Perturbative QCD

from basic principles to current applications

Marco Stratmann

BROOKHAVEN NATIONAL LABORATORY

marco@bnl.gov

Office of Science
U.S. Department of Energy

BROOKHAVEN NATIONAL LABORATORY

July 15/16th, 2013

disclaimer:

pQCD is about 40 years old – impossible to review in 3 hrs

topics & questions to be addressed

we will mainly concentrate on a few basics and their consequences for phenomenology

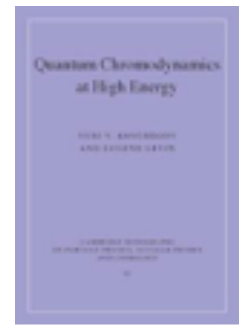
- What are the foundations of QCD?
  keywords: color; SU(3) gauge group; local gauge invariance; Feynman rules
- What are the general features of QCD?
  keywords: asymptotic freedom; infrared safety; origin of "singularities"
- How to relate QCD to experiment?
  keywords: partons; factorization; renormalization group eqs. / evolution
- How reliable is a theoretical QCD calculation?
  keywords: scale dependence; NLO; small-x; all-order resummations
- What is the status of some non-perturbative inputs
  keywords: global QCD analysis

bibliography – a personal selection

Photo by Matt Heyssler

textbooks:

- the "pink book" on QCD and Collider Physics by R.K. Ellis, W.J. Stirling, and B.R. Webber — always a good reference
- R.D. Field, Applications of pQCD — detailed examples
- Y.V. Kovchegov, E. Levin, QCD at High Energy — focus on small x physics
- J. Collins, Foundations of pQCD — focus on formal aspects of evolution



lecture notes & write-ups:

- D. Soper, Basics of QCD Perturbation Theory, hep-ph/9702203
- Collins, Soper, Sterman, Factorization of Hard Processes in QCD, hep-ph/0409313
- G. Salam, Elements of QCD for Hadron Colliders, arXiv:1011.5131
- Particle Data Group, Review of Particle Physics, pdg.lbl.gov

talks & lectures on the web:

- annual CTEQ summer school, tons of material on www.cteq.org
- annual CERN/FNAL Hadron Collider Physics School hcpss.web.cern.ch/hcpss

tentative outline of the lectures

Part 1: **the foundations**

SU(3); color algebra; gauge invariance;
QCD Lagrangian; Feynman rules

Part 2: **the QCD toolbox**

asymptotic freedom; infrared safety;
the QCD final-state; jets; factorization

Part 3: **inward bound: "femto spectroscopy"**

QCD initial-state; DIS process; partons;
factorization; renormalization group; scales;
hadron-hadron collisions

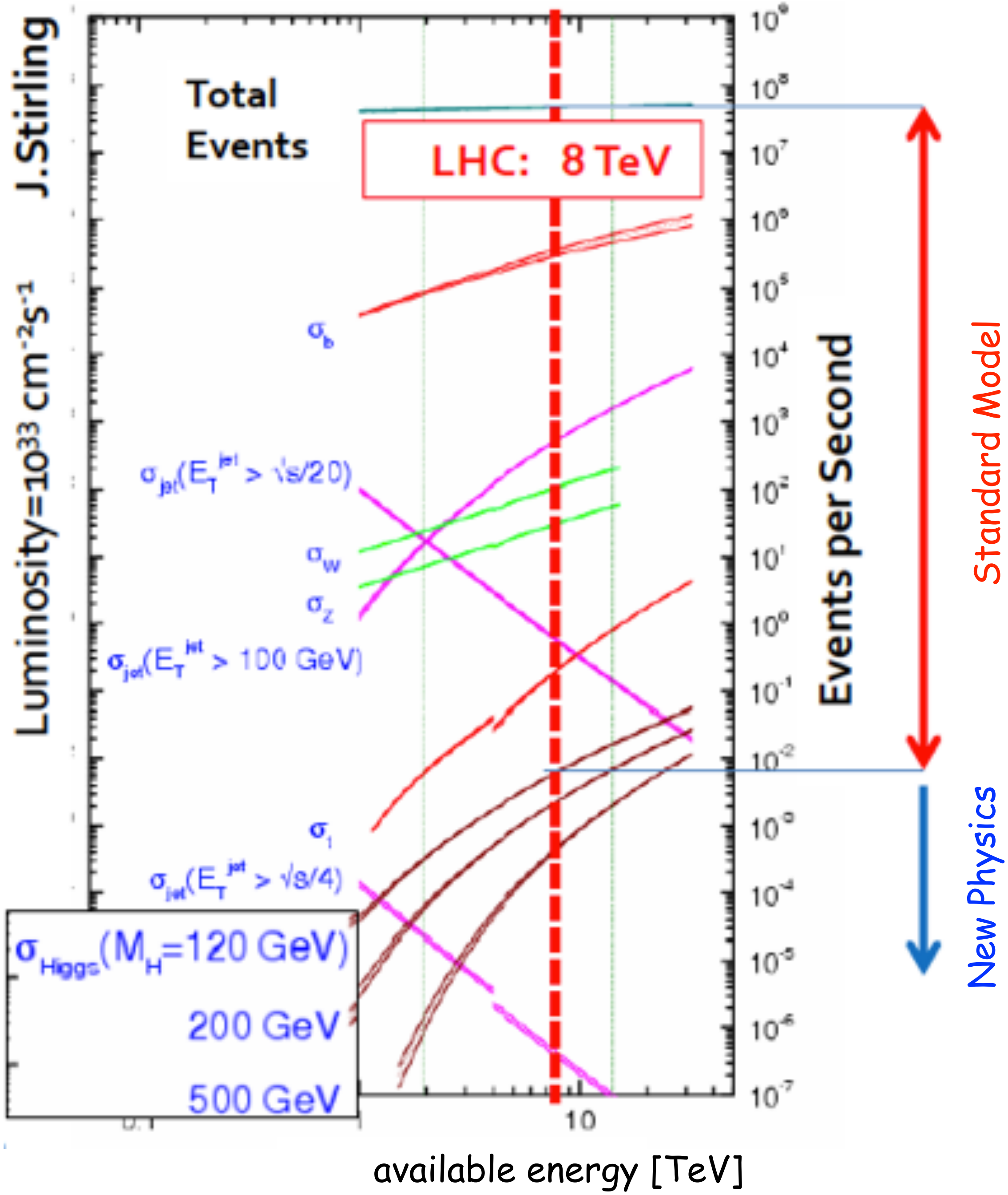
Part I

the QCD fundamentals

all about color

the concept of gauge invariance

QCD – why do we still care (or perhaps more than ever)

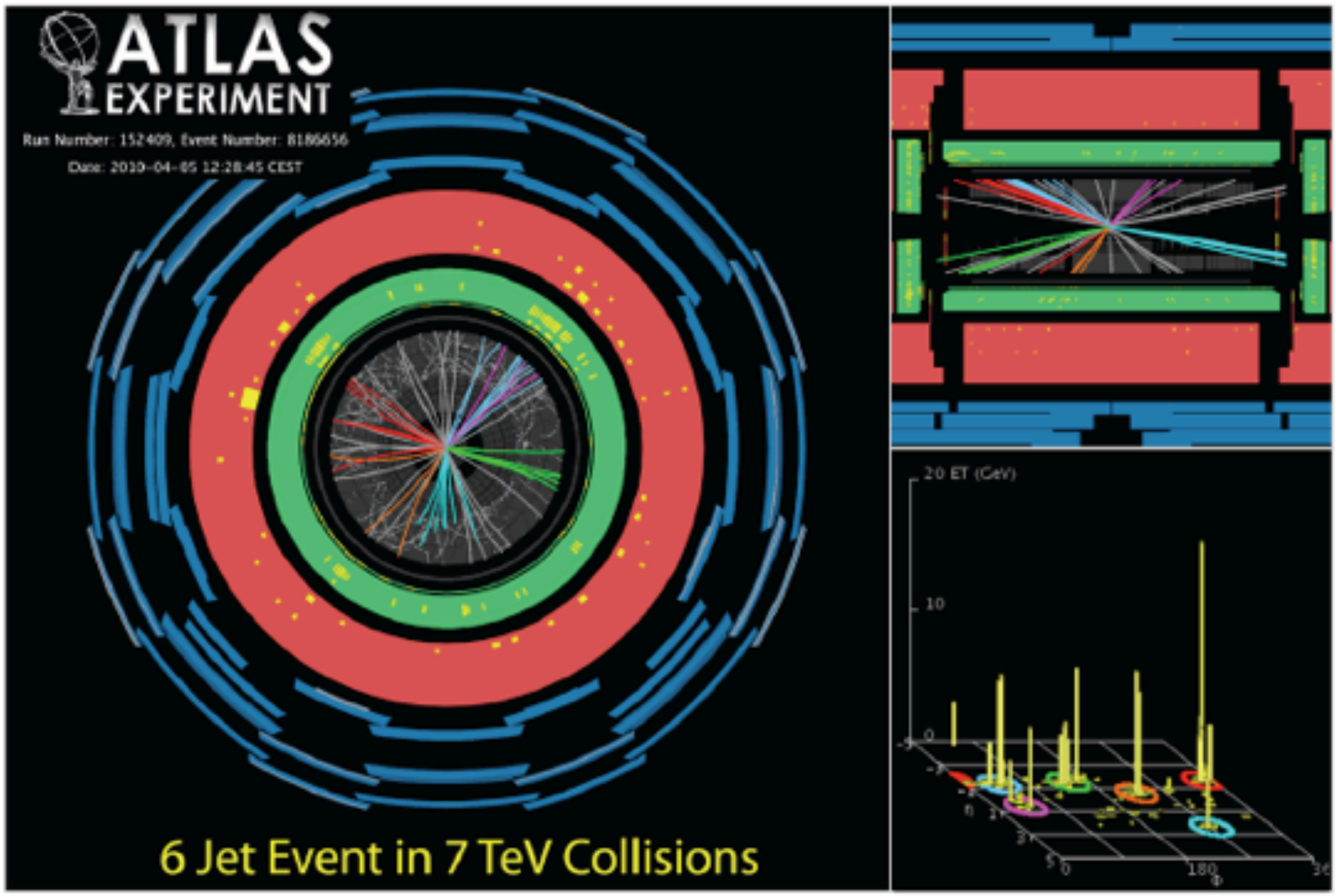


hadron colliders inevitably have to deal with QCD

discovering the Higgs or some New Physics requires a sophisticated **quantitative understanding of QCD**

achieving that can be quite a challenge ...

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + \sum_{\rm flavors} \bar{q}_i \left(i\not{D} - m\right)_{ij} q_j$$



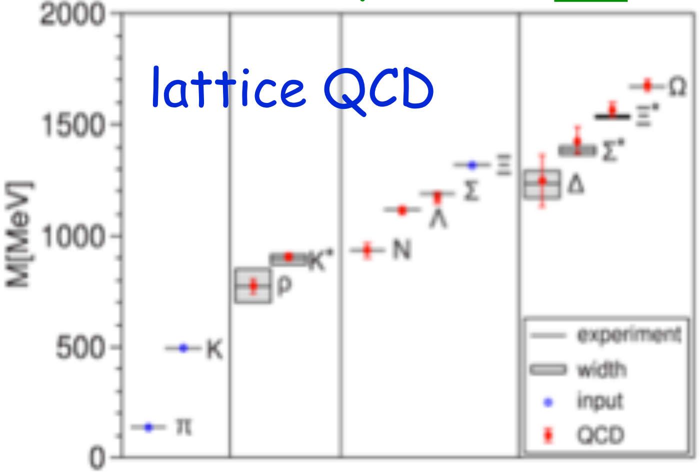
QCD – the theory of strong interactions

a simple QED-like theory, leading to extremely rich & complex phenomena

QCD – the theory of strong interactions

a simple QED-like theory, leading to extremely rich & complex phenomena

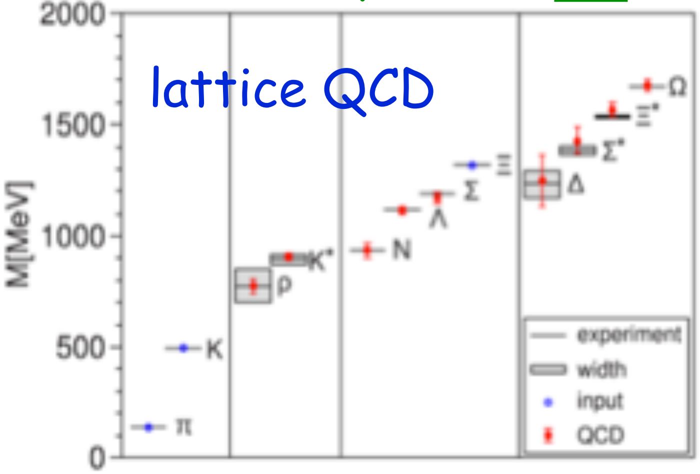
Durr et al., Science 322

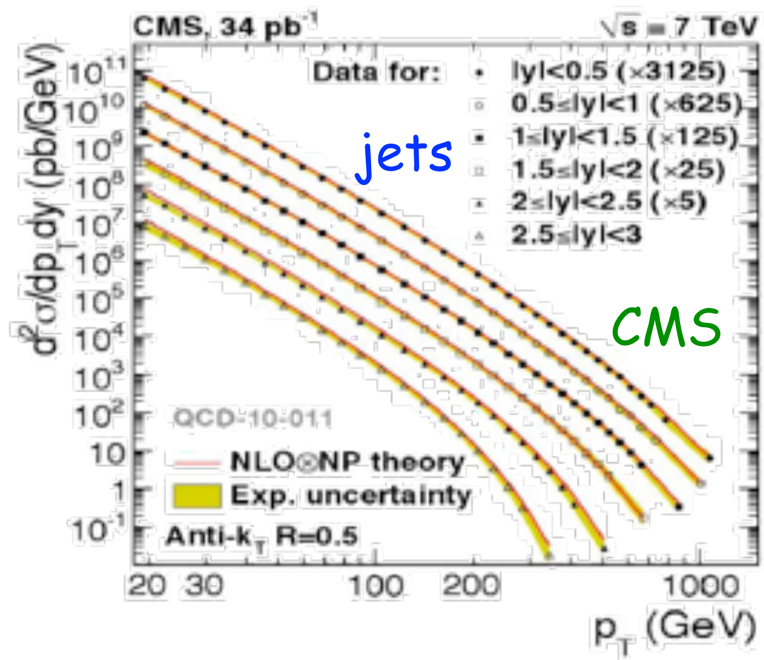


QCD – the theory of strong interactions

a simple QED-like theory, leading to extremely rich & complex phenomena

Durr et al., Science 322

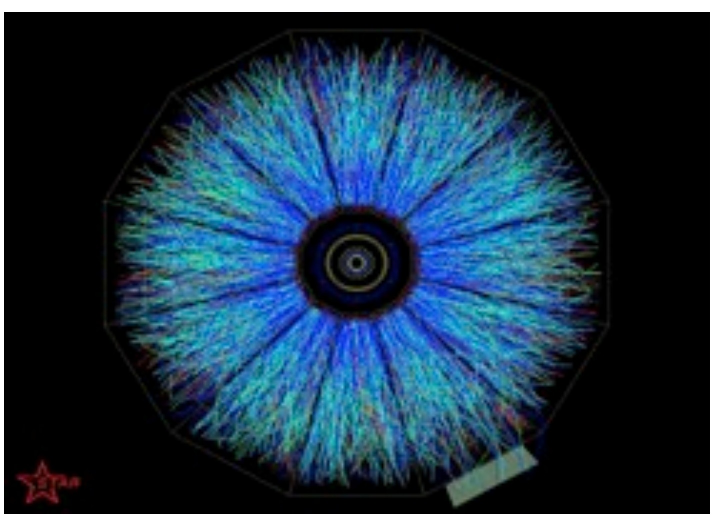




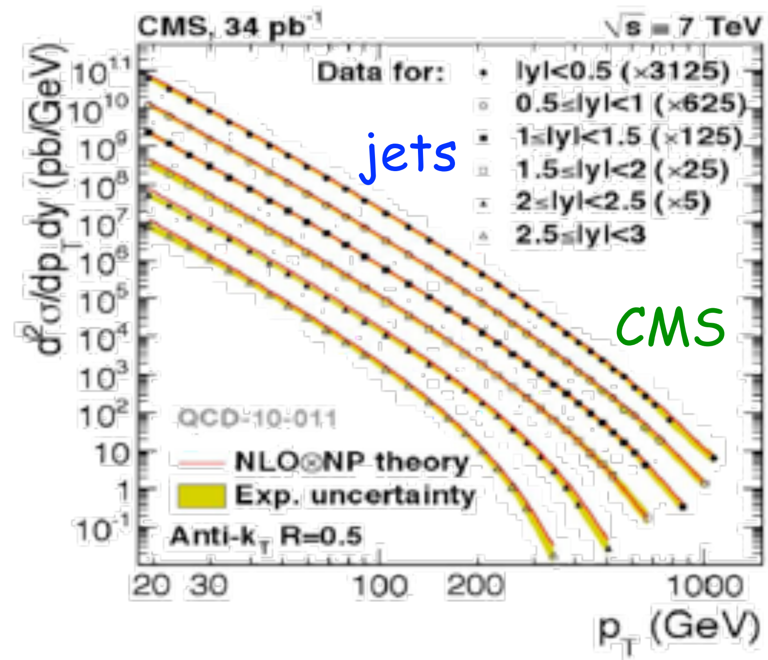
QCD – the theory of strong interactions

a simple QED-like theory, leading to extremely rich & complex phenomena

Durr et al., Science 322



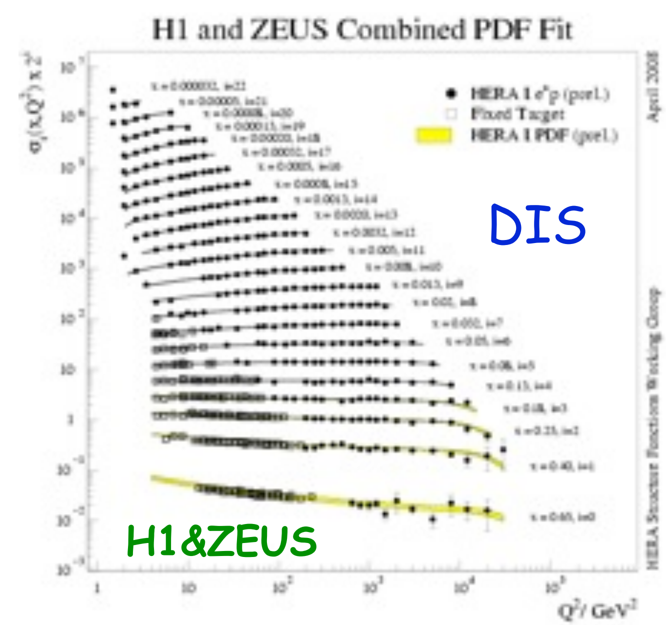
AuAu collision at STAR

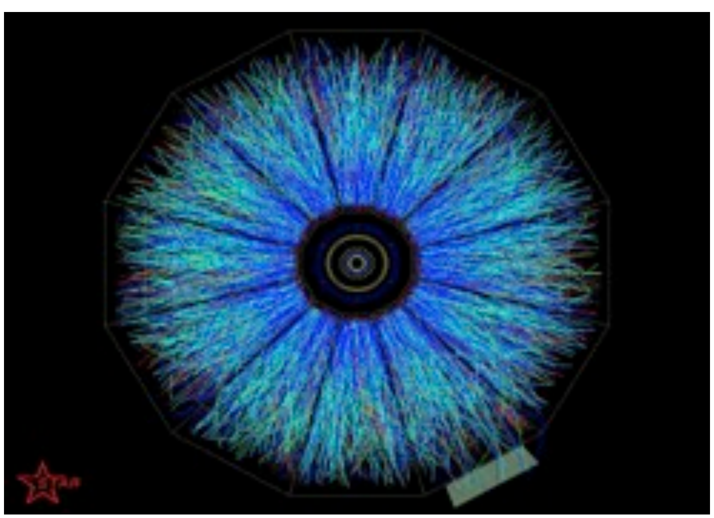


QCD – the theory of strong interactions

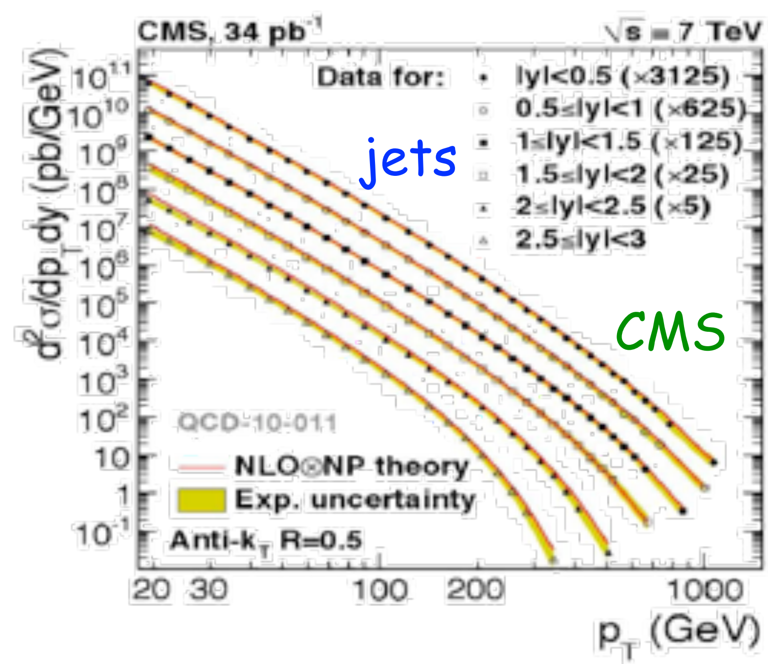
a simple QED-like theory, leading to extremely rich & complex phenomena

Durr et al., Science 322





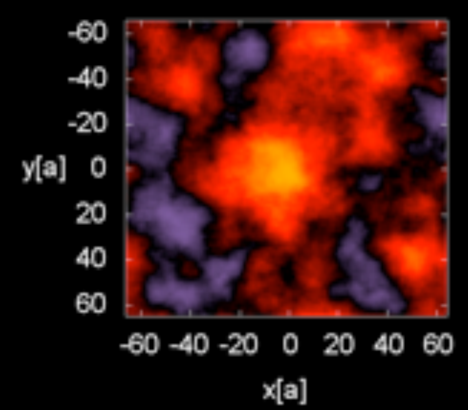
AuAu collision at STAR



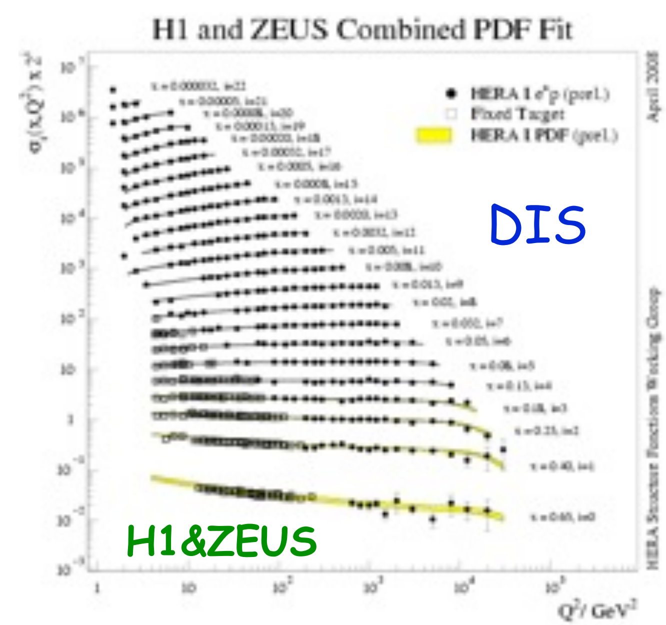
QCD – the theory of strong interactions

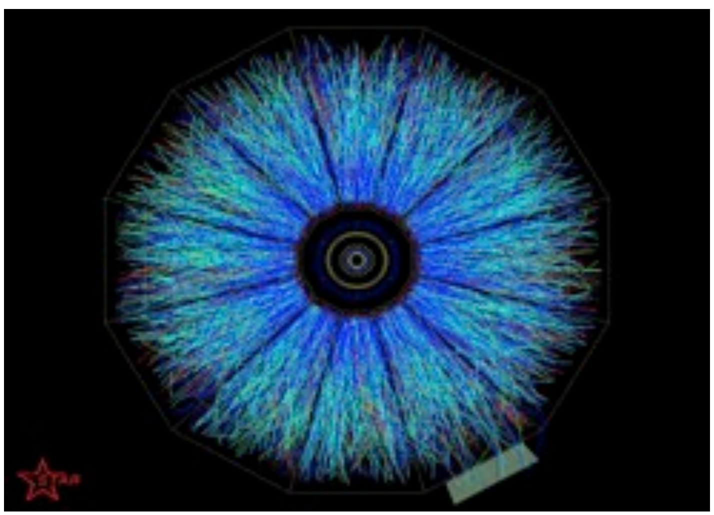
a simple QED-like theory, leading to extremely rich & complex phenomena

Durr et al., Science 322

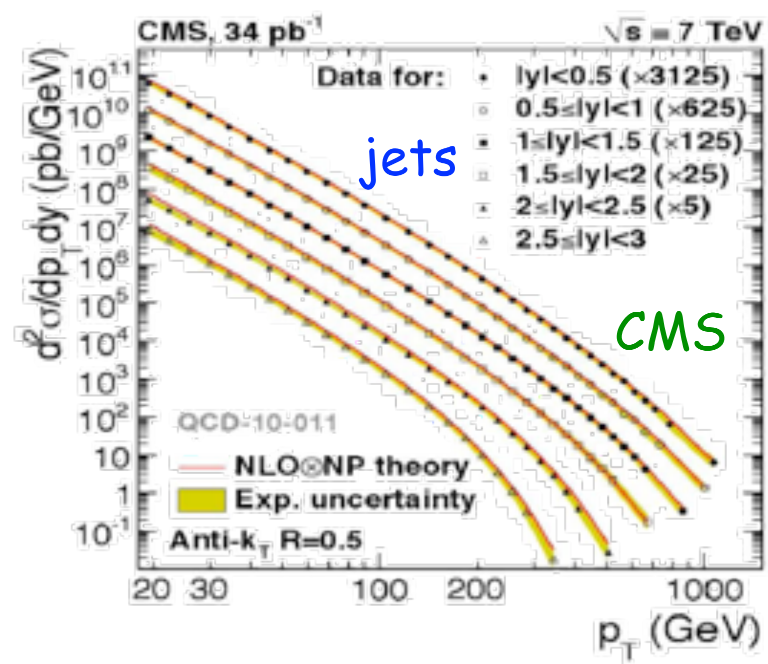


RGE evolution of gluon correlators
arXiv:1108.4764



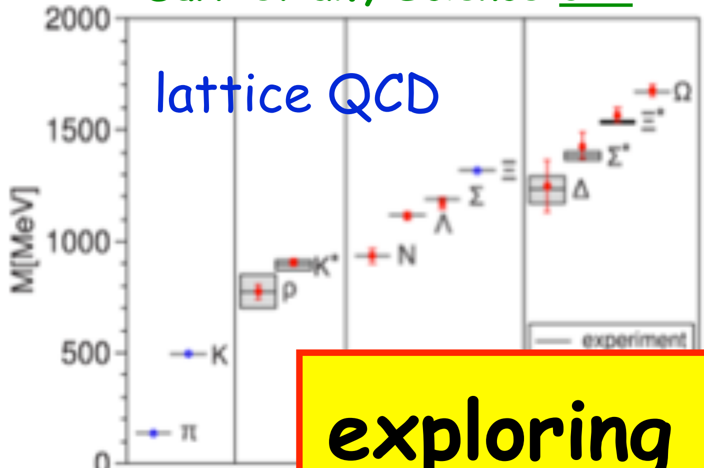


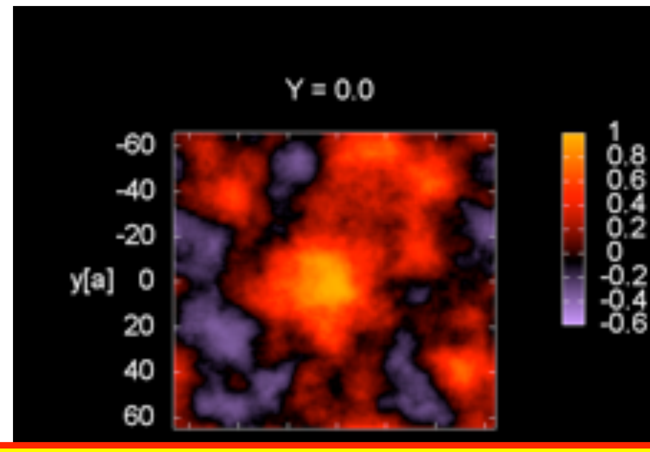
AuAu collision at STAR

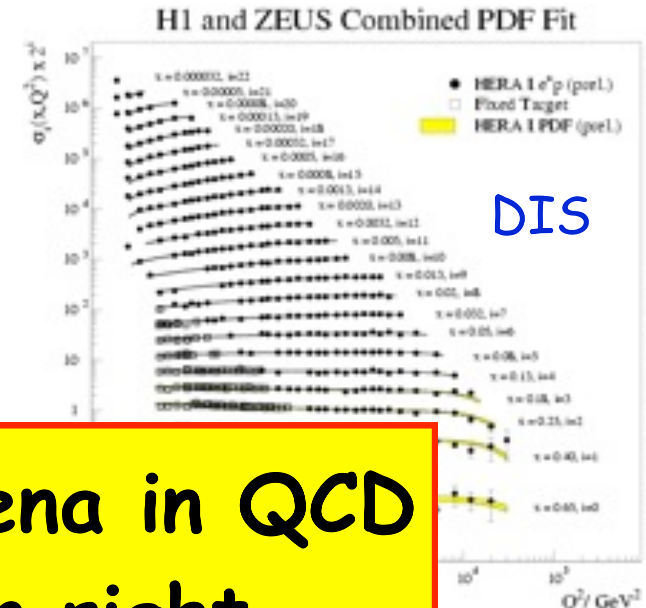


QCD – the theory of strong interactions

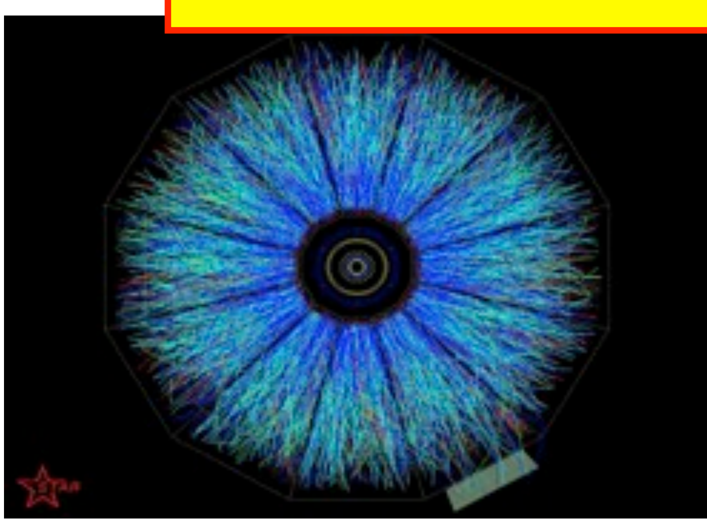
a simple QED-like theory, leading to extremely rich & complex phenomena



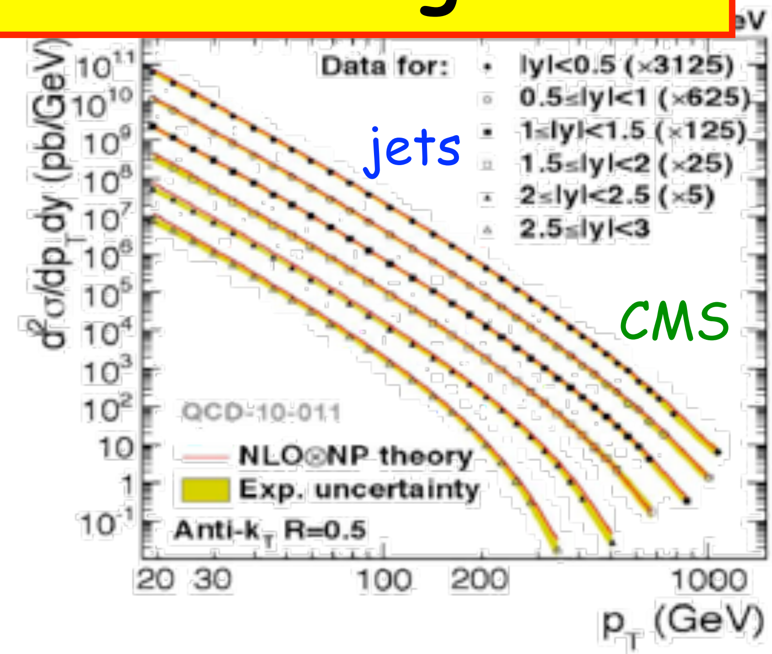




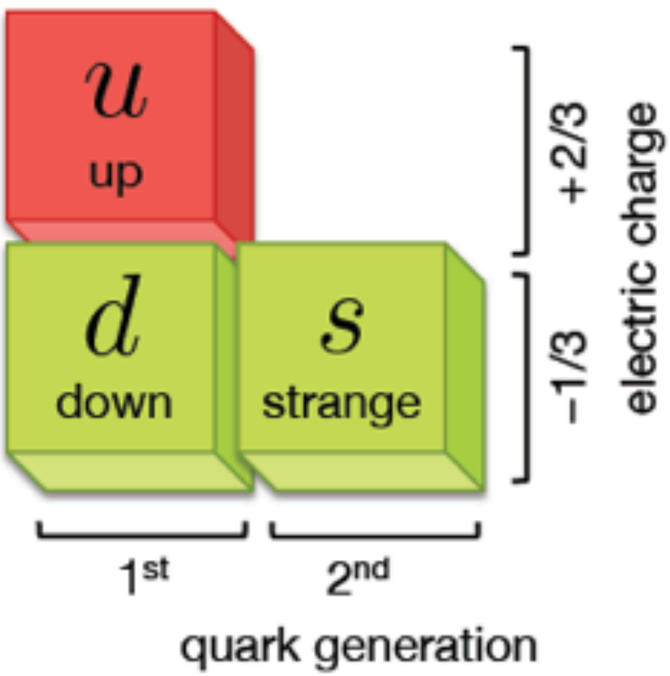
exploring all these phenomena in QCD is interesting in its own right

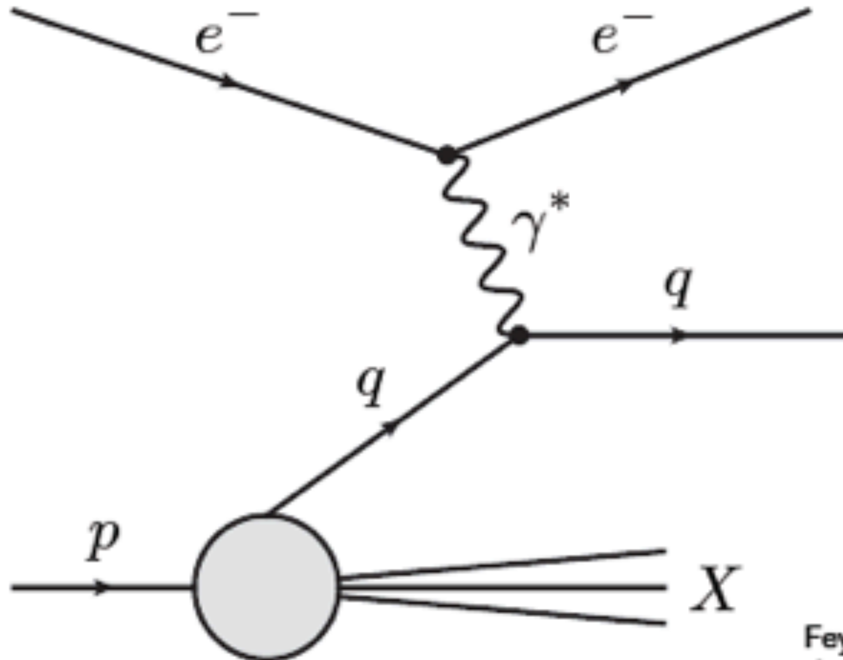


AuAu collision at STAR



QCD matter sector: Three Quarks for Muster Mark

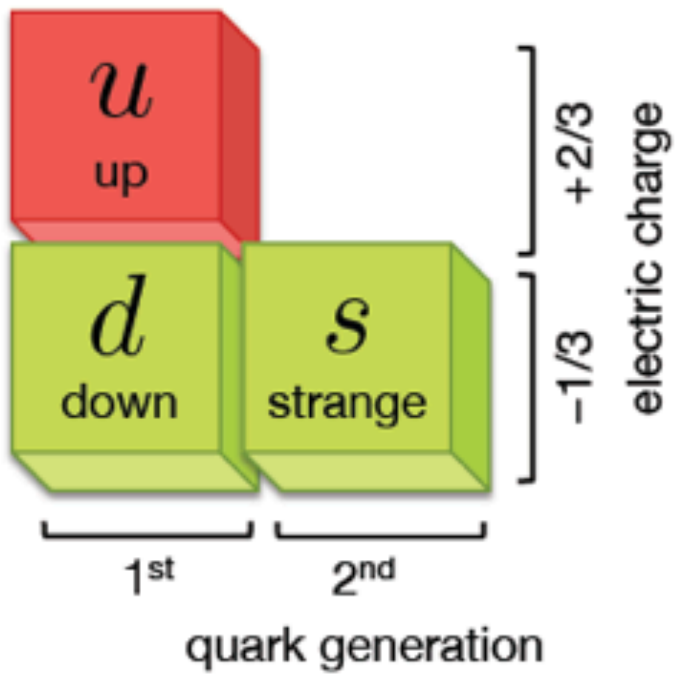


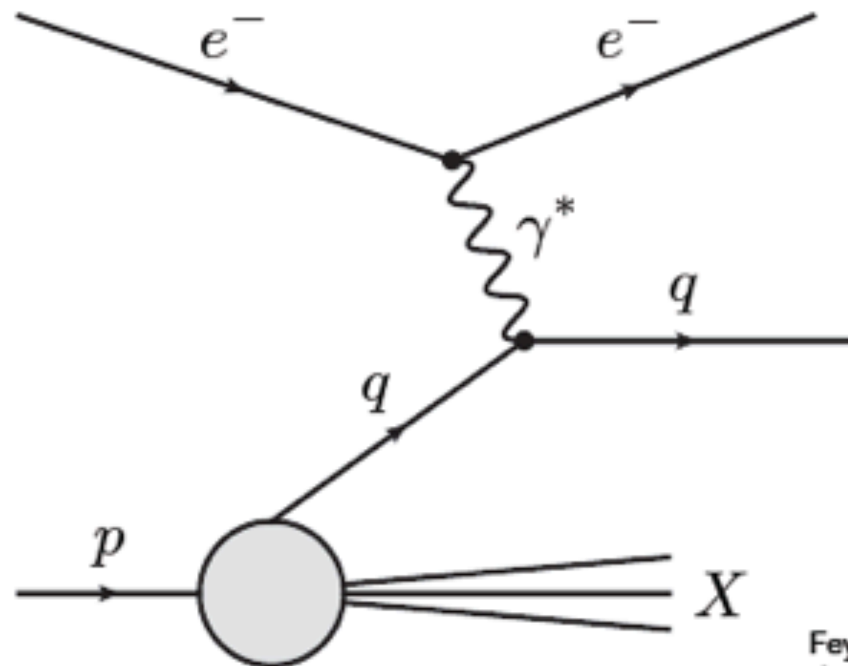


Feynman diagram describing DIS of an electron on a proton

existence of light quarks validated in deep-inelastic scattering (DIS) experiments carried out at SLAC in 1968

QCD matter sector: Three Quarks for Muster Mark





Feynman diagram describing DIS of an electron on a proton

existence of light quarks validated in deep-inelastic scattering (DIS) experiments carried out at SLAC in 1968

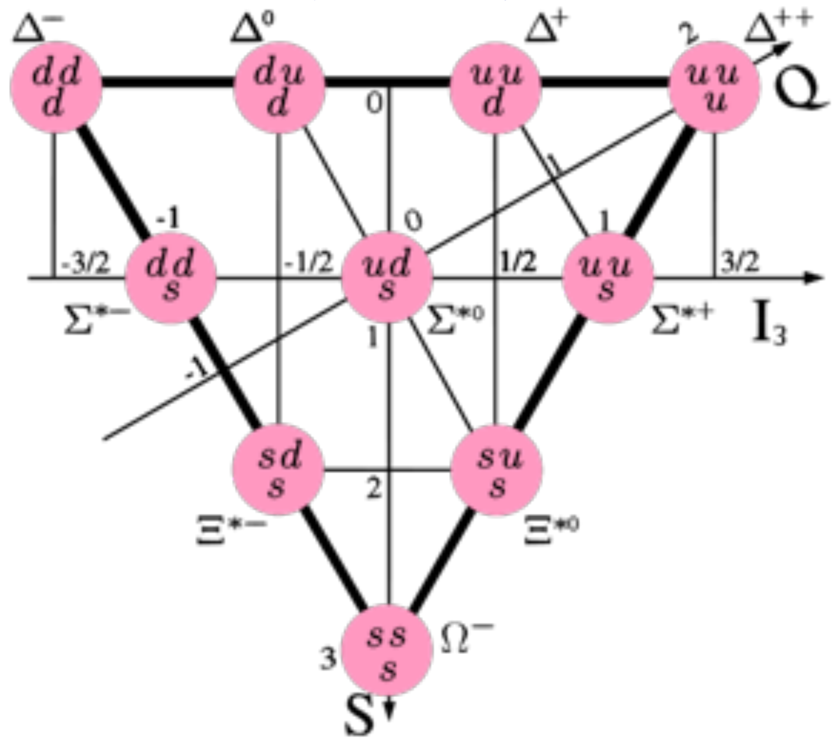
strange quarks necessary component in **quark model** to classify the observed slew of mesons/baryons Gell-Mann, Zweig (1964)

based on "**Eightfold Way**" (= $SU(3)_{flavor}$) Gell-Mann; Ne'eman (1961)

quark model: mesons and baryons

categorizes mesons (baryons) in terms of two (three) constituent quarks in **SU(3)$_{flavor}$ multiplets** = octets and decuplets

baryon decuplet



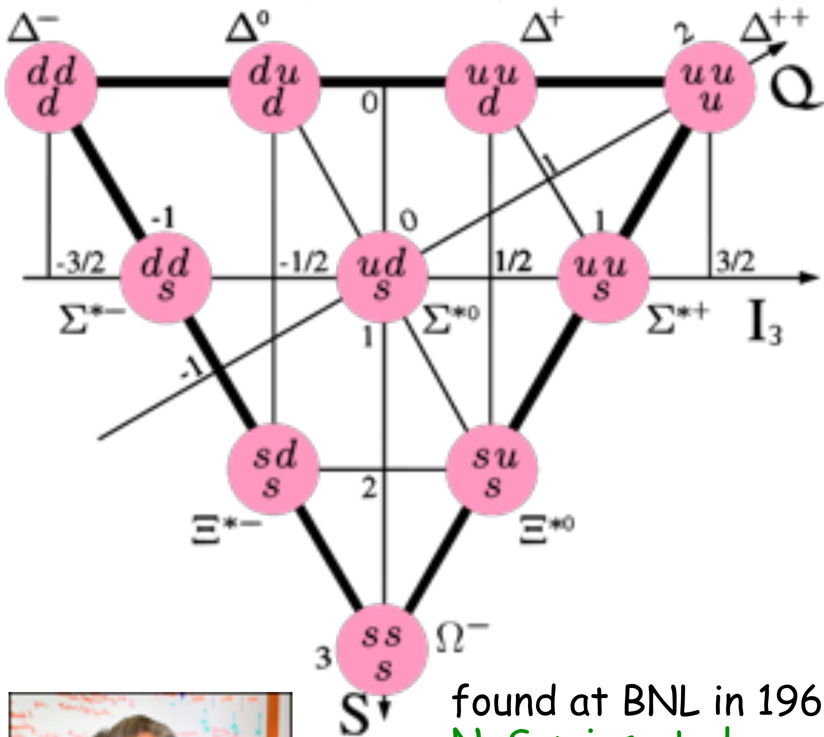
spectrum fully classified by assuming:

- quarks have spin $\frac{1}{2}$
- quarks have fractional charges
  (but combine into hadrons with integer charges)

quark model: mesons and baryons

categorizes mesons (baryons) in terms of two (three) constituent quarks in **SU(3)$_{flavor}$ multiplets** = octets and decuplets

baryon decuplet

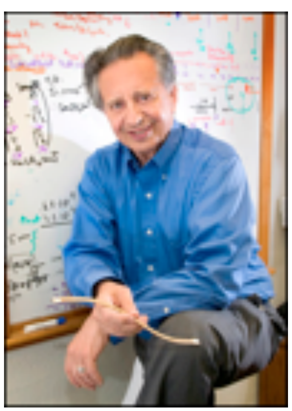


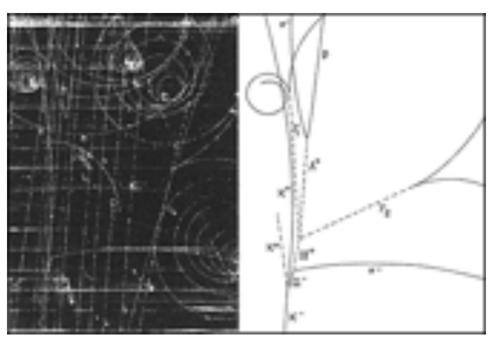
spectrum fully classified by assuming:

- quarks have spin $\frac{1}{2}$
- quarks have fractional charges
  (but combine into hadrons with integer charges)

big success: prediction of Ω^- (sss)

found at BNL in 1964
N. Samios et al.

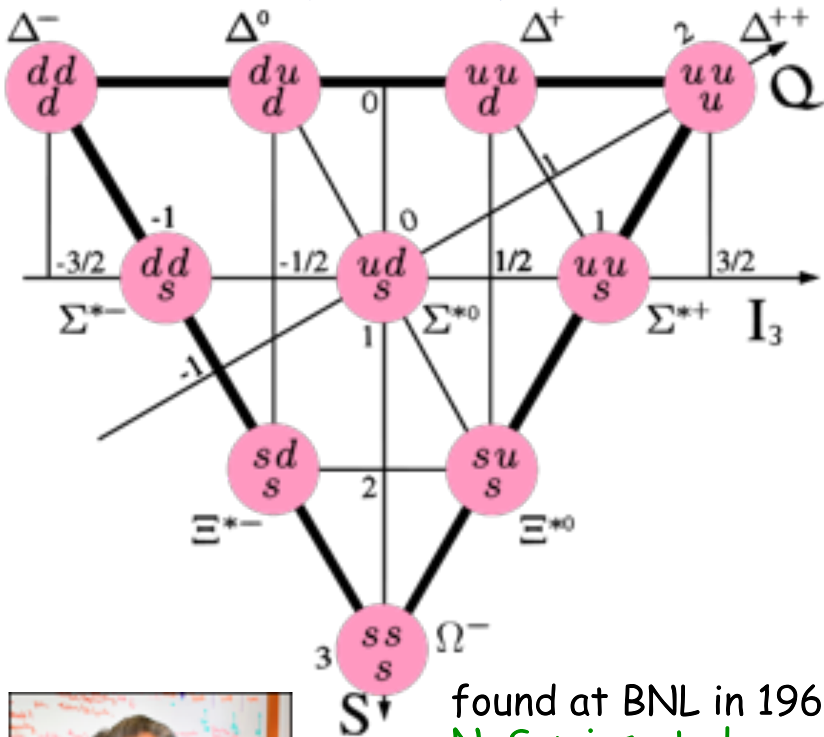


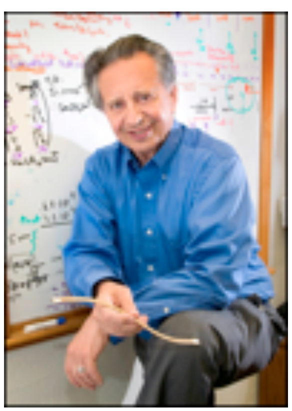


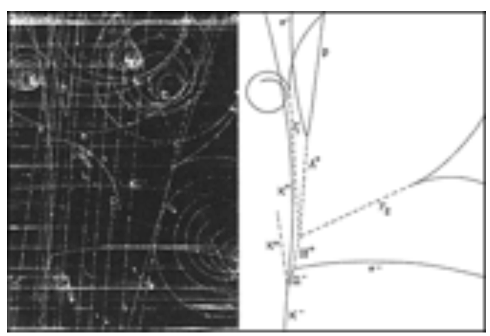
quark model: mesons and baryons

categorizes mesons (baryons) in terms of two (three) constituent quarks in **SU(3)$_{flavor}$ multiplets** = octets and decuplets

baryon decuplet







found at BNL in 1964
N. Samios et al.

spectrum fully classified by assuming:

- quarks have spin $\frac{1}{2}$
- quarks have fractional charges
  (but combine into hadrons with integer charges)

big success: prediction of Ω^- (sss)

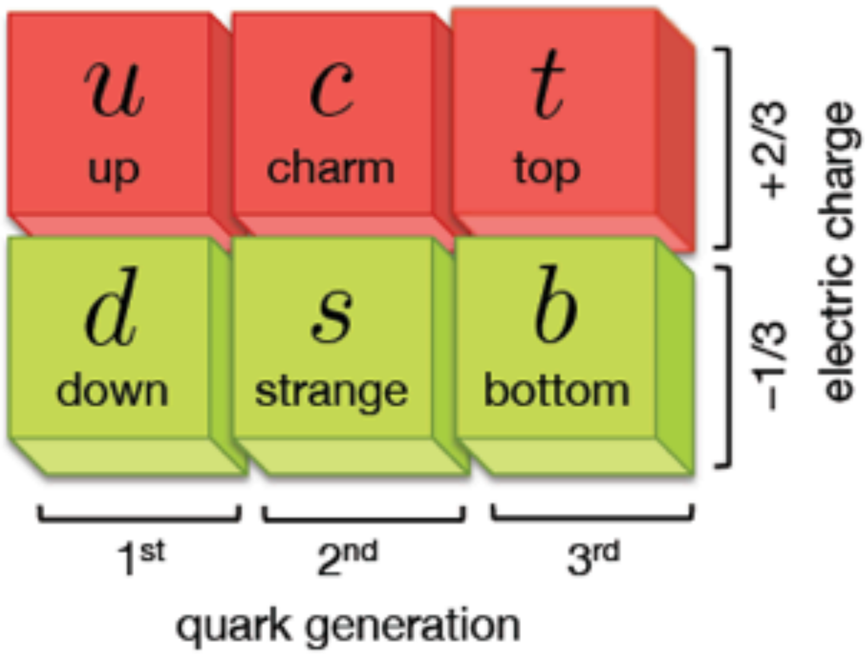
also, **first evidence of color**

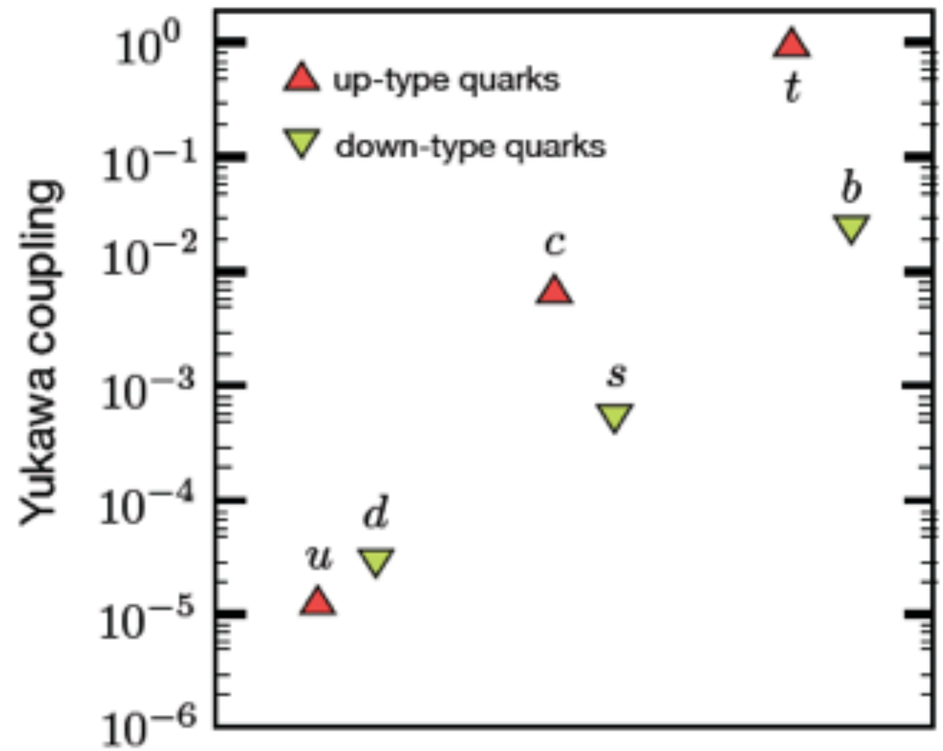
- Δ^{++} wave function |uuu> not anti-sym
  (violates Pauli principle)
- remedy: color quantum number
  but hadrons remain colorless/color singlets

$\sim \sum_{ijk} \epsilon_{ijk} |q_i q_j q_k\rangle$

$\sim \sum_i |\bar{q}_i q_i\rangle$

QCD matter sector: 3 generations

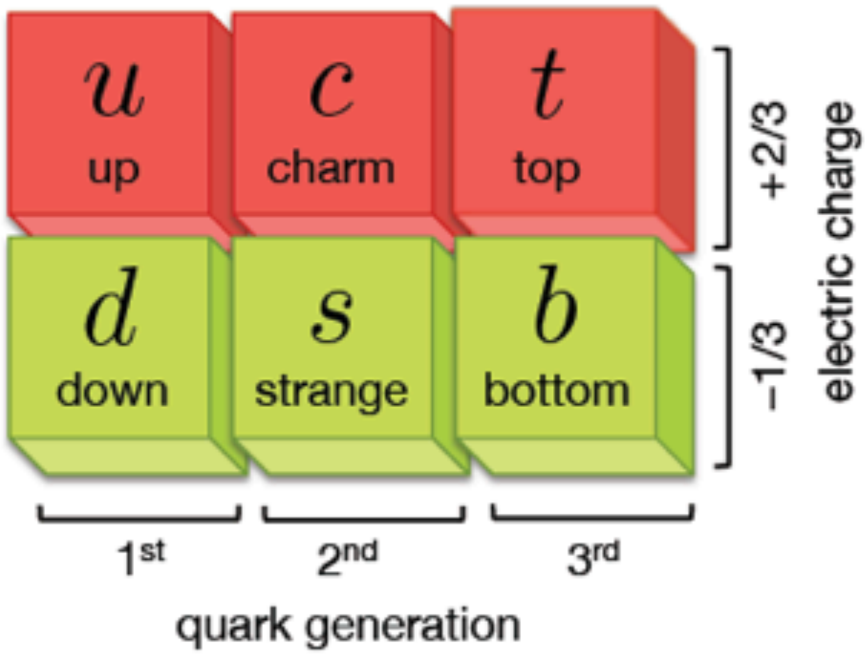


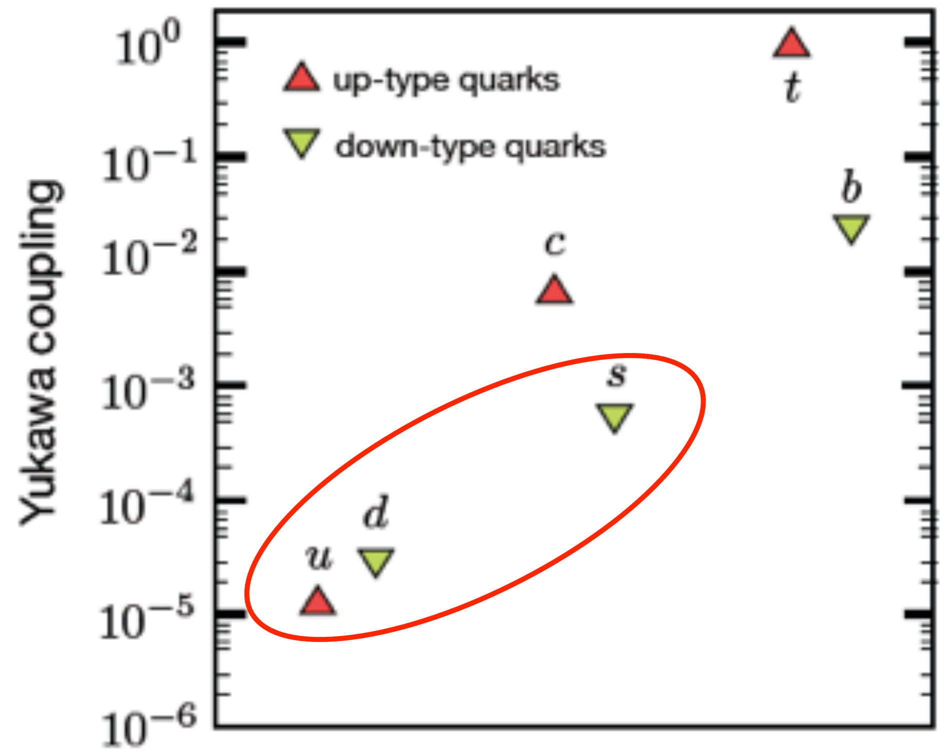


- masses of six quarks range from O(MeV) to about 175 GeV

  why the masses are split by almost six orders of magnitude remains a big mystery

QCD matter sector: 3 generations





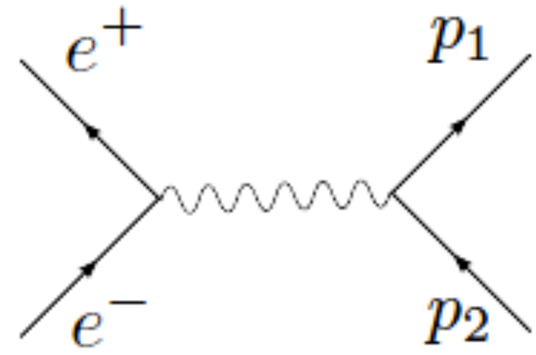
- masses of six quarks range from O(MeV) to about 175 GeV
  why the masses are split by almost six orders of magnitude remains a big mystery
- masses of u, d, s quarks are lighter than 1 GeV (proton mass)
  in the limit of vanishing u,d,s masses there is an exact $SU(3)_{flavor}$ symmetry

further evidence for color quantum number

- **color can be probed directly in e^+e^- collisions**

  idea:

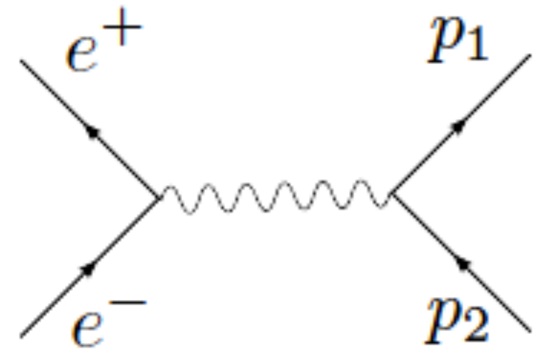
  production of fermion pairs (leptons or quarks) through a virtual photon sensitive to electric charge and number of degrees of freedom



further evidence for color quantum number

- **color can be probed directly in e^+e^- collisions**

  idea:
  production of fermion pairs (leptons or quarks) through a virtual photon sensitive to electric charge and number of degrees of freedom



- hence, investigate quarks through "**R ratio**"

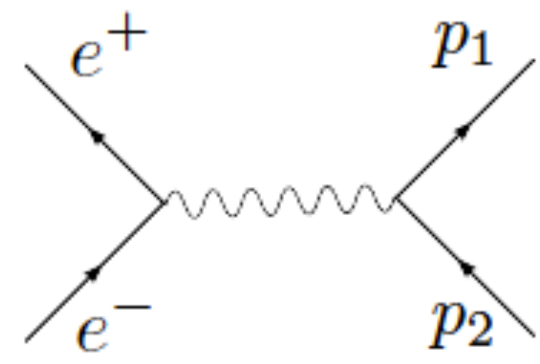
$$R \equiv \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$

assumed number of colors of quark (N_c)

further evidence for color quantum number

- **color can be probed directly in e^+e^- collisions**

  idea:
  production of fermion pairs (leptons or quarks) through a virtual photon sensitive to electric charge and number of degrees of freedom



- hence, investigate quarks through "**R ratio**"

$$R \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$

electric charge of quark [in units of e]

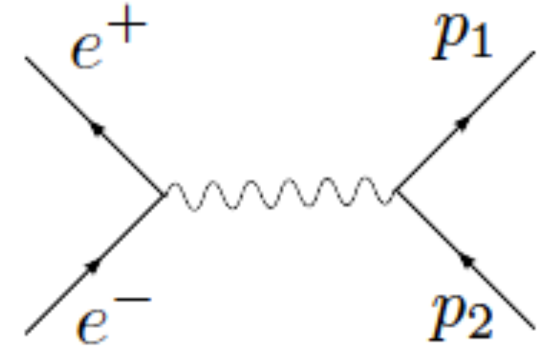
assumed number of colors of quark

  - in LO described by process $\mathbf{e^+e^- \to q\bar{q}}$

further evidence for color quantum number

- **color can be probed directly in e^+e^- collisions**

  idea:
  production of fermion pairs (leptons or quarks) through a virtual photon sensitive to electric charge and number of degrees of freedom



- hence, investigate quarks through "**R ratio**"

$$R \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$

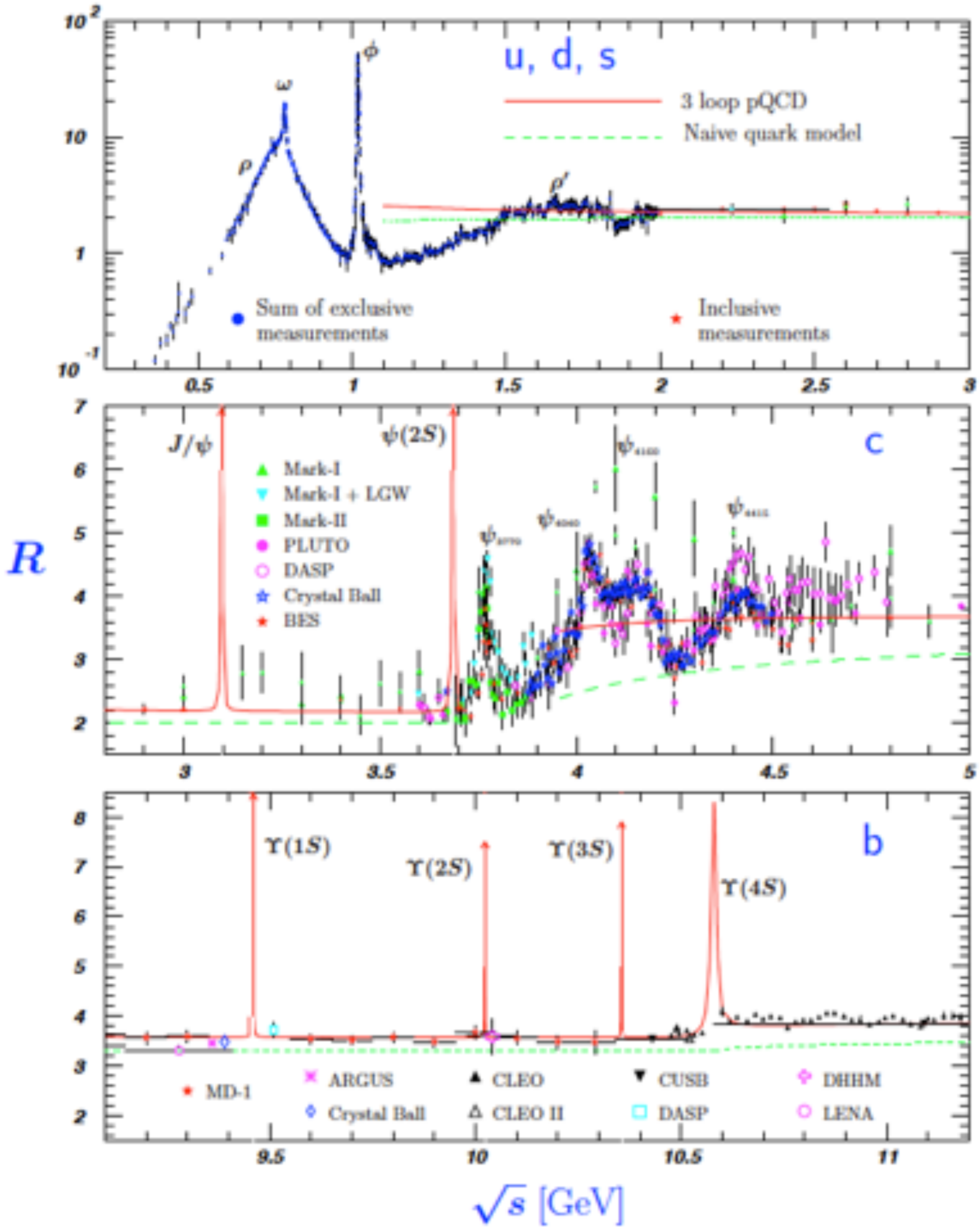
electric charge of quark [in units of e]

assumed number of colors of quark

sum over active quarks

  - in LO described by process $e^+e^- \to q\bar{q}$
  - each active quark is produced in one out of N_c colors above kinematic threshold

experimental results for R ratio



$$R_{u,d,s} = 3 \times \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$$

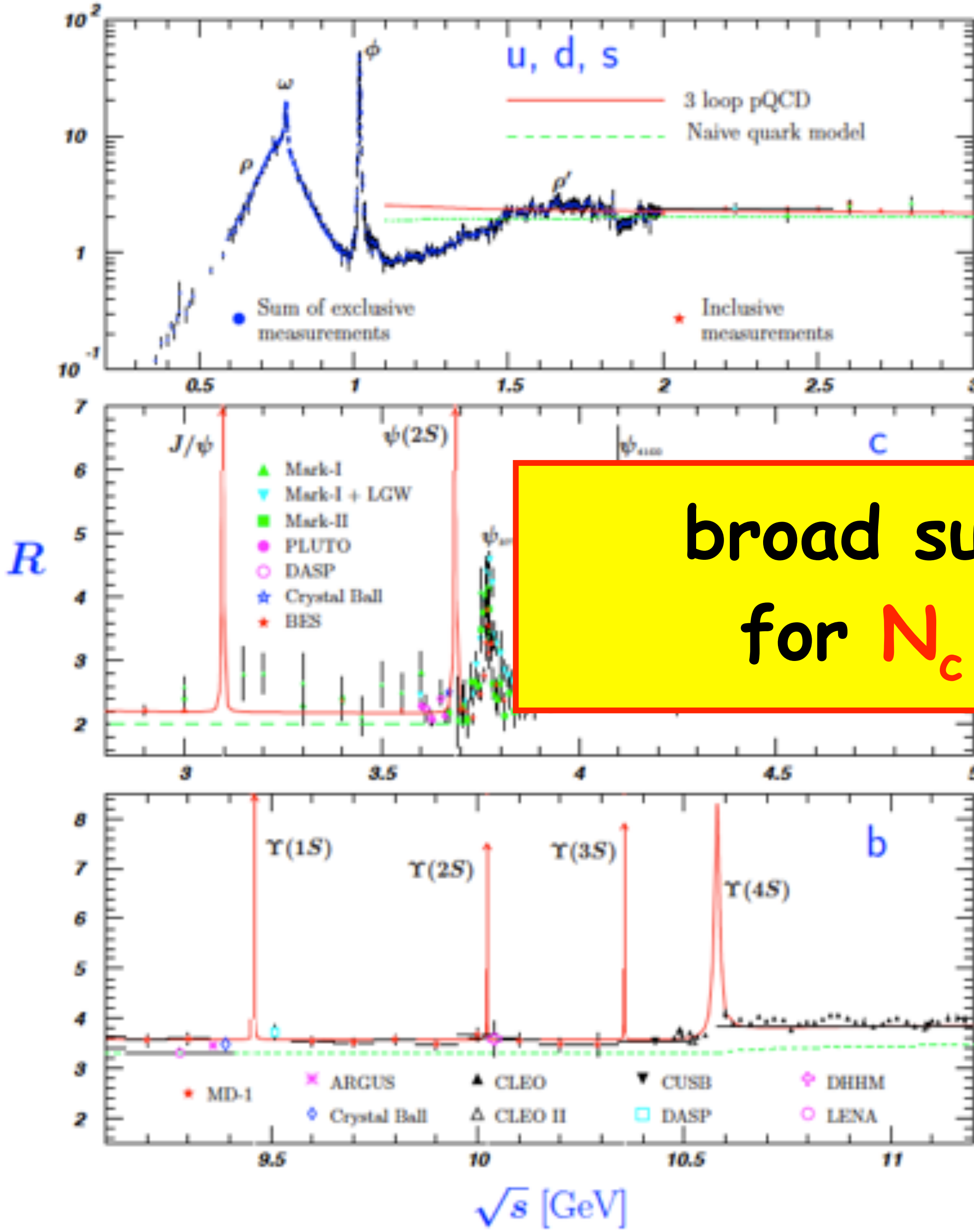
$$R_{u,d,s,c} = R_{u,d,s} + 3 \times \left(\frac{2}{3}\right)^2 = \frac{10}{3}$$

$$R_{u,d,s,c,b} = R_{u,d,s,c} + 3 \times \left(-\frac{1}{3}\right)^2 = \frac{11}{3}$$

caveats:

- higher order corrections
- mass effects near threshold

experimental results for R ratio



$$R_{u,d,s} = 3 \times \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$$

$$R_{u,d,s,c} = R_{u,d,s} + 3 \times \left(\frac{2}{3}\right)^2$$

broad support for N_c = 3

$$R_{u,d,s,c,b} = R_{u,d,s,c} + 3 \times \left(-\frac{1}{3}\right)^2 = \frac{11}{3}$$

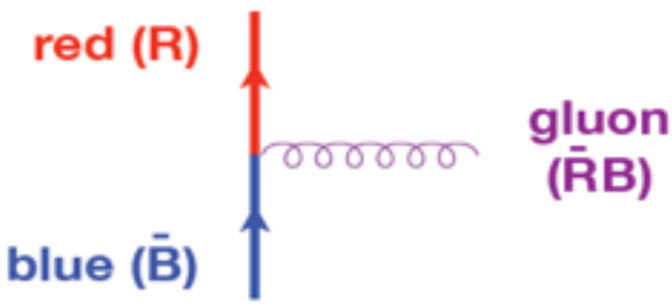
caveats:

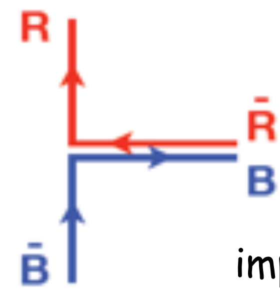
- higher order corrections
- mass effects near threshold

QCD color interactions heuristically

- QCD color quantum number is mediated by the **gluon**
  analogous to the photon in QED
- gluons are changing quarks from one color to another
  as such they must also carry a color charge (unlike the charge neutral photon in QED)

example:



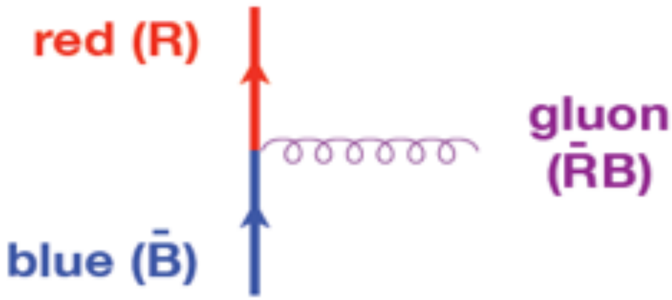


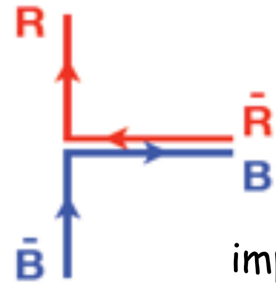
"color flow"
important calculational tool

QCD color interactions heuristically

- QCD color quantum number is mediated by the **gluon**
  analogous to the photon in QED
- gluons are changing quarks from one color to another
  as such they must also carry a color charge (unlike the charge neutral photon in QED)

example:

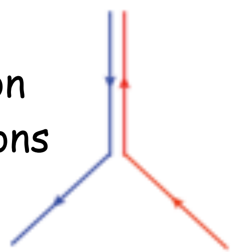




"color flow"
important calculational tool

- color charge of each gluon represented by a 3x3 matrix in color space
  conventional choice: express t^a (a=1…8) in terms of **Gell-Mann matrices**

typical color interaction between quarks and gluons

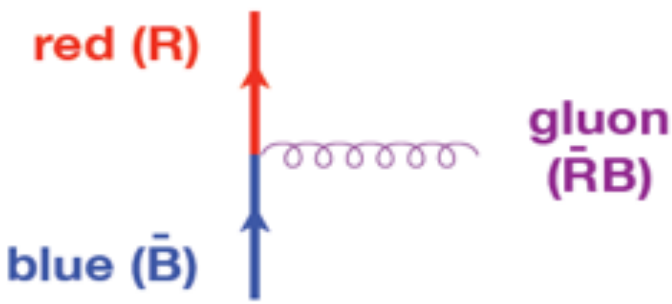


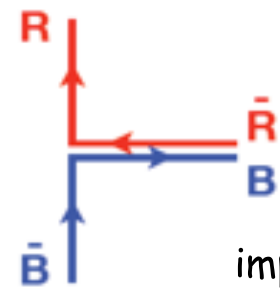
$$\underset{\bar{\psi}_i}{(1,0,0)} \underset{t^1_{ij}}{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \underset{\psi_j}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

QCD color interactions heuristically

- QCD color quantum number is mediated by the **gluon**
  analogous to the photon in QED
- gluons are changing quarks from one color to another
  as such they must also carry a color charge (unlike the charge neutral photon in QED)

example:

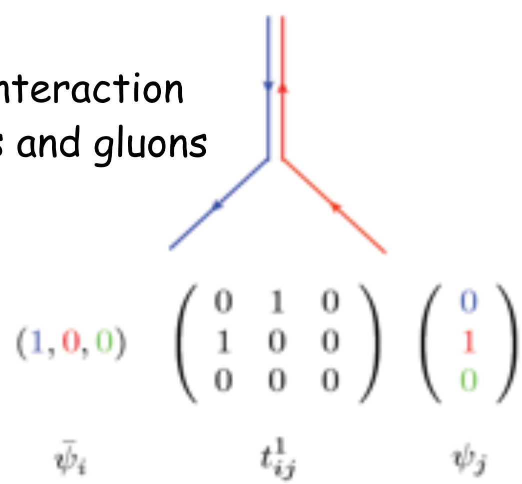




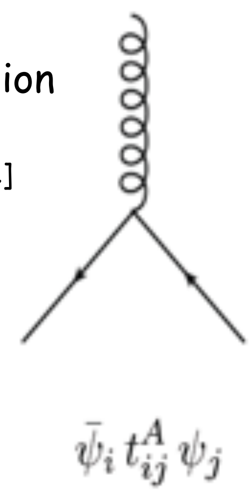
"color flow"
important calculational tool

- color charge of each gluon represented by a 3x3 matrix in color space
  conventional choice: express t^a (a=1…8) in terms of **Gell-Mann matrices**

typical color interaction
between quarks and gluons



more formal expression
as **Feynman rule**
[only color structure here]



QCD: an unbroken SU(3) Quantum Field Theory

guiding principle for all field theories: **local gauge invariance** of the underlying Lagrangian

i.e., redefining the quark and gluon fields independently at each space-time point has no impact on the physics

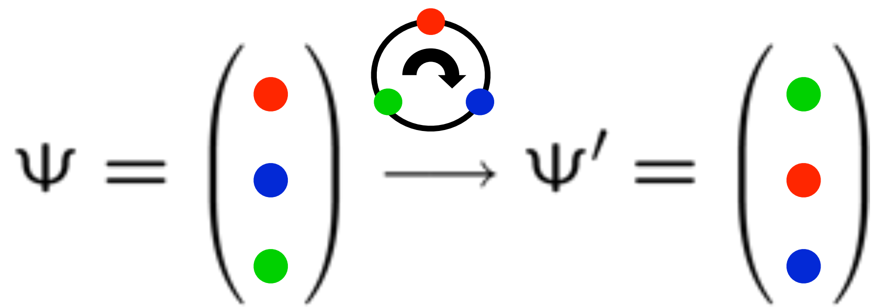
QCD: an unbroken SU(3) Quantum Field Theory

guiding principle for all field theories: **local gauge invariance** of the underlying Lagrangian

i.e., redefining the quark and gluon fields independently at each space-time point has no impact on the physics

here: local SU(3) rotations in color space

spin-$\frac{1}{2}$ quark fields
come as colors triplets
(fundamental representation)



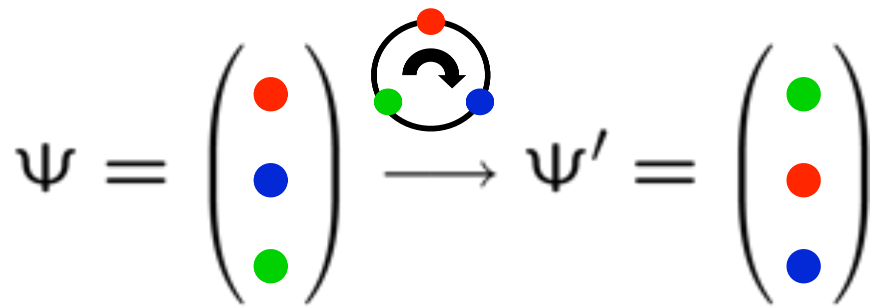
QCD: an unbroken SU(3) Quantum Field Theory

guiding principle for all field theories: local gauge invariance of the underlying Lagrangian

i.e., redefining the quark and gluon fields independently at each space-time point has no impact on the physics

here: local SU(3) rotations in color space

spin-$\frac{1}{2}$ quark fields
come as colors triplets
(fundamental representation)



local SU(3) invariance dictates:

- 8 massless spin-1 gluons
  (adjoint representation)
- all interactions between quarks and gluons (covariant derivative)

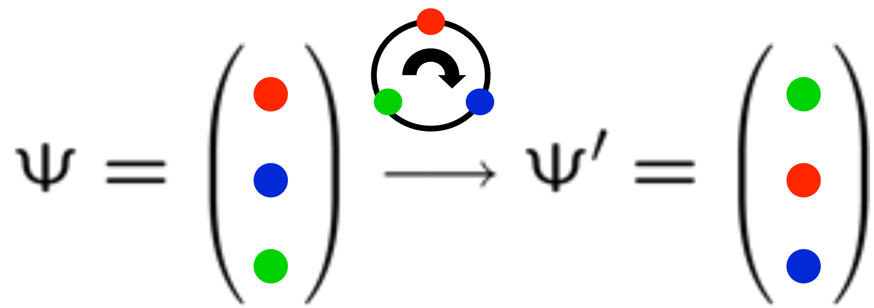
QCD: an unbroken SU(3) Quantum Field Theory

guiding principle for all field theories: local gauge invariance of the underlying Lagrangian

i.e., redefining the quark and gluon fields independently at each space-time point has no impact on the physics

here: local SU(3) rotations in color space

spin-$\frac{1}{2}$ quark fields come as colors triplets (fundamental representation)

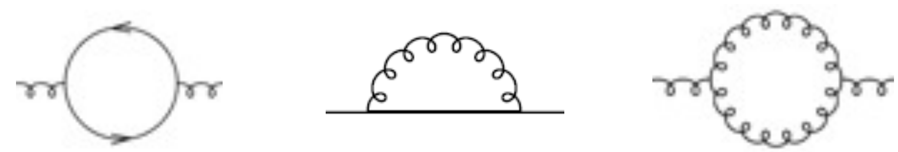


local SU(3) invariance dictates:

- 8 massless spin-1 gluons (adjoint representation)
- all interactions between quarks and gluons (covariant derivative)

non-Abelian group structure:

- Lie algebra: $[T_a, T_b] = i\, f_{abc} T_c$
- invariants ("color factors"):



$T_F = 1/2$ $\quad C_F = 4/3$ $\quad C_A = 3$

experimental support for SU(3)

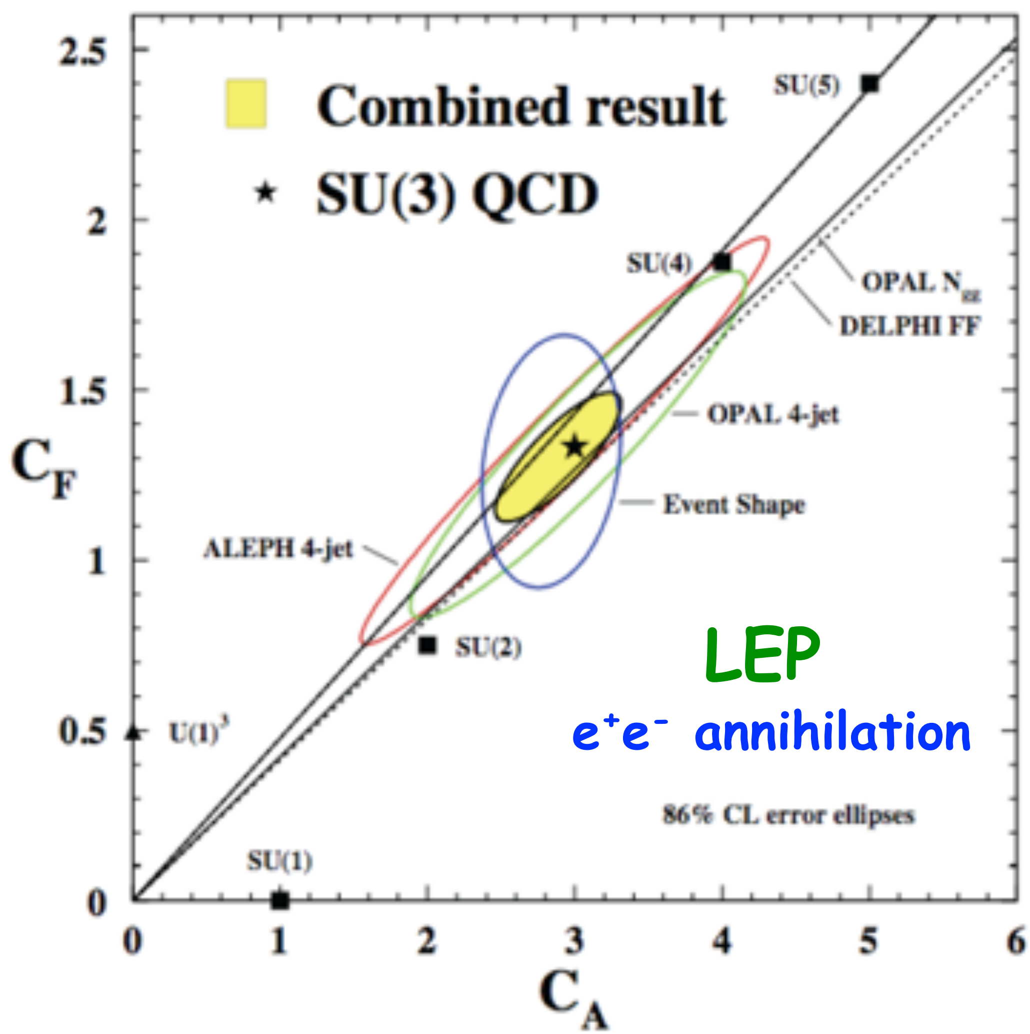
- **color factors are not just math**

  assumed group structure has
  impact on theoretical predictions

experimental support for SU(3)

- **color factors are not just math**

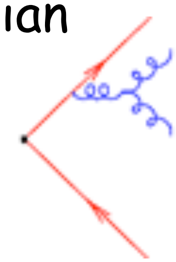
  assumed group structure has impact on theoretical predictions



- angular correlations between four jets depend on C_A/C_F and T_F/C_F

- sensitivity to non-Abelian three-gluon-vertex

  LO: Ellis, Ross, Terrano

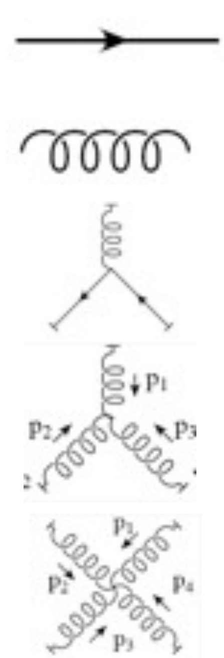


QCD Lagrangian & Feynman rules

L_{QCD} encodes all physics related to strong interactions

for perturbative calculations we simply read off the **Feynman rules**

$$
\begin{aligned}
\mathcal{L}_{\text{QCD}} &= \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi \\
&- (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
&- g\bar{\Psi}A^a_\mu T_a \gamma^\mu \Psi \\
&- \frac{1}{2}g(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f_{abc} A^{\mu b} A^{\nu c} \\
&- \frac{1}{4}g^2 f_{abc} A^b_\mu A^c_\nu f_{ade} A^{\mu d} A^{\nu e}
\end{aligned}
$$



like in QED

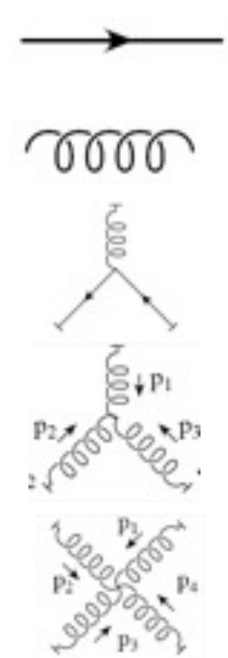
non abelian

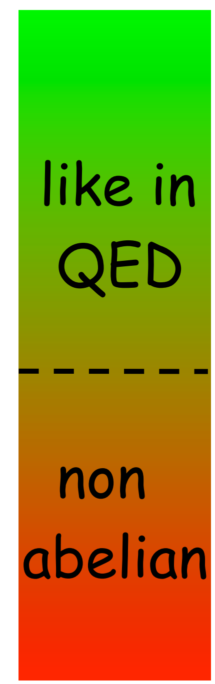
QCD Lagrangian & Feynman rules

L_{QCD} encodes all physics related to strong interactions

for perturbative calculations we simply read off the **Feynman rules**

$$\begin{aligned}
\mathcal{L}_{\rm QCD} &= \bar\Psi(i\partial_\mu\gamma^\mu - m)\Psi \\
&- (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
&- g\bar\Psi A^a_\mu T_a \gamma^\mu \Psi \\
&- \frac{1}{2} g(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f_{abc} A^{\mu b} A^{\nu c} \\
&- \frac{1}{4} g^2 f_{abc} A^b_\mu A^c_\nu f_{ade} A^{\mu d} A^{\nu e}
\end{aligned}$$

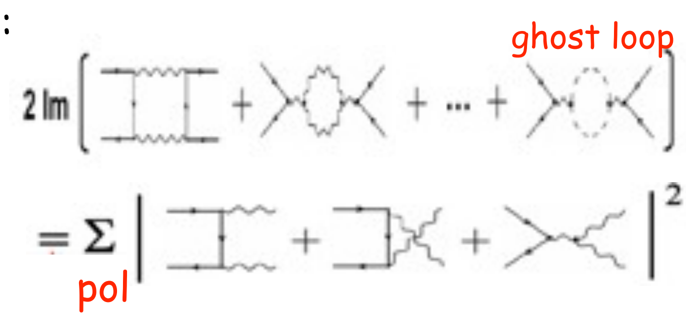




technical complications due to the gauge-fixing & ghost terms:

gauge-fixing: needed to define gluon propagator; breaks gauge-invariance but all physical results are independent of the gauge

ghosts: cancel unphysical degrees of freedom → unitarity



recall: gauge invariance in QED

$$\begin{aligned}\mathcal{L}_{\mathbf{QED}} &= \mathcal{L}_{\mathbf{Dirac}} + \mathcal{L}_{\mathbf{Maxwell}} + \mathcal{L}_{\mathbf{int}} \\ &= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma_{\mu}\Psi A^{\mu} \\ &= \bar{\Psi}(iD\!\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

recall: gauge invariance in QED

$$\mathcal{L}_{\mathrm{QED}} = \mathcal{L}_{\mathrm{Dirac}} + \mathcal{L}_{\mathrm{Maxwell}} + \mathcal{L}_{\mathrm{int}}$$

$$= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma_\mu\Psi A^\mu$$

$$= \bar{\Psi}(iD\!\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electromagnetic vector potential A_μ

field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

covariant derivative $D_\mu = \partial_\mu + iq\,A_\mu$

recall: gauge invariance in QED

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}}$$

$$= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma_\mu\Psi A^\mu$$

$$= \bar{\Psi}(iD\!\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electromagnetic vector potential A_μ

field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

covariant derivative $D_\mu = \partial_\mu + iq A_\mu$

invariant under **local gauge (phase) transformation**

$$\Psi(x) \rightarrow \Psi'(x) = e^{i\alpha(x)}\Psi(x)$$

$$A_\mu(x) \rightarrow A'_\mu = A_\mu(x) - \frac{1}{q}\partial_\mu\alpha(x)$$

- dictates interaction term
- photon mass term would violate gauge invariance $\sim m_\gamma^2 A_\mu A^\mu$

recall: gauge invariance in QED

$$\mathcal{L}_{\mathrm{QED}} = \mathcal{L}_{\mathrm{Dirac}} + \mathcal{L}_{\mathrm{Maxwell}} + \mathcal{L}_{\mathrm{int}}$$

$$= \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma_\mu\Psi A^\mu$$

$$= \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electromagnetic vector potential A_μ — photon field carries no electric charge

field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ — field strength itself gauge invariant

covariant derivative $D_\mu = \partial_\mu + iq A_\mu$ — "covariant" = $D_\mu\psi$ transforms as ψ

invariant under **local gauge (phase) transformation**

$$\Psi(x) \rightarrow \Psi'(x) = e^{i\alpha(x)}\Psi(x)$$

$$A_\mu(x) \rightarrow A'_\mu = A_\mu(x) - \frac{1}{q}\partial_\mu\alpha(x)$$

- dictates interaction term
- photon mass term would violate gauge invariance $\sim m_\gamma^2 A_\mu A^\mu$

recall: gauge invariance in QED

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L}_{Maxwell} + \mathcal{L}_{int}$$

$$= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - q\bar{\Psi}\gamma_\mu\Psi A^\mu$$

$$= \bar{\Psi}(iD\!\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electromagnetic ve

field strength tens

covariant derivative $D_\mu = \partial_\mu + iqA_\mu$

more cumbersome to demonstrate for QCD

ength itself
invariant

" =
$D_\mu\psi$ transforms as ψ

invariant under **local gauge (phase) transformation**

$$\Psi(x) \to \Psi'(x) = e^{i\alpha(x)}\Psi(x)$$

$$A_\mu(x) \to A'_\mu = A_\mu(x) - \frac{1}{q}\partial_\mu\alpha(x)$$

- dictates interaction term
- photon mass term would violate gauge invariance $\sim m_\gamma^2 A_\mu A^\mu$

one more look at the QCD Lagrangian

- Yang and Mills proposed in 1954 that the local "phase rotation" in QED could be generalized to non Abelian groups such as SU(3)

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}^{\mu\nu}_{\mathbf{a}}\mathbf{F}^{\mathbf{a}}_{\mu\nu} + \sum_{\mathbf{f}} \bar{\Psi}^{(\mathbf{f})}_{\mathbf{i}}(\mathbf{i}\not{D}_{\mathbf{ij}} - \mathbf{m_f}\,\delta_{\mathbf{ij}})\Psi^{(\mathbf{f})}_{\mathbf{j}}$$

gluon field strength
a = 1,…,8

color index
i = 1,2,3

one more look at the QCD Lagrangian

- Yang and Mills proposed in 1954 that the local "phase rotation" in QED could be generalized to non Abelian groups such as SU(3)

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{\mathbf{a}}^{\mu\nu}\mathbf{F}^{\mathbf{a}}_{\mu\nu} + \sum_{\mathbf{f}} \bar{\mathbf{\Psi}}_{\mathbf{i}}^{(\mathbf{f})}(\mathbf{i}\not{\mathbf{D}}_{\mathbf{ij}} - \mathbf{m_f}\,\delta_{\mathbf{ij}})\mathbf{\Psi}_{\mathbf{j}}^{(\mathbf{f})}$$

gluon field strength
a = 1,…,8

color index
i = 1,2,3

- color plays a crucial role (unlike QCD, field strength not gauge invariant)

$$\mathbf{F}^{\mathbf{a}}_{\mu\nu} = \partial_\mu \mathbf{A}^{\mathbf{a}}_\nu - \partial_\nu \mathbf{A}^{\mathbf{a}}_\mu - \mathbf{g_s}\mathbf{f}^{\mathbf{abc}}\mathbf{A}^{\mathbf{b}}_\mu \mathbf{A}^{\mathbf{c}}_\mu$$

QED like but field
carries color charge

one more look at the QCD Lagrangian

- Yang and Mills proposed in 1954 that the local "phase rotation" in QED could be generalized to non Abelian groups such as SU(3)

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\Psi}_i^{(f)} (i \not{D}_{ij} - m_f \delta_{ij}) \Psi_j^{(f)}$$

gluon field strength $a = 1,\ldots,8$

color index $i = 1,2,3$

- color plays a crucial role (unlike QCD, field strength not gauge invariant)

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\mu$$

QED like but field carries color charge

non Abelian part gives rise to **gluon self interactions**

one more look at the QCD Lagrangian

- Yang and Mills proposed in 1954 that the local "phase rotation" in QED could be generalized to non Abelian groups such as SU(3)

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{a}^{\mu\nu}\mathbf{F}_{\mu\nu}^{a} + \sum_{f} \bar{\Psi}_{i}^{(f)}(i\not{D}_{ij} - m_f\,\delta_{ij})\Psi_{j}^{(f)}$$

gluon field strength $a = 1,\dots,8$

color index $i = 1,2,3$

- color plays a crucial role (unlike QCD, field strength not gauge invariant)

$$\mathbf{F}_{\mu\nu}^{a} = \partial_\mu \mathbf{A}_\nu^a - \partial_\nu \mathbf{A}_\mu^a - g_s f^{abc} \mathbf{A}_\mu^b \mathbf{A}_\mu^c$$

QED like but field carries color charge

non Abelian part gives rise to **gluon self interactions**

also in the interaction "covariant derivative"

$$(\mathbf{D}_\mu)_{ij} = \partial_\mu\,\delta_{ij} + i g_s\,(t^a)_{ij}\,\mathbf{A}_\mu^a$$

8 generators

one more look at the QCD Lagrangian

- Yang and Mills proposed in 1954 that the local "phase rotation" in QED could be generalized to non Abelian groups such as SU(3)

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\Psi}_i^{(f)} (i \not{D}_{ij} - m_f \, \delta_{ij}) \Psi_j^{(f)}$$

gluon field strength $a = 1,\ldots,8$

color index $i = 1,2,3$

- color plays a crucial role (unlike QCD, field strength not gauge invariant)

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\mu$$

QED like but field carries color charge

non Abelian part gives rise to **gluon self interactions**

also in the interaction "covariant derivative"

$$(D_\mu)_{ij} = \partial_\mu \, \delta_{ij} + i g_s \, (t^a)_{ij} \, A^a_\mu$$

8 generators

- QCD interaction is flavor blind
- **coupling g_s is the only parameter** (masses have e-w origin)

take home message for part I
the foundations

QCD is based on a simple Lagrangian but has a rich phenomenology

QCD is based on the non Abelian gauge group SU(3)

- number of colors and group structure can be tested experimentally
- concept of local gauge invariance dictates interactions
- similarities to QED, yet profound differences (and more to come)
- color leads to self-interactions between "force carrying" gluons
- perturbation theory can be based on a short list of Feynman rules

color algebra decouples and can be performed separately

- color factors can be expressed in terms of two Casimirs: C_A and C_F