

Ab Initio Nuclear Structure Theory

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National Nuclear Physics Summer School
SUNY Stony Brook
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3. Recent results in light nuclei with ab initio no core methods

- Overview of ab initio approaches (GFMC, CC, NCSM, NCFC)
- Applications to light nuclei – importance of NNN
- Applications to nuclear reactions - deferred due to time limits
- Applications to non-perturbative solution of quantum field theory
- Conclusions and outlook

The Nuclear Many-Body Problem

The many-body Schroedinger equation for bound states consists of 2^A coupled second-order differential equations in $3A$ coordinates using strong (NN & NNN) and electromagnetic interactions.

Successful *ab initio* quantum many-body approaches ($A > 6$)

Stochastic approach in coordinate space
Greens Function Monte Carlo (**GFMC**)

Hamiltonian matrix in basis function space
No Core Configuration Interaction (**NCSM/NCFC**)

Cluster hierarchy in basis function space
Coupled Cluster (**CC**)

Lattice Nuclear Chiral EFT, MB Greens Function,
MB Perturbation Theory, . . . approaches

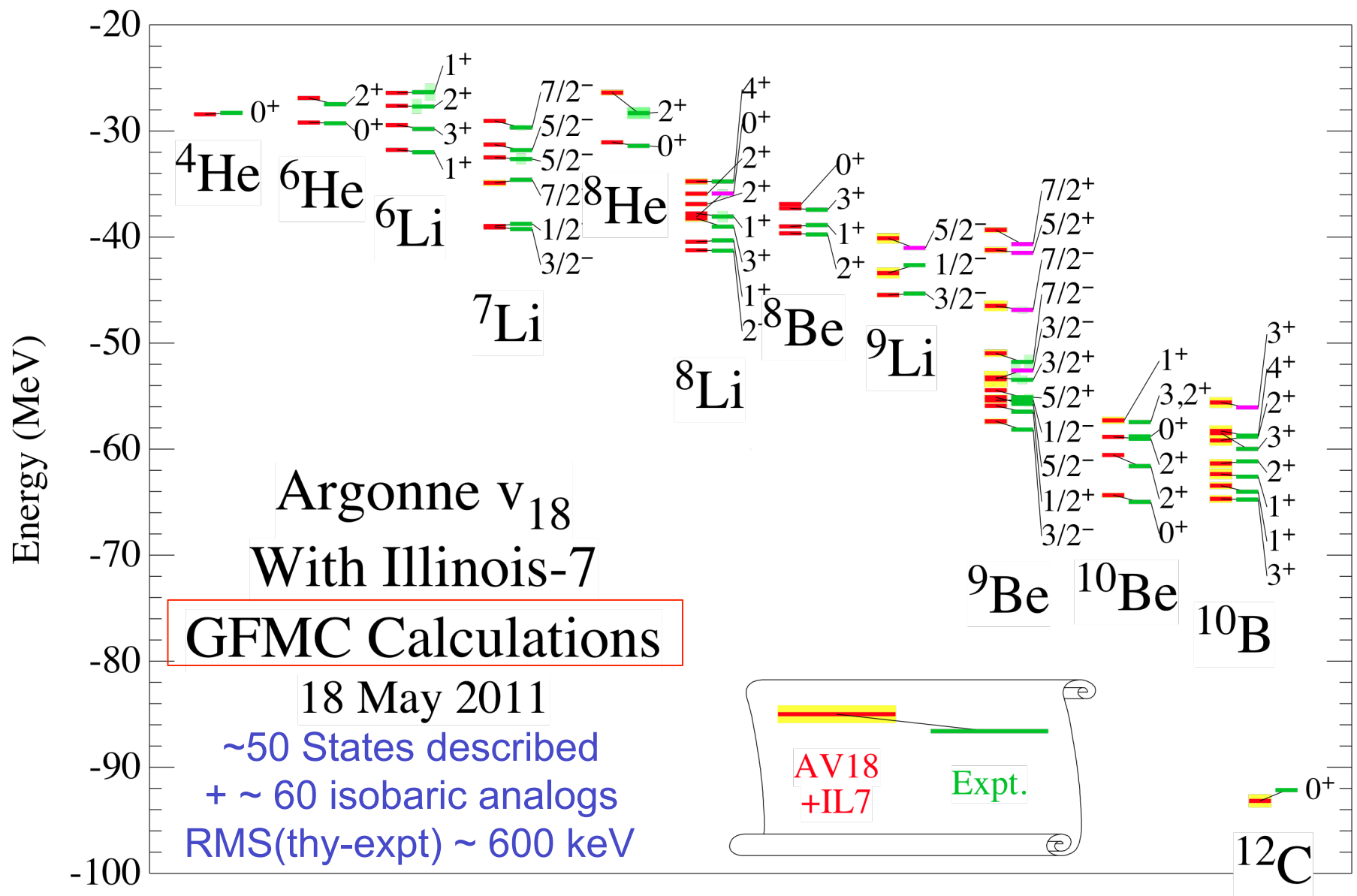
Meson Exchg interactions

Featured results here

Chiral EFT interactions

Comments

All work to preserve and exploit symmetries
Extensions of each to scattering/reactions are well-underway
They have different advantages and limitations



S. Pieper, R. Wiringa, et al

No Core Shell Model

A large sparse matrix eigenvalue problem

$$H = T_{rel} + V_{NN} + V_{3N} + \dots$$

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle$$

$$|\Psi_i\rangle = \sum_{n=0}^{\infty} A_n^i |\Phi_n\rangle$$

$$\text{Diagonalize } \{ \langle \Phi_m | H | \Phi_n \rangle \}$$

- Adopt realistic NN (and NNN) interaction(s) & renormalize as needed - retain induced many-body interactions: **Chiral EFT interactions and JISP16**
- Adopt the 3-D Harmonic Oscillator (HO) for the single-nucleon basis states, α, β, \dots
- Evaluate the nuclear Hamiltonian, H , in basis space of HO (Slater) determinants (manages the bookkeeping of anti-symmetrization)
- Diagonalize this sparse many-body H in its “m-scheme” basis where $[\alpha = (n, l, j, m_j, \tau_z)]$

$$|\Phi_n\rangle = [a_{\alpha}^+ \dots a_{\zeta}^+]_n |0\rangle$$

$$n = 1, 2, \dots, 10^{10} \text{ or more!}$$

- Evaluate observables and compare with experiment

Comments

- Straightforward but computationally demanding => new algorithms/computers
- Requires convergence assessments and extrapolation tools
- Achievable for nuclei up to $A=20$ (40) today with largest computers available

ab initio NCSM

Effective Hamiltonian for A-Particles

Okubo-Lee-Suzuki Method plus Cluster Decomposition

P. Navratil, J.P. Vary and B.R. Barrett,

Phys. Rev. Lett. **84**, 5728(2000); Phys. Rev. C **62**, 054311(2000)

C. Viazminsky and J.P. Vary, J. Math. Phys. **42**, 2055 (2001);

S. Okubo, Progr. Theor. Phys. **12** (1954) 603;

K. Suzuki and S.Y. Lee, Progr. Theor. Phys. **64**, 2091(1980);

K. Suzuki, *ibid*, **68**, 246(1982);

Review: B.R. Barrett, P. Navratil and J.P. Vary, Prog. Part. Nucl. Phys. **69**, 131 (2013)

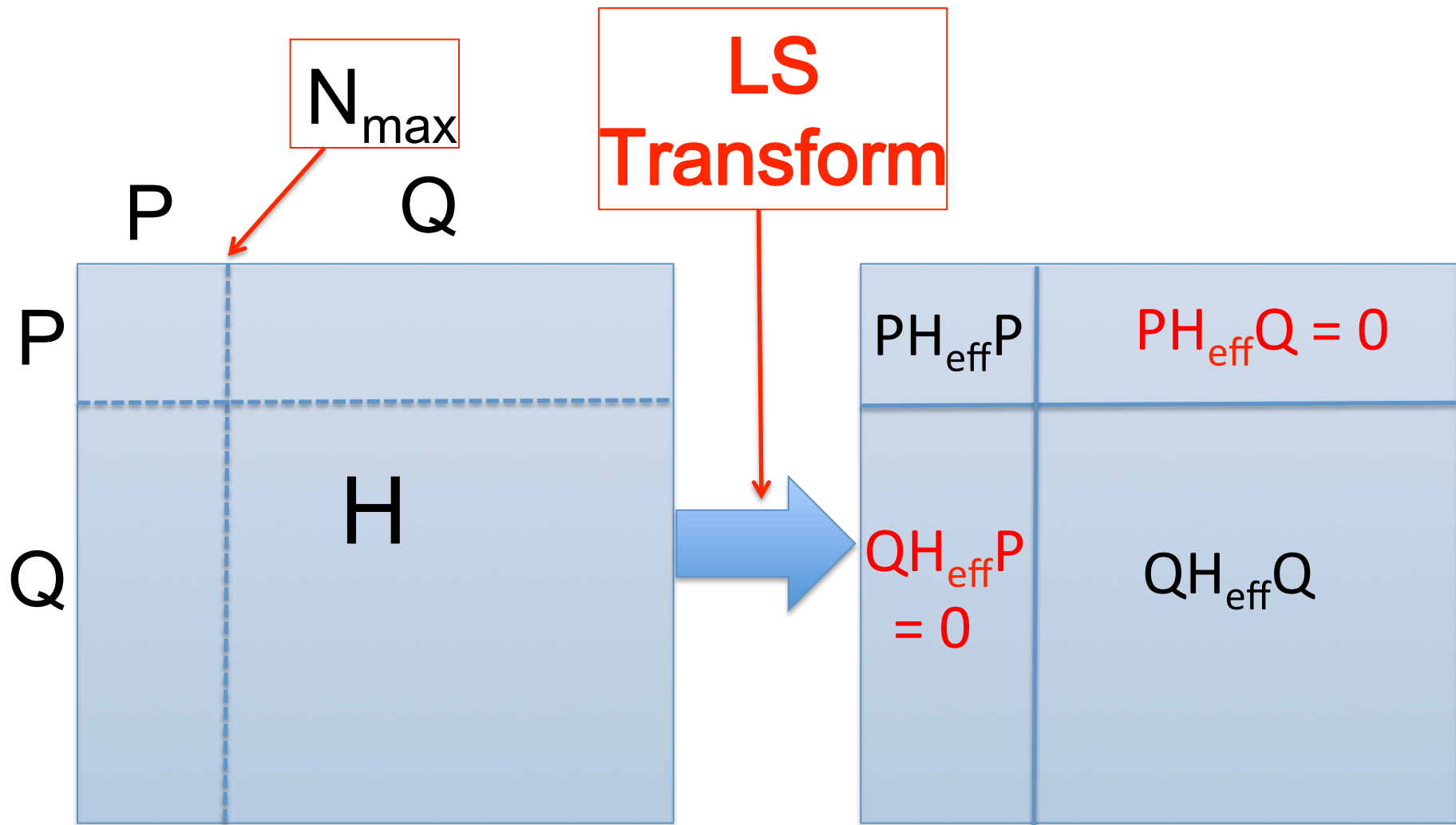
**Preserves the symmetries of the full Hamiltonian:
Rotational, translational, parity, etc., invariance**

$$H_{\mathcal{A}} = T_{rel} + V = \sum_{i < j}^{\mathcal{A}} \left[\frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + V_{ij} \right] + V_{NNN}$$

Select a finite oscillator basis space (P-space) and evaluate an a - body cluster effective Hamiltonian:

$$H_{eff} = P \left[T_{rel} + V^a (N_{max}, \hbar\Omega) \right] P$$

Guaranteed to provide exact answers as $a \rightarrow A$ or as $P \rightarrow 1$.

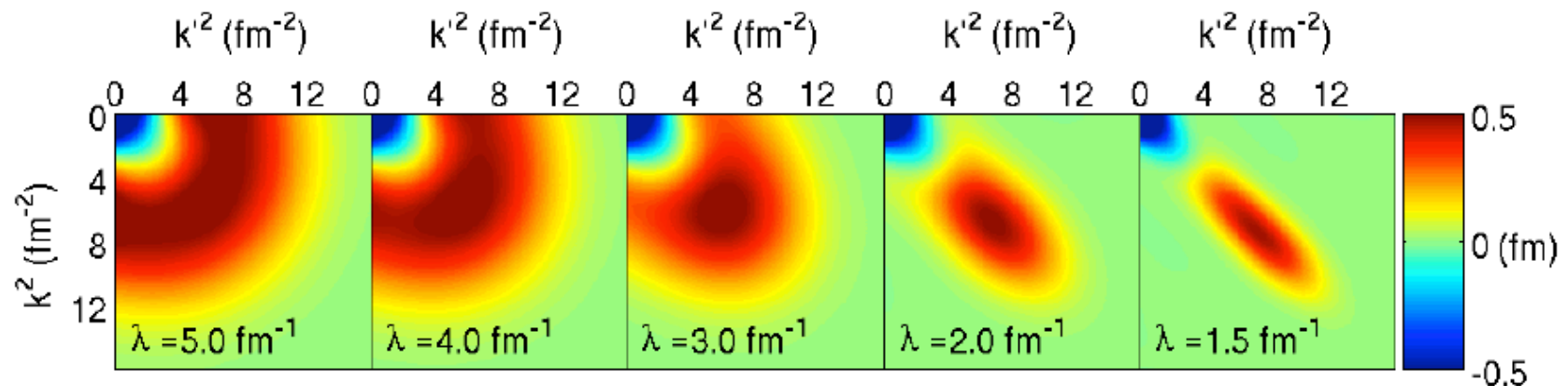


Review: B.R. Barrett, P. Navratil and J.P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013)

Similarity Renormalization Group – NN interaction

SRG evolution

Bogner, Furnstahl, Perry, PRC 75 (2007) 061001



- drives interaction towards band-diagonal structure
- SRG shifts strength between 2-body and many-body forces
- Initial chiral EFT Hamiltonian
power-counting hierarchy A -body forces

$$V_{NN} \gg V_{NNN} \gg V_{NNNN}$$

Both OLS and SRG derivations of H_{eff} will be used in the applications surveyed

Controlling the center-of-mass (cm) motion
in order to preserve Galilean invariance

Add a Lagrange multiplier term acting on the cm alone
so as not to interfere with the internal motion dynamics

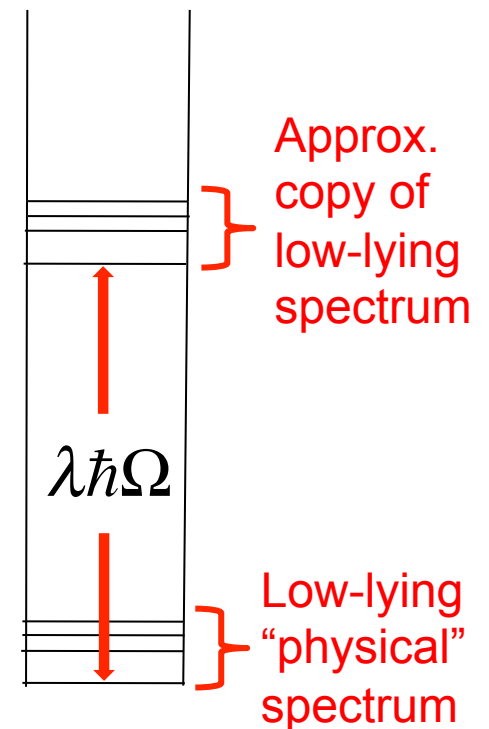
$$H_{eff} = P[T_{rel} + V^a(N_{max}, \hbar\Omega)]P$$

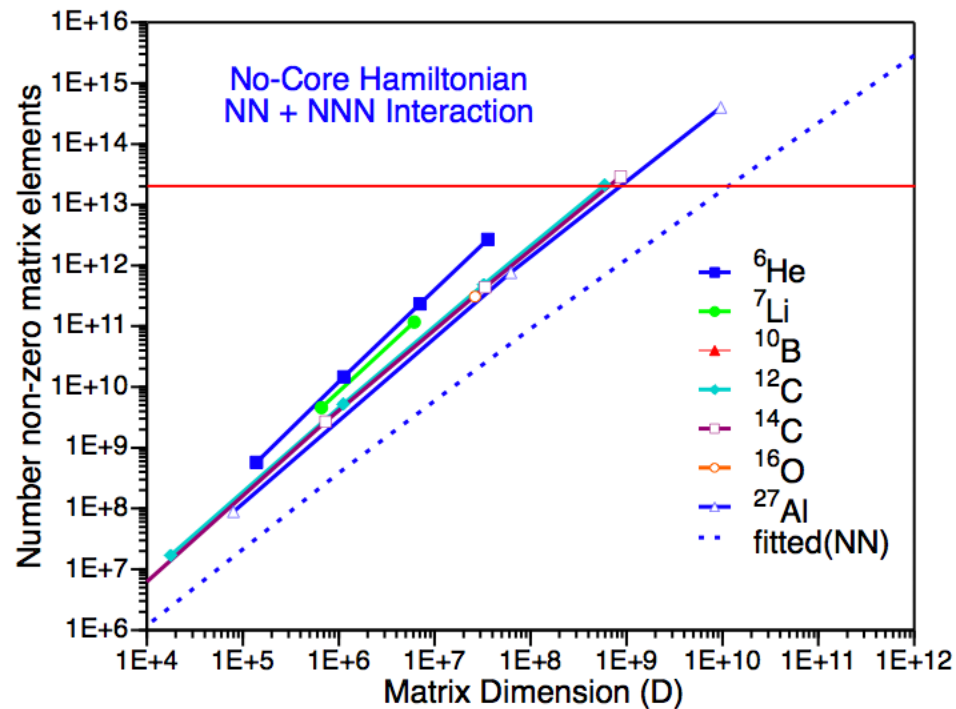
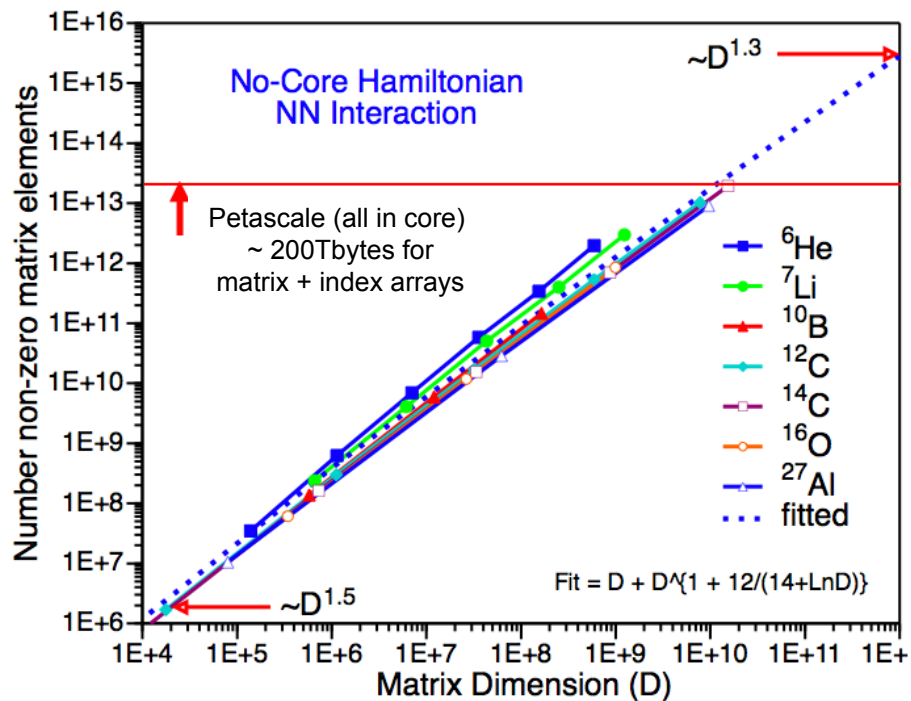
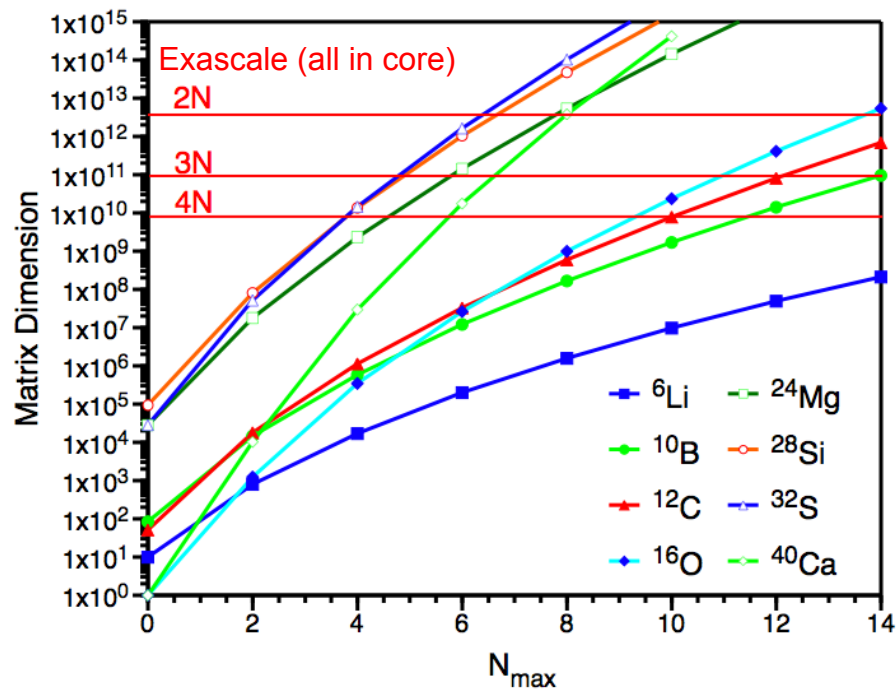
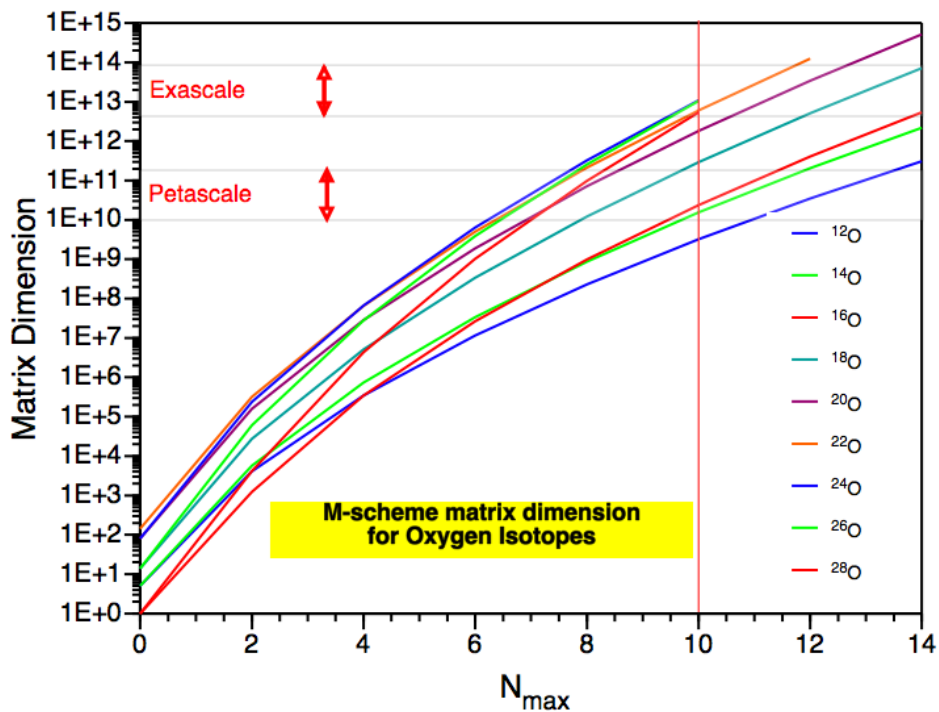
$$H = H_{eff}(N_{max}, \hbar\Omega) + \lambda H_{cm}$$

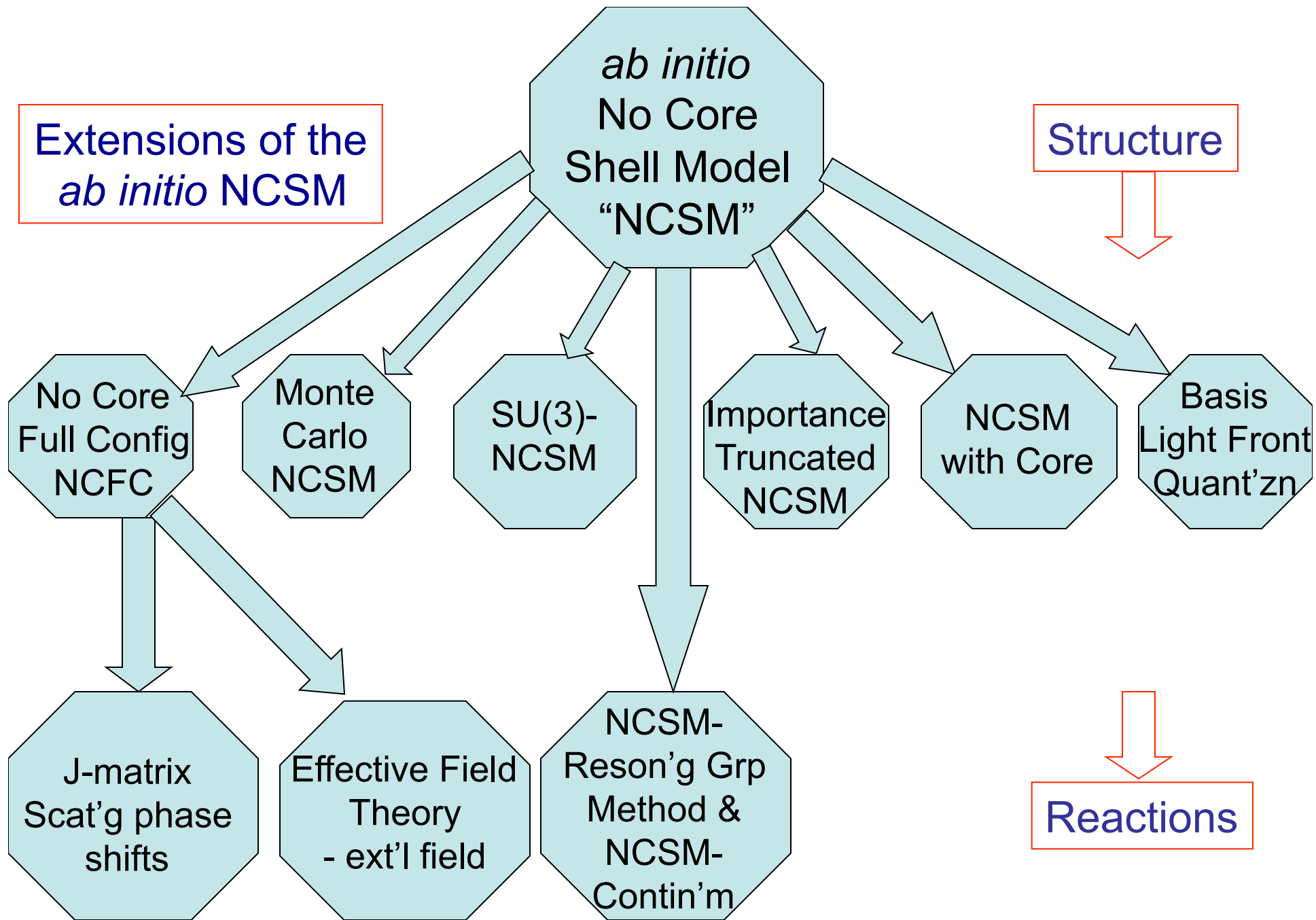
$$H_{cm} = \frac{P^2}{2M_A} + \frac{1}{2}M_A\Omega^2 R^2$$

$\lambda \sim 10$ suffices

Along with the N_{max} truncation in the HO basis,
the Lagrange multiplier term guarantees that
all low-lying solutions have wavefunctions that
factorize into a 0s HO wavefunction for the cm
times a translationally invariant wavefunction.







Extrapolating to the infinite matrix limit i.e. to the “continuum limit”

Results with both IR and UV extrapolations

References:

S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris, and J.P. Vary,
Phys. Rev. C 86, 054002 (2012); arXiv: 1205.3230

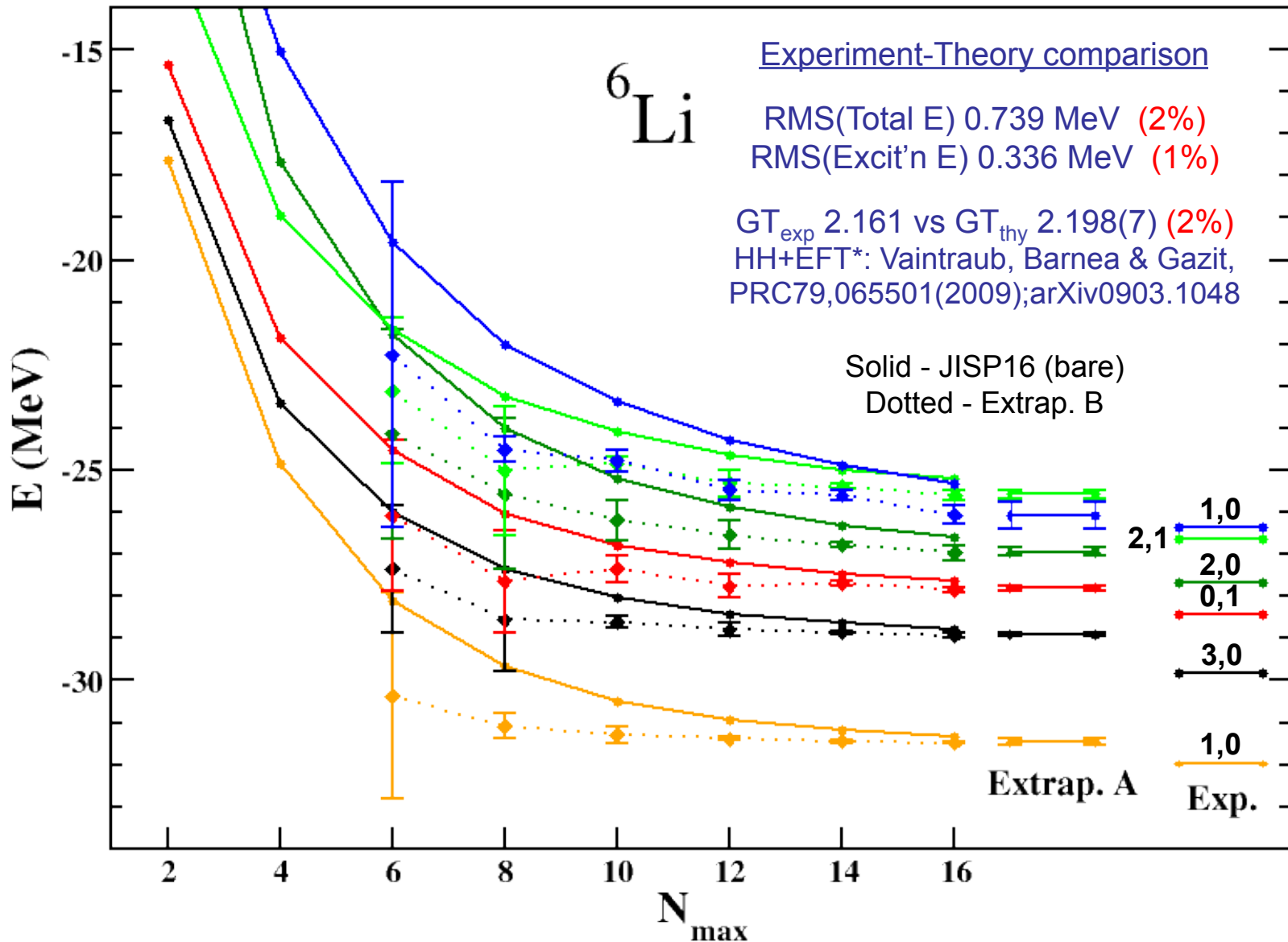
R.J. Furnstahl, G. Hagen, T. Papenbrock, Phys. Rev. C 86 (2012) 031301

E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary,
Phys. Rev. C 87, 054312(2013); arXiv 1302.5473

S.N. More, A. Ekstroem, R.J. Furnstahl, G. Hagen and T. Papenbrock,
Phys. Rev. C87, 044326 (2013); arXiv 1302.3815

=> Uncertainty Quantification

NCFC results (does not adopt a renormalization)



P. Maris, A. Shirokov and J.P. Vary, Phys. Rev. C 81, 021301(R) (2010). ArXiv 0911.2281
 C. Cockrell, J.P. Vary, P. Maris, Phys. Rev. C 86, 034325 (2012); arXiv:1201.0724

Convergence and Uncertainty Assessments: Recent Highlight

Convergence properties of *ab initio* calculations of light nuclei in a harmonic oscillator basis

Phys. Rev. C 86, 054002 (2012); arXiv:1205.3230

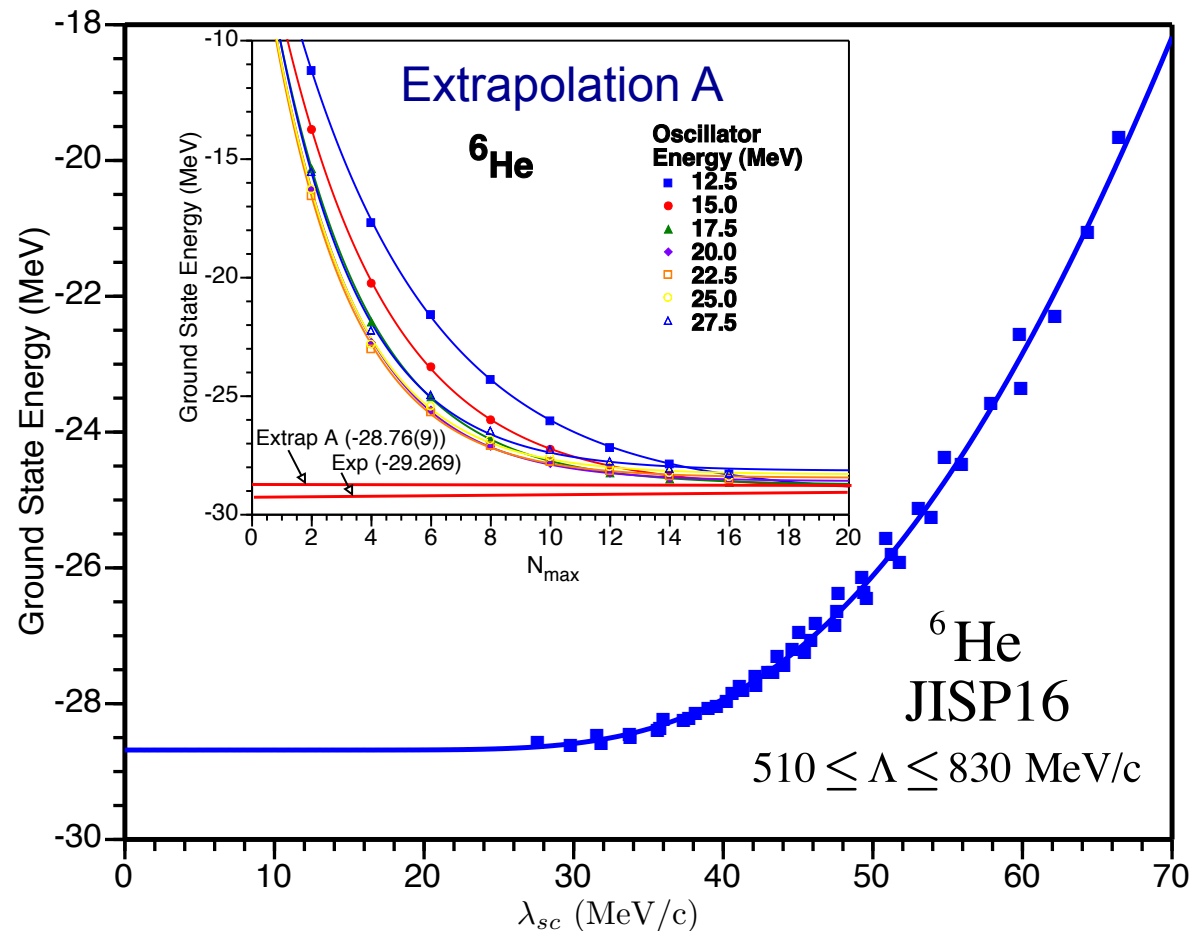
S. A. Coon^a, M. I. Avetian^a, M. K. G. Kruse^a, U. van Kolck^{a,b}, P. Maris^c, J. P. Vary^c

UV regulator:

$$\Lambda = \sqrt{(N + 3/2)m\hbar\Omega}$$

IR regulator:

$$\lambda_{sc} = \sqrt{\frac{m\hbar\Omega}{(N + 3/2)}}$$



Combined IR and UV extrapolation: HO-basis regulator definitions

	Ref. 1	Ref. 2	Ref. 3
UV: Λ	$\sqrt{(N + 3/2)m\hbar\Omega}$	$\sqrt{2(N + 3/2)m\hbar\Omega}$	$\sqrt{2(N + 3/2)m\hbar\Omega}$
IR: λ	$\sqrt{\frac{m\hbar\Omega}{(N + 3/2)}}$	$\sqrt{\frac{m\hbar\Omega}{2(N + 3/2)}}$	$\sqrt{\frac{m\hbar\Omega}{2(N + 3/2)}}$
N (p-shell)	$N_{\max} + 1$	$N_{\max} + 2$	$N_{\max} + 3$

$$E(\Lambda, \lambda) \approx E_{\infty} + B_0 e^{-2\Lambda^2/B_1^2} + B_2 e^{-2k_{\infty}/\lambda}$$

¹S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris, and J.P. Vary, Phys. Rev. C 86, 054002 (2012); arXiv: 1205.3230

²R.J. Furnstahl, G. Hagen, T. Papenbrock, Phys. Rev. C 86 (2012) 031301

³E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary, Phys. Rev. C 87, 054312(2013); arXiv 1302.5473

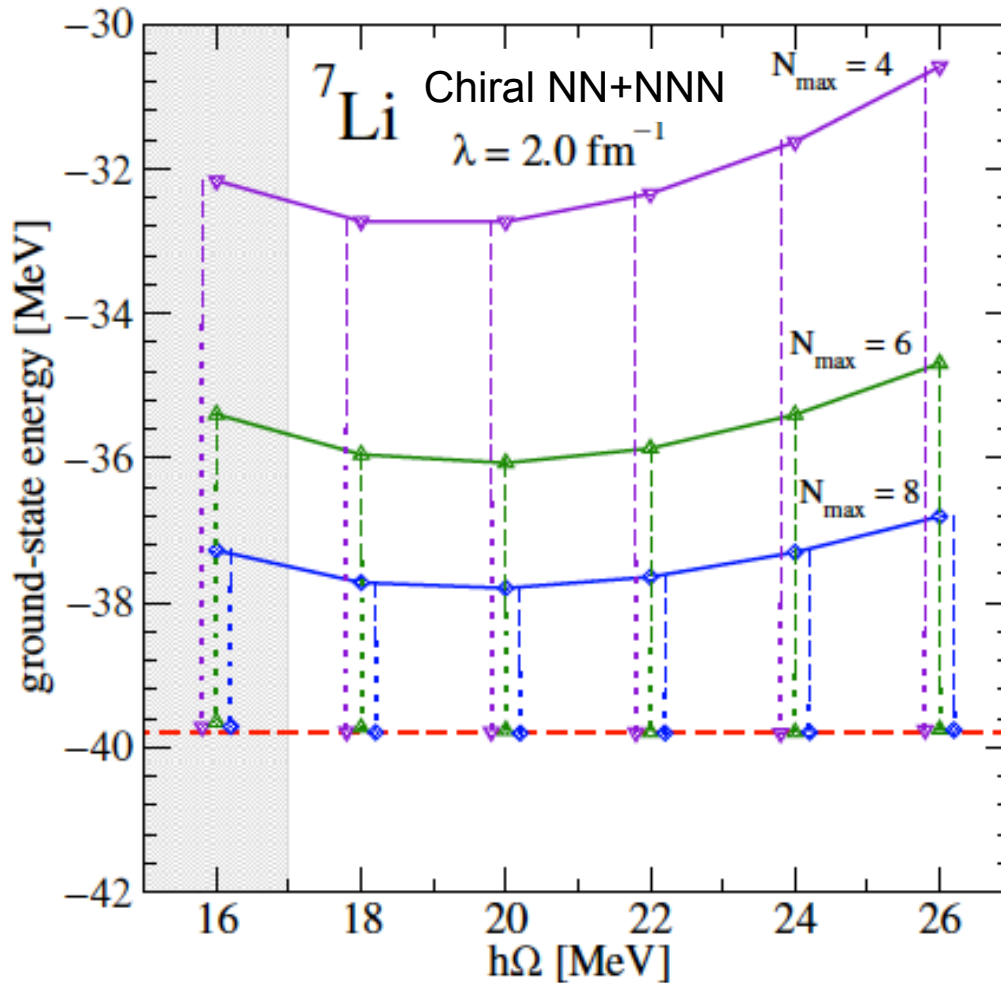
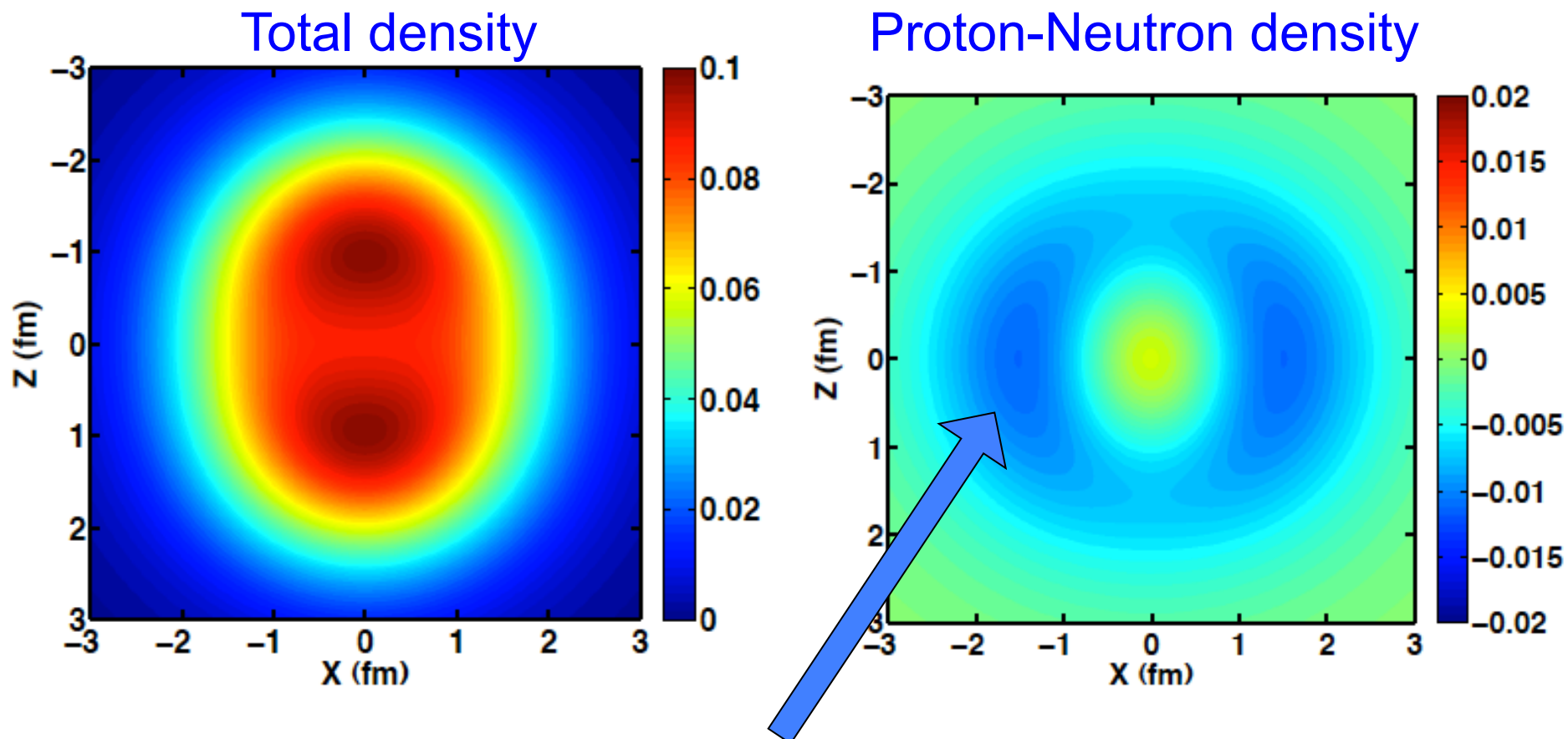


FIG. 17. (color online) Ground-state energy of ${}^7\text{Li}$ for the NN+NNN evolved Hamiltonians at $\lambda = 2.0 \text{ fm}^{-1}$, with IR (vertical dashed) and UV (vertical dotted) corrections from Eq. (5) that add to predicted E_{∞} values (points near the horizontal dashed line, which is the global E_{∞}).

E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary, Phys. Rev. C. 87, 054312 (2013); arXiv: 1302:5473

^9Be Translationally invariant gs density
Full 3D densities = rotate around the vertical axis



Shows that one neutron provides a “ring” cloud around two alpha clusters binding them together

Structure of $A = 10\text{--}13$ Nuclei with Two- Plus Three-Nucleon Interactions from Chiral Effective Field Theory

P. Navrátil,¹ V. G. Gueorguiev,^{1,*} J. P. Vary,^{1,2} W. E. Ormand,¹ and A. Nogga³

Strong correlation between c_D and c_E for exp'l properties of $A = 3$ & 4

=> Retain this correlation in applications to other systems

Range favored by various analyses & values are "natural"

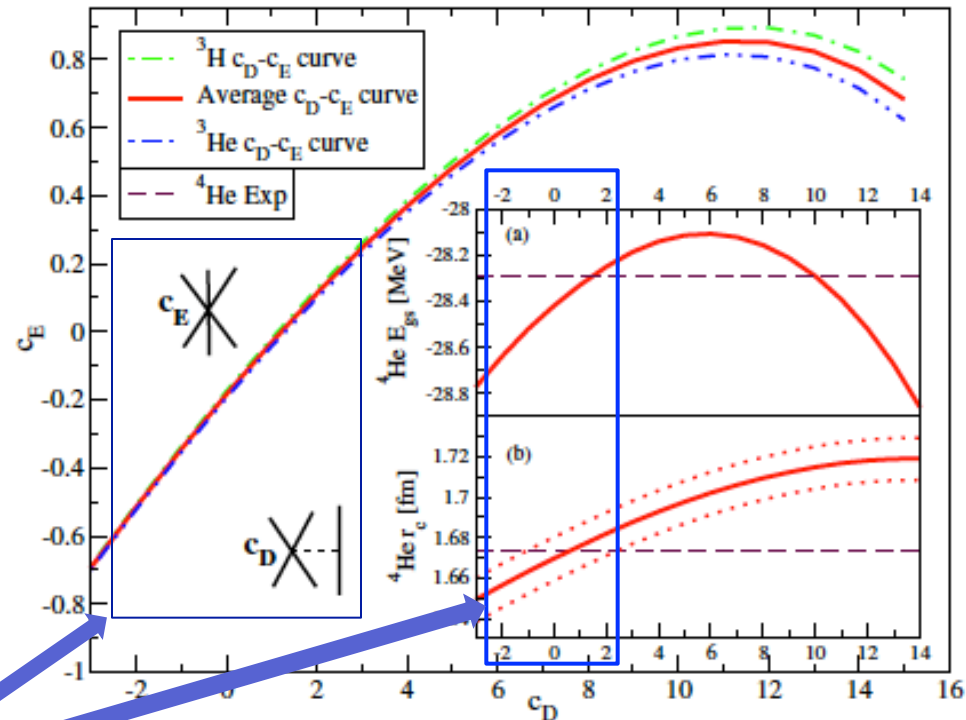
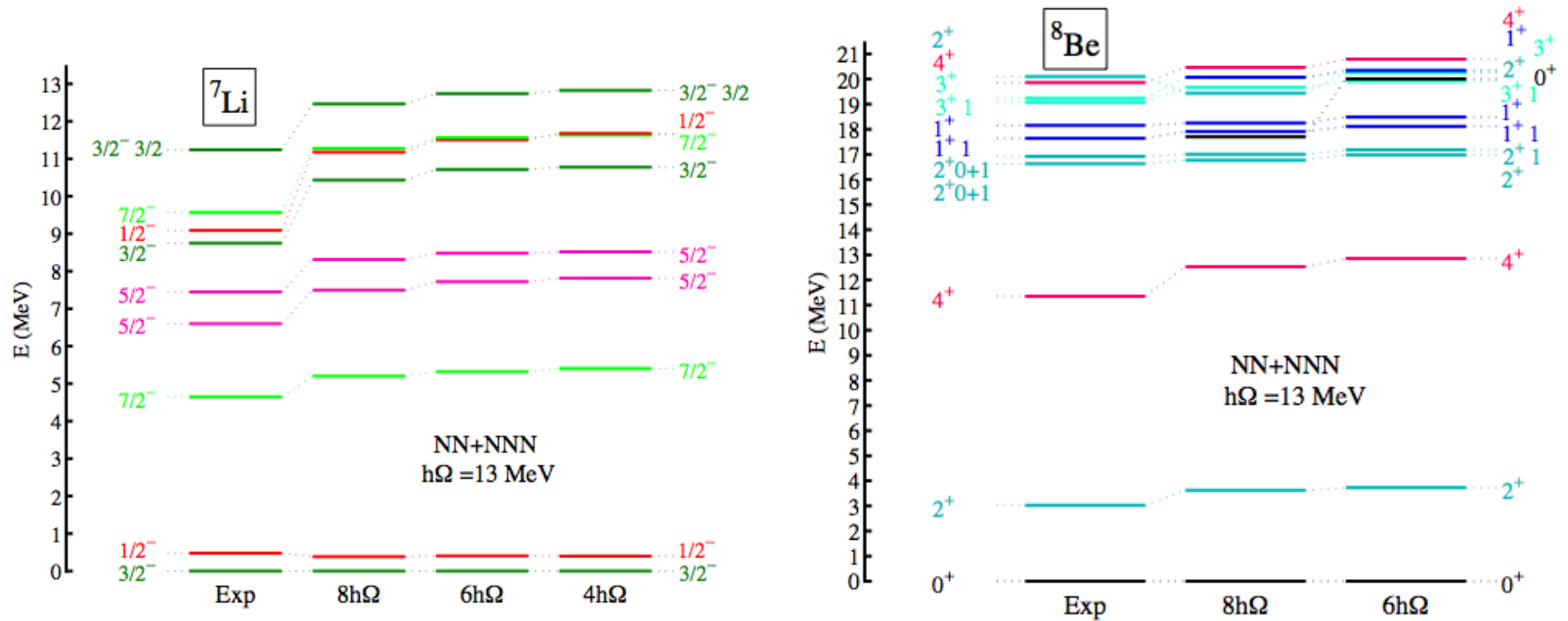


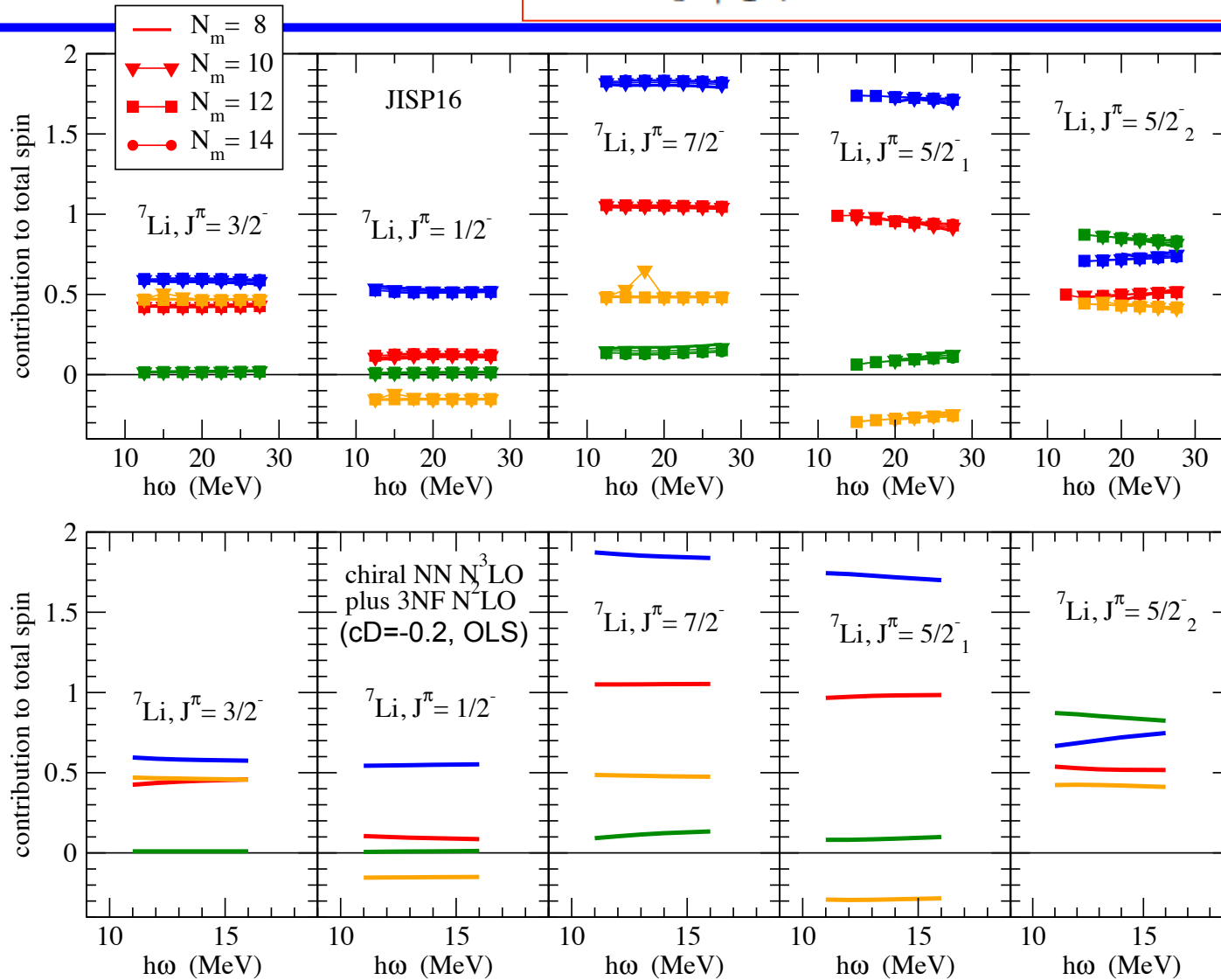
FIG. 1 (color online). Relations between c_D and c_E for which the binding energy of ${}^3\text{H}$ (8.482 MeV) and ${}^3\text{He}$ (7.718 MeV) are reproduced. (a) ${}^4\text{He}$ ground-state energy along the averaged curve. (b) ${}^4\text{He}$ charge radius r_c along the averaged curve. Dotted lines represent the r_c uncertainty due to the uncertainties in the proton charge radius.

NCSM with Chiral NN (N3LO) + NNN (N2LO, $C_D=-0.2$)
 Employs Okubo-Lee-Suzuki (OLS) renormalization



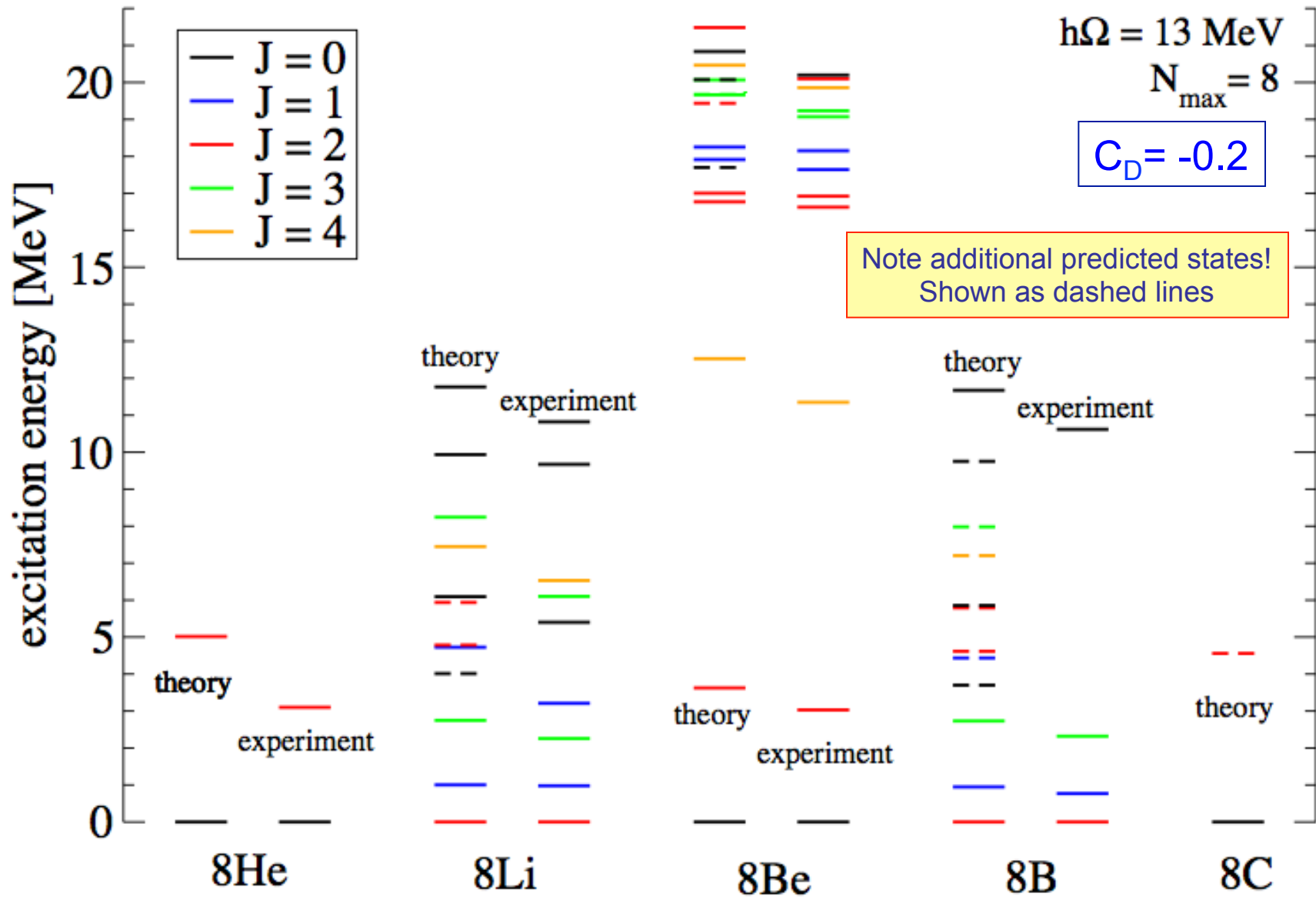
Spin components of ${}^7\text{Li}$

$$J = \frac{1}{J+1} \left(\langle \mathbf{J} \cdot \mathbf{L}_p \rangle + \langle \mathbf{J} \cdot \mathbf{L}_n \rangle + \langle \mathbf{J} \cdot \mathbf{S}_p \rangle + \langle \mathbf{J} \cdot \mathbf{S}_n \rangle \right)$$



● Very similar structure with JISP16 and chiral NN plus 3NF

Spectrum A=8 nuclei with chiral NN(N3LO) + NNN(N2LO) & OLS renormalization



^8Be

No Core CI calculations for light nuclei
with chiral 2- and 3-body forces

Pieter Maris¹, H Metin Aktulga², Sven Binder³, Angelo Calci³,
Ümit V Çatalyürek^{4,5}, Joachim Langhammer³, Esmond Ng²,
Erik Saule⁴, Robert Roth³, James P Vary¹ and Chao Yang²

CCP-2012
proceedings
(to appear).

SRG renormalization scale invariance & agreement with experiment ($cD=-0.2$)

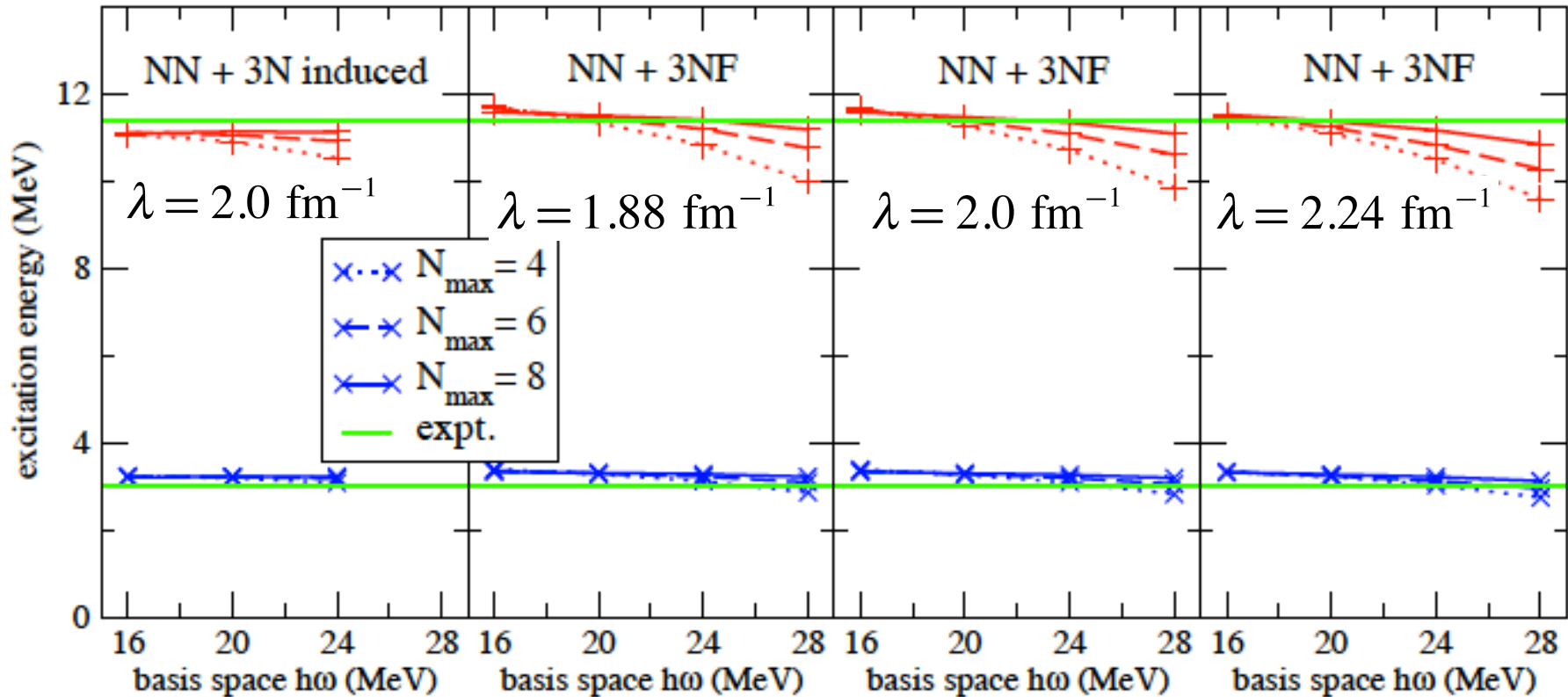


Figure 5. Excitation energies of the 2^+ (blue crosses) and 4^+ states (red pluses) for ^8Be with SRG evolved chiral $N^3\text{LO}$ 2NF plus induced 3NF at $\lambda = 2.0 \text{ fm}^{-1}$ (left-most panel) and with SRG evolved chiral $N^3\text{LO}$ 2NF plus chiral $N^2\text{LO}$ 3NF. Experimental values are indicated by the horizontal green lines.

ab initio NCSM - comparison of Chiral EFT (OLS) with JISP16

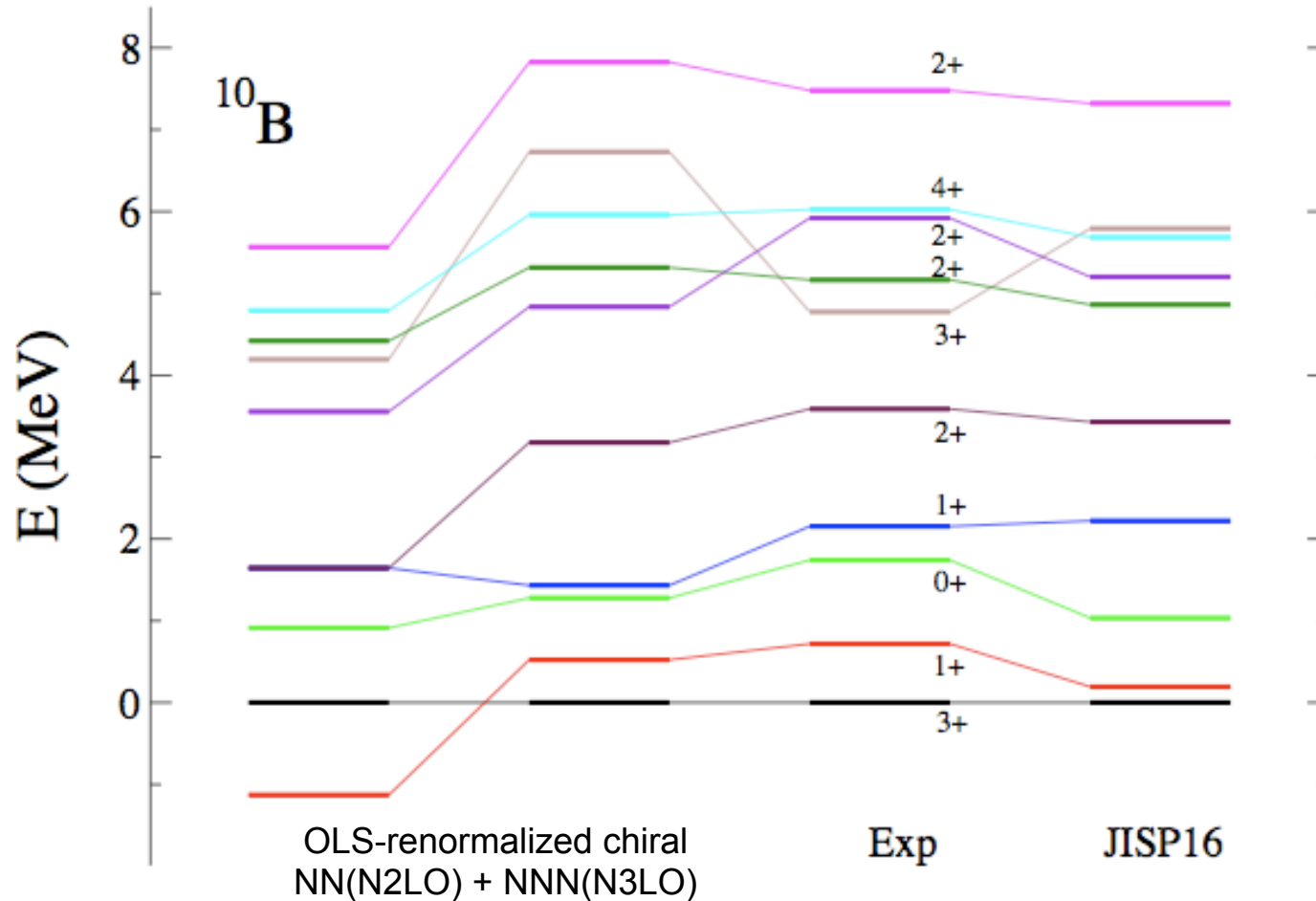
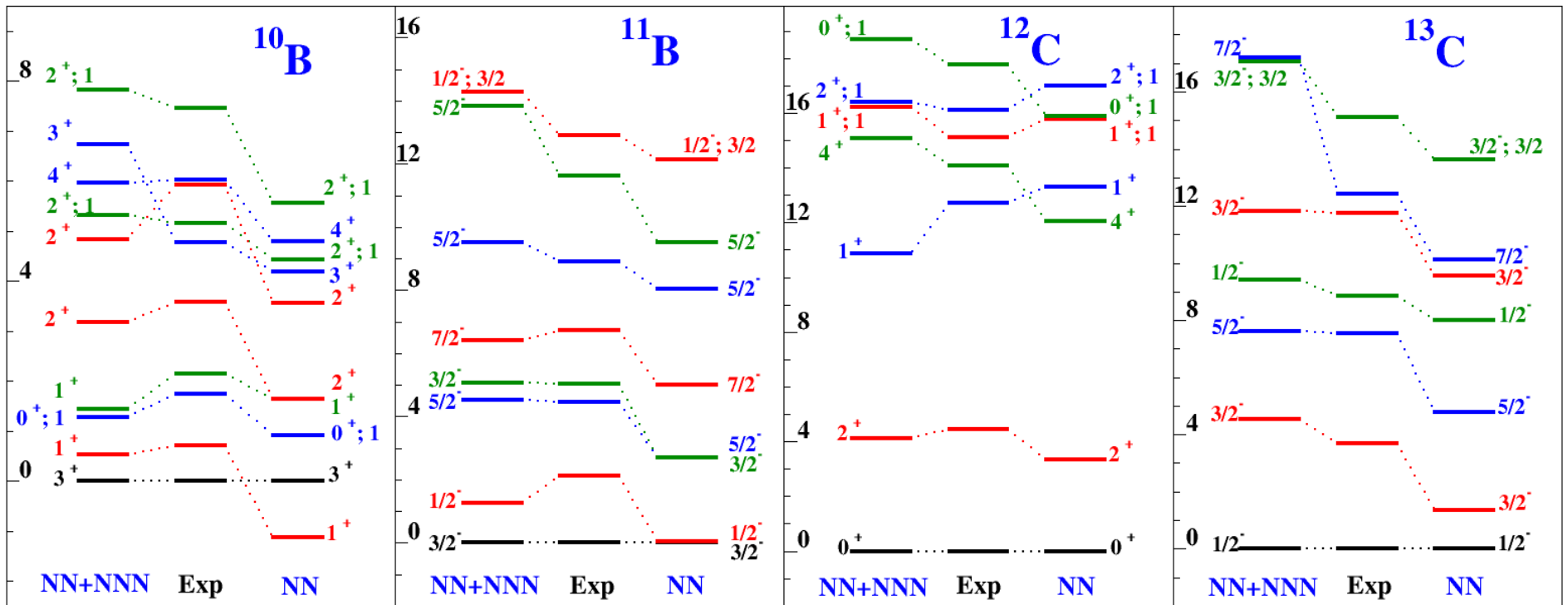


FIGURE 2. Experimental and theoretical excitation spectra of ^{10}B with respect to the lowest 3^+ state at an oscillator energy $\hbar\Omega = 14 \text{ MeV}$. The chiral effective interaction results are obtained at $N_{max} = 6$ while the JISP16 results are obtained at $N_{max} = 8$.

J.P. Vary, P. Maris, A. Negoita, P. Navratil, V.G. Gueorguiev, W. E. Ormand, A. Nogga, A. Shirokov and S. Stoica, in *Exotic Nuclei and Nuclear/Particle Astrophysics (II)*, Proceedings of the Carpathian Summer School of Physics 2007, L. Trache and S. Stoica, Editors, AIP Conference Proceedings 972, 49(2008).

ab initio NCSM with chiral NN(N3LO) + NNN(N2LO) and OLS-renormalization

$$N_{\max} = 6, c_D = -1$$

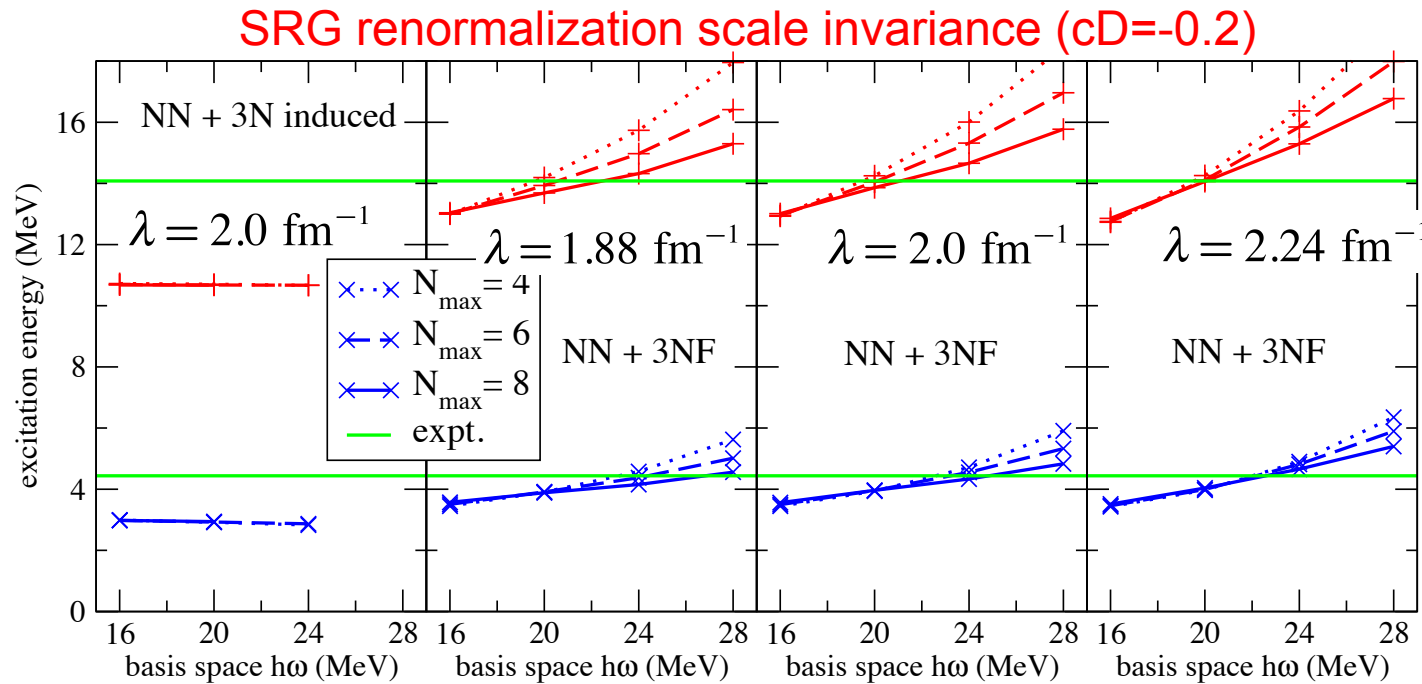


P. Navratil, V.G. Gueorguiev, J. P. Vary, W. E. Ormand and A. Nogga,
 PRL 99, 042501(2007); ArXiv: nucl-th 0701038.

Low-lying spectrum of ^{12}C

Pieter Maris¹, H Metin Aktulga², Sven Binder³, Angelo Calci³,
Ümit V Çatalyürek^{4,5}, Joachim Langhammer³, Esmond Ng²,
Erik Saule⁴, Robert Roth³, James P Vary¹ and Chao Yang²

proceedings CCP 2012, in press



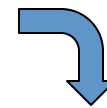
- Qualitative agreement with data
- Not converged with explicit 3NF, despite weak N_{max} dependence
- Ratio's of excitation energies, quadrupole moments and B(E2)'s in agreement with rotational model

^{14}C beta decay - detailed results and estimated corrections due to chiral 2-body currents

TABLE I. Decomposition of p -shell contributions to M_{GT} in the LS scheme for the beta decay of ^{14}C without and with 3NF. The 3NF is included at two values of c_D where $c_D \simeq -0.2$ is preferred by the ^3H lifetime and $c_D \simeq -2.0$ is preferred by the ^{14}C lifetime. The calculations are performed in the $N_{\text{max}} = 8$ basis space with $\hbar\Omega = 14$ MeV.

(m_l, m_s)	NN only	NN + 3NF $c_D = -0.2$	NN + 3NF $c_D = -2.0$
$(1, +\frac{1}{2})$	0.015	0.009	0.009
$(1, -\frac{1}{2})$	-0.176	-0.296	-0.280
$(0, +\frac{1}{2})$	0.307	0.277	0.283
$(0, -\frac{1}{2})$	0.307	0.277	0.283
$(-1, +\frac{1}{2})$	-0.176	-0.296	-0.280
$(-1, -\frac{1}{2})$	0.015	0.009	0.009
Subtotal	0.292	-0.019	0.024
Total sum	0.275	-0.063	-0.013

Table I from:
P. Maris, J.P. Vary,
P. Navratil, W.E. Ormand,
H. Nam and D.J. Dean,
Phys. Rev. Lett. 106,
202502 (2011)



Tritium half-life		
c_D	= -0.20	-2.0
Thy/Exp. =	1.00	0.80

2-body current quenching (est' d)* x 0.75 => **-0.047** x 0.93 => **-0.012**

Preliminary

*J. Menéndez, D. Gazit and A. Schwenk, Phys.Rev.Lett. 107 (2011) 062501 (estimated using their effective density-dependent 1-body operator)



Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



First observation of ^{14}F

V.Z. Goldberg^{a,*}, B.T. Roeder^a, G.V. Rogachev^b, G.G. Chubarian^a, E.D. Johnson^b, C. Fu^c,
 A.A. Alharbi^{a,1}, M.L. Avila^b, A. Banu^a, M. McCleskey^a, J.P. Mitchell^b, E. Simmons^a,
 G. Tabacaru^a, L. Trache^a, R.E. Tribble^a

^a Cyclotron Institute, Texas A&M University, College Station, TX 77843-3366, USA

^b Department of Physics, Florida State University, Tallahassee, FL 32306-4350, USA

^c Indiana University, Bloomington, IN 47408, USA

TAMU Cyclotron Institute

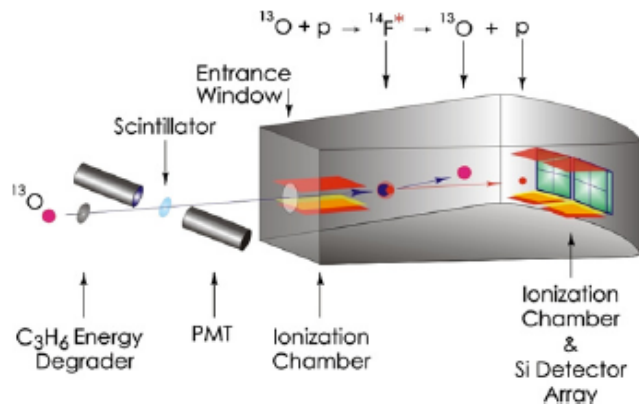


Fig. 1. (Color online.) The setup for the ^{14}F experiment. The “gray box” is the scattering chamber. See explanation in the text.

ab initio predictions in close agreement with experiment

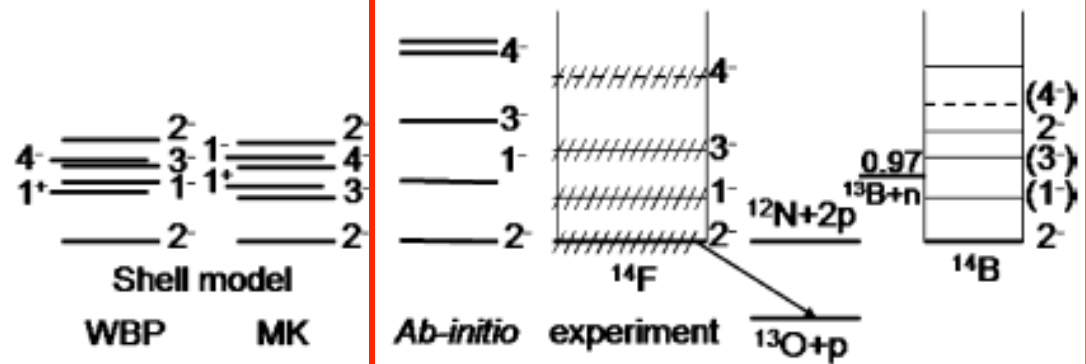
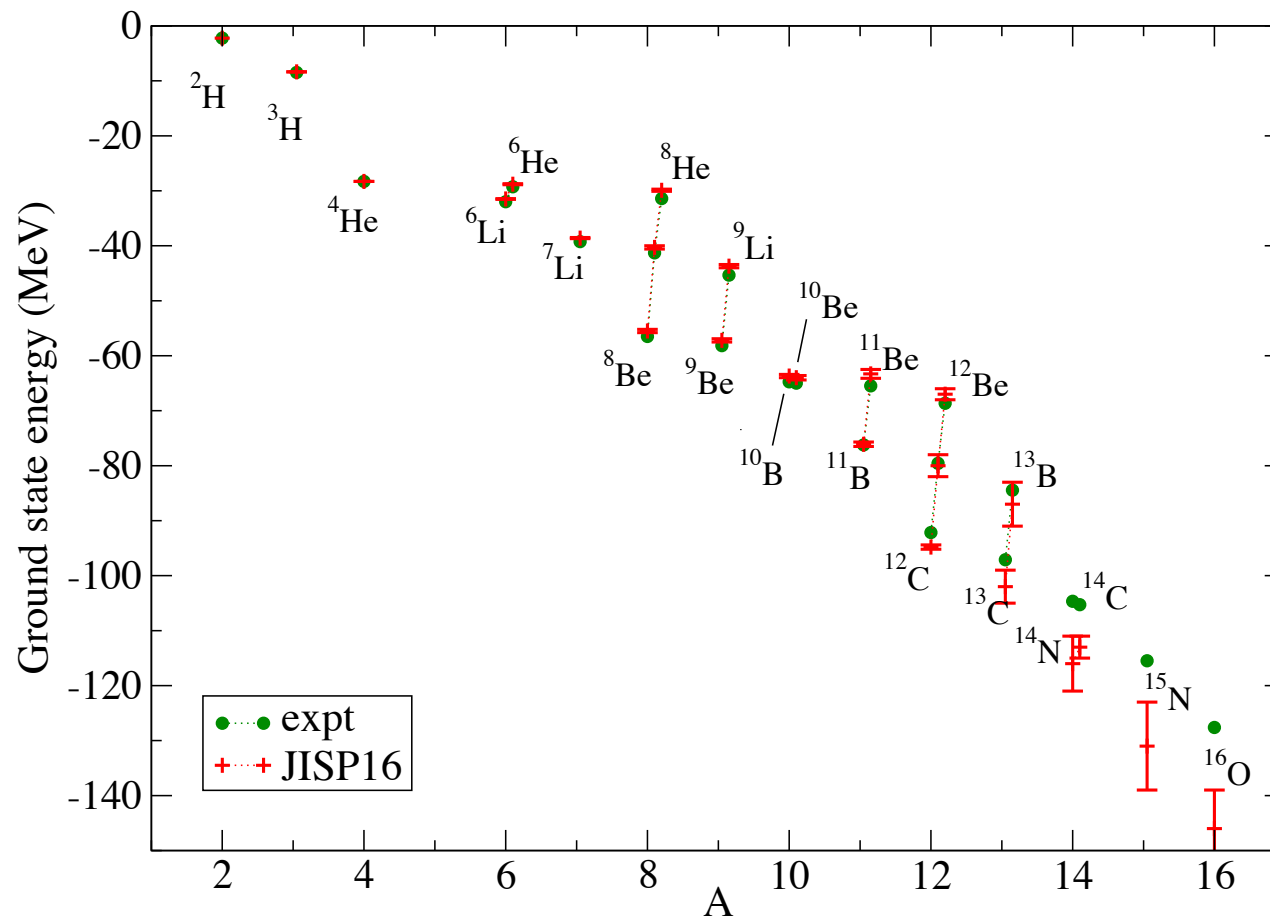


Fig. 6. ^{14}F level scheme from this work compared with shell-model calculations, *ab-initio* calculations [3] and the ^{14}B level scheme [16]. The shell model calculations were performed with the WBP [21] and MK [22] residual interactions using the code COSMO [23].

Ground state energy of *p*-shell nuclei with JISP16

Maris, Vary, IJMPE, in press

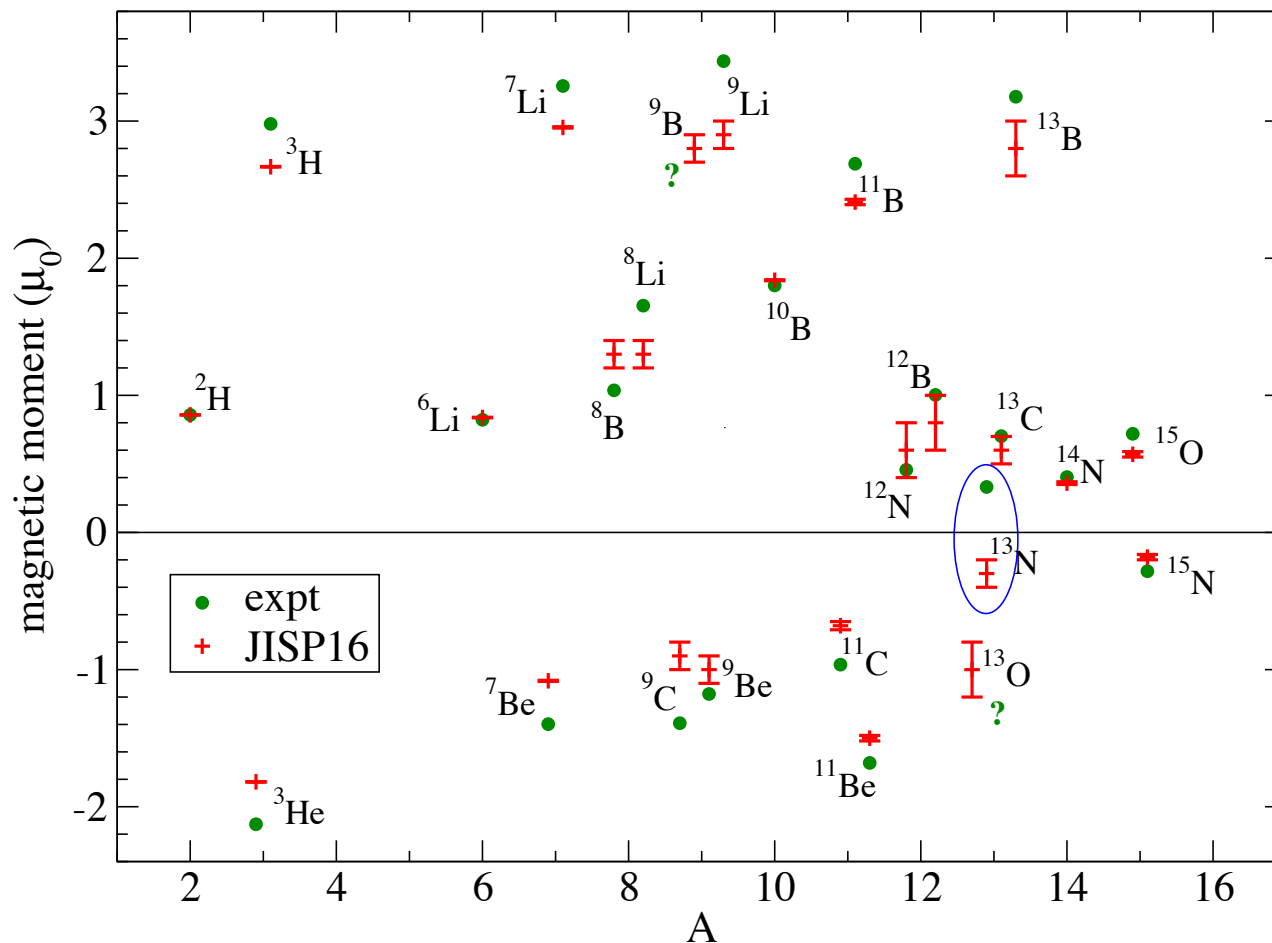


- ^{10}B – most likely JISP16 produces correct 3^+ ground state, but extrapolation of 1^+ states not reliable due to mixing of two 1^+ states
- ^{11}Be – expt. observed parity inversion within error estimates of extrapolation
- ^{12}B and ^{12}N – unclear whether gs is 1^+ or 2^+ (expt. at $E_x = 1$ MeV) with JISP16

Ground state magnetic moments with JISP16

Maris, Vary, IJMPE, in press

$$\mu = \frac{1}{J+1} \left(\langle \mathbf{J} \cdot \mathbf{L}_p \rangle + 5.586 \langle \mathbf{J} \cdot \mathbf{S}_p \rangle - 3.826 \langle \mathbf{J} \cdot \mathbf{S}_n \rangle \right) \mu_0$$



- Good agreement with data, given that we do not have any meson-exchange currents



Emergence of rotational bands in *ab initio* no-core configuration interaction calculations of light nuclei

M.A. Caprio^{a,*}, P. Maris^b, J.P. Vary^b

^a Department of Physics, University of Notre Dame, Notre Dame, IN 46556-5670, USA

^b Department of Physics and Astronomy, Iowa State University, Ames, IA 50011-3160, USA

Both natural and unnatural parity bands identified
 Employed JISP16 interaction; $N_{\max} = 10 - 7$

K=1/2 bands include Coriolis decoupling parameter:

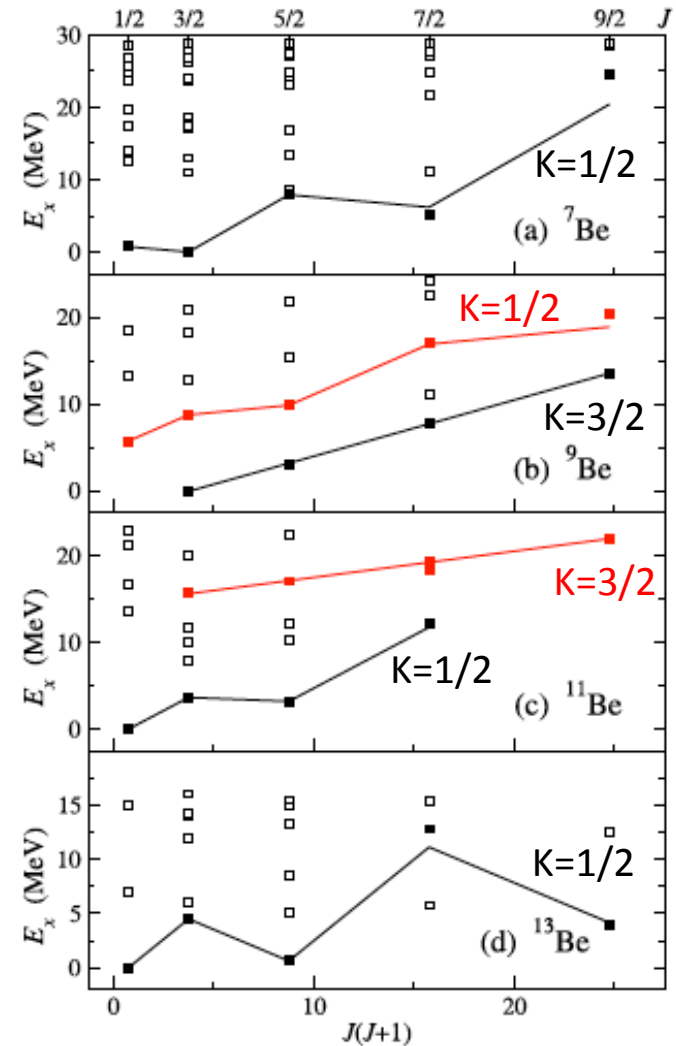
$$E(J) = E_0 + A \left[J(J+1) + a(-)^{J+1/2} \left(J + \frac{1}{2} \right) \right],$$

$$Q(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_0,$$

$$B(E2; J_i \rightarrow J_f) = \frac{5}{16\pi} (J_i K 20 | J_f K)^2 (eQ_0)^2.$$

Fig. 1. Excitation energies obtained for states in the natural parity spaces of the odd-mass Be isotopes: (a) ⁷Be, (b) ⁹Be, (c) ¹¹Be, and (d) ¹³Be. Energies are plotted with respect to $J(J+1)$ to facilitate identification of rotational energy patterns, while the J values themselves are indicated at top. Filled symbols indicate candidate rotational bandmembers (black for yrast states and red for excited states, in the web version of this Letter). The lines indicate the corresponding best fits for rotational energies. Where quadrupole transition strengths indicate significant two-state mixing (see text), more than one state of a given J is indicated as a bandmember.

Black line: Yrast band in collective model fit
 Red line: excited band in collective model fit



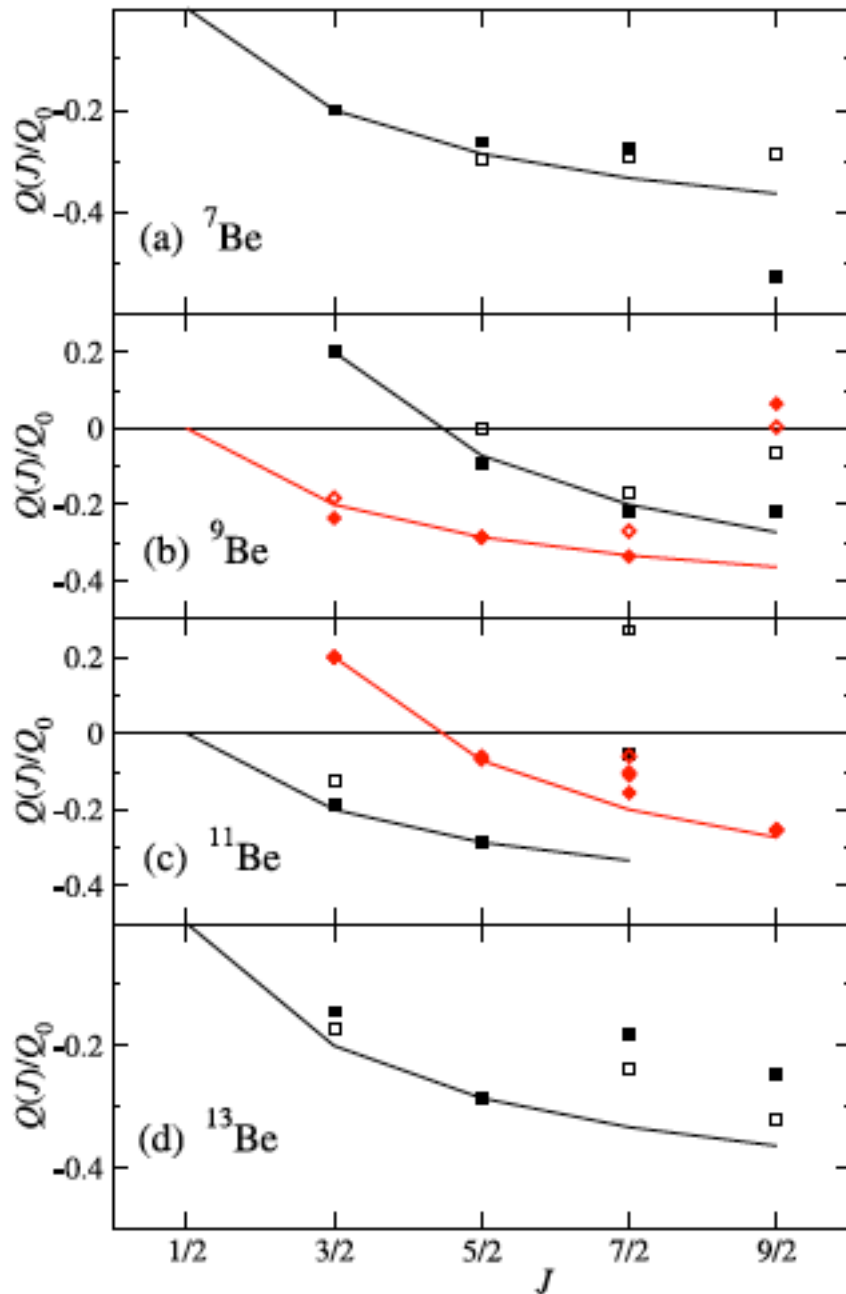


Fig. 3. Quadrupole moments calculated for candidate bandmembers in the *natural* parity spaces of the odd-mass Be isotopes: (a) ${}^7\text{Be}$, (b) ${}^9\text{Be}$, (c) ${}^{11}\text{Be}$, and (d) ${}^{13}\text{Be}$. The states are as identified in Fig. 1 and are shown as black squares for yrast states or red diamonds for excited states (color in the web version of this Letter). Filled symbols indicate proton quadrupole moments, and open symbols indicate neutron quadrupole moments. The curves indicate the theoretical values for a $K = 1/2$ or $K = 3/2$ rotational band, as appropriate, given by (4). Quadrupole moments are normalized to Q_0 , which is defined by either the $J = 3/2$ or $J = 5/2$ bandmember (see text).

Note:

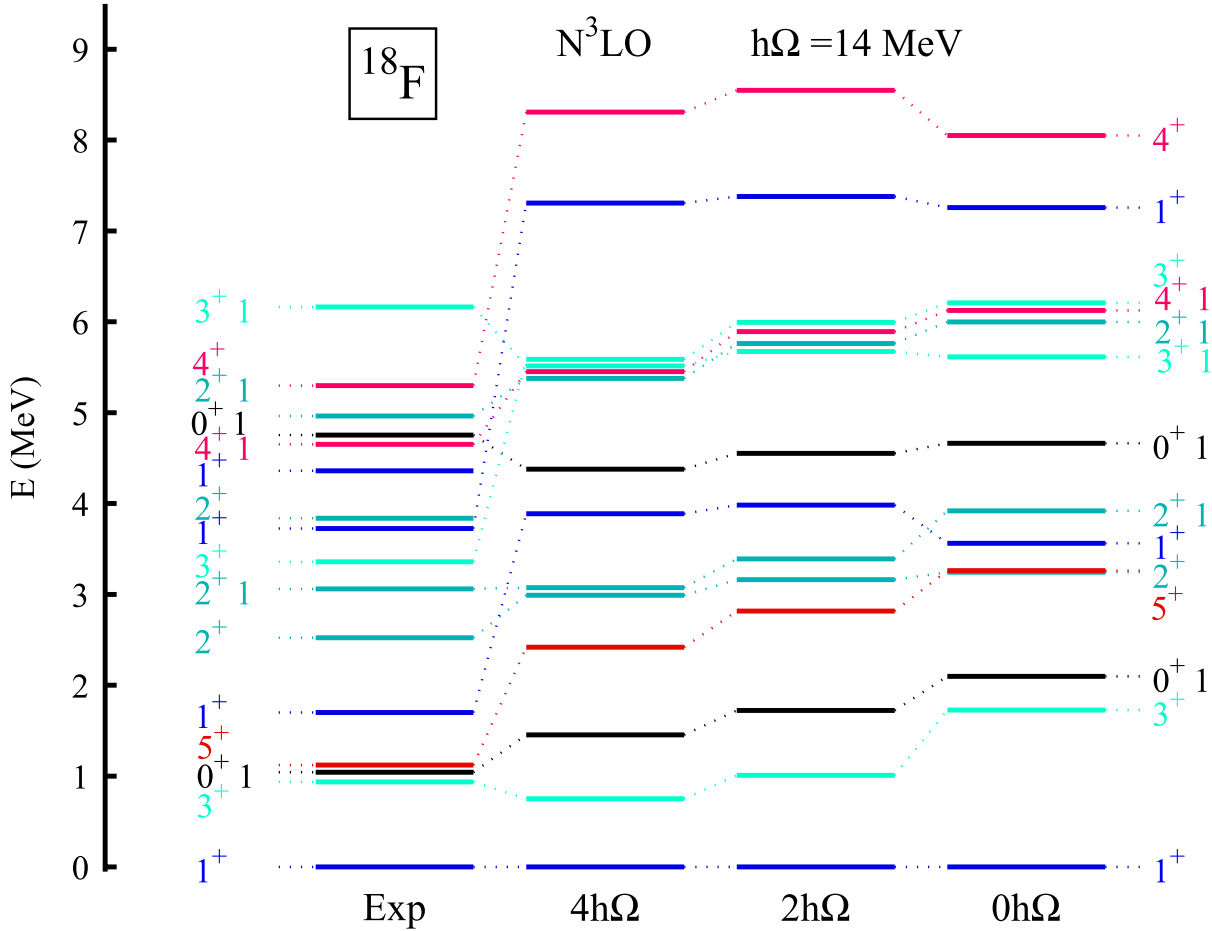
Although Q , $B(E2)$ are slowly converging, the ratios within a rotational band appear remarkably stable

Next challenge: Investigate same phenomena with Chiral EFT interactions

M.A. Caprio, P. Maris and J.P. Vary,
Phys. Lett. B 719, 179 (2013)

NCSM with Chiral NN (N3LO) without NNN interaction and OLS renormalization

B.R. Barrett et al. / Progress in Particle and Nuclear Physics 69 (2013) 131–181



Effective interactions in *sd*-shell from *ab-initio* shell model with a core

Preliminary Results

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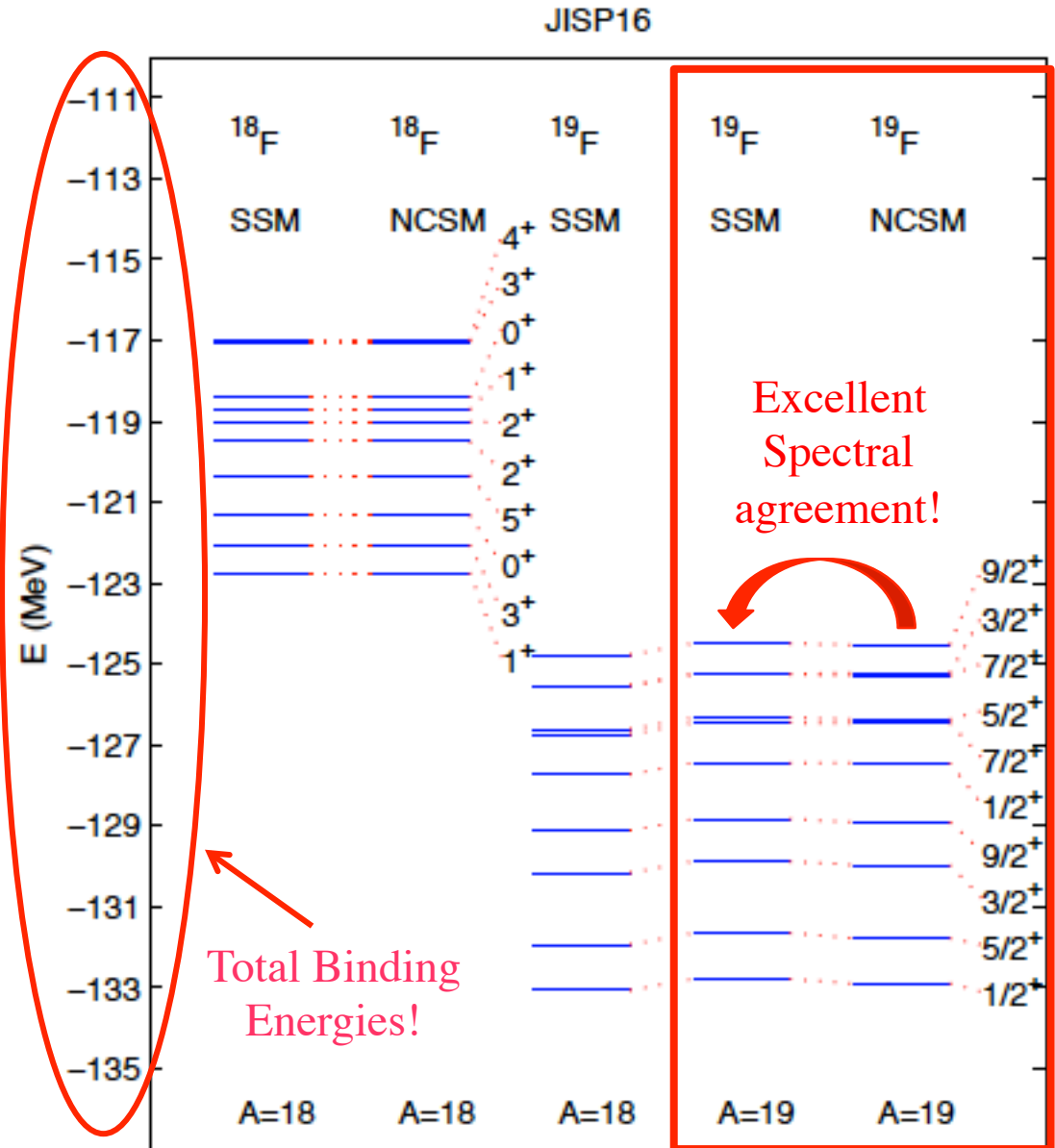
Aim: Regain valence-core separation
but retain full *ab initio* NCSM

=> “Double OLS” Approach

Now extend to *s-d* shell the
successful *p*-shell applications

p-shell application:

A. F. Lisetskiy, B. R. Barrett,
M. K. G. Kruse, P. Navratil,
I. Stetcu, J. P. Vary,
Phys. Rev. C. 78, 044302 (2008);
arXiv:0808.2187



Outstanding challenges for the *ab initio* No-Core Shell Model (NCSM) and No-Core Full Configuration (NCFC)

- Described the cluster states, such as the Hoyle state, in light nuclei
- Explain the shift between the positive and negative parity spectra in p-shell nuclei
- Explain the intruder negative parity ground state in ${}^9\text{B}$
- Explain the location of the $(1+,0)$ excited state in ${}^{12}\text{C}$
- Describe collective rotational bands in light nuclei – even the non-cluster cases
- Explain GTs to excited states in $A = 14$ (A. Negret, et al, Phys. Rev. Lett. 97, 062502 (2006))
- Include non-resonant continuum states and extend to reactions between light nuclei
- Extend the methods, while retaining precision, into the sd shell and beyond
- Continue to develop the methods for non-perturbative solutions of quantum field theory

Under what conditions do we need quarks & gluons to describe nuclear structure and nuclear reactions?

1. Spin crisis in the proton
2. Proton RMS radius
3. DIS on nuclei – e.g. Bjorken $x > 1$
4. Nuclear Equation of State
5. $Q > 1 \text{ GeV}/c$

Applications to Relativistic Quantum Field Theory QED (new) and QCD (under development)

J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng and C. Yang,
“Hamiltonian light-front field theory in a basis function approach”,
Phys. Rev. C 81, 035205 (2010); arXiv nucl-th 0905.1411

H. Honkanen, P. Maris, J. P. Vary and S. J. Brodsky,
“Electron in a transverse harmonic cavity”,
Phys. Rev. Lett. 106, 061603 (2011); arXiv: 1008.0068

Pieter Maris, Paul Wiecki, Yang Li, Xingbo Zhao and James P. Vary,
“Bound state calculations in QED and QCD using Basis Light-Front Quantization,”
Acta Phys. Polon. Supp. 6, 321(2013).

X. Zhao, A. Ilderton, P. Maris and J.P. Vary,
“Scattering in Time-dependent Basis Light Front Quantization,”
arXiv 1303.3237

Basis Light-Front Quantization Approach

[Dirac 1949]

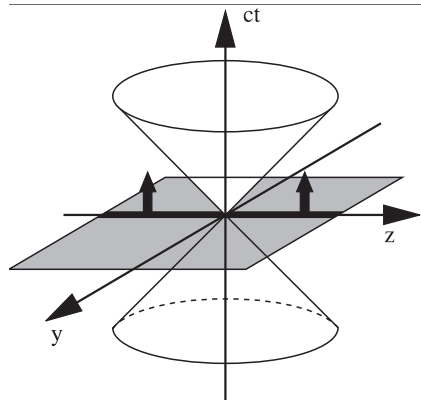
- Basic idea: solve generalized wave eq. for quantum field evolution

equal time quantization

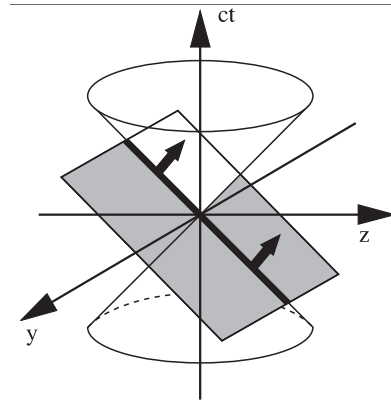


light front quantization

$$i \frac{\partial}{\partial t} |\varphi(t)\rangle = H |\varphi(t)\rangle$$



$$i \frac{\partial}{\partial x^+} |\varphi(x^+)\rangle = P_+ |\varphi(x^+)\rangle$$



- **Time:** $t \equiv x^0$

$$t \equiv x^+ = x^0 + x^3$$

- **Hamiltonian:** $H \equiv P^0$

$$H \equiv P_+ = \frac{P^0 - P^3}{2}$$

- **On-shell condition:** $P^0 = \sqrt{m^2 + P_\perp^2 + P_3^2}$

$$P_+ = \frac{m^2 + P_\perp^2}{2P^+}$$

Discretized Light Cone Quantization

Pauli & Brodsky c1985



Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} [f_{\alpha}(\vec{x})a_{\alpha}^{+} + f_{\alpha}^{*}(\vec{x})a_{\alpha}]$$

where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_{\alpha}(\vec{x})f_{\alpha'}^{*}(\vec{x})d^3x = \delta_{\alpha\alpha'}$

Complete: $\sum_{\alpha} f_{\alpha}(\vec{x})f_{\alpha}^{*}(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_{\alpha}(\vec{x})$ and our initial choice is

$$f_{\alpha}(\vec{x}) = Ne^{ik^{+}x^{-}} \Psi_{n,m}(\rho, \varphi) = Ne^{ik^{+}x^{-}} f_{n,m}(\rho) \chi_m(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Steps to implement BLFQ

- Enumerate Fock-space basis subject to symmetry constraints
- Evaluate/renormalize/store H in that basis
- Diagonalize (Lanczos)
- Iterate previous two steps for sector-dep. renormalization
- Evaluate observables using eigenvectors (LF amplitudes)
- Repeat previous 4 steps for new regulator(s)
- Extrapolate to infinite matrix limit & remove all regulators
- Compare with experiment or predict new experimental results

Above achieved for QED test case – electron in a trap

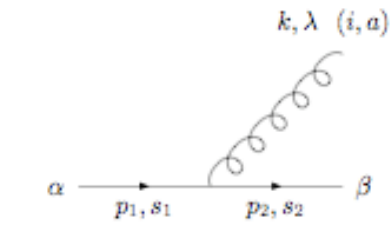
H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky,

Phys. Rev. Lett. 106, 061603 (2011)

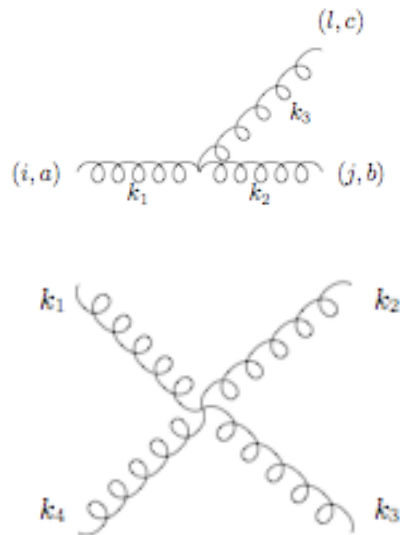
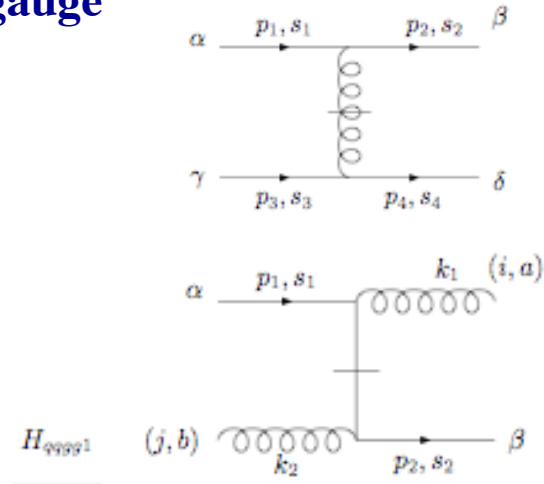
Improvements: trap independence, (m,e) renormalization, . . .

X. Zhao, H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, in prep'n

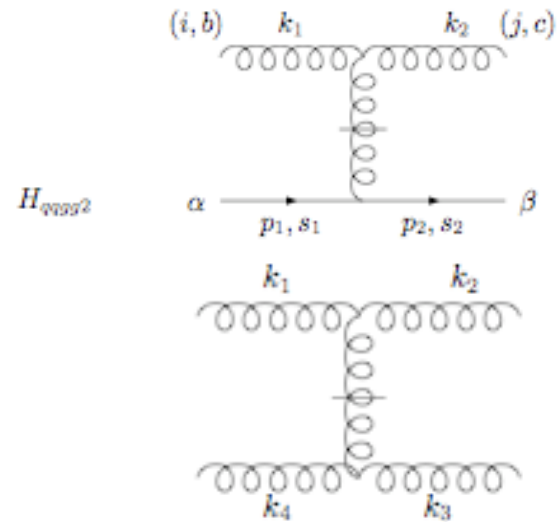
Light Front (LF) Hamiltonian defined by its elementary vertices in LF gauge



QED & QCD

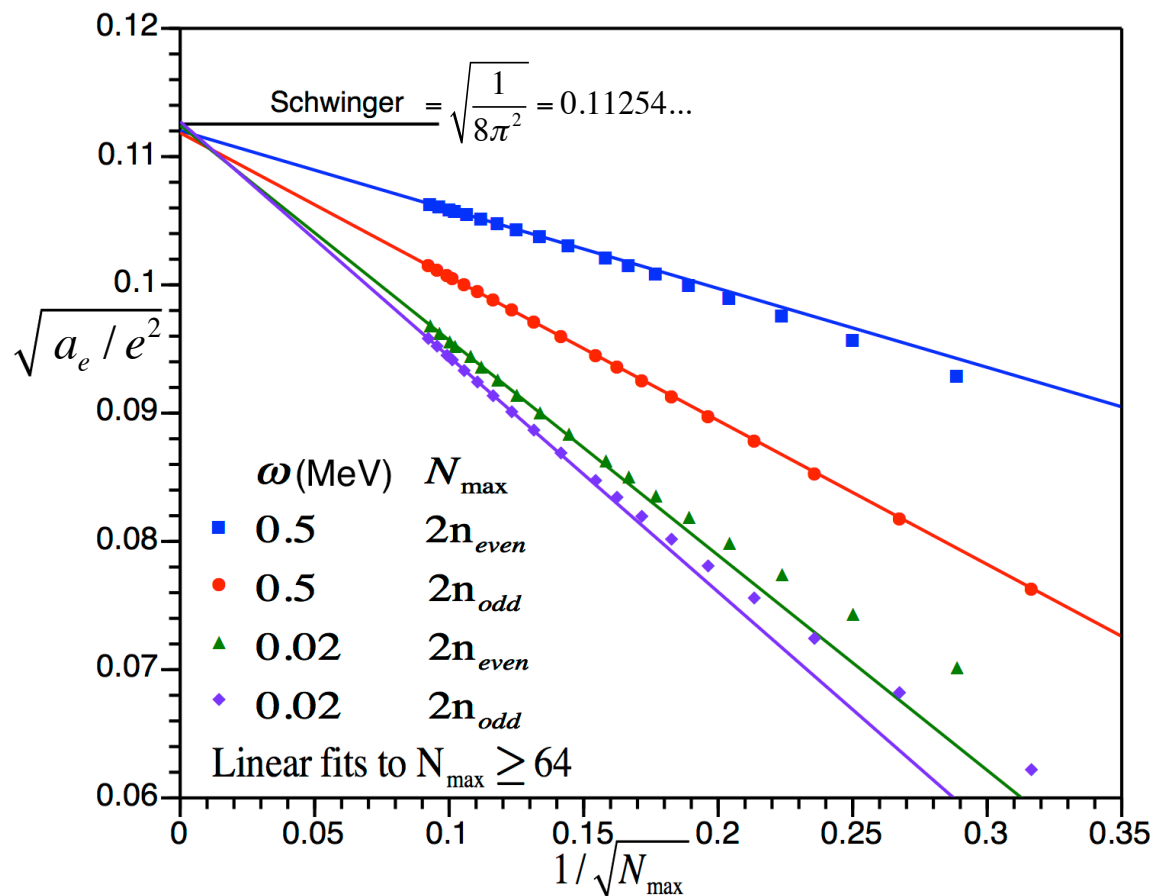


QCD



Numerical Results for Electron g-2

[X. Zhao, H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, in preparation as major update to:
H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, Phys. Rev. Lett. 106, 061603 (2011)]

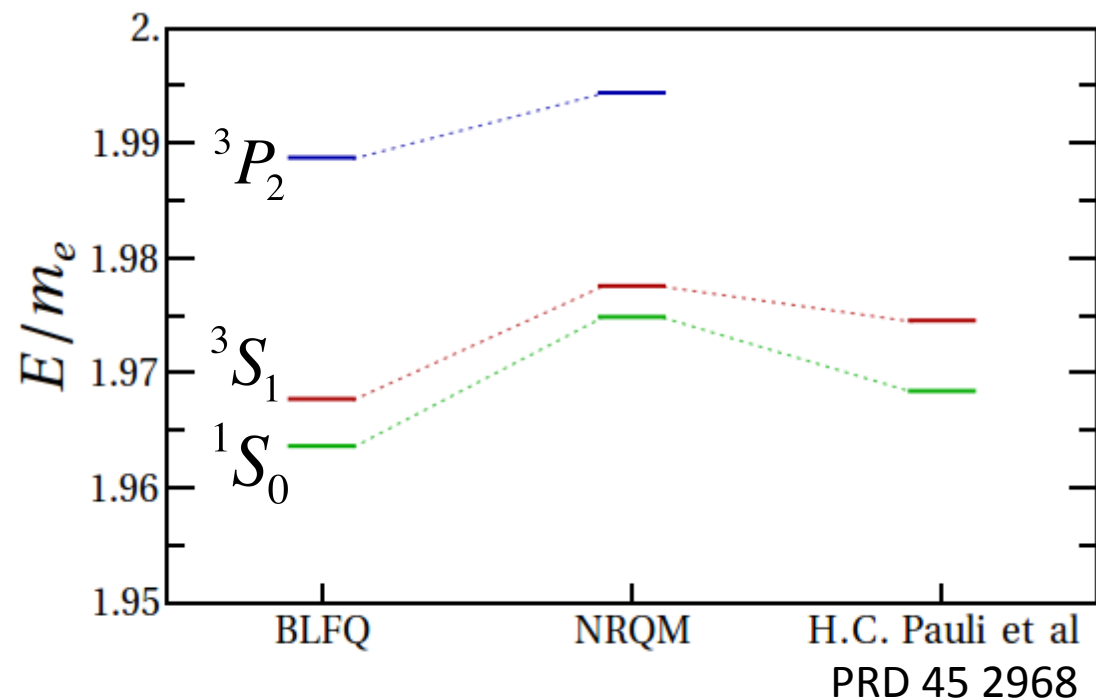


- As $N_{\max} \rightarrow \infty$, results approach Schwinger result
- Less than **1%** deviation from Schwinger's result (by linear extrapl.)
- Convergence over **wide** range of ω 's (by a factor of 25!)

Applications to QED and QCD bound states (positronium, mesons, baryons) are underway:
Paul Wiecki, NTSE-2013 Proceedings, to appear
Yang Li, NTSE-2013 Proceedings, to appear

Positronium with $\alpha = 0.3$

Involves three extrapolations to remove regulators



Outstanding Challenges

- ❖ improve NN + NNN + NNNN interactions/renormalization
develop effective operators beyond the Hamiltonian
tests of fundamental symmetries
- ❖ proceed to heavier systems - breaking out of the p-shell
extend quantum many-body methods
- ❖ achieve higher precision
quantify the uncertainties - justified through simulations
global dependencies mapped out
- ❖ evaluate more complex projectile-target reactions
- ❖ achieve efficient use of computational resources – improve
scalability, load-balance, I/O, inter-process communications
- ❖ build a community to develop/sustain open libraries of codes/data,
develop/implement provenance framework/practices

clusters, halo nuclei, continuum coupling, delta-full EFT,
EFT for all observables, energy density functional (EDF), nuclear EOS,
dark matter searches, double beta-decay, nuclear parton distribution
functions (PDFs), . . .

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Markus Kortelainen, Witek Nazarewicz,
Gaute Hagen, Thomas Papenbrock
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Russia: Vladimir Karmanov

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Questions?