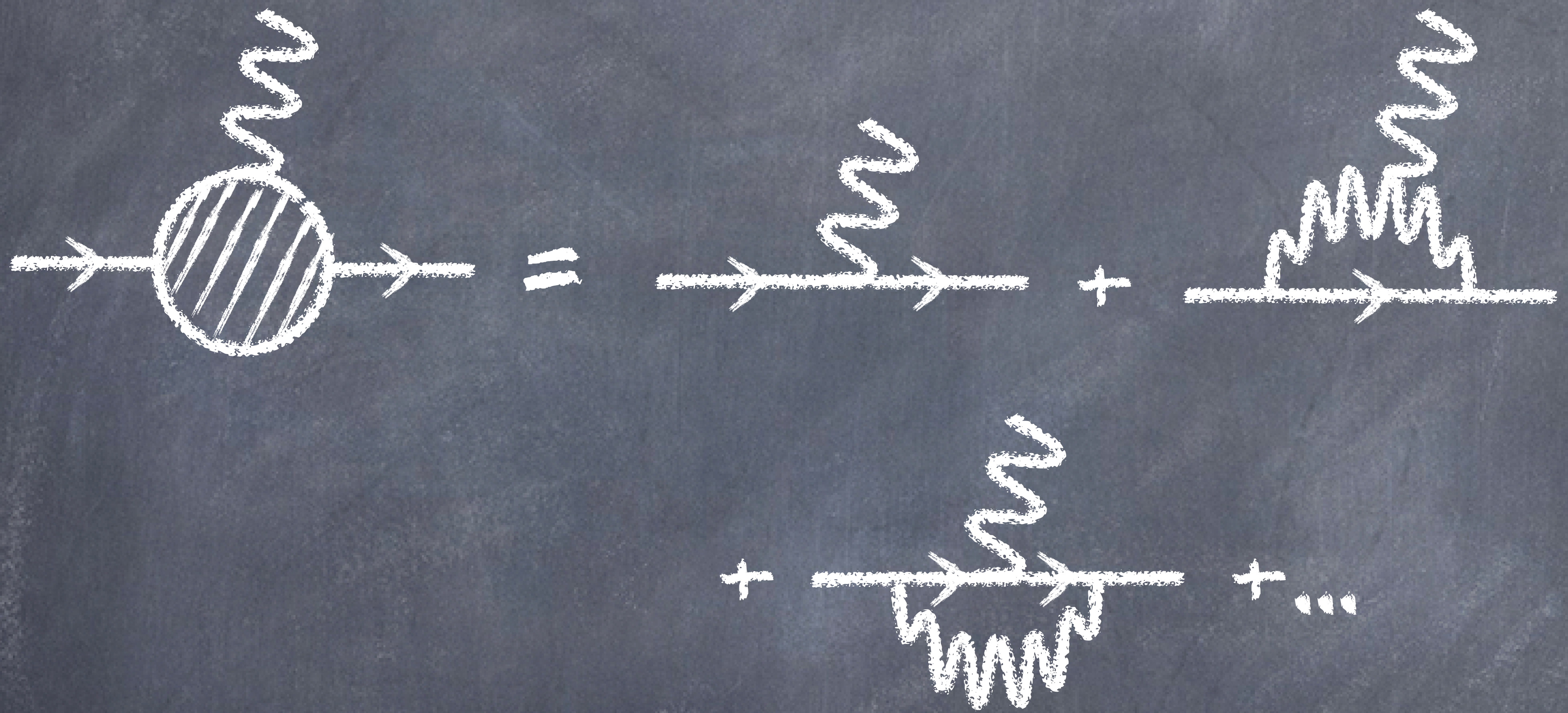
The gluon propagator



The quark propagator



The quark-gluon vertex



Asymptotic Freedom

Running of the QCD coupling takes into account the renormalization of the gluon propagator, the vertex, and the quark lines.

The result is an effective description valid around a specified renormalization scale M.

$$\beta(g) = M\frac{\partial}{\partial M}g(M)$$

$$\approx -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{4}{3}\sum_{fermions}\mu_f - \frac{1}{3}\sum_{scalars}\mu_s\right).$$

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Exercise: The one-loop beta function is negative in QCD.

Hence, the QCD coupling decreases at high energies. This is asymptotic freedom.

(Politzer; Gross, Wilczek - 1973)

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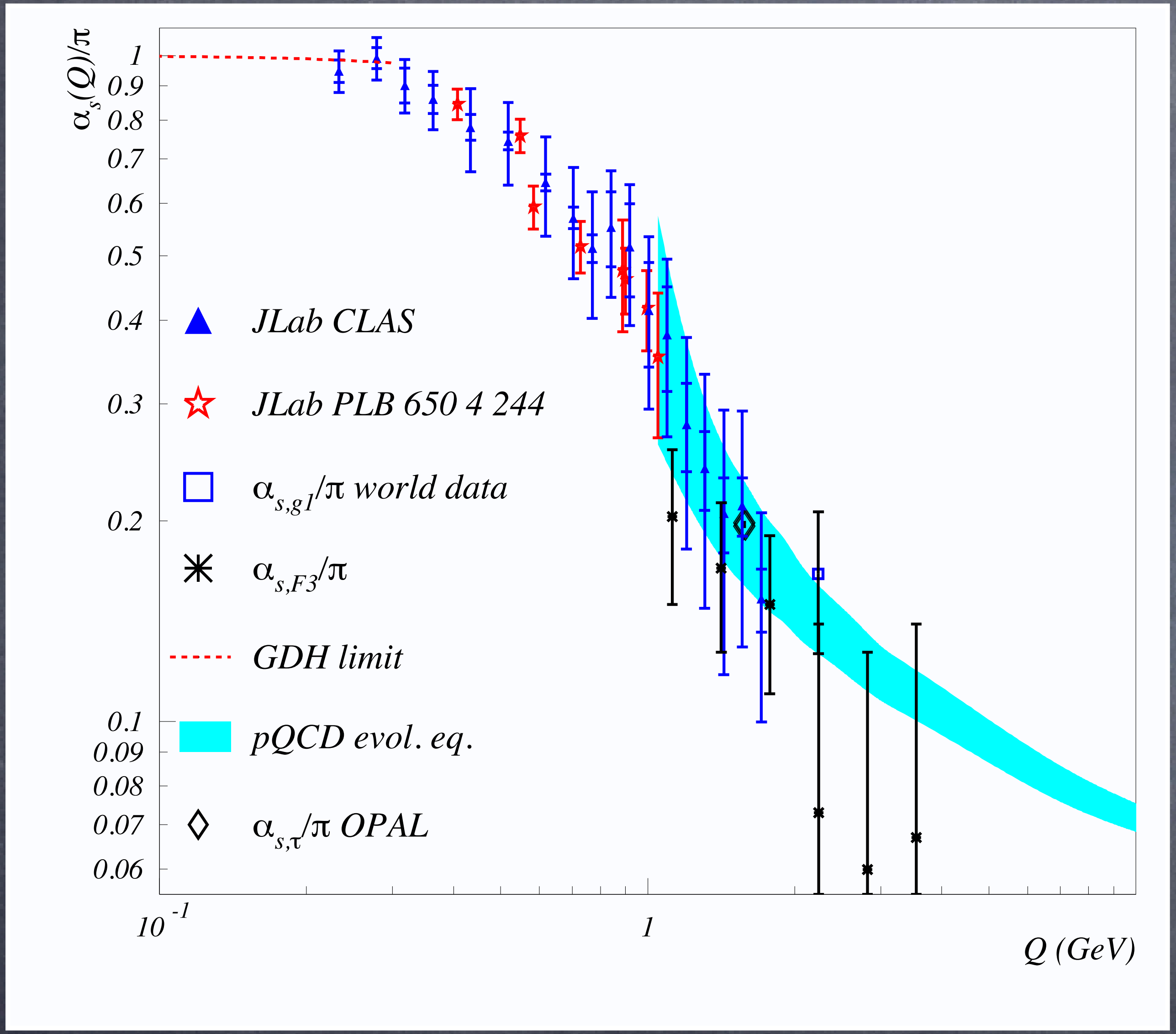
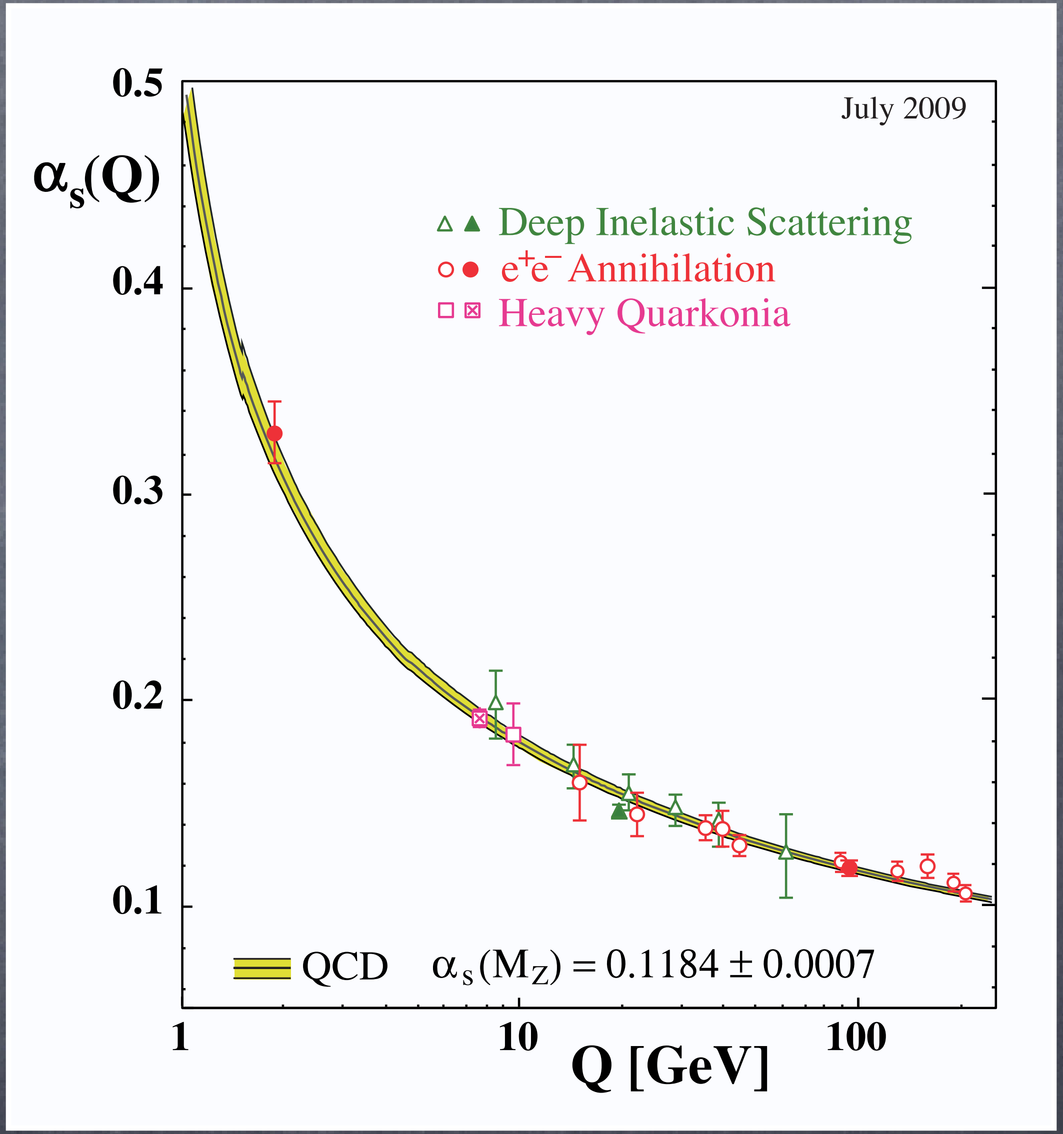
Question: What about $SU(2)_W$?

Asymptotic Freedom



from CLAS spin structure function data-Deur, Burkert, Chen, Korsch arxiv:0803.4119



2011 PDG

Where are the resonances?

Perturbative QCD predicts smoothly-varying cross sections down to some scale Λ_{QCD}. It does not (easily) predict the resonances observed in scattering experiments. Confinement in hadronic states is a nonperturbative phenomenon.

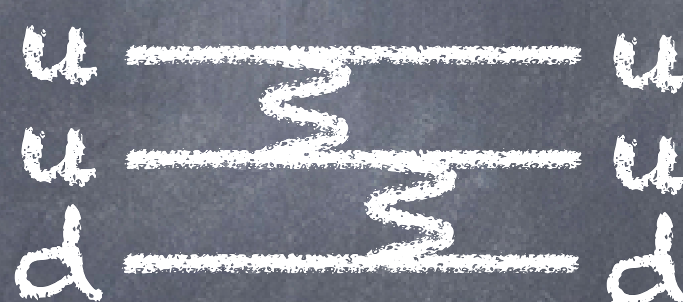
Confinement

There are no asymptotic colored states in QCD. Color charge is confined.

$SU(3)_C$

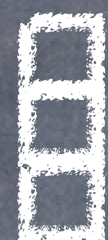
q □

Proton $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array}$



$SU(3)_C$ singlet: completely antisymmetric

Confinement

Proton 

SU(3)
singlet

Interpolating op for proton:
Only keeping track of color
(Ignoring spinor structure)

U=3x3 unitary
matrix

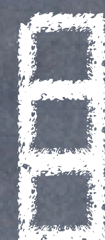
$$P \equiv \epsilon_{ijk} u_i u_j d_k$$

$$\xrightarrow{\text{SU(3)}} \epsilon_{ijk}(U_{il}u_l)(U_{jm}u_m)(U_{kn}d_n)$$

$$= (\det U)\epsilon_{lmn} u_l u_m d_n$$

$$= P$$

Confinement

Proton 

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Exercise

Hint: $\det U = \epsilon_{ijk}\epsilon_{lmn} U_{il} U_{jm} U_{kn} = 1$

Confinement

Meson interpolating operators can be made from a quark and an antiquark field

$SU(3)_C$

$u \quad \square$

$\bar{d} \quad \overline{\square} = \ydiagram{1,1}$

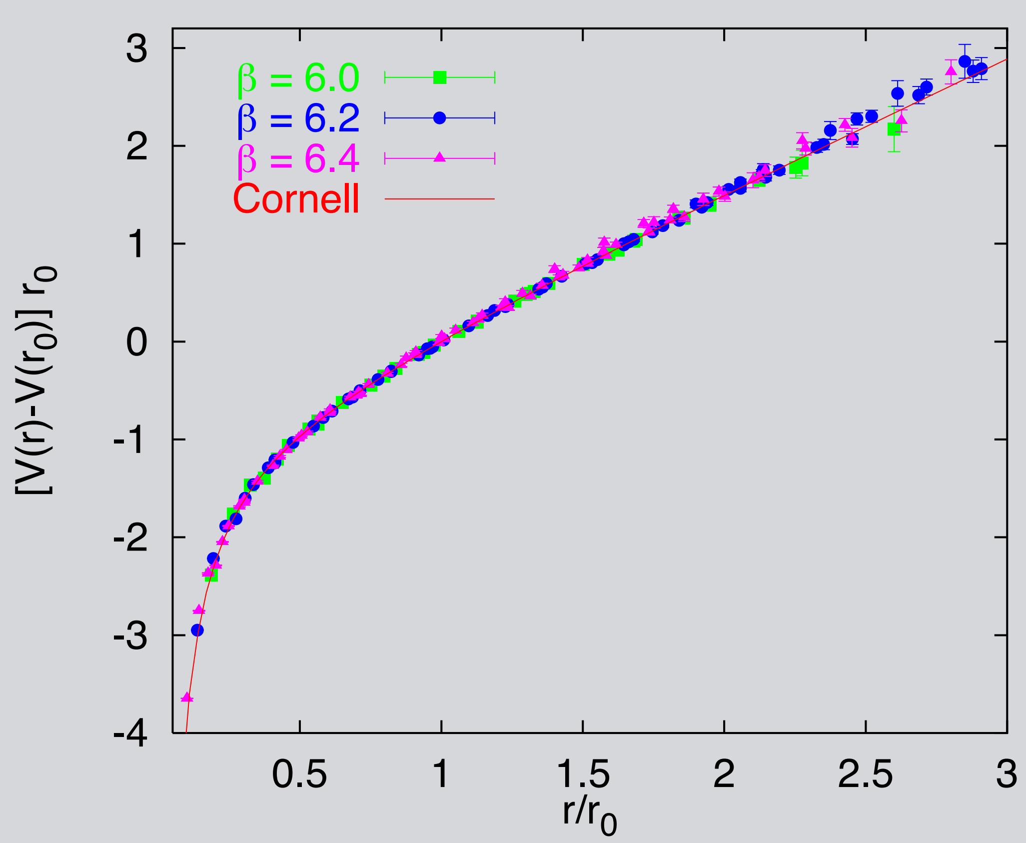
Pion $\ydiagram{1,1,1}$

$\square \times \overline{\square} = \ydiagram{1,1,1} + \ydiagram{2,1}$

$3 \times \bar{3} = 1 + 8$

Confinement

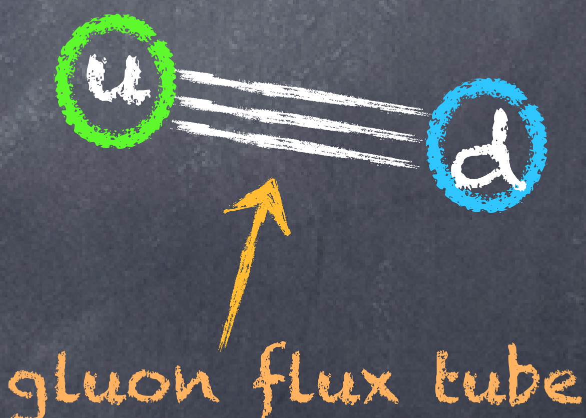
Static quark potential



The quenched Wilson action $SU(3)$ potential, normalised to $V(r_0) = 0$.

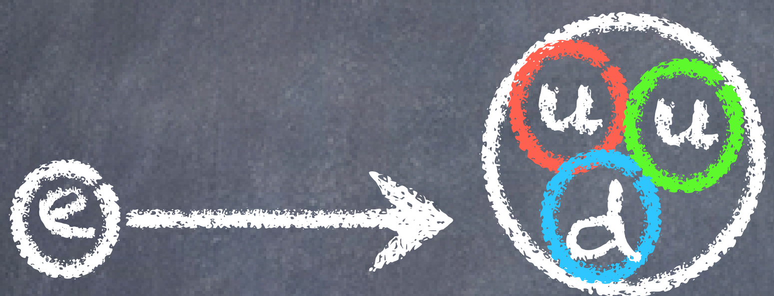
Bali, hep-ph/0001312

Linear potential
→ constant force



Confinement

So, are quarks confined?



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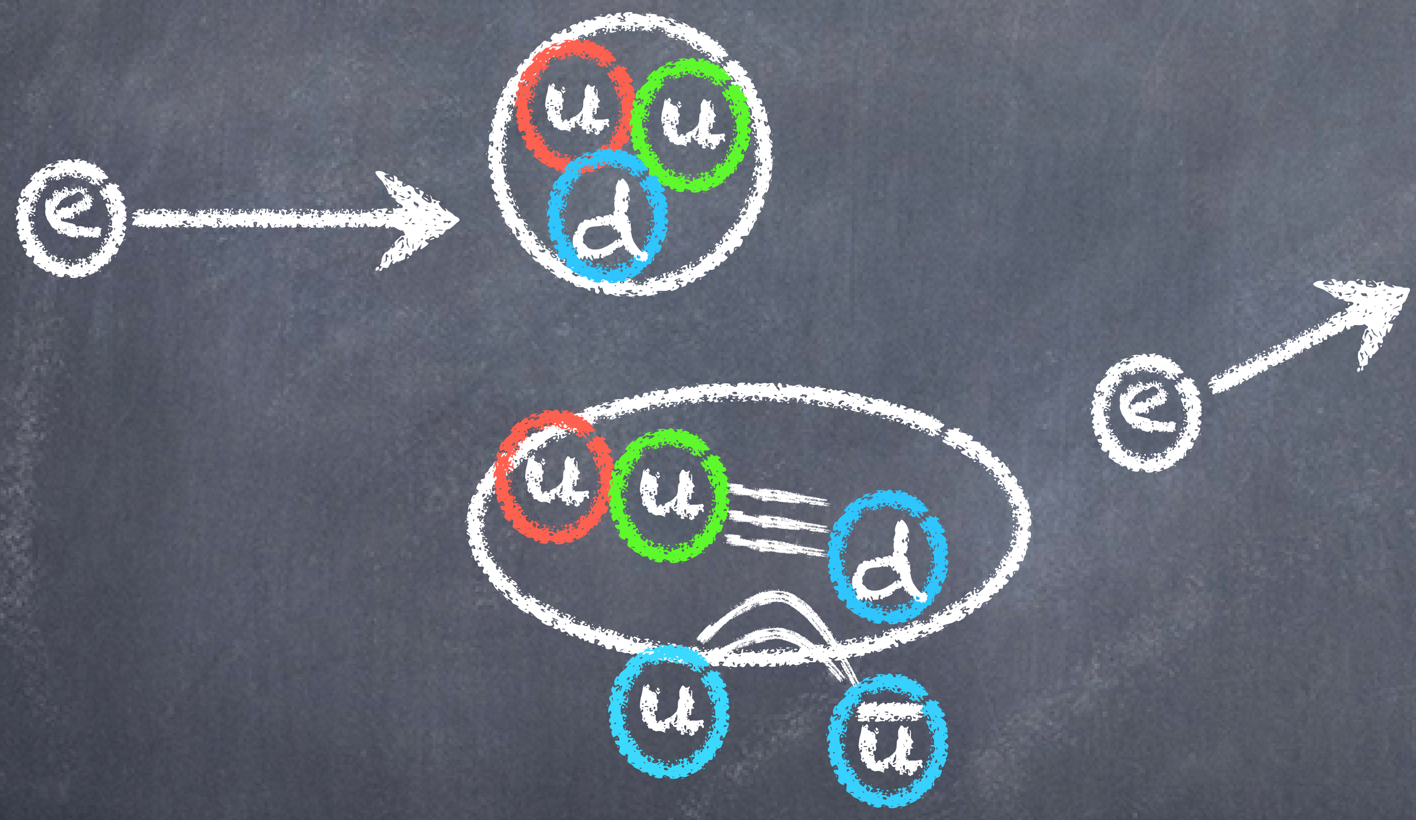

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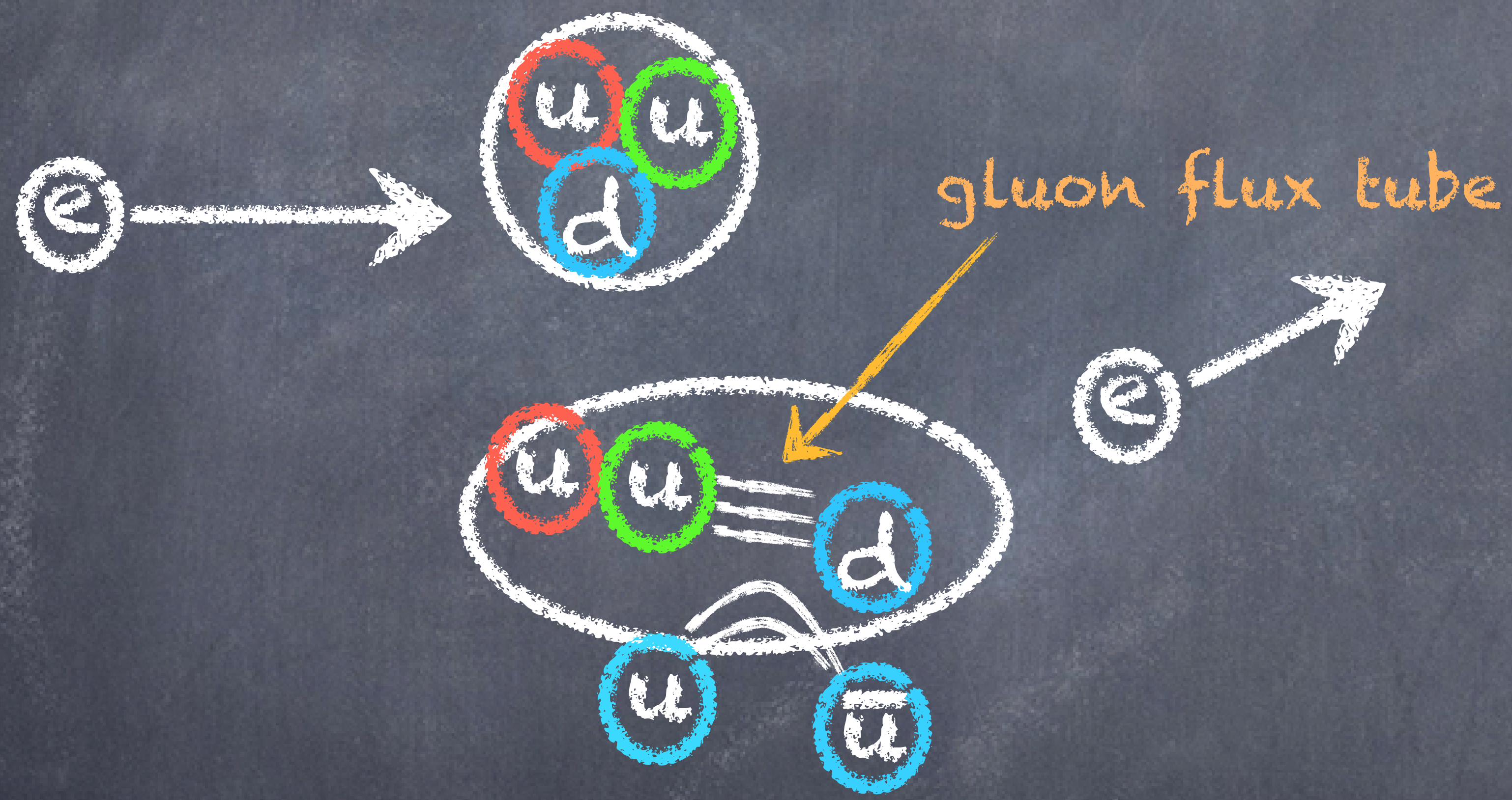

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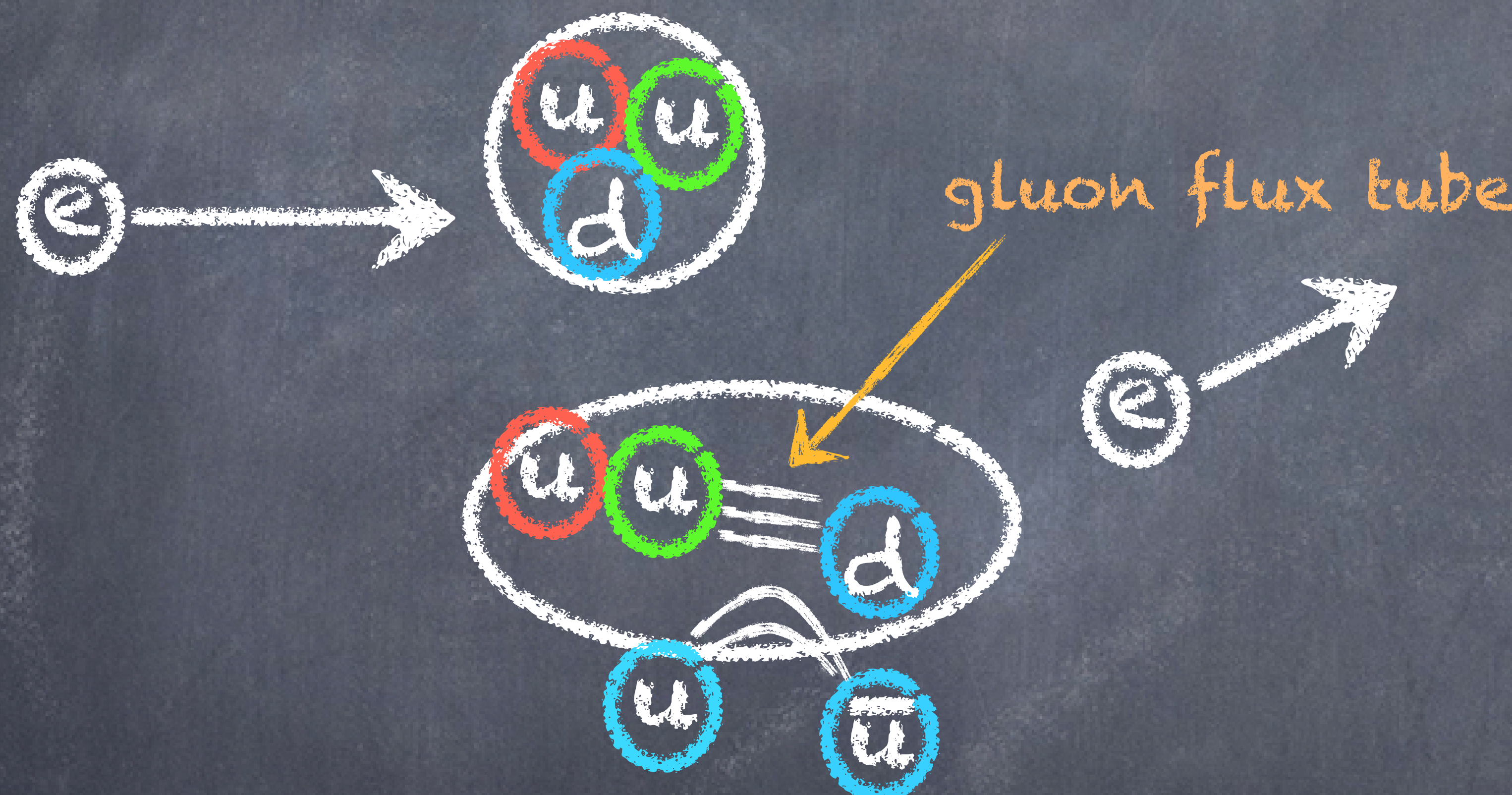

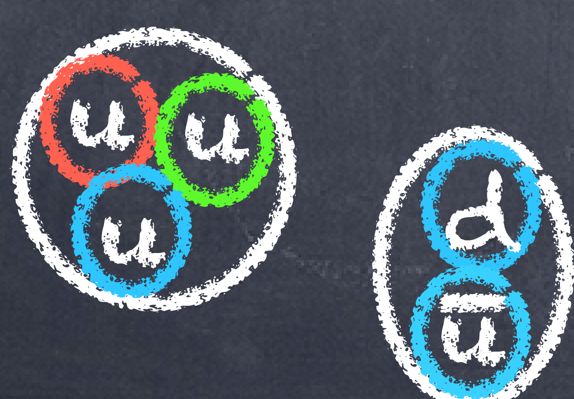

The down quark has been liberated from the proton!

Confinement

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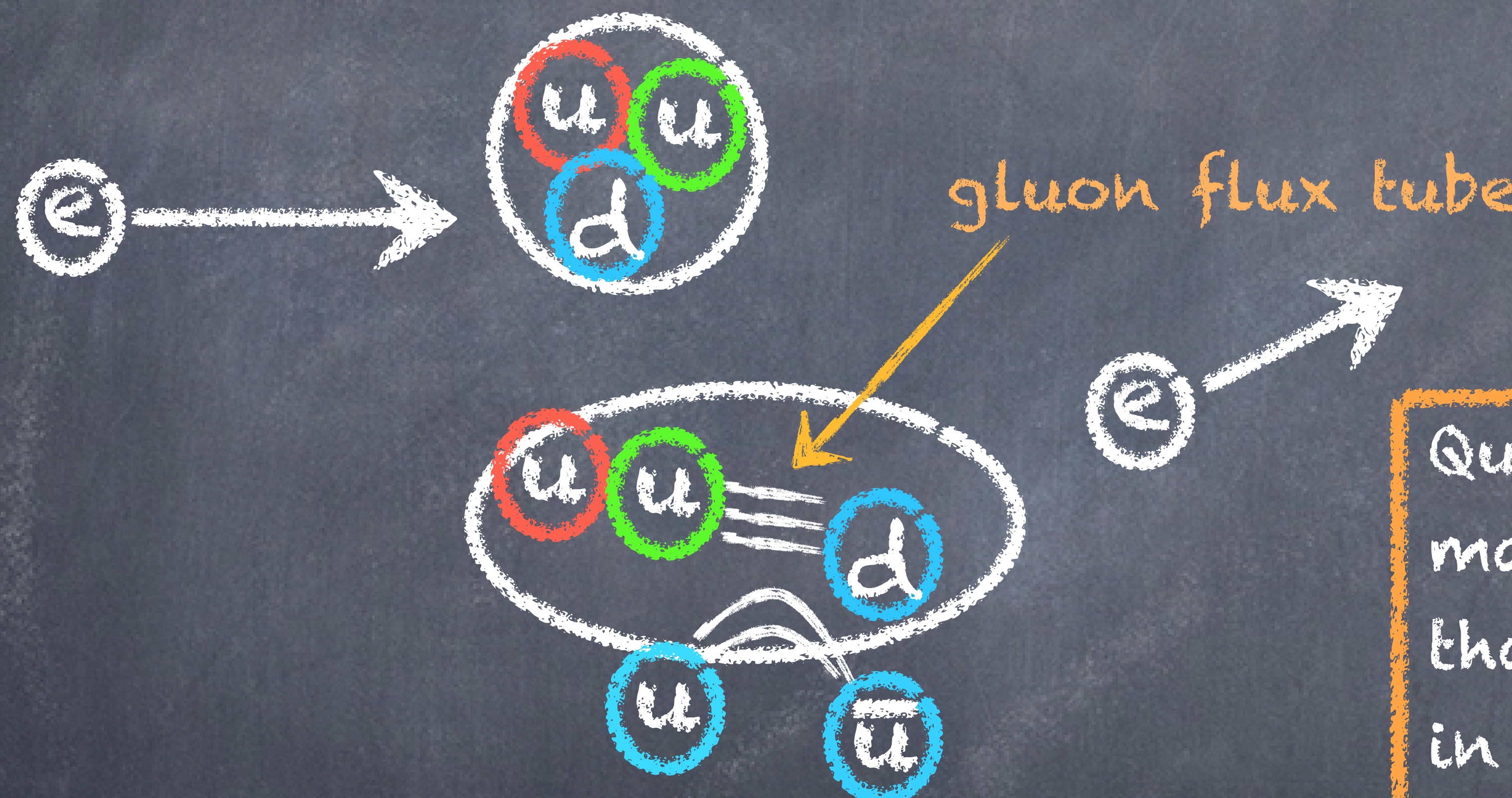
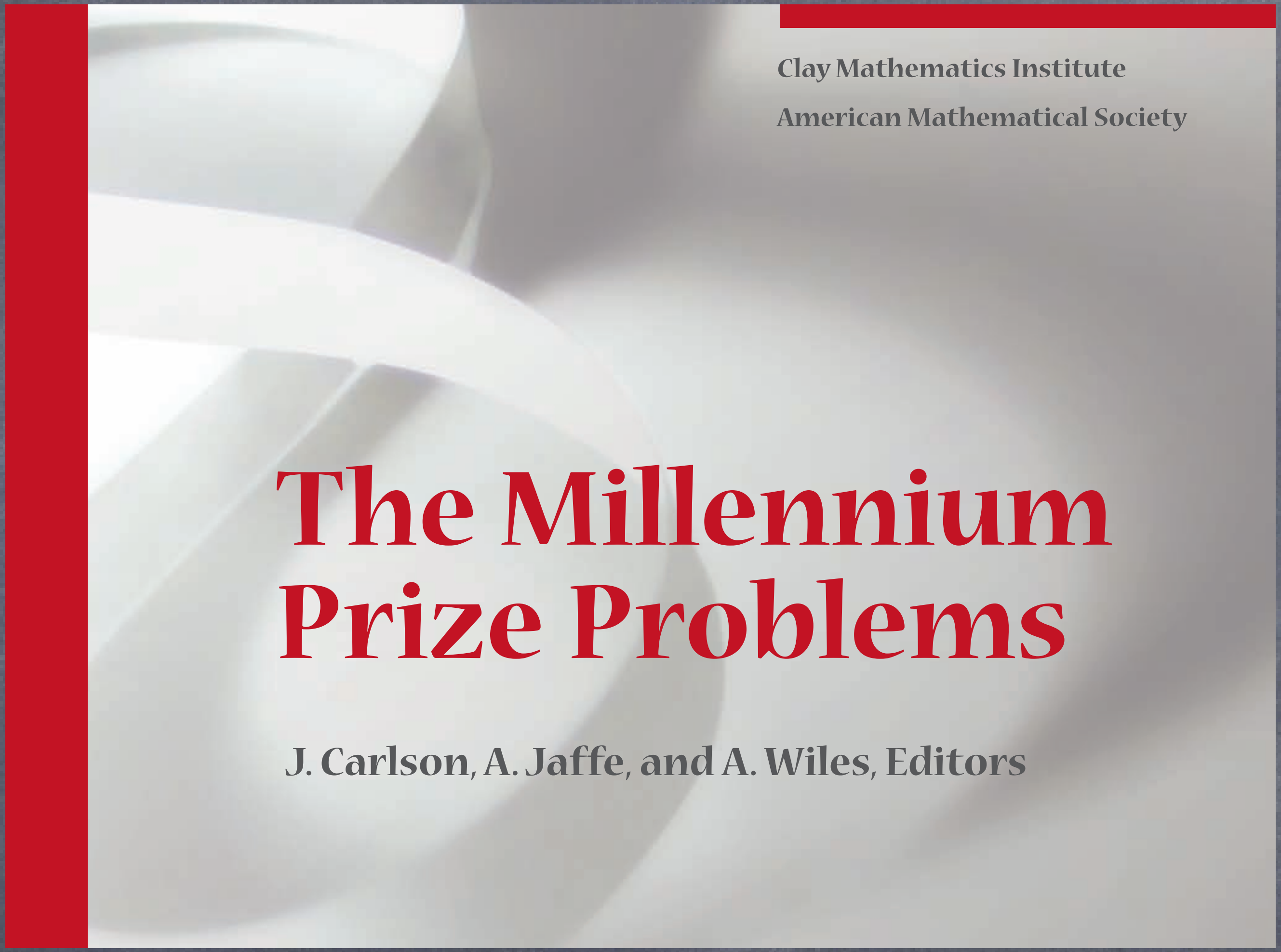


Question: What if the quark masses were all much larger than the (energy density)$^{1/4}$ in the gluon flux tube?

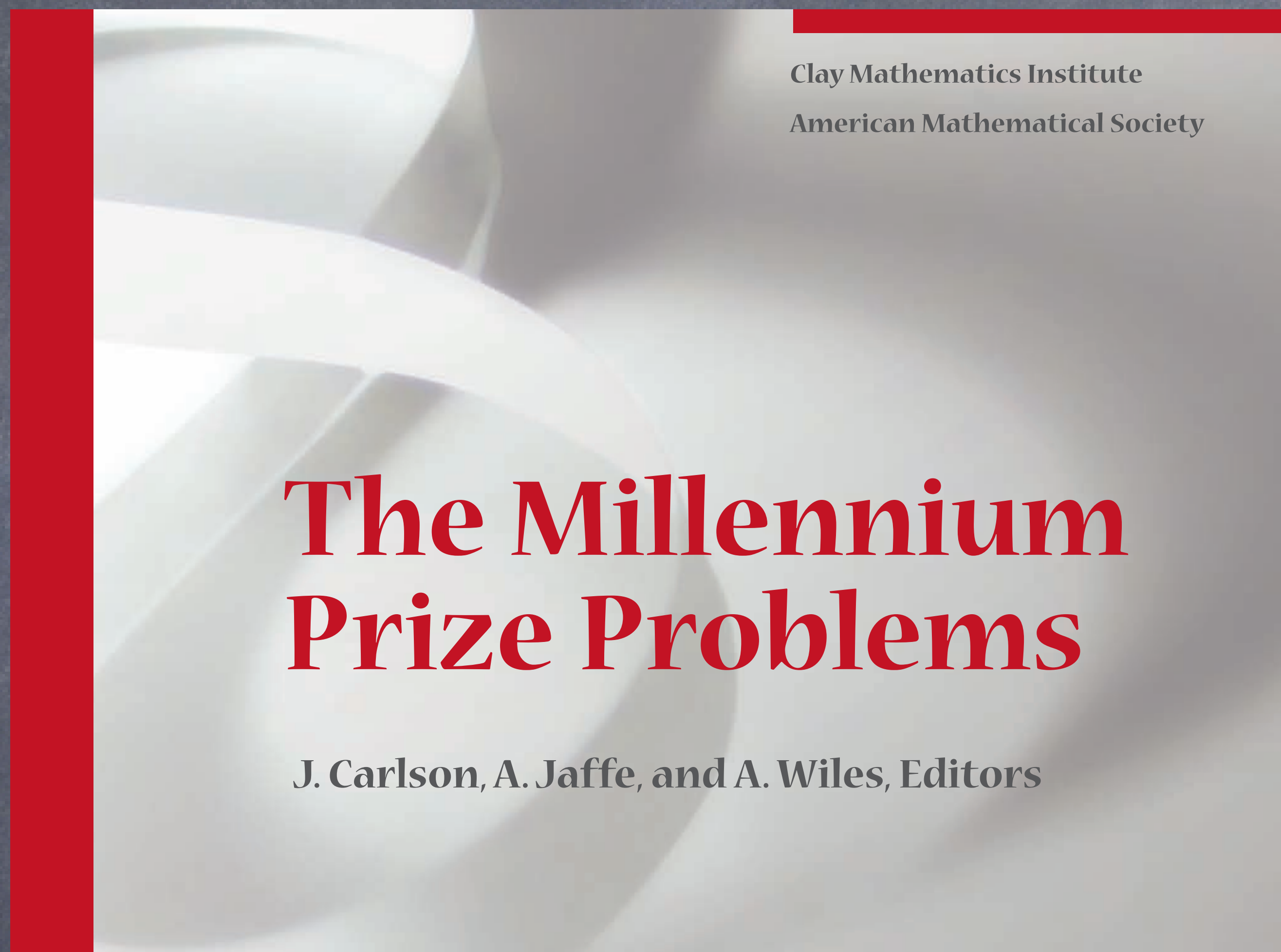
The down quark has been liberated from the proton!

Confinement



Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [45, 35].

Confinement



What physical evidence is there for the mass gap in QCD?

Question

Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [45, 35].

Quark-Hadron Duality

Poggio-Quinn-Weinberg (1976):

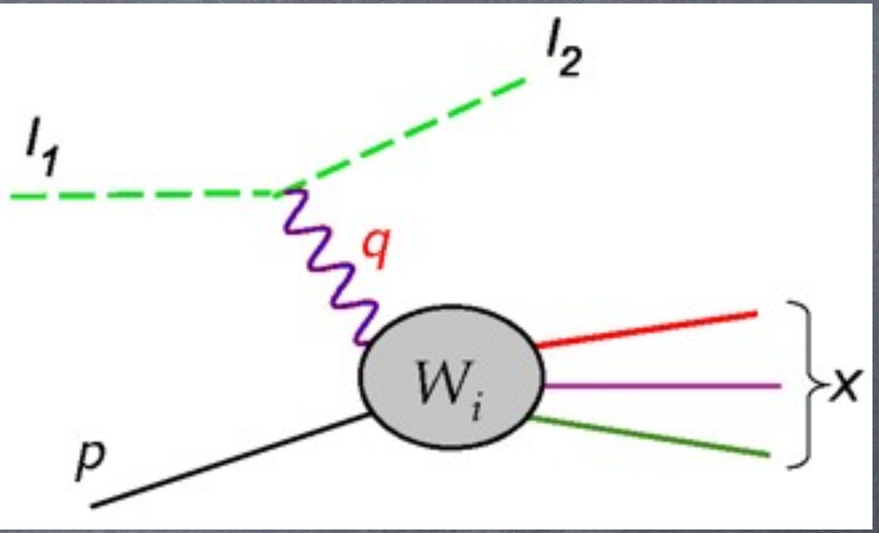
Argued that certain inclusive hadronic cross sections, averaged with appropriate weighting factors over appropriately high energy ranges, could be calculated perturbatively in terms of quarks and gluons.

This is called global quark-hadron duality.

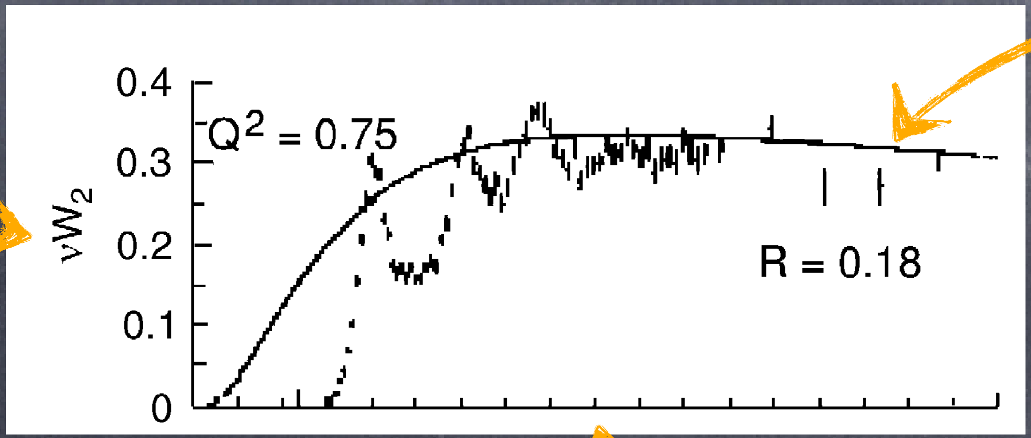
Quark-Hadron Duality

Bloom-Gilman Duality -1970

Inclusive cross sections in inelastic electron-proton scattering follow scaling relations (on average), even in resonance region.



scaling curve



~F_2 structure function

kinematic variable ω'

Quark-Hadron Duality

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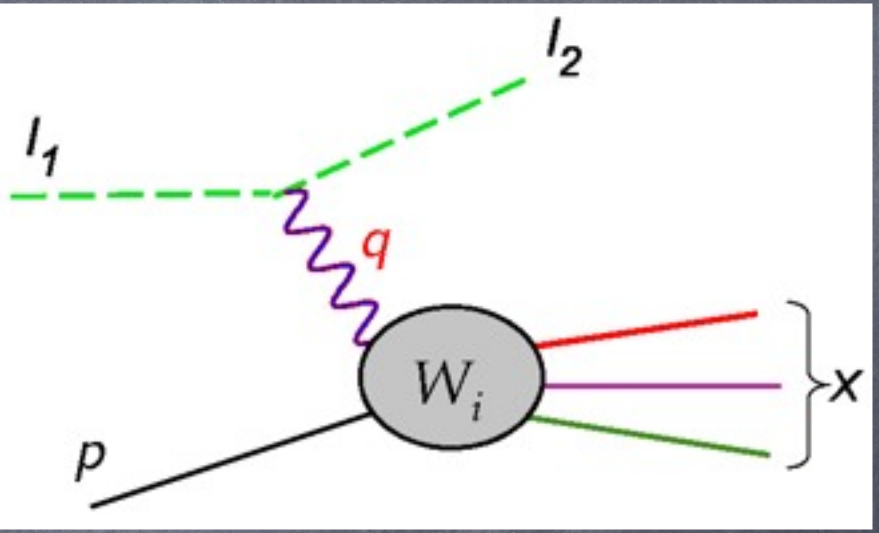

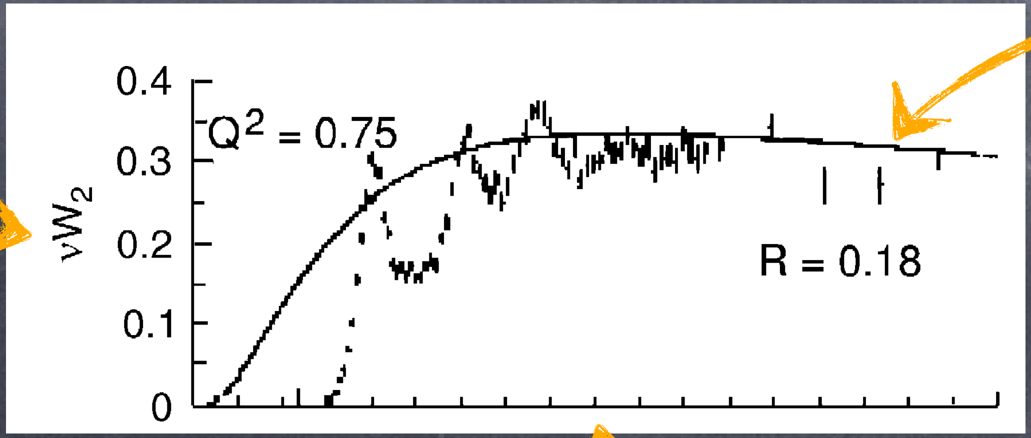

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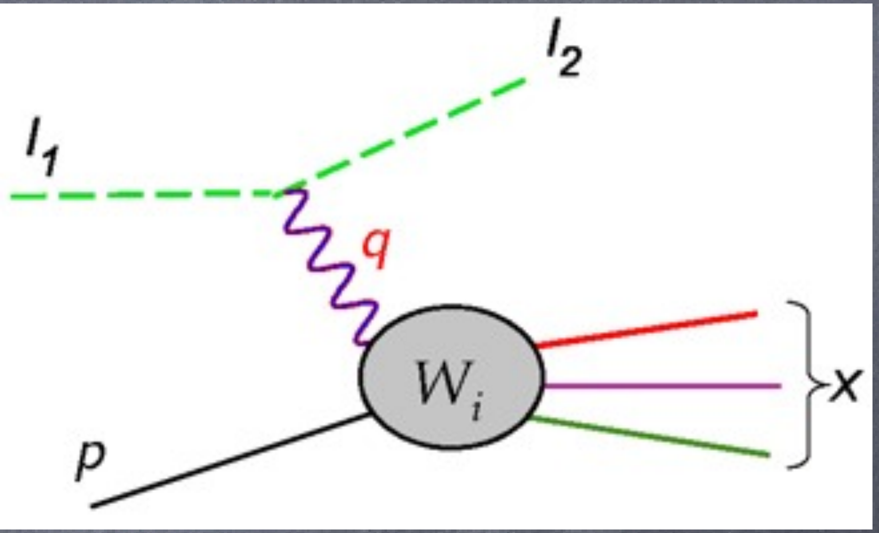
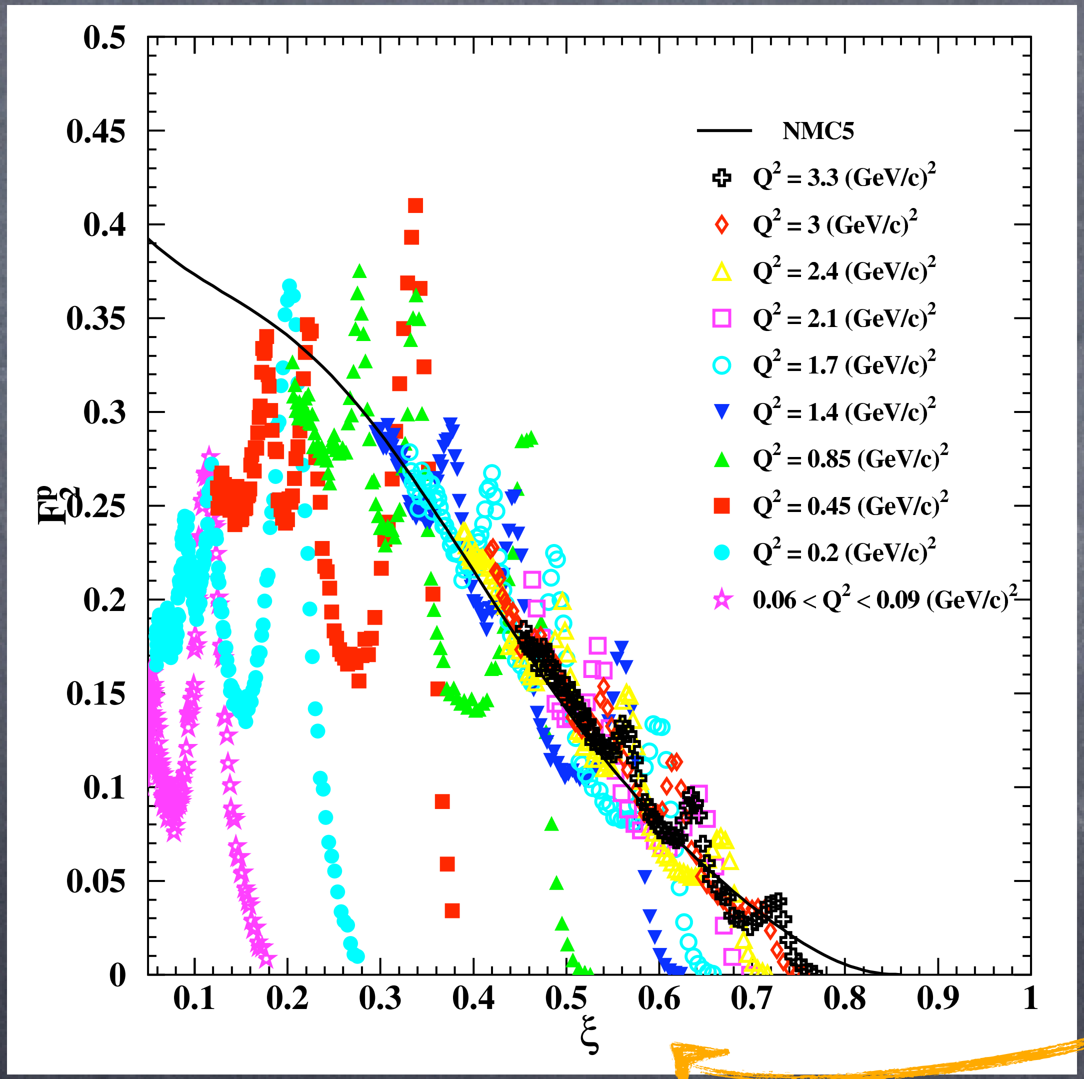


Quark-Hadron Duality

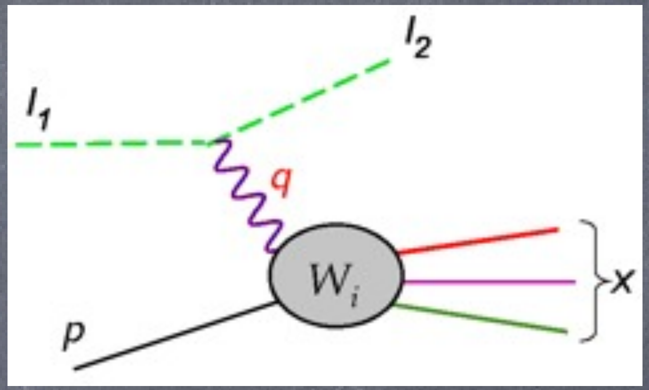
Bloom-Gilman Duality -1970

Modern Duality Data



JLab Hall C

Niculescu et al. - 2000



"Nachtmann scaling variable"

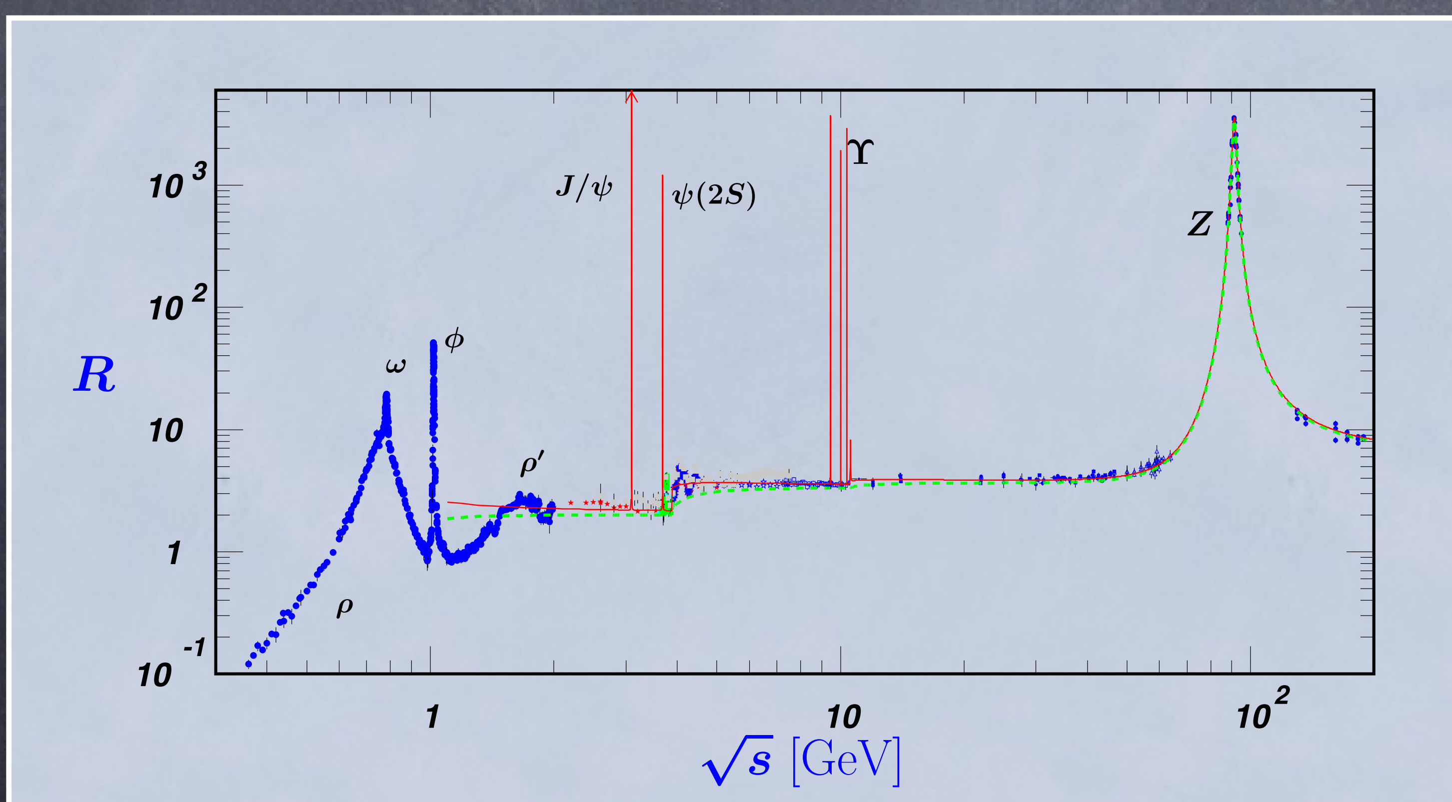
$\xi \sim x = q^2/2q \cdot p$

at large q^2

Quark-Hadron Duality

Consider $e^+e^- \to q\bar{q}$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_c \sum_q e_q^2$$



Exercise:

$R(u,d,s) \approx 2$

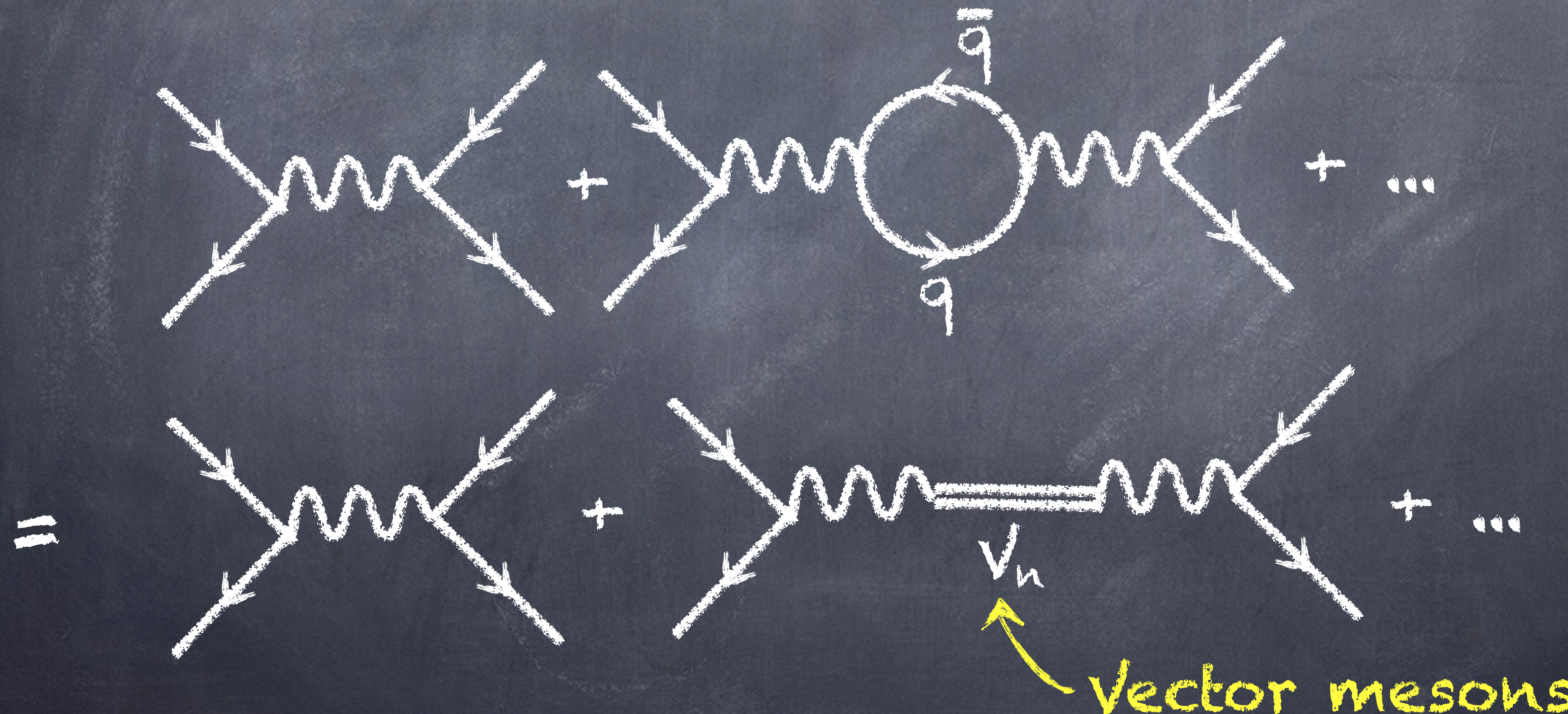
$R(u,d,s,c) \approx 10/3$

$R(u,d,s,c,b) \approx 11/3$

2008 PDG

Quark-Hadron Duality

Consider elastic electron-positron scattering:



Quark-Hadron Duality

Peskin & Schroeder, Ch 18

Optical Theorem:

$$\sigma(e^+e^- \to \text{anything}) = \frac{1}{2s}\mathrm{Im}\,\mathcal{M}(e^+e^- \to e^+e^-)$$

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Quark-Hadron Duality

Peskin & Schroeder, Ch 18

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Quark-Hadron Duality and the Operator Product Expansion

Shifman, "The Quark-Hadron Duality" - 2003

$$
\begin{aligned}
i\Pi^{\mu\nu}(q) &= \int d^4x\, e^{iq\cdot x} \langle 0|T\{J^\mu(x)J^\nu(0)\}|0\rangle \\
&= \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\Pi(q^2)
\end{aligned}
$$

At short distances we can try to expand perturbatively in local operators.

Operator Product Expansion:

$$J^\mu(x)J^\nu(0) \sim C_1^{\mu\nu}(x)\cdot 1 + C_{\overline{q}q}^{\mu\nu}(x)\overline{q}q(0) + C_{F^2}^{\mu\nu}(x)(F_{\alpha\beta}^a)^2(0) + \cdots$$

Fourier transform, expand Π in powers of $1/q^2$

Quark-Hadron Duality

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Perturbative QCD:

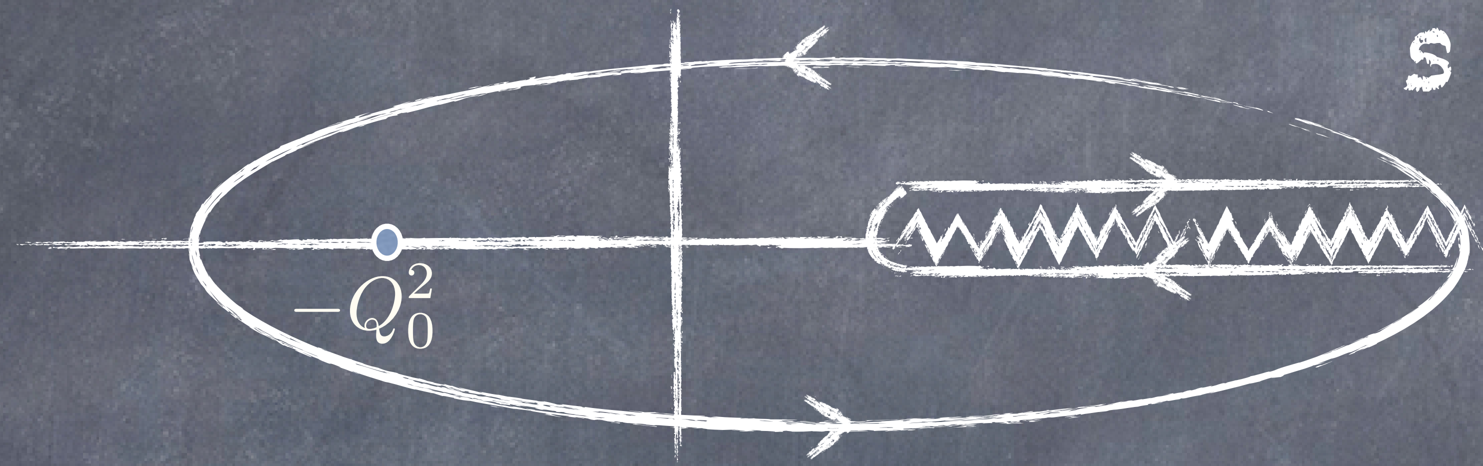
$$\Pi(s) = -\frac{N_c}{12\pi^2}\ln(-s/M^2) + \ldots$$

Resonance model:

$$\Pi(s) = \sum_{V_n} \frac{F_{V_n}^2}{s - m_{V_n}^2 + i\Gamma_{V_n} m_{V_n}} + \ldots$$

Vector mesons

Quark-Hadron Duality



$$I_n = -4\pi\alpha \oint \frac{ds}{2\pi i} \frac{1}{(s+Q_0^2)^{n+1}} \Pi(s)$$

by Cauchy's theorem

$$= \frac{1}{n!} \frac{d^n}{ds^n} \Pi(s) \Big|_{s=-Q_0^2}$$

from discontinuity across cut

$$= -4\pi\alpha \int \frac{ds}{2\pi} \frac{1}{(s+Q_0^2)^{n+1}} 2\,\mathrm{Im}\,\Pi(s)$$

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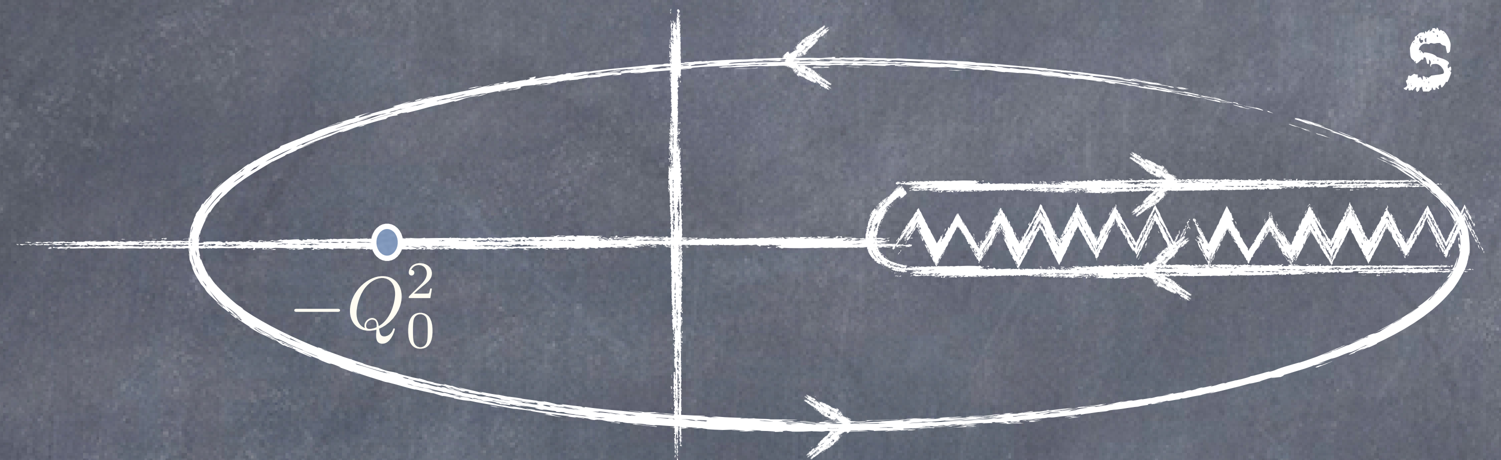
$$= \frac{1}{\pi} \int_0^\infty ds \frac{s}{(s+Q_0^2)^{n+1}} \sigma(s)$$

Quark-Hadron Duality



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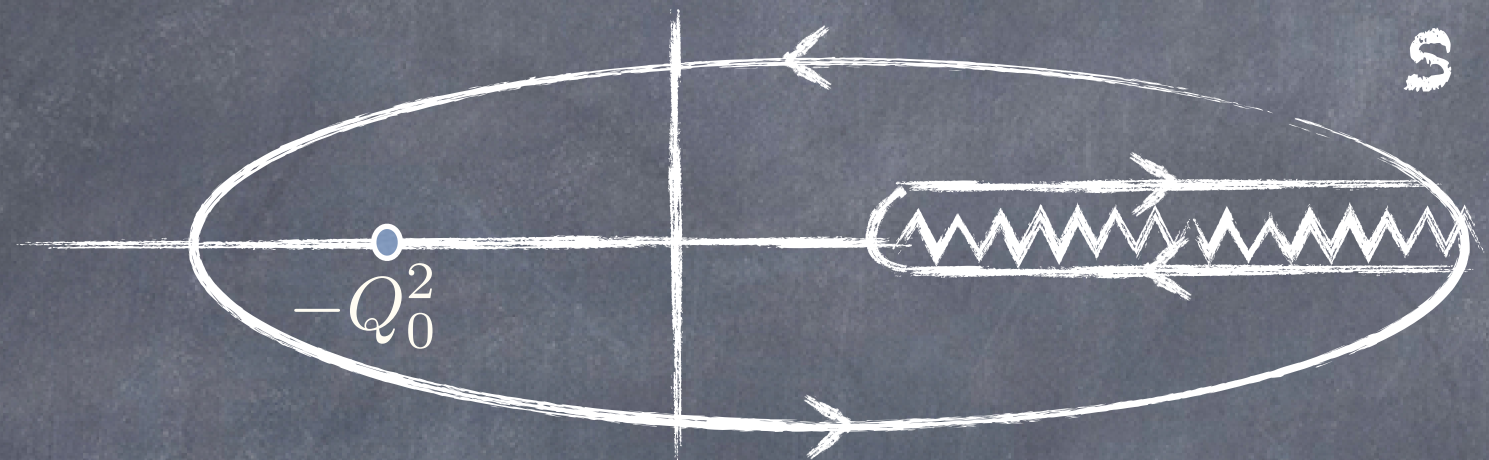

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$\sigma(e^+e^- \to \text{hadrons})$

Quark-Hadron Duality

The resulting relations between $\sigma(e^+e^- \to \mathrm{hadrons})$ and perturbative OPE coefficients are called ITEP Sum Rules (Novikov,Shifman,Vainshtein,Voloshin,Zakharov)

At sufficiently high s, the OPE is relatively accurate.

At smaller s, resonances dominate but averages over resonances still agree roughly with the perturbative results.

Dualities Lecture 1 Summary

Dualities exist when there are multiple descriptions of the same physics.

The high-energy (>2 GeV) quark/gluon regime and low-energy (<2 GeV) resonance regime can sometimes be connected by quark-hadron duality.

One can understand aspects of quark-hadron duality by way of the Operator Product Expansion, which also helps to identify sources of duality violations.