Accelerator Physics

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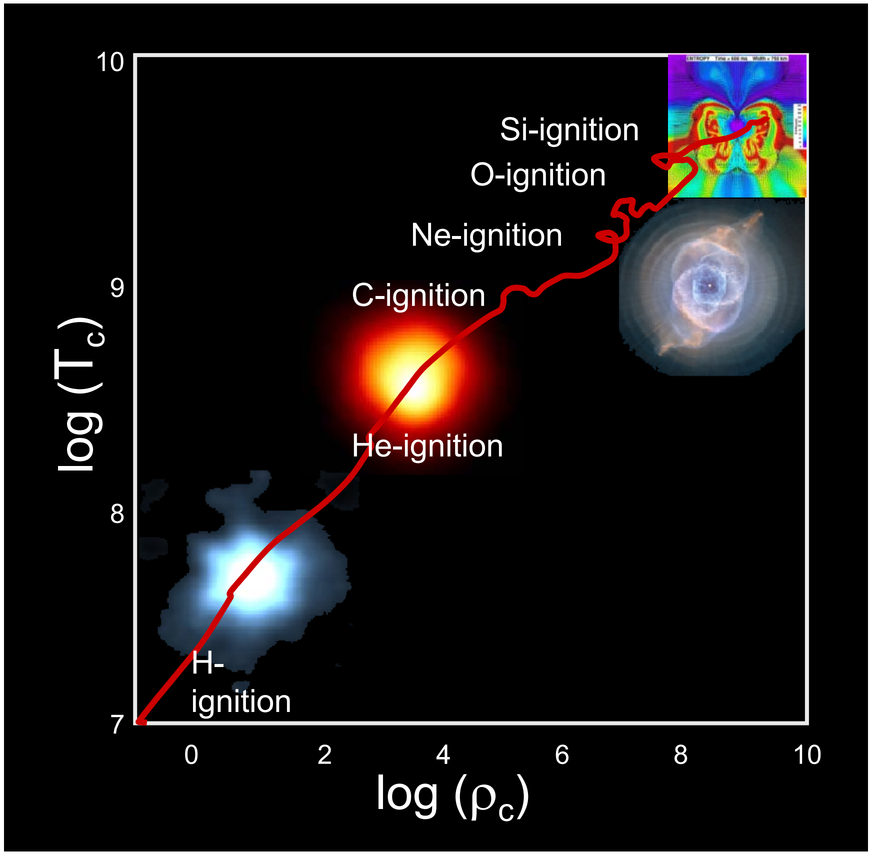
Accelerators Applications

- Accelerators are designed and optimized for the science goals studied at the facility
- Various types: primary accelerators, drivers (produce secondary particles: rare isotopes, neutrinos, neutrons, x-rays…), postaccelerators..
  - Electrostatic: Pelletrons, Cockcroft Walton Generators, High voltage platforms …
  - Electromagnetic: Cyclotrons, Linear Accelerator, Synchrotron, + storage rings…
- Main Parameters
  - Top Energy
  - Particle type and range (electrons, protons, heavy ions, rare isotopes)
  - Intensity or Beam Power

e.g. FRIB: **200MeV/u** for Uranium, 8.4pμA
heavy ions ^{16}O-^{238}U for fragmentation (+lighter ion option)
particles /sec = dN/dt = I/Qe (1 pμA == 6x10^{12} /s)
Power = dN/dt [pμA] x Particle Energy [MeV]= 200*8.4*238=**400kW**

Applications of Accelerators in Nuclear Physics [1]

Accelerators are tailored to the experiments they support

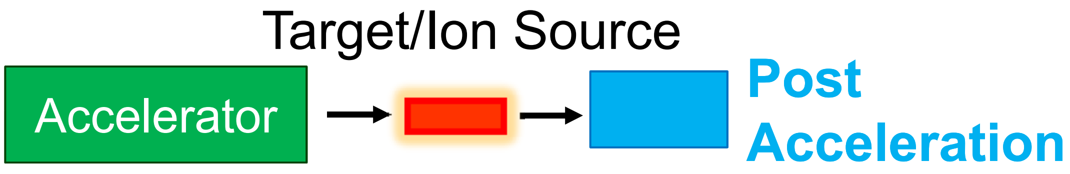


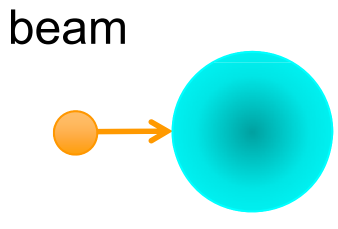
Low Energy Nuclear Physics

- Electrostatic Accelerators: few keV – few MeV (total energy)
  - » Extremely low energy accelerators (aimed to measure solar fusion cross sections), keV to a few MeV, underground
  - » Low energy accelerator, stable beams (e.g. Notre Dame), electrostatic accelerators, < 5MeV
- Cyclotrons, linacs, SC-Linacs, …: >100keV/nuc – tens of MeV/nuc
  - Stable Beam Facilities (Atlas, 88-Inch, Texas A&M..)
  - Radioactive Ion Beam postaccelerators (Rex-Isolde, ISAC, ReA, CARIBU..)

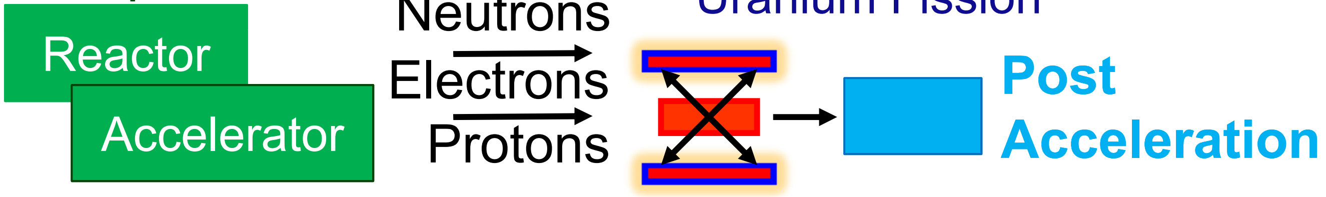
Rare Isotope Beam Production Techniques

- Target spallation and fragmentation by light ions (ISOL – Isotope separation on line)

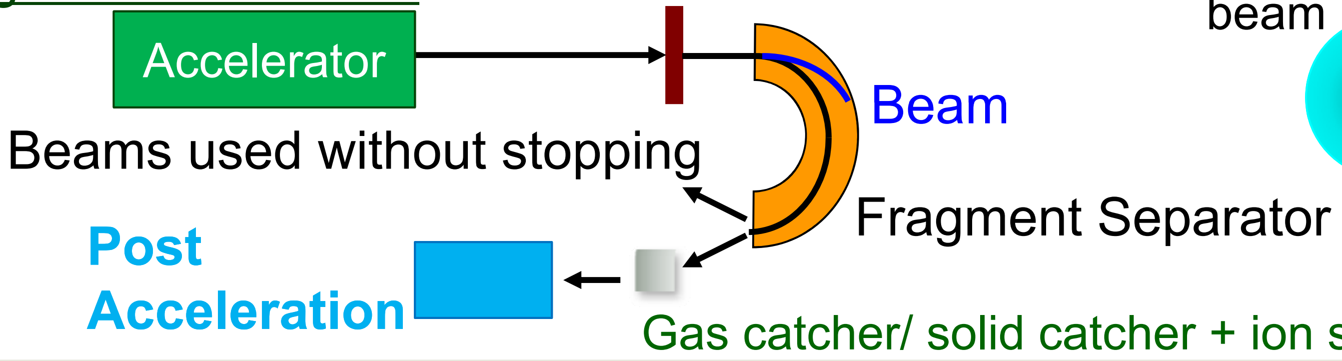


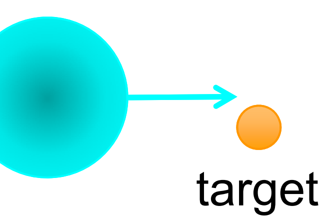


- Photon or particle induced fission



- In-flight Separation following nucleon transfer, fusion, projectile fragmentation/fission





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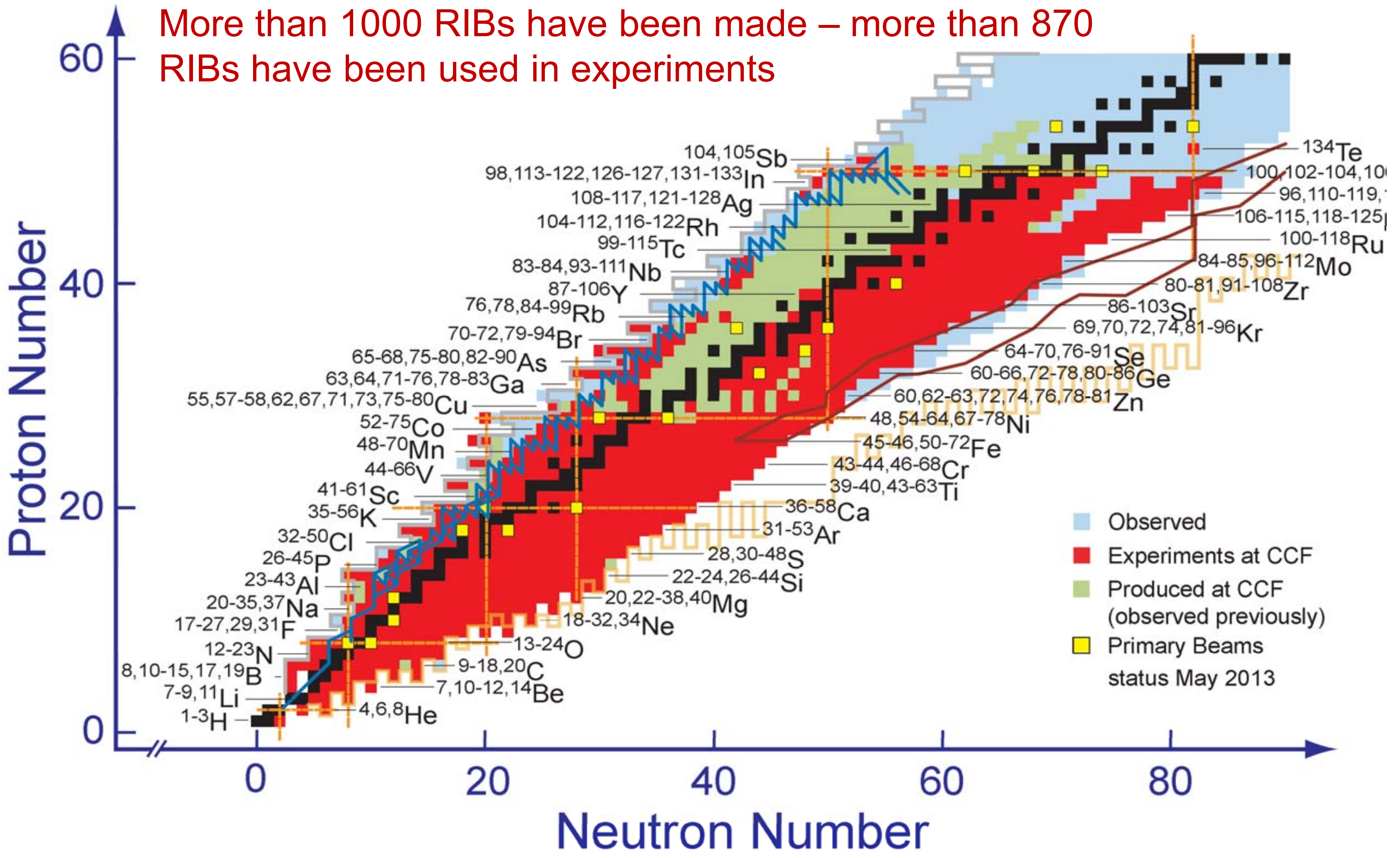
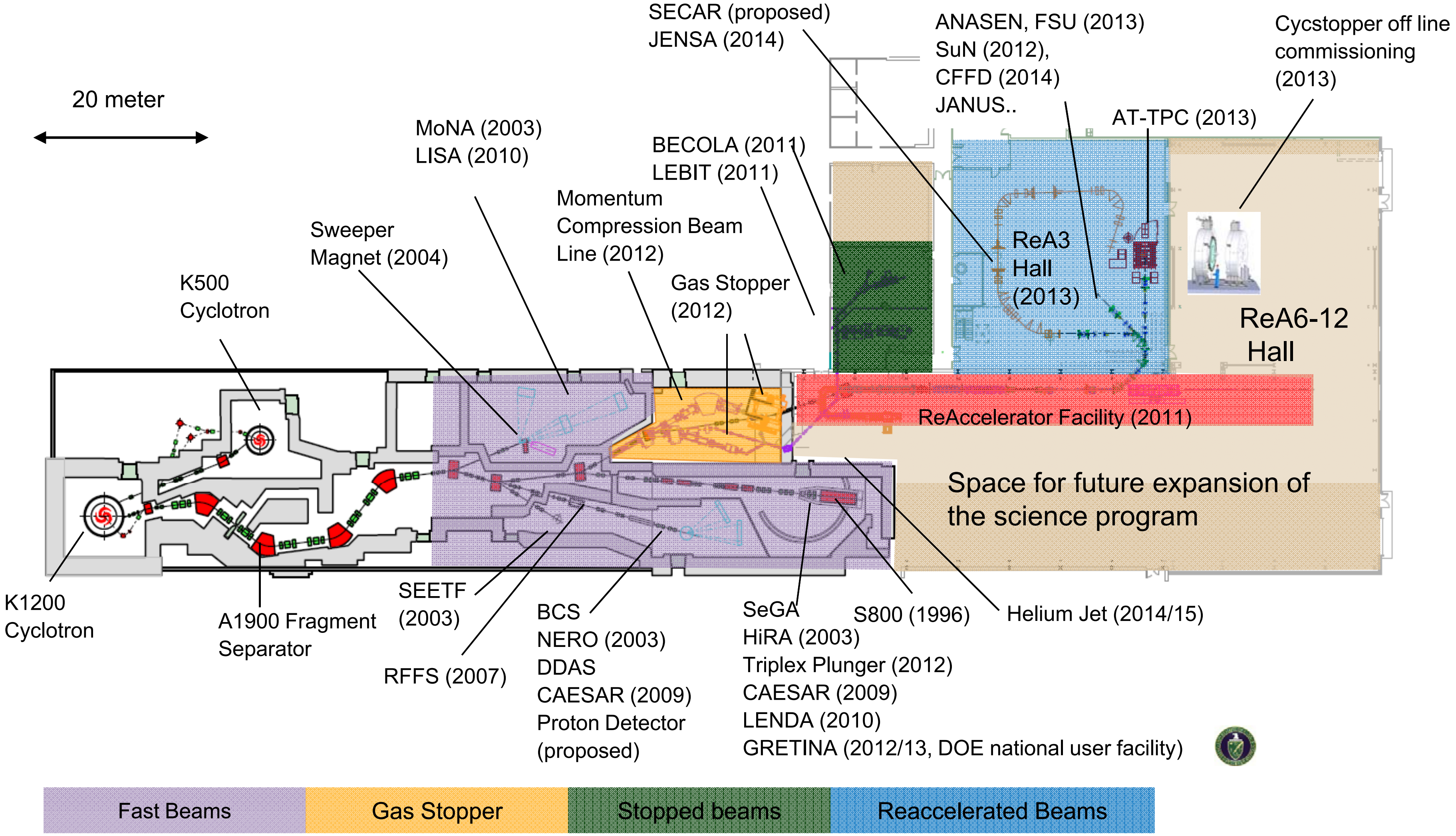
Exotic Beams Produced in Flight at NSCL’s CCF

Experienced group of PhD beam physicists delivers rare isotope beams to users



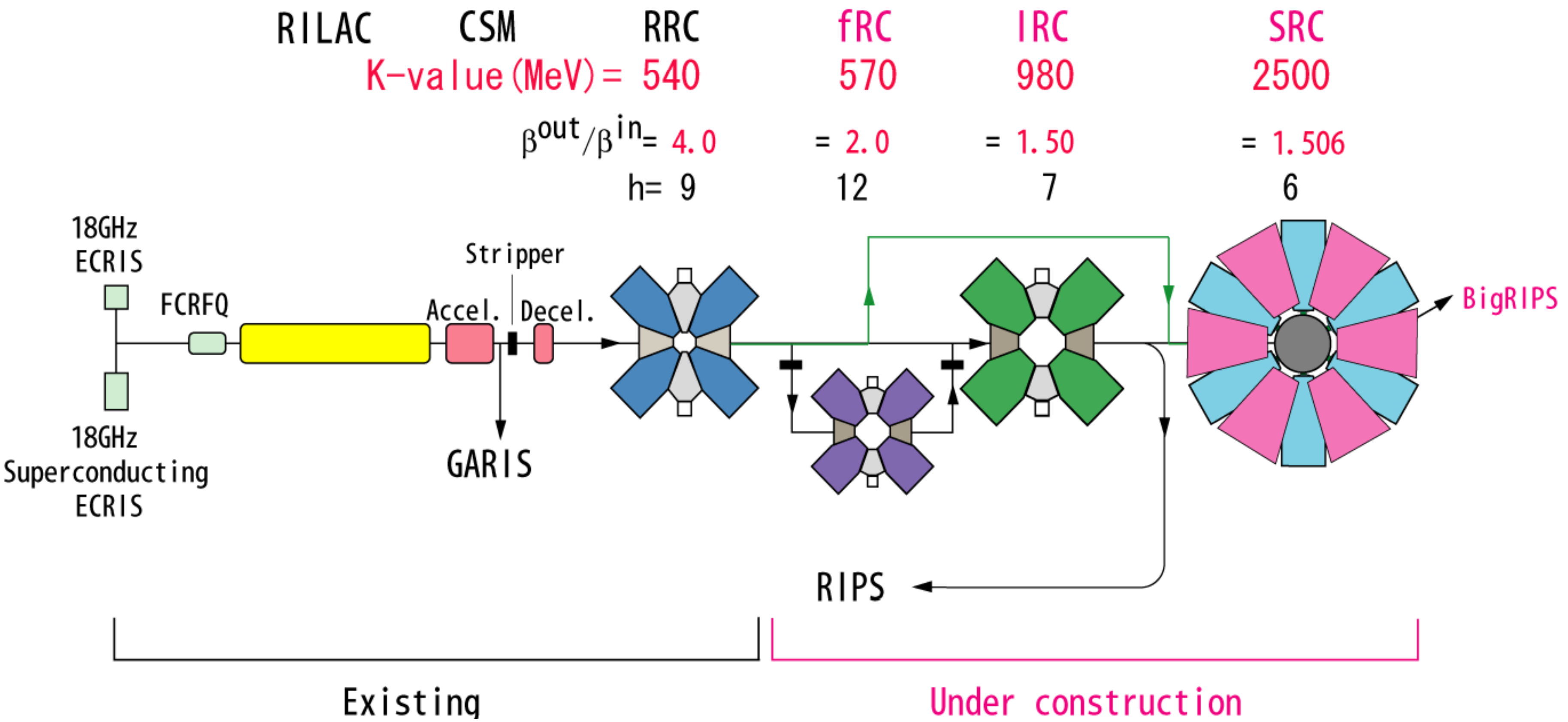
More than 1000 RIBs have been made – more than 870 RIBs have been used in experiments

In-flight fragmentation CCF facility at MSU (up to 1kW beam power, ^{16}O – ^{238} U)



Applications of Accelerators in Nuclear Physics [2]

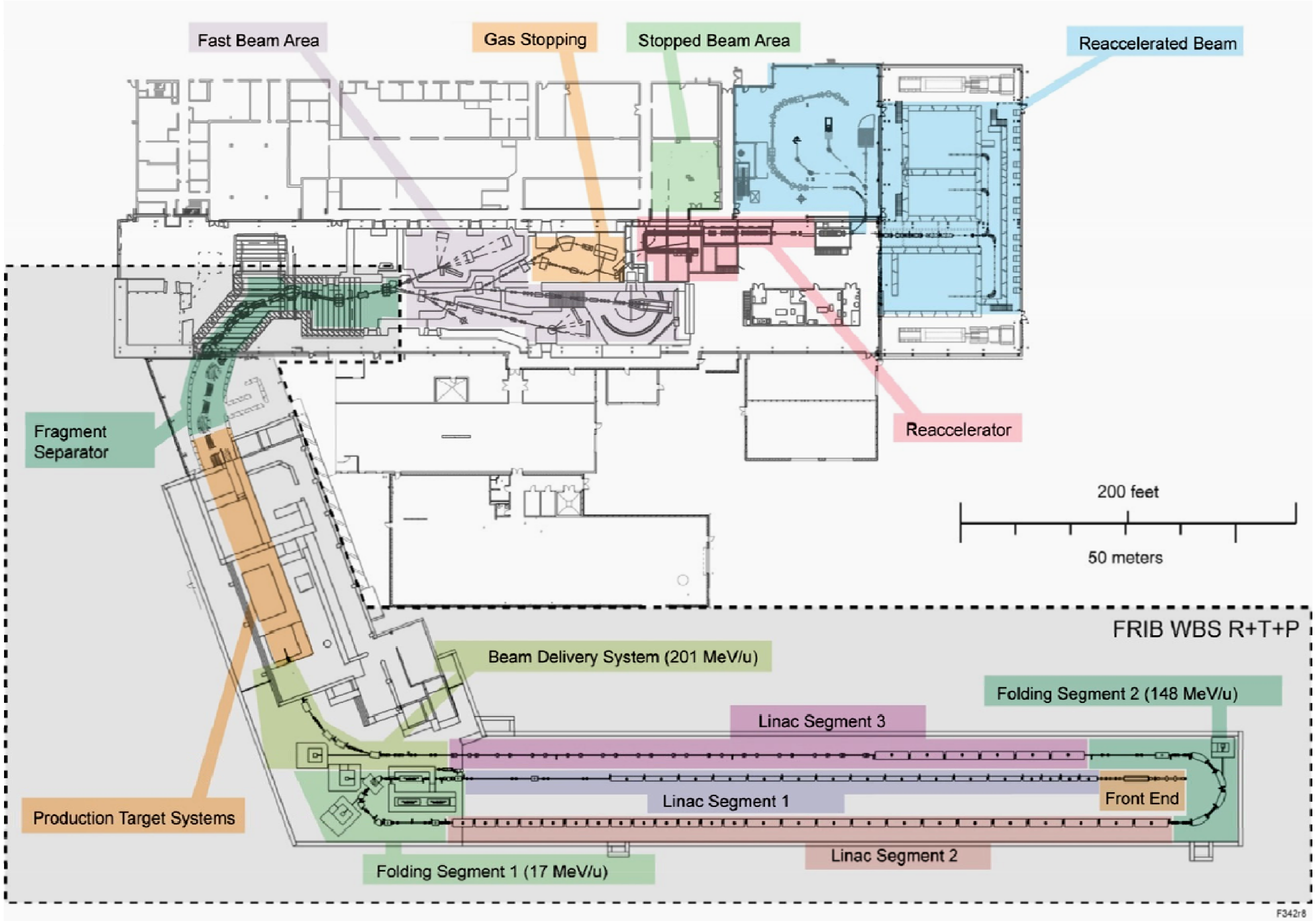
- Accelerators as driver to produce rare isotope beams: Cyclotrons



U^{35+} ~ 15pμA (525 eμA), Injector → 350 Mev/u U ~ 0.5 to 1pμA

Applications of Accelerators in Nuclear Physics [2]

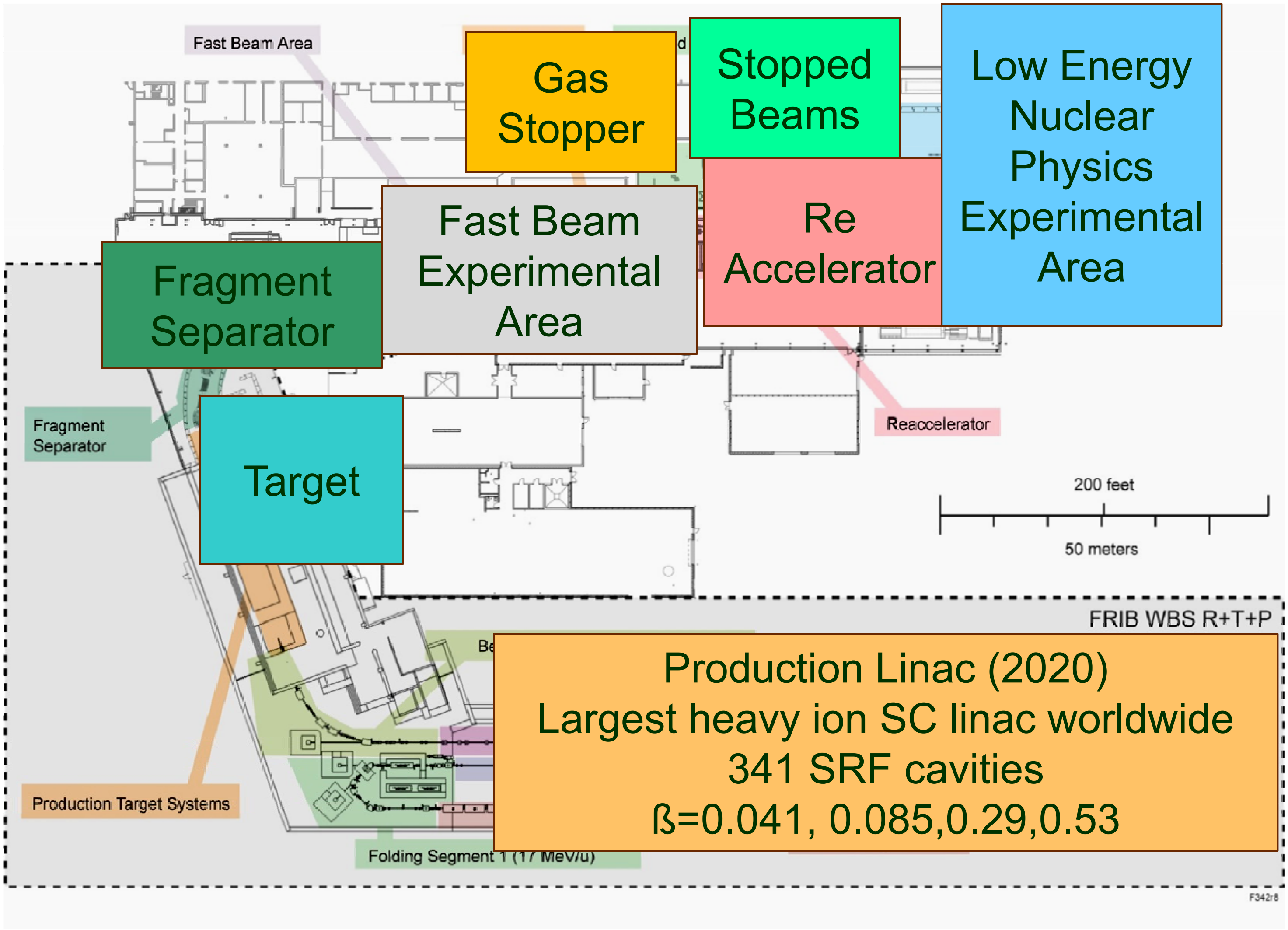
- FRIB: in-flight fragmentation: fast, stopped, reaccelerated beams



200MeV/u for Uranium, 8.4pμA, **400kW on beam power for all ions**

Applications of Accelerators in Nuclear Physics [2]

- FRIB: in-flight fragmentation: fast, stopped, reaccelerated beams



Gas Stopper

Stopped Beams

Low Energy Nuclear Physics Experimental Area

Fragment Separator

Fast Beam Experimental Area

Re Accelerator

Target

Production Linac (2020)
Largest heavy ion SC linac worldwide
341 SRF cavities
ß=0.041, 0.085,0.29,0.53

Applications of Accelerators in Nuclear Physics and Particle Physics [3]

Relativistic heavy ions and particle physics

GeV to TeV/ nucleon of protons or heavy ions (RHIC and LHC, colliders): LHC: 7TeV proton energy in each collider ring, 2.47 TeV lead

Fixed target: GeV electrons (JLAB), protons (Fermilab)

RHIC
- 2 independent intersecting storage rings with 6 interaction points
- Can circulate heavy ions or protons

Chain of accelerators:
- 3 injectors (High charge state injector (EBIS source+linac), Tandem injector, proton linac)
- Booster Synchrotron
- Alternating Gradient Synchrotron
- RHIC 2.4 miles circumference storage ring

K-factor: corresponds to the maximum kinetic energy a proton can reach/transport for the accelerator's maximum magnetic rigidity, Bρ

- Magnetic rigidity:

$$e \cdot Q \cdot v \cdot B = \omega^2 \cdot \rho \cdot M, \quad \omega = \frac{v}{\rho}$$

$$B \cdot \rho = \frac{M \cdot v}{e \cdot Q} = \frac{p}{e \cdot Q}$$

$$E = \frac{1}{2} \cdot M \cdot v^2 = \frac{e^2 Q^2}{M} \cdot \frac{\rho^2 \cdot B^2}{2} = K \cdot \frac{Q^2}{A}$$

$$B\rho = 8 \cdot Tm, U^{80+}, \max energy\,?$$

$$\frac{K}{A} = \frac{48.3 \cdot MeV}{(Tm)^2} \cdot \frac{80^2}{238} (8Tm)^2$$

$$\frac{K}{A} = 345 MeV / nuc$$

Or for heavy ions substitute m_0 with µA and e with eQ

K-factor: corresponds to the maximum kinetic energy a proton can reach/transport for the accelerator's maximum magnetic rigidity, Bρ

- Magnetic rigidity:

$$e\cdot Q\cdot v\cdot B=\omega^2\cdot\rho\cdot M,\quad \omega=\frac{v}{\rho}$$

$$B\cdot\rho=\frac{M\cdot v}{e\cdot Q}=\frac{p}{e\cdot Q}$$

$$E=\frac{1}{2}\cdot M\cdot v^2=\frac{e^2Q^2}{M}\cdot\frac{\rho^2\cdot B^2}{2}=K\cdot\frac{Q^2}{A}$$

$$K_p=\frac{e^2}{2\cdot m_o}\cdot(B\rho)^2=\frac{e^2\cdot c^2}{2\cdot m_o\cdot c^2}\cdot(B\rho)^2=\frac{e^2\cdot c^2}{2\cdot 938MeV}\cdot\frac{MeV}{MeV}(B\rho)^2$$

$$K_p=\frac{47.9\cdot MeV}{[Tm]^2}\cdot(B\rho)^2 \longrightarrow \frac{K}{A}=\frac{48.3\cdot[MeV]}{[Tm]^2}\cdot\frac{Q^2}{A}(B\rho)^2$$

$$B\rho=8\cdot Tm, U^{80+}, \text{max } energy\,?$$

$$\frac{K}{A}=\frac{48.3\cdot MeV}{(Tm)^2}\cdot\frac{80^2}{238}(8Tm)^2$$

$$\frac{K}{A}=345MeV/nuc$$

Or for heavy ions substitute m_0 with μA and e with eQ

Electrostatic focusing versus magnetic focusing

$$F_{electric} = e \cdot Q \cdot E$$

$$F_{magnetic} = e \cdot Q \cdot v \times B$$

$$v_z = \sqrt{\frac{2eQ}{m} V_z}, t = \frac{L}{v_z}$$

$$m \frac{\partial^2 x}{\partial t^2} = eQE_x$$

$$x = \frac{1}{2} \cdot \frac{eQ}{m} \cdot E_x \cdot t^2 = \frac{L^2}{2} \cdot \frac{eQ}{mv_z^2} \cdot E_x = \frac{L^2}{2} \cdot \frac{eQ}{m} \cdot E_x \cdot \frac{m}{2eQV_z}$$

Focusing strength for magnetic elements increases with velocity, therefore for higher energy beams magnets need to be used (electrostatic elements are only used up to a few MeV total beam energy)

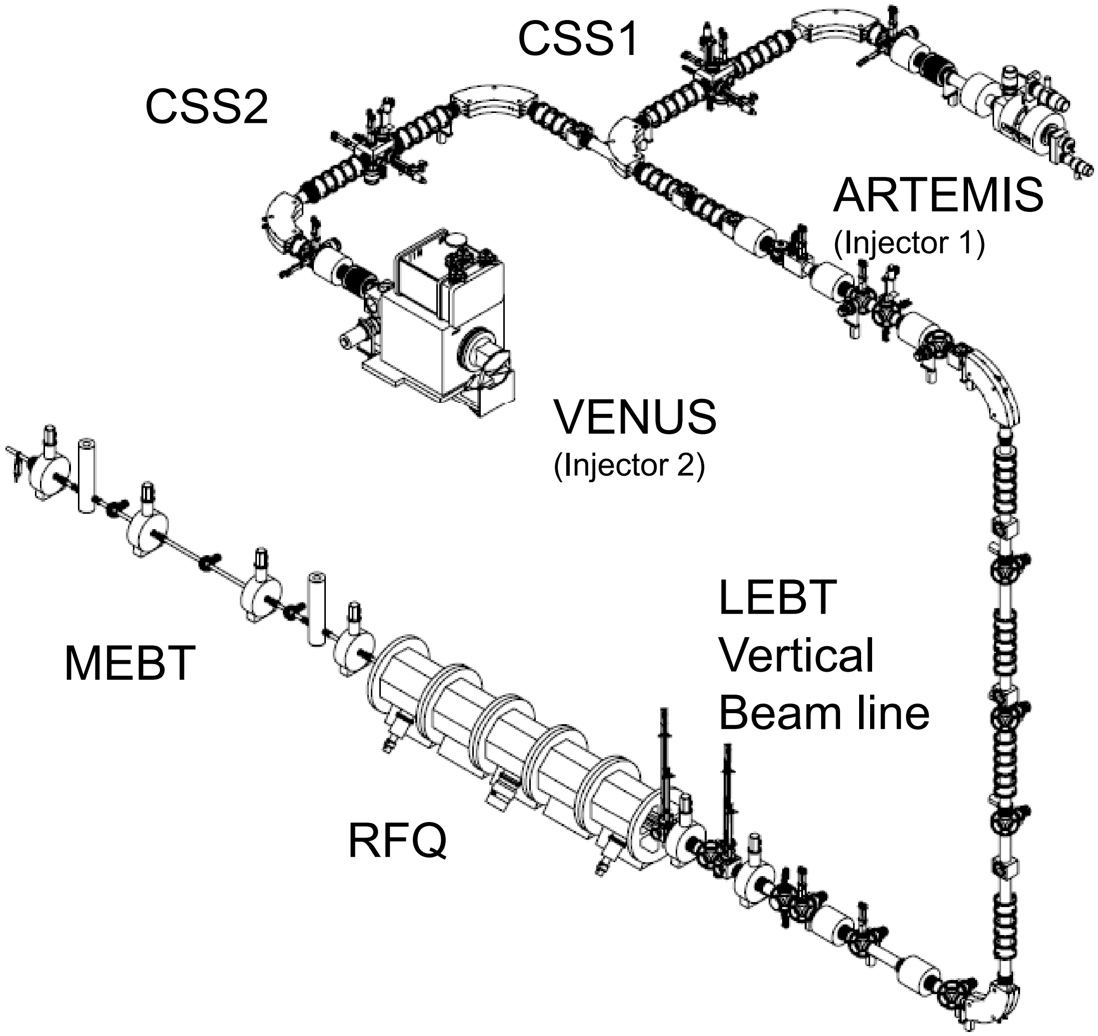
Electrostatic focusing is Q/A independent, used for injection lines (e.g. post accelerators, 2 charge state injection, e.g. FRIB front end…)

FRIB front end accelerates and transports two charge states simultaneously to the SC heavy ion linac

Beam is very slow 12keV/u (required injection energy for the RFQ linac)

Combines magnetic elements and electrostatic focusing elements to select and transport two charge states

Other uses for electrostatic devices: injector lines for charge breeders in post accelerators , electron microscopes, focusing elements electrostatic accelerators….



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A Brief Accelerator History

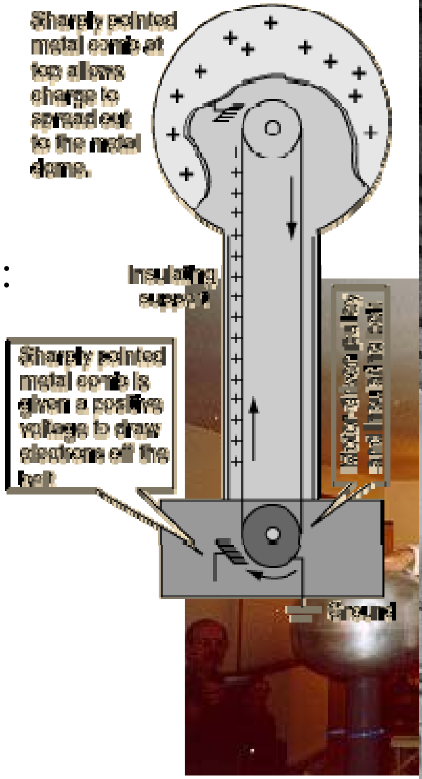
- DC Acceleration

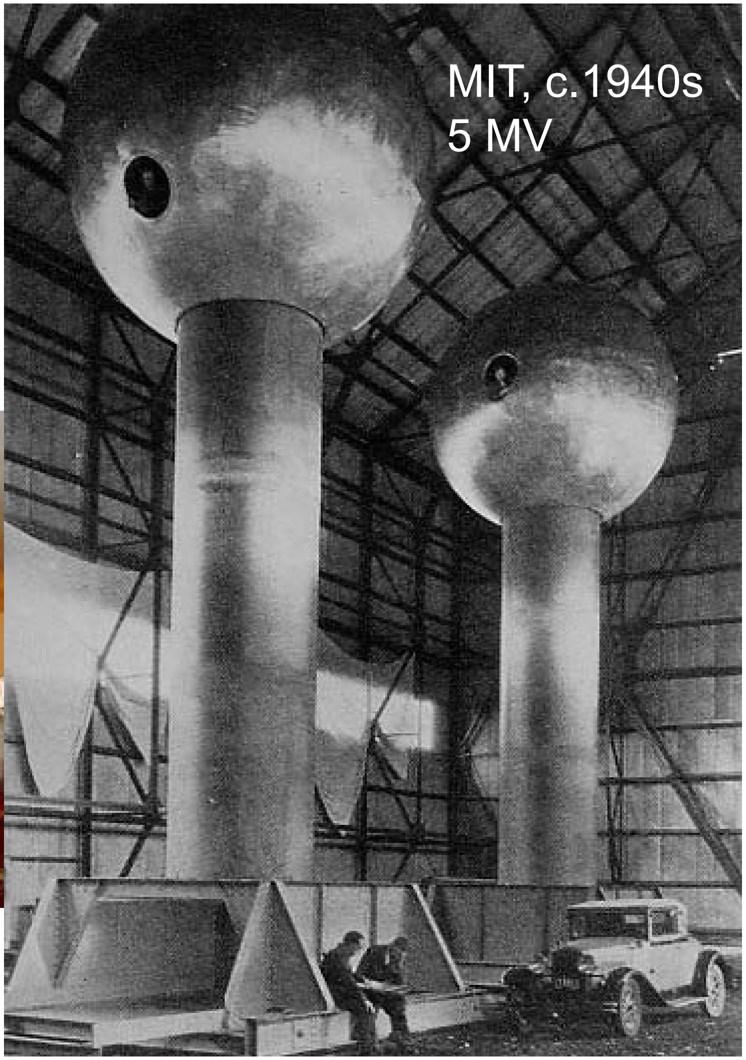
1927: Lord Rutherford requested a "copious supply" of projectiles more energetic than natural alpha and beta particles. At the opening of the resulting High Tension Laboratory, Rutherford went on to reiterate the goal:

> "What we require is an apparatus to give us a potential of the order of 10 million volts which can be safely accommodated in a reasonably sized room and operated by a few kilowatts of power. We require too an exhausted tube capable of withstanding this voltage… I see no reason why such a requirement cannot be made practical."

Van de Graaff

(1929)



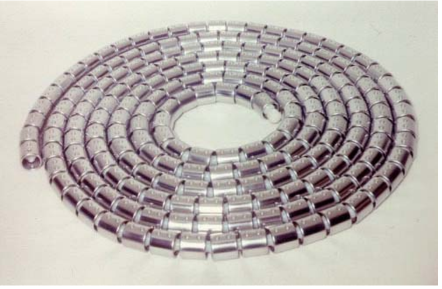

MIT, c.1940s
5 MV

http://libraries.mit.edu/archives/exhibits/van-de-graaff/

DC Acceleration: Pelletrons/Tandems

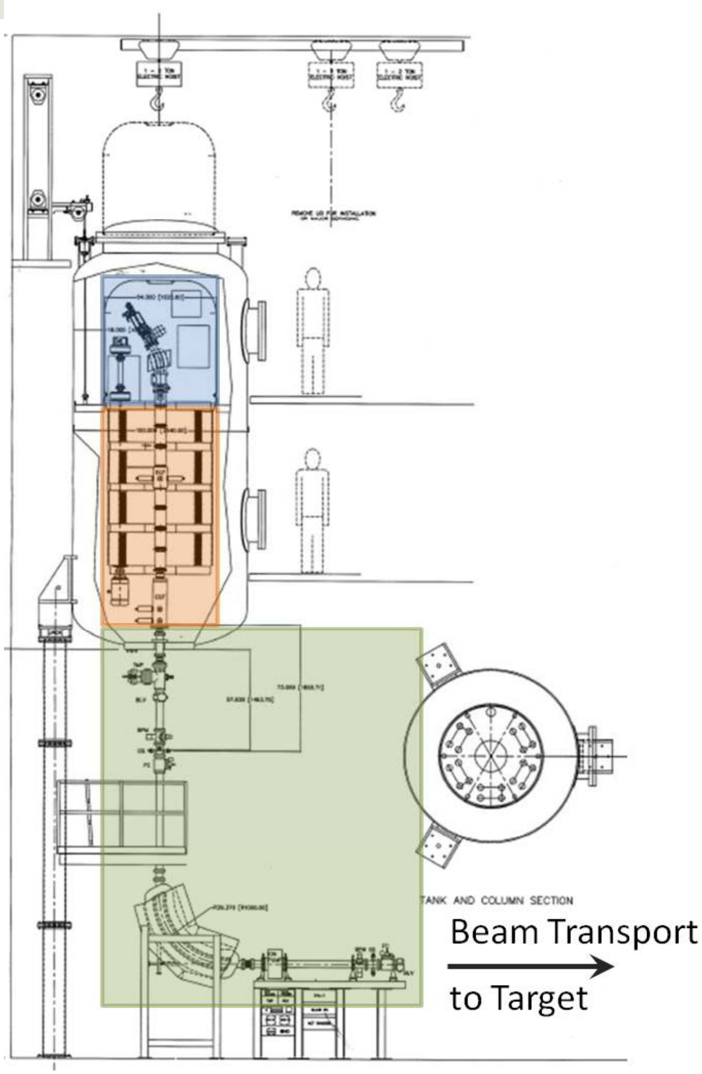
Pelletrons and Tandems (injector or stand alone)

- HV Terminal
- Charging belts
- Pressure Tank (SF6)
- High energy stability
- Limited voltage typically few MV (25MV max)



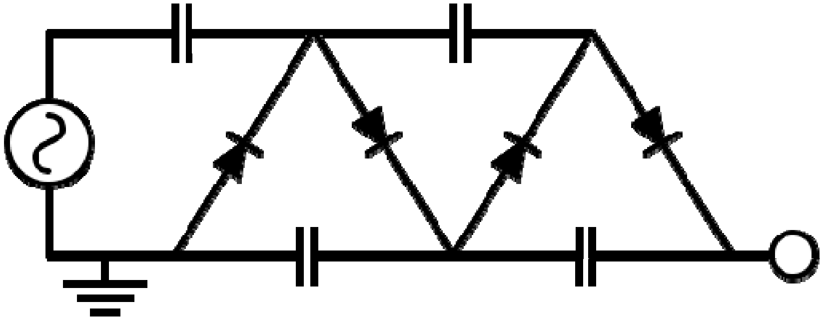
SF_6 filled pressure tank 5.8MV/m

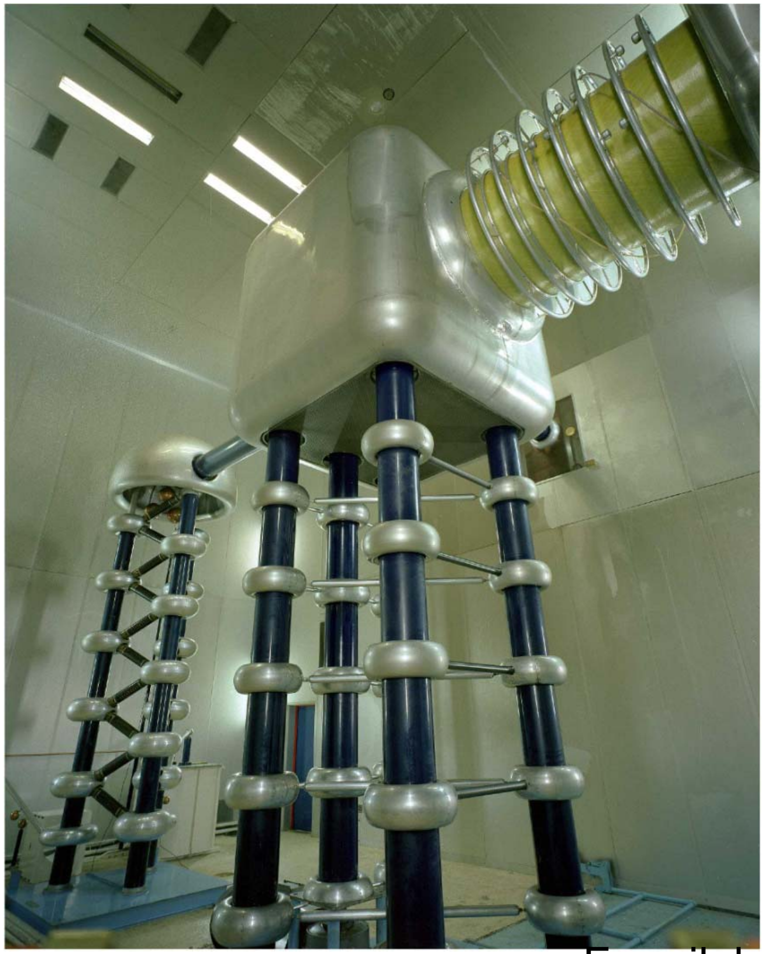
vacuum/insulator interface 2MV/m



Cockcroft and Walton

- Voltage Multiplier





Fermilab

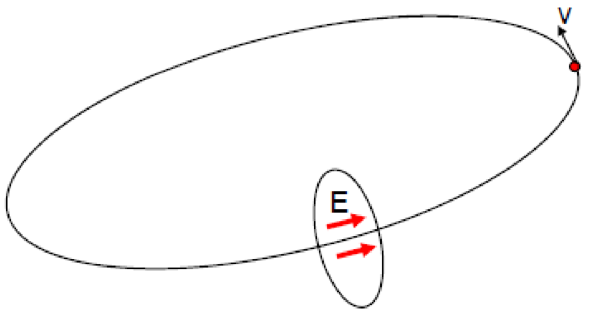
- Series of diodes and capacitors, converts AC power to DC power through several stages
- Used also switching power supplies
- Traditional injector for high energy accelerators
- Today mainly replaced by RFQ (Radio Frequency Quadrupole linacs)

How can we get to higher energies?

- DC accelerators are limited by a maximum size and a maximum break down voltage
- By using a time varying field and adding drifts that are matched in length to the growing velocity of the particles and rf frequency, one can apply relatively low accelerating voltages at each acceleration step

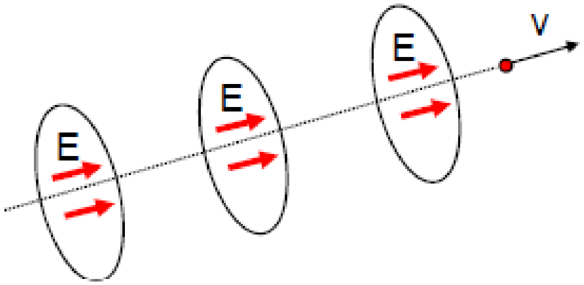
Circular Accelerator

Use one or a small number of radiofrequency accelerating cavities and make use of repeated passage through them



Linear Accelerator

Use many accelerating cavities through which the particle passes only once.



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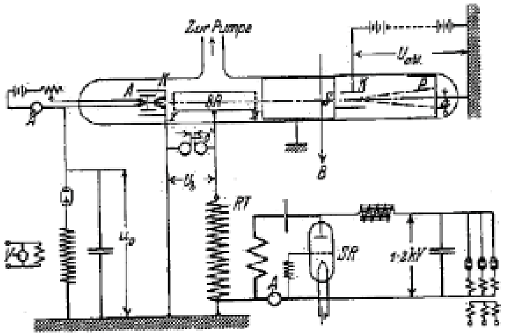
- First principles were developed in the 1930's
- Wiederoe 1929: First linear drift tube as demonstration experiment
  50 keV; accelerated heavy ions (K+, Na+), utilized oscillating voltage of 25 kV @ 1 MHz
- Lawrence 1930 read (looked at) Wiederoe's paper, was aiming to make the design more compact by adding a magnetic bending field



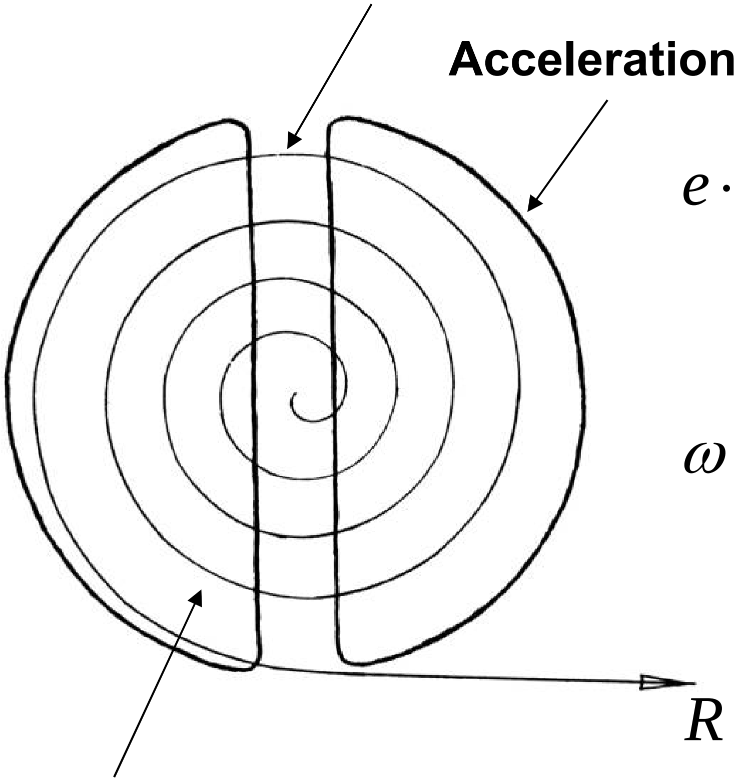
Cyclotron acceleration

5 inch diameter wax vacuum seal!

80keV protons

Dummy Dee at ground

Acceleration while crossing the gap



$$e \cdot Q \cdot v \cdot B = \omega^2 \cdot R \cdot M \, ,$$

$$\omega = \frac{e \cdot Q \cdot B}{M}$$

Rotation frequency is independent of the velocity

$$R = \frac{M \cdot v}{e \cdot Q \cdot B}$$

Radius grows with growing velocity

Energy gain depends quadratic on Q

$$E = \frac{1}{2} \cdot M \cdot v^2 = \frac{e^2 Q^2}{M} \cdot \frac{R^2 \cdot B^2}{2} = K \cdot \frac{Q^2}{A}$$

Acceleration Dee, connected to RF, 10s of keV* Q per gap

Cyclotrons can have many Dees

Linear accelerators

- Create a series of resonant cavities for one time path
- Match drift spaces with growing velocity

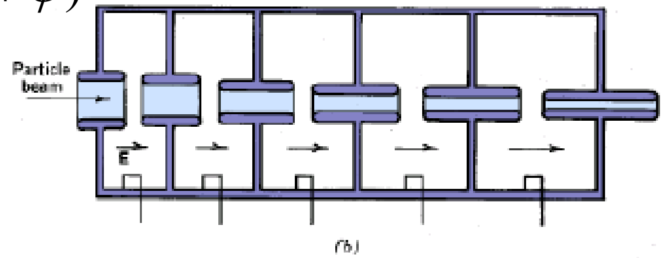
DTL tank Fermilab
Room temperature drift tube linac

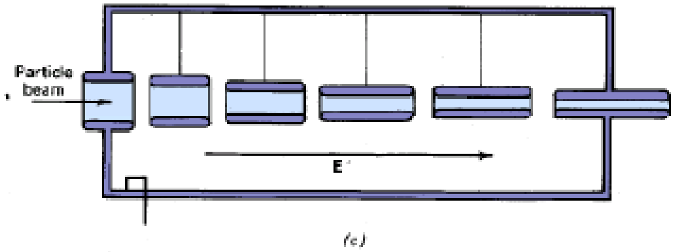
$$\beta = \frac{v}{c}, \lambda = \frac{c}{f}$$

$$L = vt = \frac{v}{c} \cdot c \cdot \frac{\tau}{2} = \frac{\beta\lambda}{2}$$

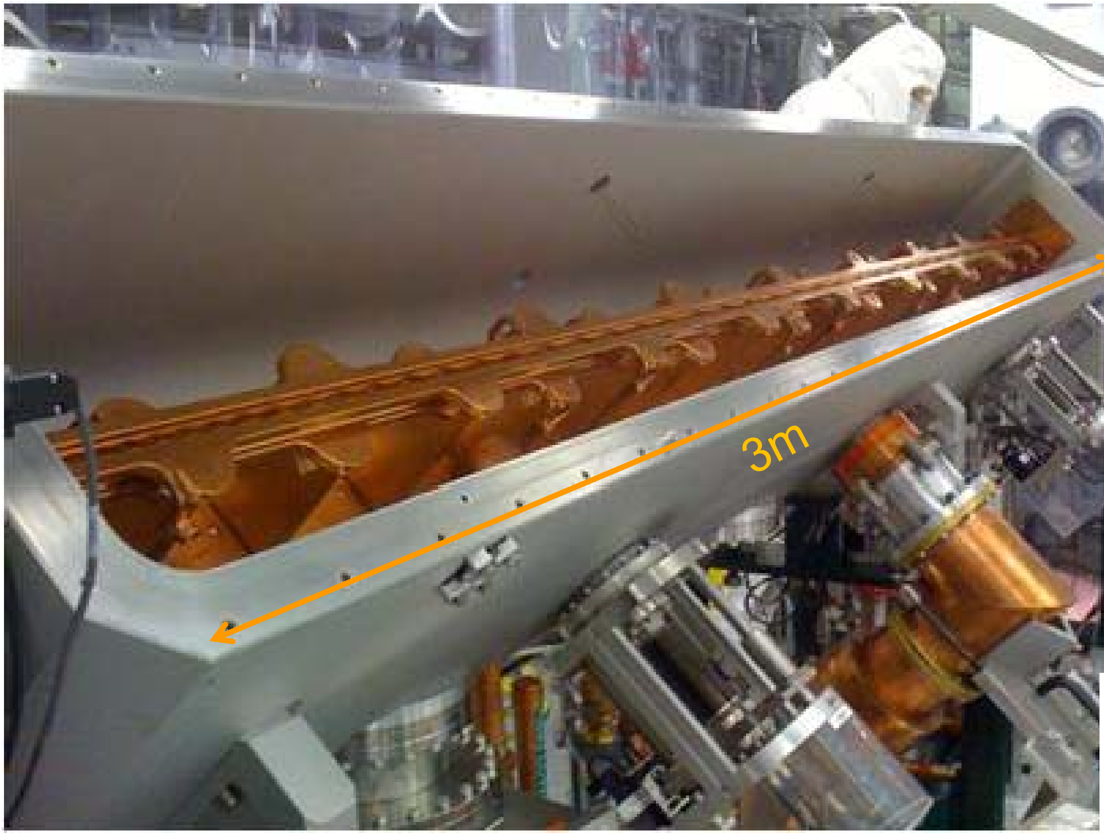
$$E_z(0,t) = E_0 \cos(\omega t + \varphi)$$

Voltages switches polarity as particles travel through drift tube (π-mode)
So the drift tubes should be βλ/2 apart, bunches are βλ apart



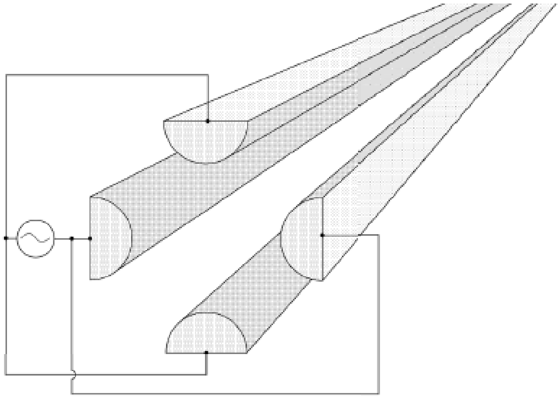


Radio Frequency Quadrupole Linac

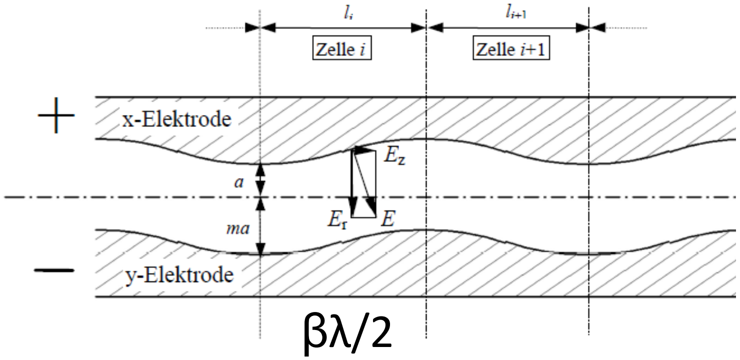


92 cells acceleration cells ReA RFQ accelerator at MSU, energy gain 12keV/u to 600keV/u

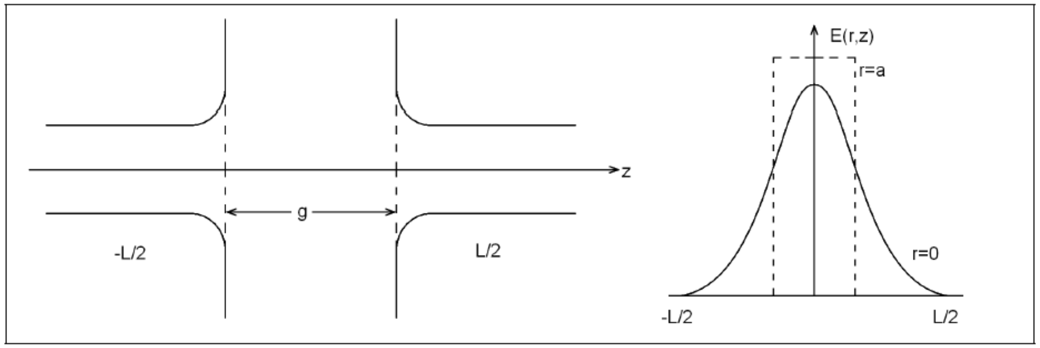
Electrostatic Quadrupole Transport channel



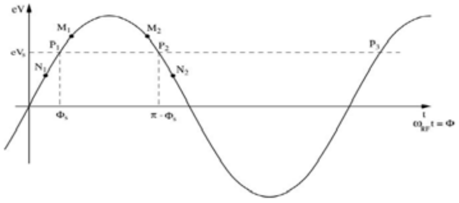
Acceleration is added as perturbation



Linear accelerators



$$E_z = E_z(t) = E_0 \cos(\omega t + \phi)$$



At t=0, the particle is in the center of the cavity, and has a phase phi relative to the maximum field

$$\Delta W = q\int_{-L/2}^{L/2} E_z dz = qE_0 \int_{-L/2}^{L/2} \cos(\omega t(z) + \phi) dz$$

$$\omega t \approx \omega \frac{z}{v} = \frac{2\pi z}{\beta\lambda} = kz$$

$$\Delta W = qE_0 \int_{-L/2}^{L/2} (\cos \omega t \cos \phi - \sin \omega t \sin \phi) dz$$

$$\Delta W = qE_0 \cos\phi \int_{-L/2}^{L/2} \cos\left(\frac{2\pi z}{\beta\lambda}\right) dz - qE_0 \sin\phi \cancel{\int_{-L/2}^{L/2} \sin\left(\frac{2\pi z}{\beta\lambda}\right)} dz$$

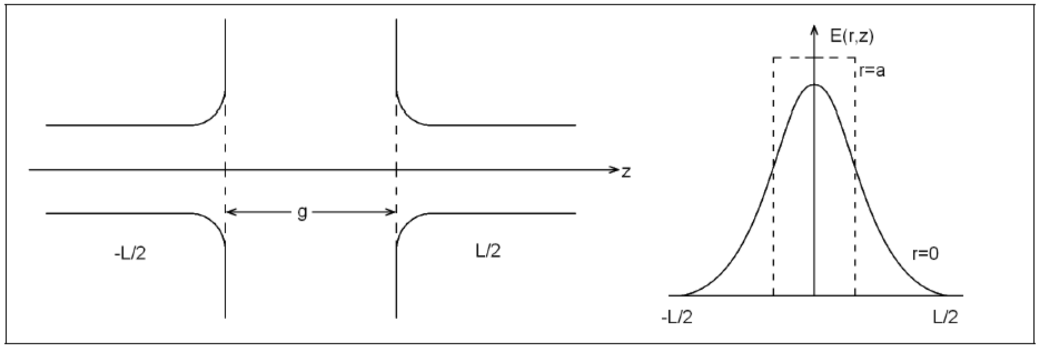
$$\Delta W = qE_0 \cos\phi \frac{\beta\lambda}{2\pi}\left[\sin \frac{2\pi z}{\beta\lambda}\right]_{-L/2}^{L/2}$$

Energy gain depends linear on q

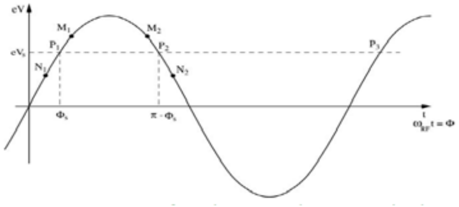
$$\boxed{\begin{gathered}\Delta W = qV_0 T \cos\phi \\ T = \frac{\sin(\pi L / \beta\lambda)}{\pi L / \beta\lambda}\end{gathered}}$$

Transit Time Factor TTF

Linear accelerators



$$E_z = E_z(t) = E_0 \cos(\omega t + \phi)$$



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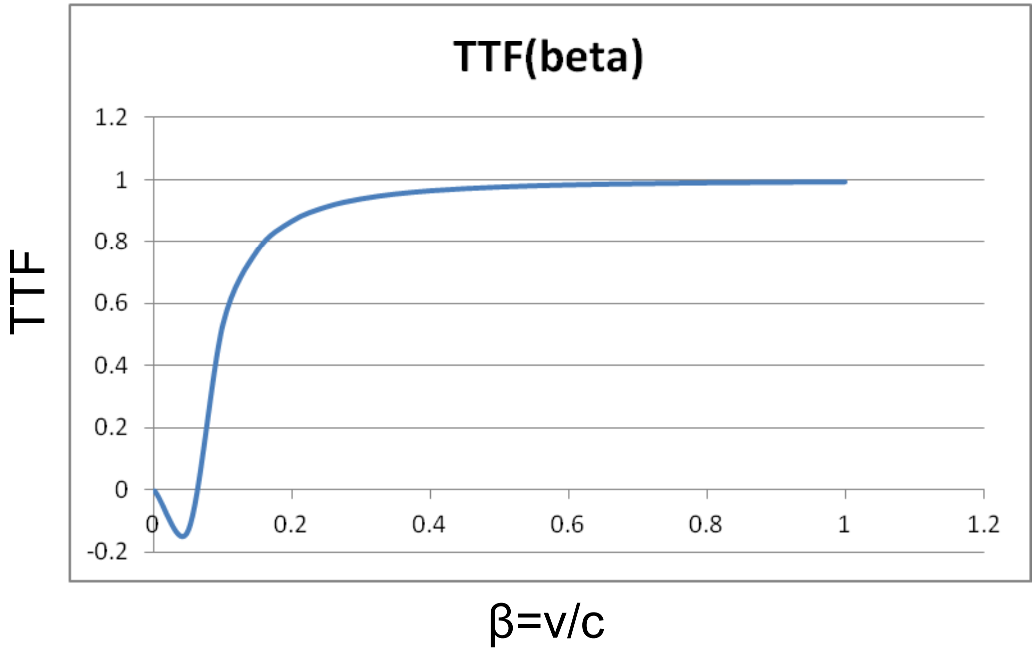


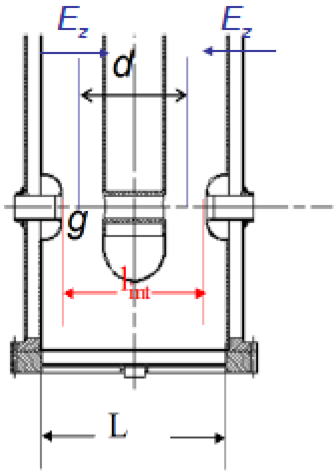
$$\Delta W = qV_0 T \cos\phi$$

$$T = \frac{\sin(\pi L / \beta\lambda)}{\pi L / \beta\lambda}$$

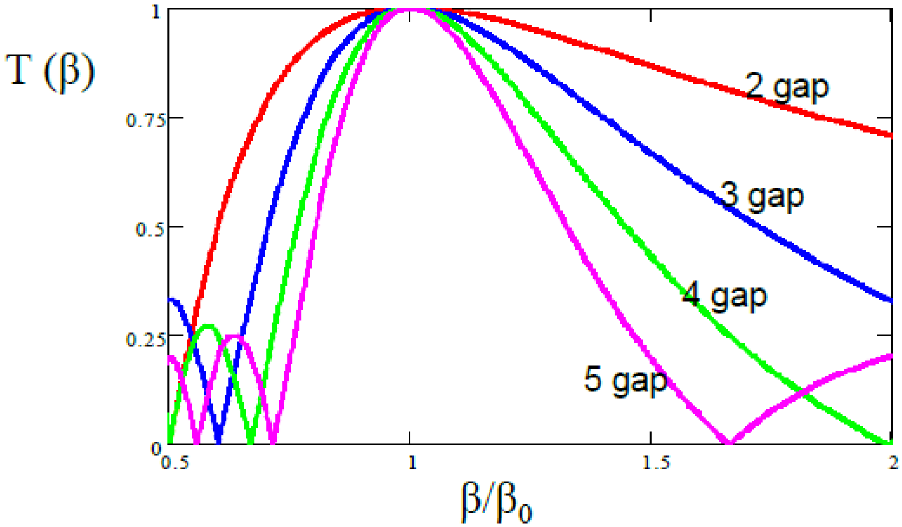
Transit Time Factor TTF

TTF for 2 or more gaps

naked β=0.041 SC cavity



Normalized transit time factor curves vs. normalized velocity, for cavities with different number of gap



$$d \approx \frac{\beta\lambda}{2}$$

$$\Delta W = qE_0TL\cos\phi$$

$$T = \frac{sin\pi g/\beta\lambda}{\pi g/\beta\lambda}\sin\frac{\pi\beta_s}{2\beta}$$

For more than 2 equal gaps in π mode, the formulas change only in the 2nd order term

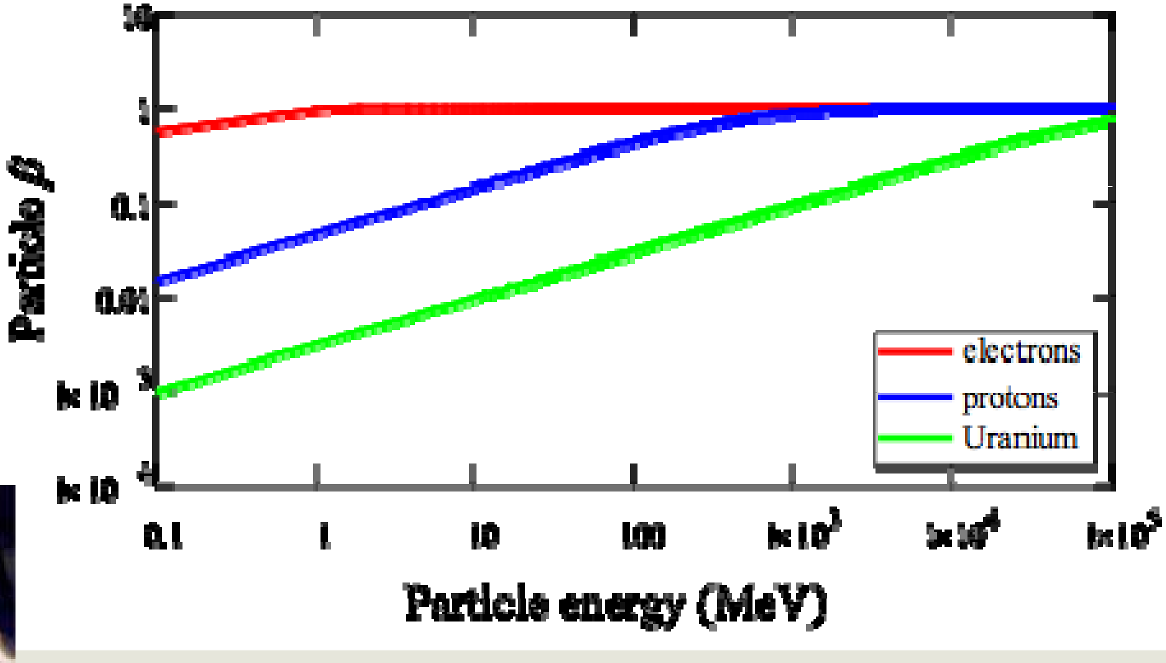
- SC linac have typically individually phased cavities with a few gaps
- The more gaps, the narrower the velocity acceptance
- The larger the number of gaps the larger the energy gain, but longer cavity structures are needed
- The individual accelerator arrangement will depend upon the evolution of the particle velocity along the system

Different Arrangements for Different Particles

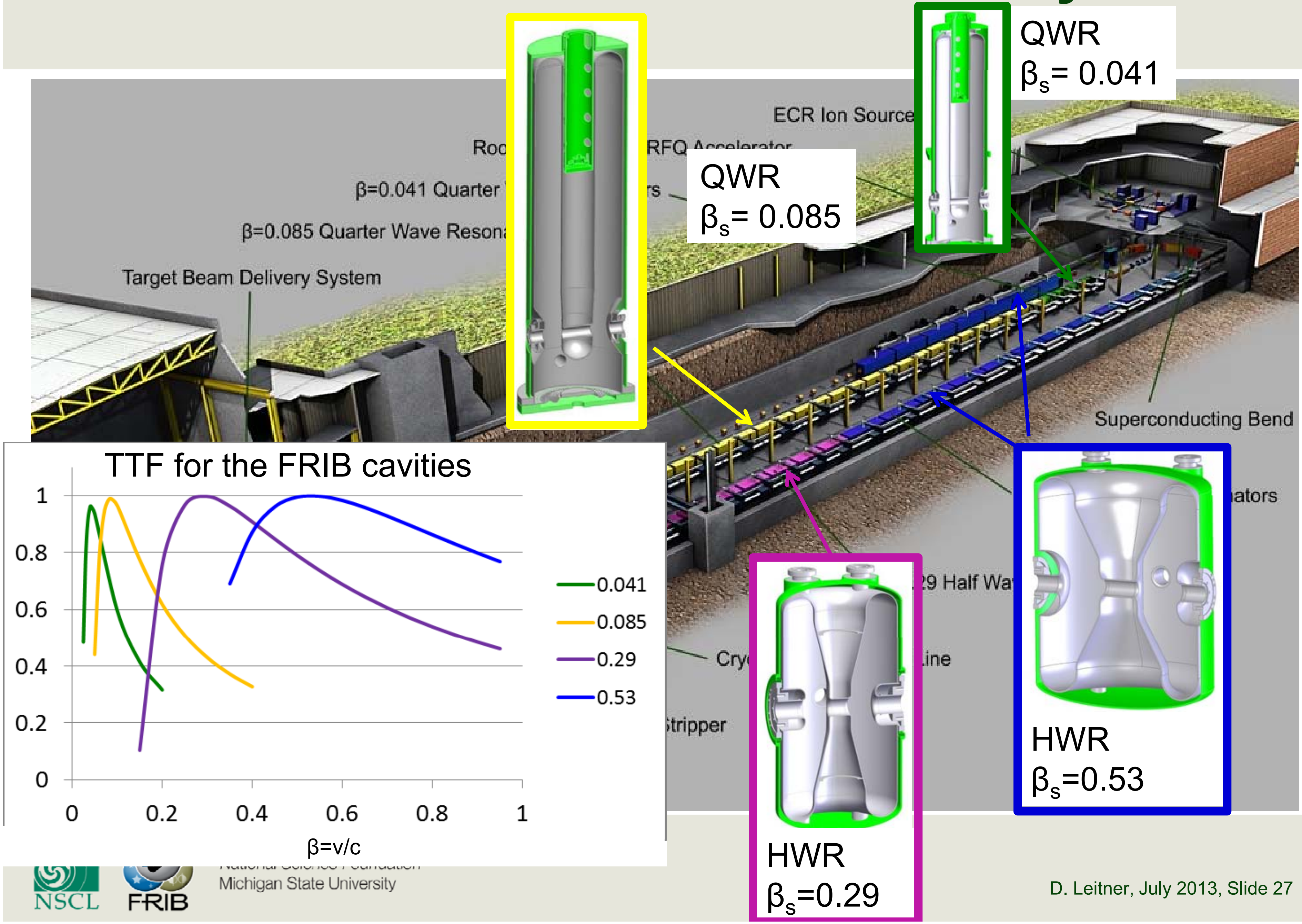
- Accelerating system used will depend upon the evolution of the particle velocity along the system
  - electrons reach a constant velocity at relatively low energy
    - thus, can use one type of resonator
  - heavy particles reach a constant velocity only at very high energy
    - thus, may need different types of resonators, optimized for different velocities

Particles rest mass:

•*e*	*0.511 MeV*
•*p*	*938 MeV*
•^{239}U	*~220000 MeV*



FRIB driver linac accelerates heavy ions



Normal vs. Superconducting Cavities

DTL tank - Fermilab

Normal conducting

Cu cavity @ 300K

$R_s \sim 10^{-3}\ \Omega$

$Q \sim 10^4$

LNL PIAVE 80 MHz, β =0.047 QWR

Superconducting

Nb Cavity @ 4.2K

$R_s \sim 10^{-8}\ \Omega$

$Q \sim 10^9$

Superconductivity allows

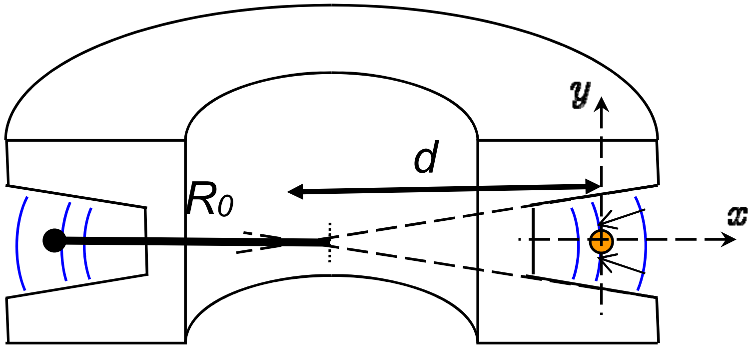
- ***great reduction of rf power consumption even considering cryogenics (1W at 4.2K ~ 300W at 300K)***
- ***the use of short cavities with wide velocity acceptance***

A. Facco –FRIB and INFN   SRF Low-beta Accelerating Cavities for FRIB   MSU 4/10/2011

Keeping Focused
Weak Focusing

- Without focusing the particles do not experience any restoring force in y-direction, spiral out of control
- Early circular accelerators used what is now called “weak focusing
- The magnetic field decreases with radius and thus provide restoring force
- Classic cyclotron were build this way, but $$\omega = \frac{e \cdot Q \cdot B}{M}$$

Frequency changes!



$$B = B_0 \left(\frac{R_0}{r} \right)^n$$

n is determined by adjusting the opening angle between the poles

$$n \approx \frac{R_0}{d}$$

must have
$0 \leq n \leq 1$
for stability

$d = \infty,\ n = 0$

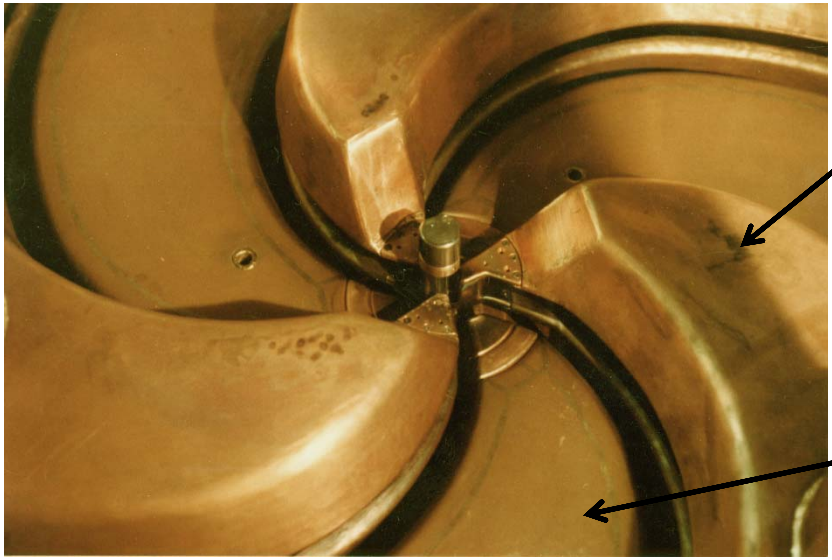
$d = R_0,\ n = 1$

Path to higher energies
strong focusing (alternate gradient focusing)

- In the classic cyclotron focusing requires $\frac{\partial B}{\partial R} < 0$

  but isochronicity requires $\frac{\partial B}{\partial R} > 0$ in order to maintain $\omega = \frac{e \cdot Q \cdot B}{M}$

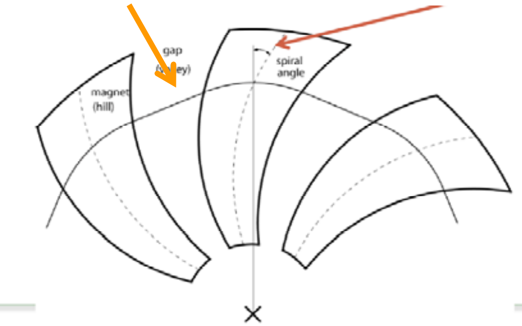
- Sector focusing cyclotron : B increases with R and has spiral magnet boundaries, azimuthally varying B-field



RF Cavities

Magnet Valleys

Magnet Hills

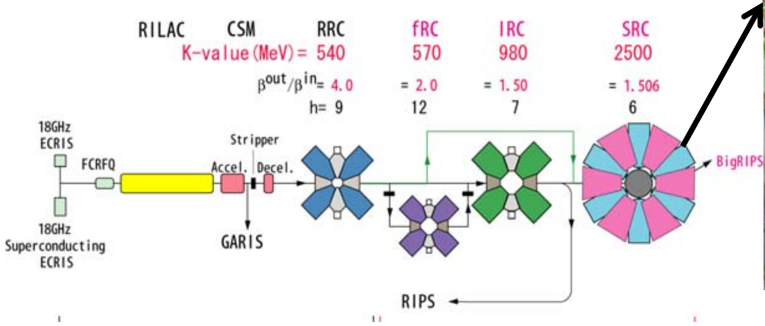


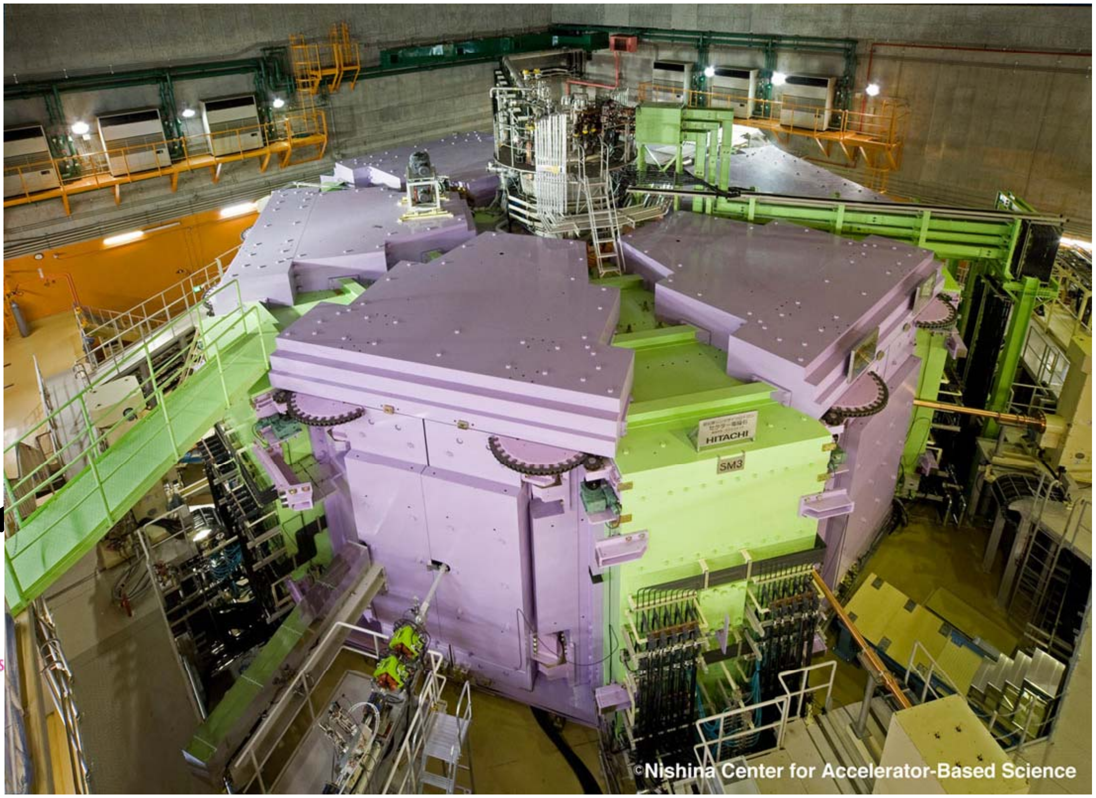
Position of the Sector Magnets

NSCL CCF K1200 superconducting sector focusing cyclotron Center Region, MSU

RIKEN RI Beam Factory

- RIKEN SRC, K2500, biggest SC cyclotron: 440 MeV/nucleon for light ions and 350 MeV/nucleon for heavy ions (Uranium)
- 4 RF cavities (Dee)
- 800 tons/SC sector magnet
- 8300 tons total weight
- Last acceleration station for the RIBF RIKEN.





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Linear Restoring Forces [1]

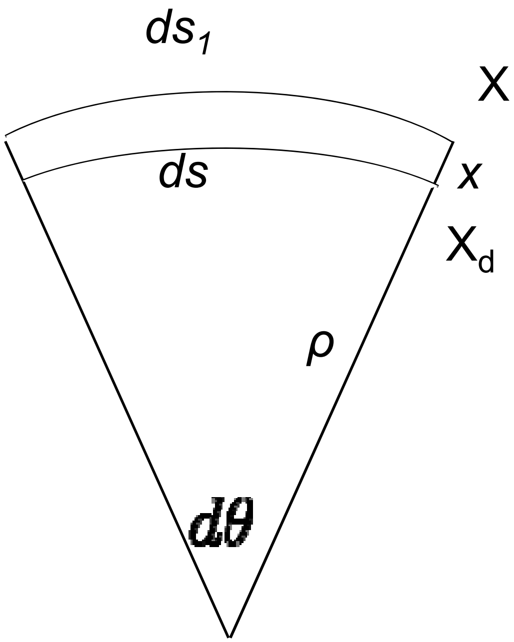
- Wish to look at motion “near” the ideal trajectory of the accelerator system
- Assume linear guide fields
- $B_x(0,0)=0$, $B_y(0,0)=B_0$, first order expansion of fields

$$\frac{dx}{dt} = \frac{dx}{ds}\cdot\frac{ds}{dt} = x' \cdot v_s$$

$$ds = d\theta \cdot \rho$$

$$ds_1 = d\theta \cdot (\rho + x)$$

$$\frac{ds_1}{ds} = \frac{d\theta \cdot (\rho + x)}{d\theta \cdot \rho} = 1 + \frac{x}{\rho}$$



$$B_y = B_0 + \frac{dB_y(s)}{dx}\cdot x = B_0 + B' \cdot x$$

$$B_x = \frac{dB_x(s)}{dy}\cdot y = B' \cdot y$$

$$\frac{dB_x(s)}{dy} = \frac{dB_y(s)}{dx} = B', \quad \vec{\nabla} \times \vec{B} = 0$$

$$X = (X_d + x)$$

X_d…… design trajectory
X……...actual trajectory

Linear Restoring Forces [2]

- Wish to look at motion “near” the ideal trajectory of the accelerator system
- Assume linear guide fields
- $B_x(0,0)=0$, $B_y(0,0)=B_{0,}$ first order expansion of fields

- Look at radial motion:

$$\frac{dx}{dt} = \frac{dx}{ds}\cdot\frac{ds}{dt} = x'\cdot v_s$$

$$\frac{d^2x}{dt^2} = x'\cdot v_s^2$$

$$\frac{v_{s1}}{v_s} = 1+\frac{x}{\rho}$$

$$B_0\rho = \frac{p}{e}$$

$$\begin{aligned}
\gamma m\frac{d^2(X_d)}{dt^2} &= -ev_sB_0 && [1]\\
\gamma m\frac{d^2(X_d+x)}{dt^2} &= -ev_{s1}B_y(X) && [2]\\
\gamma m(X_d''+x'')v_s^2 &= -ev_{s1}B_y(X)\\
\gamma m v_s x'' &= -e\frac{v_{s1}}{v_s}B_y + eB_0\\
\gamma m v_s x'' &= -e\left[B_y\left(1+\frac{x}{\rho}\right) - B_0\right]\\
x'' &= -\frac{e}{p}\left[(B_y - B_0) + B_y\frac{x}{\rho}\right]\\
&\approx -\frac{1}{B\rho}\left[B'x + B_0\frac{x}{\rho}\right]
\end{aligned}$$

Hill's Equation

- Now, for vertical motion:

$B_y = B_0 + B'x$
$B_x = B'y$

$$\begin{aligned}
\gamma m \frac{d^2(Y_d)}{dt^2} &= 0 \\
\gamma m \frac{d^2(Y_d + y)}{dt^2} &= e v_{s1} B_x(Y) \\
\gamma m v_s^2 y'' &= e v_{s1} B_x(Y) \\
\gamma m v_s y'' &= e \frac{v_{s1}}{v_s} B_x \\
\gamma m v_s y'' &= e B_x \left(1 + \frac{x}{\rho}\right) \\
y'' &= \frac{e}{p} \left[B_x (1 + \frac{x}{\rho}) \right] \\
&\approx \left(\frac{B'}{B\rho}\right) y
\end{aligned}$$

$B_0 \rho = \frac{p}{e}$

- So we have,
  - to lowest order,

$$\boxed{\begin{aligned}
x'' + \left(\frac{B'}{B\rho} + \frac{1}{\rho^2}\right) x &= 0 \\
y'' - \left(\frac{B'}{B\rho}\right) y &= 0
\end{aligned}}$$

Hill' s Equation

General Form: $\longrightarrow$ $x'' + K(s)x = 0$

- As accelerate, scale *K* with momentum; becomes purely geometrical

Piecewise Method of Solution

- Hill's Equation: $x'' + K(s)x = 0$
  - Though *K*(*s*) changes along the design trajectory, it is typically constant, in a piecewise fashion, through individual elements (drift, sector mag, quad, edge, ...)

- K = 0: drift $x'' = 0 \longrightarrow x(s) = x_0 + x_0' s$

- K > 0: Quad, Gradient Magnet, edge, ... $x(s) = x_0 \cos(\sqrt{K}\ s) + \frac{x_0'}{\sqrt{K}} \sin(\sqrt{K}\ s)$

- K < 0: $x(s) = x_0 \cosh(\sqrt{|K|}s) + \frac{x_0'}{\sqrt{|K|}} \sinh(\sqrt{|K|}s)$

Here, *x* refers to horizontal or vertical motion, with relevant value of *K*

Piecewise Method -- Matrix Formalism

- Write solution to each piece in matrix form
  - for each, assume *K* = const. from *s*=0 to *s*=*L*

- *K* = 0: drift

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

- *K* > 0: Quad, Gradient Magnet, edge, ...

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

- *K* < 0:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cosh(\sqrt{|K|}L) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}L) \\ \sqrt{|K|}\sinh(\sqrt{|K|}L) & \cosh(\sqrt{|K|}L) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

“Thin Lens” Quadrupole

- If the quadrupole magnet is short enough, particle’s offset through the quad does not change by much, but the slope of the trajectory does -- acts like a “thin lens” in geometrical optics
- Take limit as *L* --> 0, while *KL* remains finite

$$\begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}$$

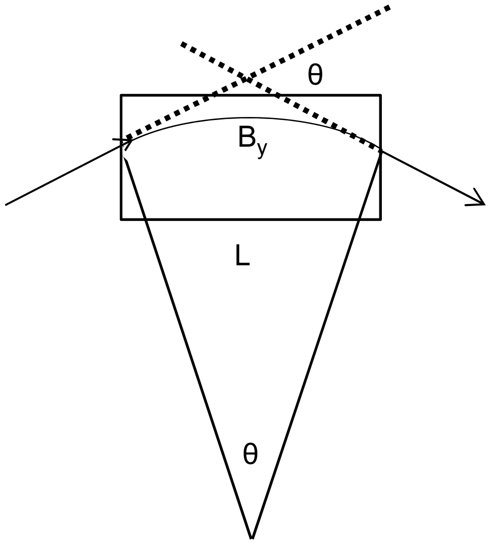
  - (similarly, for defocusing quadrupole)

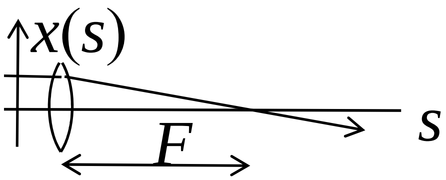
- Valid approx., if *F* >> *L*

Change of the transverse directory

$$\Delta x' = \theta = -\frac{L}{\rho} = -L\frac{eB_y}{p} = -\frac{B'L}{B\rho}\cdot x$$

$$KL = \frac{B'L}{B\rho} = \frac{1}{F}$$



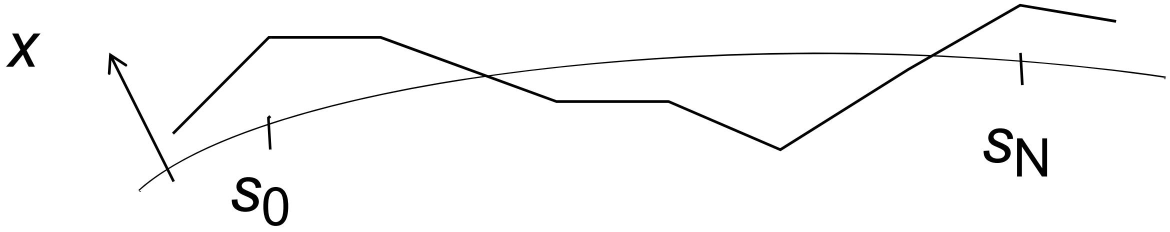


Piecewise Method -- Matrix Formalism

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in} = \begin{pmatrix} 1+\frac{L}{f} & L \\ \frac{L}{-f^2} & 1-\frac{L}{f} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

- Arbitrary trajectory, relative to the design trajectory, can be computed *via* matrix multiplication

$$\begin{pmatrix} x_N \\ x'_N \end{pmatrix} = M_N M_{N-1} \cdots M_2 M_1 \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$



→ Matrix formalism (using beam envelope) is the basis of now-standard computer codes for analysis: TRANSPORT, MAD, DIMAD, TRACE, TRACE3D, COSY

Stability Criterion

- For single pass through a short system of elements, above may be all we need to know to describe the system. But, suppose the "system" is very long and made of many repetitions of the same type of elements (or, perhaps the "repetition" is a complete circular accelerator, for instance) -- how to show that the motion is stable for many (infinite?) passages?
- Look at matrix describing motion for one passage:

$$M = M_N M_{N-1} \cdots M_2 M_1$$

- We want:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_k = M^k \begin{pmatrix} x \\ x' \end{pmatrix}_0 \text{ finite as } k \to \infty \text{ for arbitrary } \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Stability Criterion

V = eigenvector
λ = eigenvalue

$$X_k = M^k X_0 = M^k(AV_1 + BV_2) = A\lambda_1^k V_1 + B\lambda_2^k V_2$$

$$\det M = 1 = \lambda_1\lambda_2 \rightarrow \lambda_2 = 1/\lambda_1 \rightarrow \lambda = e^{\pm i\mu}$$

If μ is imaginary, then repeated application of *M* gives exponential growth; if μ real, gives oscillatory solutions…

$$\text{characteristic equation: } \det(M - \lambda I) = 0$$

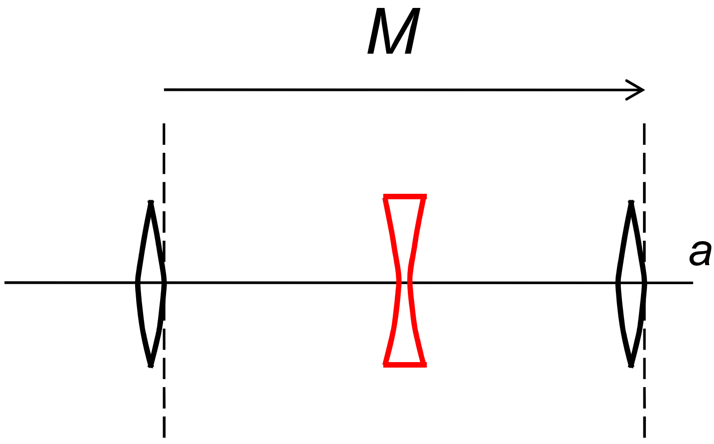
$$\text{if } M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } (a-\lambda)(d-\lambda) - bc = 0$$

$$\rightarrow \quad \begin{aligned} &\lambda^2 - (a+d)\lambda + (ad - bc) = 0 \\ &\lambda^2 - trM\lambda + 1 = 0 \\ &\lambda + 1/\lambda = trM \\ &e^{i\mu} + e^{-i\mu} = 2\cos\mu = trM \end{aligned}$$

So, μ real (stability) $\rightarrow |trM| < 2$

Example: Application to FODO system

$$
\begin{aligned}
M &= \begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/F & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & L \\ -1/F & 1 - L/F \end{pmatrix} \begin{pmatrix} 1 & L \\ 1/F & 1 + L/F \end{pmatrix} \\
&= \begin{pmatrix} 1 + L/F & 2L + L^2/F \\ -L/F^2 & 1 - L/F - L^2/F^2 \end{pmatrix}
\end{aligned}
$$



So, $trM = 2 - L^2/F^2$ and thus, for stability,

and repeat…

$$-2 < 2 - L^2/F^2 < 2$$

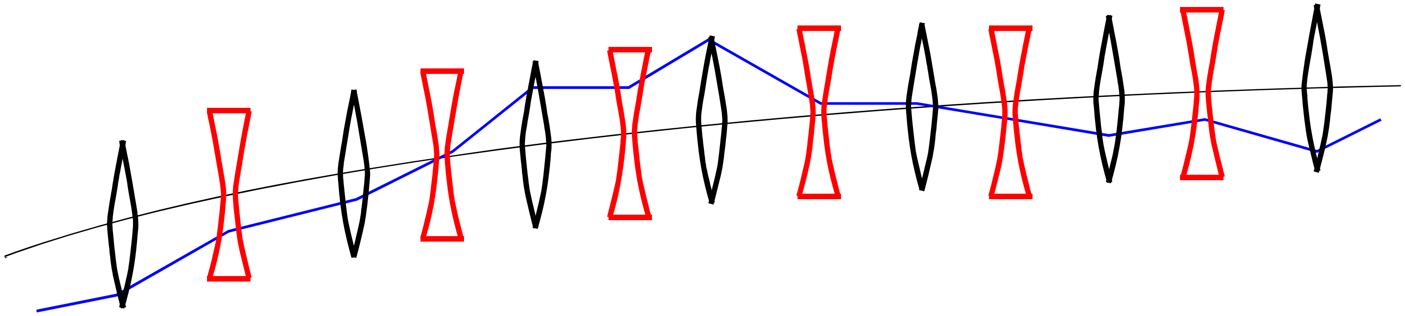
$$-4 < -L^2/F^2 < 0$$

$$F > L/2$$

Motion is stable provided that the lens spacing is less than twice the focal length

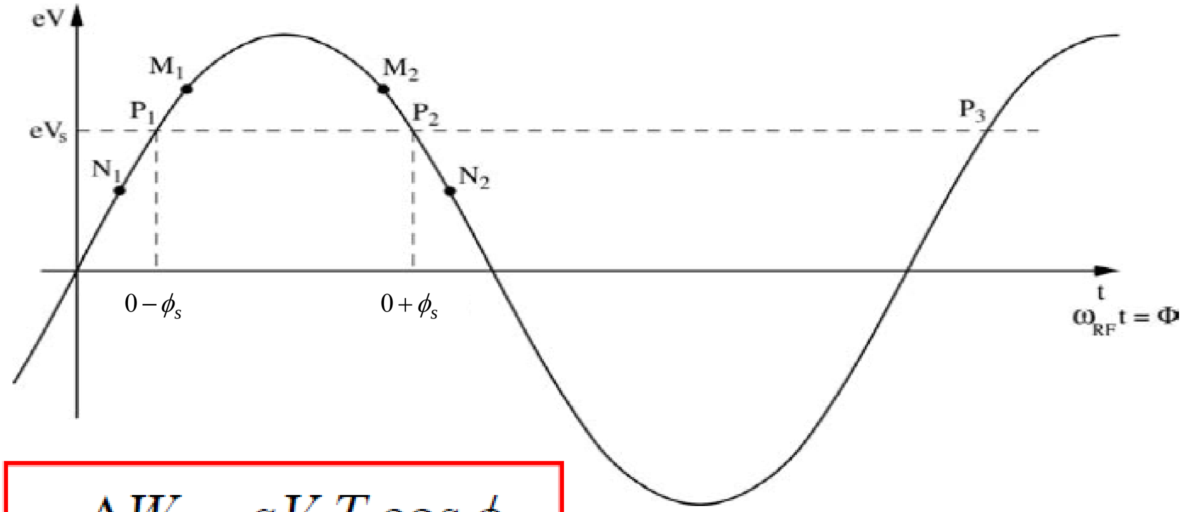
Can now make LARGE accelerators!

- Since the lens spacing can be made arbitrarily short, with corresponding focusing field strengths, then in principal can make beam transport systems (and linacs and synchrotrons, for instance) of arbitrary size



- Instrumental in paving the way for very large accelerators, both linacs and especially synchrotrons, where the bending and focusing functions can be separated into distinct magnet types

What about longitudinal stability?



$$\Delta W = qV_0 T\cos\phi$$

$$T = \frac{\sin(\pi L/\beta\lambda)}{\pi L/\beta\lambda}$$

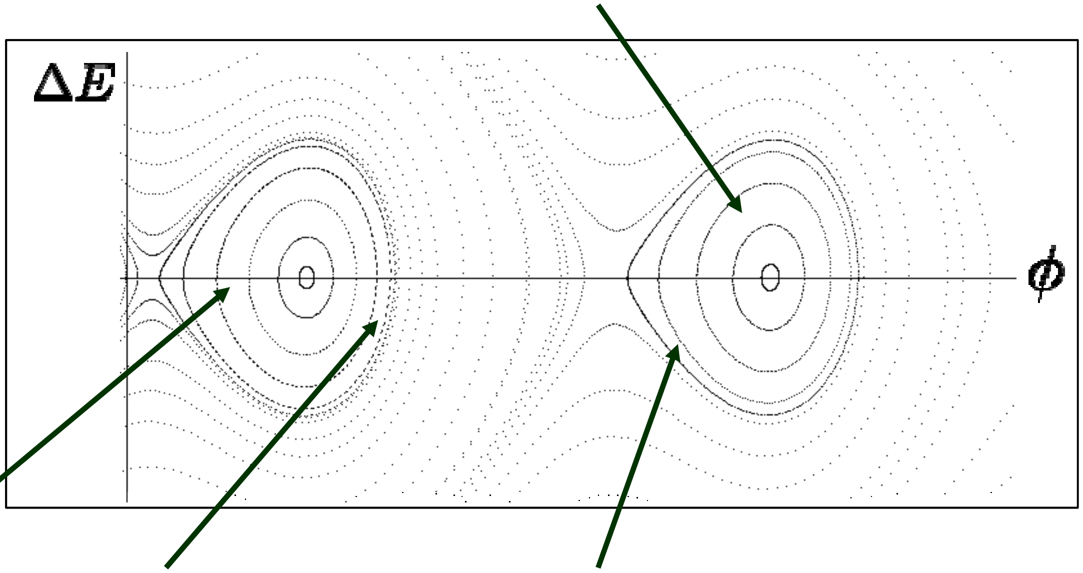
is the energy gain in one gap for the particle to reach the next gap with the same RF phase: P1 ,P2, …… are fixed points in phase. P1 and P2 would allow for the same acceleration, but operation point P2 would be unstable

N1 arrives early at the cavity (get less acceleration), M1 will arrive late at the cavity (get more acceleration, P1 is the synchronous particle. So M1 and N1 will move towards P1(stable).
(M2& N2will go away from P2(unstable))

Longitudinal beam dynamics

If the ideal particle has energy E_s and arrives at phase φ_s ...

particles arriving nearby in phase, and nearby in energy $E = E_s + \Delta E$ will oscillate about this ideal condition, particles are transported in bunches



"Buckets"

Stable Phase Space regions

Separatrix

Acknowledgements and additional info

- Thanks to Mike Syphers, Brad Sherrill, and the FRIB team for sharing class slides!
- Slides were also used from the MSU course “Accelerator Physics of FRIB -- PHY905”, 2011
- Want to learn more?
  - US Particle Accelerator School! Held twice yearly at venues across the country; offers graduate credit at major universities for courses in accelerator physics and technology (lecture notes!)

Current Program

USPAS sponsored by the University of Texas at Austin January 16-27, 2012 held in Austin, Texas

View program details

Next Program

USPAS sponsored by Michigan State University June 18-29, 2012 held in Grand Rapids, Michigan

Travel grants for the International Particle Accelerator Conference series are available to student applicants. Grants include reimbursement of the student registration fee and funds toward travel and accommodation expenses. The web site containing information on the IPAC'12 Student Grant Program and Poster Prizes will be available in mid-October, 2011.

2011 USPAS Achievement Prize Recipients Announced

USPAS in the News

- US-CERN-Japan-Russia International Accelerator School
- USPAS Prize for Achievement in Accelerator Physics and Technology
- Internships
- Job Opportunities

Need the help of an Academic Advisor?

Would you like advice as to which USPAS course to take? Contact Bill Barletta, the USPAS Director at uspas@fnal.gov

Foreign Nationals

The Fermilab Visa Office will help USPAS participants with their visa application process. Contact uspas@fnal.gov for more information.

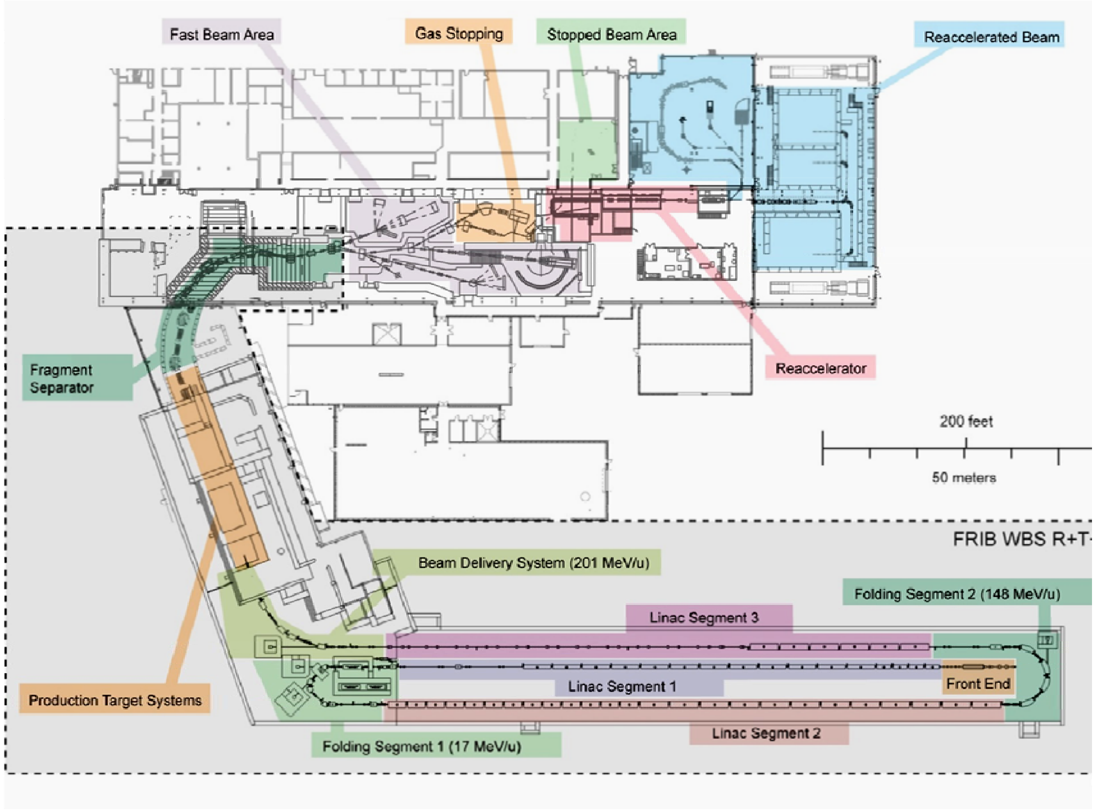
http://uspas.fnal.gov

Interested in joining the FRIB team? http://frib.msu.edu/ !

Interested in becoming involved in the evolving user community? http://fribusers.org/

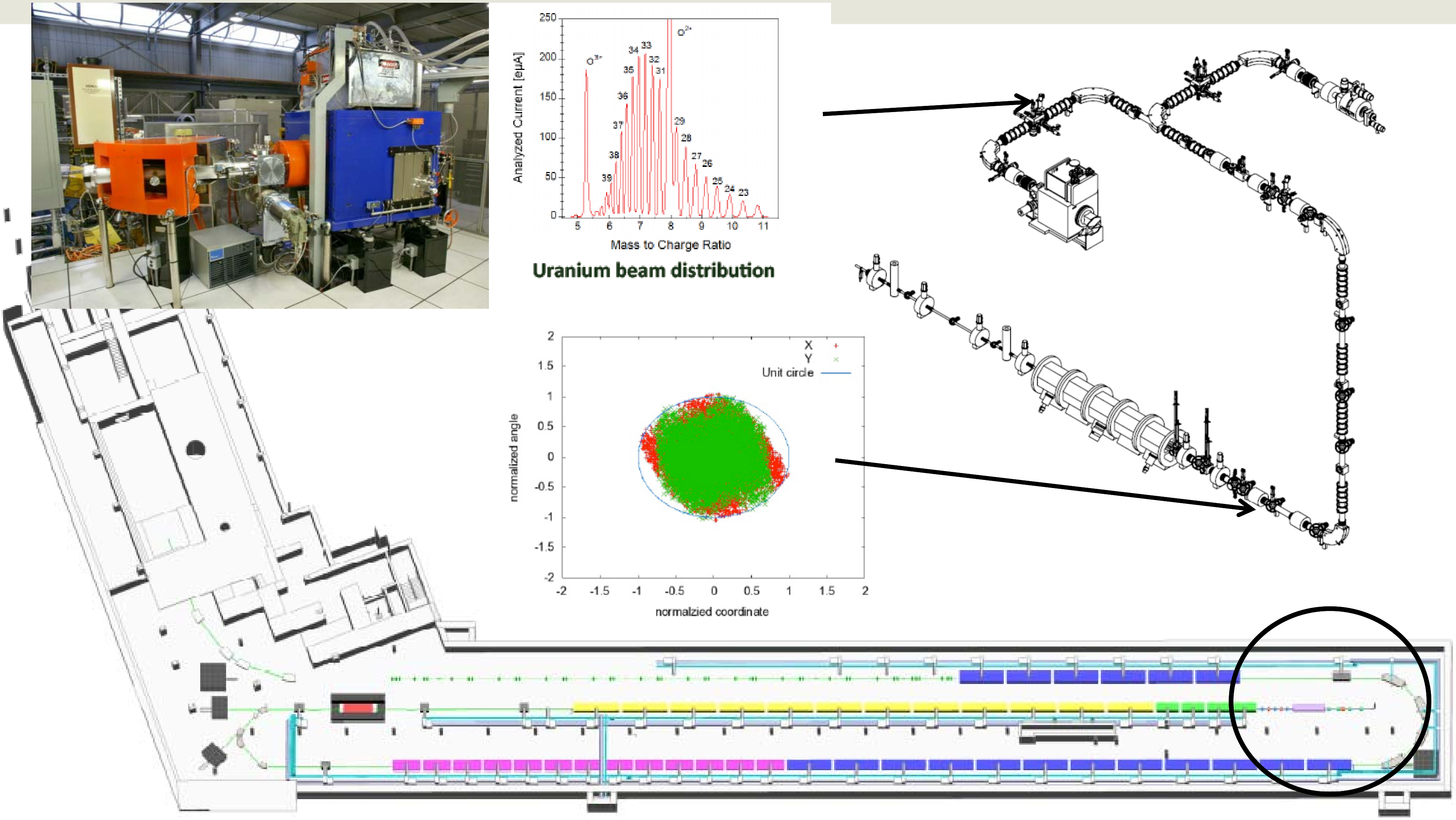
Appendix

Finally a quick tour an accelerator under construction



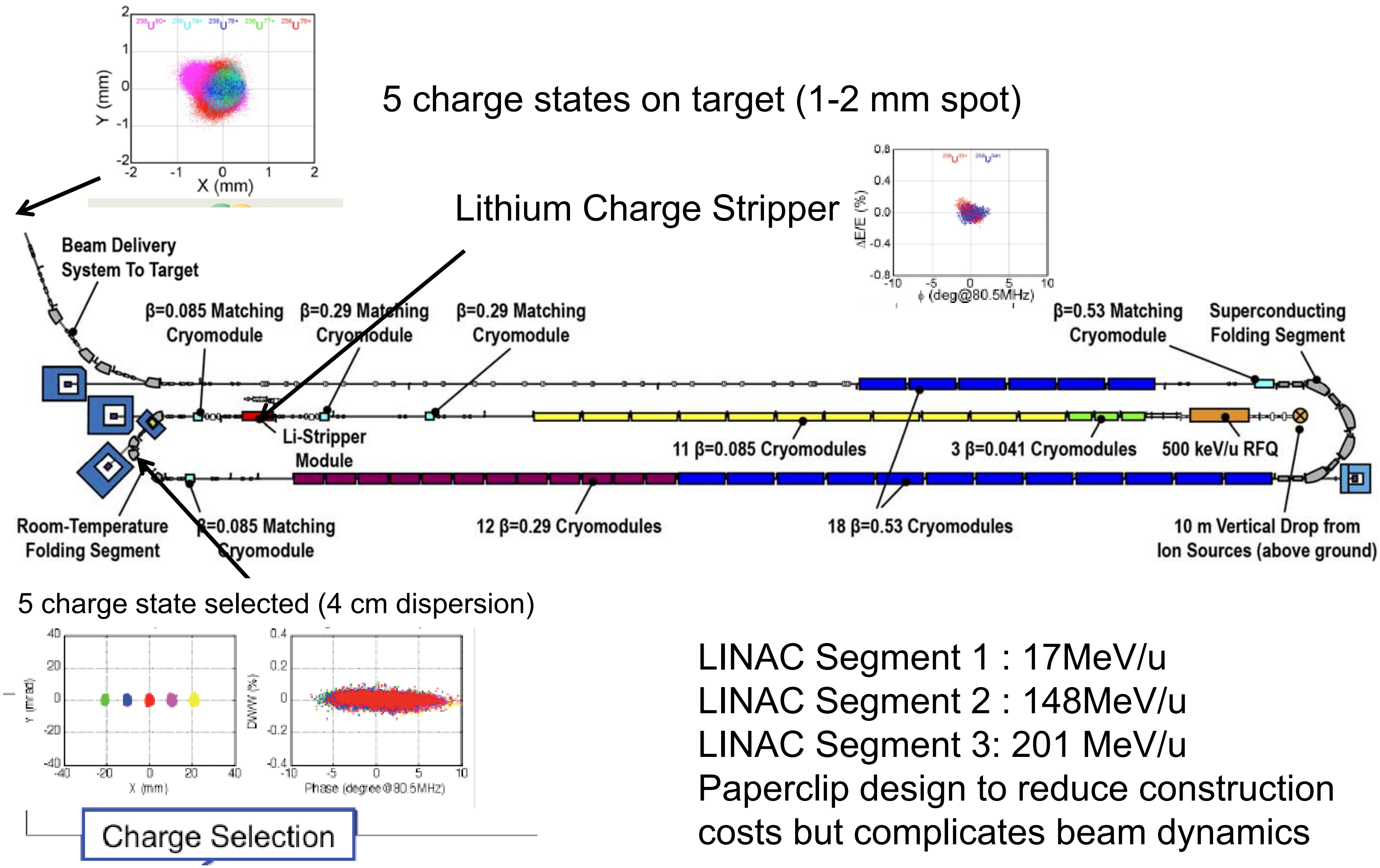
- High power CW superconducting driver linac (400 kW beam power)
  - Advanced ECR ion sources + multiple charge state acceleration
  - 5 charge states in the LINAC
  - Accelerates beams from p (600 MeV/u) to U (200 MeV/u)
- High Power Target
- Fast Beams (about 150MeV/u), direct from the fragment separator
- Stopped Beams
- Efficient post acceleration from 1+ and n+ ion sources
  - 3MeV/u Astrophysics
  - 12MeV/u Nuclear Physics

FRIB front end produces and pre-accelerates the primary beam



Uranium beam distribution

Double folded heavy ion driver linac



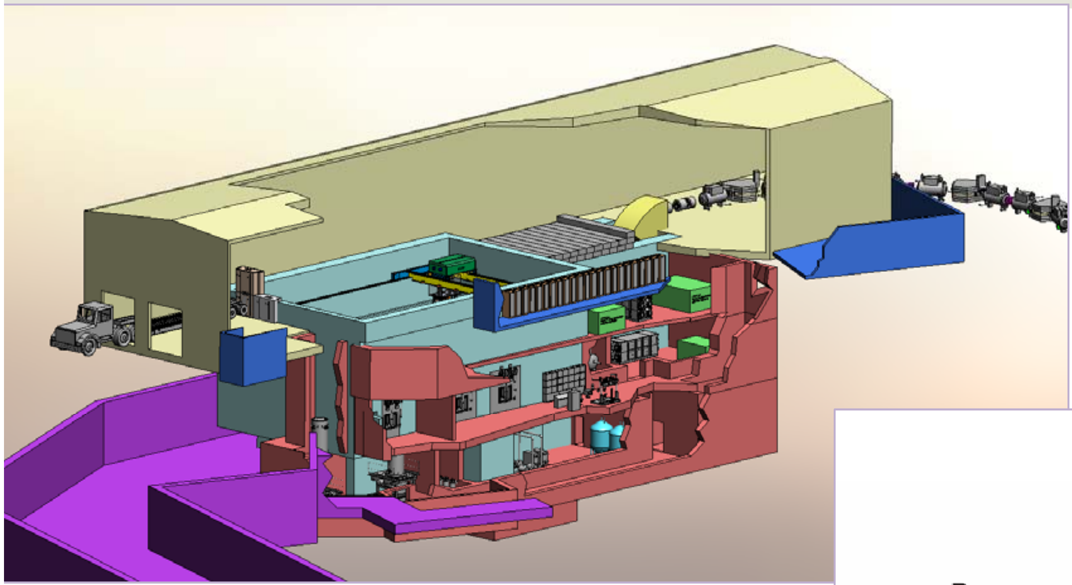
5 charge states on target (1-2 mm spot)

Lithium Charge Stripper

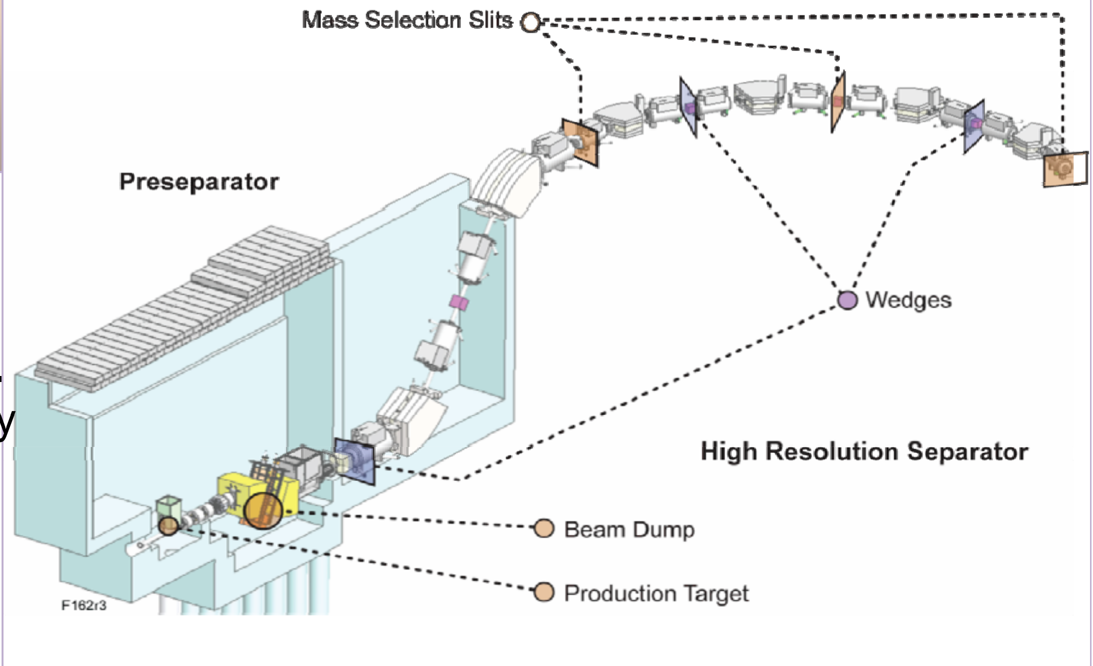
5 charge state selected (4 cm dispersion)

LINAC Segment 1 : 17MeV/u
LINAC Segment 2 : 148MeV/u
LINAC Segment 3: 201 MeV/u
Paperclip design to reduce construction costs but complicates beam dynamics

Production target and fragment separator



Prompt radiation at 30 cm distance from target (neutrons, protons): 100 Mrem/h (10^8 rem/h)
Total weight of steel shielding in target hot cell: 3900 tons

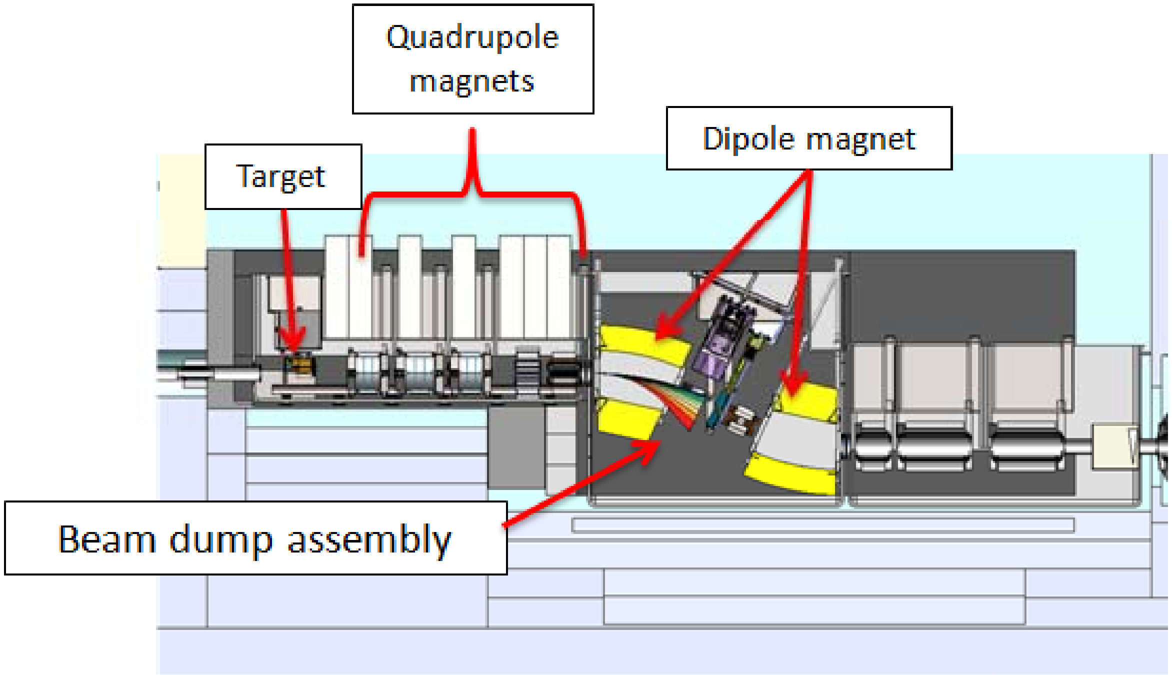


3 separation stages provide high purity.

400 kW beam power **^{238}U is 5×10^{13} ions/s.** Many experiments will be performed at very rare isotopes with beam rates of less than **10^{-3}/s.** This means a suppression of unwanted ions by factors of **10^{17}** or higher are required.

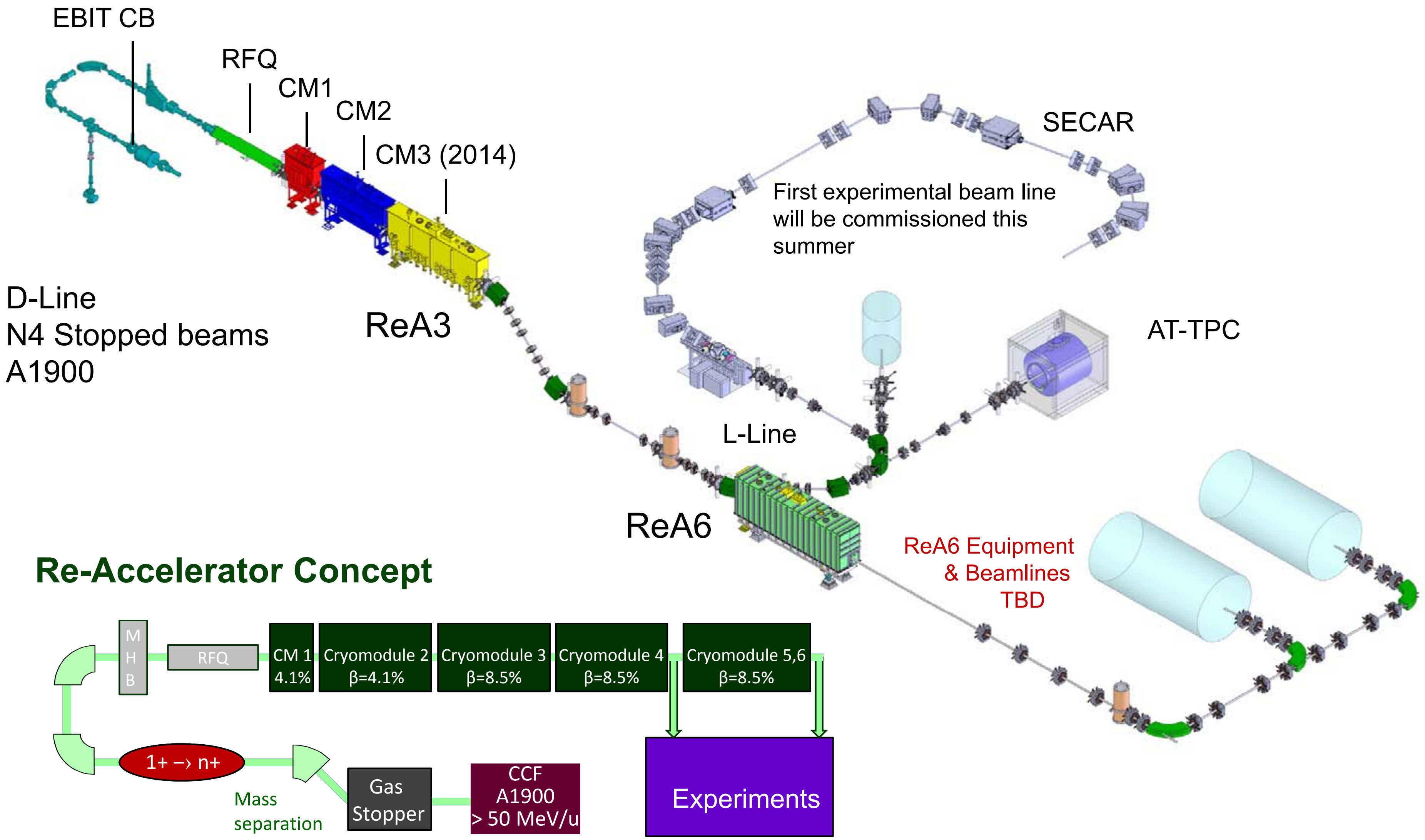
Production target

- Power density in target ~ 20 - 60 MW/cm^3
  - Caused by 1 mm beam spot on target and high energy loss of heavy ions in matter (up to 100 kW deposited in target)
  - Mitigated by multiple fast spinning (5000 rpm) graphite disks as targets
- Target lifetime at full power expected to be 2 weeks
  - Target needs to be replaced with remote-handling equipment



Prompt radiation at 30 cm distance from target (neutrons, protons): 100 Mrem/h (10^8 rem/h)

Re-Accelerator facility is a state-of-the-art RIB post-accelerator and the first coupled to a fragmentation facility

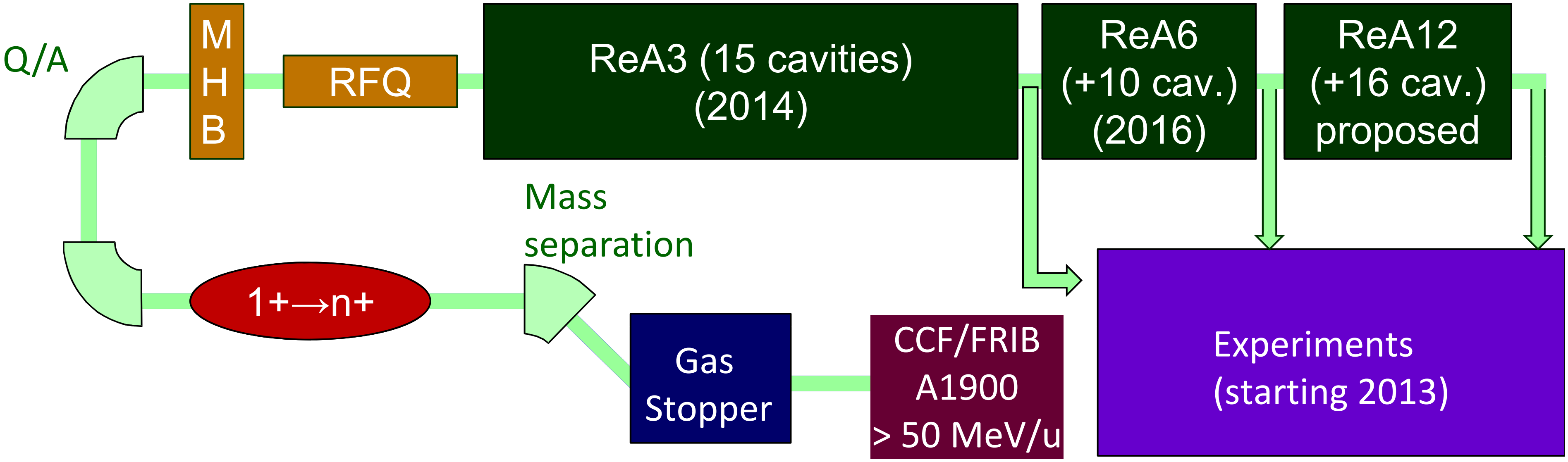


First experimental beam line will be commissioned this summer

Re-Accelerator Concept

Leitner, State-of –the-art RIB post accelerators, NIMB, accepted (2013)

ReAccelerator is a state-of-the-art compact SC linac wide energy range



Q/A

MHB

RFQ

ReA3 (15 cavities) (2014)

ReA6 (+10 cav.) (2016)

ReA12 (+16 cav.) proposed

Mass separation

1+→n+

Gas Stopper

CCF/FRIB A1900 > 50 MeV/u

Experiments (starting 2013)

Requirements		
Energy variability while preserving beam properties	300keV/u – 12MeV/u	Independently phased and tuned SC RF cavities and rebunchers
Ion efficiency for all elements	> 20 %	EBIT charge breeder + high efficiency LINAC
Beam rate capabilities	10^8 ions/sec	Hybrid EBIS/T charge breeder
High beam purity		A1900, EBIT CB, Q/A
Low energy spread, short pulse length	1keV/u, 1nsec	Multiharmonic external buncher and tight phase control in SRF linac

2020