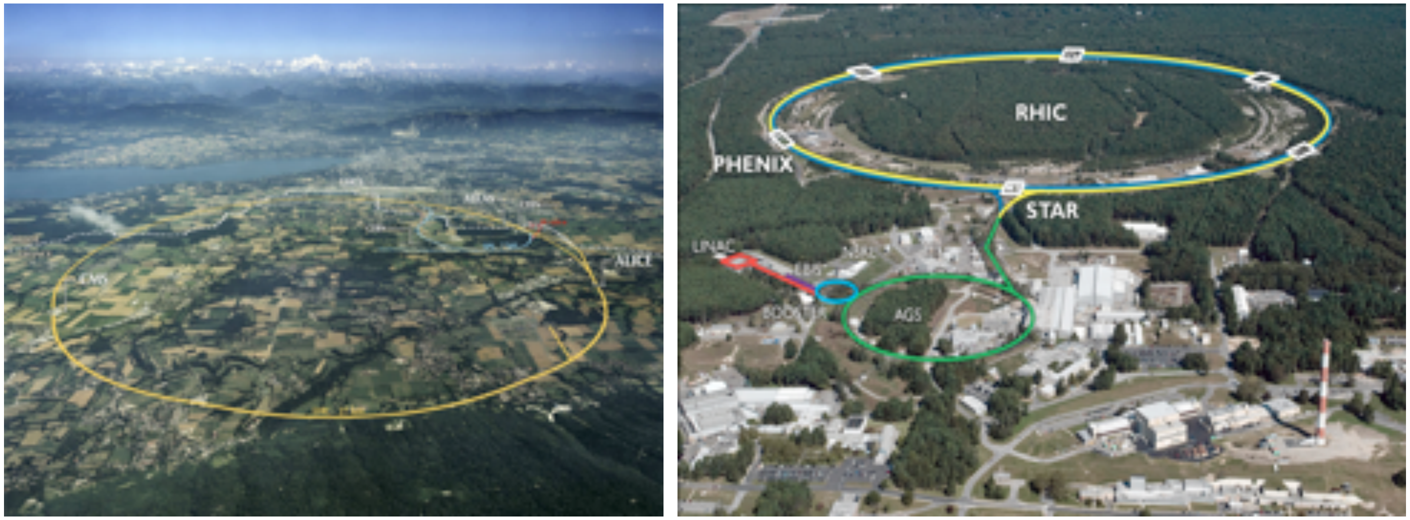
early microscopes



the World's most powerful microscopes

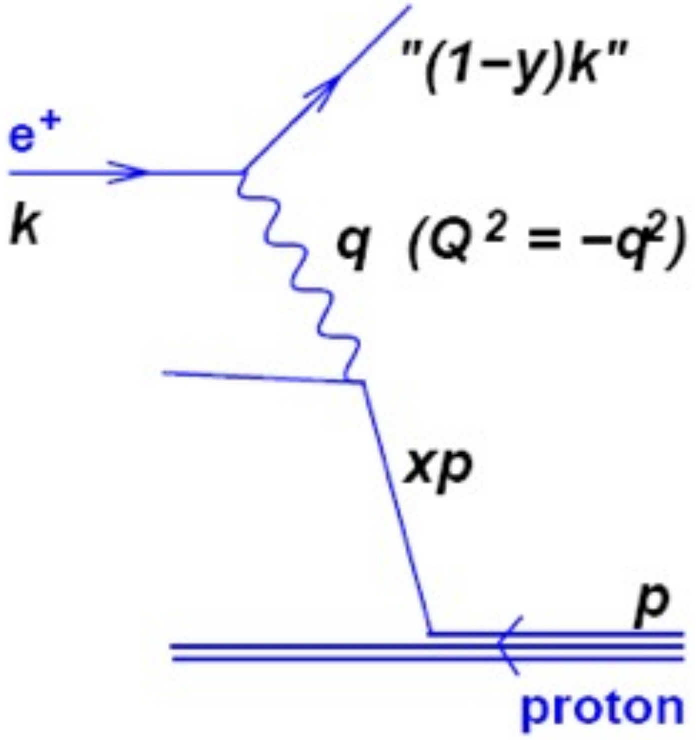
Part III

inward bound: "femto-spectroscopy"

QCD initial state, partons, DIS, factorization, renormalization group, hadron-hadron collisions

partons in the initial state: the DIS process

start with the simplest process: **deep-inelastic scattering**



relevant kinematics:

$$x = \frac{Q^2}{2p \cdot q} \qquad y = \frac{p \cdot q}{p \cdot k} \qquad Q^2 = xys$$

- Q^2: photon virtuality ↔ resolution $r \sim 1/Q$ at which the proton is probed
- **x**: long. momentum fraction of struck parton in the proton
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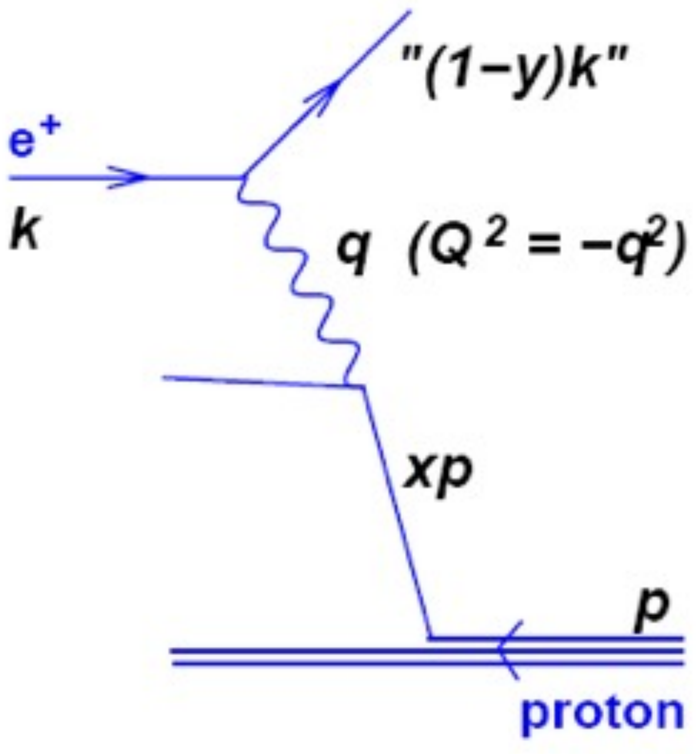

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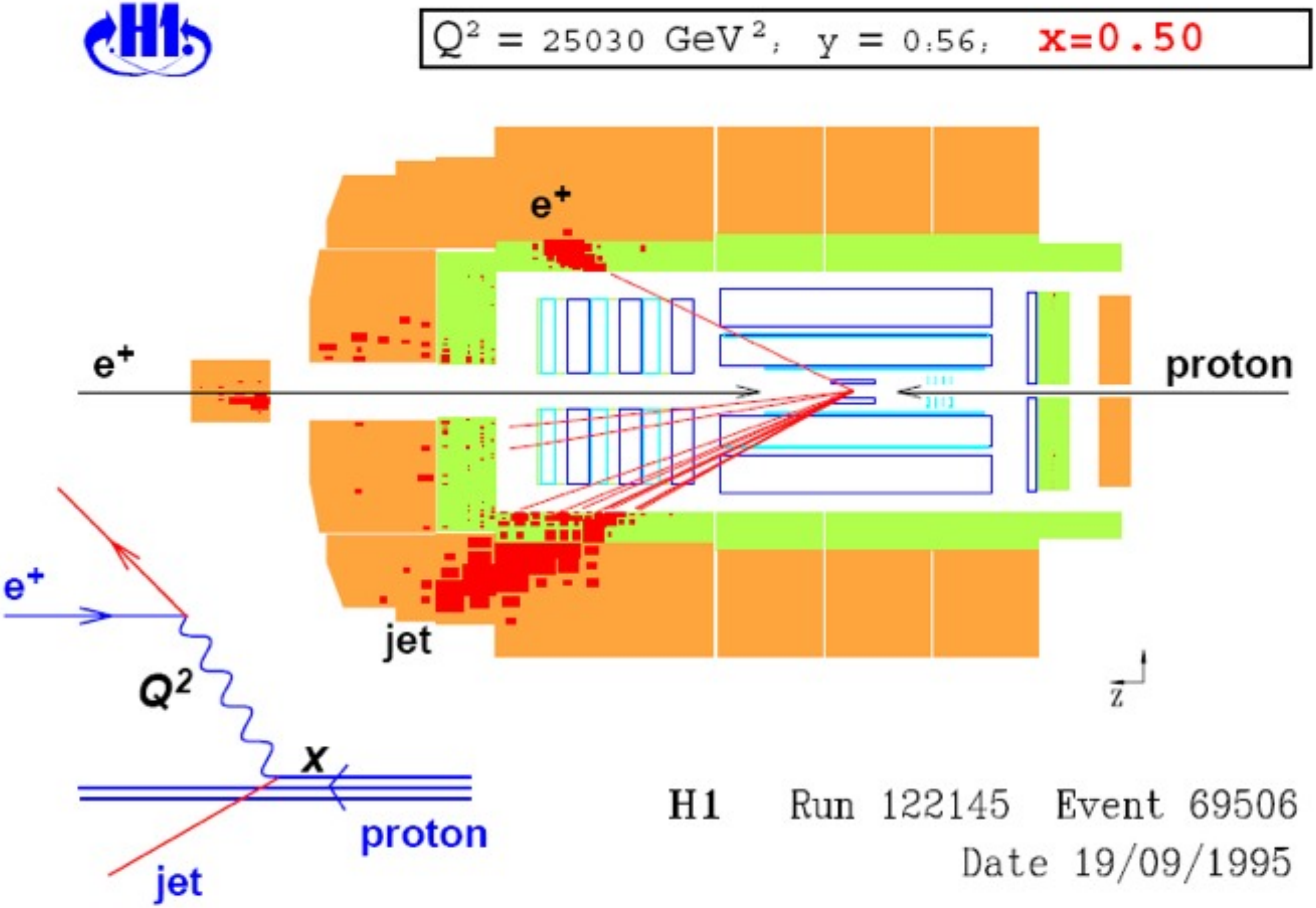
"deep-inelastic": $Q^2 \gg 1$ GeV^2

"scaling limit": $Q^2 \to \infty$, x fixed

resolution: $\frac{\hbar}{Q} \approx \frac{2 \times 10^{-16}\mathrm{m}}{Q[\mathrm{GeV}]}$

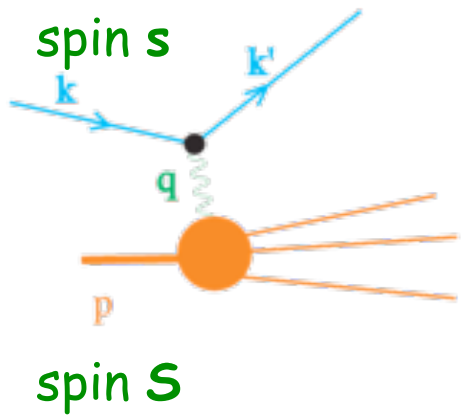
$r \sim 1/Q$

a typical DIS event



analysis of DIS: 1st steps

electroweak theory tells us how the virtual vector boson (here γ^*) couples:



$$d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q,s)\boxed{W_{\mu\nu}(p,q,S)}$$

leptonic
tensor
from QED

hadronic tensor
contains information
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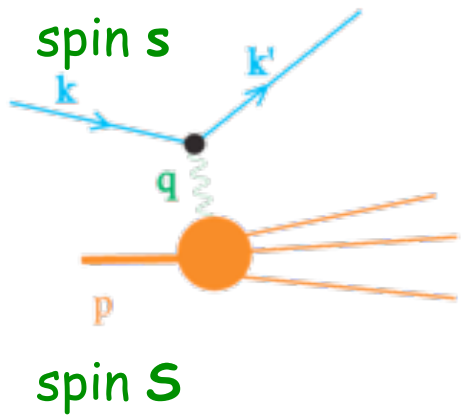

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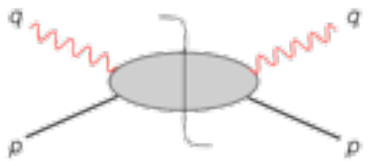
$$\mathcal{W}^{\mu\nu}(P,q,S) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\,\langle P,S|\,J_\mu(z)\,J_\nu(0)\,|P,S\rangle$$



$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)F_2(x,Q^2)$$

$$+\,i\,M\,\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{P\cdot q}g_1(x,Q^2) + \frac{S_\sigma(P\cdot q)-P_\sigma(S\cdot q)}{(P\cdot q)^2}g_2(x,Q^2)\right]$$

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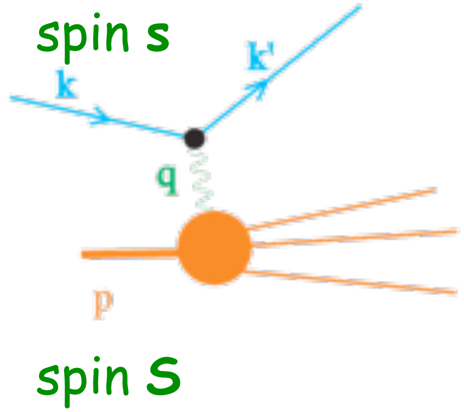

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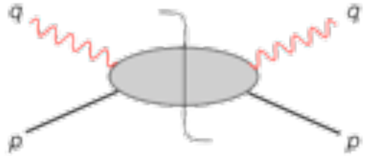

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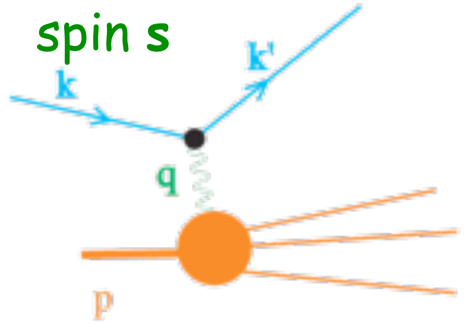

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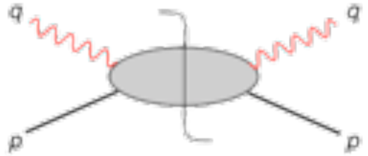

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pol. structure fcts. $g_{1,2}$ – measure W(P,q,S) – W(P,q,-S) !

DIS in the naïve parton model

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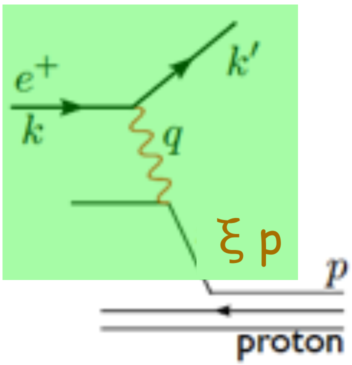

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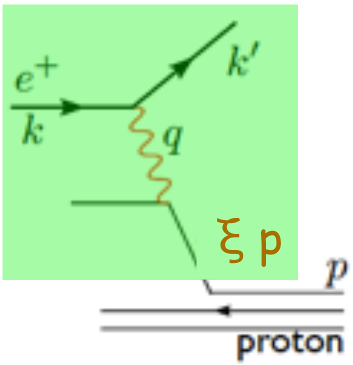

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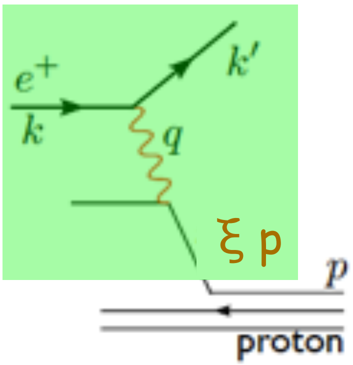

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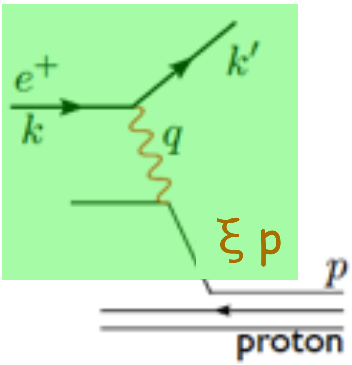

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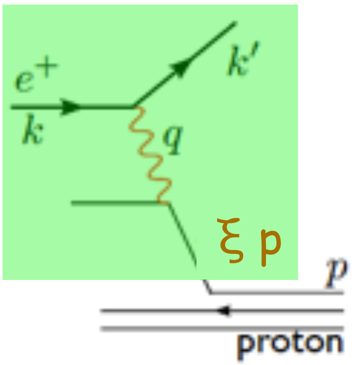

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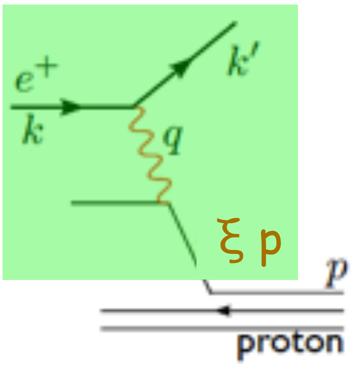

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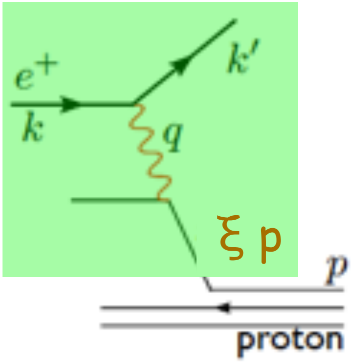

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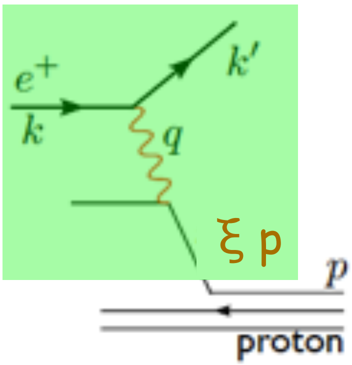

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this implies that ξ is equal to Bjorken x

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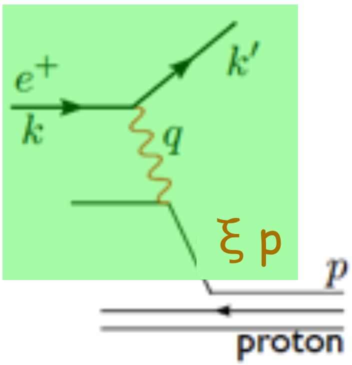

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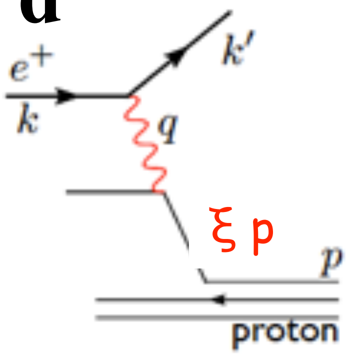

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$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}[1+(1-y)^2]\frac{1}{2}e_q^2\delta(x-\xi)$$

to what one obtains with the hadronic tensor (on the quark level)

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1(x) + \frac{(1-y)}{x}(F_2(x) - 2xF_1(x))\right]$$

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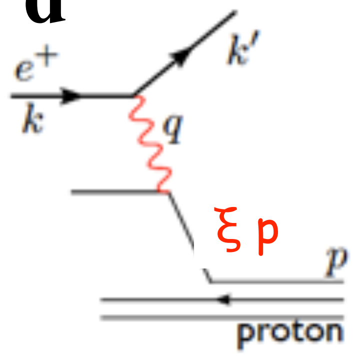

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Callan Gross relation

reflects spin 1/2 nature of quarks

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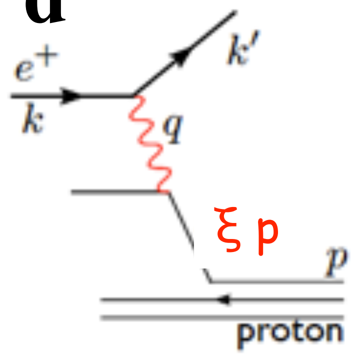

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proton structure functions then obtained by weighting the quark str. fct. with the **parton distribution functions** (probability to find a quark with momentum ξ)

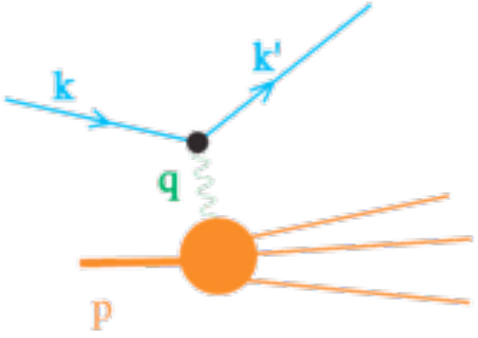
$$F_2 = 2xF_1 = \sum_{q,q'}\int_0^1 d\xi\, q(\xi)\, xe_q^2\,\delta(x-\xi)$$

$$= \sum_{q,q'} e_q^2\, x\, q(x)$$

DIS measures the charged-weighted sum of quarks and antiquarks

“scaling” - no dependence on scale Q

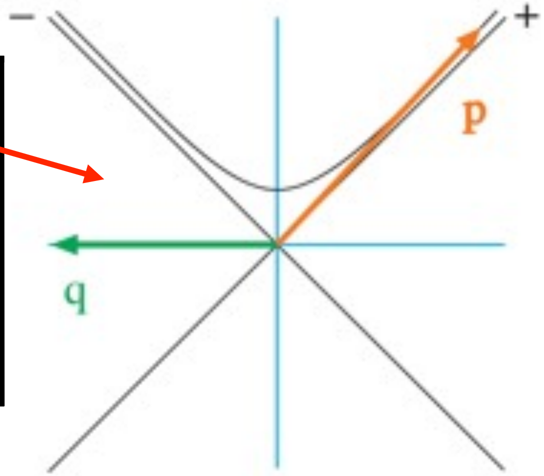
space-time picture of DIS



this can be best understood in a reference frame
where the proton moves very fast and $Q \gg m_h$ is big

(recall light-cone kinematics from part II)

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



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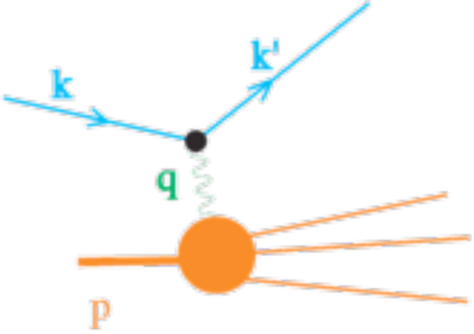

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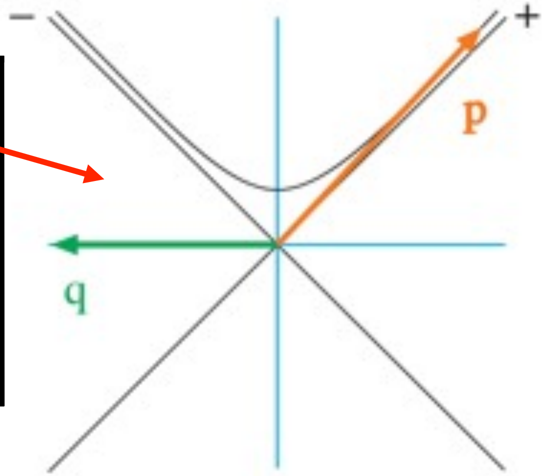
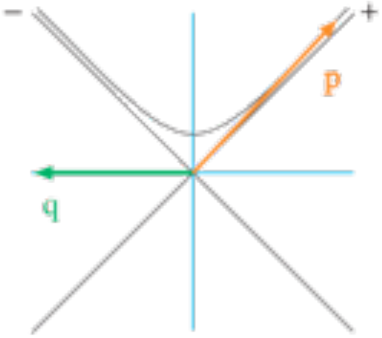


Lorentz boost

in general $(a^+, a^-, \vec{a}_T) \rightarrow (e^{\omega} a^+, e^{-\omega} a^-, \vec{a}_T) = (a'^+, a'^-, \vec{a}')$

here: $e^{\omega} = Q/(x m_h)$

space-time picture of DIS – cont'd



simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:

rest frame: $\Delta x^+ \sim \Delta x^- \sim \frac{1}{m}$

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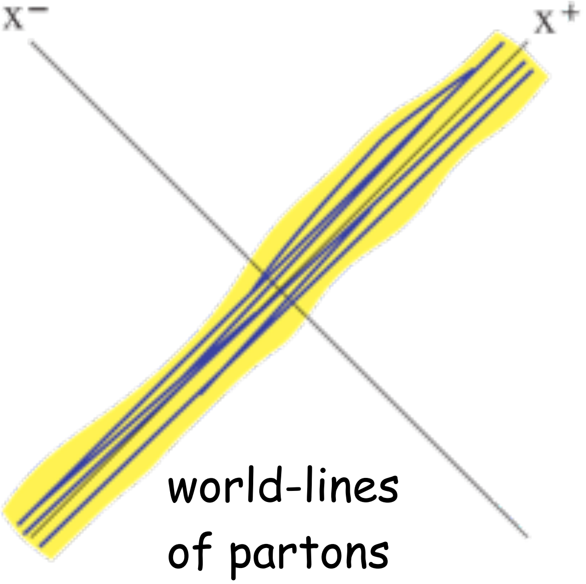

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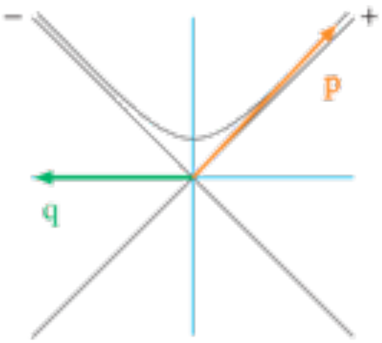

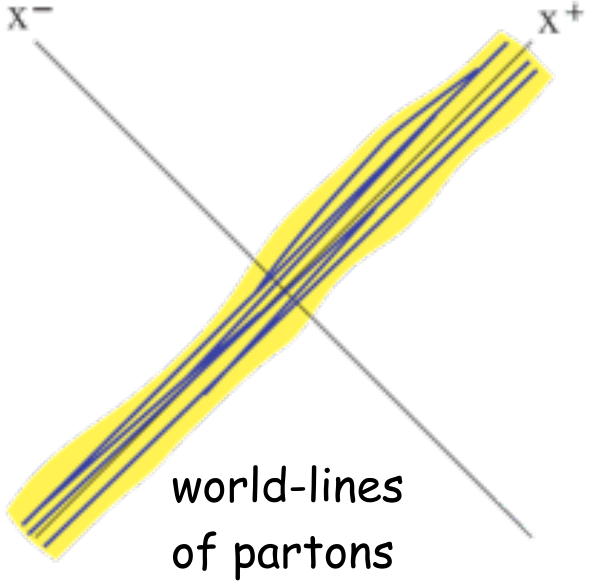

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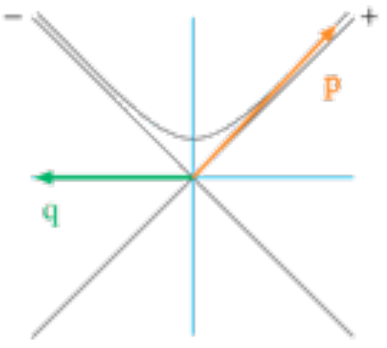

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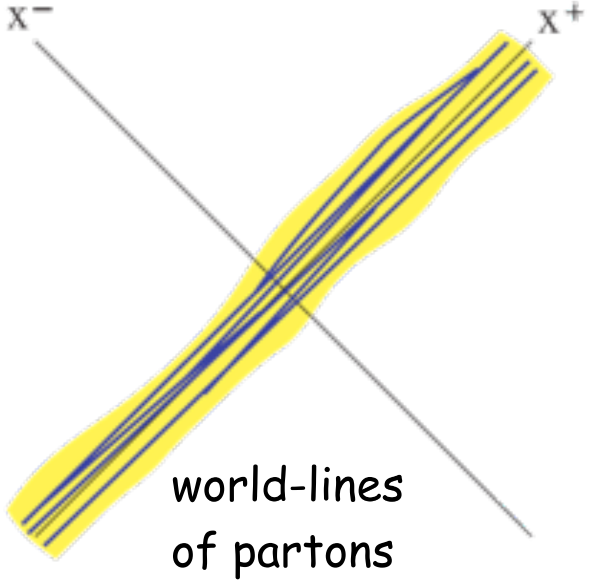


interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?

foundation of naïve Parton Model

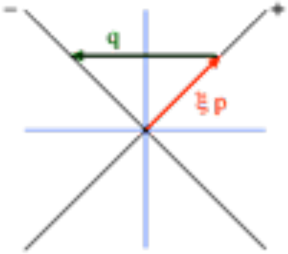
Feynman;
Bjorken, Paschos

Breit frame:

proton moves very fast and $Q \gg m_h$ is big

$$(p^+, p^-, \vec{p}_T) = \frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0}) \quad (q^+, q^-, \vec{q}_T) = \frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$$

struck quark
on-shell



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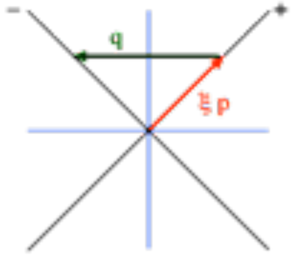

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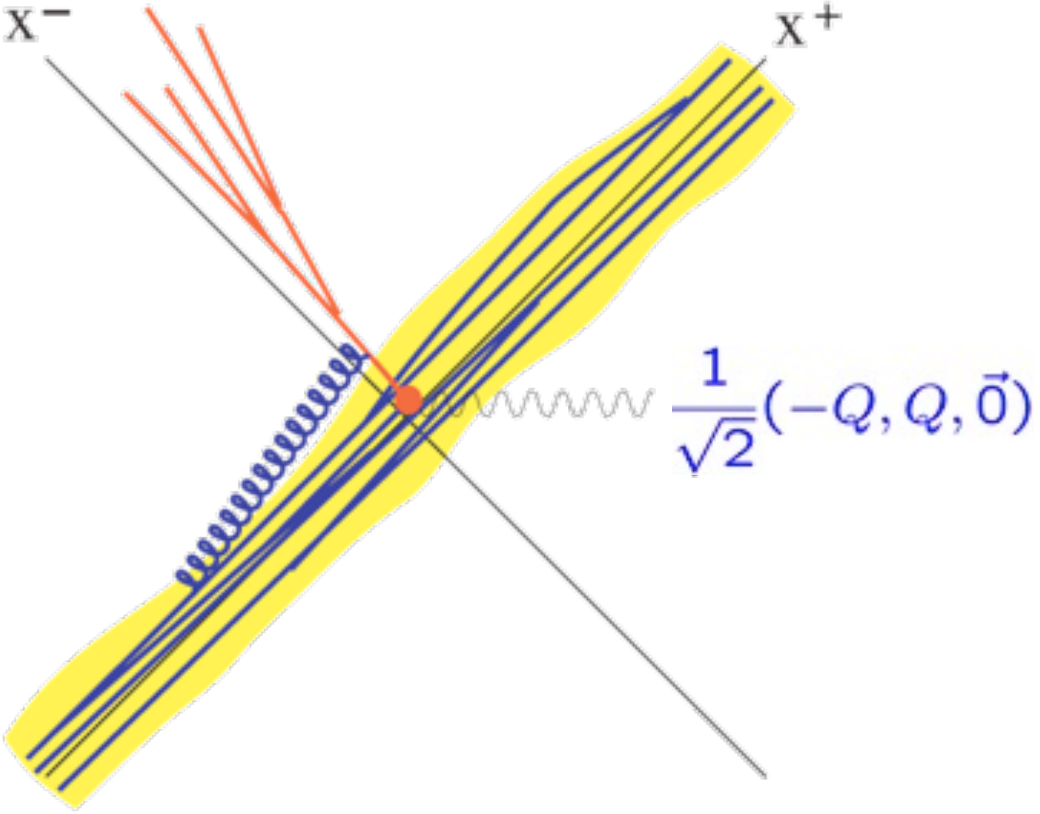

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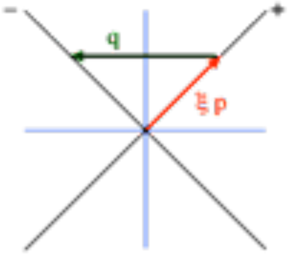

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interactions of
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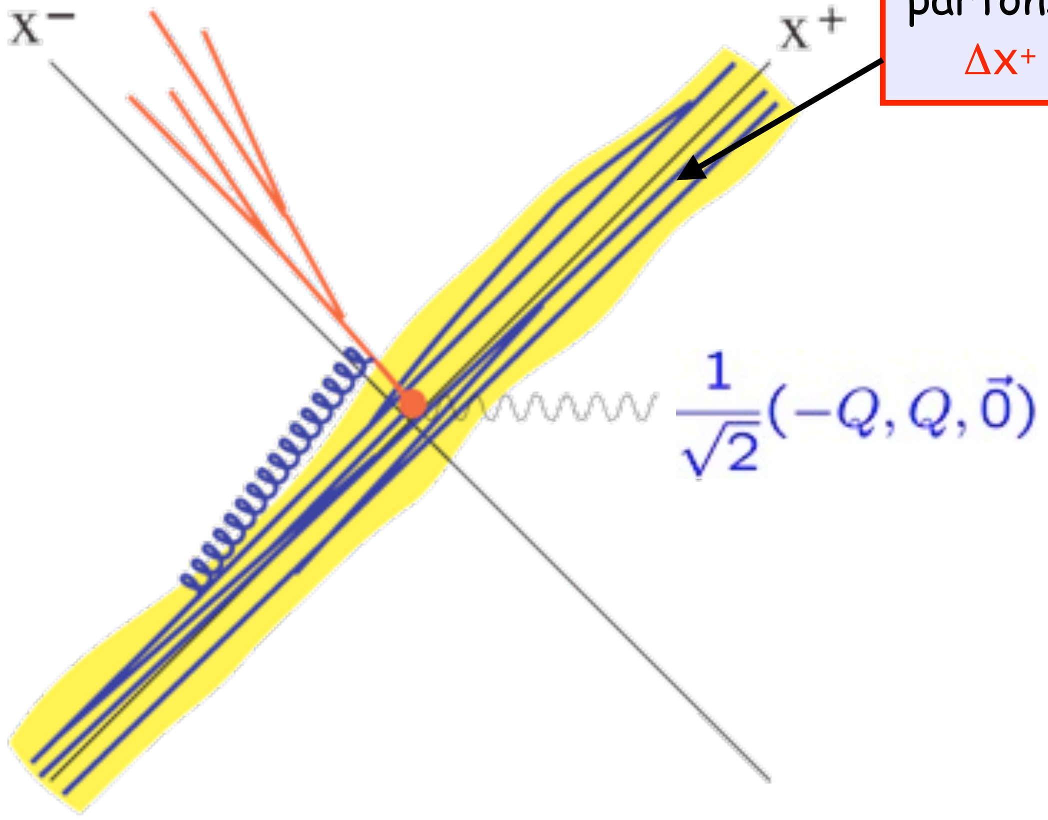

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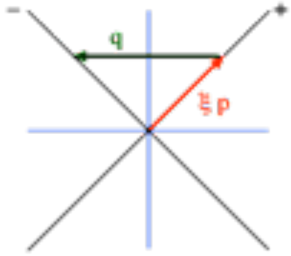

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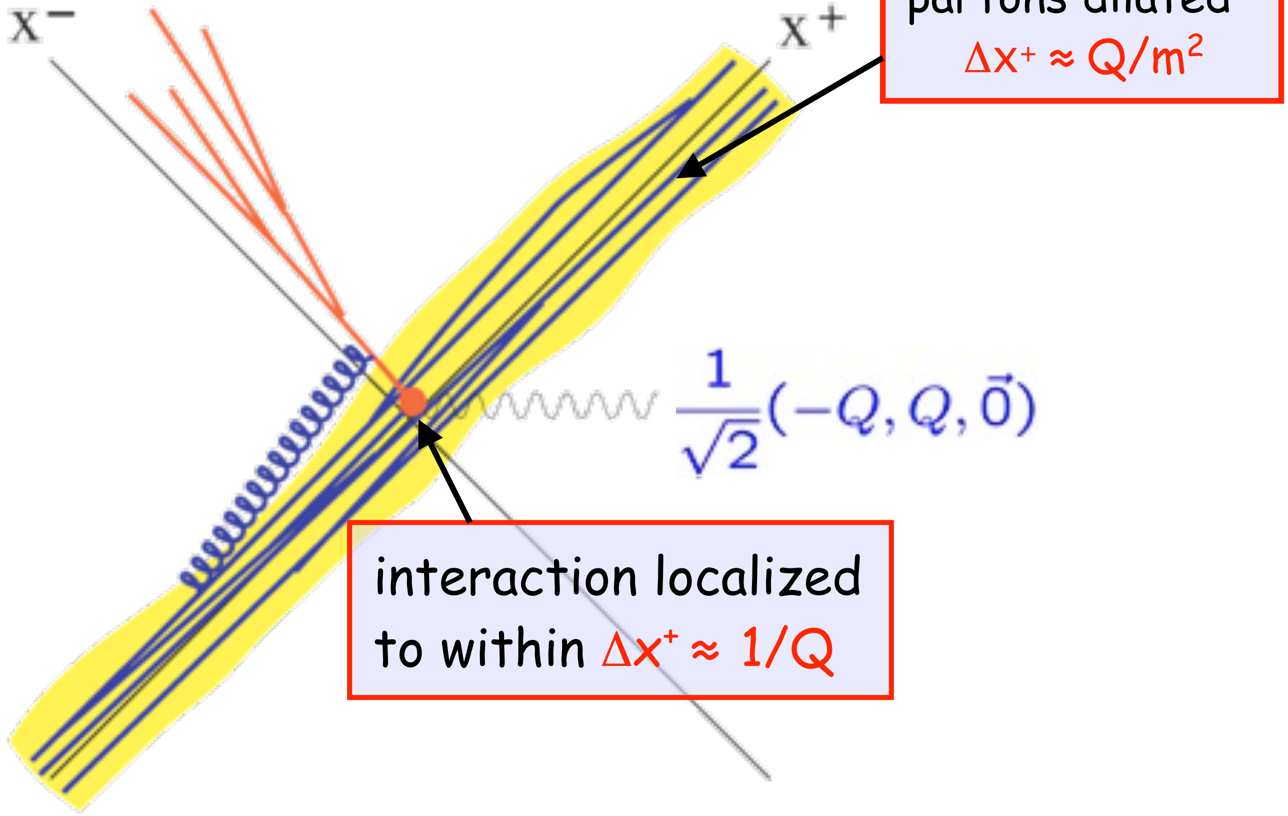

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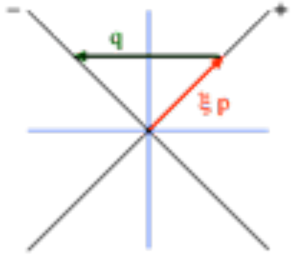

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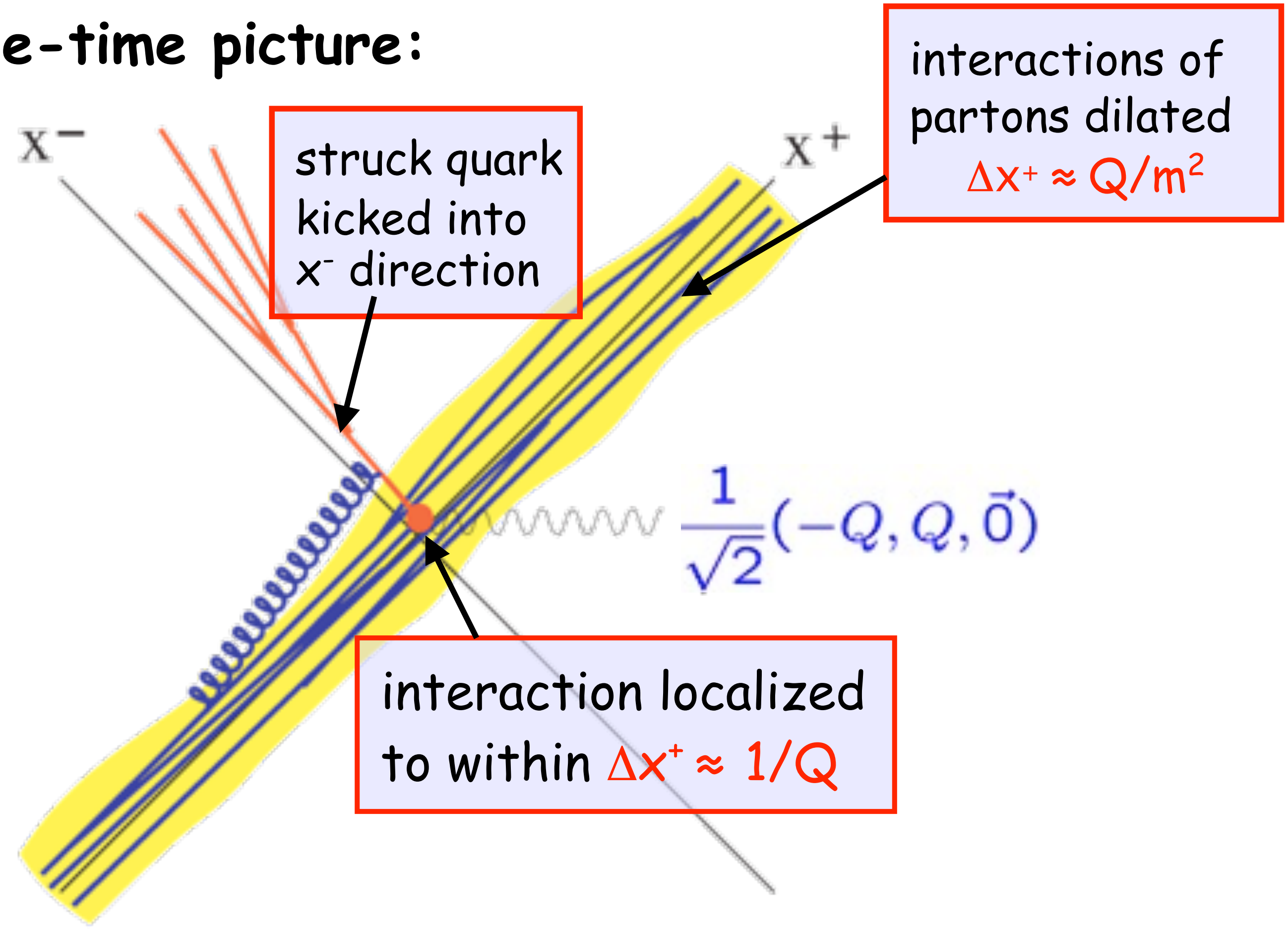

struck quark kicked into x^- direction

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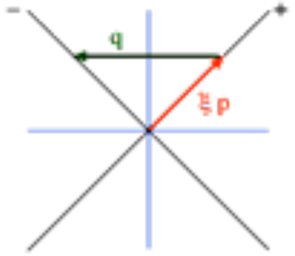

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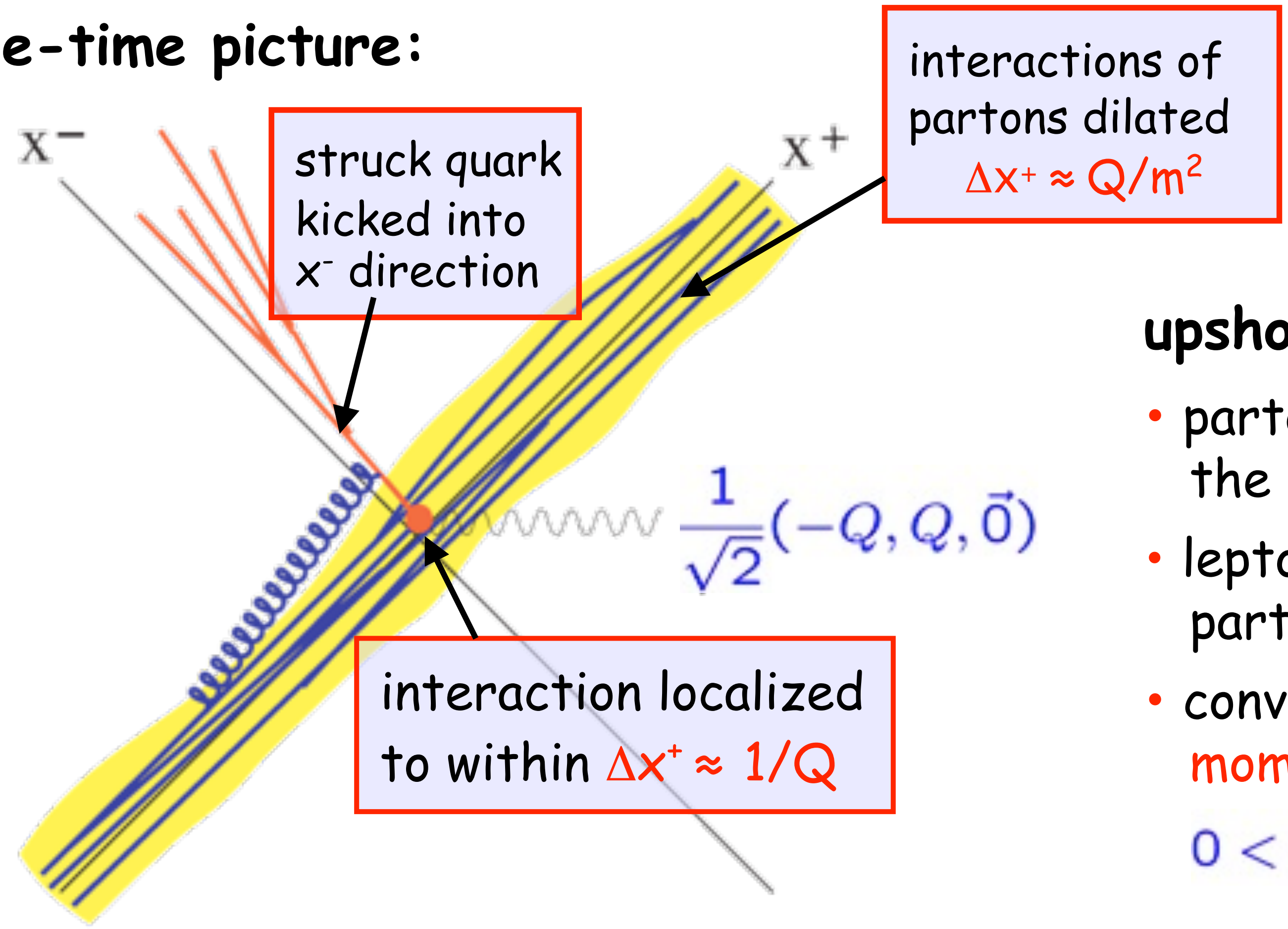

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interactions of partons dilated $\Delta x^+ \approx Q/m^2$

interaction localized to within $\Delta x^+ \approx 1/Q$

upshot:

- partons are free during the hard interaction
- lepton scatters off free partons incoherently
- convenient to introduce momentum fractions

$0 < \xi_i \equiv p_i^+/p^+ < 1$

sum rules and isospin

for the quark distributions in a proton there are several sum rules to obey

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

momentum sum rule

quarks share proton momentum

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

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flavor sum rules

conservation of quantum numbers

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flavor sum rules

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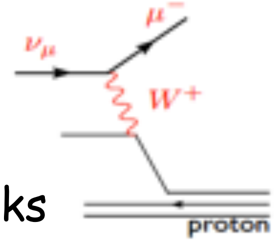
isospin symmetry relates a neutron to a proton (just u and d interchanged)

$$F_2^n(x) = x \left(\frac{1}{9} d_n(x) + \frac{4}{9} u_n(x) \right) = x \left(\frac{4}{9} d_p(x) + \frac{1}{9} u_p(x) \right)$$

- measuring both allows to determine u^p and d^p separately
- note: CC DIS couples to weak charges and separates quarks and antiquarks

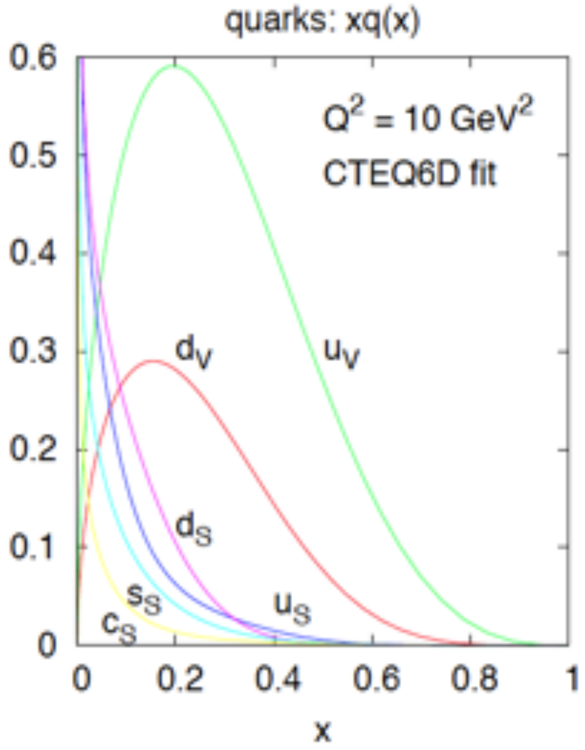


momentum sum rule in the naïve parton model

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

half of the momentum is missing



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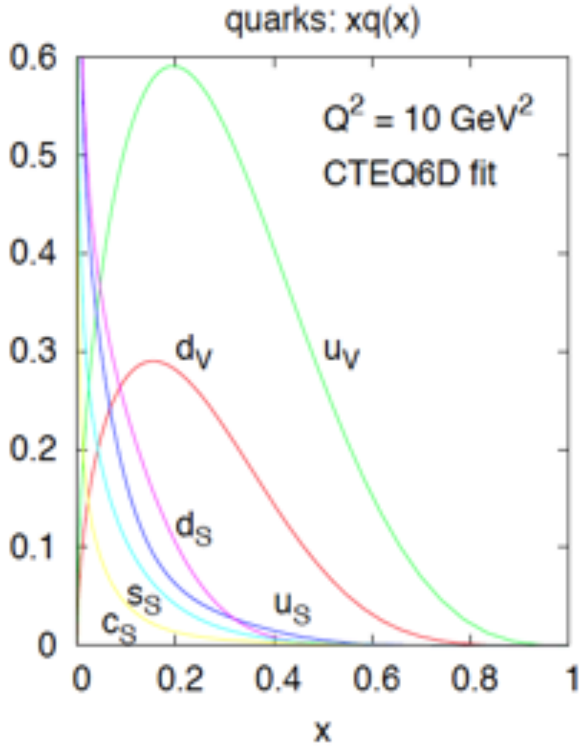

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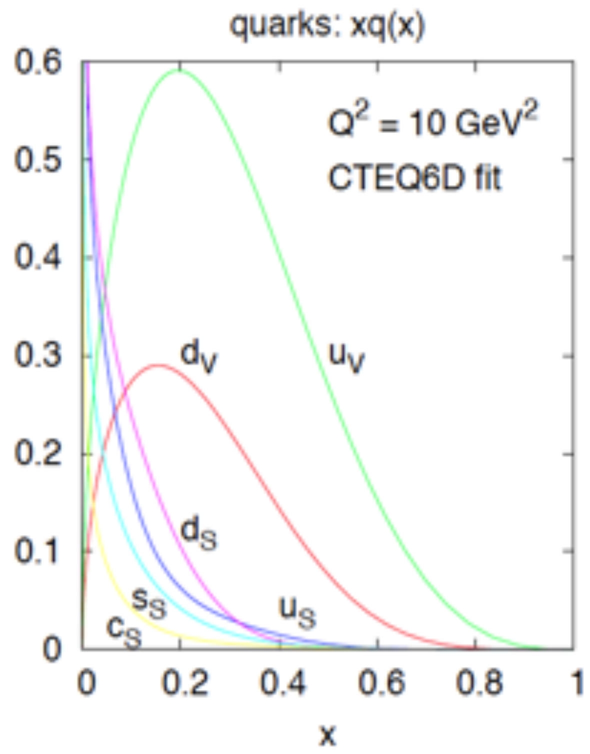

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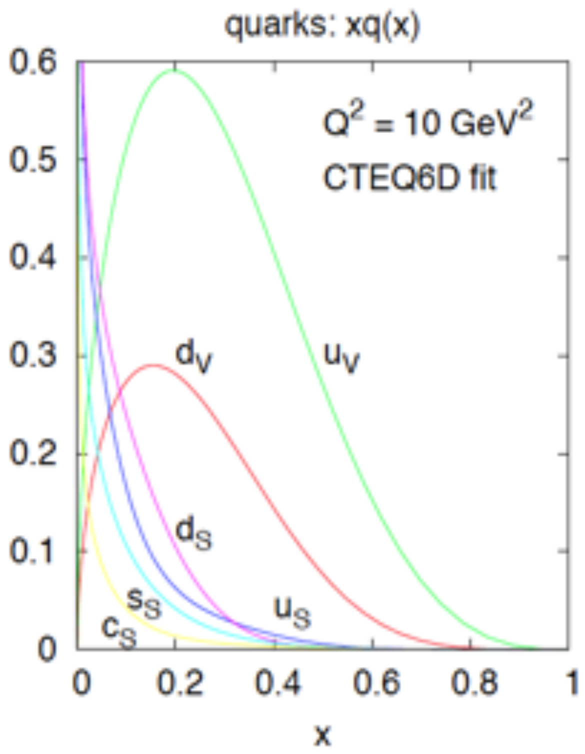

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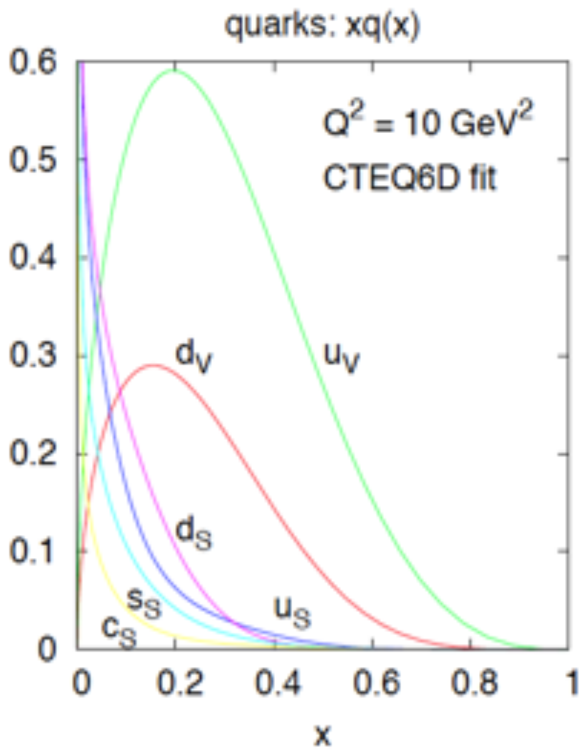

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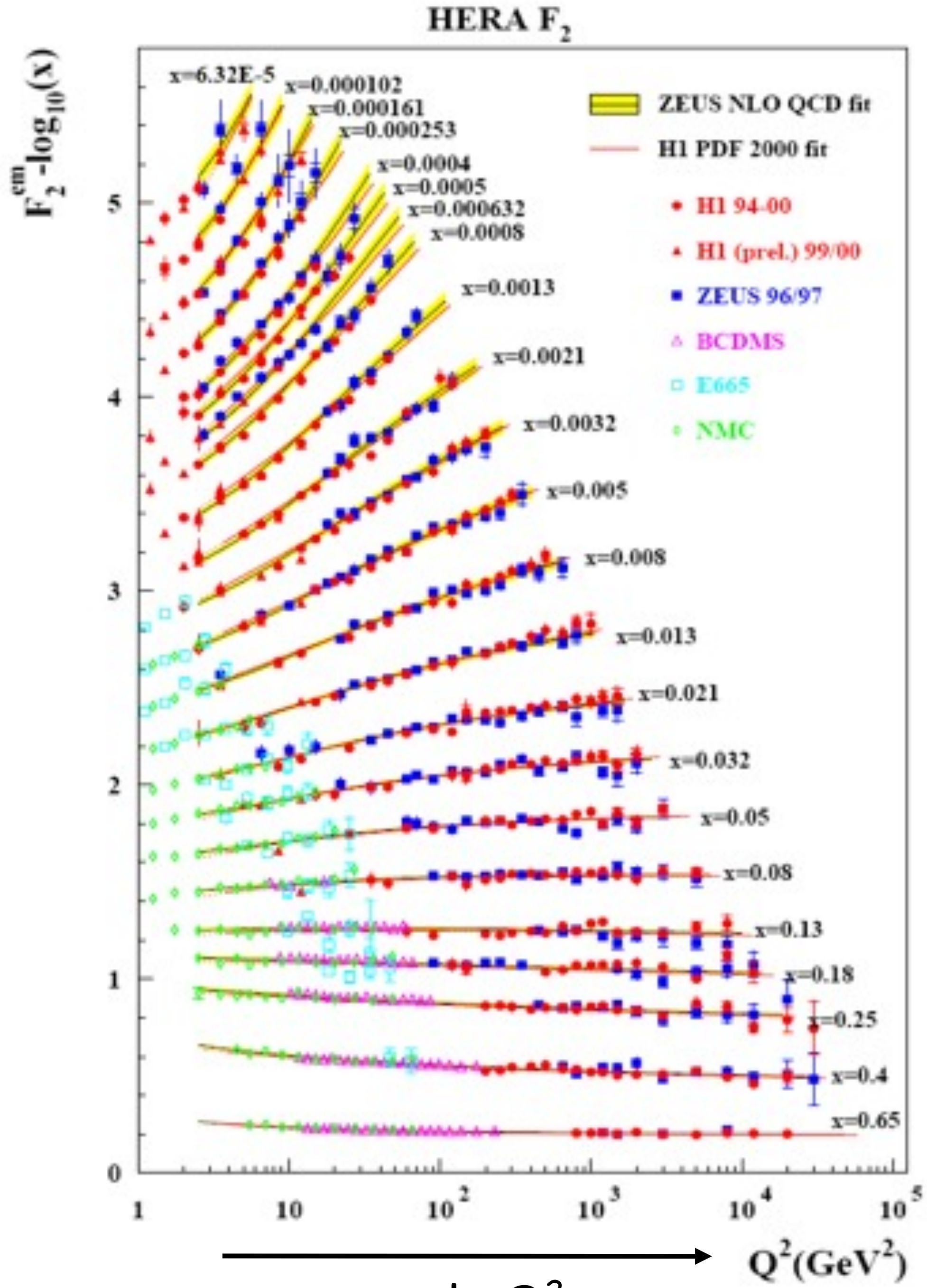
gluons will enter the game and everything will become scale dependent

Naïve parton model vs. experiment



scale Q^2

find strong scaling violations

Naïve parton model vs. experiment

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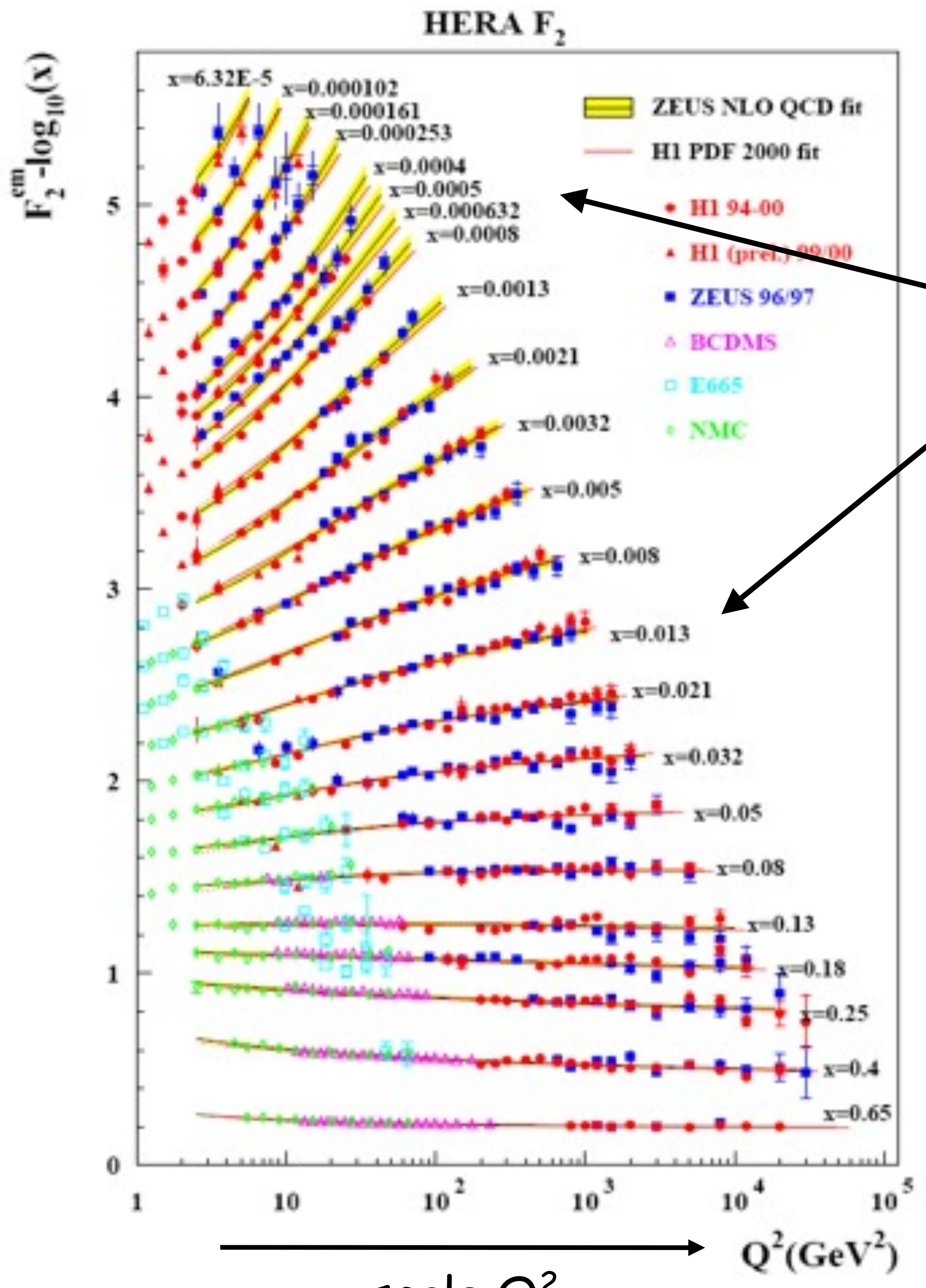

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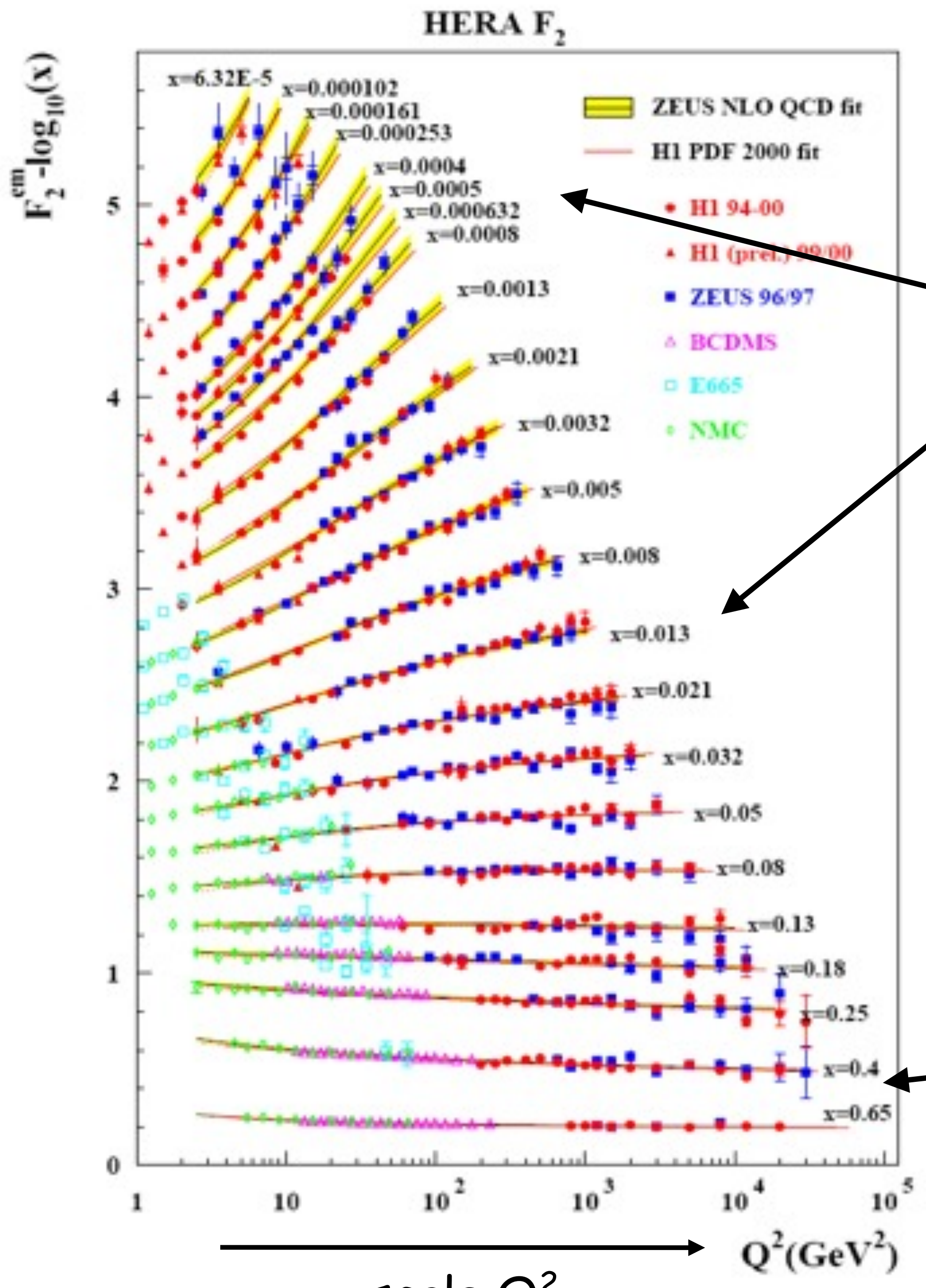

significant rise at small x

decrease at high x

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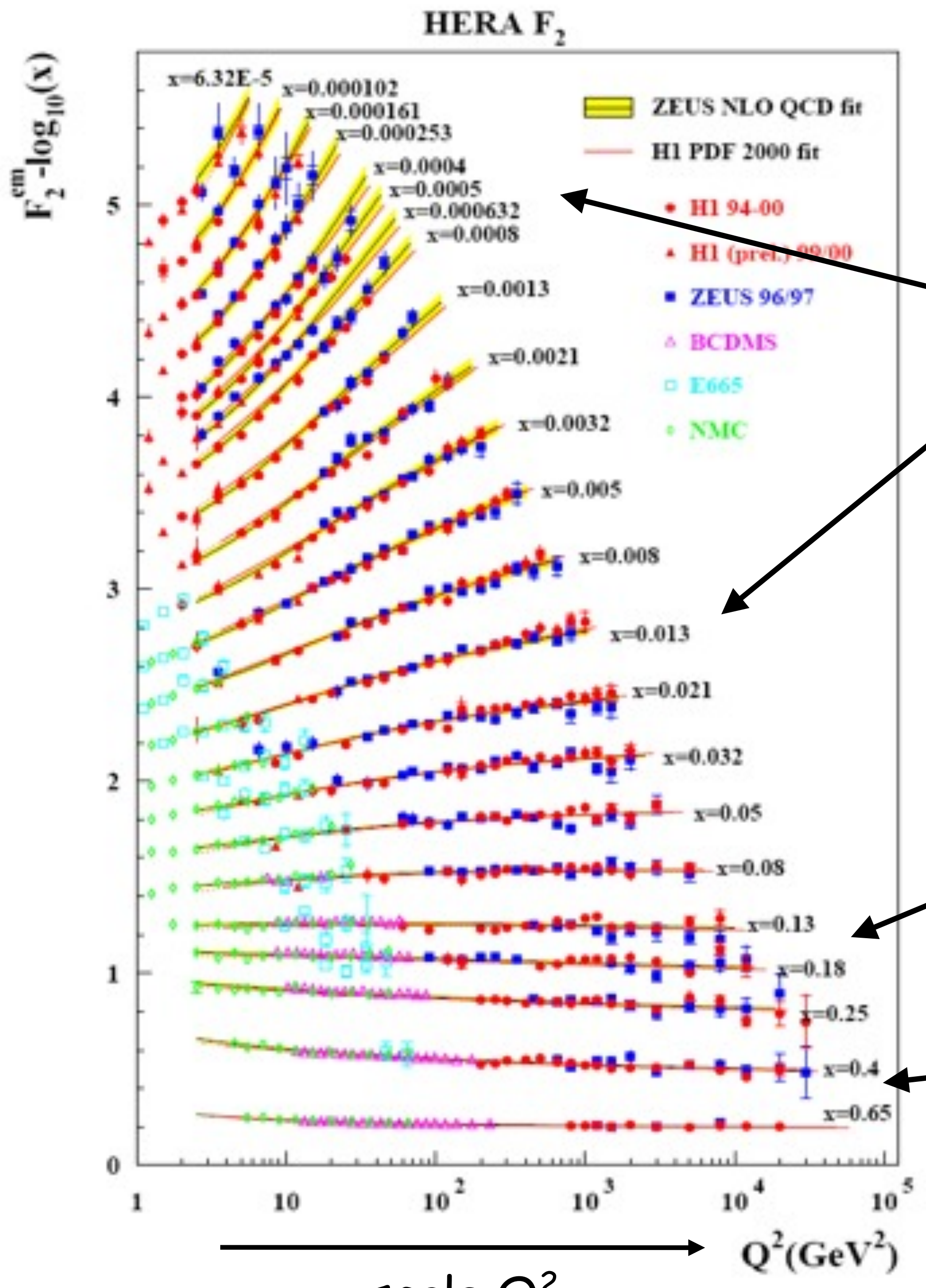


scale Q^2

find strong scaling violations

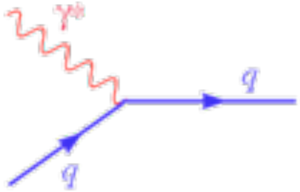
significant rise at small x

approximate scaling only around x ≃ 0.15

decrease at high x

DIS in the QCD improved parton model

we got a long way (parton model) without invoking QCD



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- this leads to similar problems already encountered in e^+e^-

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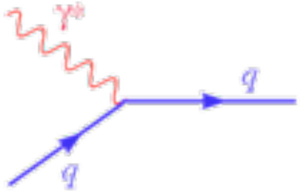

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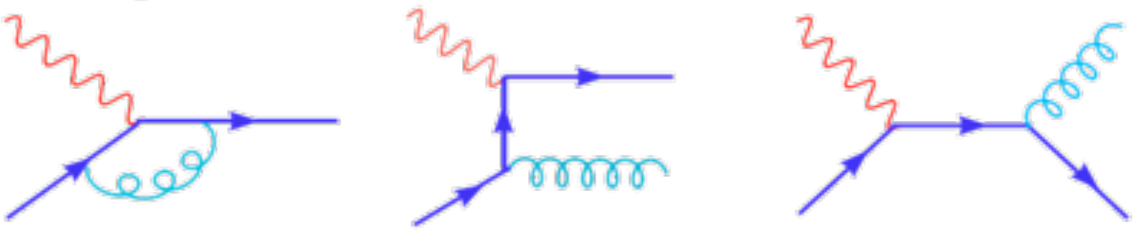
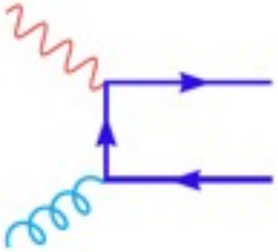


α_s corrections to the LO process



photon-gluon fusion

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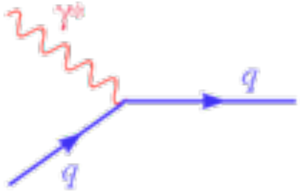

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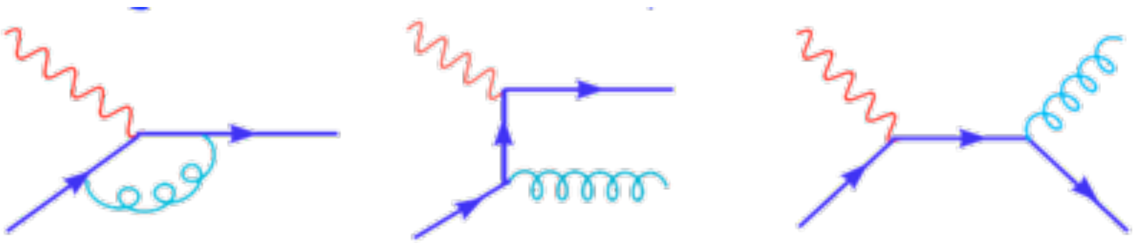

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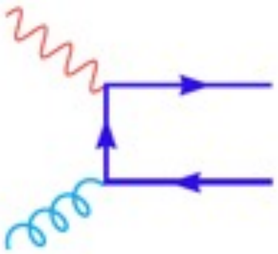


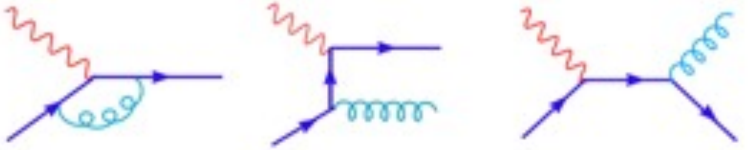
photon-gluon fusion

caveat: have to expect divergencies (recall 2nd part)
related to soft/collinear emission or from loops

we cannot calculate with infinities → introduce a "**regulator**"
and remove it in the end

general structure of the O(α_s) corrections

using small (artificial) quark/gluon masses as regulator we obtain:



$$\left.\frac{d^2\hat{\sigma}}{dx dQ^2}\right|_{F_2} \equiv \hat{F}_2^q$$
$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

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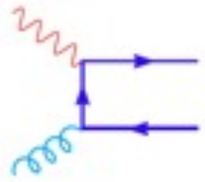

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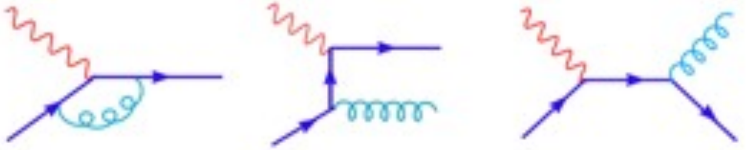

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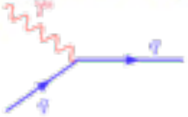

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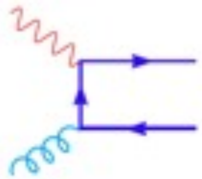

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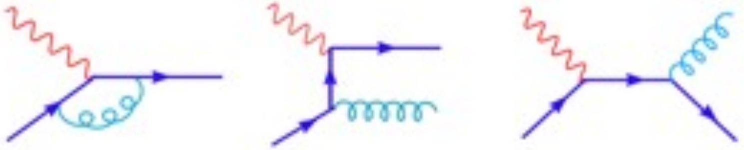

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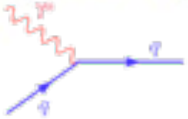

large logarithms
(collinear emission)

$$\left.\frac{d^2\hat{\sigma}}{dxdQ^2}\right|_{F_2} \equiv \hat{F}_2^g$$

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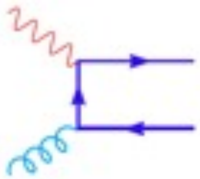
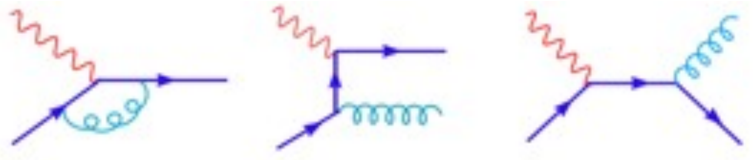


general structure of the O(α_s) corrections

using small (artificial) quark/gluon masses as regulator we obtain:



LO

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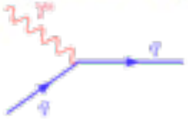

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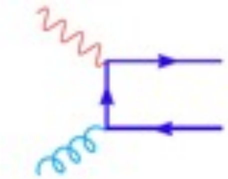

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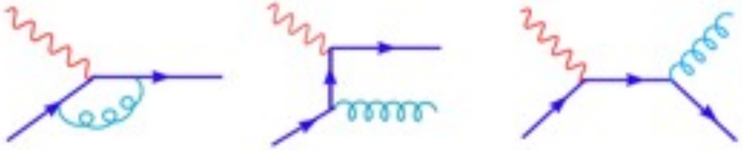

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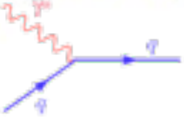

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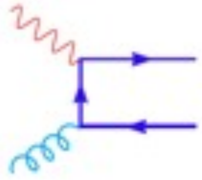
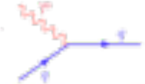


to see what happens to the logs we have to convolute our results with the PDFs

factorization of collinear singularities

for the quark part we obtain:



$$F_2(x,Q^2) \;=\; x \sum_{a=q,\bar{q}} e_q^2 \Big[f_{a,0}(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi}$$

$$f_{a,0}(x) \Big[P_{qq}\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{m_g^2} + C_2^q\left(\frac{x}{\xi}\right) \Big] \Big]$$

similarly for
the gluonic part

from

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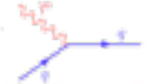

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$f_{a,0}(x)$: unmeasurable "bare" (= infinite) parton densities; need to be re-defined (= renormalized) to make them physical

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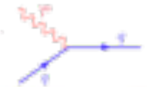

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at order α_s: (can be generalized to all orders)

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factorization of collinear singularities

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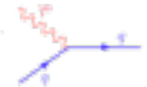

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absorbs all long-distance singularities
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factorization of collinear singularities

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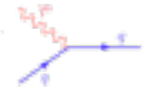

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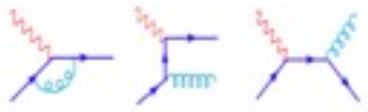
from 

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physical/renormalized densities: not calculable in pQCD but **universal**

general structure of a factorized cross section

putting everything together, keeping only terms up to α_s:

$$\begin{aligned} F_2(x,Q^2) \;=\; & x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \\ & \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right] \end{aligned}$$

general structure of a factorized cross section

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both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

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short-distance "Wilson coefficient"

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yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

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yet another scale: μ_r
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choice of the factorization scheme

this result is readily extended to hadron-hadron collisions

lesson: theorists are not afraid of infinities

JOAN CARTIER

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

universal PDFs → key to predictive power of pQCD

once PDFs are extracted from one set of experiments, e.g. DIS, we can use them to **predict cross sections** in, say, hadron-hadron collisions

parton densities are **universal**

→ there must be a process-independent **precise definition**

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small print: we need to specify a common factorization scheme for short- and long-distance physics (= choice of z_{ij} in our result for F_2)

standard choice: **modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme** (closely linked to dim. regularization; used in all PDF fits)

less often used: **DIS scheme** = "maximal" subtraction where all $O(\alpha_s)$ corrections in DIS are absorbed into PDFs (nice for DIS but a bit awkward for other processes)

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Bardeen, Buras,
Duke, Muta

classic (but old-fashioned) definition of PDFs through their Mellin moments in Wilson-Zimmermann's operator product expansion (OPE)

PDFs as bi-local operators

Curci, Furmanski,
Petronzio; Collins, Soper
see, e.g., D. Soper,
hep-lat/9609018

more physical formulation in Bjorken-x space:
matrix elements of bi-local operators on the light-cone

for quarks: (similar for gluons; easy to include spin $\gamma^{+} \rightarrow \gamma^{+}\gamma_{5}$)

$$f_a(\xi,\mu_f) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\overline{\Psi}_a(0,y^-,\vec{0})\gamma^+ \mathcal{F}\Psi_a(0)|p\rangle_{\overline{\mathrm{MS}}}$$

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annihilates
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recreates quark
at $x^+=0$ and $x^-=y^-$

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Fourier transform
→ momentum $\xi\, p^+$

recreates quark
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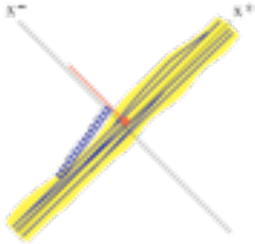
- in general we need a "**gauge link**" for a gauge invariant definition:


Foundations of
Perturbative QCD

$$\mathcal{F} = \mathcal{P}\exp\left(-ig\int_0^{y^-} dz^- A_c^+(0,z^-,\vec{0})T_c\right)$$



crucial role for a special class of "**transverse-momentum dep. PDFs**"
describing phenomena with transverse polarization **("Sivers function",** …)

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Curci, Furmanski, Petronzio; Collins, Soper
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Fourier transform
→ momentum ξ p^+

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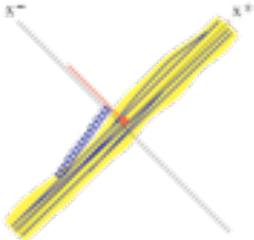

crucial role for a special class of "**transverse-momentum dep. PDFs**"
describing phenomena with transverse polarization (**"Sivers function"**, …)

- interpretation as number operator only in "**$A^+= 0$ gauge**"

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Fourier transform → momentum ξ p^+

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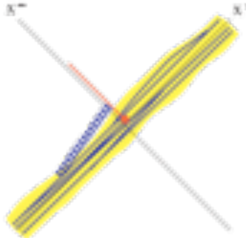
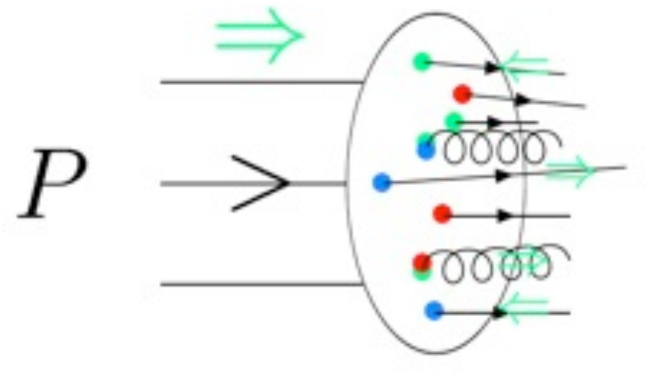


crucial role for a special class of "**transverse-momentum dep. PDFs**" describing phenomena with transverse polarization (**"Sivers function", …**)

- interpretation as number operator only in "**$A^+=0$ gauge**"
- turn into local operators (→ lattice QCD) if taking moments $\int_0^1 d\xi\, \xi^n$

pictorial representation of PDFs

suppose we could take a snapshot of a nucleon with positive helicity



question: how many constituents (quark, anti-quarks, gluons) have momenta between xP and (x+dx)P and how many have the same/opposite helicity?

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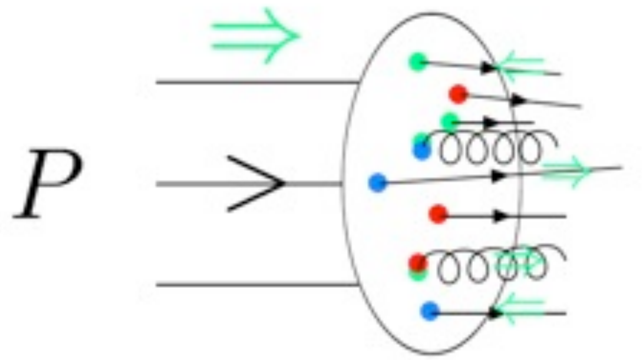

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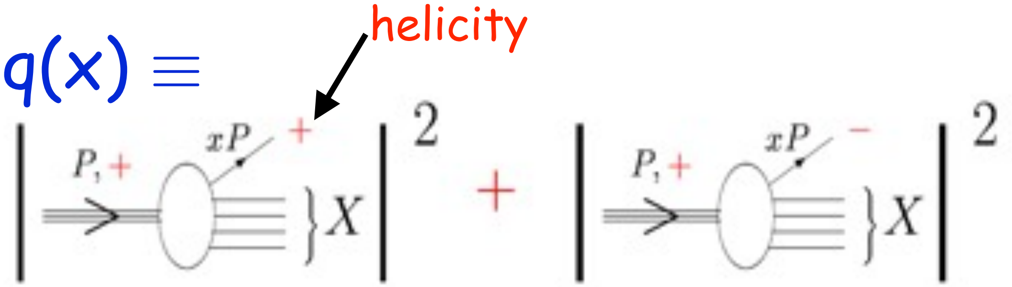

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→ LHC phenomenology, etc.

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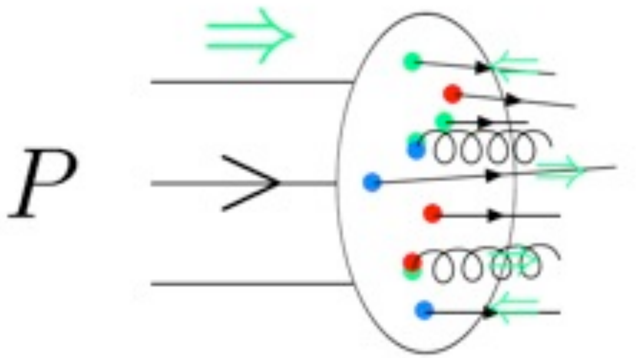

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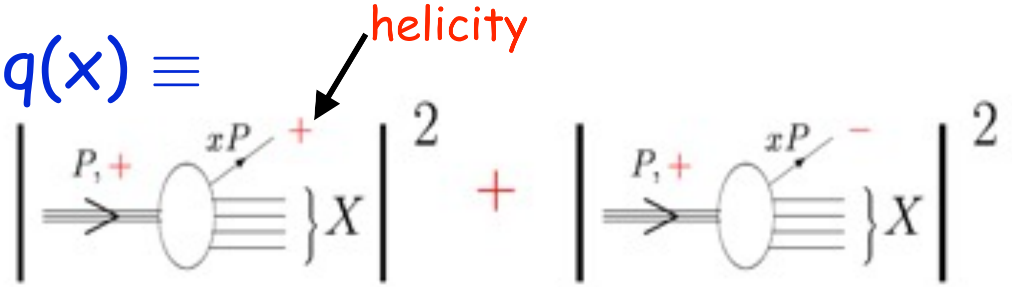

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Δq(x) ≡

Δg(x) =

helicity-dep. PDFs

→ spin of the nucleon

towards renormalization group equations

so far: infinities related to long-time/distance physics (soft/collinear emissions)

these singularities cancel for infrared safe observables
or can be systematically removed (factorization) by "hiding" them
in some non-perturbative parton or fragmentation functions

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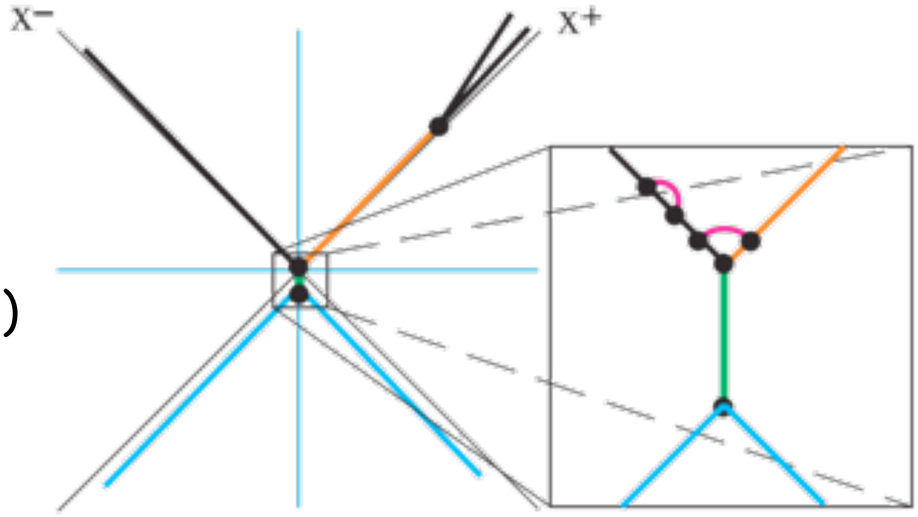

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loop momenta can be very large (=infinite)
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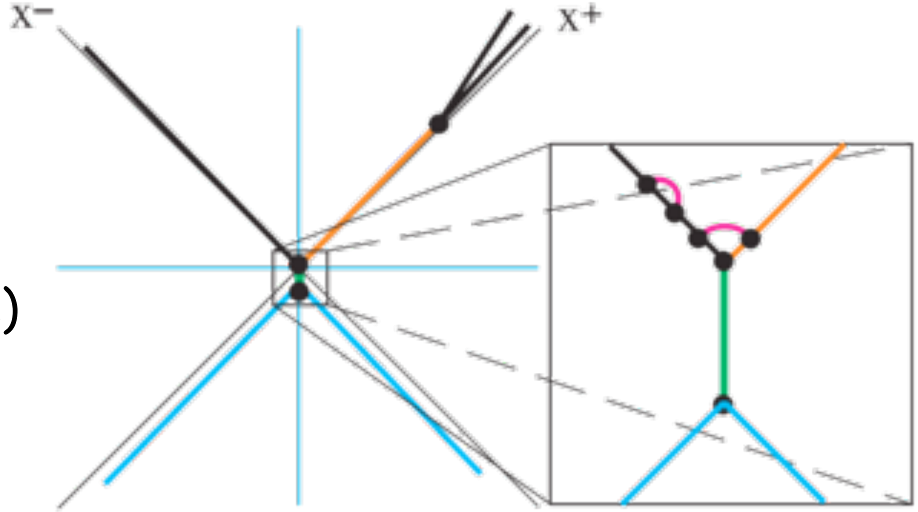

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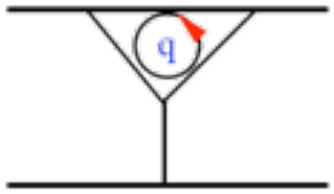
again, we need a suitable regulator for
divergent loop integrations:

UV cut-off vs. **dim. regularization**

intuitive;
not beyond NLO

involved;
works to all orders

$$\int_0^\infty d^4q$$



the importance of scales

factorization and renormalization play similar roles
at opposite ends of the energy range of pQCD

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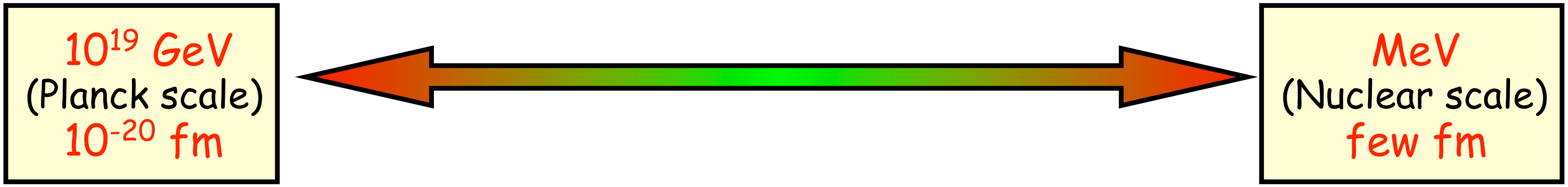

10^{19} GeV
(Planck scale)
10^{-20} fm

MeV
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few fm

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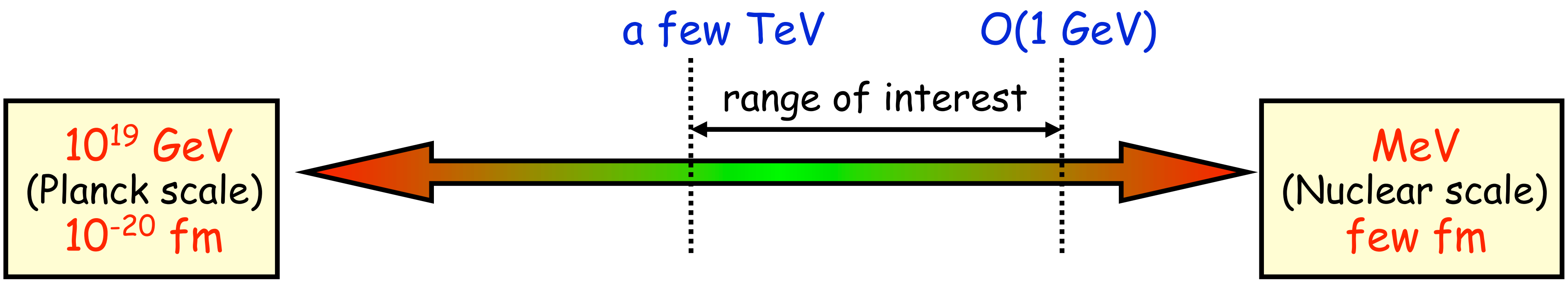

a few TeV

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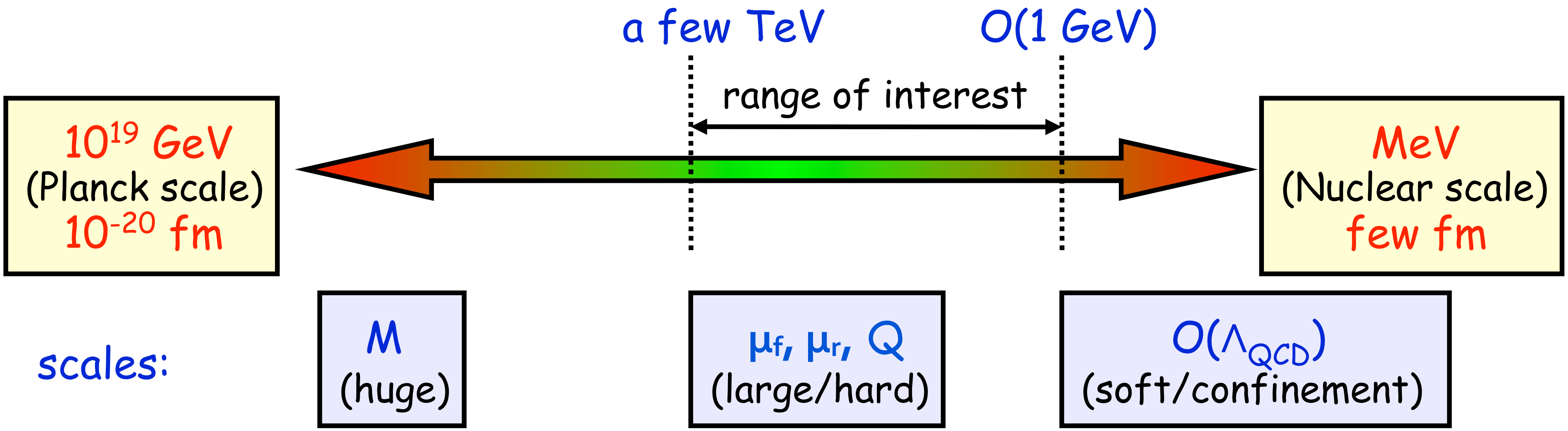

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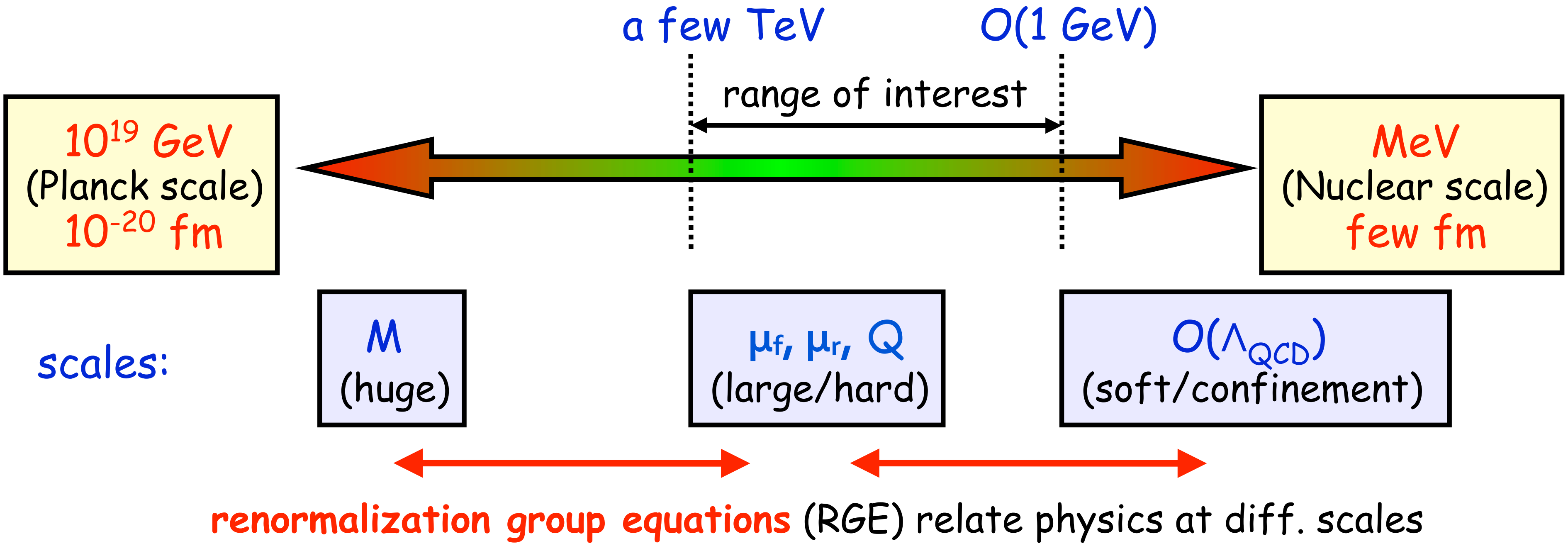

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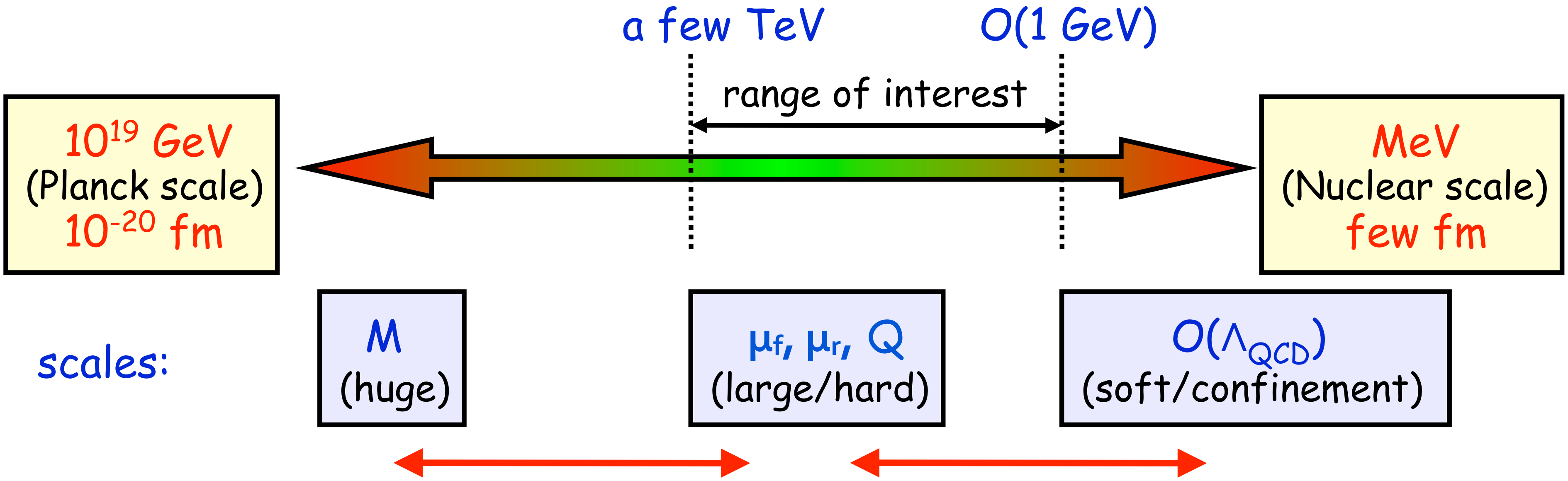

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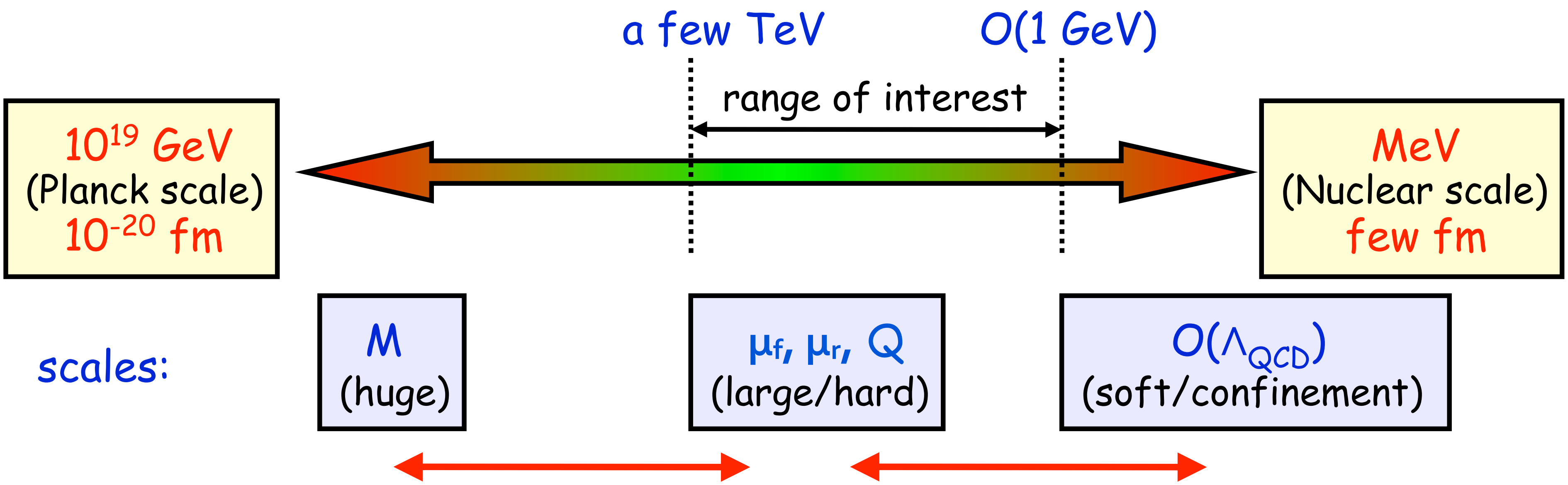

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IR/collinear factorization
hides non-perturbative QCD at confinement scale in
$f_a(x,\mu_f)$, $\Delta f_a(x,\mu_f)$, $D_a^H(z,\mu_f)$, …

RGE: the swiss army knife of pQCD

we use α_s (and f_a, D_c^H) to absorb UV (IR) divergencies

→ **we cannot predict their values within pQCD**

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both scale parameters μ_f and μ_r are not intrinsic to QCD

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renormalization group equations

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all we need is a reference measurement at some scale μ_0

scale evolution of α_s and parton densities

simplest example of RGE: running coupling α_s derived from $\frac{d\sigma}{d\ln\mu_r} = 0$

→ recall part II $\quad \frac{da_s}{d\ln\mu^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + \ldots \qquad a_s \equiv \frac{\alpha_s}{4\pi}$

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scale dependence of PDFs: more complicated

simplified example:
F_2 for one quark flavor

$$\underbrace{F_2(x,Q^2)}_{\text{physical}} = \underbrace{q(x,\mu_f)}_{\text{quark pdf}} \otimes \underbrace{\hat{F}_2(x,\frac{Q}{\mu_f})}_{\text{hard cross section}}$$

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$$\int_0^1 dx x^{n-1} \left[\int_x^1 \frac{dy}{y} f(y) g\left(\frac{x}{y}\right) \right] =$$
$$\int_0^1 dx x^{n-1} \int_0^1 dy \int_0^1 dz\, \delta(x - zy) f(y) g(z) = f(n)\, g(n)$$

simplest example of DGLAP evolution

Dokshitzer; Gribov, Lipatov; Altarelli, Parisi

now we can compute $\dfrac{dF_2(x,Q^2)}{d\ln\mu_f}=0$

↔ $$\frac{dq(n,\mu_f)}{d\ln\mu_f}\hat{F}_2(n,\frac{Q}{\mu_f})+q(n,\mu_f)\frac{d\hat{F}_2(n,\frac{Q}{\mu_f})}{d\ln\mu_f}=0$$

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→ **once we know the PDFs at a scale μ_0 we can predict them at $\mu > \mu_0$**

factorization → evolution → resummation

physical interpretation of the evolution eqs.:

RGE resums collinear emissions to all orders

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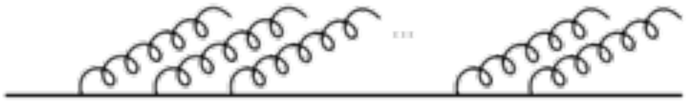

$$\exp[\ldots] = 1 + \frac{\alpha_s}{2\pi} P_{qq}(n) \ln \frac{\mu_f}{\mu_0} + \frac{1}{2} \left[\frac{\alpha_s}{2\pi} P_{qq}(n) \ln \frac{\mu_f}{\mu_0} \right]^2 + \ldots$$

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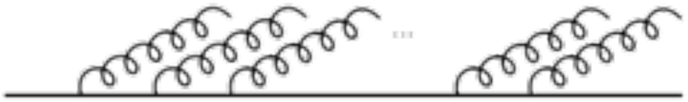

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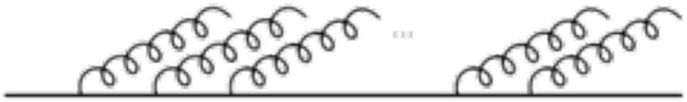

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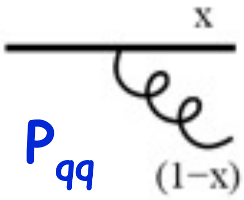
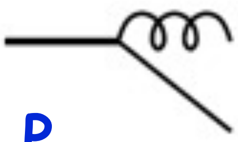
- the physical meaning of the splitting functions is easy:

$P_{ij}(x)$: probability that a parton j splits collinearly into a parton i (and something) carrying a momentum fraction x



P_{qq}



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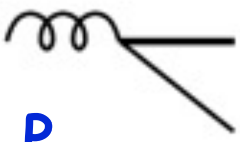

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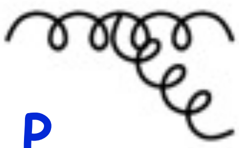
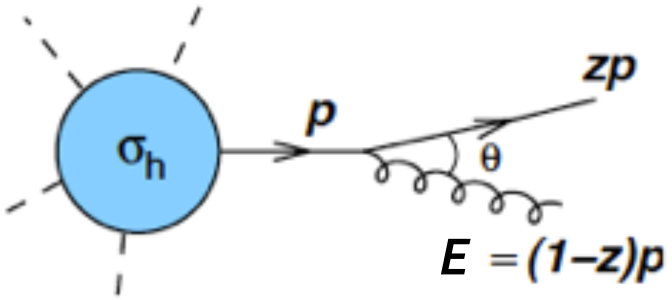


P_{gg}

factorization recap: final-state vs initial-state

recall what we learned for **final-state radiation**

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta^2}{\theta^2}$$



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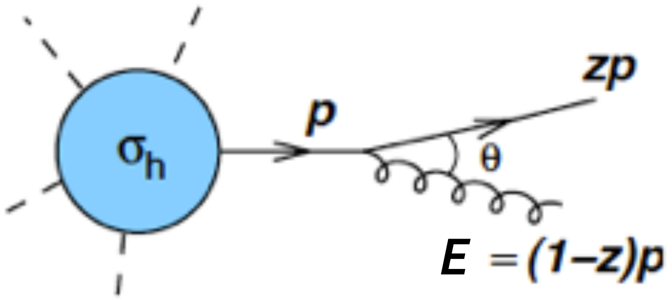

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and rewrite in terms of new variable k_T

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

where we have used

$$\mathbf{E} = (\mathbf{1} - \mathbf{z})\mathbf{p}$$
$$\mathbf{k_T} = \mathbf{E} \sin\theta \simeq \mathbf{E}\theta$$

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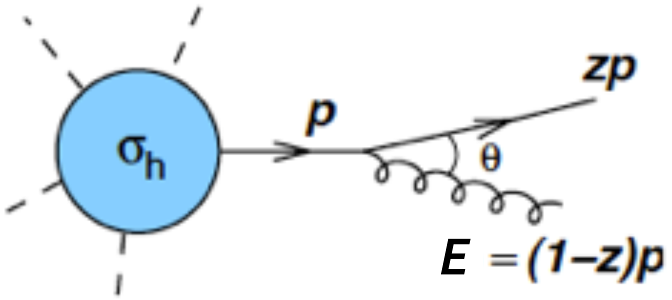

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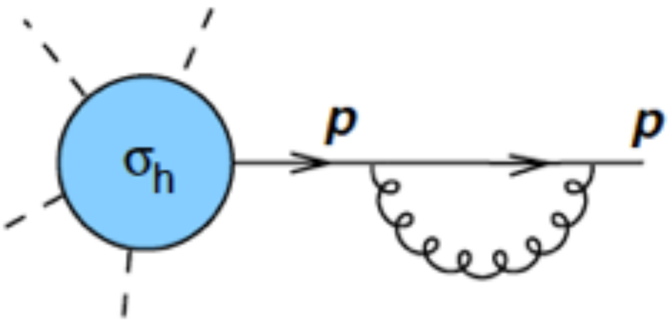
KLN: if we avoid distinguishing quark and collinear quark-gluon final-states (like for **jets**) divergencies cancel against virtual corrections

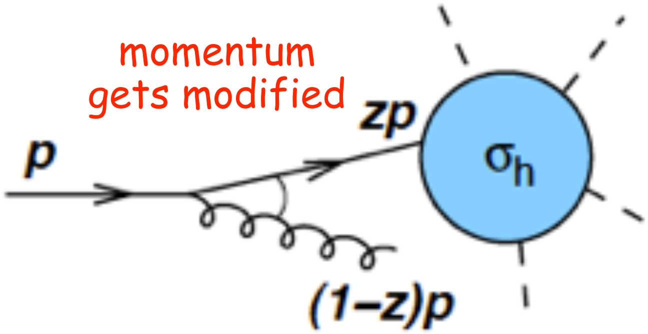


$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

factorization recap: initial-state peculiarities

initial-state radiation: crucial difference – hard scattering happens after splitting

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



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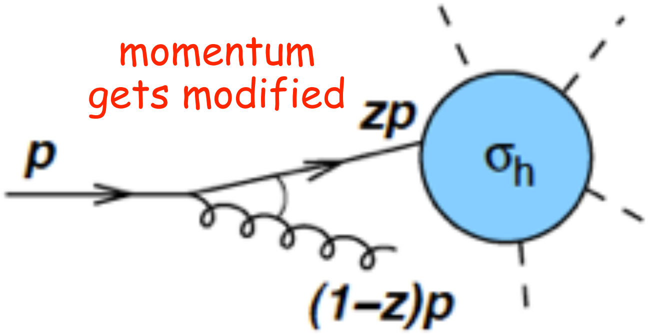
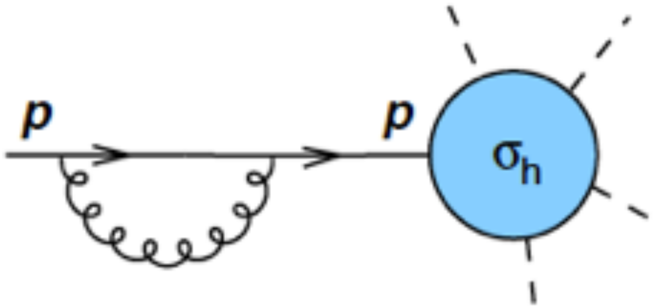
$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$


momentum gets modified

but for the virtual piece the momentum is unchanged



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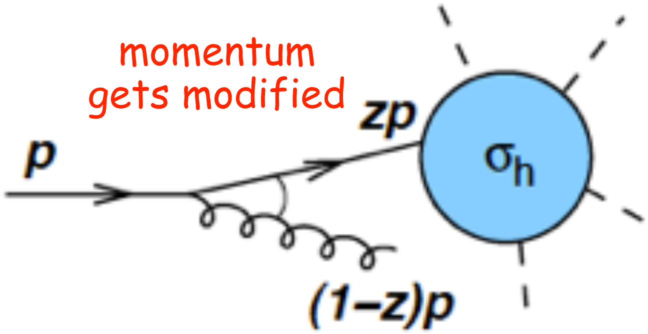

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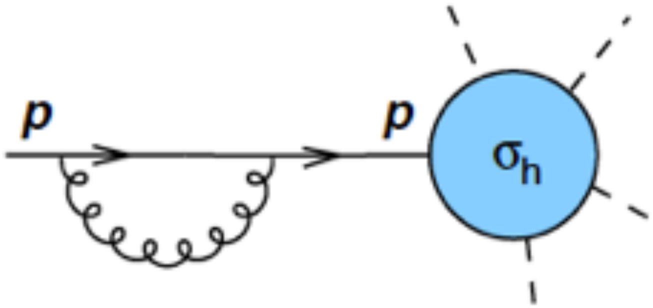

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

hence, the sum receives two contributions with different momenta

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int\frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

disclaimer: we assume that k_T << Q (large) to ignore other transverse momenta

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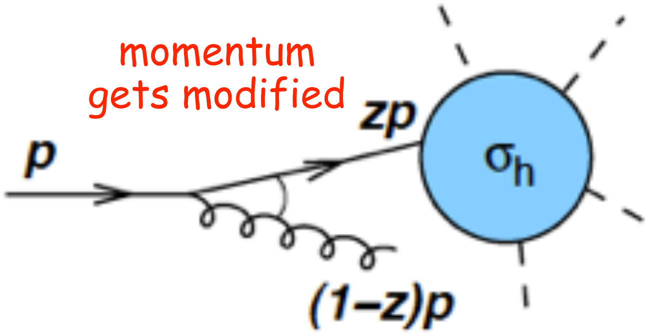

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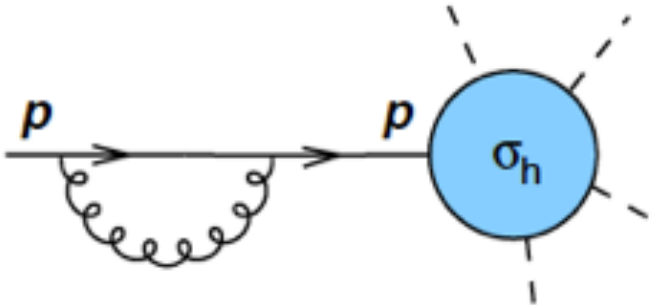

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leads to uncanceled
collinear singularity

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factorization revisited: collinear singularity

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- z=1: soft divergence cancels (KLN) as $\sigma_\mathbf{h}(\mathbf{zp}) - \sigma_\mathbf{h}(\mathbf{p}) \to \mathbf{0}$
- arbitrary z: $\sigma_\mathbf{h}(\mathbf{zp}) - \sigma_\mathbf{h}(\mathbf{p}) \neq \mathbf{0}$ but z integration is finite
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cross sections with incoming partons not collinear safe

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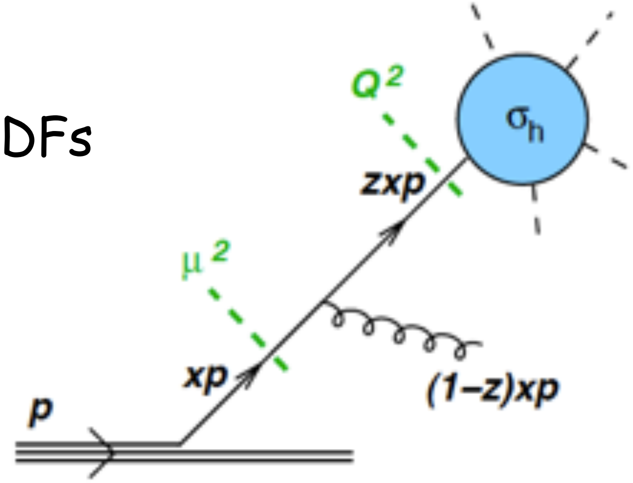
reflects collinear singularity

cross sections with incoming partons not collinear safe

factorization = collinear "cut-off"

- absorb divergent small k_T region in non-perturbative PDFs

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} [\sigma_h(zxp) - \sigma_h(xp)]\, q(x, \mu^2)}_{\text{finite}}$$



anatomy of splitting functions

splitting functions may receive two kinds of contributions:

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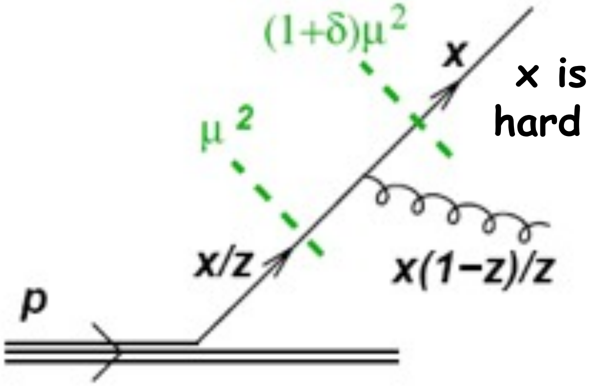

x is fixed by
hard scattering

real emission
“something happens”

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz P_{qq}(z)\frac{q(x/z,\mu^2)}{z}$$

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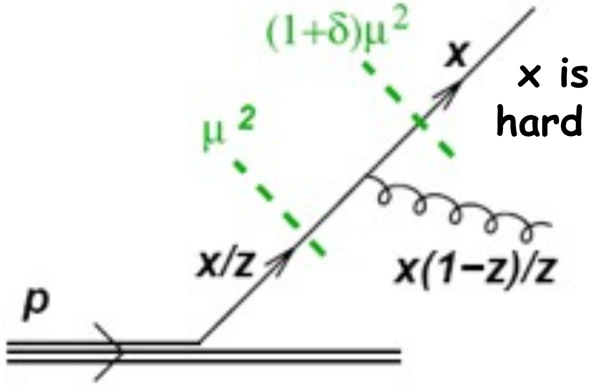

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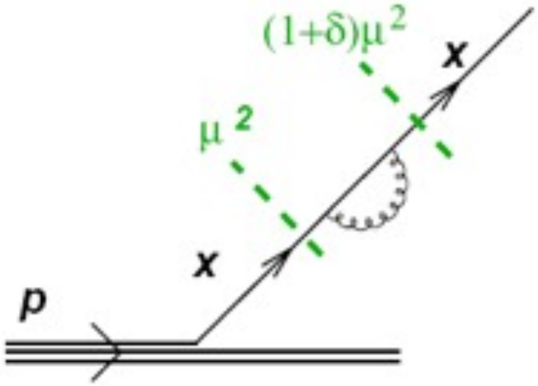
+



virtual emission
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$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz P_{qq}(z)\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz P_{qq}(z) q(x,\mu^2)$$

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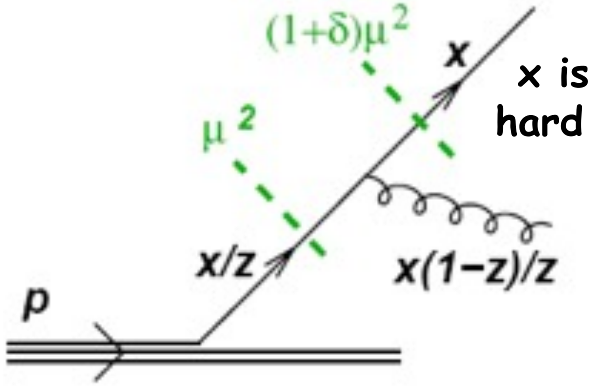

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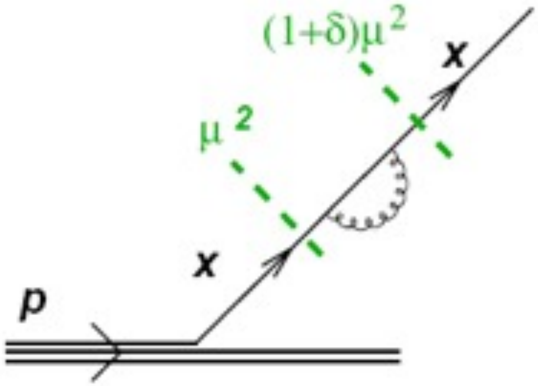

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combine !

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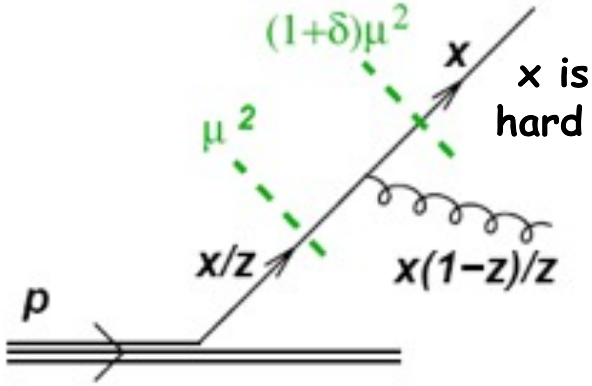

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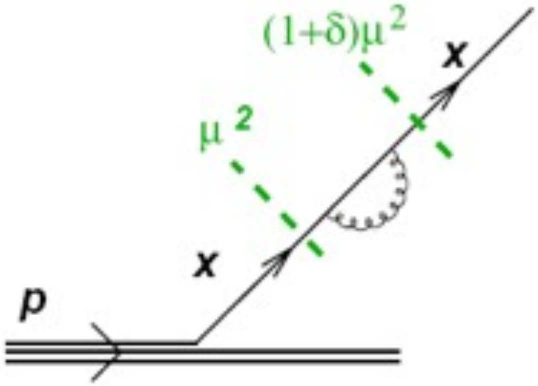

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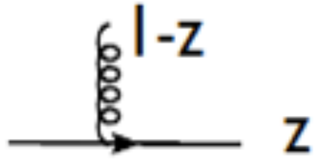
involves "**plus distribution**" $\int_0^1 dz\,[g(z)]_+ f(z) \equiv \int_0^1 dz\, g(z)\,[f(z) - f(1)]$

condition: f(z) sufficiently smooth for z→ 1

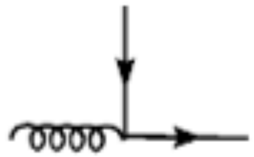
properties of LO splitting functions

in general, quarks and gluons can split into quarks and gluons -> 4 functions

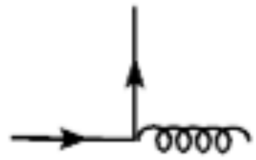
$$P^{(0)}_{qq} = P^{(0)}_{\bar{q}\bar{q}} = C_F\left[\frac{\mathbf{1+z^2}}{\mathbf{(1-z)}_+} + \frac{3}{2}\delta(1-z)\right]$$



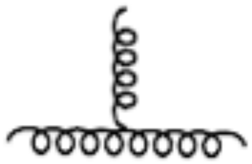
$$P^{(0)}_{qg} = P^{(0)}_{\bar{q}g} = T_R\left(z^2+(1-z)\right)$$



$$P^{(0)}_{gq} = P^{(0)}_{g\bar{q}} = C_F\frac{1+(1-z)^2}{z}$$



$$P^{(0)}_{gg} = 2C_A\left[z\left(\frac{1}{1-z}\right)_+ + \frac{1-z}{z} + z(1-z) + b_0\delta(1-z)\right]$$



in higher orders more complicated, as $\mathbf{P_{q_iq_j} \neq 0}$ arise

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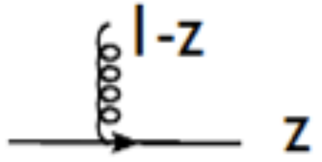

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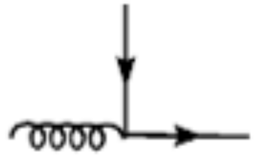

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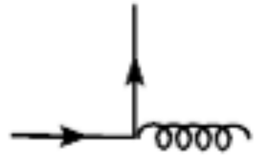

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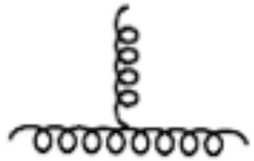

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soft gluon divergence (z=1)
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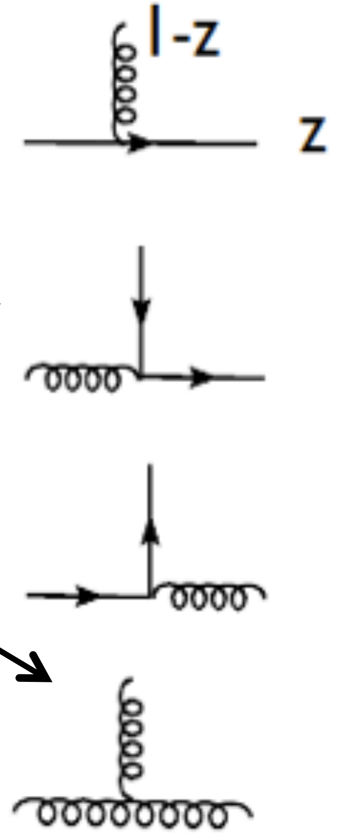
symmetric under
z -> (1-z)
except virtuals



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reaching for precision

$$
\begin{aligned}
P^{(0)}_{\mathrm{ns}}(x) &= C_F\left(2p_{\mathrm{qq}}(x)+3\delta(1-x)\right)\\
P^{(0)}_{\mathrm{ps}}(x) &= 0\\
P^{(0)}_{\mathrm{qg}}(x) &= 2n_f\,p_{\mathrm{qg}}(x)\\
P^{(0)}_{\mathrm{gq}}(x) &= 2C_F\,p_{\mathrm{gq}}(x)\\
P^{(0)}_{\mathrm{gg}}(x) &= C_A\left(4p_{\mathrm{gg}}(x)+\frac{11}{3}\delta(1-x)\right)-\frac{2}{3}n_f\,\delta(1-x)
\end{aligned}
$$

LO: 1973

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$$
\begin{aligned}
P_{\mathrm{ns}}^{(1)+}(x) &= 4C_AC_F\Big(p_{\mathrm{qq}}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}\mathrm{H}_0+\mathrm{H}_{0,0}\Big]+p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0}-\mathrm{H}_{0,0}\Big]\\
&\quad+\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\mathrm{qq}}(x)\Big[\frac{5}{9}+\frac{1}{3}\mathrm{H}_0\Big]+\frac{2}{3}(1-x)\\
&\quad+\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4C_F{}^2\Big(2p_{\mathrm{qq}}(x)\Big[\mathrm{H}_{1,0}-\frac{3}{4}\mathrm{H}_0+\mathrm{H}_2\Big]-2p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0}\\
&\quad-\mathrm{H}_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}\mathrm{H}_0\Big]-\mathrm{H}_0-(1+x)\mathrm{H}_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big)\\
P_{\mathrm{ns}}^{(1)-}(x) &= P_{\mathrm{ns}}^{(1)+}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0}-\mathrm{H}_{0,0}\Big]-2(1-x)\\
&\quad-(1+x)\mathrm{H}_0\Big)
\end{aligned}
$$

$$
P_{\mathrm{ps}}^{(1)}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x}-2+6x-4\mathrm{H}_0+x^2\Big[\frac{8}{3}\mathrm{H}_0-\frac{56}{9}\Big]+(1+x)\Big[5\mathrm{H}_0-2\mathrm{H}_{0,0}\Big]\Big)
$$

$$
\begin{aligned}
P_{\mathrm{qg}}^{(1)}(x) &= 4C_An_f\Big(\frac{20}{9}\frac{1}{x}-2+25x-2p_{\mathrm{qg}}(-x)\mathrm{H}_{-1,0}-2p_{\mathrm{qg}}(x)\mathrm{H}_{1,1}+x^2\Big[\frac{44}{3}\mathrm{H}_0-\frac{218}{9}\Big]\\
&+4(1-x)\Big[\mathrm{H}_{0,0}-2\mathrm{H}_0+x\mathrm{H}_1\Big]-4\zeta_2x-6\mathrm{H}_{0,0}+9\mathrm{H}_0\Big)+4C_Fn_f\Big(2p_{\mathrm{qg}}(x)\Big[\mathrm{H}_{1,0}+\mathrm{H}_{1,1}+\mathrm{H}_2\\
&-\zeta_2\Big]+4x^2\Big[\mathrm{H}_0+\mathrm{H}_{0,0}+\frac{5}{2}\Big]+2(1-x)\Big[\mathrm{H}_0+\mathrm{H}_{0,0}-2x\mathrm{H}_1+\frac{29}{4}\Big]-\frac{15}{2}-\mathrm{H}_{0,0}-\frac{1}{2}\mathrm{H}_0\Big)
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{gq}}^{(1)}(x) &= 4C_AC_F\Big(\frac{1}{x}+2p_{\mathrm{gq}}(x)\Big[\mathrm{H}_{1,0}+\mathrm{H}_{1,1}+\mathrm{H}_2-\frac{11}{6}\mathrm{H}_1\Big]-x^2\Big[\frac{8}{3}\mathrm{H}_0-\frac{44}{9}\Big]+4\zeta_2-2\\
&-7\mathrm{H}_0+2\mathrm{H}_{0,0}-2\mathrm{H}_1x+(1+x)\Big[2\mathrm{H}_{0,0}-5\mathrm{H}_0+\frac{37}{9}\Big]-2p_{\mathrm{gq}}(-x)\mathrm{H}_{-1,0}\Big)-4C_Fn_f\Big(\frac{2}{3}x\\
&-p_{\mathrm{gq}}(x)\Big[\frac{2}{3}\mathrm{H}_1-\frac{10}{9}\Big]\Big)+4C_F{}^2\Big(p_{\mathrm{gq}}(x)\Big[3\mathrm{H}_1-2\mathrm{H}_{1,1}\Big]+(1+x)\Big[\mathrm{H}_{0,0}-\frac{7}{2}+\frac{7}{2}\mathrm{H}_0\Big]-3\mathrm{H}_{0,0}\\
&+1-\frac{3}{2}\mathrm{H}_0+2\mathrm{H}_1x\Big)
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{gg}}^{(1)}(x) &= 4C_An_f\Big(1-x-\frac{10}{9}p_{\mathrm{gg}}(x)-\frac{13}{9}\Big(\frac{1}{x}-x^2\Big)-\frac{2}{3}(1+x)\mathrm{H}_0-\frac{2}{3}\delta(1-x)\Big)+4C_A{}^2\Big(27\\
&+(1+x)\Big[\frac{11}{3}\mathrm{H}_0+8\mathrm{H}_{0,0}-\frac{27}{2}\Big]+2p_{\mathrm{gg}}(-x)\Big[\mathrm{H}_{0,0}-2\mathrm{H}_{-1,0}-\zeta_2\Big]-\frac{67}{9}\Big(\frac{1}{x}-x^2\Big)-12\mathrm{H}_0\\
&-\frac{44}{3}x^2\mathrm{H}_0+2p_{\mathrm{gg}}(x)\Big[\frac{67}{18}-\zeta_2+\mathrm{H}_{0,0}+2\mathrm{H}_{1,0}+2\mathrm{H}_2\Big]+\delta(1-x)\Big[\frac{8}{3}+3\zeta_3\Big]\Big)+4C_Fn_f\Big(2\mathrm{H}_0\\
&+\frac{2}{3}\frac{1}{x}+\frac{10}{3}x^2-12+(1+x)\Big[4-5\mathrm{H}_0-2\mathrm{H}_{0,0}\Big]-\frac{1}{2}\delta(1-
\end{aligned}
$$

NLO: 1980

Curci, Furmanski, Petronzio;
Floratos et al., …

P_{ij} @ NNLO: a landmark calculation

10000 diagrams, 10^5 integrals, 10 man years, and several CPU years later:

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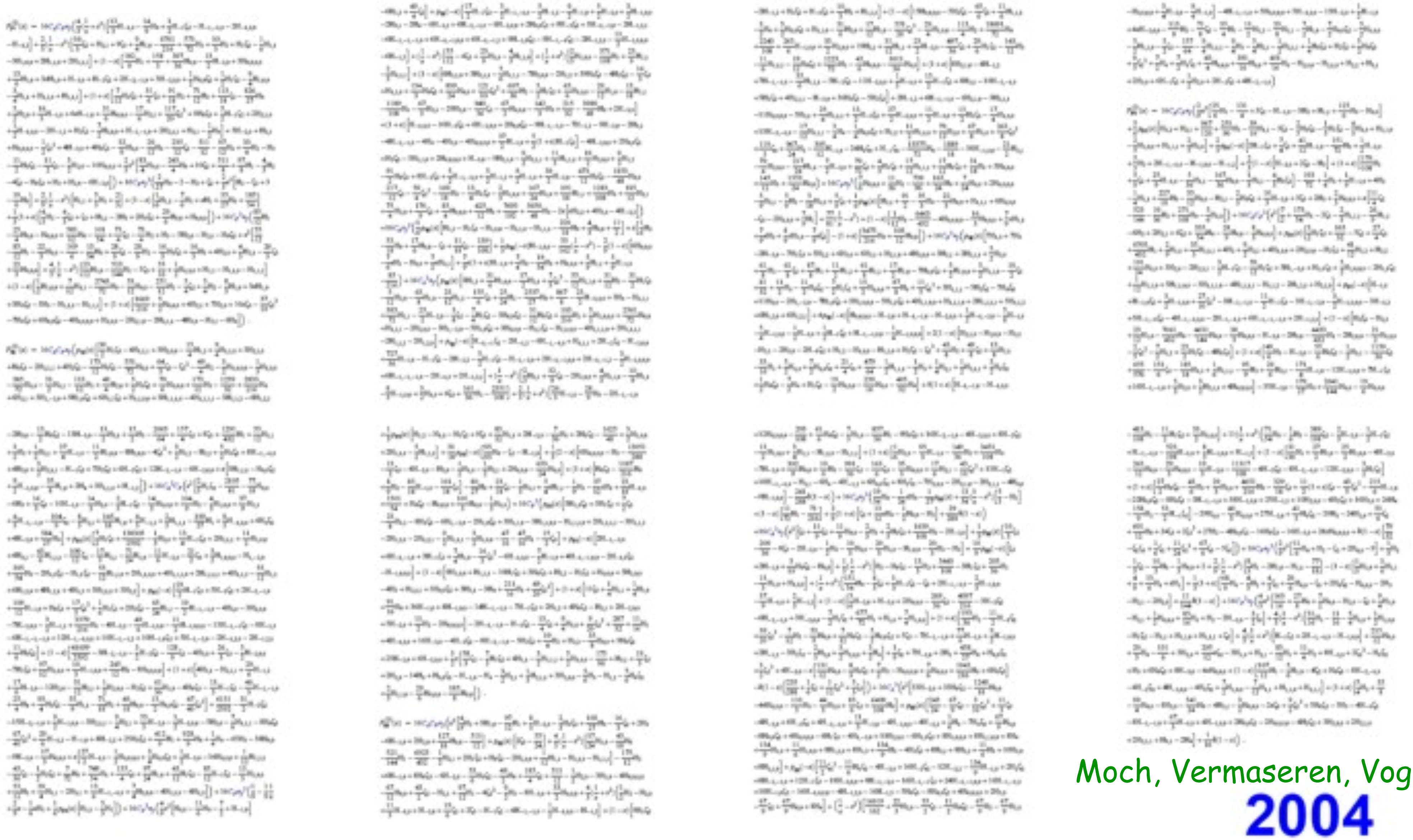

Moch, Vermaseren, Vogt

2004

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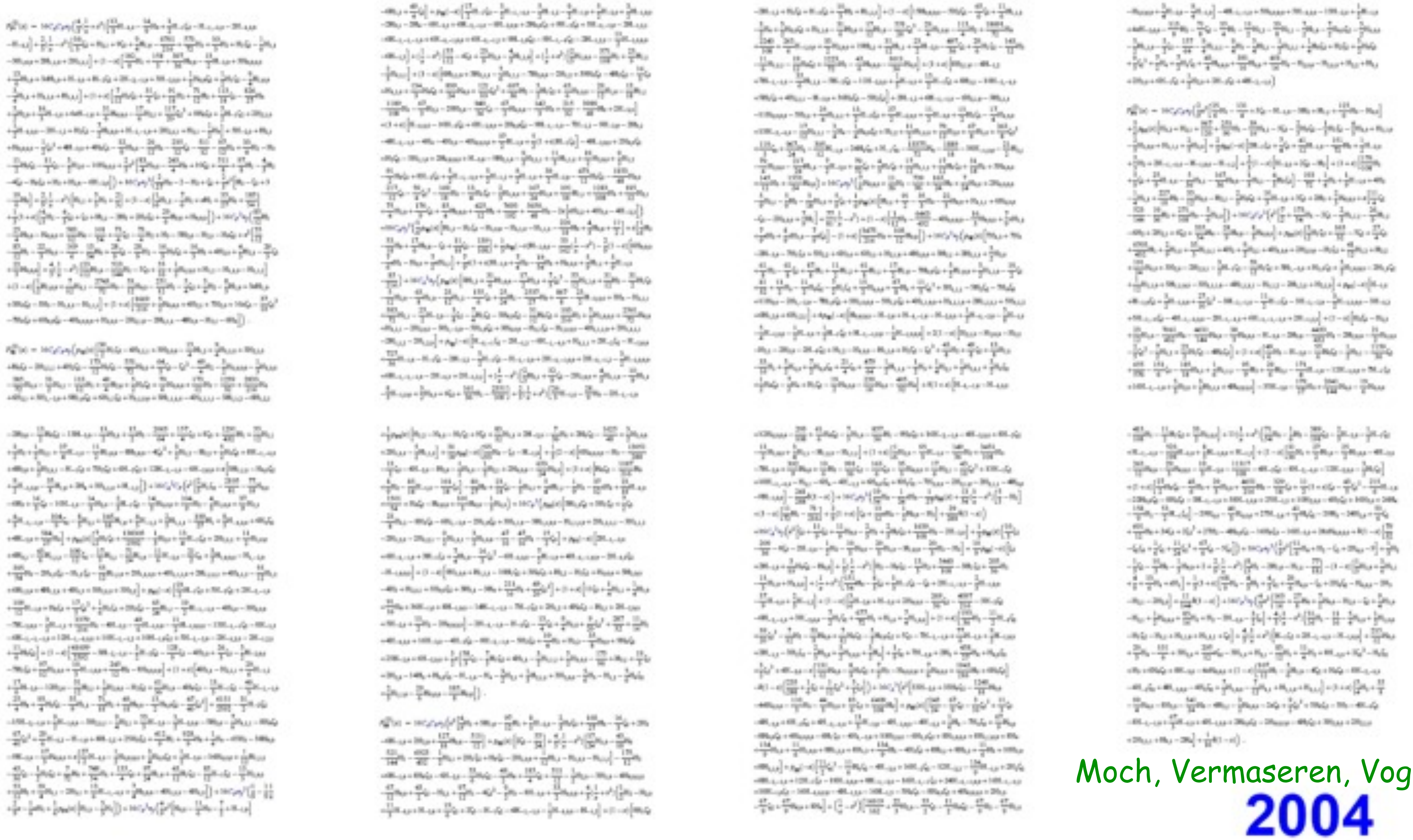

Moch, Vermaseren, Vogt

2004

NNLO the new emerging standard in QCD – essential for precision physics

DGLAP evolution in full glory

taking quarks and gluons together: coupled integro-differential equations

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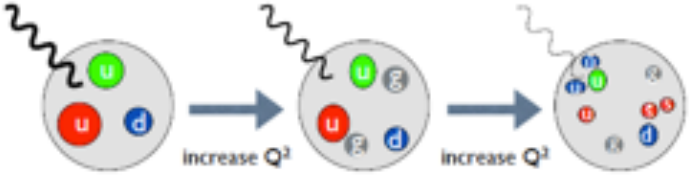
best solved in Mellin moment space: set of ordinary differential eqs.;
no closed solution in exp. form beyond LO (commutators of P matrices!)



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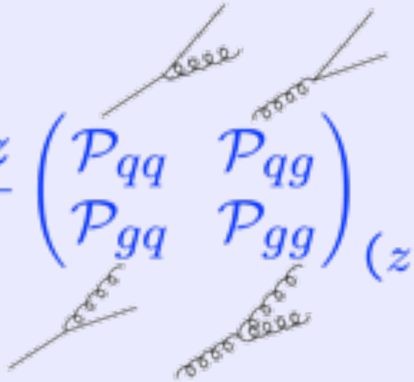

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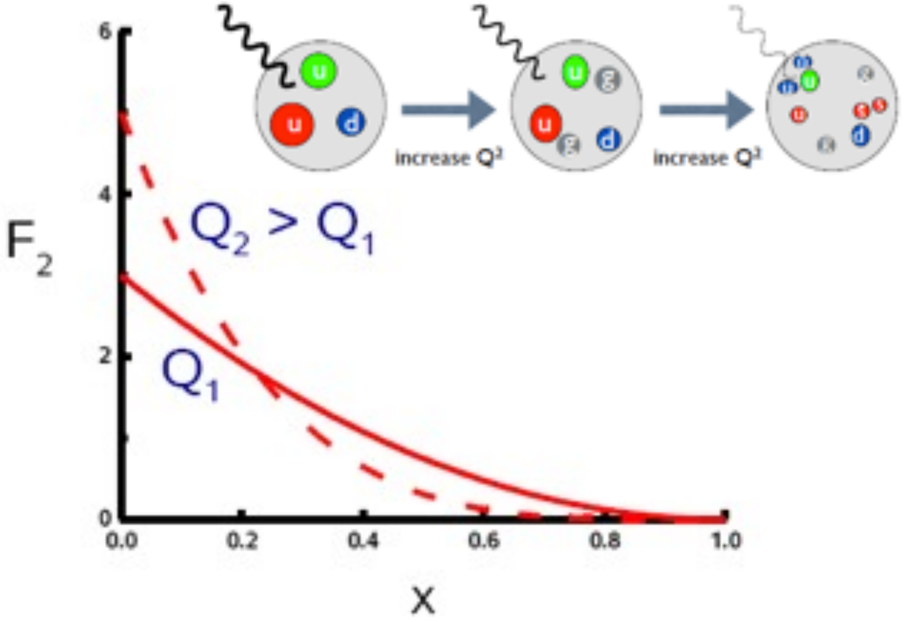
- small x increase



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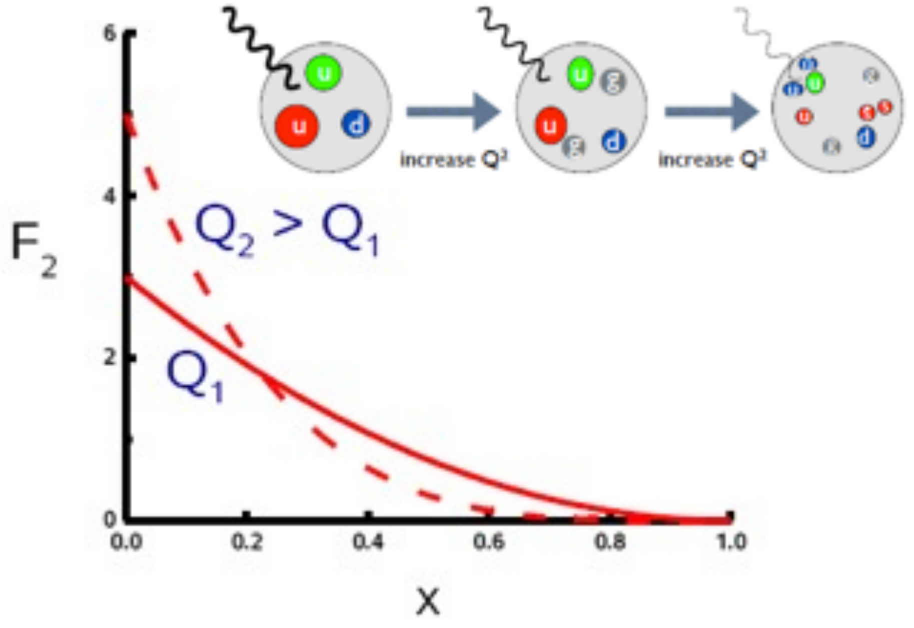
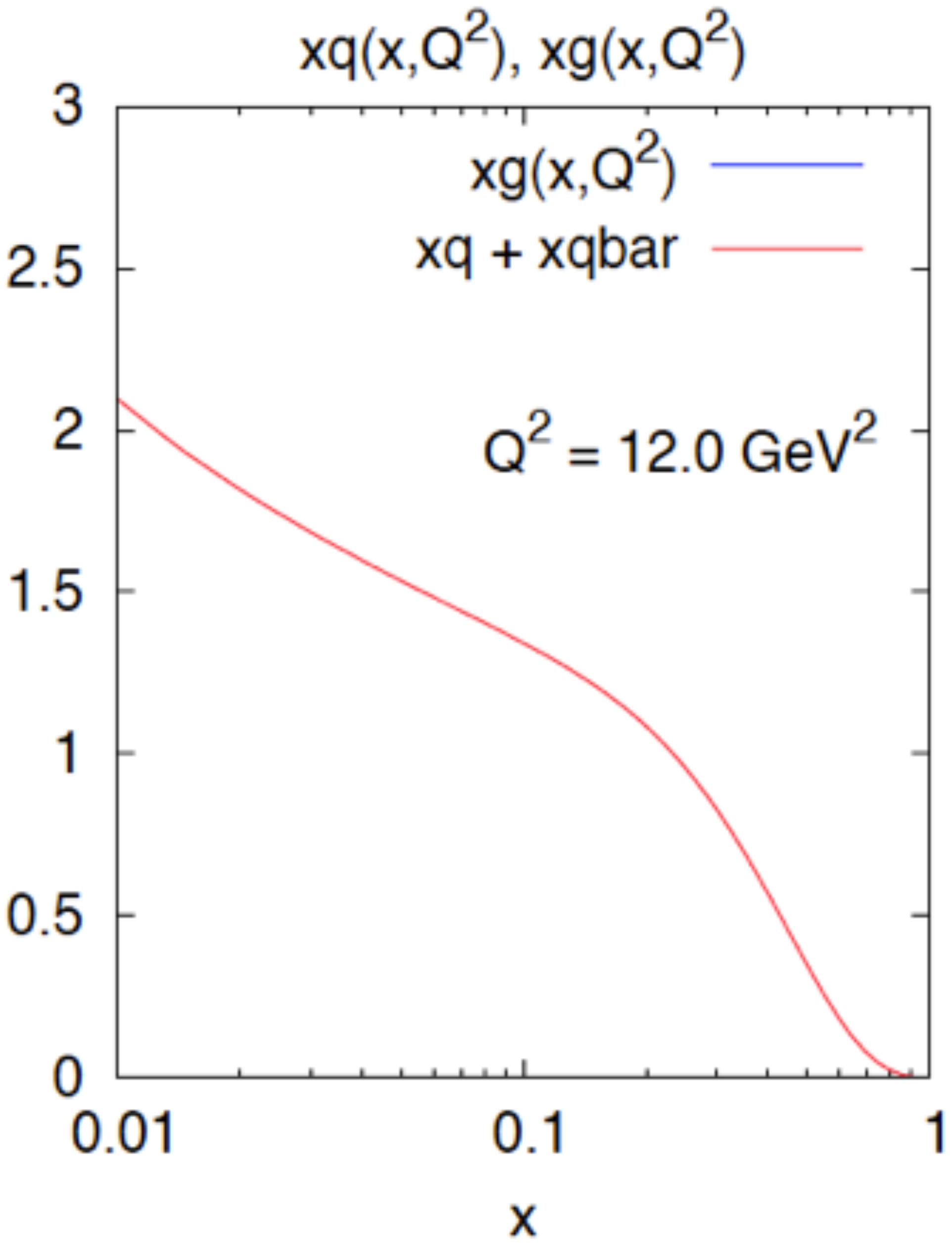
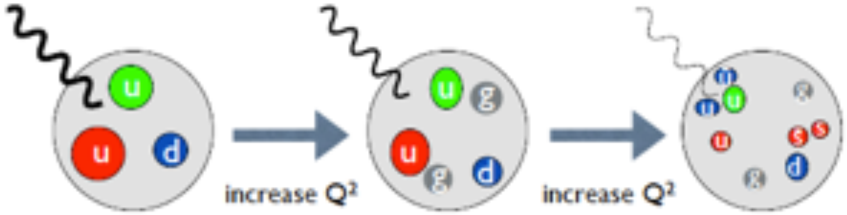


DGLAP evolution at work: toy example





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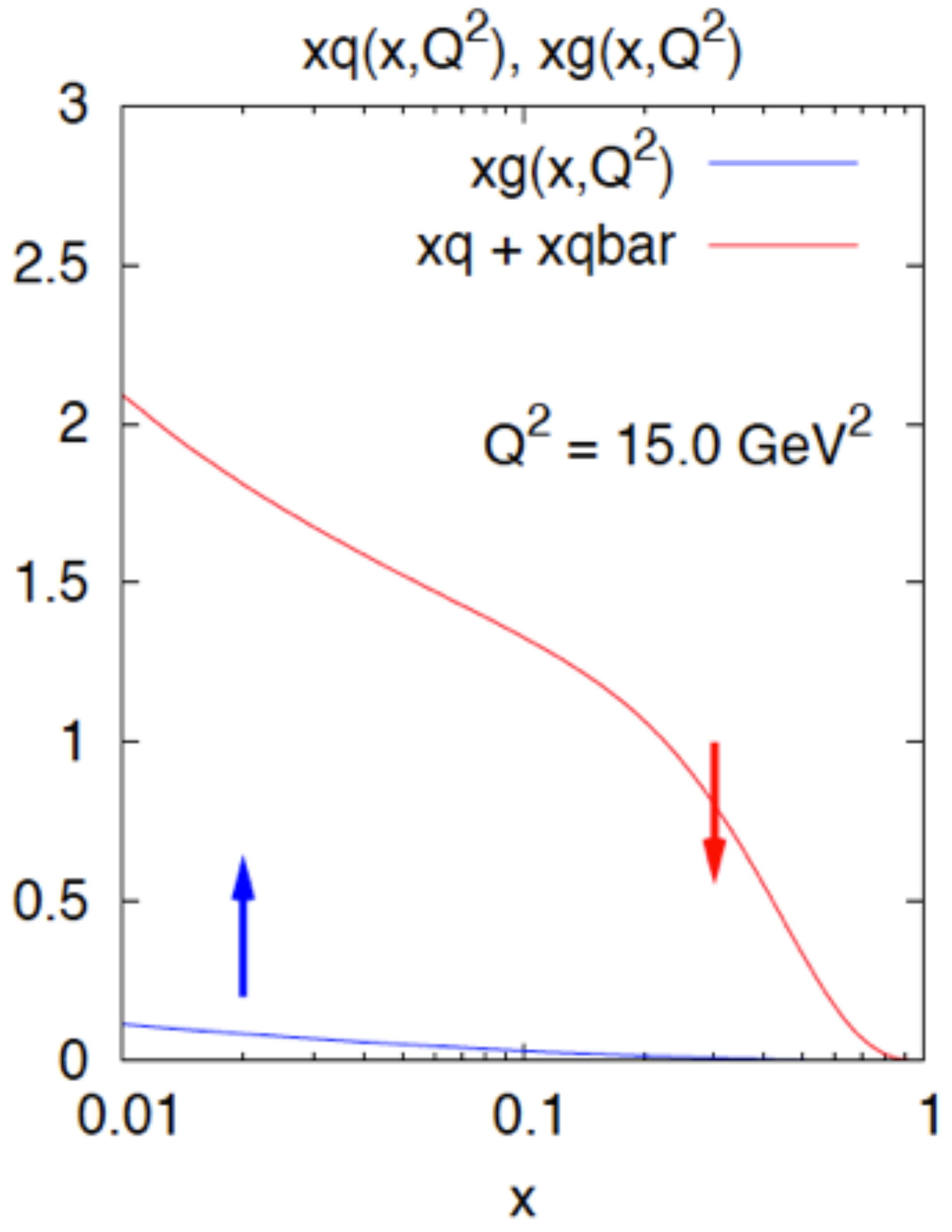

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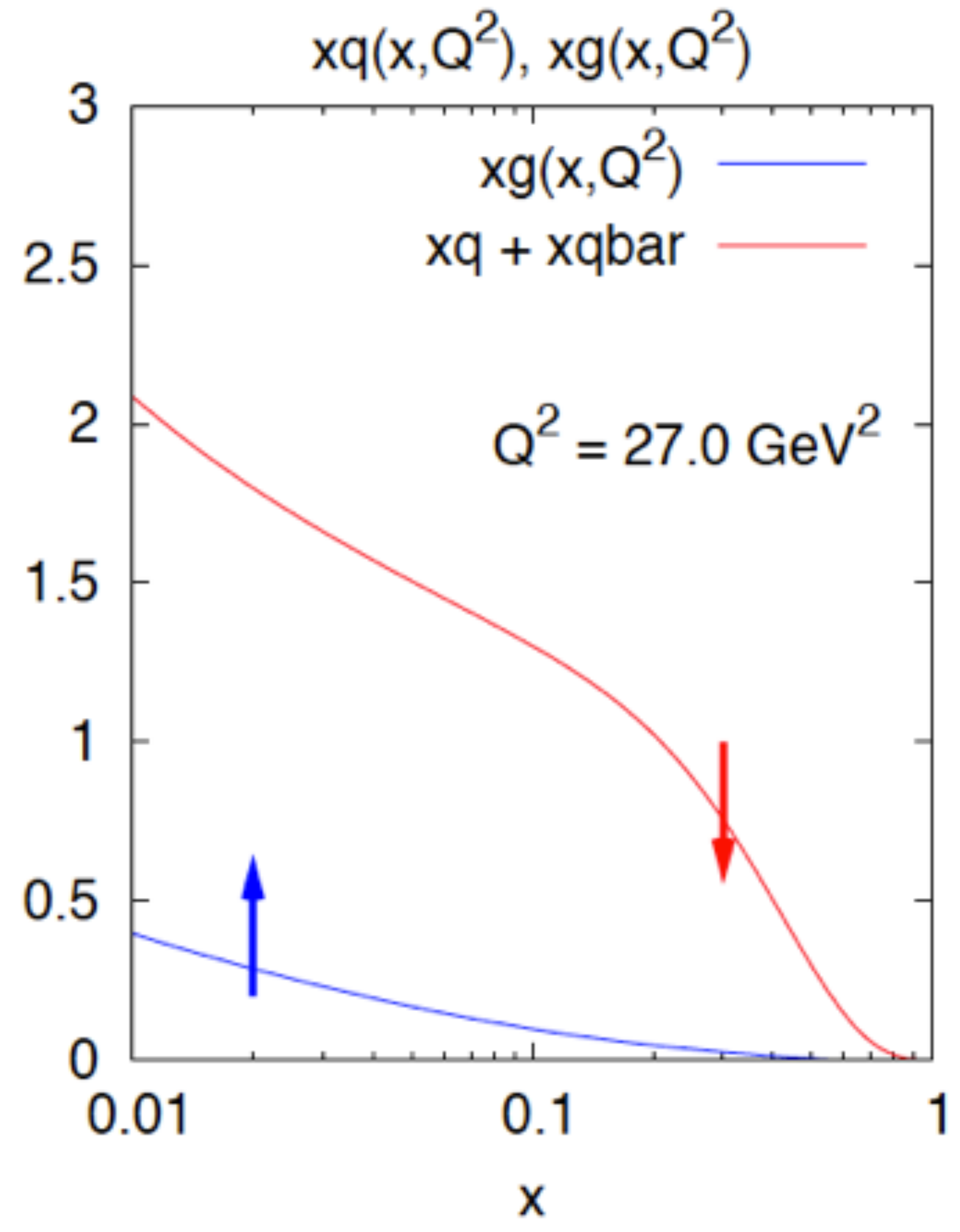

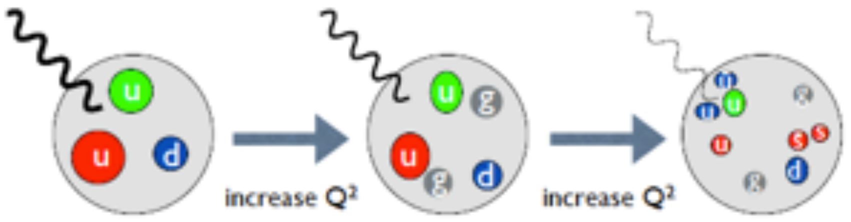

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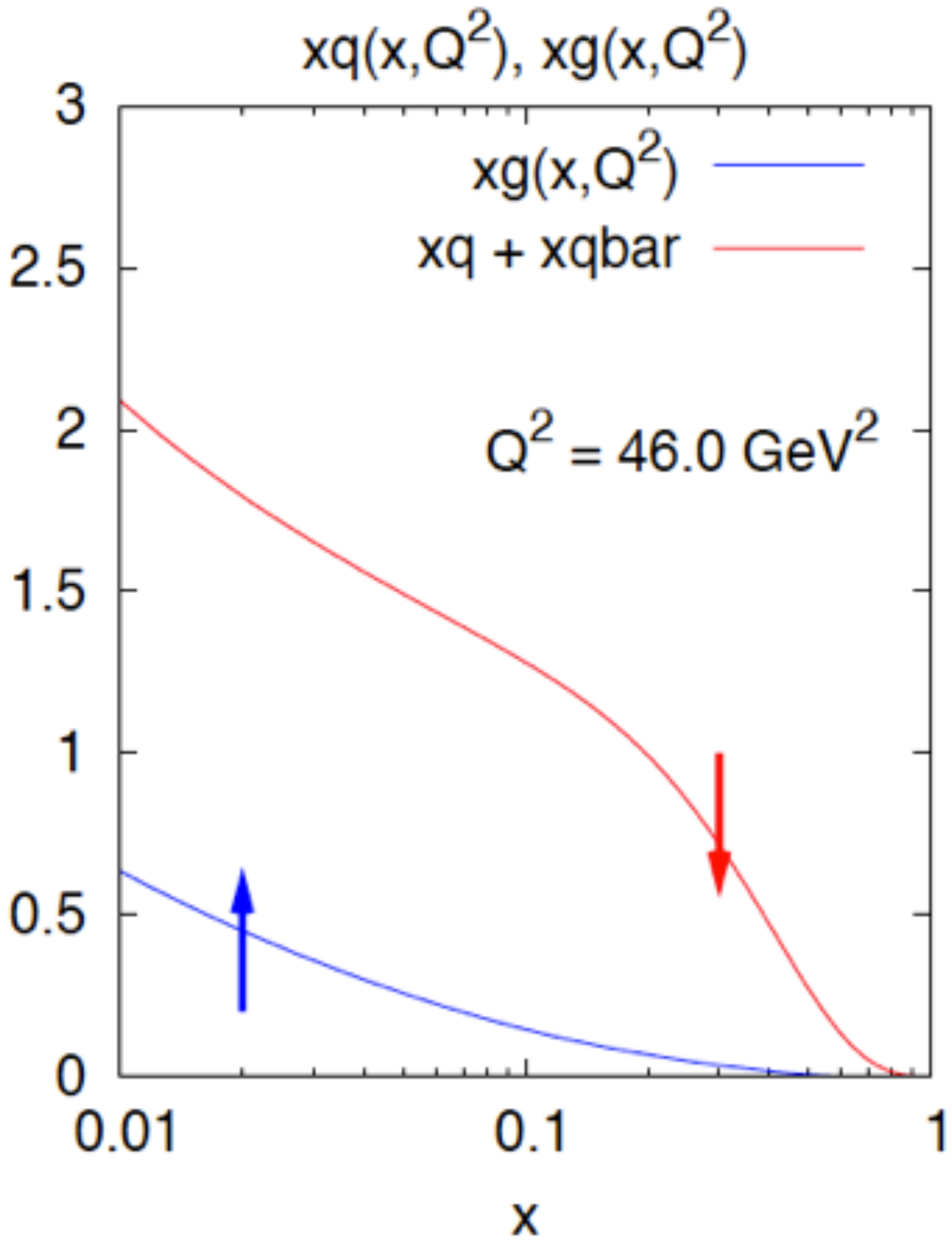

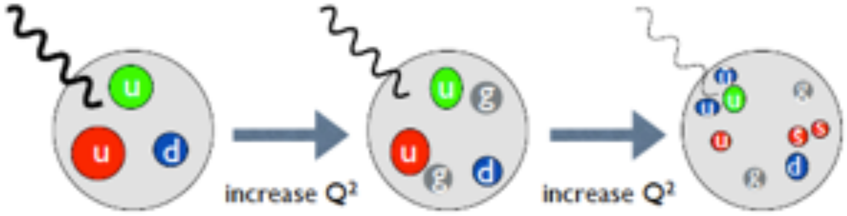

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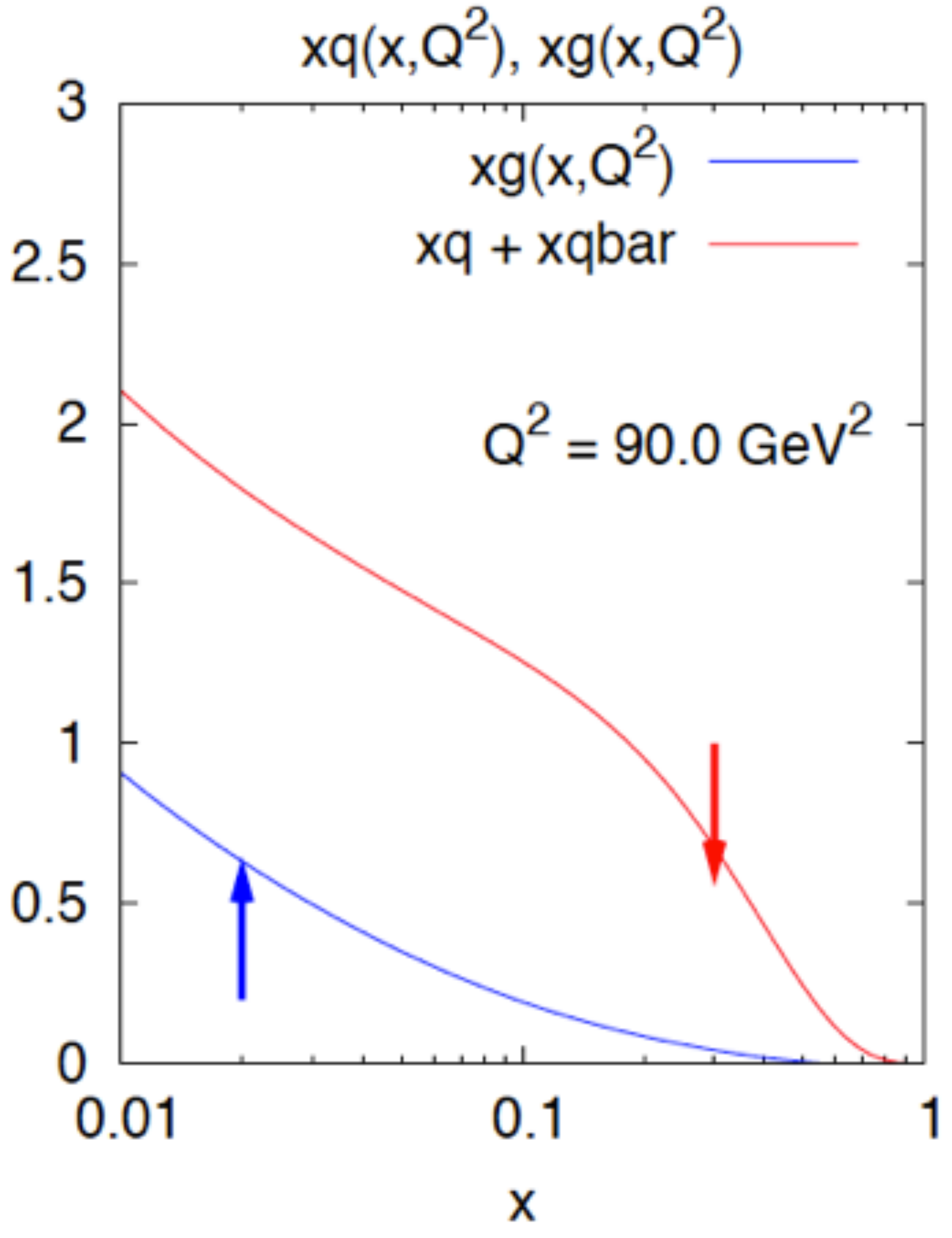

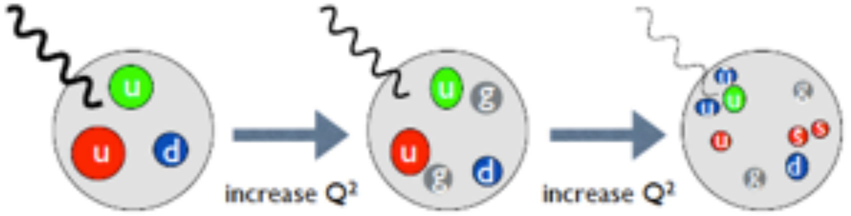

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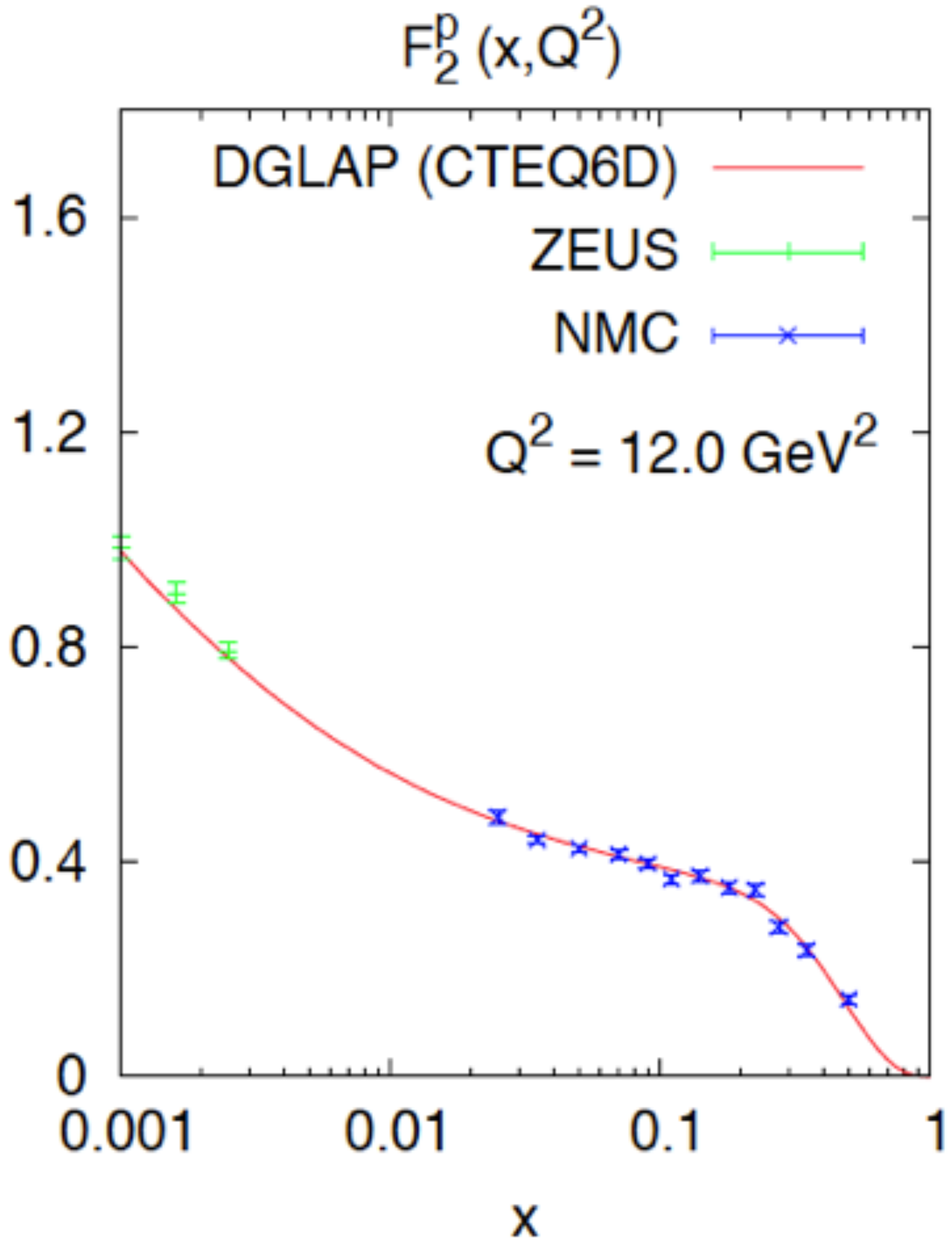

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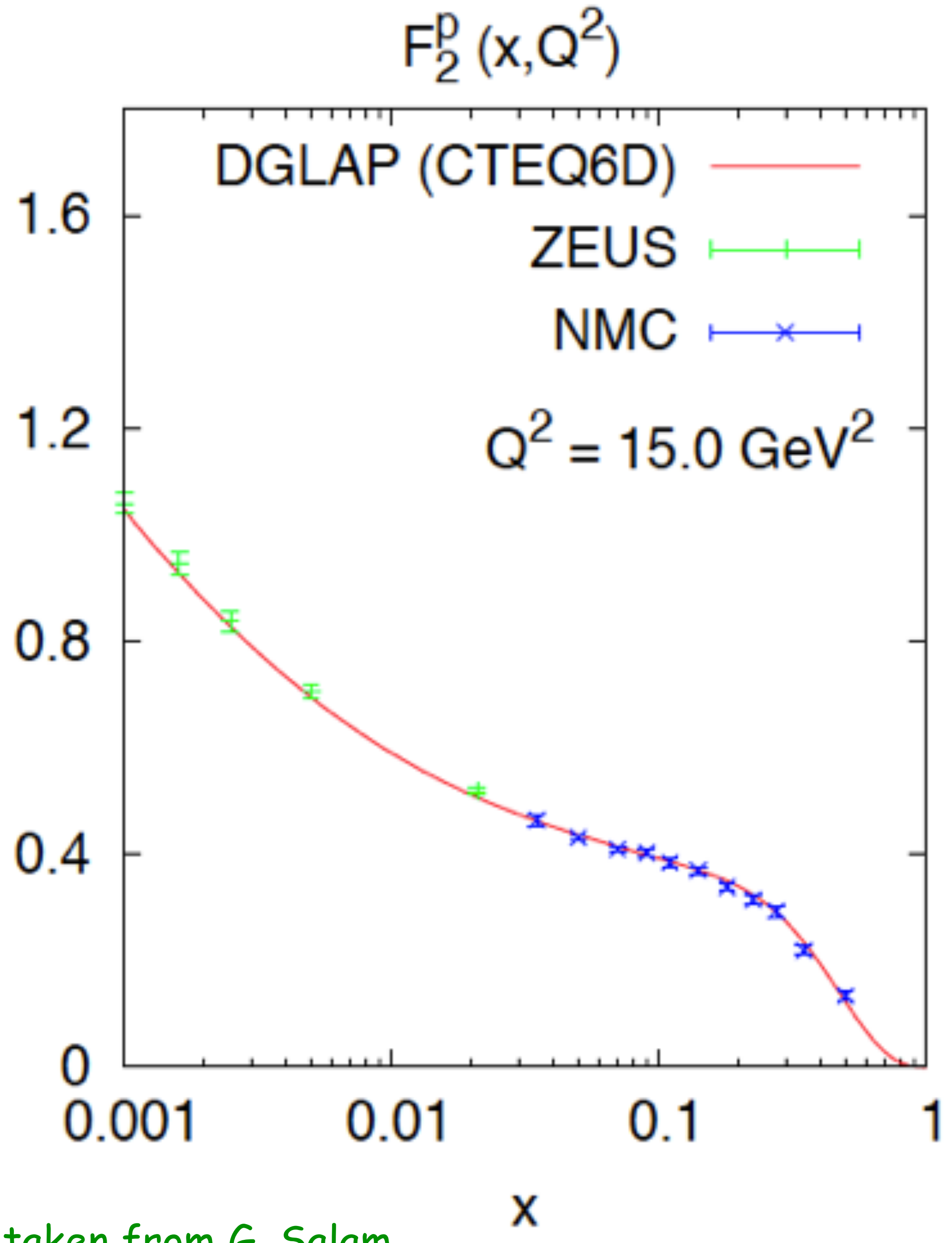

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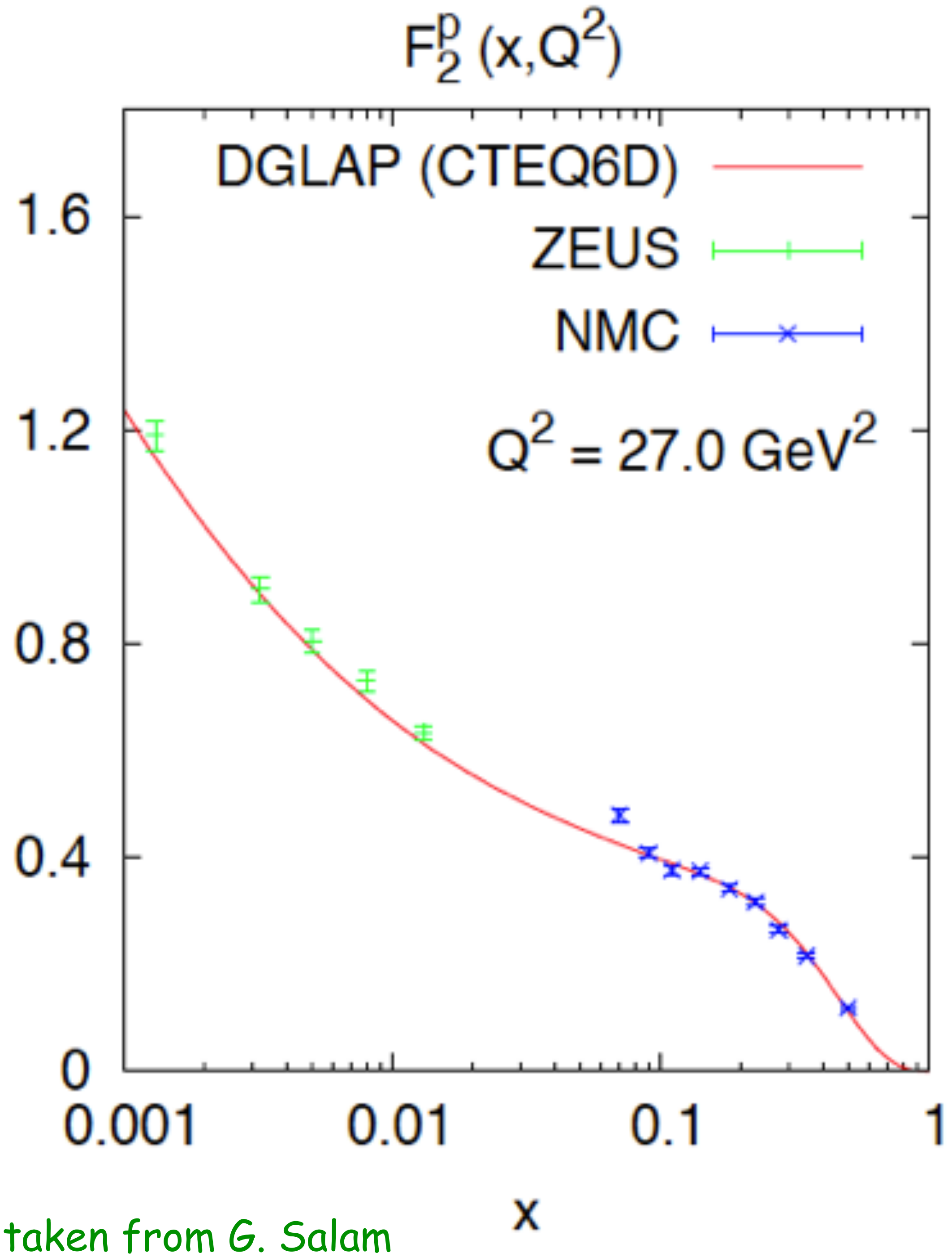

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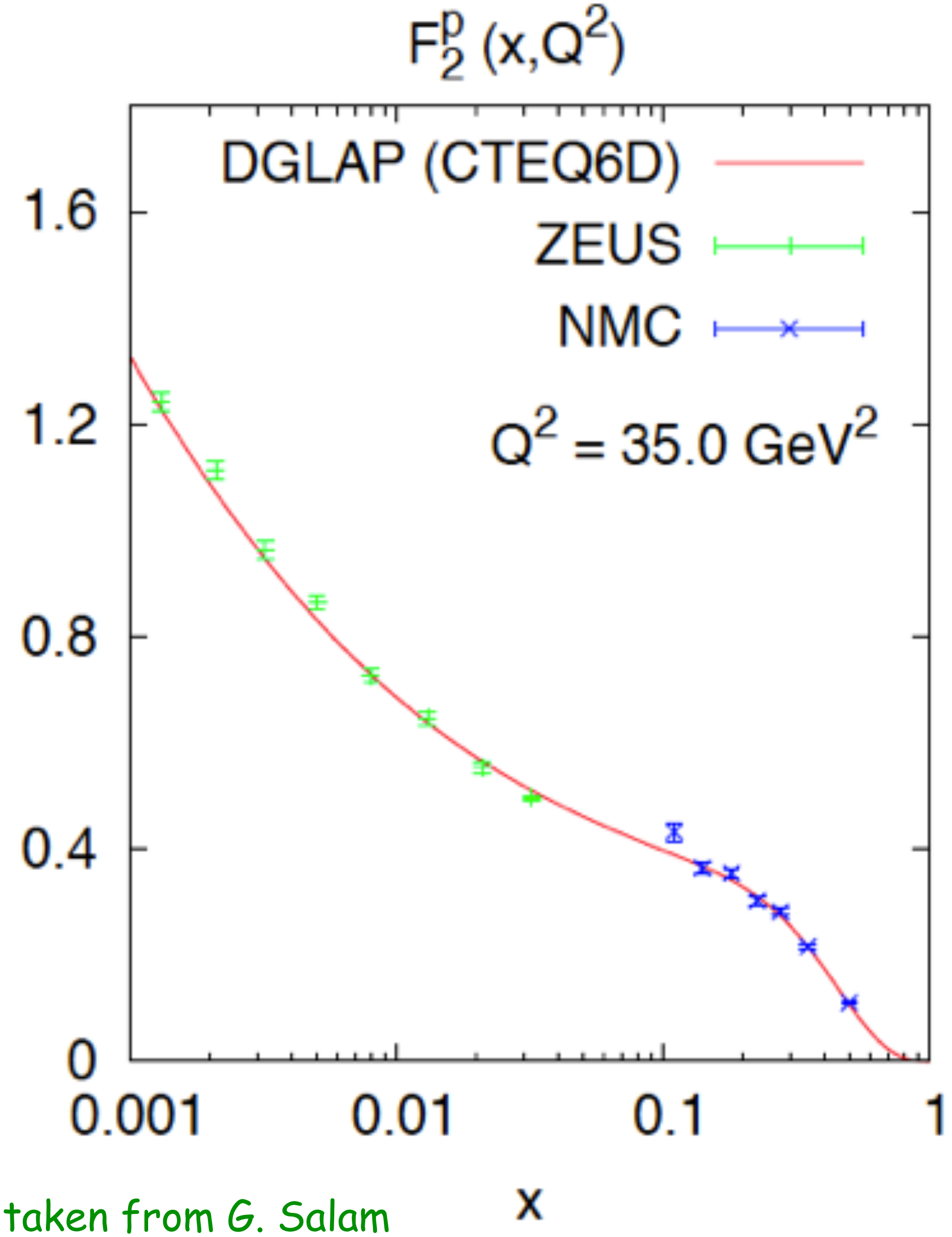

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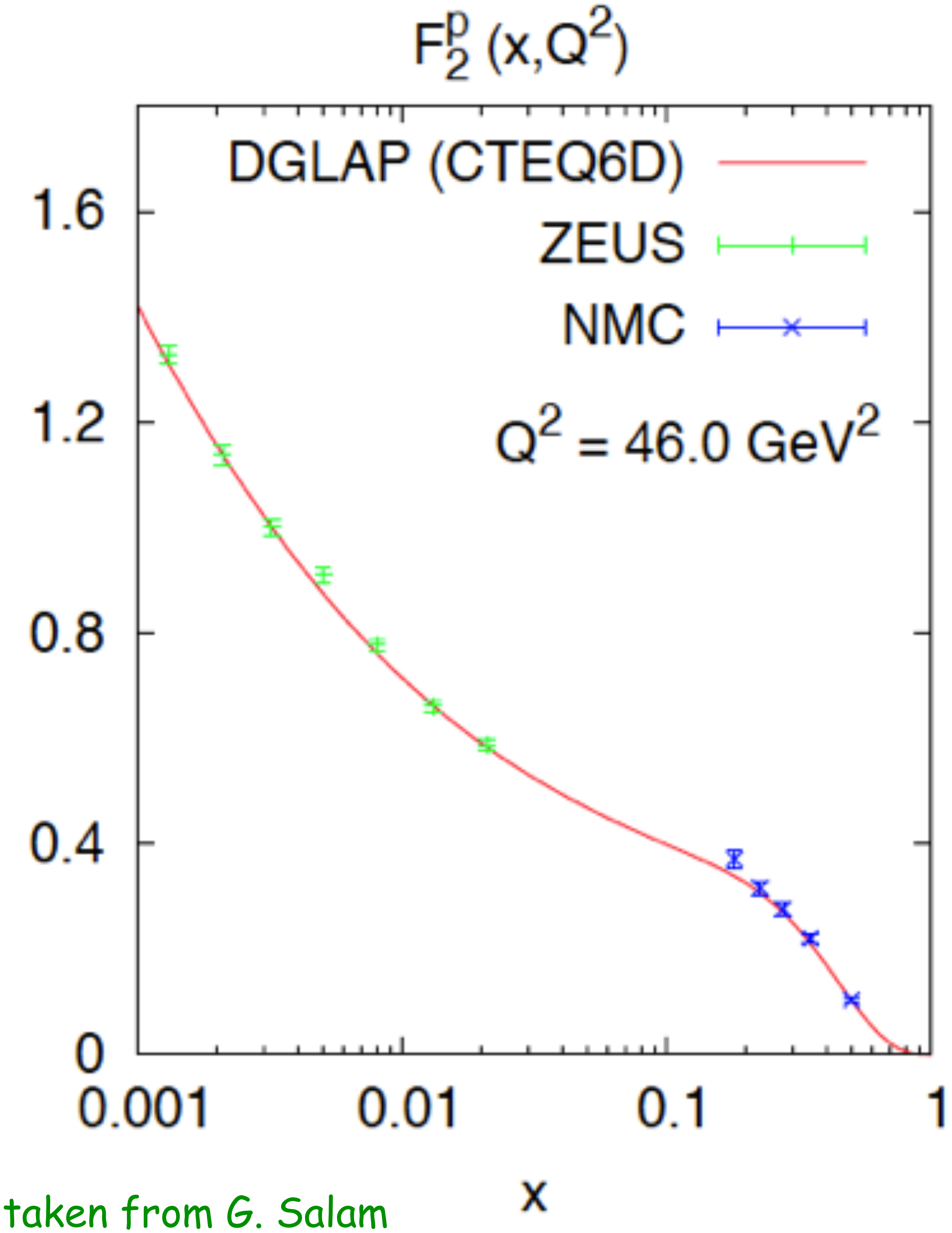

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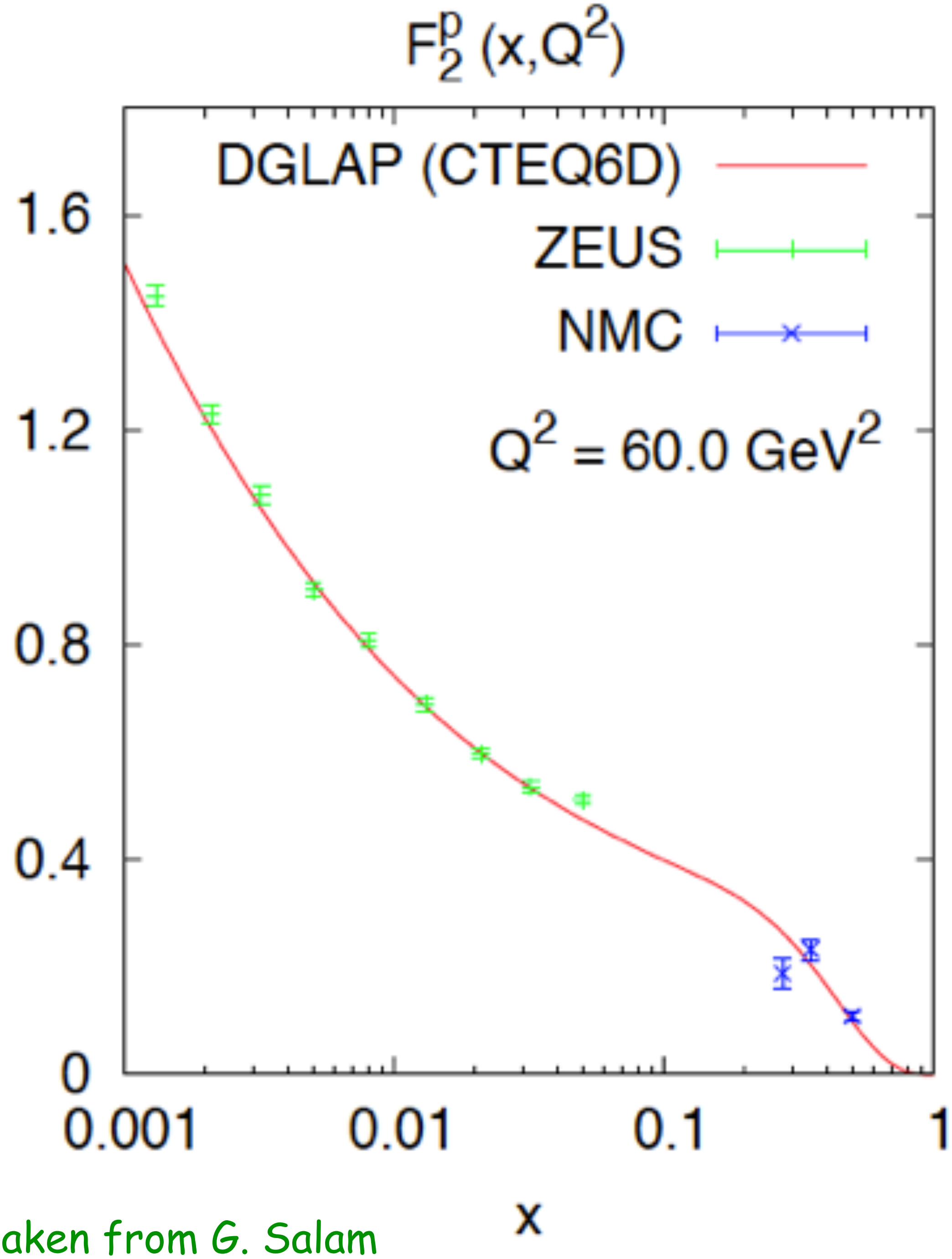

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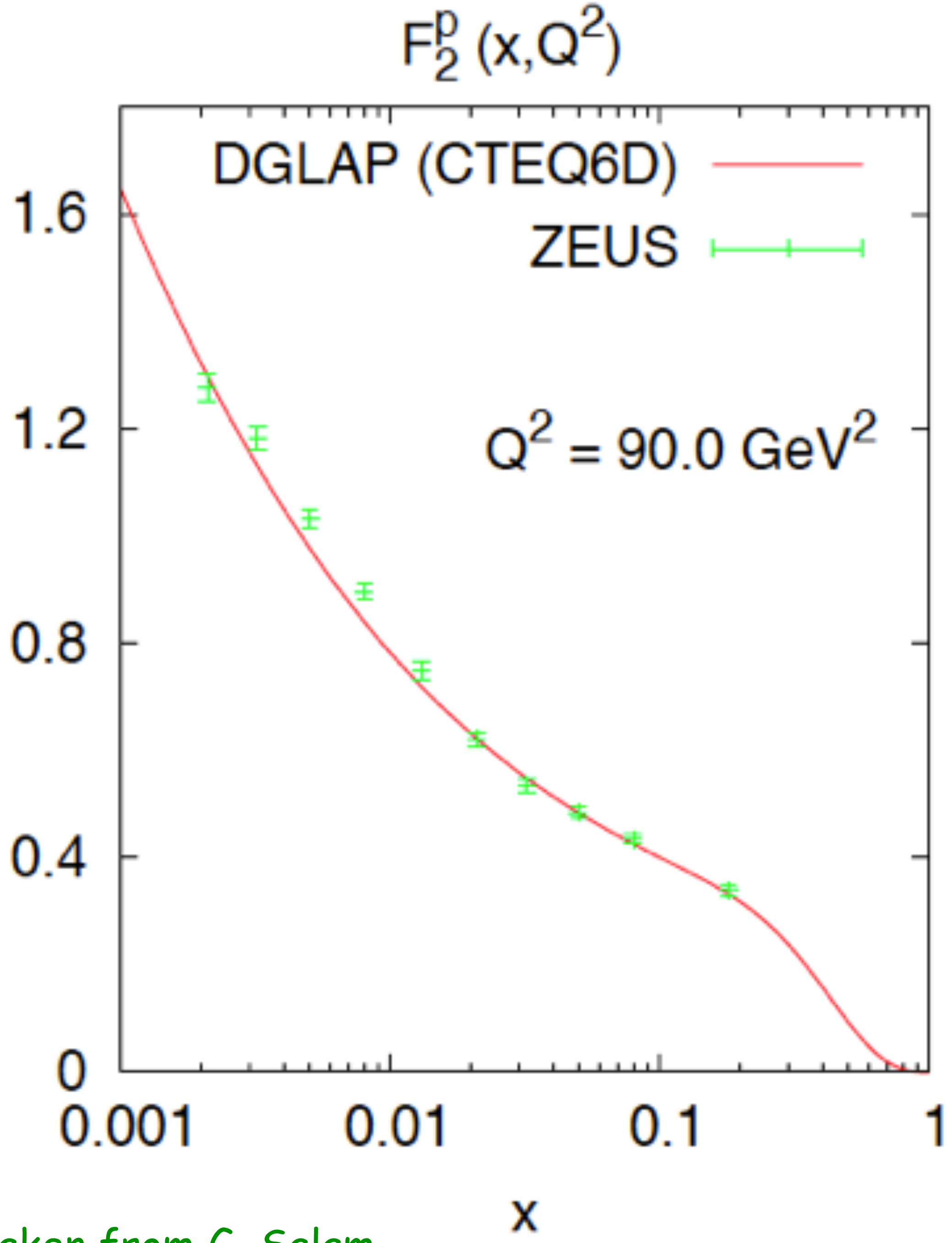

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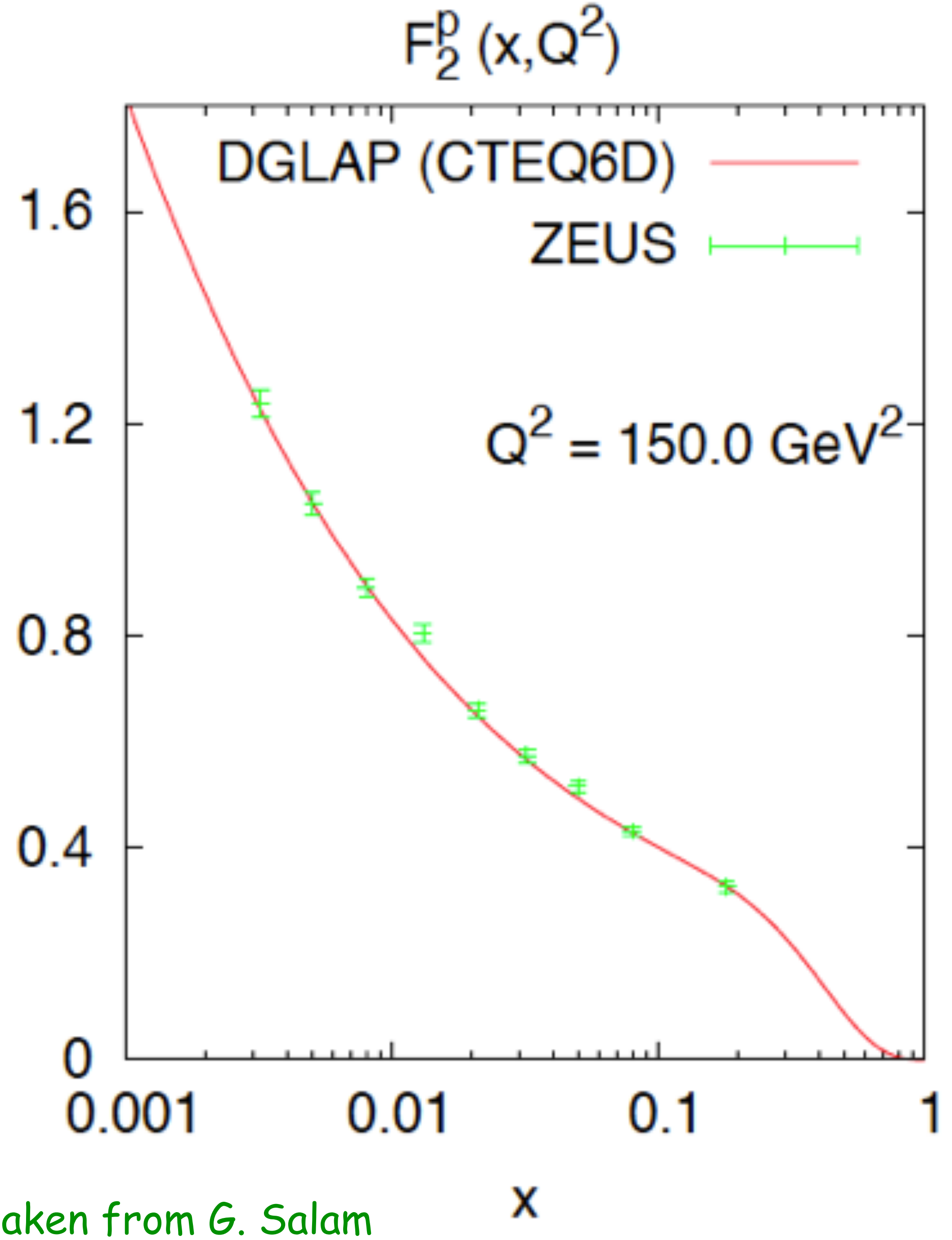

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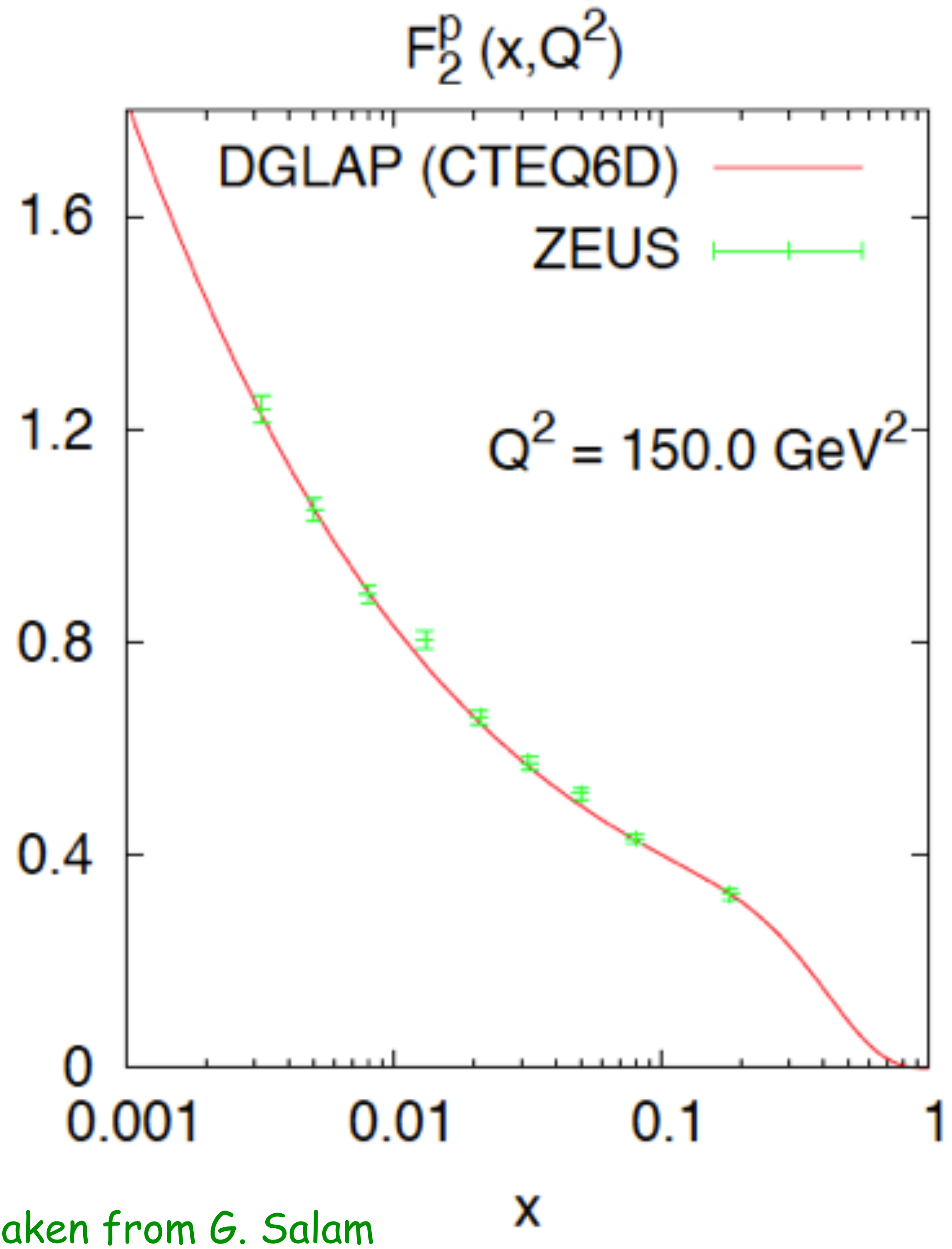

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major success of pQCD and DGLAP evolution

factorization in hadron-hadron collisions

What happens when two hadrons collide ?

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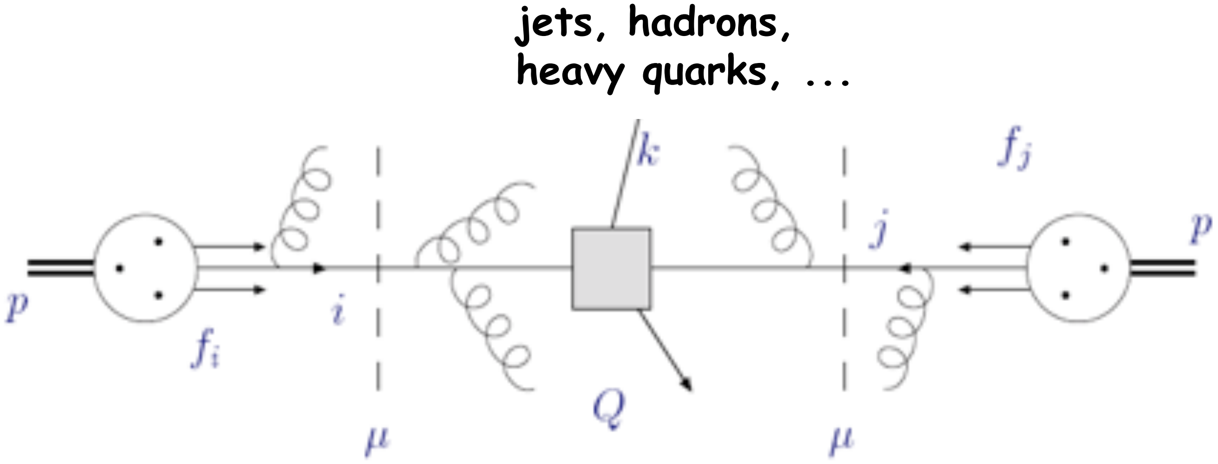

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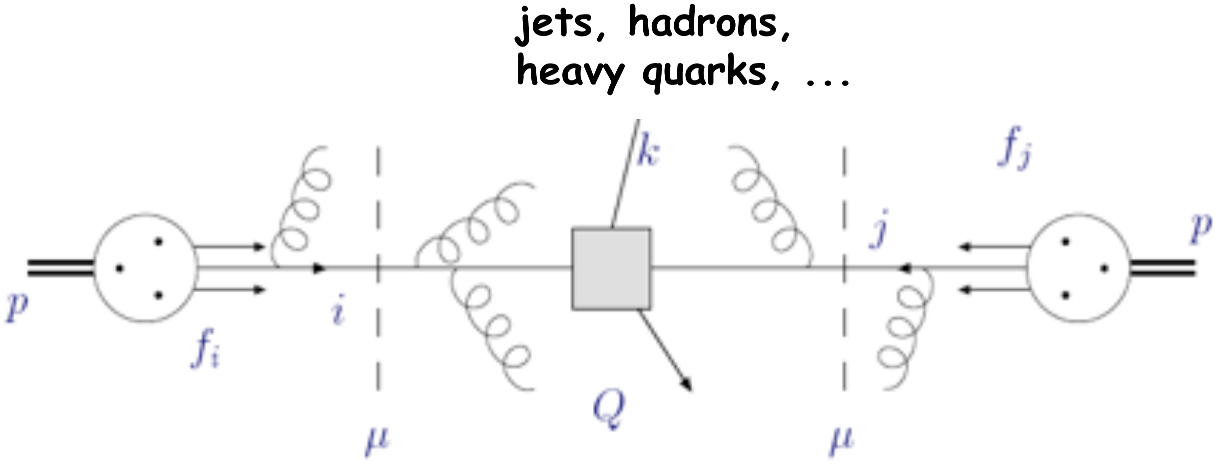

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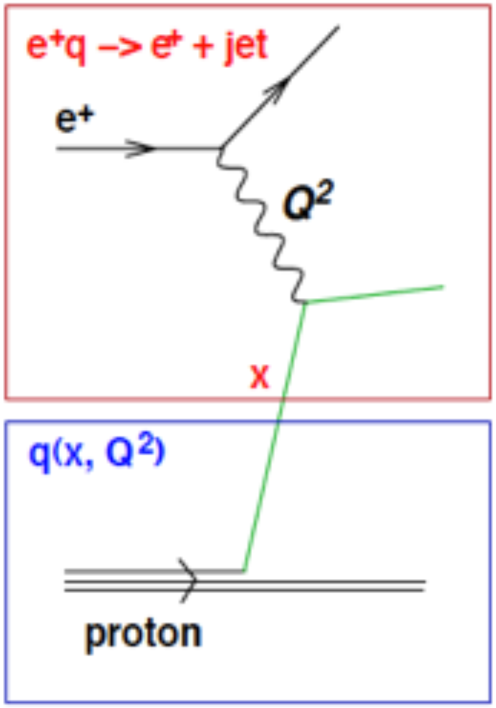

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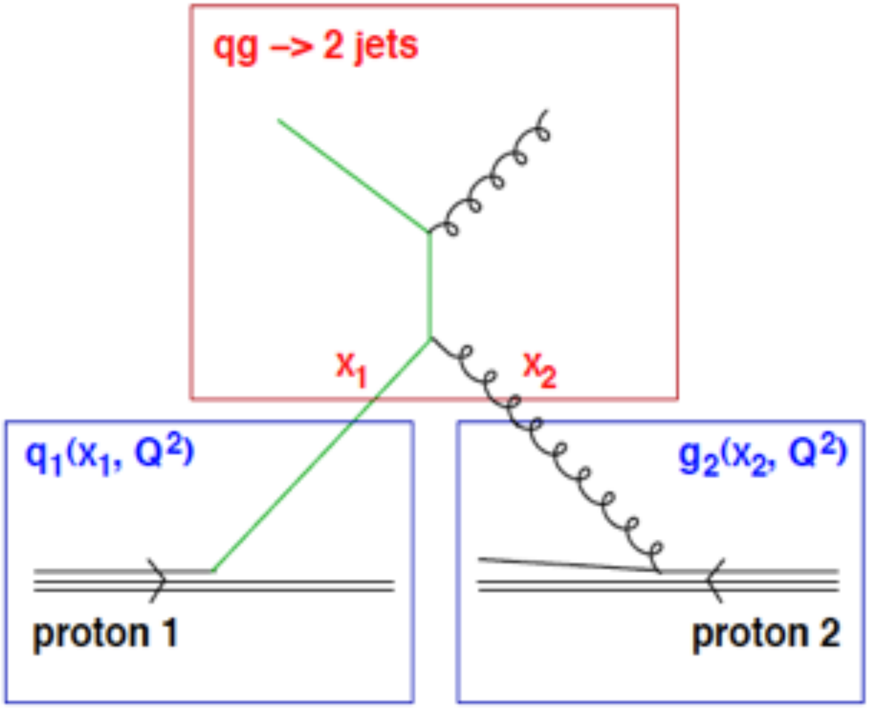

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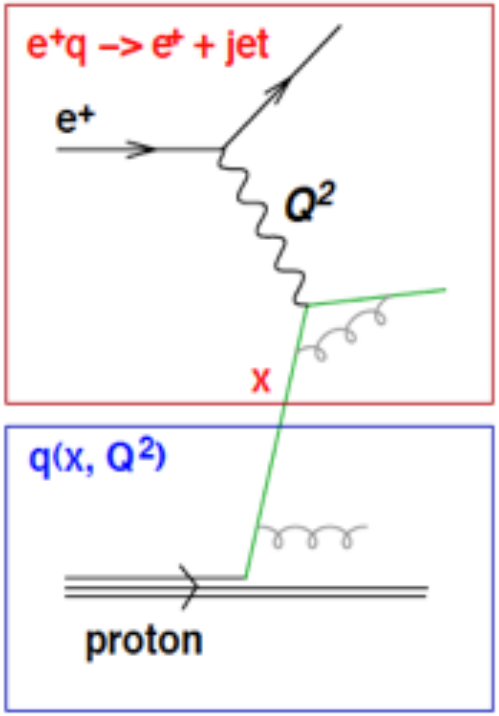

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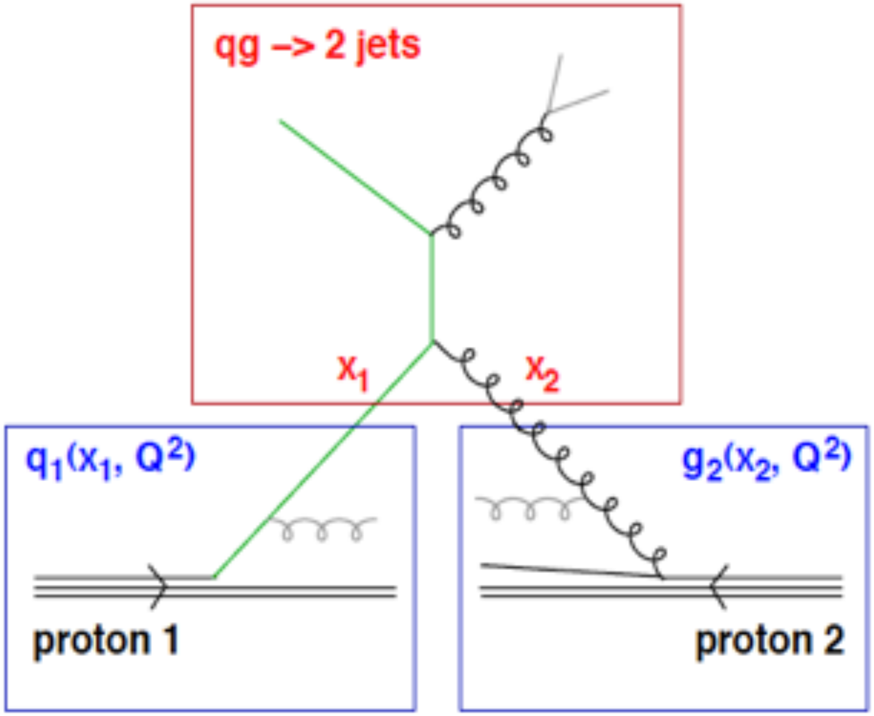

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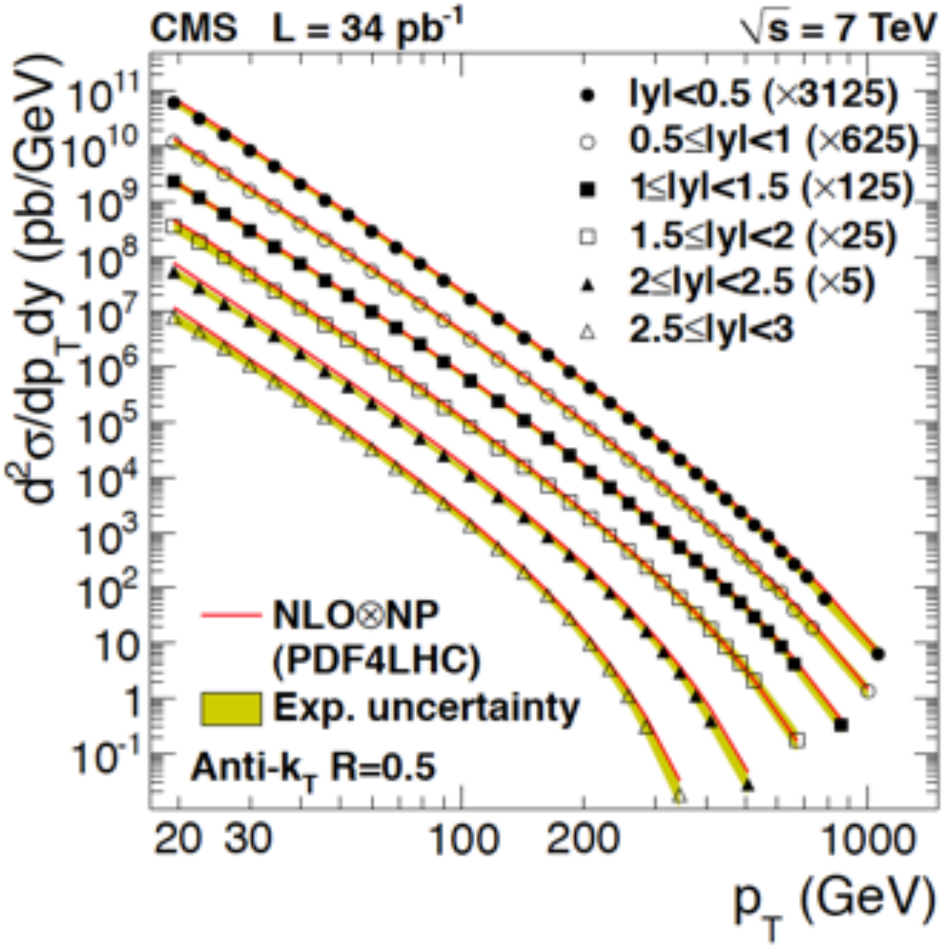
factorization: so far a success story



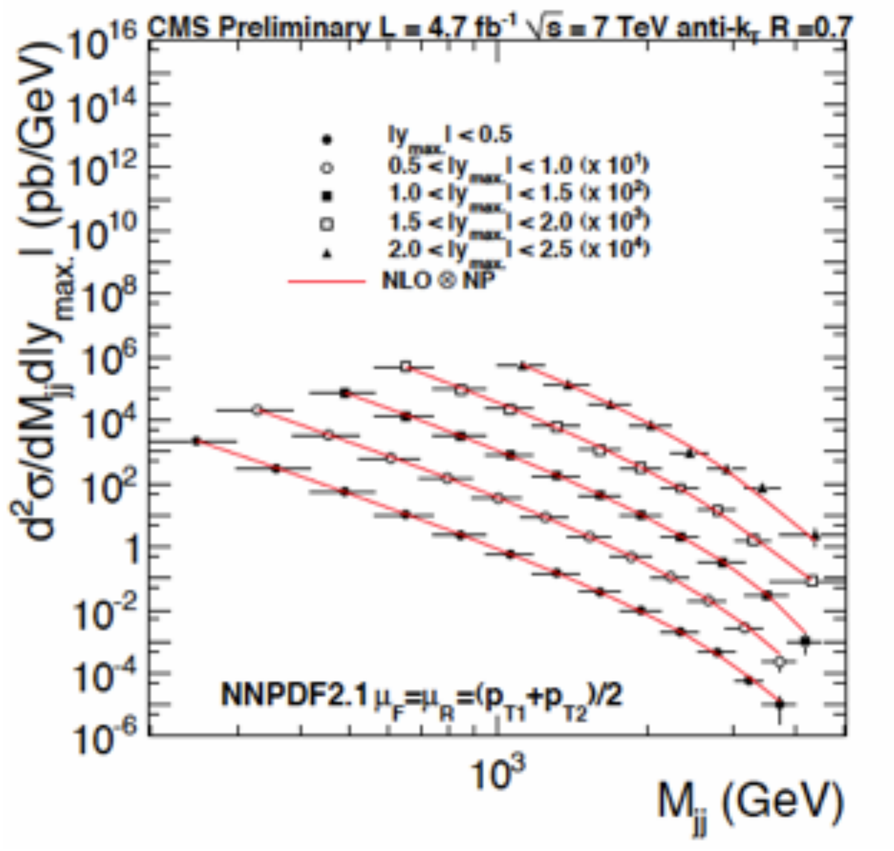
two recent examples from the LHC:

1-jet and di-jet cross sections

many other final-states available

$$y = \ln \tan \frac{\theta}{2} \sim \frac{1}{2} \ln \frac{x_1}{x_2} \qquad M = \sqrt{x_1 x_2 s}$$

$$x_1 = \frac{M}{\sqrt{s}} e^{+y} \qquad x_2 = \frac{M}{\sqrt{s}} e^{-y}$$



results now start to being used in global fits to constrain PDFs

particularly sensitive to gluons

$$gg \rightarrow gg \qquad gq \rightarrow gq$$

proofs of factorization

- to prove the validity of **factorization to all orders** of pQCD is a highly theoretical and technical matter

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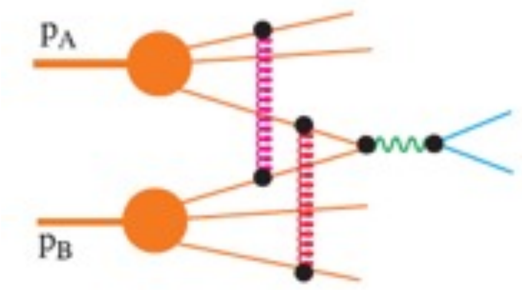

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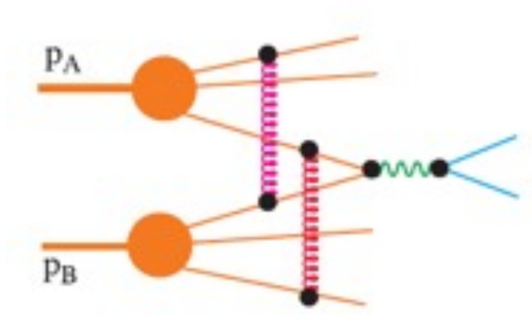

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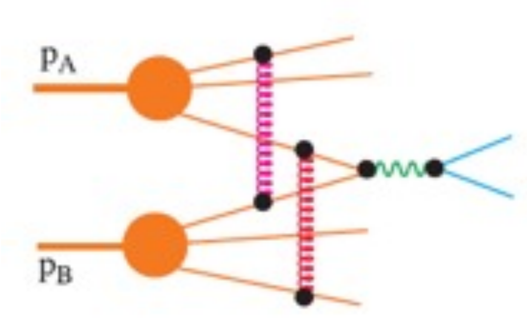

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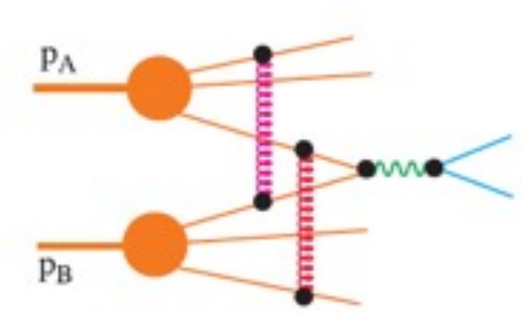

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recall: the renormalizibility of a non-abelian gauge theory like QCD was demonstrated by 't Hooft and Veltman


1999

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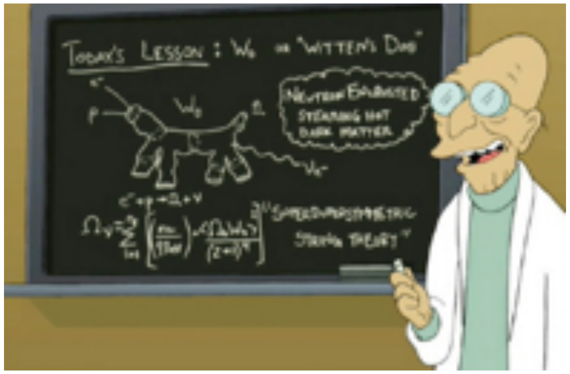
keys to resolve the apparent dilemma:

- asymptotic freedom
- infrared safety
- factorization theorems & renormalizibility

to take home from this part of the lectures



- factorization = isolating and absorbing long-distance singularities accompanying identified hadrons into parton densities (initial state) and fragmentation fcts. (final state)
- factorization and renormalization introduce arbitrary scales
  → powerful concept of renormalization group equations
  → α_s, PDFs, frag. fcts. depend on energy/resolution
- PDFs (and frag. fcts) have definitions as bilocal operators
- hard hadron-hadron interactions factorize as well: $f \otimes f \otimes d\sigma$
- strict proofs of factorization only for limited class of processes

pQCD: a tool for the most violent collisions

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high-p_T jet: factorization!

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“soft stuff”: difficult!

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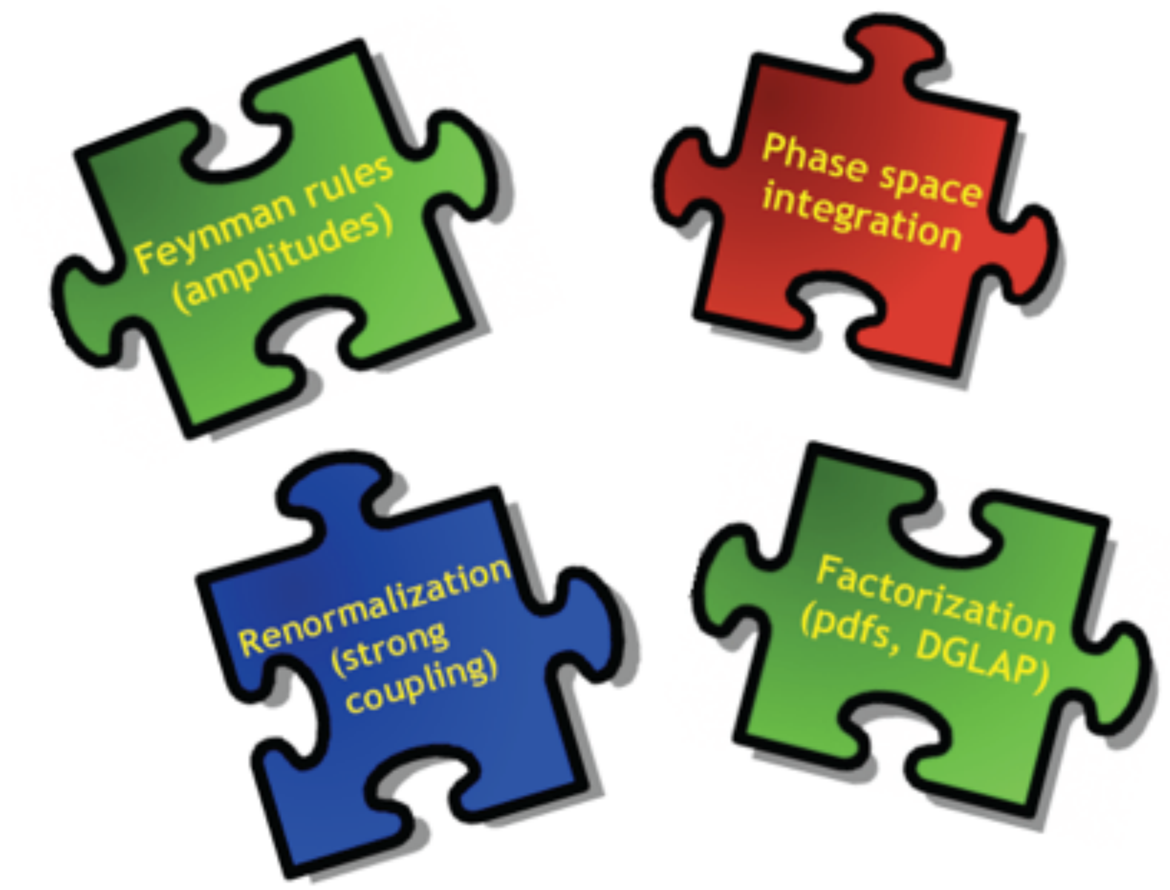
“underlying event”: more than difficult



unofficial Part IV

some applications & advanced topics

scales and theoretical uncertainties; Drell-Yan process
small-x physics; global QCD analysis; resummations

Start your
business right
with Precision
Calculations
advise!

1

the Whys and Hows of NLO Calculations & Beyond

why go beyond LO (and even NLO)?

recall factorization theorem for hadronic processes:

$$d\sigma = \sum_{ij} \int dx_i dx_j \, f_i(x_i, \mu^2) \, f_j(x_j, \mu^2) \, d\hat{\sigma}_{ij}(\alpha_s(\mu_r), Q^2, \mu^2, x_i, x_j)$$

non-perturbative but universal PDFs $\xleftrightarrow[\text{by } \mu]{\text{linked}}$ **hard scattering of two partons → pQCD**

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caveat: we work with a perturbative series truncated at LO, NLO, NNLO, …

→ at any fixed order N there will be a **residual scale dependence** in our theoretical prediction

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simplest example:

$e^+e^- \to$ hadrons

$$\frac{d}{d\ln\mu_r} \sum_{n=1}^{N} c_n(\mu_r)\alpha_s^n(\mu_r) \sim \mathcal{O}\left(\alpha_s^{N+1}(\mu_r)\right)$$

applies in general also for μ_f

uncertainty is formally of higher order

-> gets smaller if higher orders are known

explicit example: scale dependence of e^+e^- --> jets

recall: at NLO we have $\sigma^{\mathrm{NLO}}(\mu_R) = \sigma_{q\bar{q}}\,(1 + c_1\,\alpha_s(\mu_R))$

LO result

NLO coefficient
independent of scale

all **scale uncertainty**
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NLO coefficient
independent of scale

all **scale uncertainty**
from strong coupling

suppose we want to choose a different scale Q - what do we need to do?

recall: $\alpha_s(\mu_r^2) = \dfrac{\alpha_s(Q^2)}{1 + 2b_0\alpha_s(Q^2)\ln(\mu_r/Q)}$ $\underset{\text{expand}}{\overset{\text{coupling small}}{\approx}} \alpha_s(Q^2) - 2b_0\, \alpha_s^2(Q^2)\, \ln(\mu_r/Q)$

explicit example: scale dependence of e^+e^- --> jets

recall: at NLO we have $\sigma^{\text{NLO}}(\mu_R) = \sigma_{q\bar{q}}\,(1 + c_1\,\alpha_s(\mu_R))$

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plug back into σ^{NLO} $= \sigma_{q\bar{q}}\left(1 + c_1\,\alpha_s(Q) - 2c_1 b_0 \ln\frac{\mu_R}{Q}\,\alpha_s^2(Q) + \mathcal{O}\left(\alpha_s^3\right)\right)$

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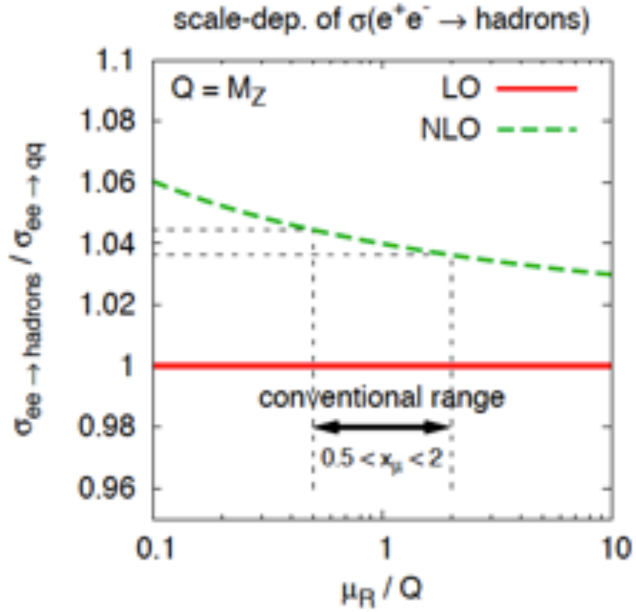

LO is a pure el-mag process, no α_s , no scales

explicit example - cont’d

next calculate full NNLO result:

$$\sigma^{\mathrm{NNLO}}(\mu_R) = \sigma_{q\bar{q}}\ \left[1\ +\ c_1\,\alpha_s(\mu_R) + c_2(\mu_R)\alpha_s^2(\mu_R)\right]$$

NNLO term starts to
depend on the scale

explicit example - cont’d

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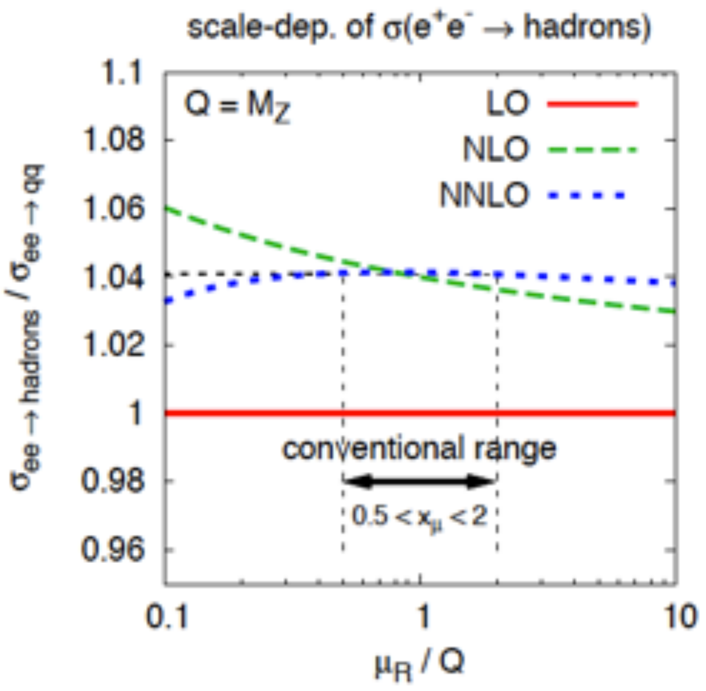

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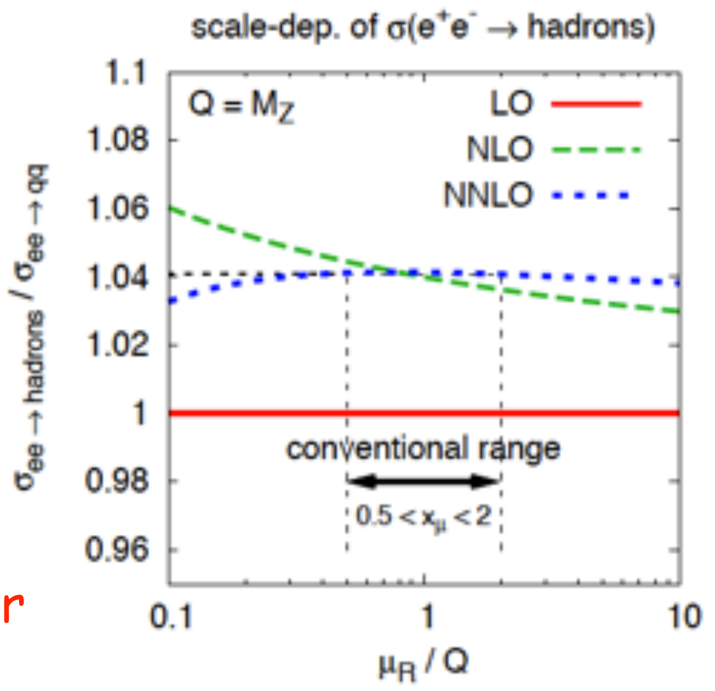

explicit example - cont'd

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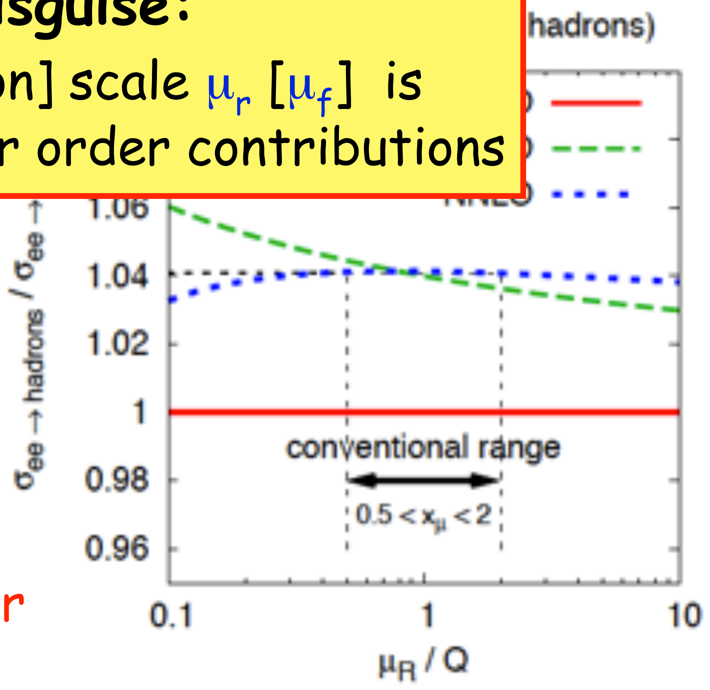
in fact c_2 must (and will !) cancel the scale ambiguity found at NLO:

scale "ambiguity" is a blessing in disguise:
varying the renormalization [factorization] scale μ_r [μ_f] is
a way of guessing the uncalculated higher order contributions



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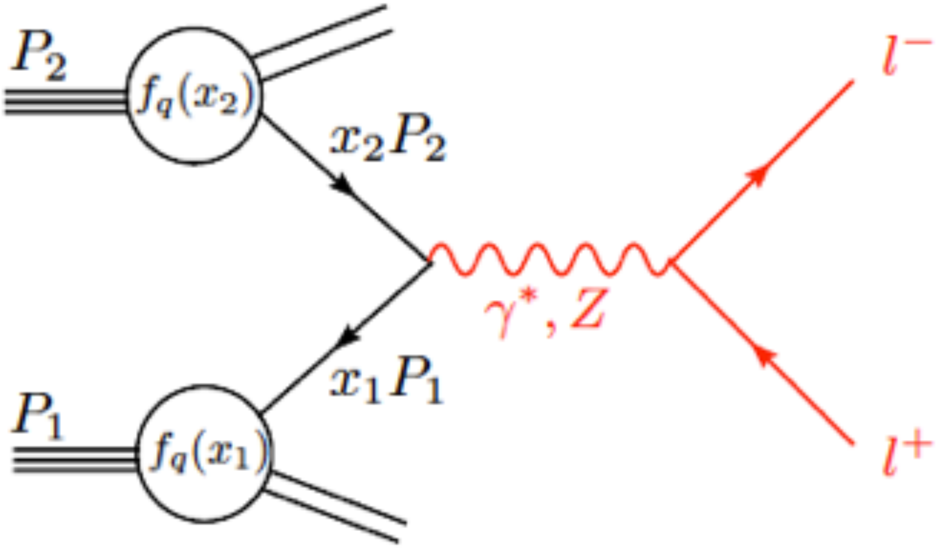
example from hadronic collisions

take the "classic" Drell Yan process



- dominated by quarks in the initial-state
- at LO no colored particles in the final-state
- clean experimental signature
- at LO an electromagnetic process (low rate)
- one of the best studied processes (known to NNLO)

as "clean" as it can get at a hadron collider

uncertainties for the Drell Yan process – cont'd

at NLO:

LO piece

$$\sigma^{\mathrm{NLO}}_{pp\to Z} = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1,\mu_F^2)\, f_j(x_2,\mu_F^2)\, [\hat{\sigma}_{0,ij\to Z}(x_1,x_2) + \\ + \alpha_s(\mu_R)\hat{\sigma}_{1,ij\to Z}(x_1,x_2,\mu_F)]$$

- no α_s at LO but μ_F appears in PDFs
- α_s enters at NLO and hence μ_R
- NLO terms reduce dep. on μ_F

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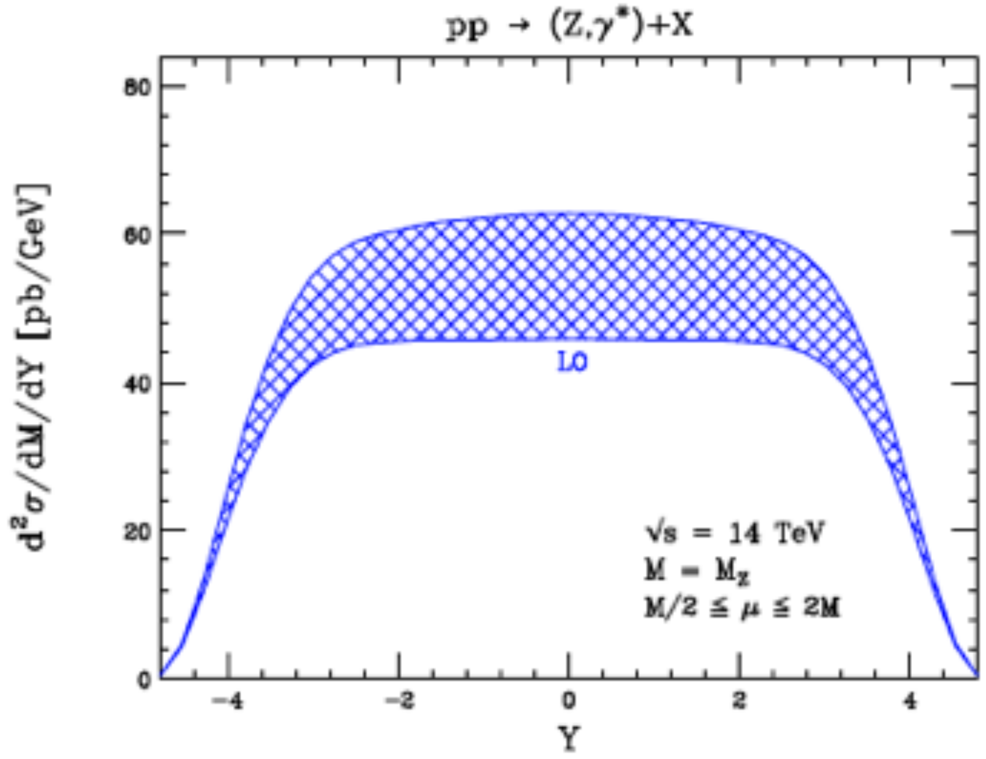

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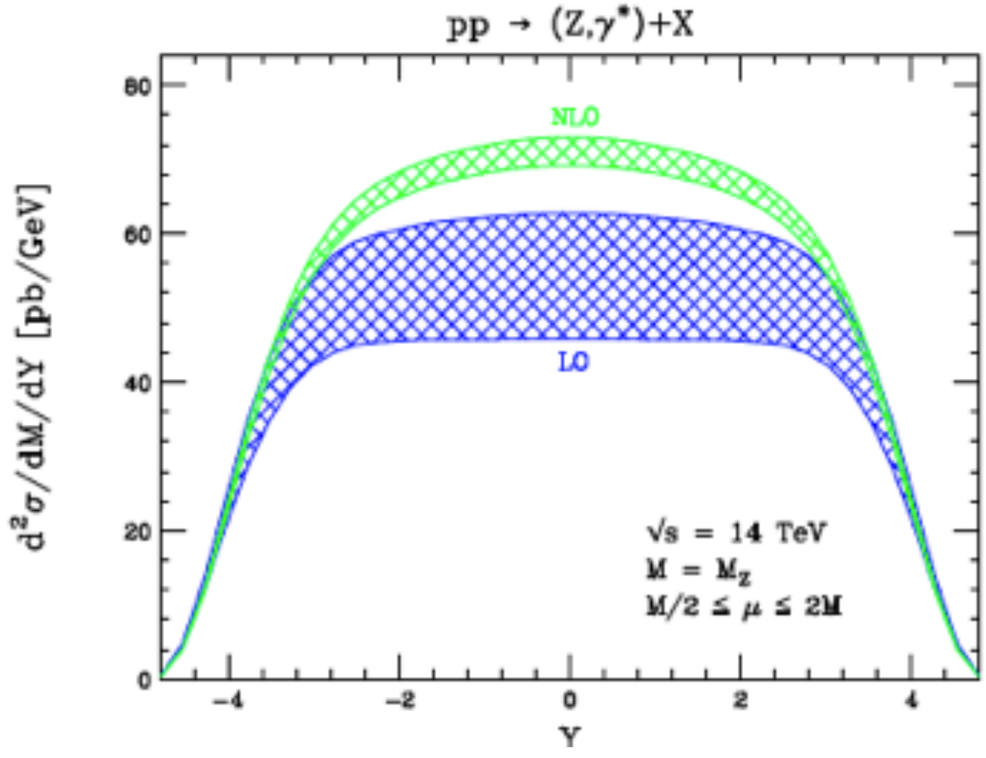

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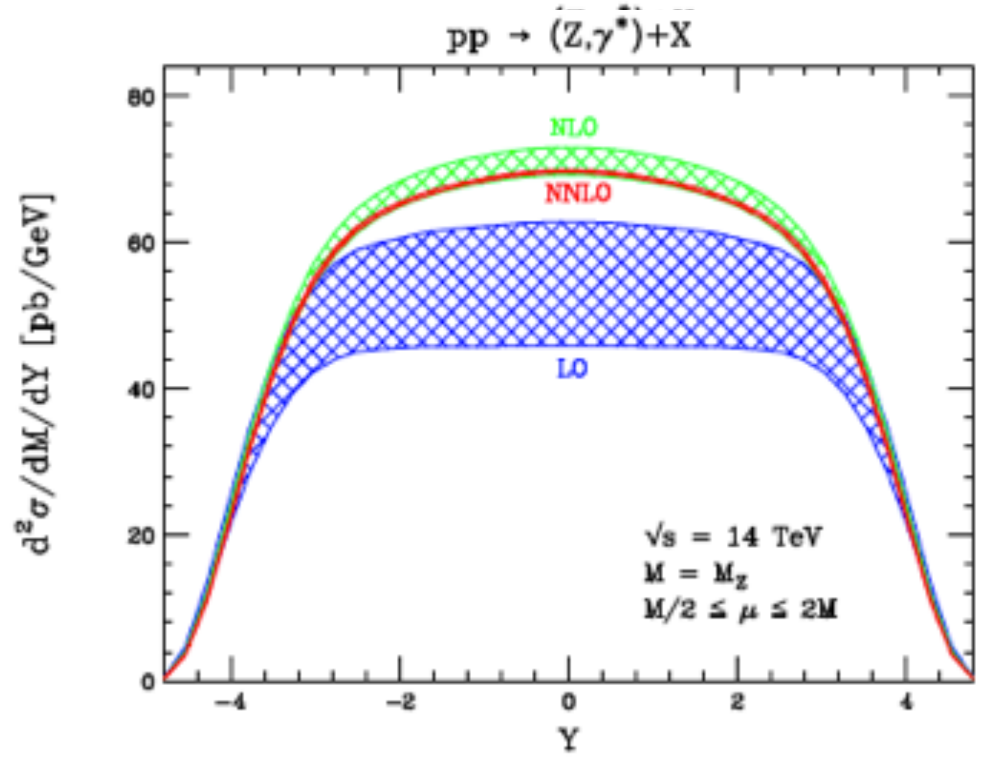

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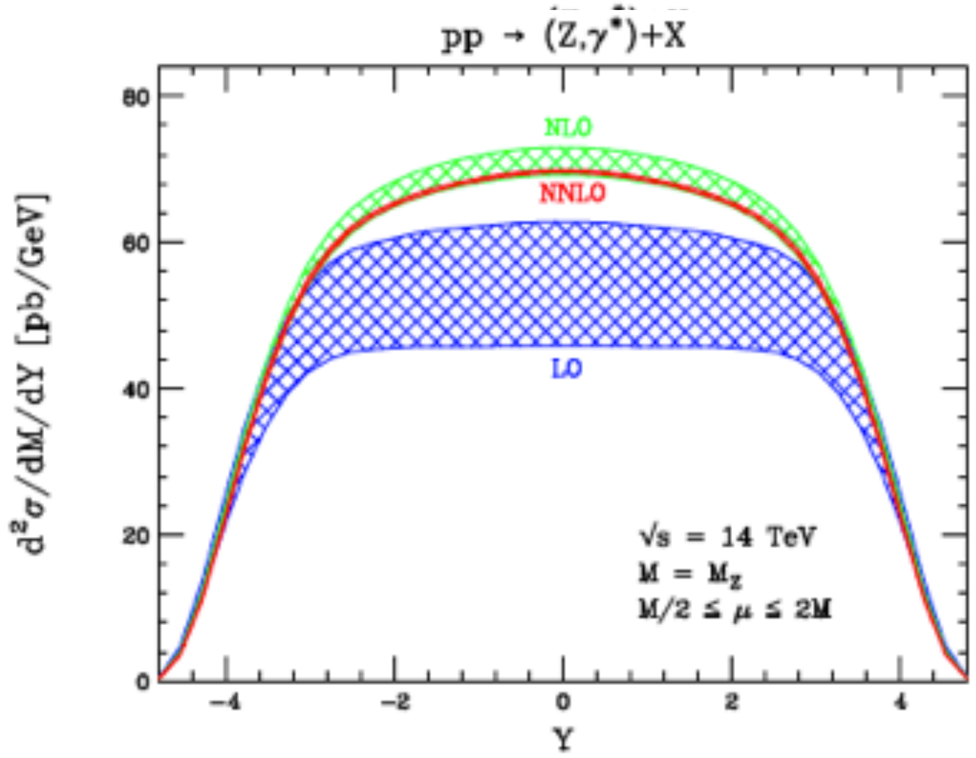
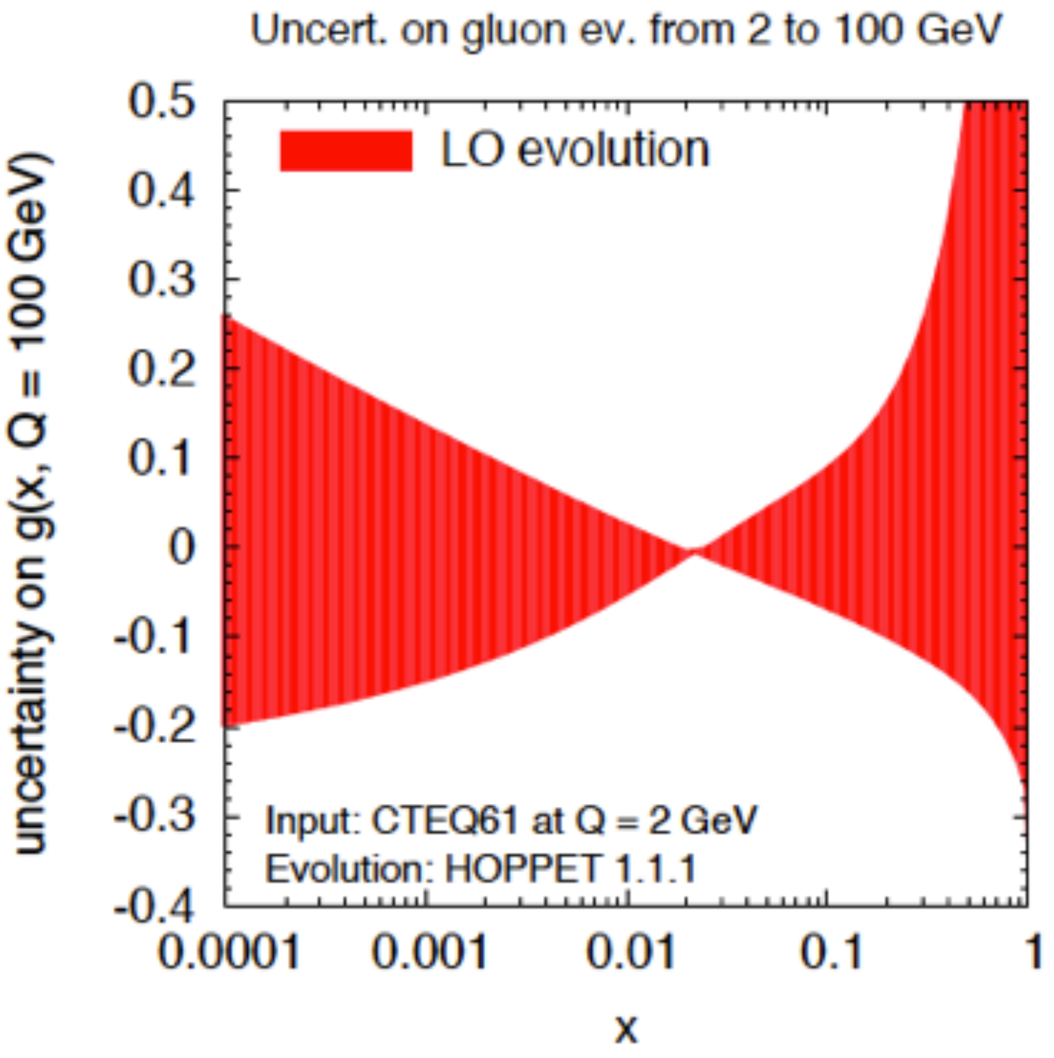


perturbative accuracy of O(percent) achieved

changing scales in DGLAP evolution

estimate by G. Salam: vary the scale of α_s in the DGLAP kernel



- about 30% in LO

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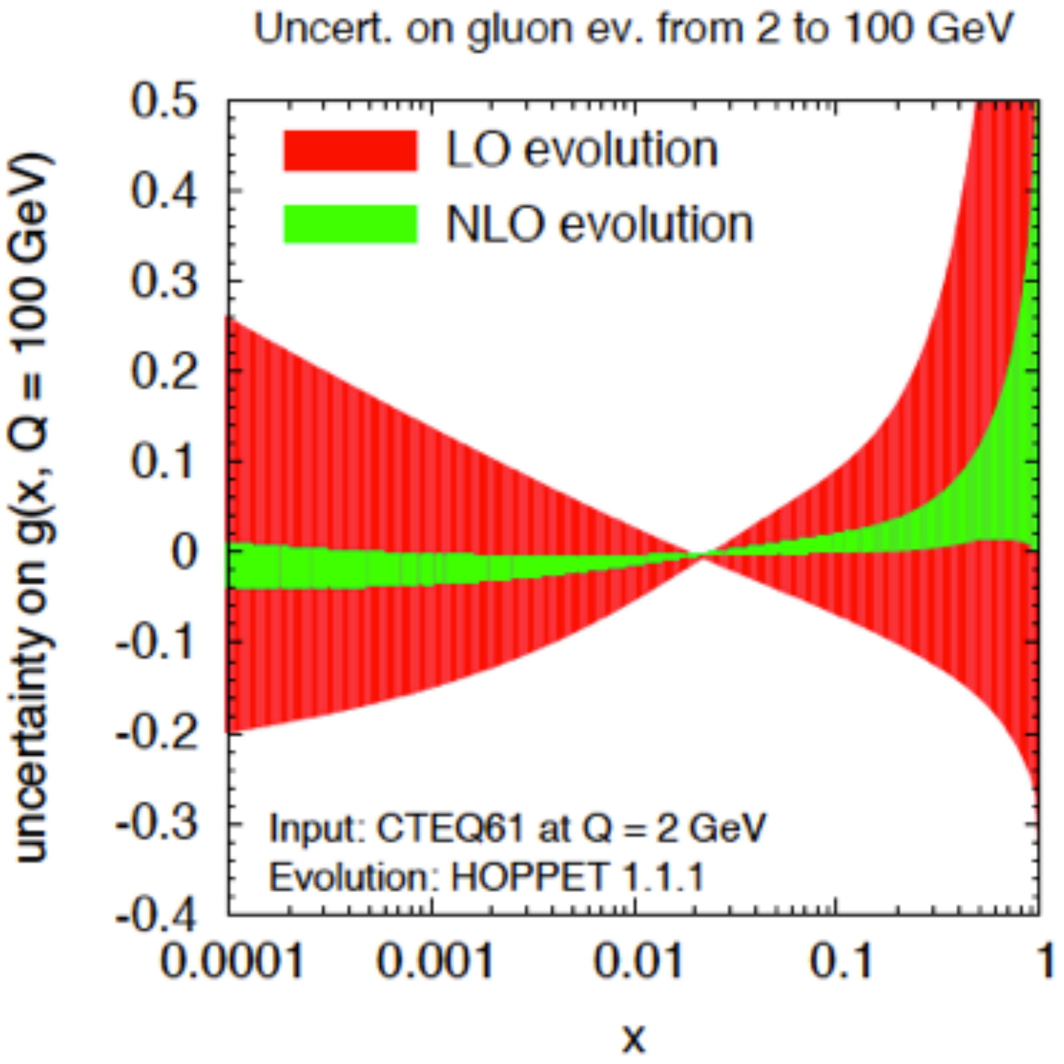

- about 30% in LO
- down to about 5% in NLO

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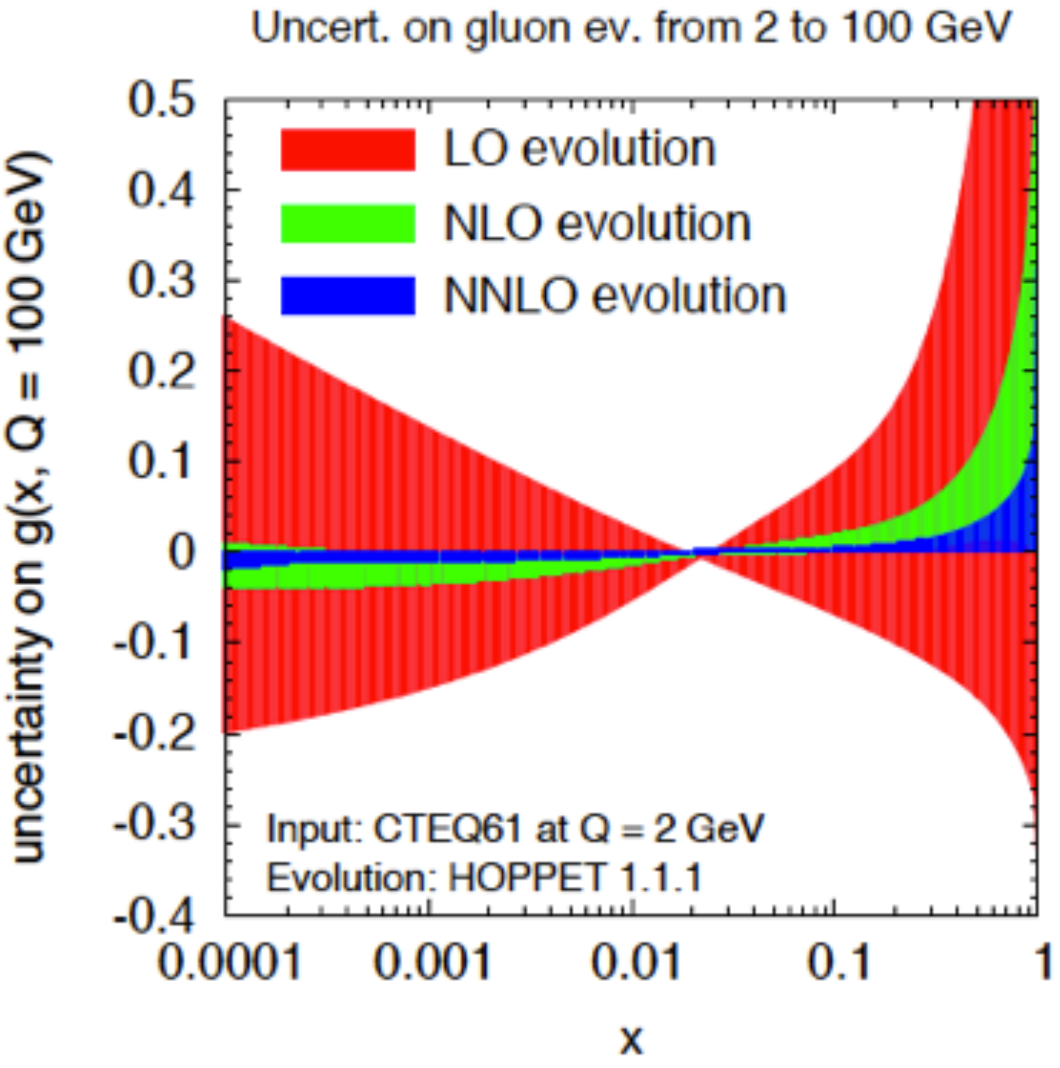

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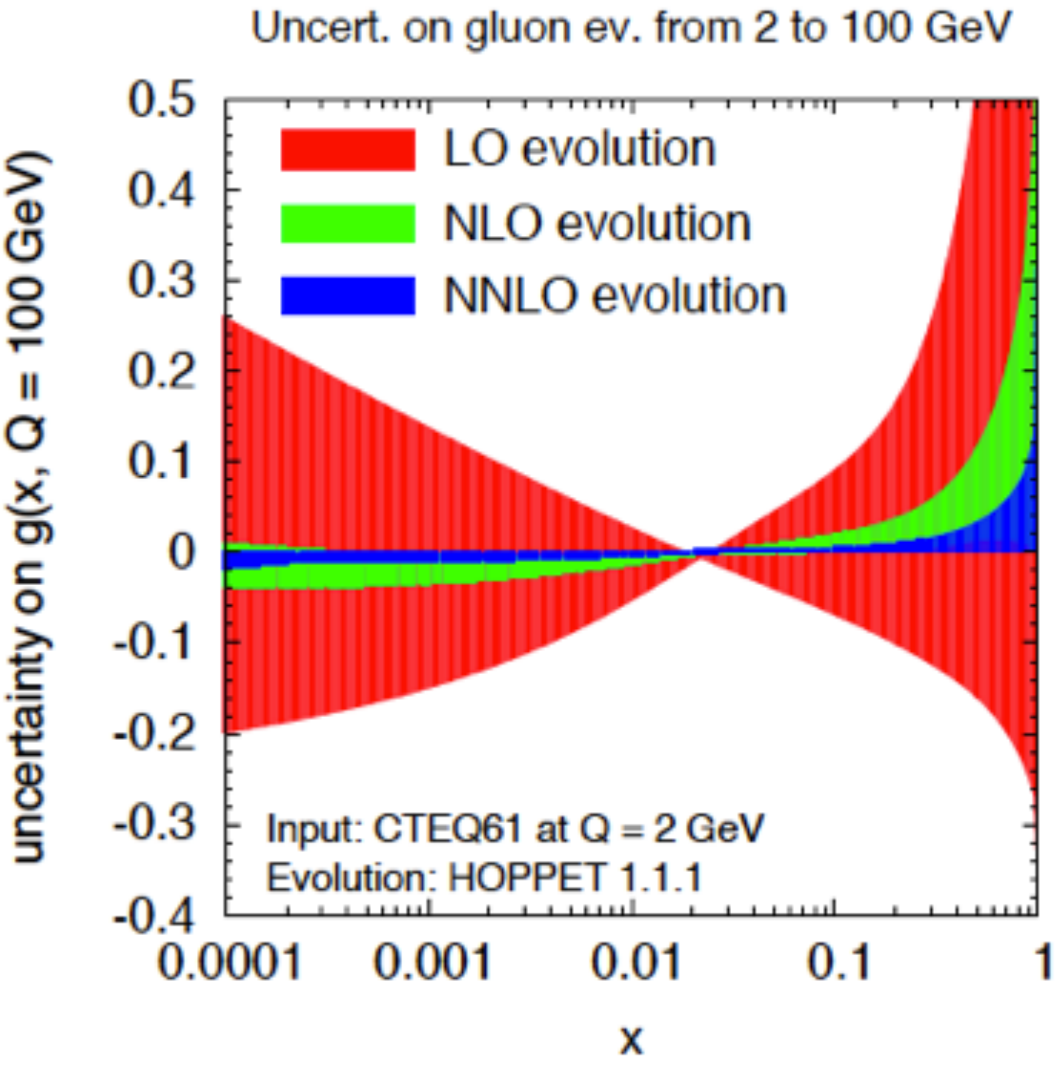

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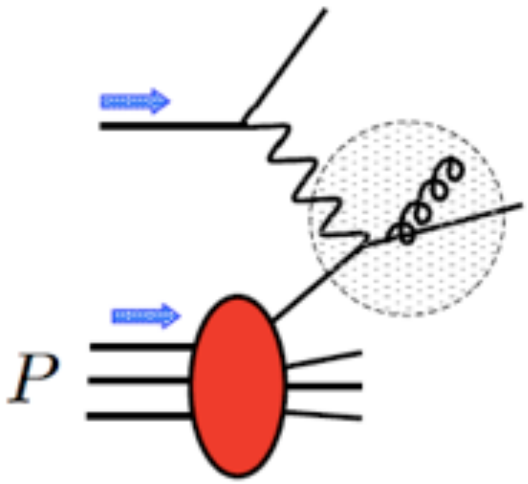
  which is about the precision of the HERA DIS data

2

Anatomy of a Global QCD Analysis

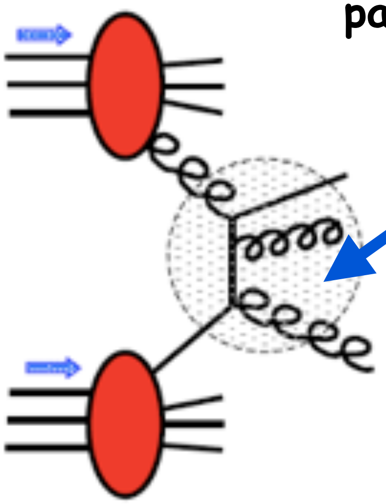
how to determine PDFs from data?

probes:



DIS

hard scale Q



parton cross section calculable

hadron-hadron

hard scale p_T

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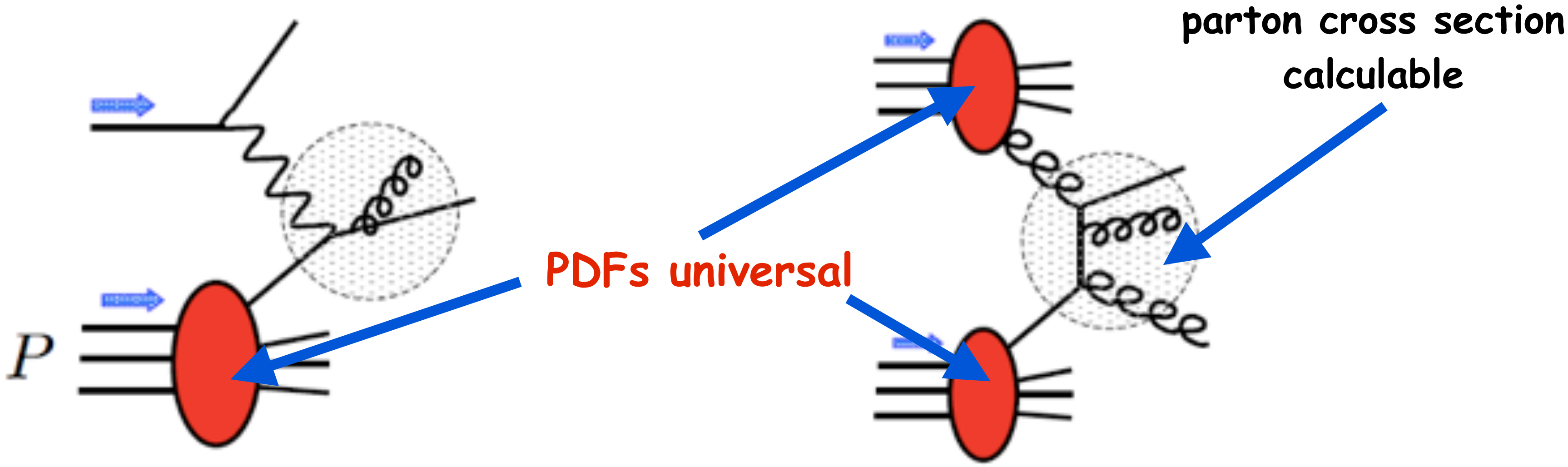

PDFs universal

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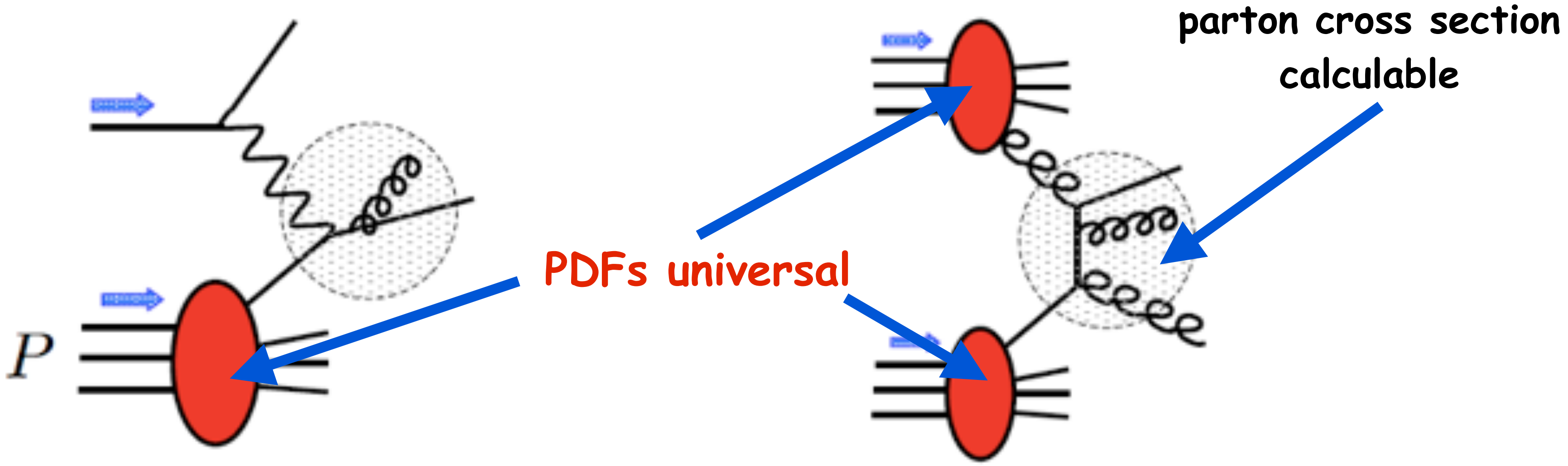

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PDFs universal

DIS

hard scale Q

hadron-hadron

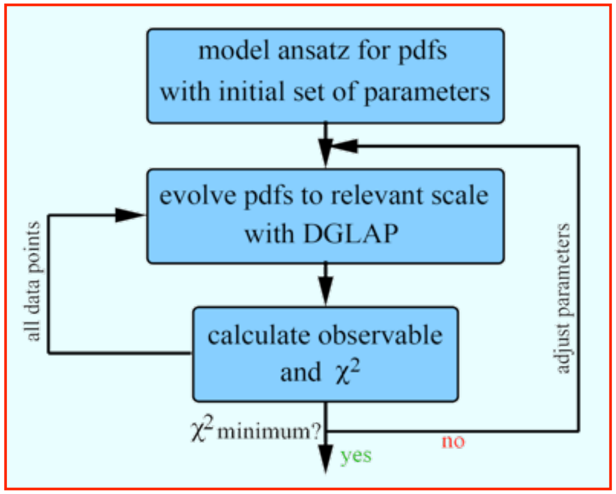
hard scale p_T

task: extract PDFs and their uncertainties (assume factorization)

- all processes tied together: universality of pdfs & Q^2 - evolution
- each reaction provides insights into different aspects and kinematics
- need at least NLO accuracy for quantitative analyses
- information on PDFs "hidden" inside complicated (multi-)convolutions

anatomy of global PDF analyses

obtain PDFs
through global χ^2 optimization

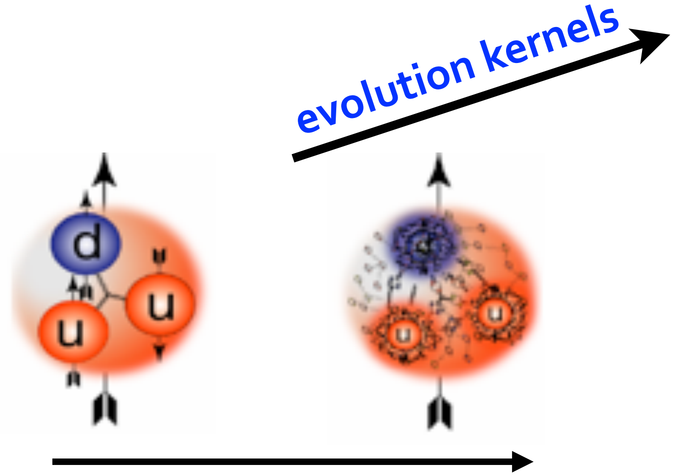


set of **optimum parameters**
for *assumed* functional form

computational challenge:

- up to O(20-30) parameters
- many sources of uncertainties
- very time-consuming NLO expressions

anatomy of global QCD analyses



evolution kernels

"resolution scale" μ

obtain PDFs
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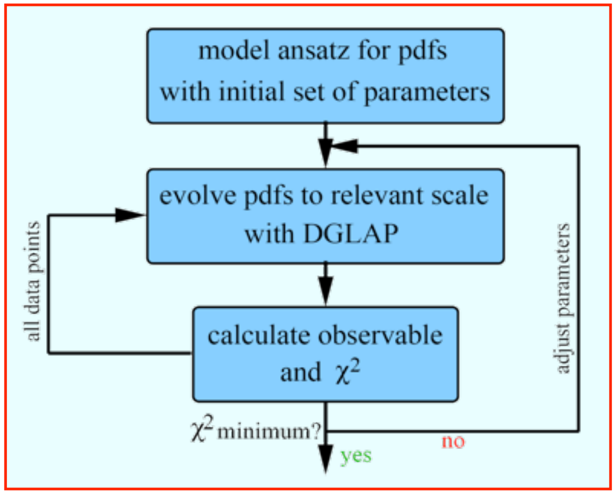

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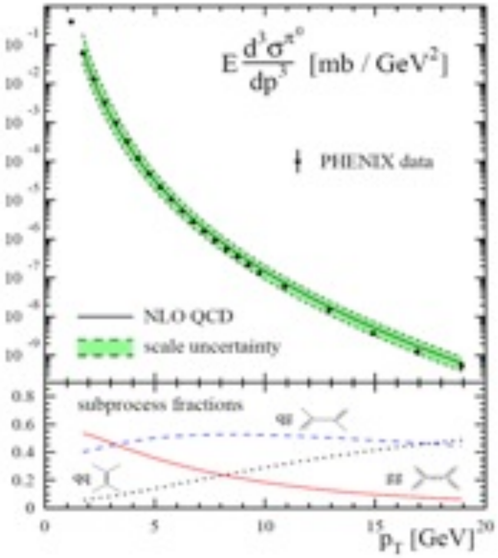

cross sections at NLO

obtain PDFs
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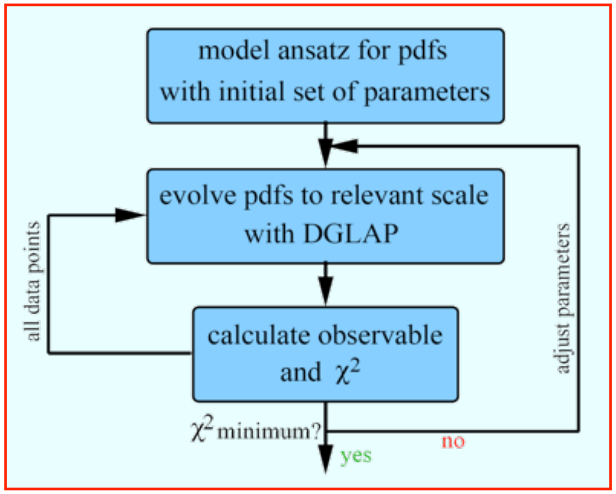

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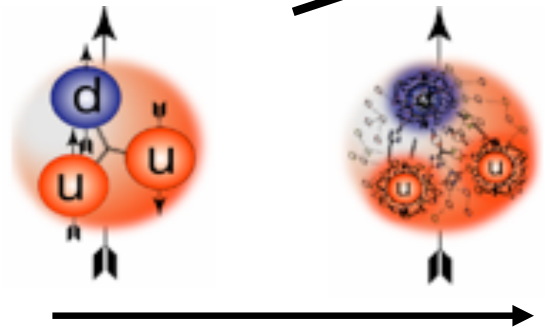

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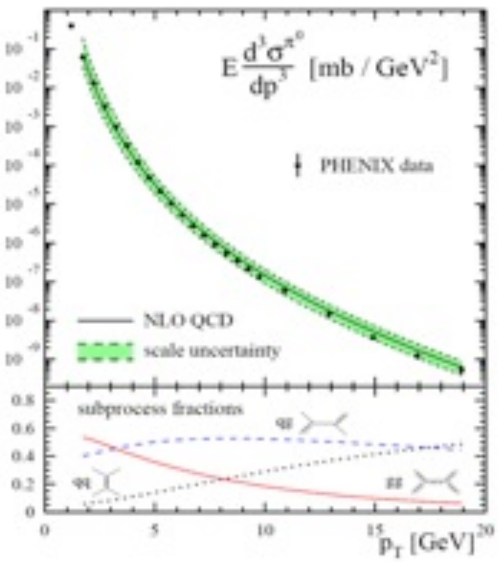
anatomy of global QCD analyses



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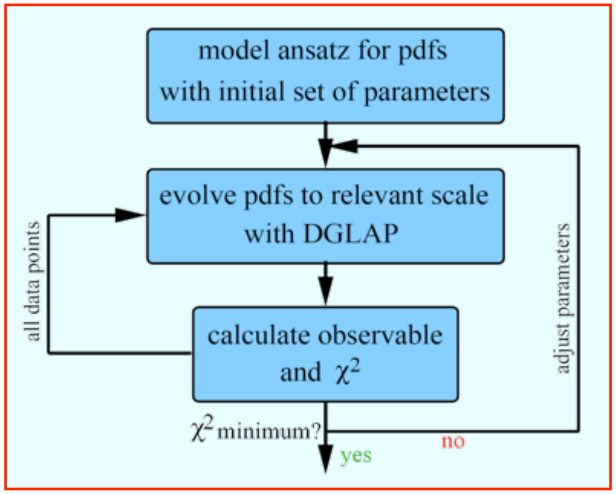
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evolve pdfs to relevant scale with DGLAP

calculate observable and χ^2

χ^2 minimum?

yes

no

all data points

adjust parameters

set of **optimum parameters**
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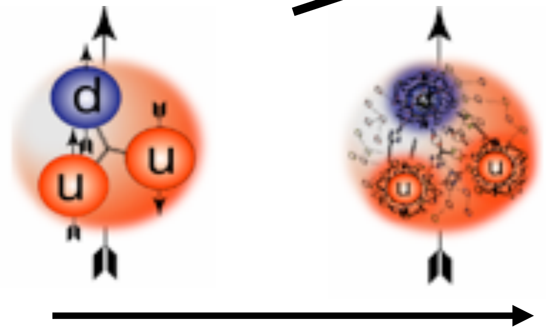
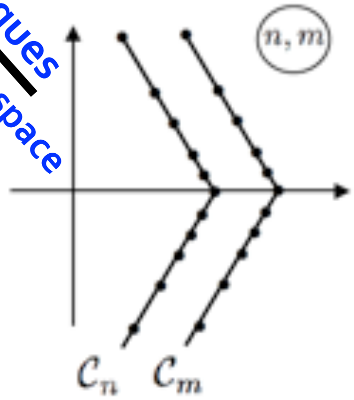
evolution kernels



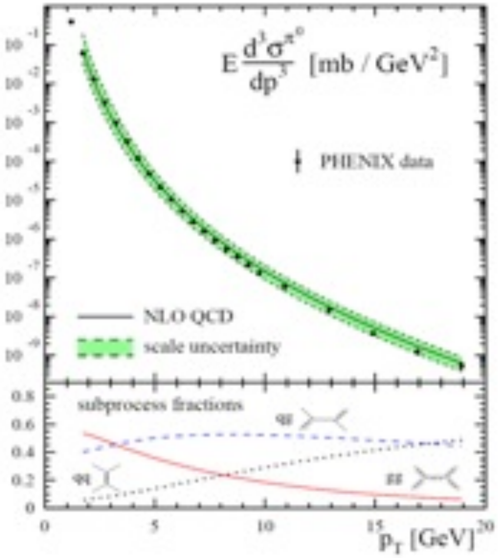
"resolution scale" μ

novel techniques

e.g. in complex Mellin space



anatomy of global QCD analyses



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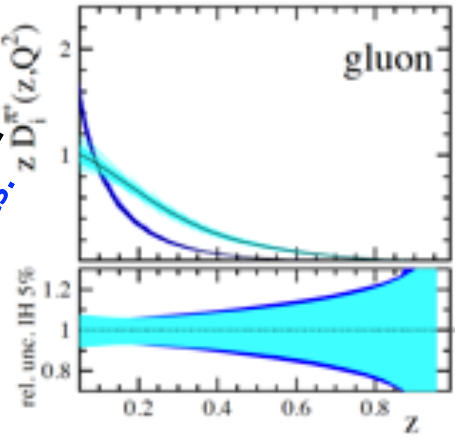
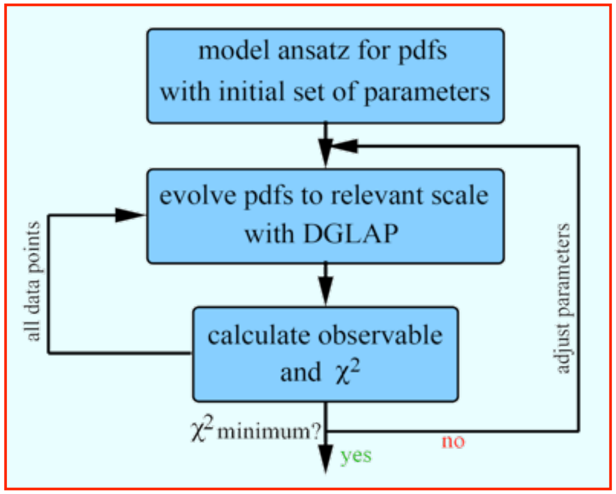


non-pert. inputs
e.g. frag. fcts.

obtain PDFs
through global χ² optimization



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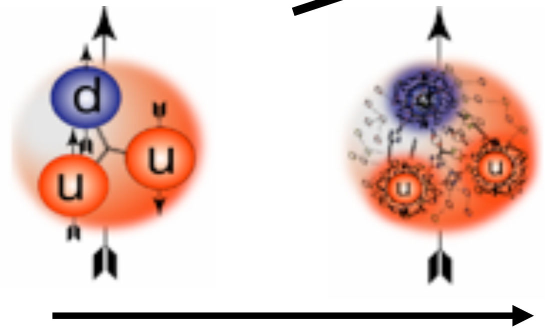

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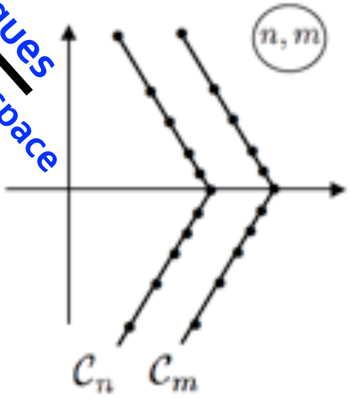
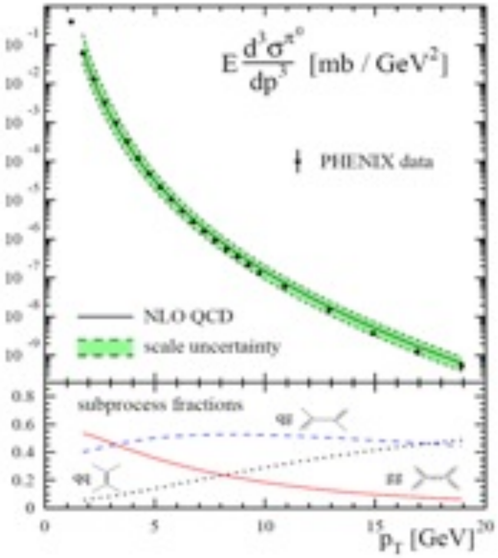


anatomy of global QCD analyses



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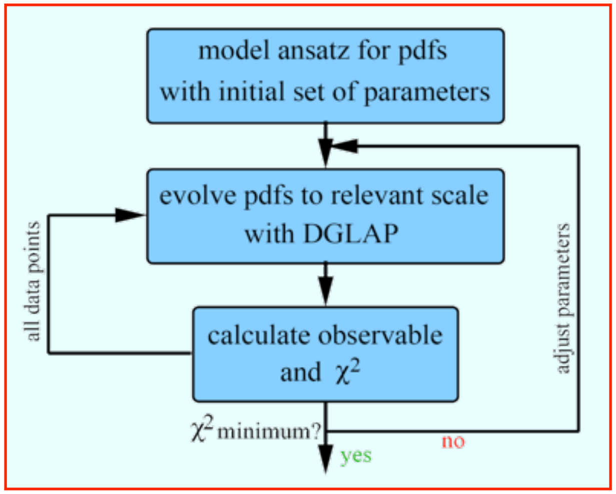
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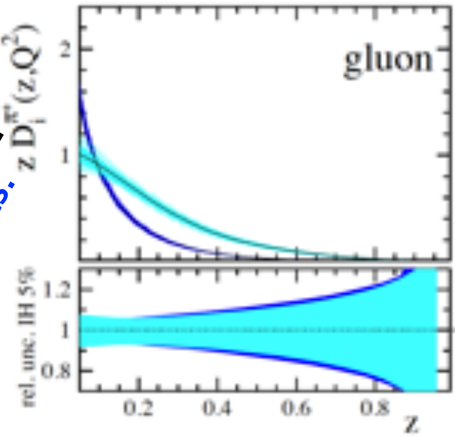
plus a prescription to
estimate & propagate
uncertainties

non-pert. inputs
e.g. frag. fcts.



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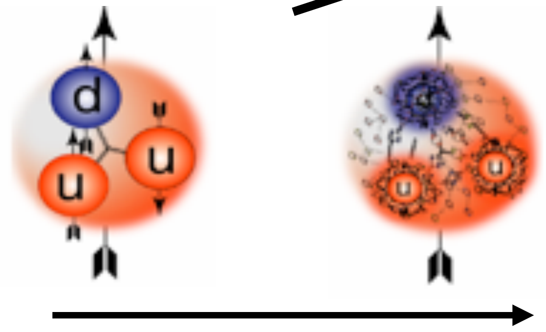

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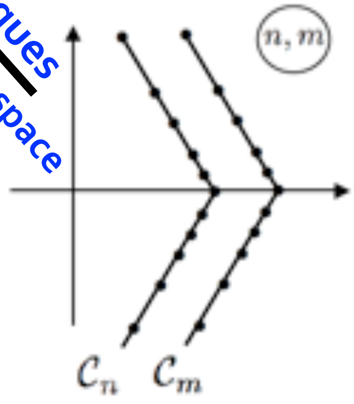

global analysis: computational challenge

- one has to deal with **O(2800) data points** from many processes and experiments
- need to determine **O(20-30) parameters** describing PDFs at μ_0
- NLO expressions often very complicated → computing time becomes excessive
  → develop **sophisticated algorithms & techniques,** e.g., based on Mellin moments

Kosower; Vogt; Vogelsang, MS

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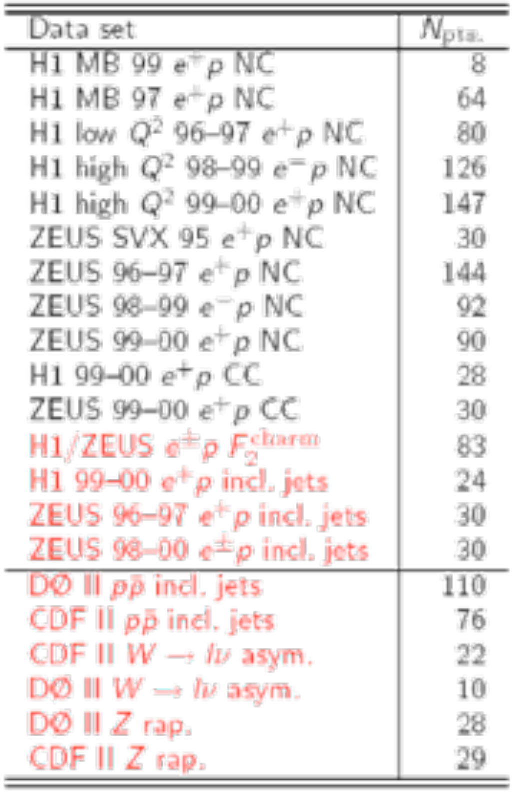
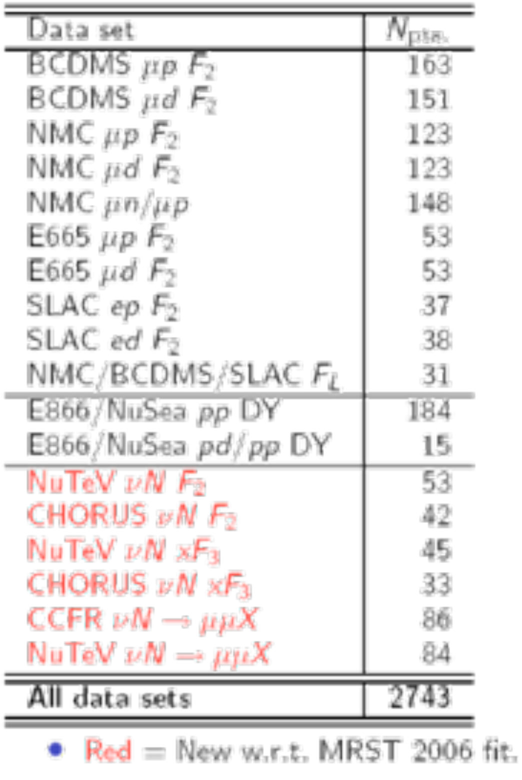
  Kosower; Vogt; Vogelsang, MS

data sets & (x,Q^2) coverage used in MSTW fit
Martin, Stirling, Thorne, Watt, arXiv:0901.0002

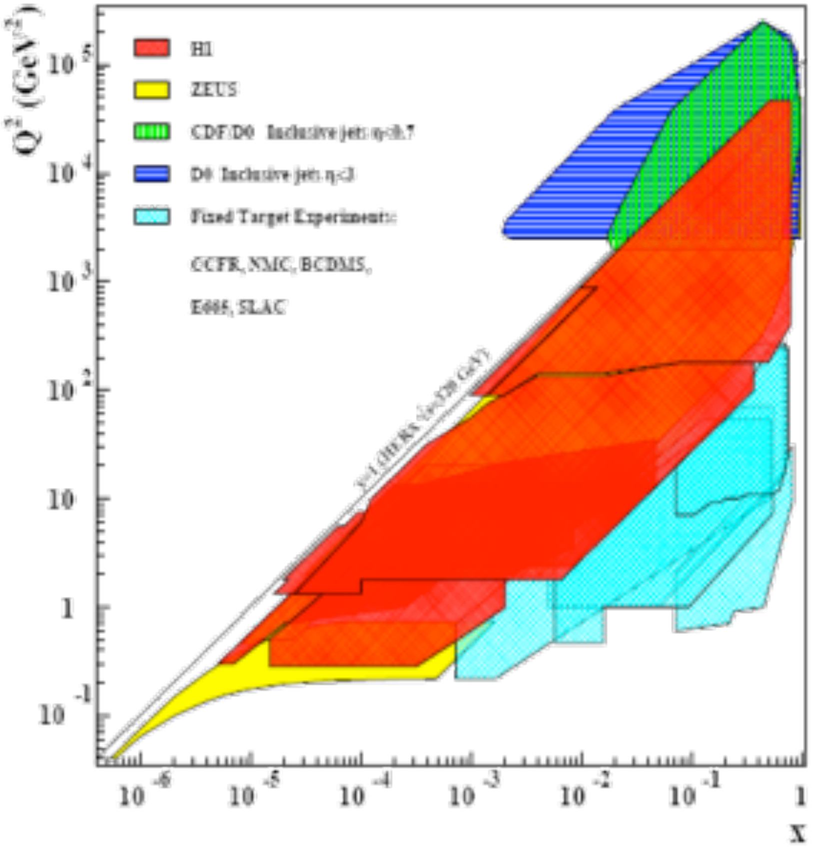


Data set	$N_{\rm pts.}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^\pm p$ $F_2^{\rm charm}$	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^\pm p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29



Data set	$N_{\rm pts.}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	2743

- Red = New w.r.t. MRST 2006 fit.

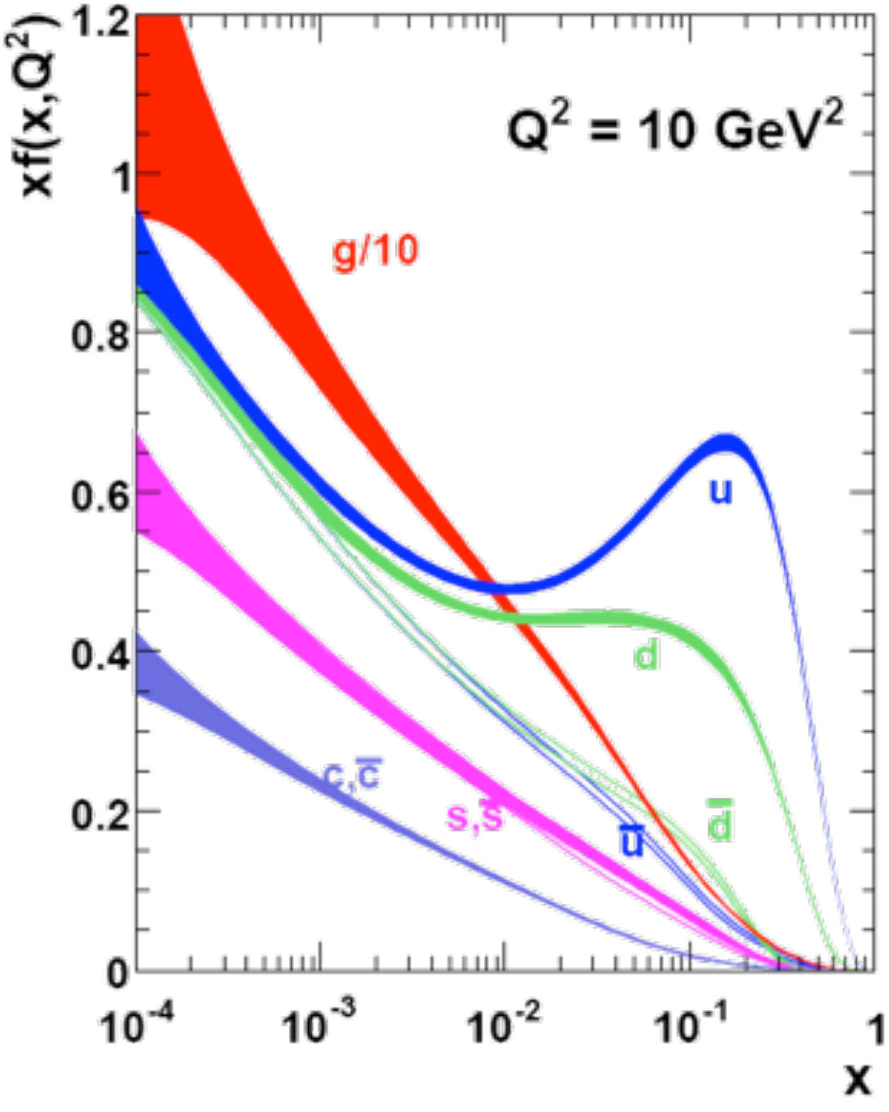


which data sets determine which partons

Process	Subprocess	Partons	x range
$\ell^\pm\{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})N \to \mu^-(\mu^+)X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu)X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

Martin, Stirling, Thorne, Watt, arXiv:0901.0002

NLO fit, 68% C.L.



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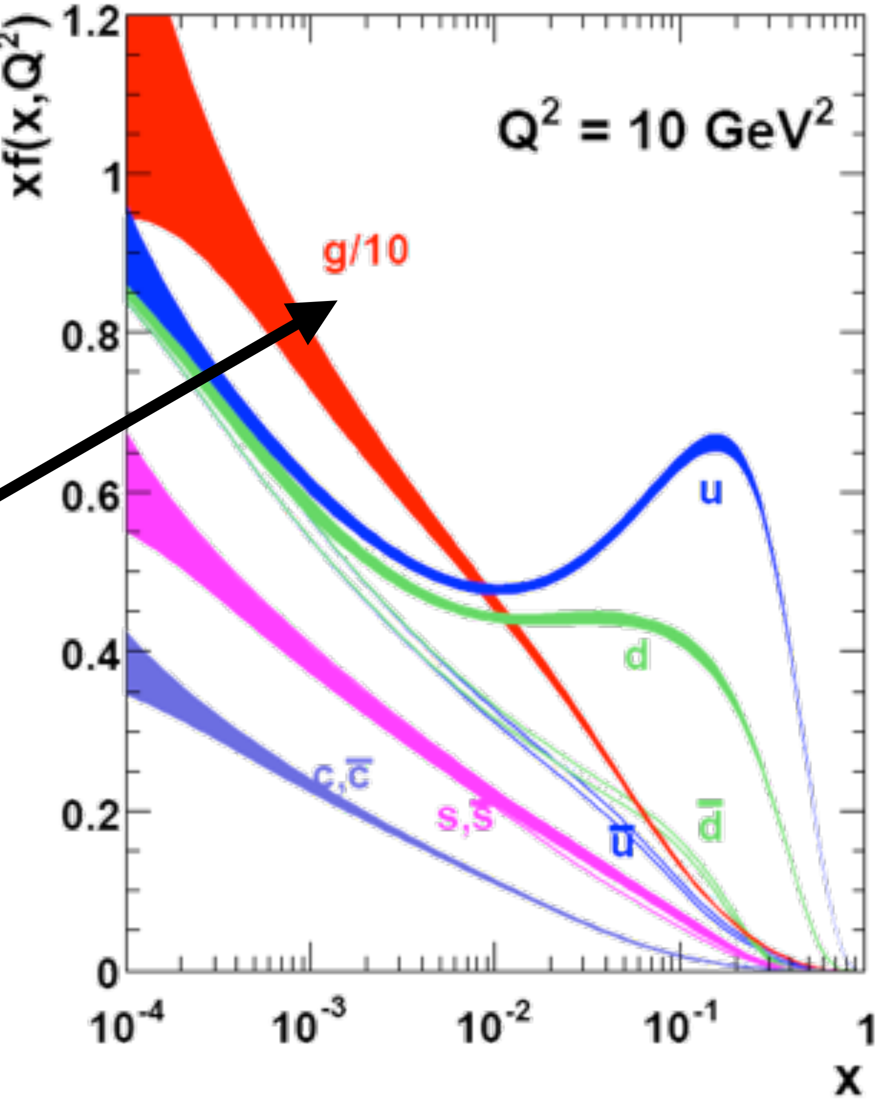


- notice the huge gluon distribution
- quality of the fit:

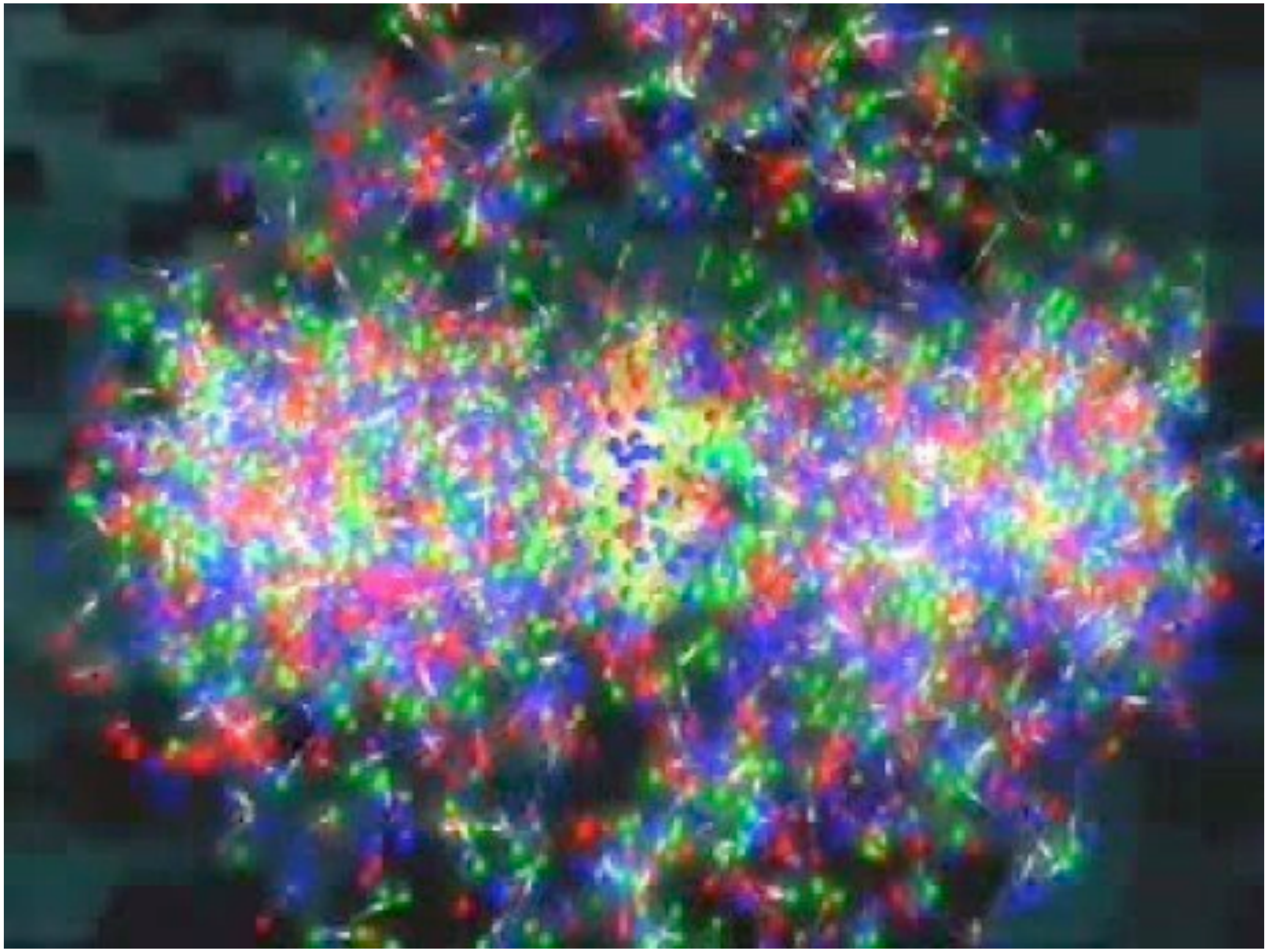
χ^2/ #data pts.
- 2543/2699 NLO
- 3066/2598 LO

interplay of many data sets crucial


PARTON
Drive carefully
Burial place of
James Clerk Maxwell

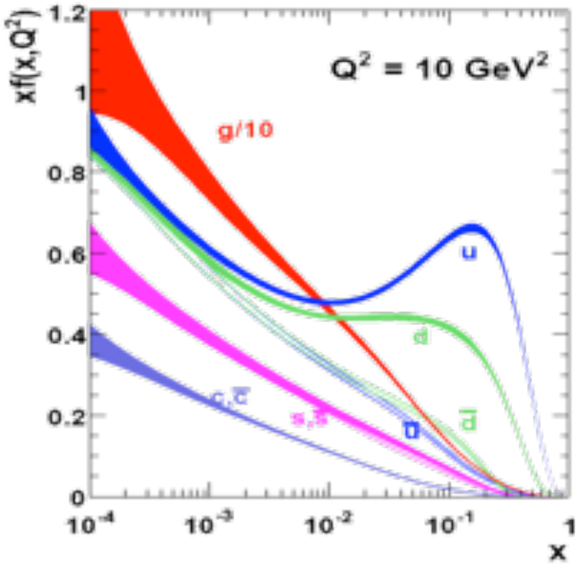
from R.D. Ball

3



when there is not enough room: gluons at small x

what drives the growth of the gluon density



observe that only 2 splitting fcts are singular at small x

$$P_{gq}(x)\big|_{x\to 0} \approx \frac{2C_F}{x} \qquad P_{gg}(x)\big|_{x\to 0} \approx \frac{2C_A}{x}$$

-> small x region dominated by gluons

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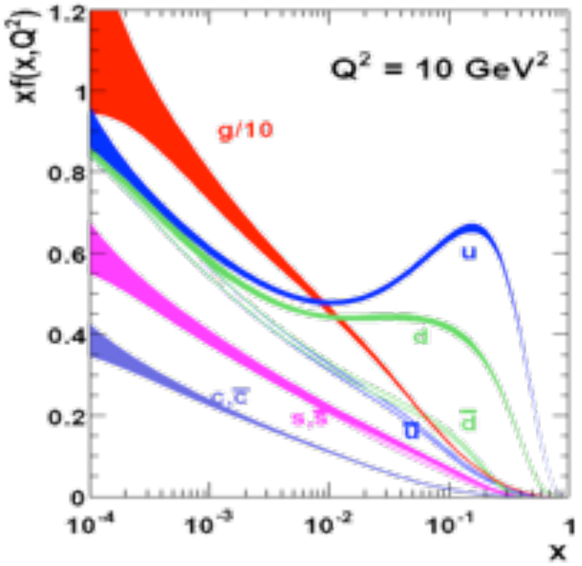

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- write down **"gluon-only" DGLAP equation** **only valid for small x and large Q^2**

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\frac{2C_A}{z} g(x/z,\mu^2)$$

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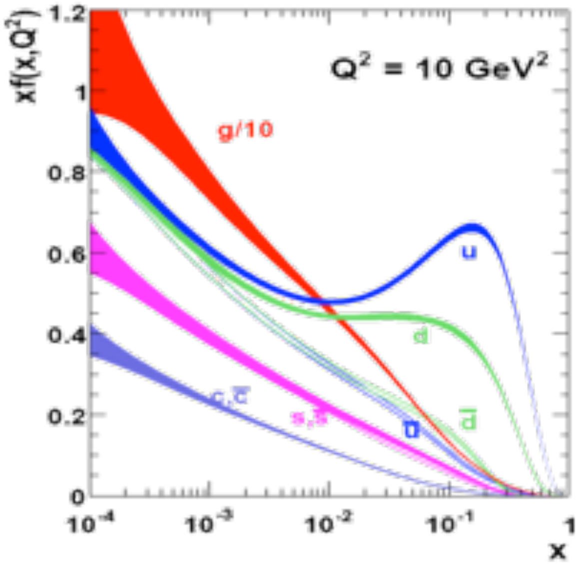

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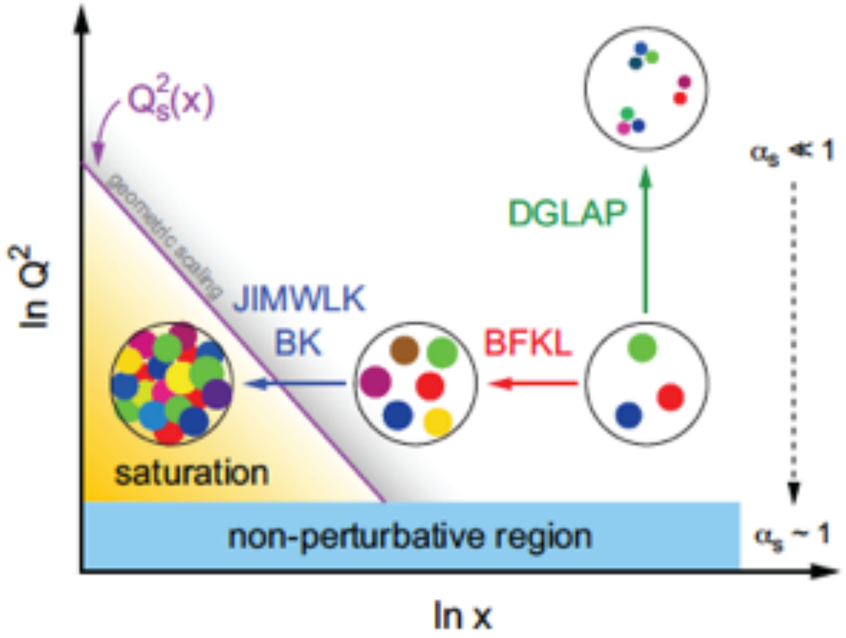
- for fixed coupling this leads to **"double logarithmic approximation"**

$$xg(x,Q^2) \sim \exp\left(2\sqrt{\frac{\alpha_S C_A}{\pi}\log(1/x)\log(Q^2/Q_0^2)}\right)$$

predicts rise that is faster than $\log^a(1/x)$ but slower than $(1/x)^a$

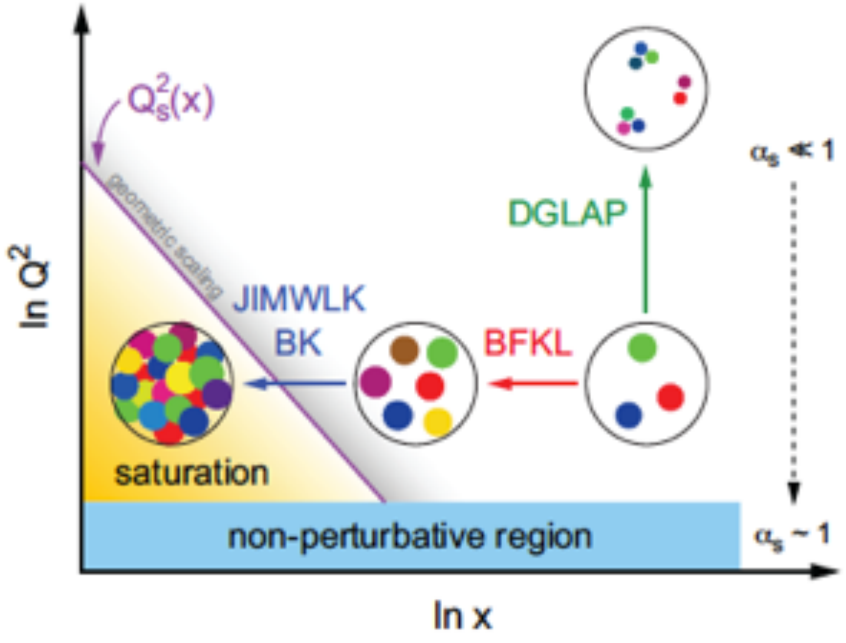
gluon occupancy



- DGLAP predicts an increase of gluons at small x but proton becomes more dilute as Q^2 increases

transverse size of partons ≈ 1/Q

gluon occupancy



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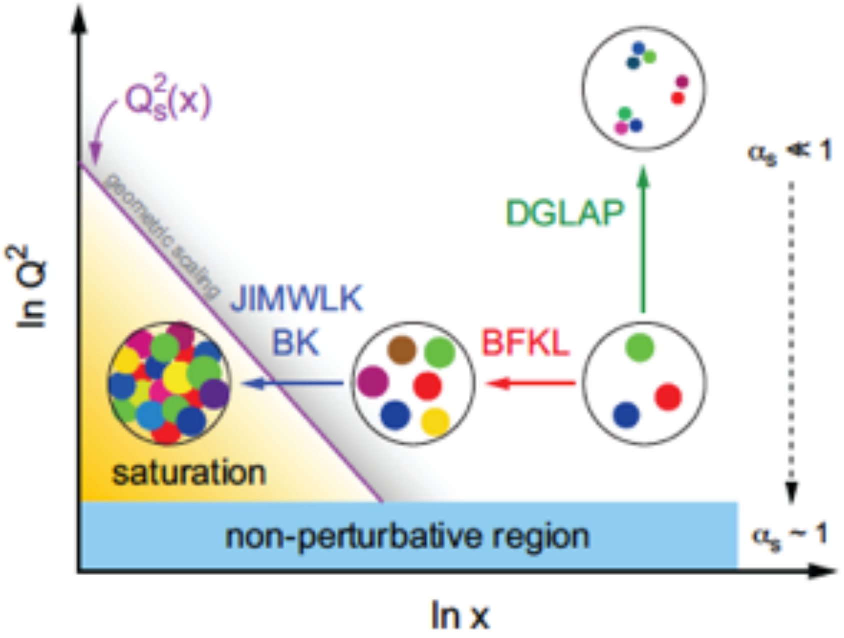

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"high-energy (Regge) limit of QCD"

- aim to resum terms ≈ **α_s log(1/x)**
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  much faster than in DGLAP

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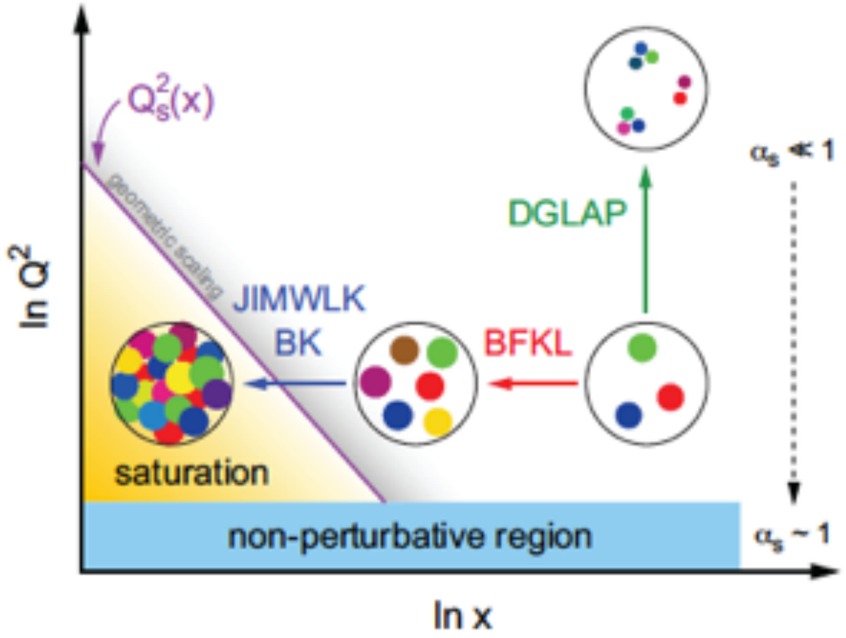

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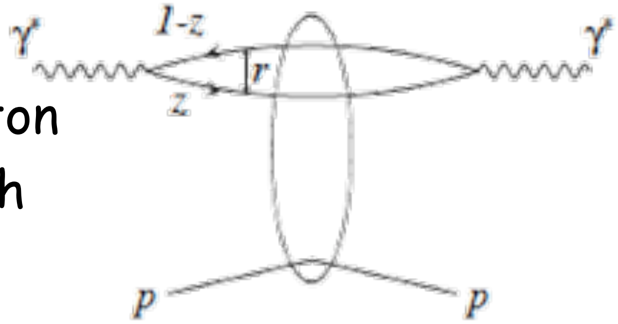
BIG problem

- proton quickly fills up with gluons (transverse size now fixed !)
- hadronic cross sections violate $\ln^2 s$ bound (Froissart-Martin) and grow like a power

color dipole model

make progress by viewing, e.g., DIS from a "different angle"



DIS in the **proton rest frame** can be viewed as the photon splitting into a quark-antiquark pair ("**color dipole**") which scatters off the proton (= "slow" gluon field)

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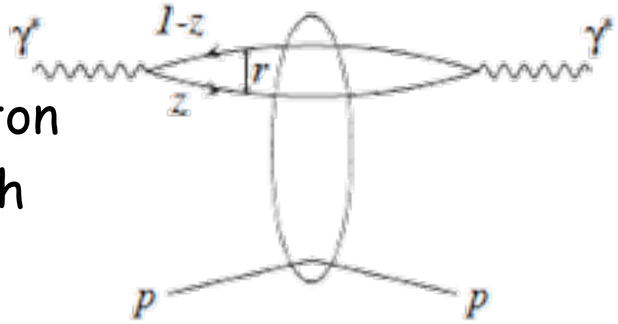

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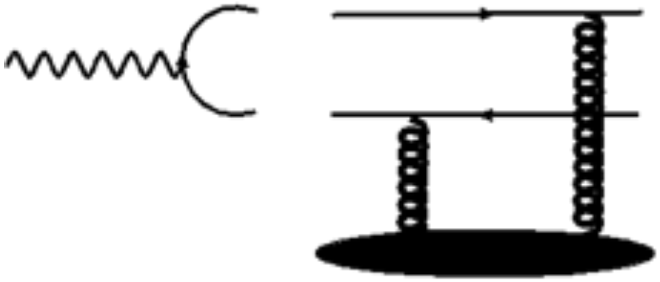

= **probability of photon fluctuating into qq-pair** ⊗ **probability of dipole scattering on the target**

QED **QCD**

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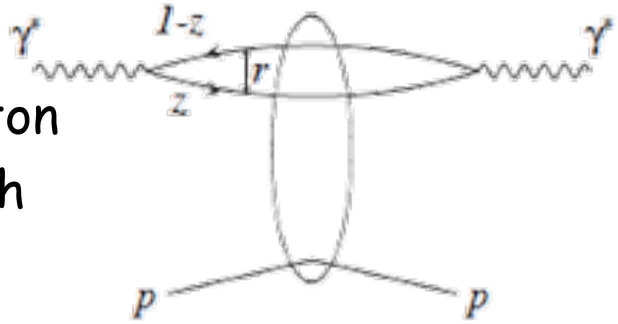

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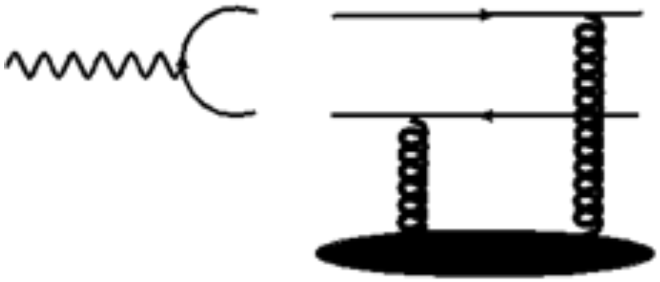

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- energy dependence of N described by Balitsky-Kovchegov equation

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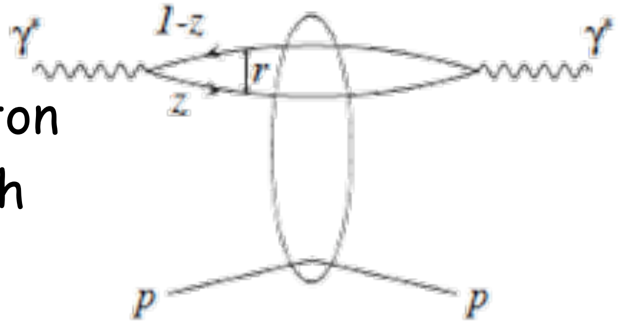

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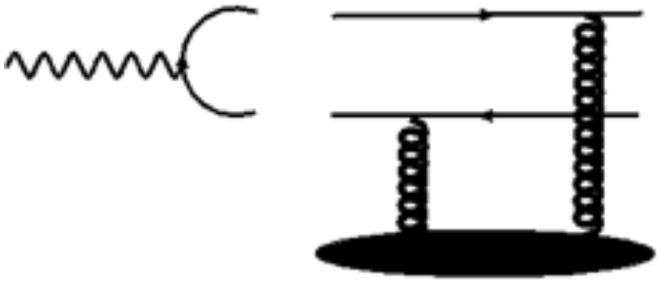

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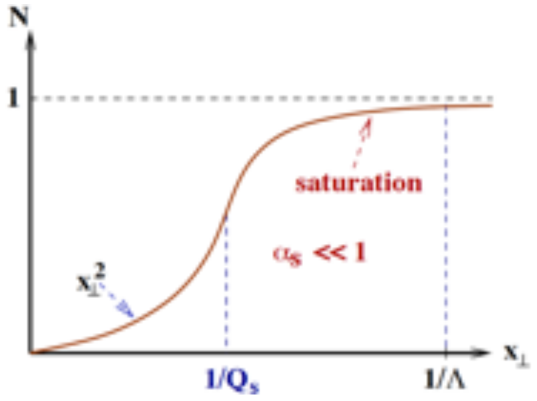
QED **QCD**

- introduces **dipole-nucleon scattering amplitude N** as fund. building block
- energy dependence of N described by Balitsky-Kovchegov equation
  - **non-linear** -> includes multiple scatterings for unitarization
  - generates saturation scale Q_s
  - suited to treat collective phenomena (shadowing, diffration)
  - impact parameter dependence



4

when N^xLO is not enough: all order resummations

when a N^xLO calculation is not good enough

observation: fixed N^xLO order QCD calculations are not necessarily reliable
this often happens at low energy fixed-target experiments
and can be an issue also at colliders, even the LHC

reason: structure of the perturbative series and IR cancellation

at partonic threshold / near exclusive boundary:

- just enough energy to produce, e.g., high-p_T parton
- "inhibited" radiation (general phenomenon for gauge theories)

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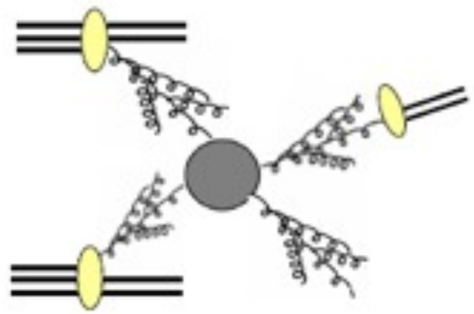
simple example:
Drell-Yan process

$$z \equiv \frac{Q^2}{\hat{s}} = 1 \qquad \propto \alpha_s^k \frac{\ln^{2k-1}(1-z)}{1-z}$$

"imbalance" of real and virtual contributions: IR cancellation leaves large log's

all order structure of partonic cross sections



let's consider pp scattering:

logarithms related to partonic threshold $\hat{x}_T = \frac{2p_T}{\sqrt{\hat{s}}} \rightarrow 1$

general structure of partonic cross sections at the kth order:

$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\text{Born}}}{dp_T} \left[1 + \underbrace{\mathcal{A}_1 \, \alpha_s \ln^2\left(1 - \hat{x}_T^2\right) + \mathcal{B}_1 \, \alpha_s \ln\left(1 - \hat{x}_T^2\right)}_{\text{NLO}} \right.$$

$$\left. + \ldots + \mathcal{A}_k \, \alpha_s^k \ln^{2k}\left(1 - \hat{x}_T^2\right) + \ldots \right] + \ldots$$

"threshold logarithms"

all order structure of partonic cross sections

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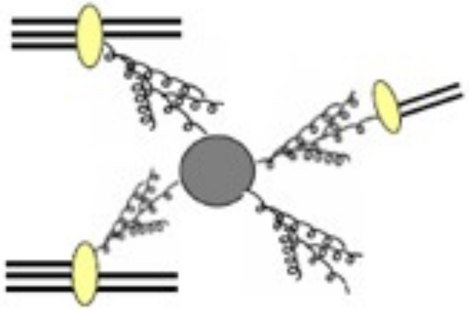

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where relevant? … convolution with steeply falling parton luminosity L_{ab}:

$$d\sigma \propto \sum_{a,b} \int_\tau^1 \frac{dz}{z} \underset{\uparrow}{\mathcal{L}_{ab}}\left(\frac{\tau}{z}\right) d\hat{\sigma}_{ab}(z)$$

large at small τ/z

z = 1 emphasized, in particular as τ → 1

→ important for fixed target phenomenology: threshold region more relevant (large τ)

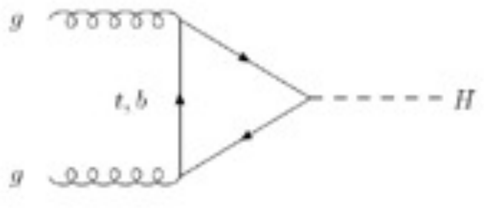
resummations – how are they done

$\alpha_s^k \ln^{2k}(1-\hat{x}_T^2)$ **may spoil perturbative series - unless taken into account to all orders**

resummation of such terms has reached a high level of sophistication

Sterman; Catani, Trentadue; Laenen, Oderda, Sterman; Catani et al.; Sterman, Vogelsang; Kidonakis, Owens; ...

- worked out for most processes of interest at least to NLL
- well defined class of higher-order corrections
- often of much phenomenological relevance even for high mass particle production at the LHC



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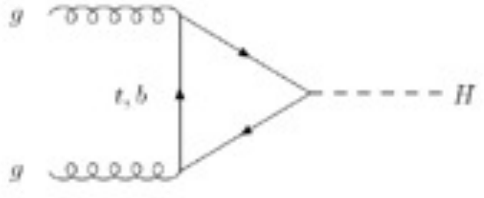

resummation (= exponentiation) occurs when "right" moments are taken:

Mellin moments for threshold logs $\alpha_s^k \ln^{2k}(1-\hat{x}_T^2) \rightarrow \alpha_s^k \ln^{2k}(N)$

- fixed order calculations needed to determine "coefficients"
- the more orders are known, the more subleading logs can be resummed

resummations – terminology

resummations – terminology

Fixed order calculation

resummations – terminology

Fixed order calculation

LO

resummations – terminology

Fixed order calculation

LO				
NLO	$\alpha_s \mathbf{L}^2$	$\alpha_s \mathbf{L}$	α_s	$+ \dots$

resummations – terminology

Fixed order calculation

LO					
NLO	$\alpha_s \mathbf{L}^2$	$\alpha_s \mathbf{L}$	α_s		$+ \dots$
NNLO	$\alpha_s^2 \mathbf{L}^4$	$\alpha_s^2 \mathbf{L}^3$	$\alpha_s^2 \mathbf{L}^2$	$\alpha_s^2 \mathbf{L}$	$+ \dots$

resummations – terminology

Fixed order calculation

LO					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		$+ \dots$
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$+ \dots$
	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$+ \dots$
	$\alpha_s^4 L^8$	$\alpha_s^4 L^7$	$\alpha_s^4 L^6$	$\alpha_s^4 L^5$	$+ \dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

resummations – terminology

Fixed order calculation

LO					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		$+ \dots$
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$+ \dots$
	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$+ \dots$
	$\alpha_s^4 L^8$	$\alpha_s^4 L^7$	$\alpha_s^4 L^6$	$\alpha_s^4 L^5$	$+ \dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$+ \dots$

resummations – terminology

Fixed order calculation

Resummation

LO					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		$+ \dots$
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$+ \dots$
	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$+ \dots$
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	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
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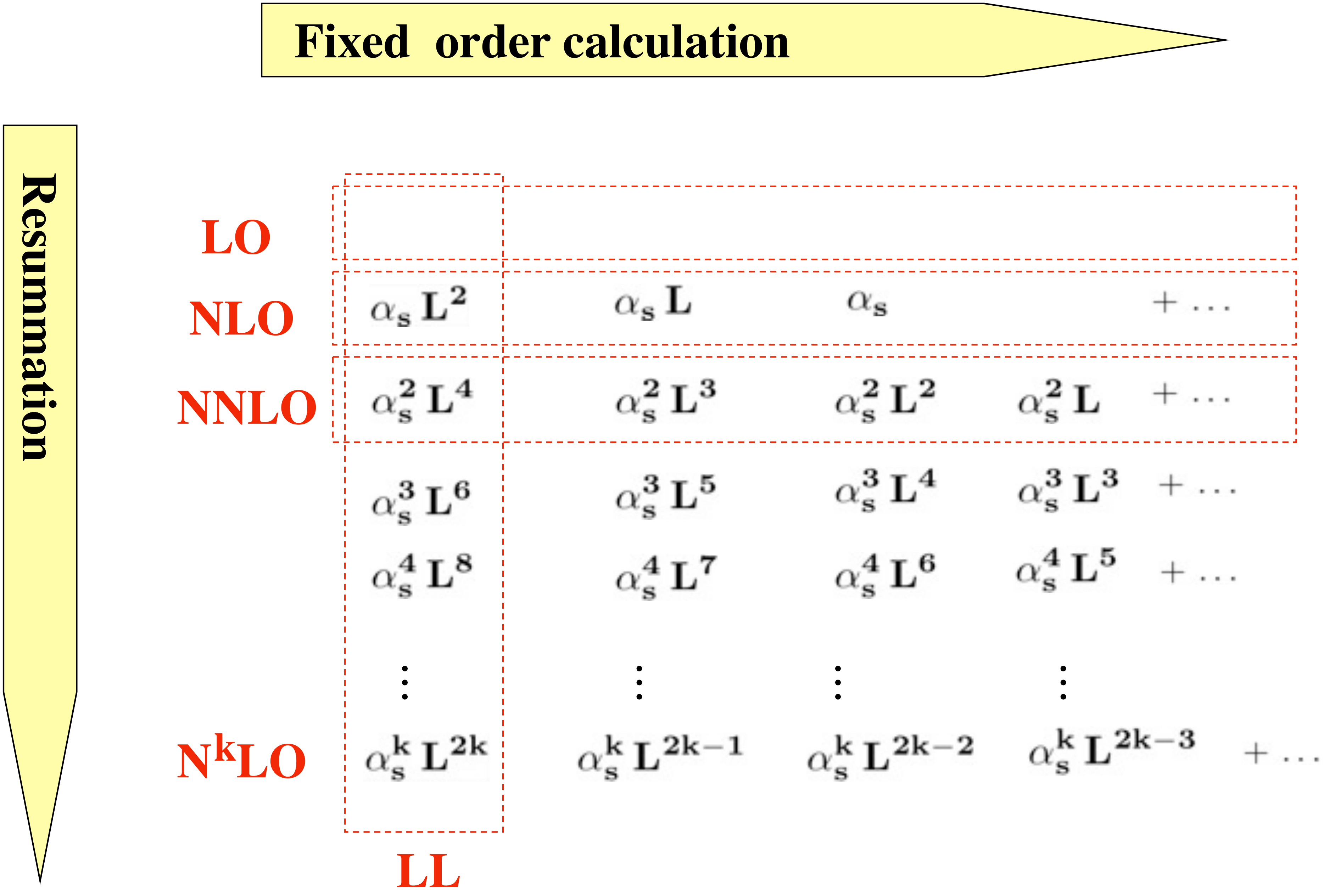
resummations – terminology



Fixed order calculation

Resummation

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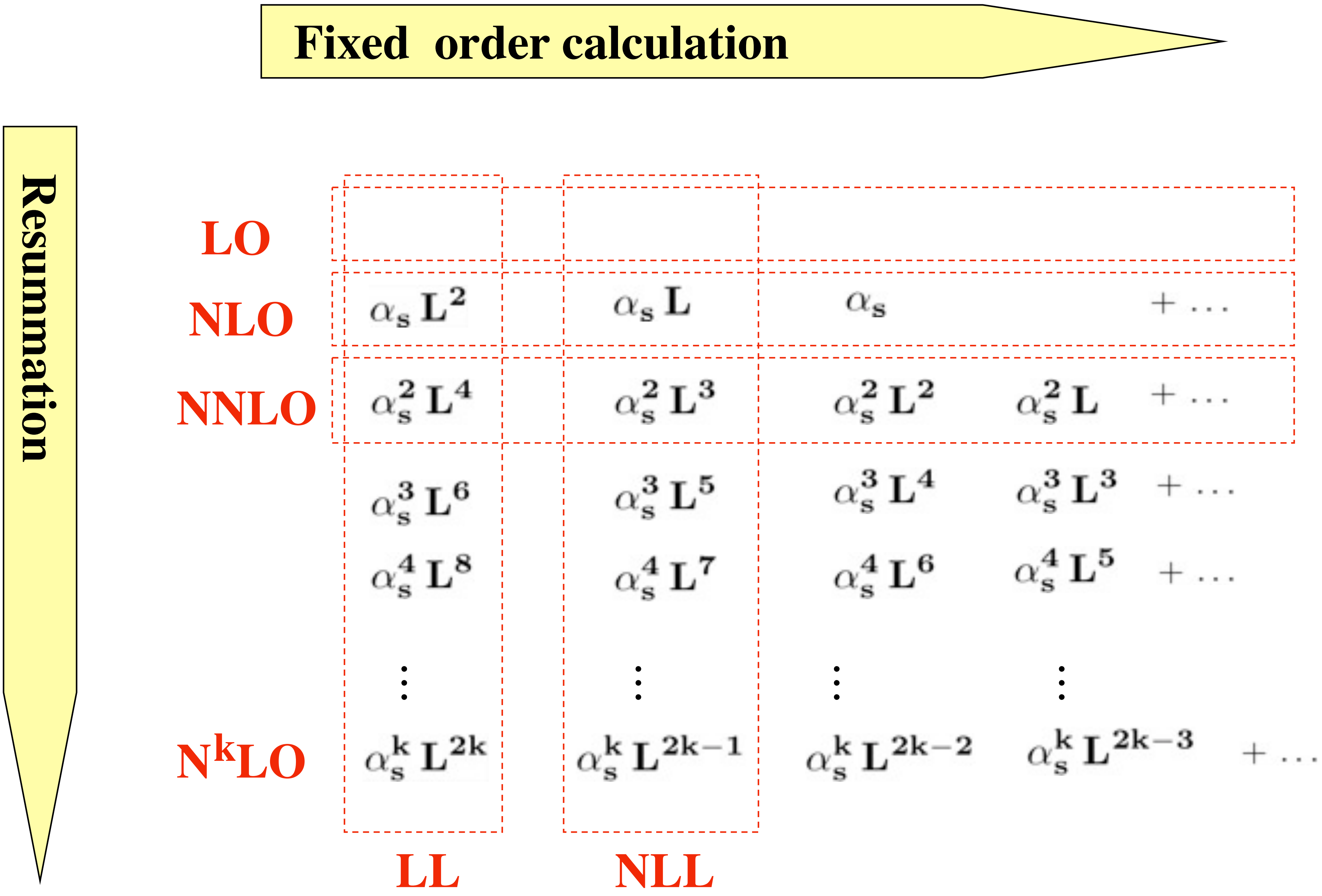
	LL				

resummations – terminology



Fixed order calculation

Resummation

LO					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		$+ \dots$
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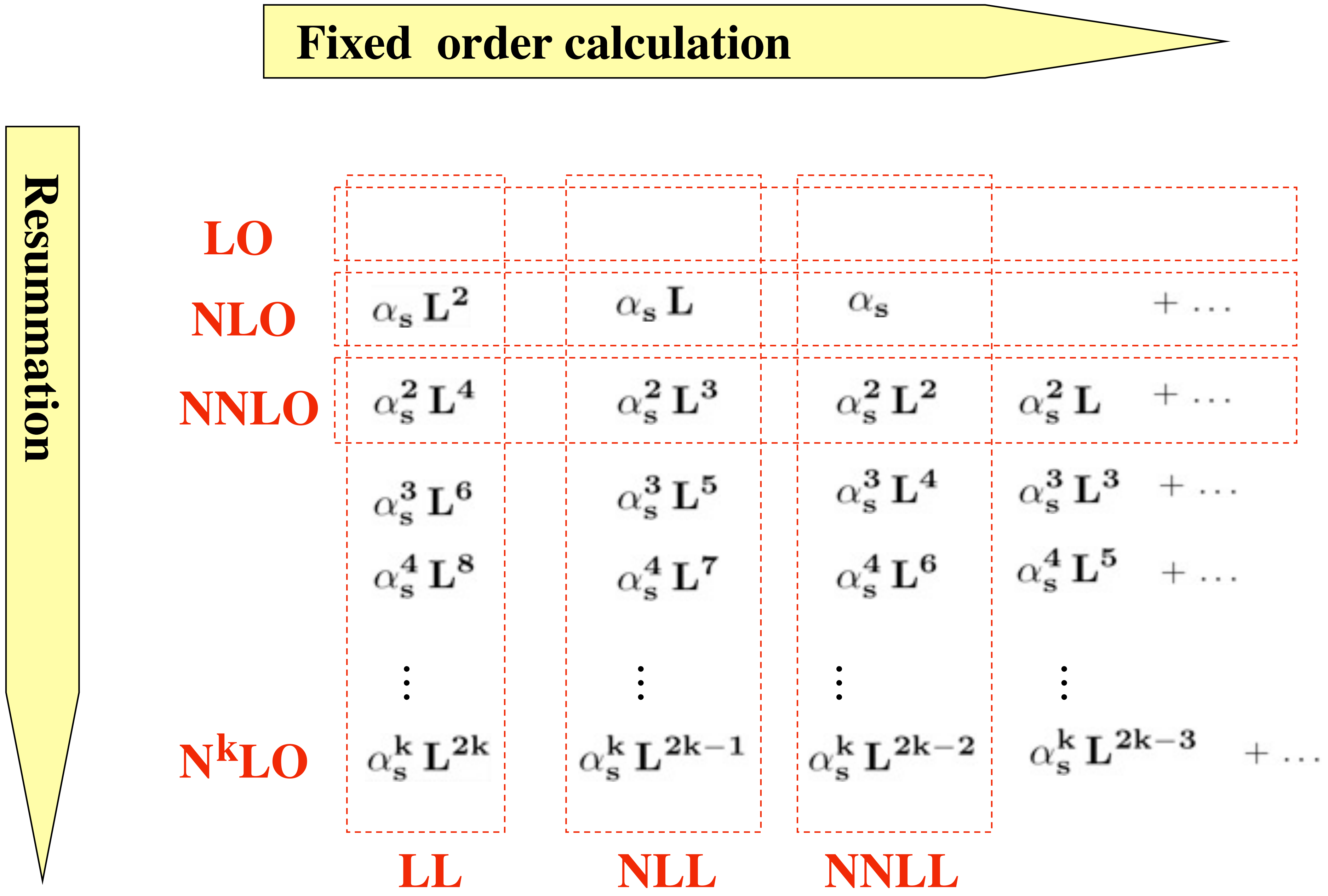
resummations – terminology



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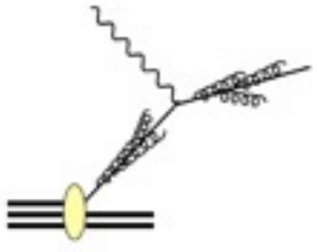
	LL	NLL	NNLL		

some leading log exponents

(assuming fixed α_s for simplicity)

color factors for soft gluon radiation matter:

DIS



unobserved parton
Sudakov "suppression"

$$\exp\left[\frac{C_F\,\alpha_s}{\pi}\ln^2(N) - \frac{C_F\,\alpha_s}{\pi}\frac{1}{2}\ln^2(N)\right]$$

moderate enhancement, unless x_{Bj} large

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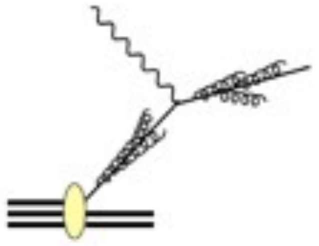

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prompt
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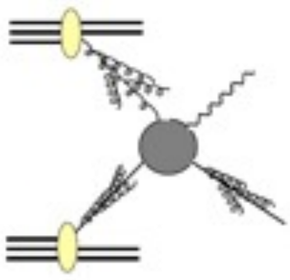

$q\bar{q} \to \gamma g$ $$\exp\left[\left(C_F + C_F - \frac{1}{2}C_A\right)\frac{\alpha_s}{\pi}\ln^2(N)\right]$$

$qg \to \gamma q$ $$\exp\left[\left(C_F + C_A - \frac{1}{2}C_F\right)\frac{\alpha_s}{\pi}\ln^2(N)\right]$$

exponents positive → enhancement

some leading log exponents

(assuming fixed α_s for simplicity)

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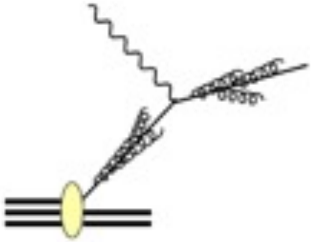

unobserved parton Sudakov "suppression"

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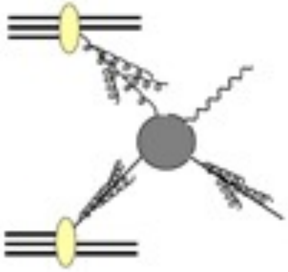
moderate enhancement, unless x_{Bj} large

prompt photons



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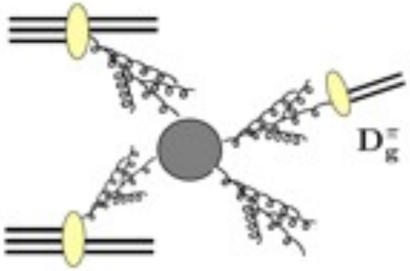
exponents positive ⟶ enhancement

inclusive hadrons



e.g.

observed partons

unobserved

$gg \to gg$ $$\exp\left[\left(C_A + C_A + C_A - \frac{1}{2}C_A\right)\frac{\alpha_s}{\pi}\ln^2(N)\right]$$

expect much larger enhancement

resummations: window to non-perturbative regime

important technical issue:

resummations are sensitive to strong coupling regime

→ need some "minimal prescription" to avoid Landau pole (where $\alpha_s \to \infty$)

Catani, Mangano, Nason, Trentadue:

define resummed result such that series is asymptotic
w/o factorial growth associated with power corrections
[achieved by particular choice of Mellin contour]

→ power corrections may be added afterwards if pheno. needed
studying power corrections prior to resummations makes no sense

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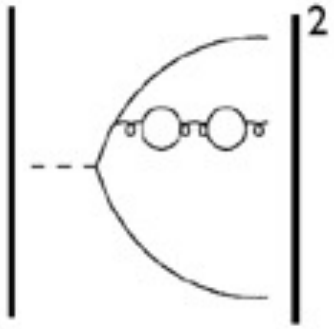
window to the non-perturbative regime so far little explored

“convergence” of an asymptotic series

see, “Renormalons” review by M. Beneke, hep-ph/9807443

suppose we keep calculating
higher and higher orders



$\rightarrow \alpha_s^{n+1} \beta_0^n n!$

factorial growth

→ **big trouble**: the perturbative series is not convergent but only asymptotic

“convergence” of an asymptotic series

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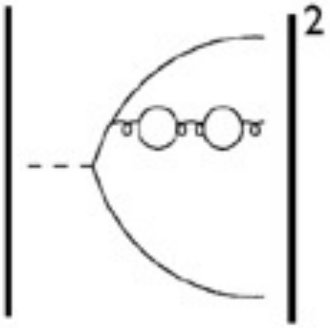


$\rightarrow \alpha_s^{n+1}\,\beta_0^n\,n!$

factorial growth

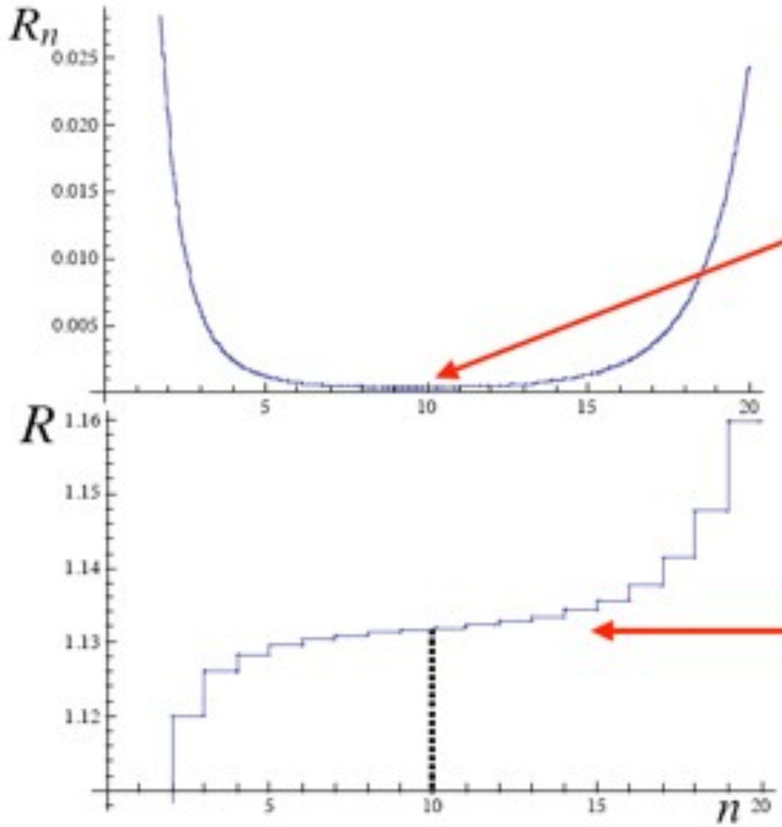
→ **big trouble**: the perturbative series is not convergent but only asymptotic

illustration:

try resumming

$$R = \sum_{n=0}^{\infty} \alpha_s^n n!$$

[with $\alpha_s = 0.1$]



minimal term

$R_{min} = 1/\alpha_s$

asymptotic value of the sum:

$$R_{asymp} = \sum_{n=0}^{n_{min}} \alpha_s^n n!$$

taken from M. Cacciari

pQCD – non-perturbative bridge

- "renormalon ambiguity" ↔ incompleteness of pQCD series

  → we can only define what the sum of the perturbative series is like truncating it at the minimal term

pQCD – non-perturbative bridge

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- what is missing is a genuine ambiguity
  - → eventually lifted by non-perturbative (NP) corrections:

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- QCD: NP corrections are power suppressed:

$$\boxed{R^{NP} = \exp\left(- p \ln \frac{Q^2}{\Lambda^2}\right) = \left(\frac{\Lambda^2}{Q^2}\right)^p}$$

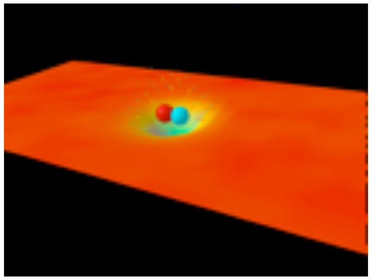
the value of p depends on the process and can sometimes be predicted

SUMMARY & OUTLOOK

QCD: the most perfect gauge theory (so far)

- simple $\mathcal{L}$ but rich & complex phenomenology; few parameters
- in principle complete up to the Planck scale
  (issue: CP, axions?)
- highly non-trivial ground state responsible for all the structure in the visible universe
- **emergent phenomena**: confinement, chiral symmetry breaking, hadrons

- **confinement**


D. Leinweber

non-perturbative
structure of hadrons

e.g. through lattice QCD

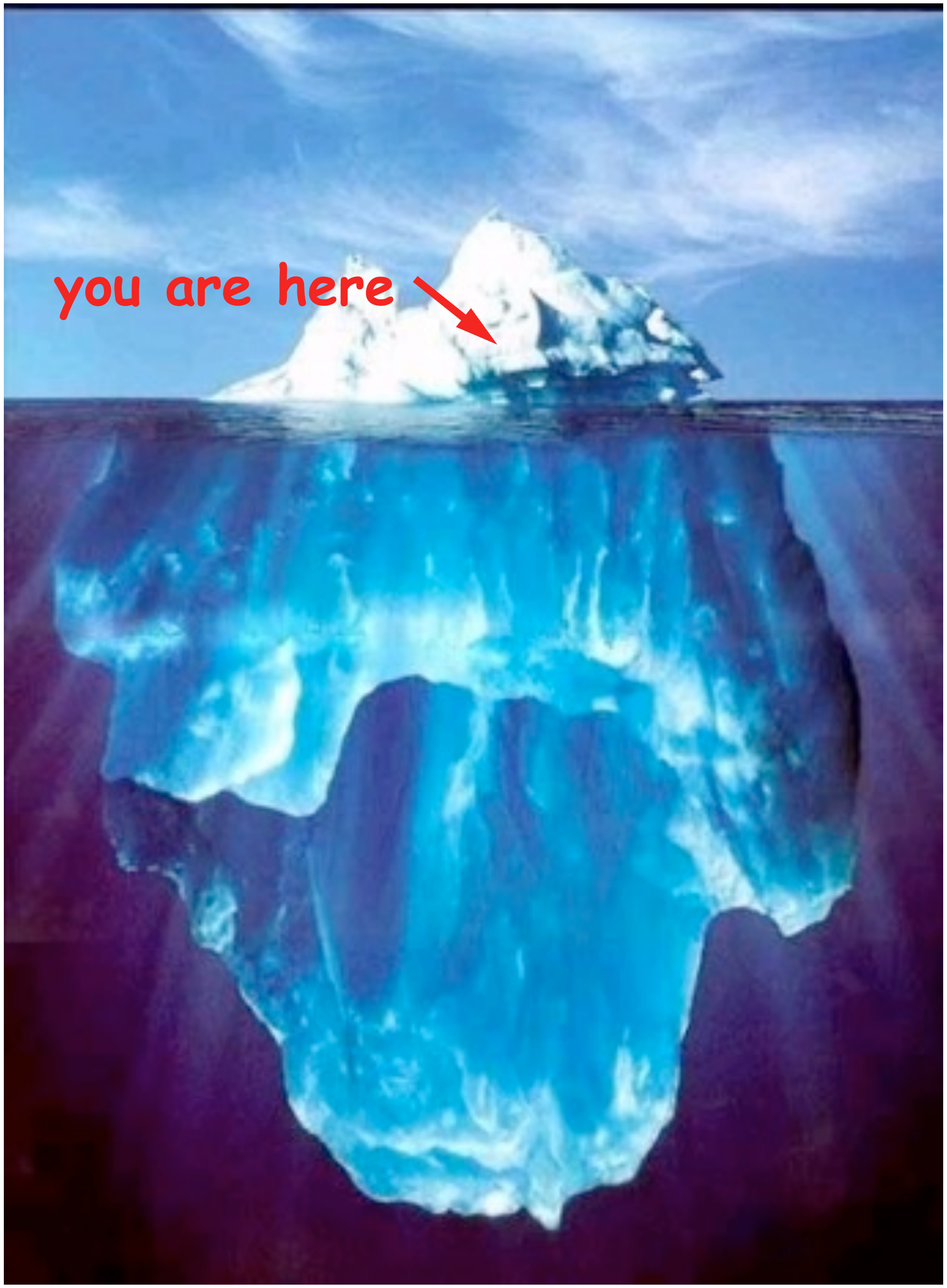
interplay between
High Energy and
Hadron Physics

asymptotic freedom

hard scattering
cross sections
and
renormalization group

perturbative methods

we have just explored the
tip of the iceberg



you are here

we have just explored the
tip of the iceberg

enjoy the other lectures !