Non-perturbative QCD for Nuclear Physicists

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LECTURE III

Lecture III:

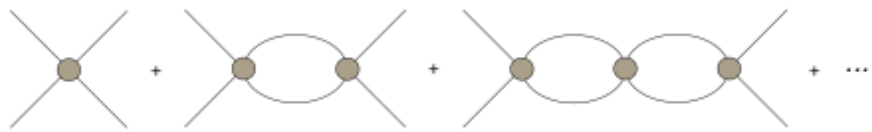
- Contact EFT
- Physics

Contact EFT of interacting fermions
(nucleons or atoms)

☆ Consider NR scattering theory in d=4

$$S = e^{i\delta(p)} = 1 + i\frac{Mp}{2\pi}\mathcal{A}_2(p)$$

$$\mathcal{A}_2(p) = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$ =



Assume finite range interactions: *Effective Range Theory*

$$p\cot\delta(p) = -\frac{1}{a} + \frac{1}{2}\Lambda^2\sum_{n=0}^{\infty} r_n\left(\frac{p^2}{\Lambda^2}\right)^{n+1} = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + \ldots$$

Generally two scenarios:

"Natural" $|a| \sim \Lambda^{-1} \quad , \quad |r_n| \sim \Lambda^{-1}$

$$\mathcal{A}_2 = -\frac{4\pi a}{M}[1 - iap + (ar_0/2 - a^2)p^2 + O(p^3/\Lambda^3)]$$

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$$\mathcal{A}_2 = -\frac{4\pi a}{M}[1 - iap + (ar_0/2 - a^2)p^2 + O(p^3/\Lambda^3)]$$

“Unnatural” $|a| \gg \Lambda^{-1} \quad , \quad |r_n| \sim \Lambda^{-1}$

$$\mathcal{A}_2 = -\frac{4\pi}{M}\frac{1}{(1/a + ip)}[1 + \frac{r_0/2}{(1/a + ip)}p^2 + \frac{(r_0/2)^2}{(1/a + ip)^2}p^4 + \frac{(r_1/2\Lambda^2)}{(1/a + ip)}p^4 + \ldots]$$

Interacting Fermions (nucleons or atoms)

Assume: finite range interaction in four space-time dimensions

$$\mathcal{L}_{EFT} = \quad N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N + C_0 (N^\dagger N)^2 + \frac{C_2}{8} \left[(NN)^\dagger (N \overleftrightarrow{\nabla}^2 N) + h.c \right] + \ldots$$

Isospin and Parity invariant

Interacting Fermions (nucleons or atoms)

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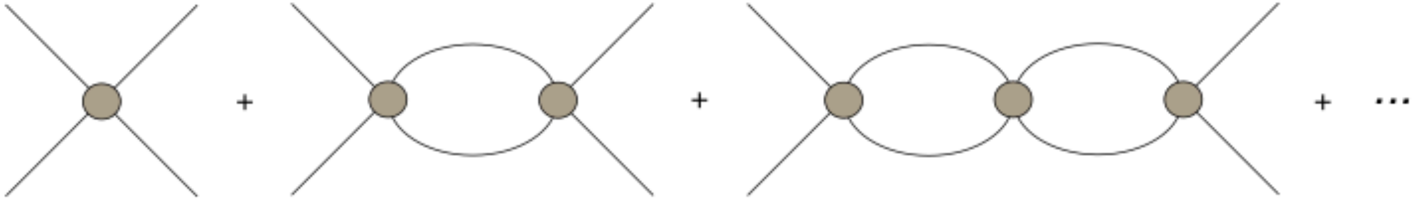
Isospin and Parity invariant

Note: can choose alternate basis:

$$C_0 \, \left(N^T \mathcal{P}_x N\right)^\dagger \left(N^T \mathcal{P}_x N\right)$$

Projection operator onto given channel: $\mathcal{P}_x$

Can solve exactly (formally)

$\mathcal{A}_2 =$ 

$$\mathcal{A}_2(p) \;=\; -\frac{\sum C_{2n}\, p^{2n}}{1 - I_0(p)\sum C_{2n}\, p^{2n}}$$

Dimensional regularization: $\epsilon \equiv 4 - D$

$$\begin{aligned} I_n \equiv \quad & i(\mu/2)^{\epsilon}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\,\frac{\mathbf{q}^{2n}}{\left(E/2 + q_0 - \frac{\mathbf{q}^2}{2M} + i\epsilon\right)\left(E/2 - q_0 - \frac{\mathbf{q}^2}{2M} + i\epsilon\right)} \\ = \quad & M(\mu/2)^{\epsilon}\int \frac{\mathrm{d}^{(D-1)}\mathbf{q}}{(2\pi)^{(D-1)}}\,\mathbf{q}^{2n}\left(\frac{1}{p^2 - \mathbf{q}^2 + i\epsilon}\right) \\ = \quad & -Mp^{2n}(-p^2 - i\epsilon)^{(D-3)/2}\Gamma\left(\frac{3-D}{2}\right)\frac{(\mu/2)^{\epsilon}}{(4\pi)^{(D-1)/2}} \end{aligned}$$

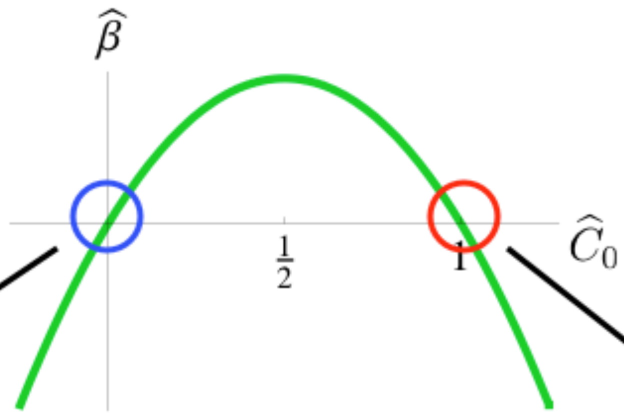
Renormalization group interpretation

Define:

$$\hat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 \;=\; \frac{\mu}{\mu - 1/a}$$

$$\hat{\beta}_0 \;=\; \mu \frac{d}{d\mu} \hat{C}_0 \;=\; -\hat{C}_0(\hat{C}_0 - 1)$$

$$C_{2n} \;\sim\; \frac{4\pi}{M\Lambda} \frac{1}{\Lambda^{2n}}$$



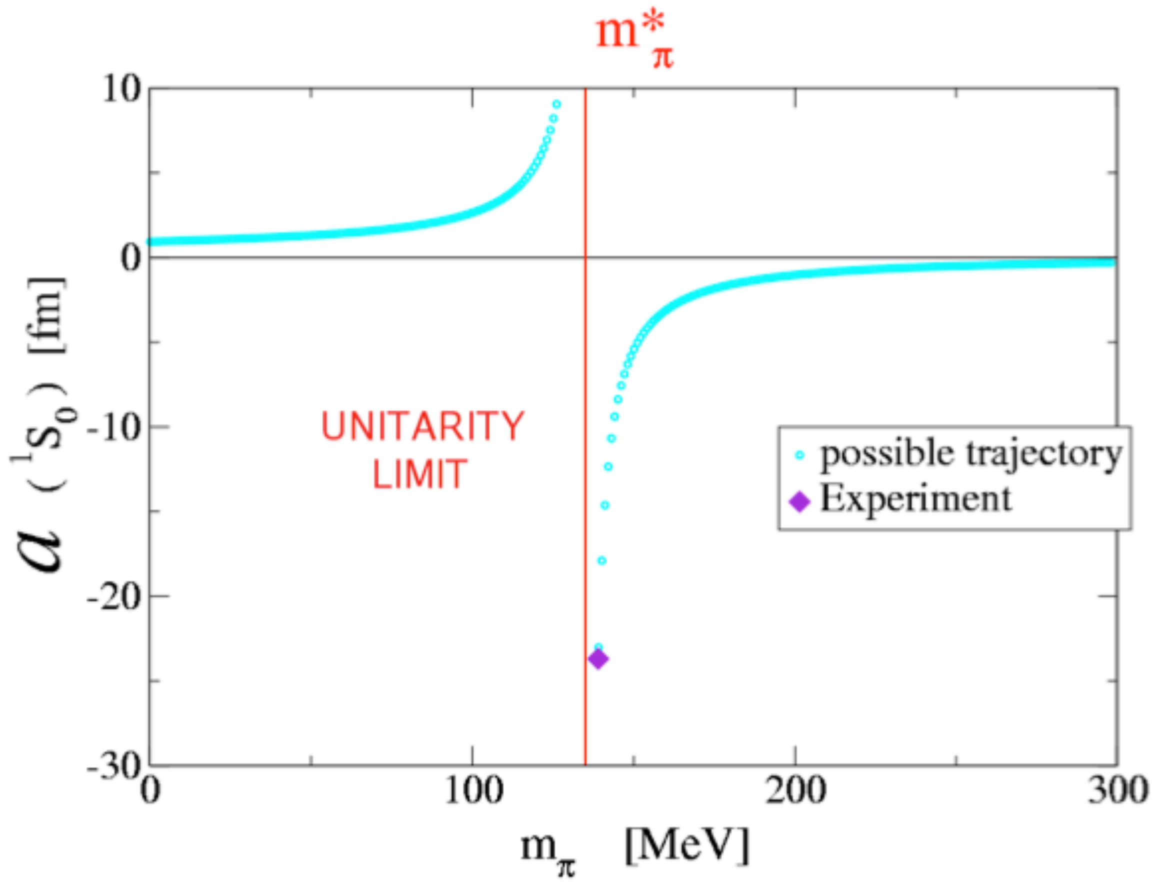
$$C_{2n}(\mu) \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}}$$

Trivial IR fixed point:
"natural case"

Nontrivial UV fixed point:
"unnatural case"

Why is nuclear physics near this UV fixed point??

Lattice QCD can help answer this question!

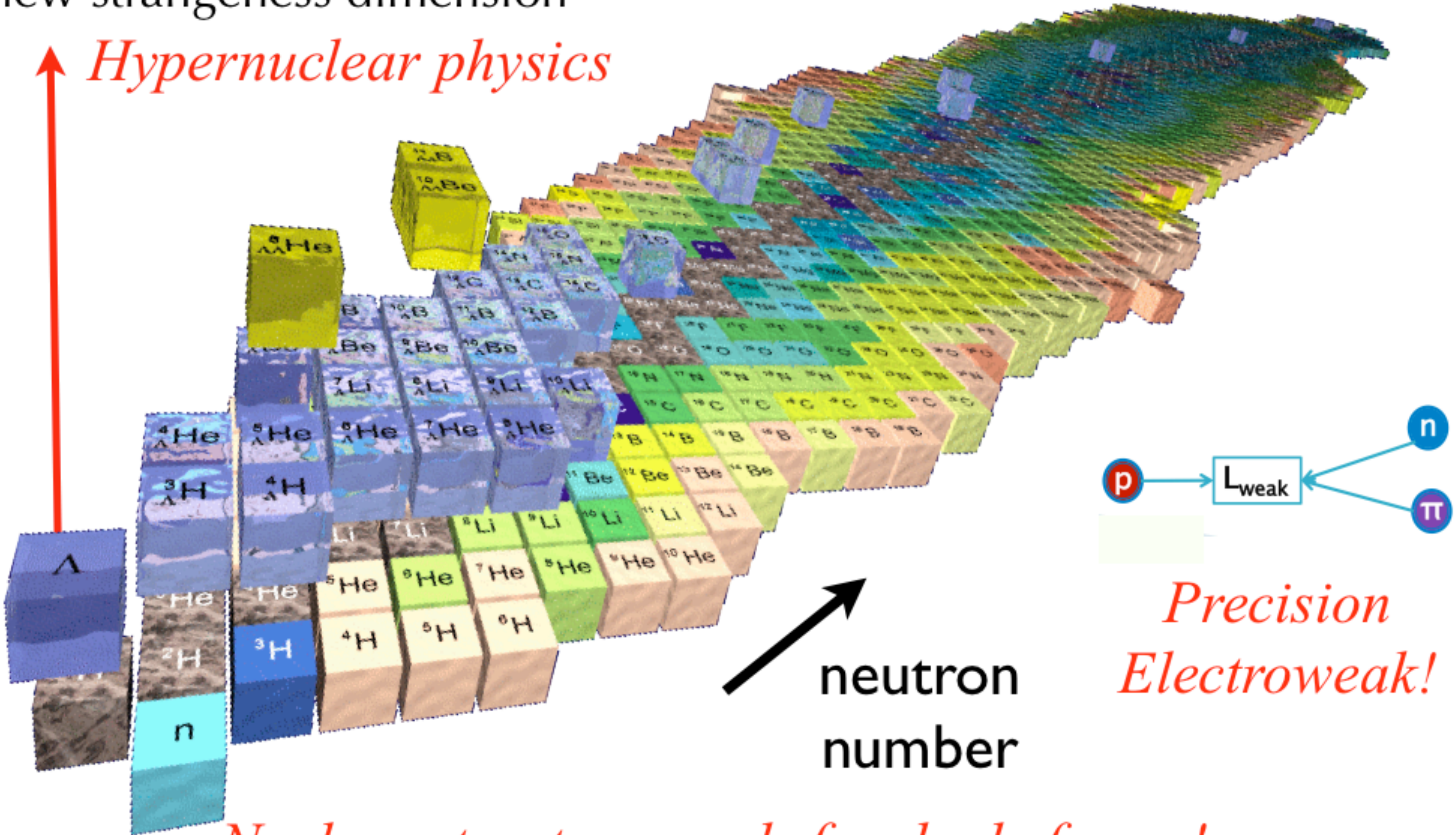


$$a_s^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi} \Lambda_{QCD}$$

★ Interesting physics that is difficult to measure!

new strangeness dimension

Hypernuclear physics



Precision Electroweak!

Nuclear structure needs few-body forces!

$e.g.\ nnn$

★ Interesting physics that is difficult to measure!

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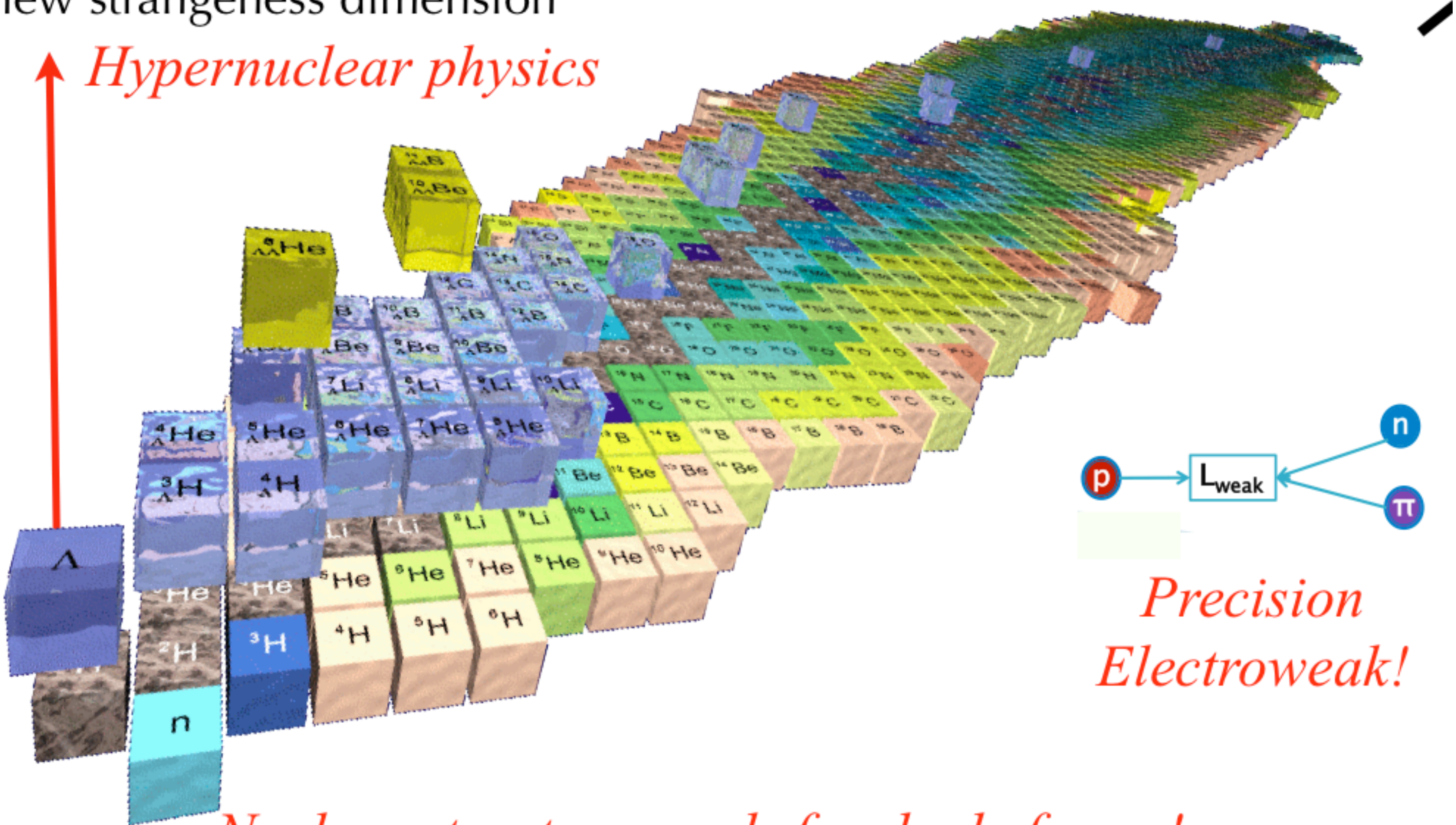
Hypernuclear physics



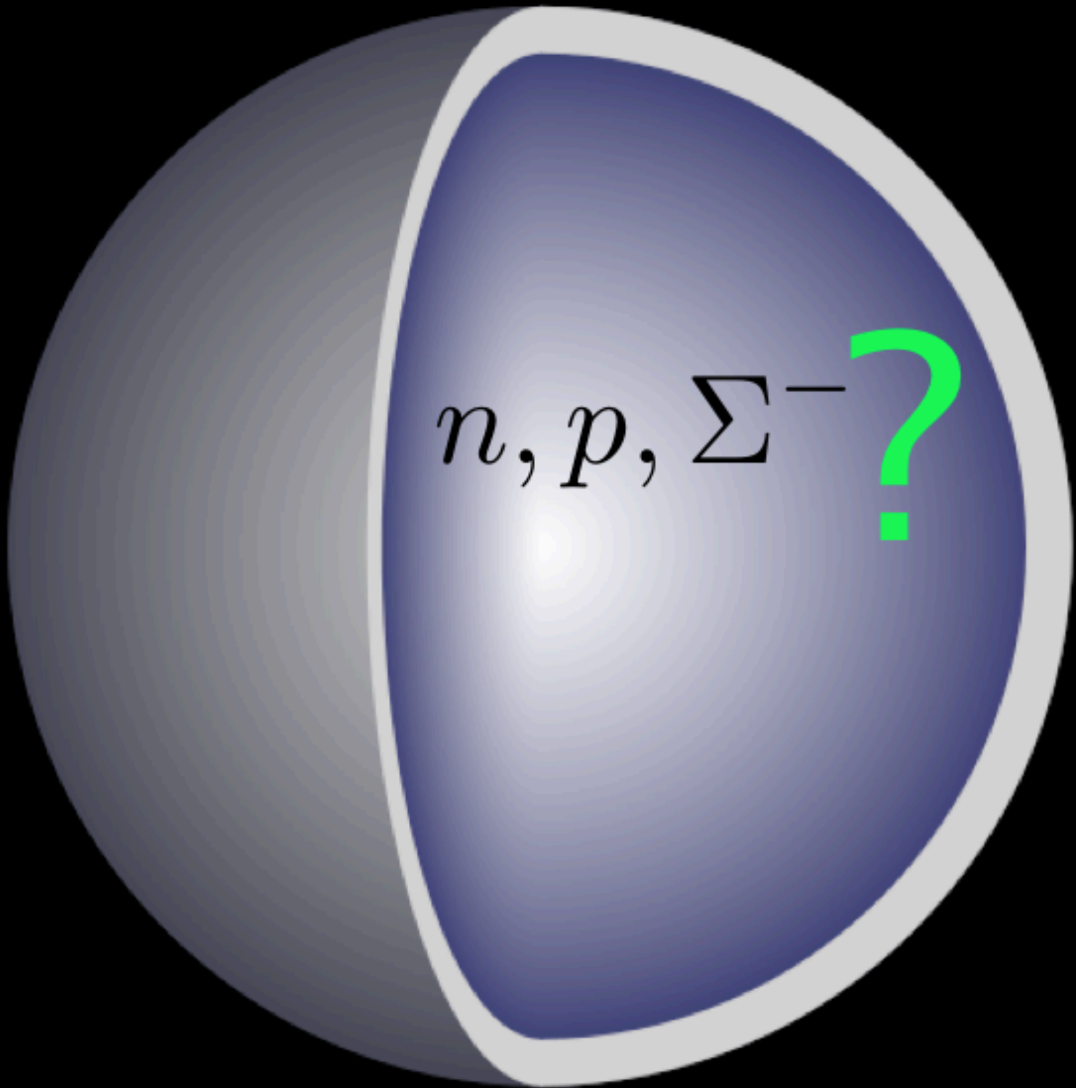
Precision Electroweak!

Nuclear structure needs few-body forces!

$e.g.\ nnn$

<u>Neutron stars</u>: the size of Vancouver but heavier than the sun!

Neutron Star Core



Neutron Star Core

★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s

Nuclear fine-tunings!

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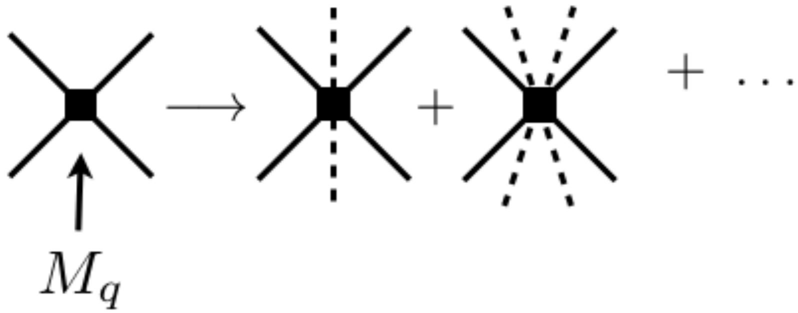
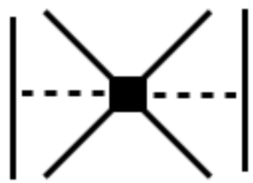
Nuclear fine-tunings!

* Calculation of nuclear forces requires these knobs!





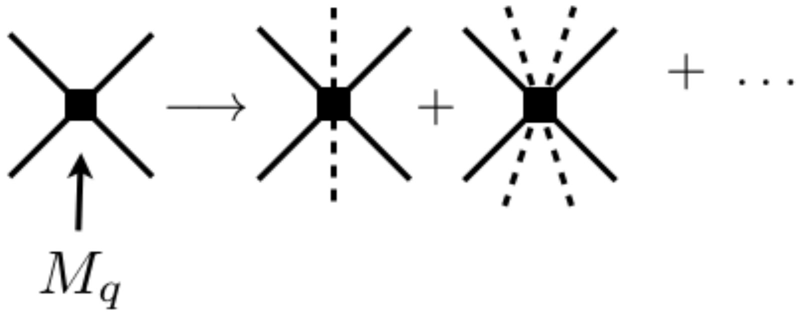
Four-body force

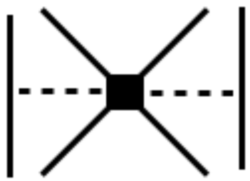
★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s m_c

Nuclear fine-tunings!

✳ Calculation of nuclear forces requires these knobs!





Four-body force

✳ Interactions of nuclei with dark matter

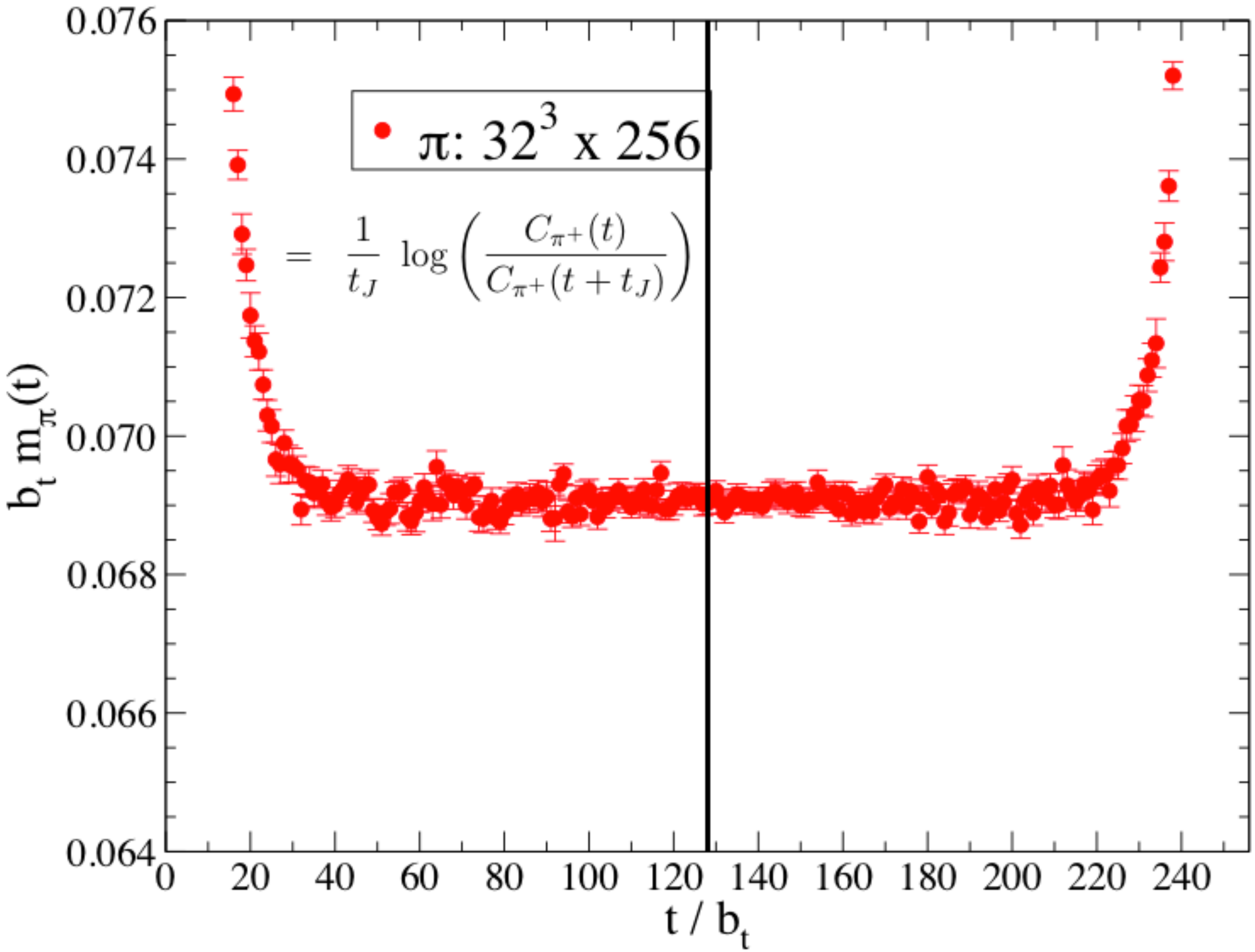
Why is lattice QCD for nuclear physics hard ?

- Signal/noise (sign problem) and statistics
- Number of contractions

[Detmold,Orginos(2012),Doi,Endres(2012)]

Signal/Noise Problem

$$C_{\pi^+}(t) \;=\; \sum_{\mathbf{x}} \langle 0|\; \pi^-(\mathbf{x},t)\; \pi^+(\mathbf{0},0)\; |0\rangle \longrightarrow e^{-m_\pi t} \quad \ldots$$



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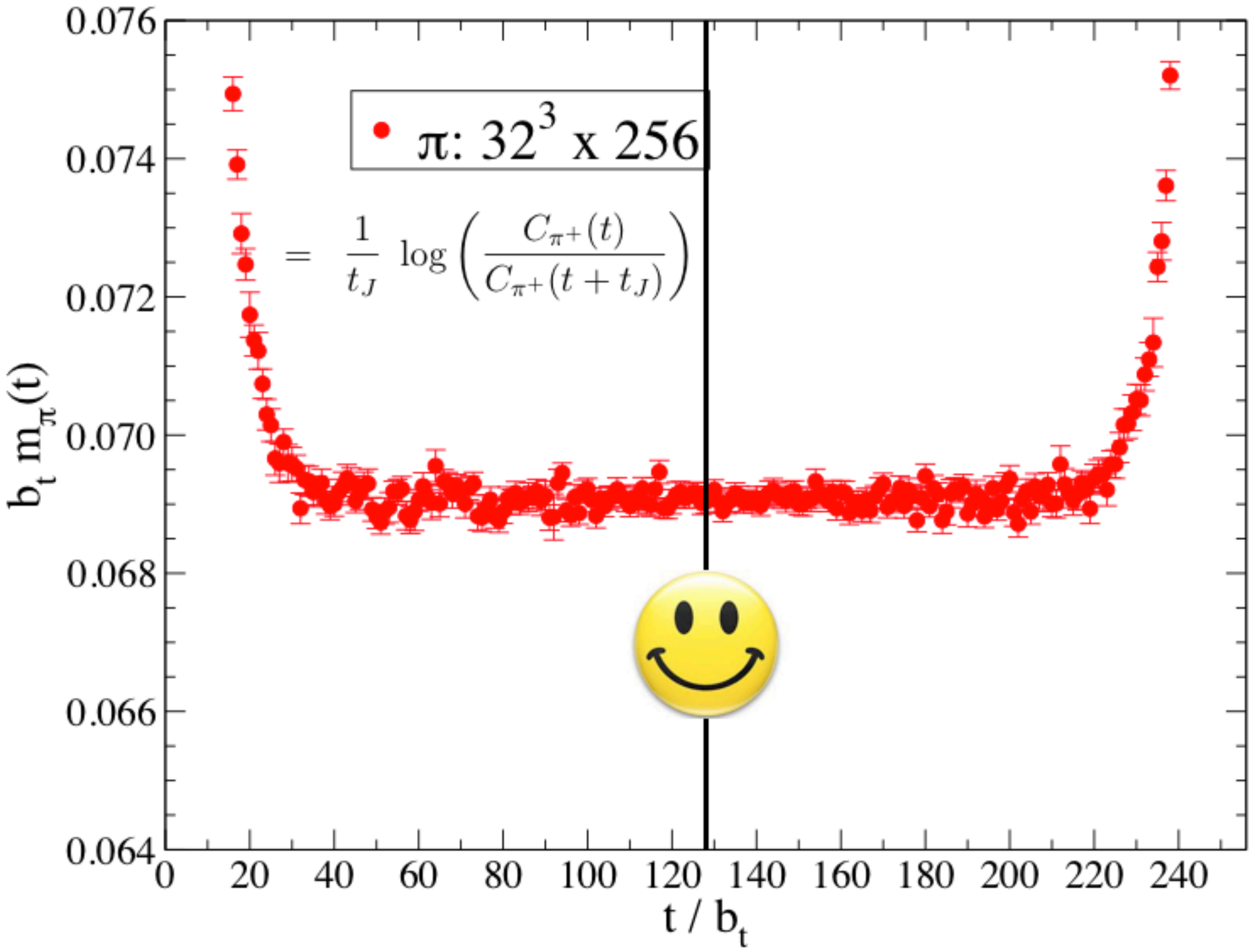
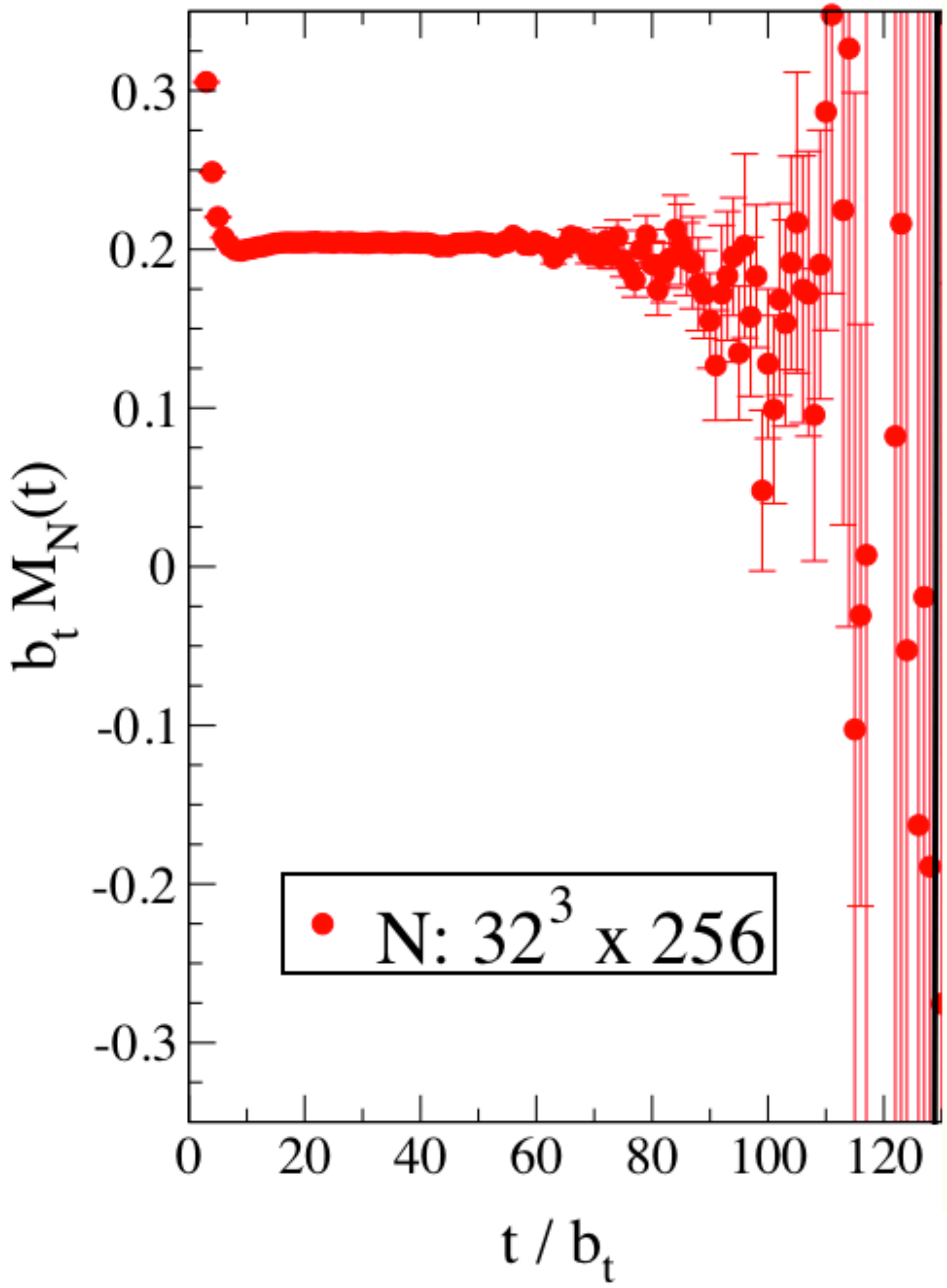
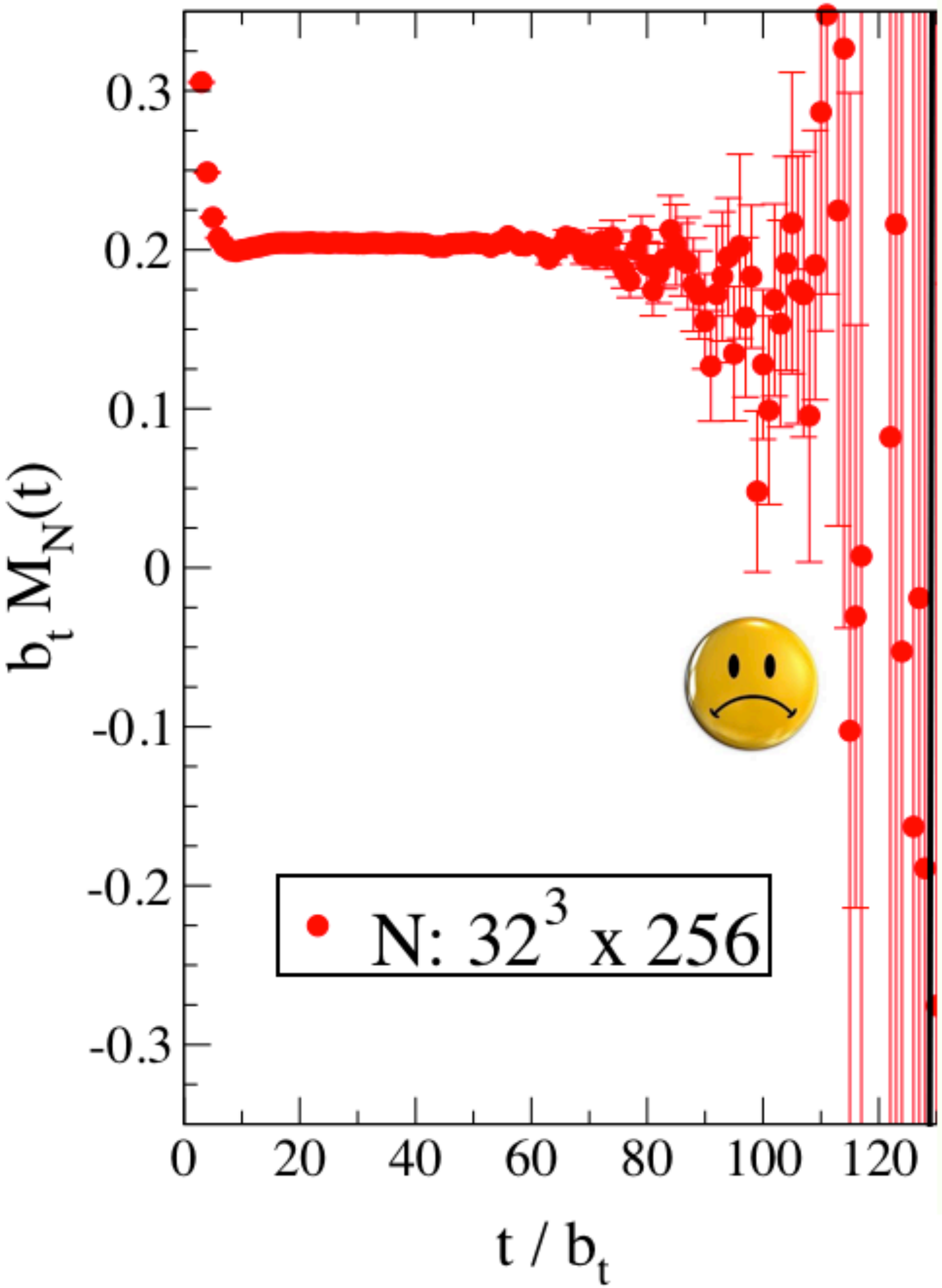


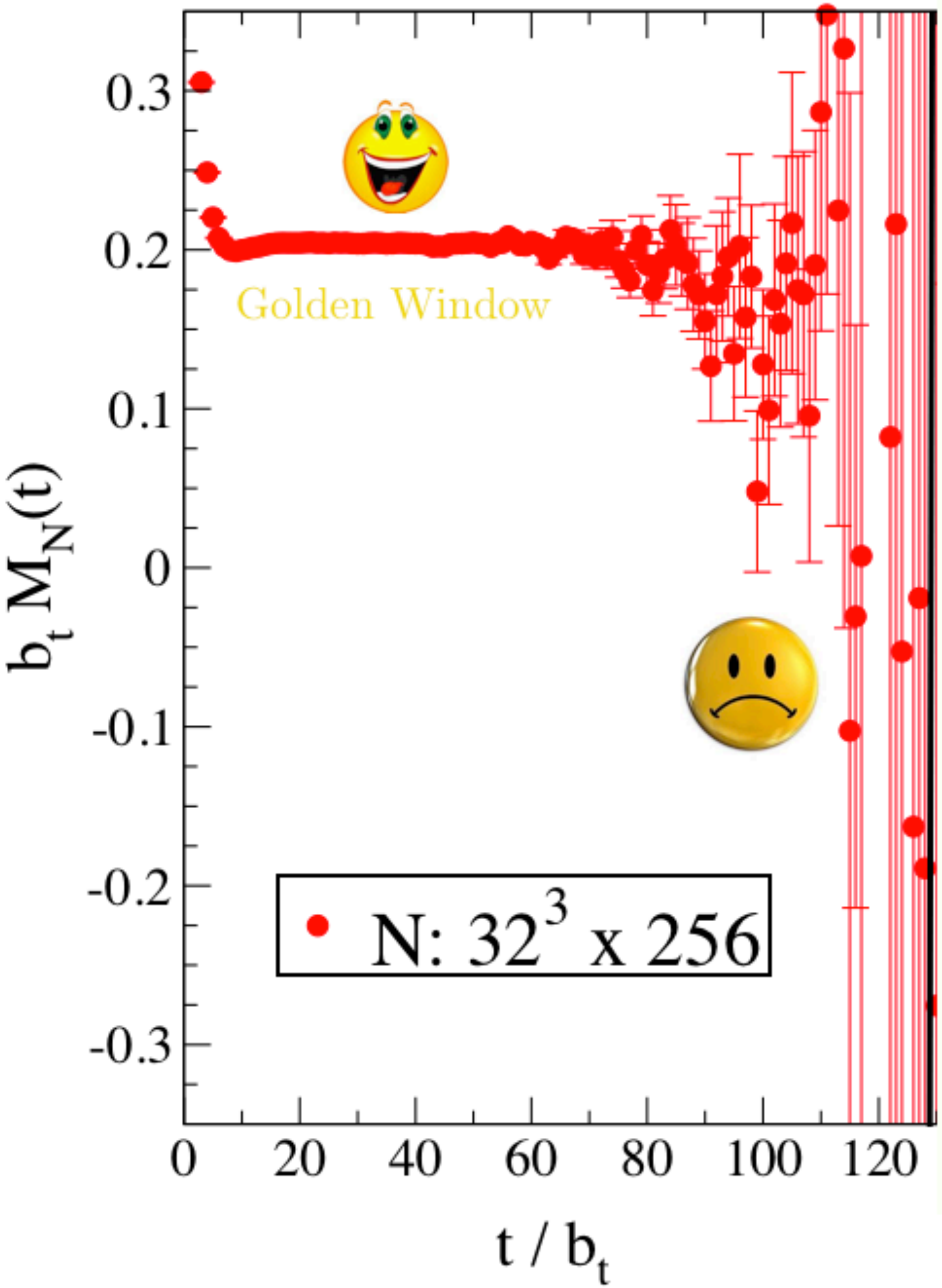
pions are easy! (i.e. cheap)



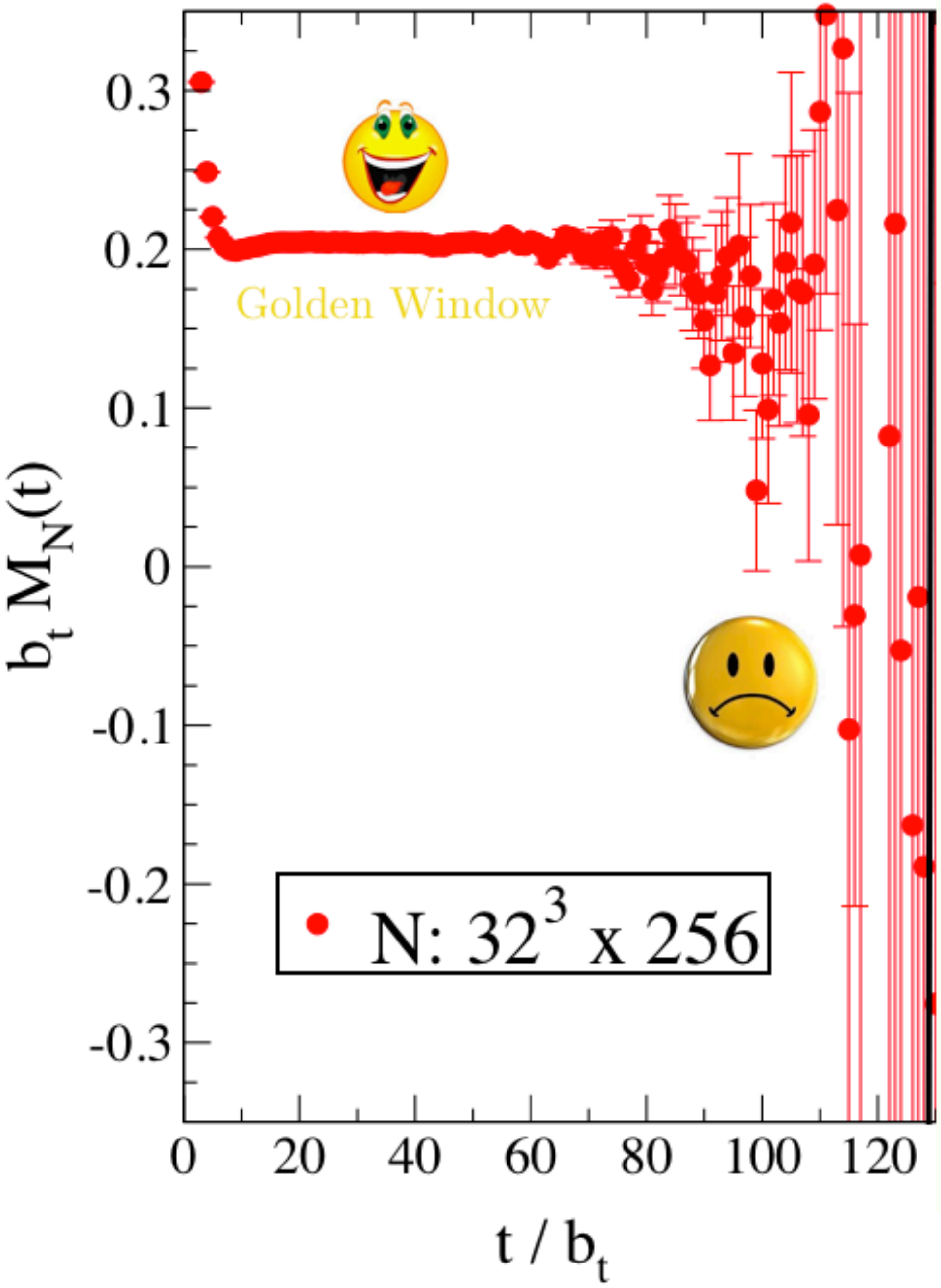
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
N: 32^3 x 256

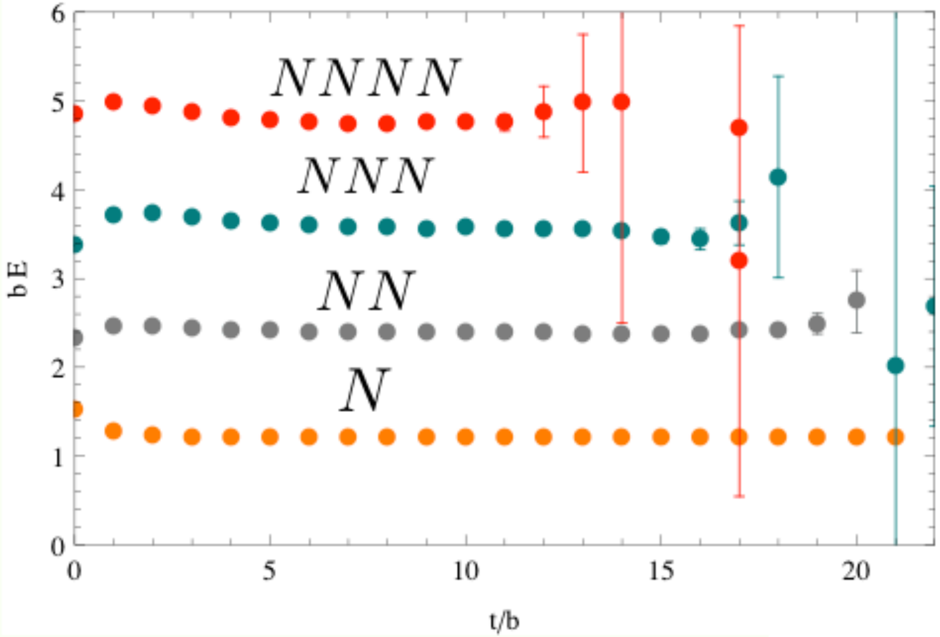


0.3
0.2
0.1
0
-0.1
-0.2
-0.3
$b_t M_N(t)$
0
20
40
60
80
100
120
t / b_t
N: 32^3 x 256



0.3
0.2
0.1
0
-0.1
-0.2
-0.3
$b_t M_N(t)$
Golden Window
N: 32^3 x 256
0
20
40
60
80
100
120
t / b_t

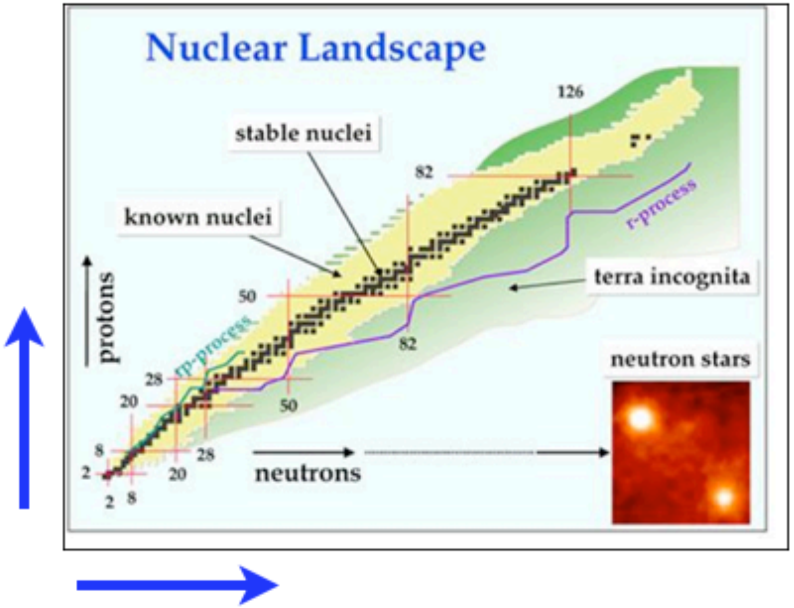


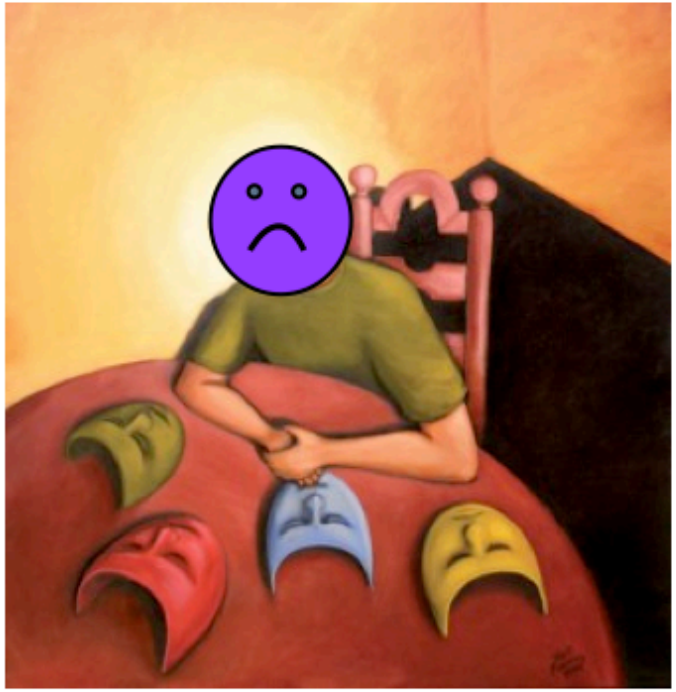


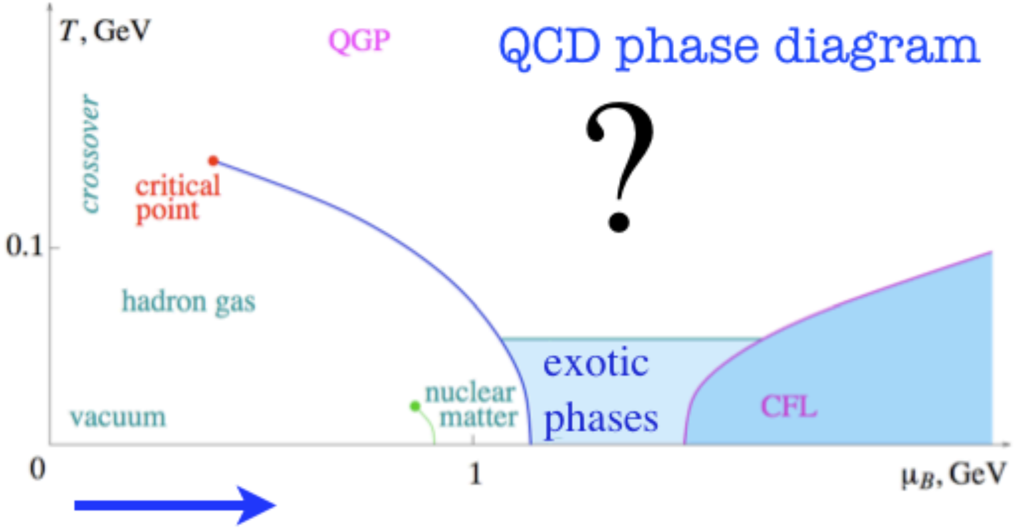
$$\frac{\text{noise}}{\text{signal}} \sim \frac{1}{\sqrt{N}}\, e^{A\left(m_p - \frac{3}{2} m_\pi\right)t}$$

baryons are hard! (i.e. costly)

✓ Signal/noise problem and sign problem are the same!

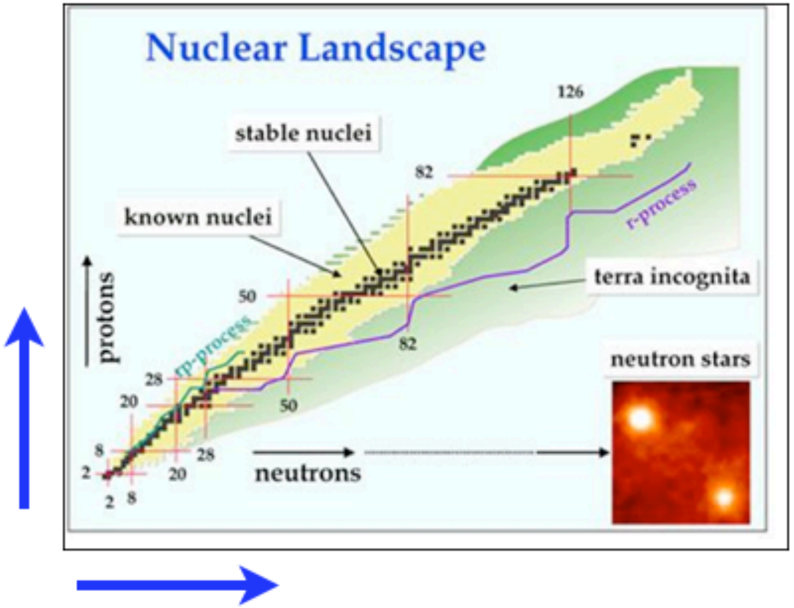


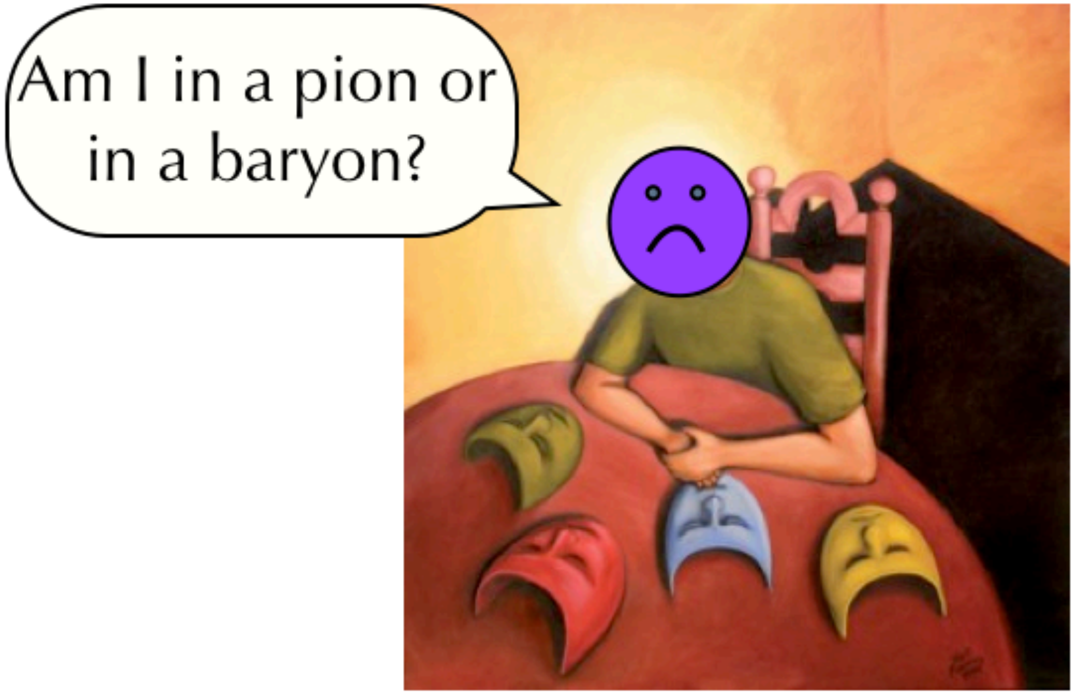


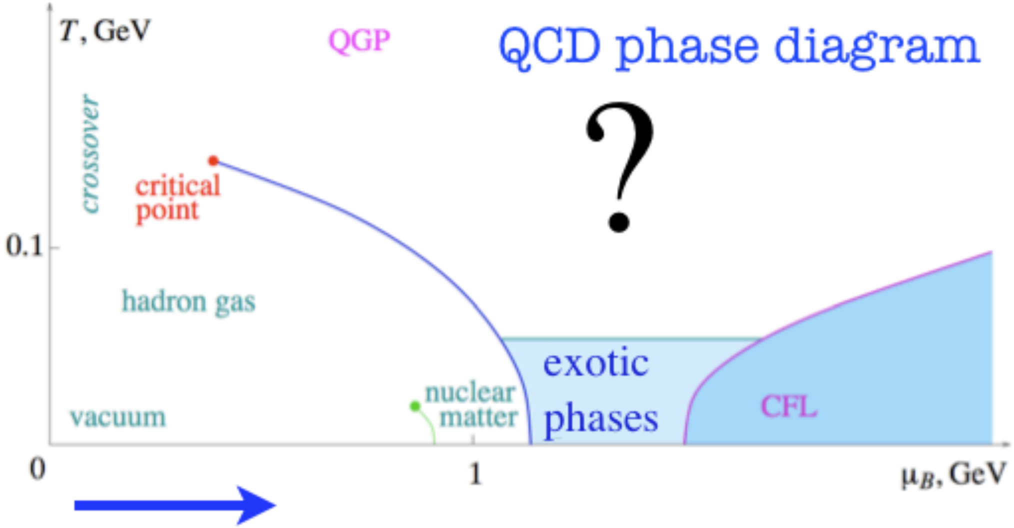


quark identity crisis!

✓ Signal/noise problem and sign problem are the same!



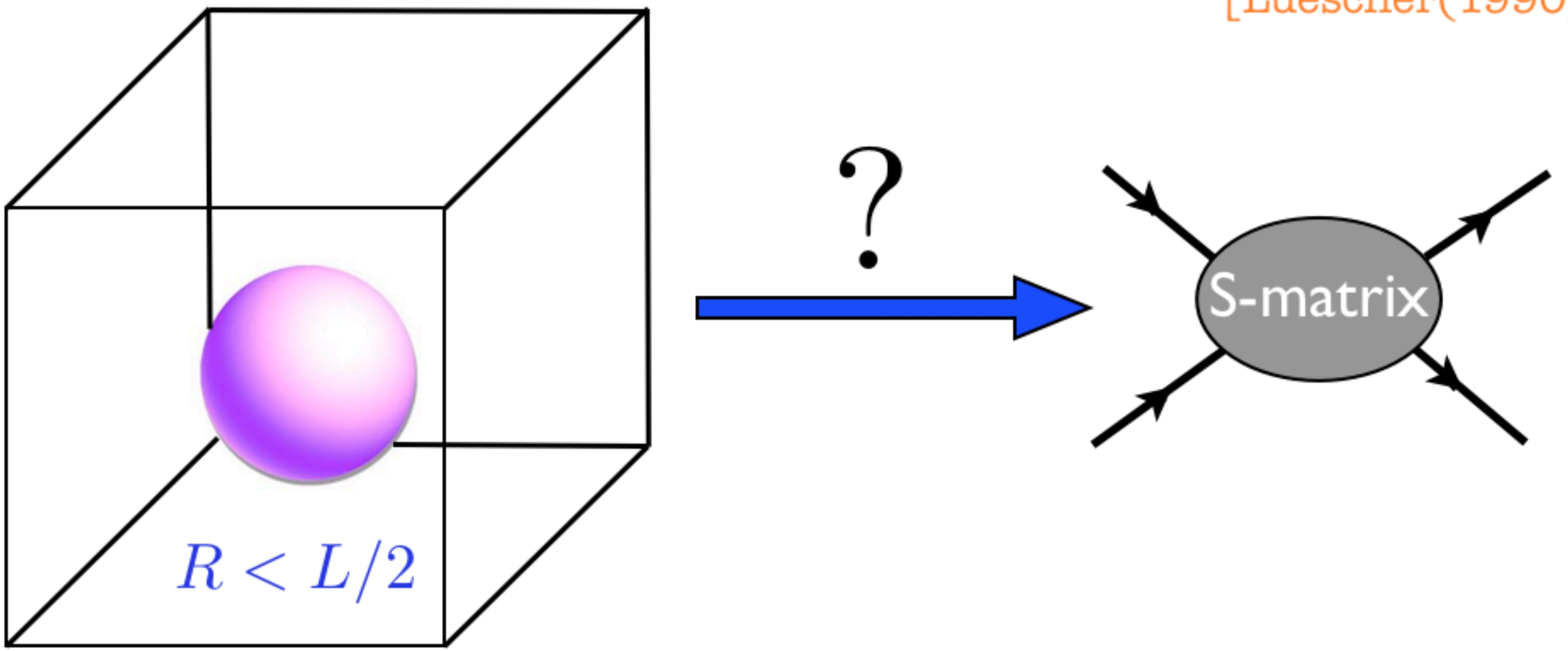




quark identity crisis!

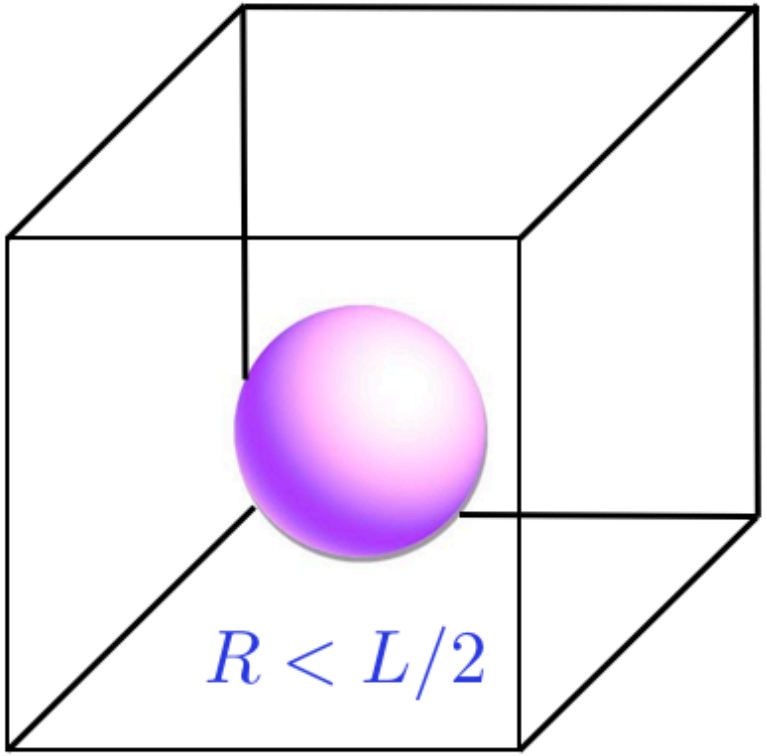
Scattering in a finite volume

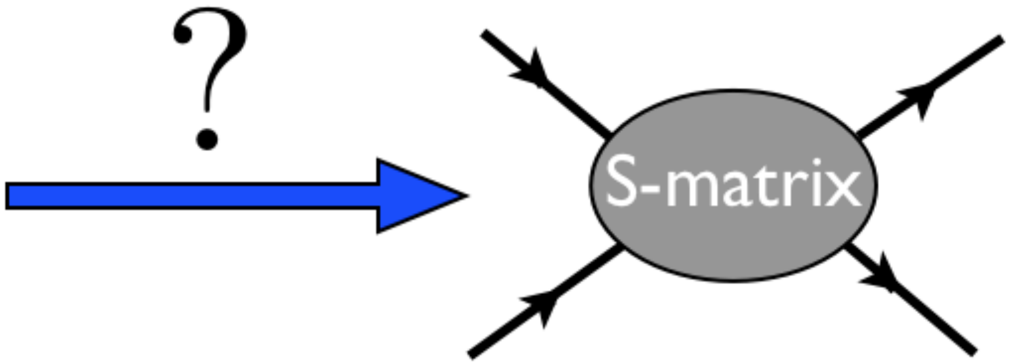
[Luescher(1990)]



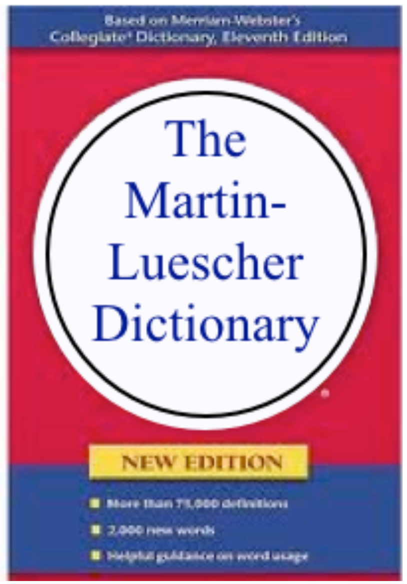
SCATTERING IN A FINITE VOLUME

[Luescher(1990)]





$$q \cot \delta(q) \;=\; \frac{1}{\pi L} \lim_{\Lambda \to \infty} \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - q^2 \left(\frac{L}{2\pi}\right)^2} - 4\pi\Lambda$$



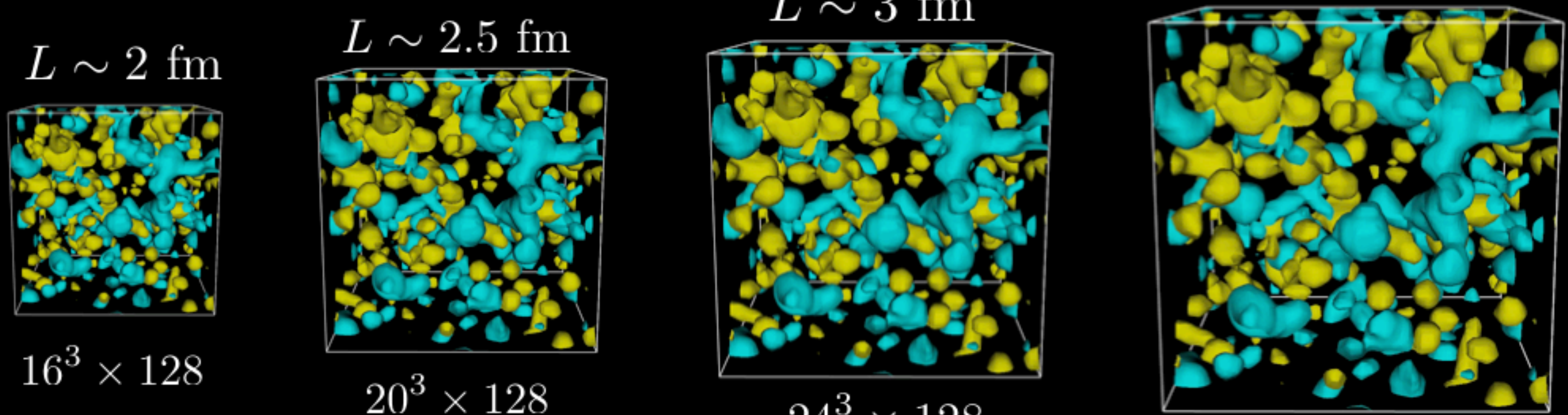
NPLQCD

$N_f = 2 + 1$ *Anisotropic Clover*

Jefferson Lab

$m_\pi \sim 390$ MeV $\qquad b_s \sim 0.123$ fm

$L \sim 2$ fm $\qquad L \sim 2.5$ fm $\qquad L \sim 3$ fm $\qquad L \sim 4$ fm

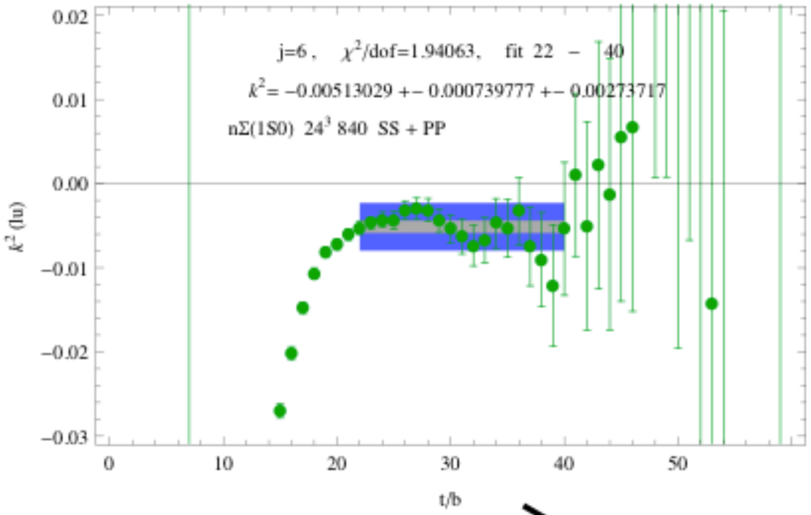


$16^3 \times 128 \qquad 20^3 \times 128 \qquad 24^3 \times 128 \qquad 32^3 \times 256$

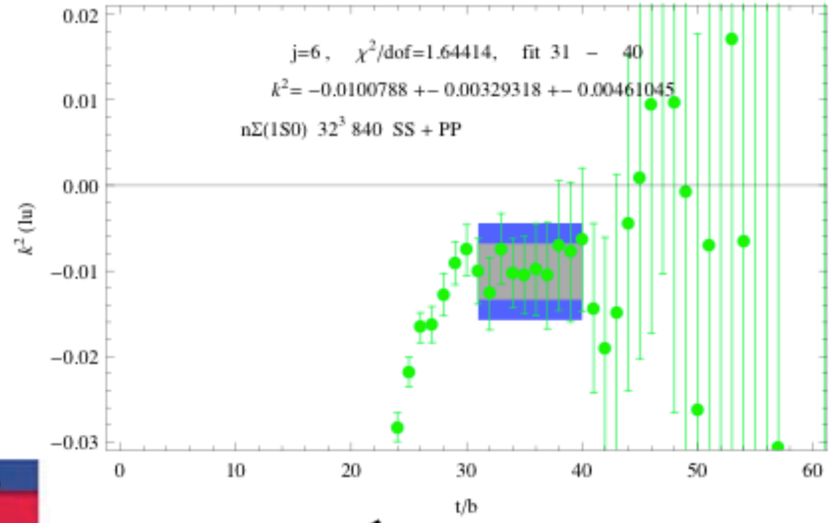
$b_s / b_t \sim 3.5$

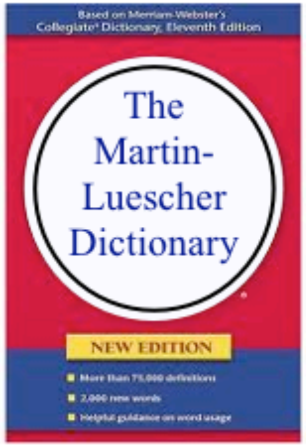
$^1S_0 \; n\Sigma^-$

$24^3 \times 128$



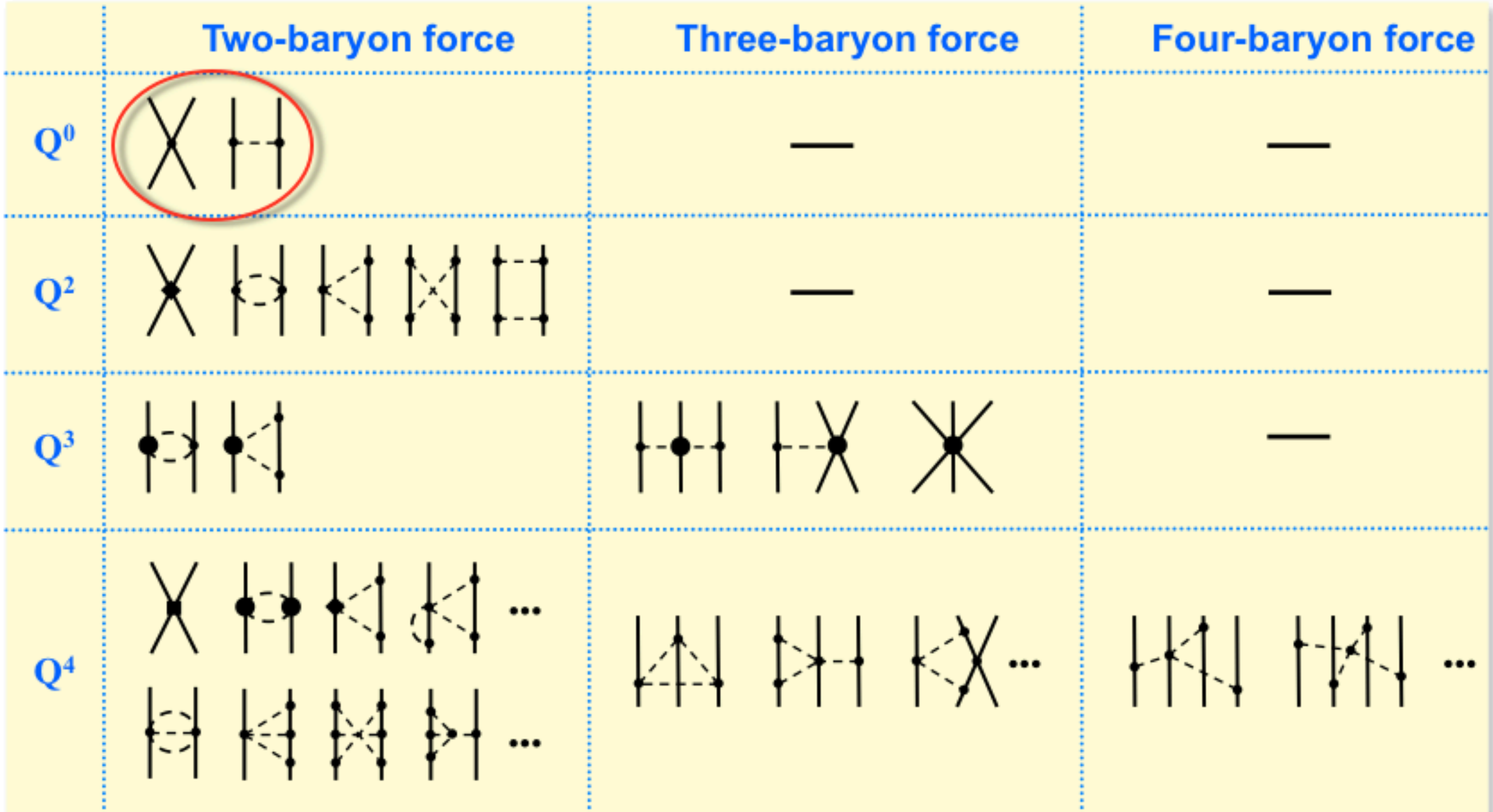
$32^3 \times 256$





$$B_{n\Sigma} \;=\; \frac{\gamma^2}{2\mu_{n\Sigma}} = 25 \pm 9.3 \pm 11 \;\; \mathrm{MeV}$$

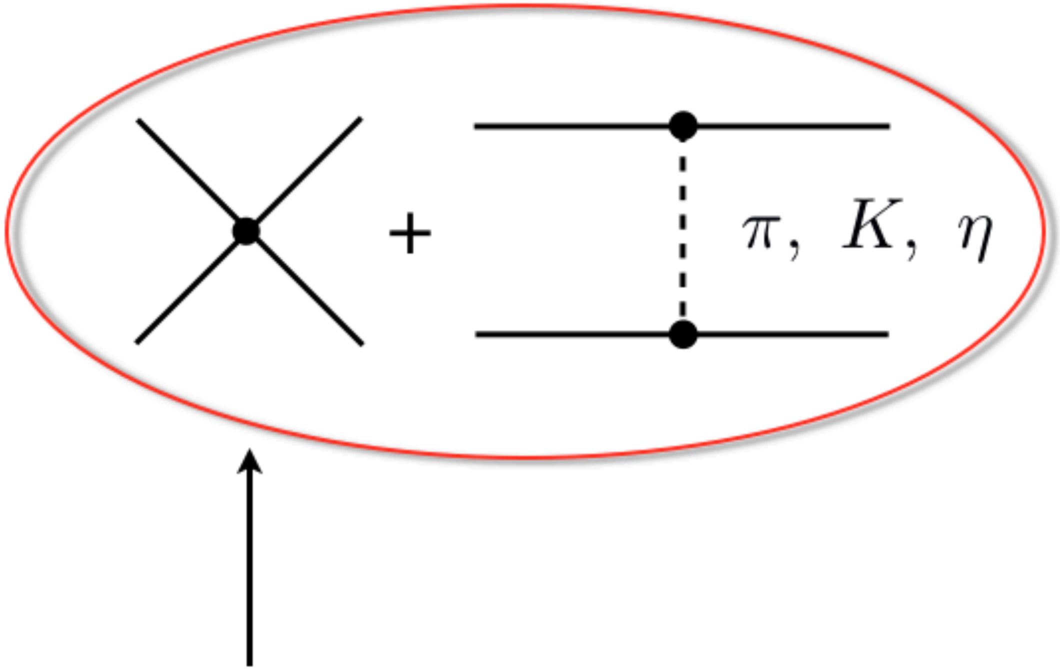
Nuclear Effective Field Theory



2 baryon force ≫ 3 baryon force ≫ 4 baryon force . . .

Match to Effective Field Theory!

LO potential:



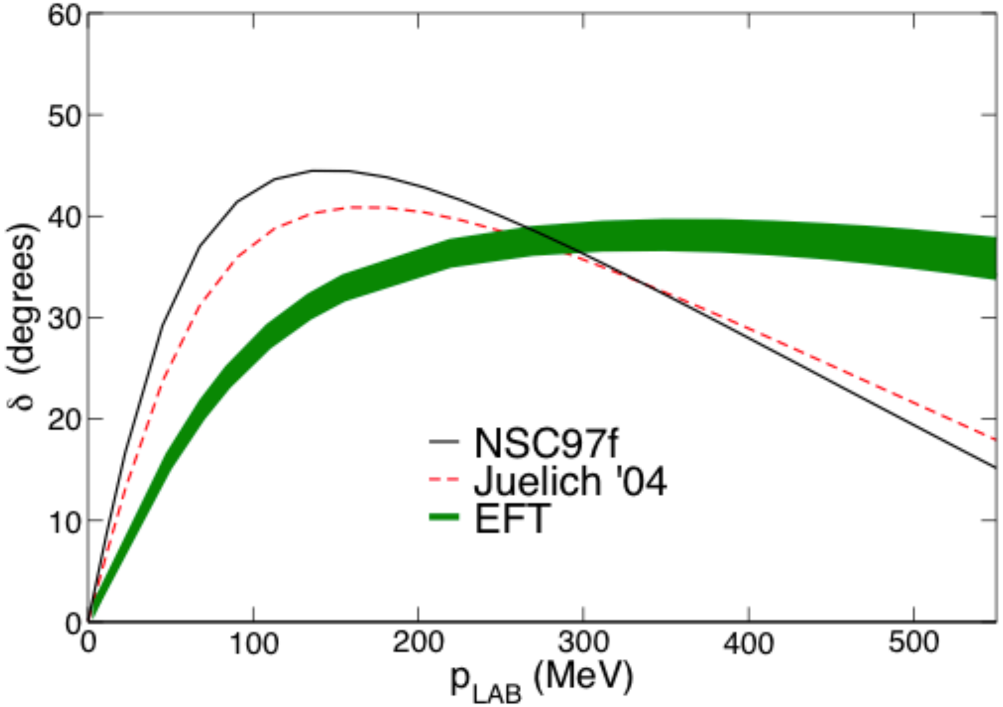
Fit coupling to binding energy: $B_{n\Sigma^-}$

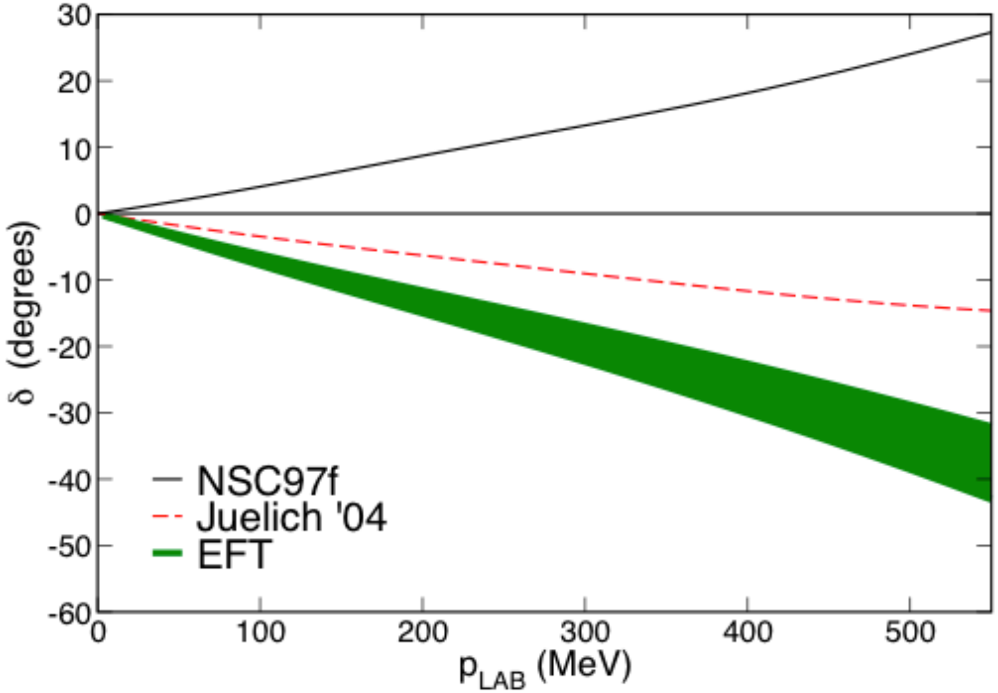
Now we have LO potential at ALL pion masses!

$^{1}S_{0}\ n\Sigma^{-}$

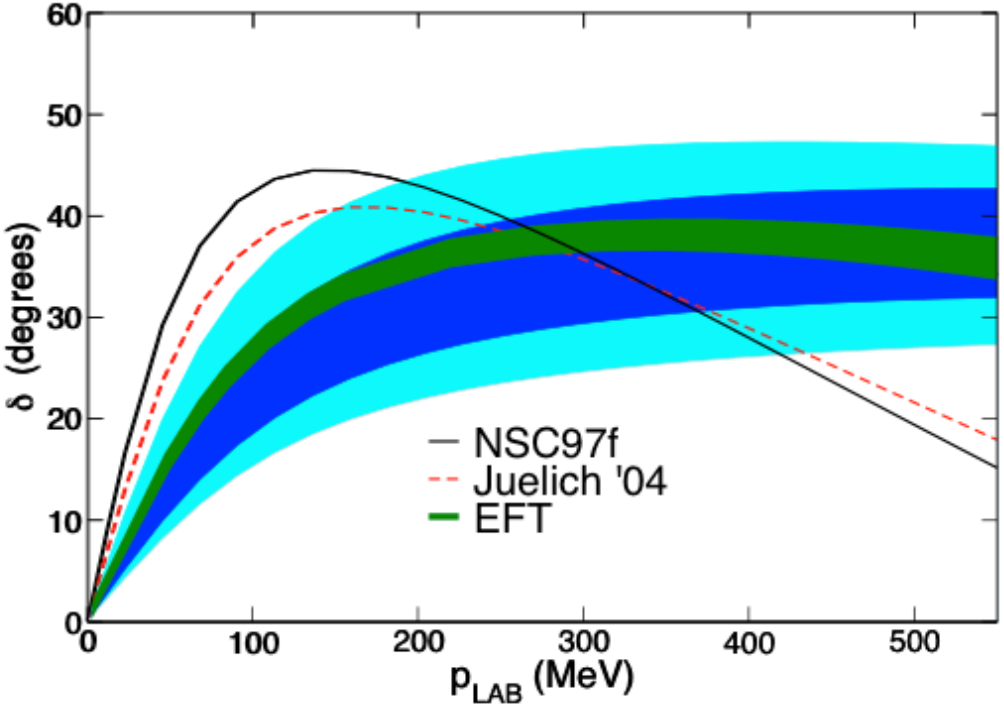
NPLQCD

$^{3}S_{1}\ n\Sigma^{-}$

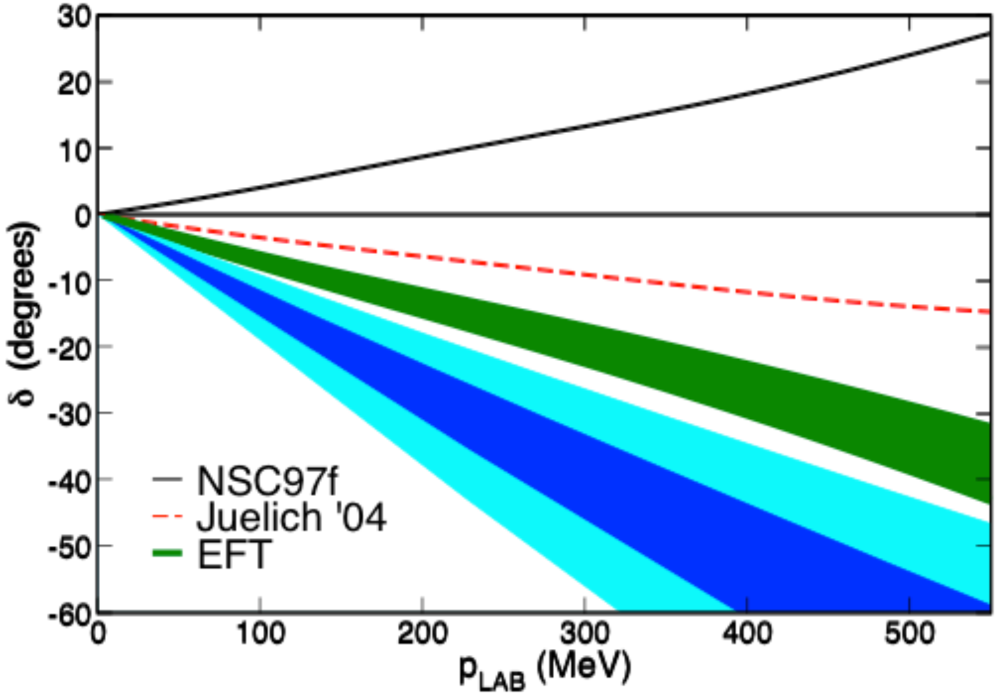




$^1S_0\ n\Sigma^-$



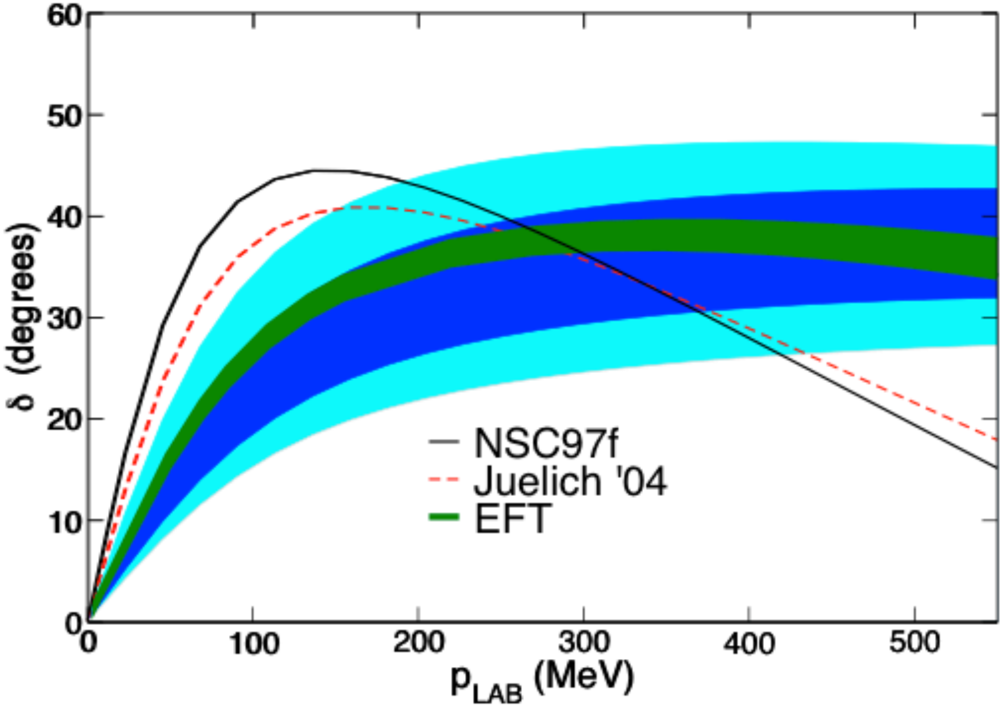
$^3S_1\ n\Sigma^-$

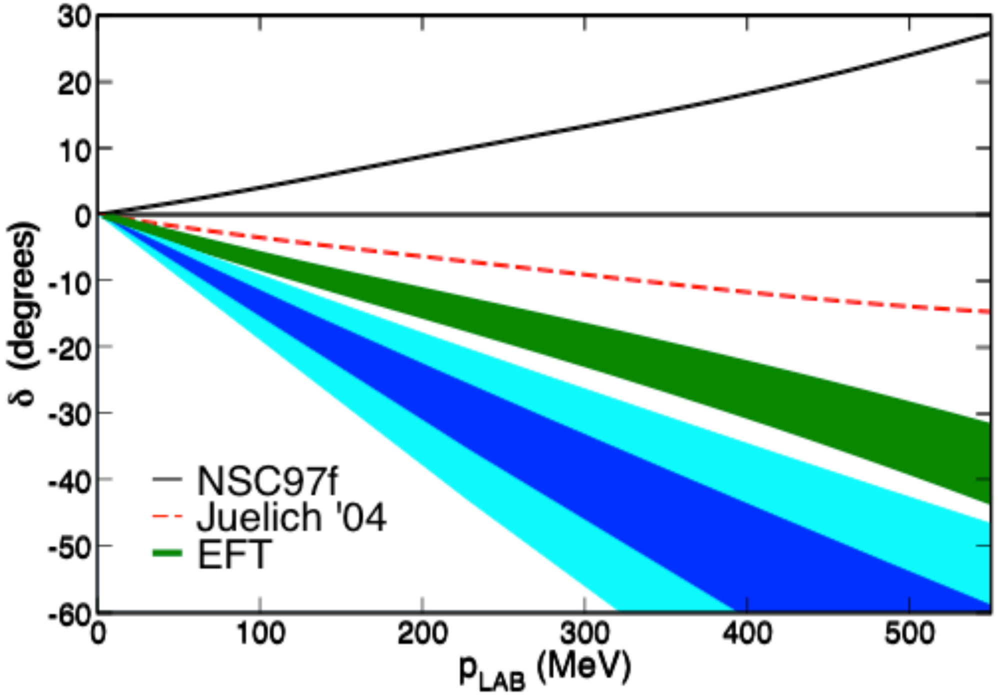


${}^{1}S_{0}\ n\Sigma^{-}$

NPLQCD

${}^{3}S_{1}\ n\Sigma^{-}$

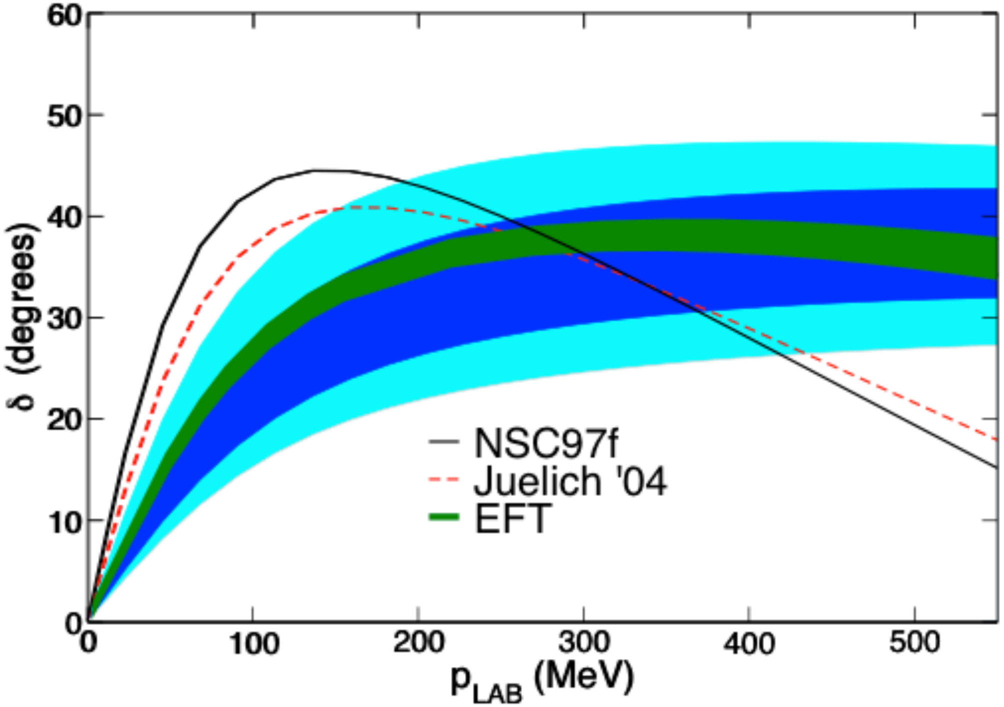


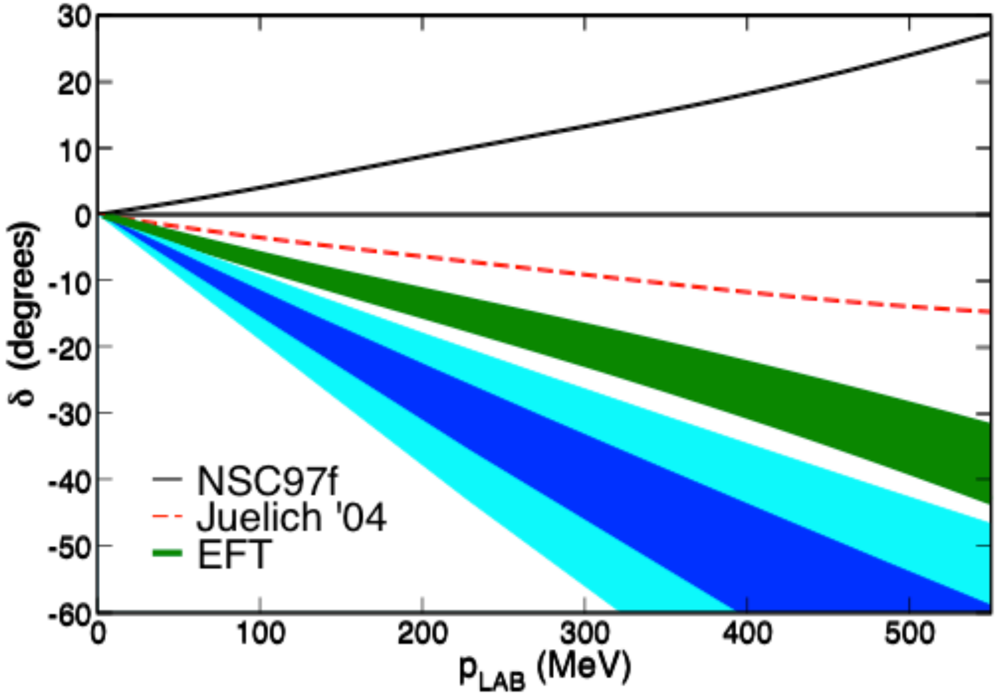


★ First predictions for nuclear physics from lattice QCD

$^1S_0\ n\Sigma^-$

$^3S_1\ n\Sigma^-$





- ★ First predictions for nuclear physics from lattice QCD
- ★ Very nearly competitive with experiment
- ★ Relevant for equation of state of dense matter

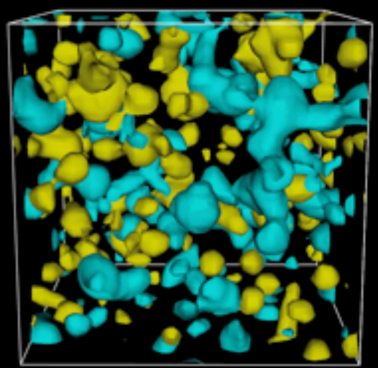
$SU(3)$ *Isotropic Clover*

$N_f = 3$

$m_\pi \sim 800$ MeV

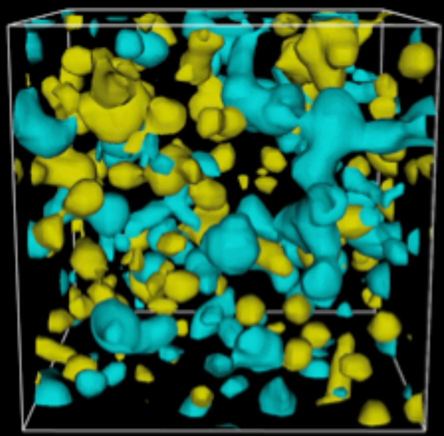
$b \sim 0.145$ fm

$L \sim 3.4$ fm



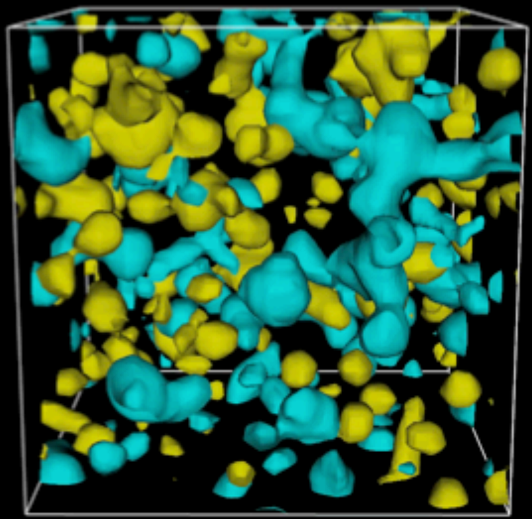
$24^3 \times 48$

$L \sim 4.5$ fm



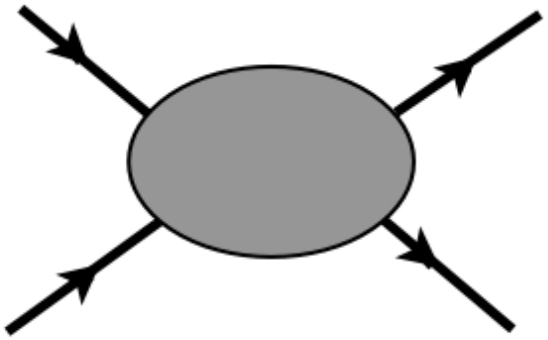
$32^3 \times 48$

$L \sim 6.7$ fm



$48^3 \times 64$

Nucleon-nucleon scattering



$$k\cot\delta = -\frac{1}{a} + \frac{1}{2}r|\mathbf{k}|^2 + P|\mathbf{k}|^4 + \mathcal{O}\left(|\mathbf{k}|^6\right)$$

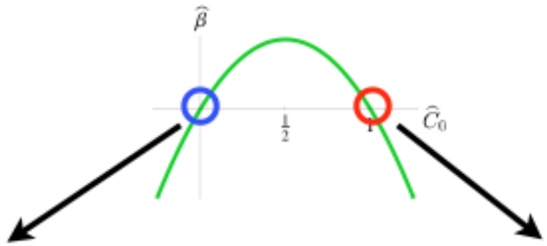
effective range: range of interaction

scattering length: unbounded

EXPERIMENT:

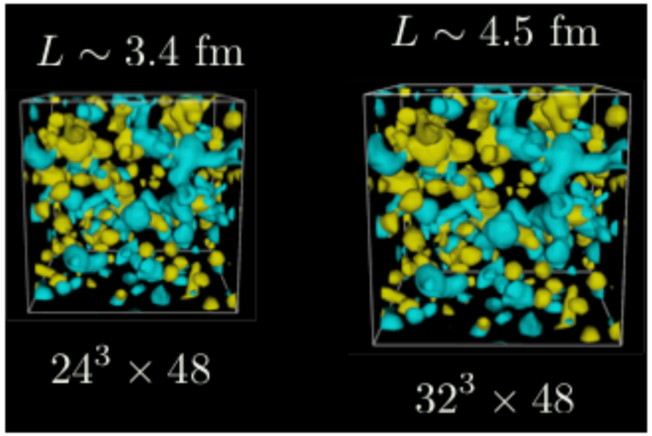
$a^{(^1S_0)} = -23.71$ fm		$a^{(^3S_1)} = 5.43$ fm	
$r^{(^1S_0)} = 2.73$ fm		$r^{(^3S_1)} = 1.75$ fm	

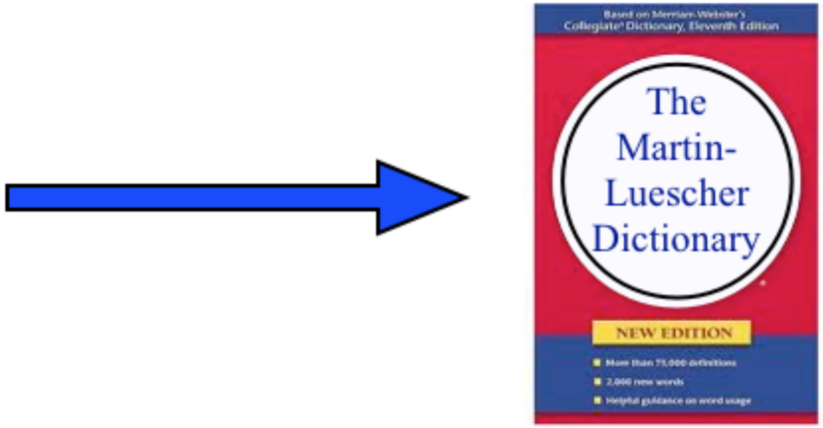
$$a^{(^1S_0)} \gg \Lambda_{QCD}^{-1}$$

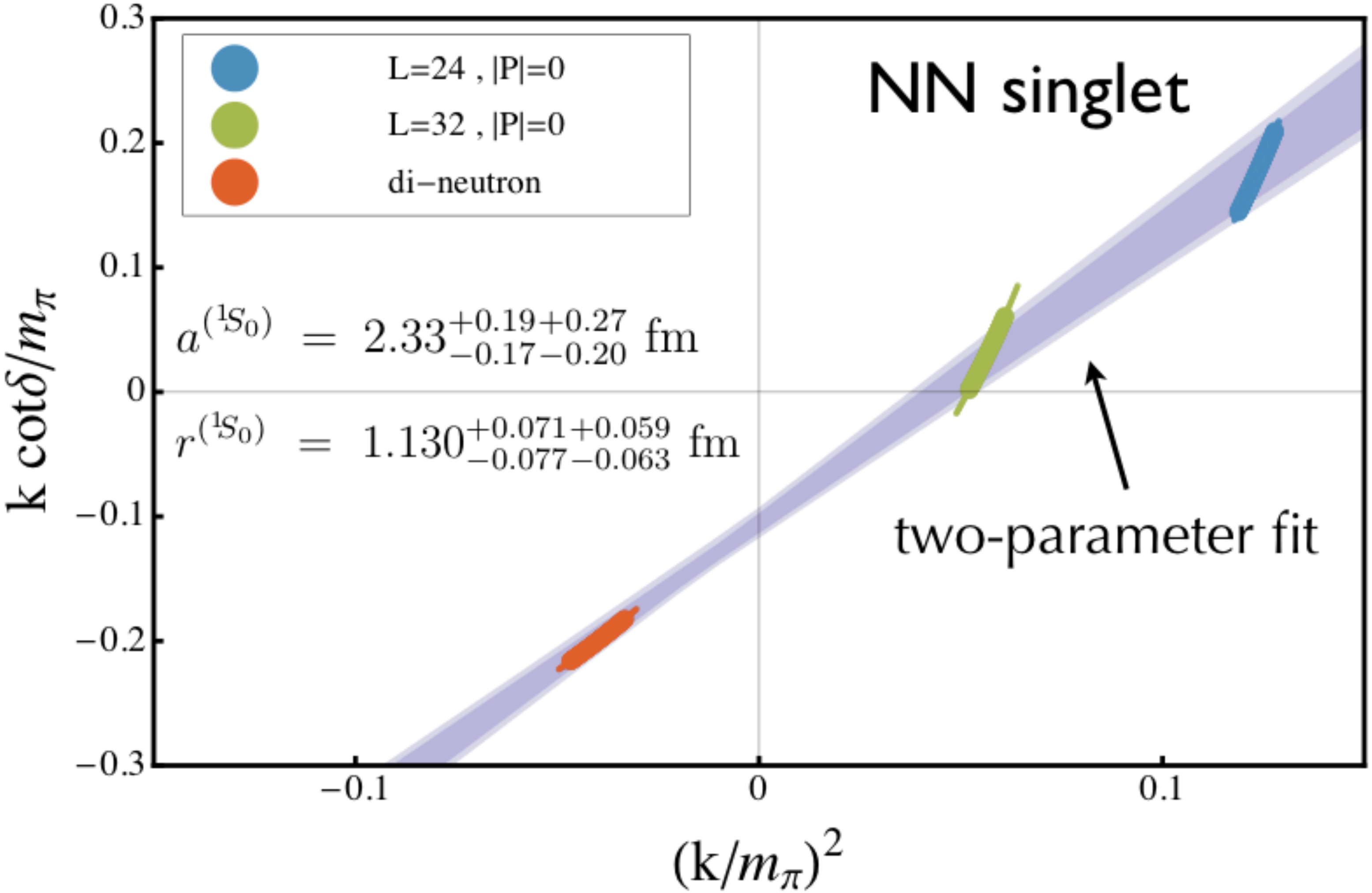


Trivial IR fixed point:
"natural case"

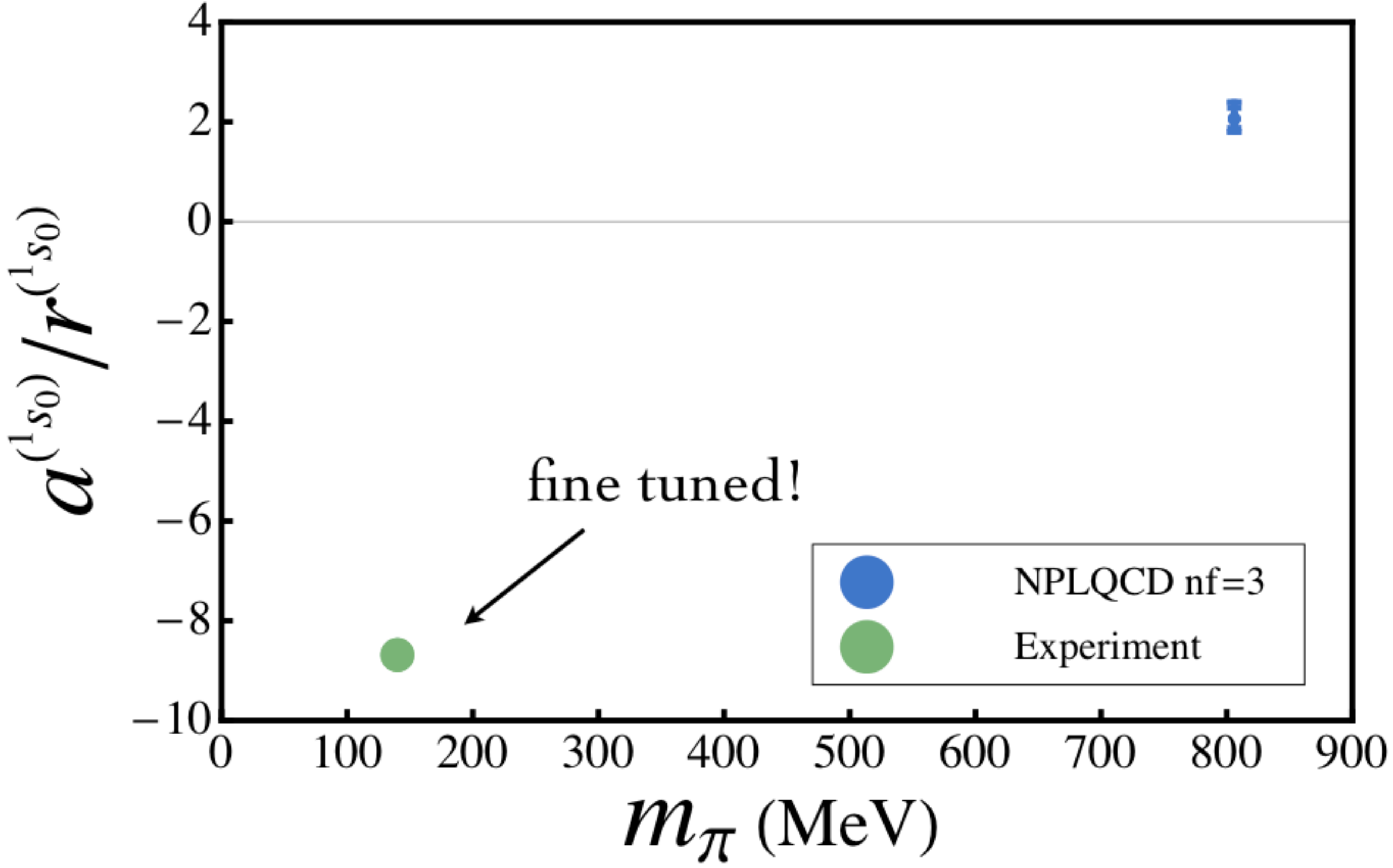
Nontrivial UV fixed point:
"unnatural case"


$L \sim 3.4$ fm
$L \sim 4.5$ fm
$24^3 \times 48$
$32^3 \times 48$


The Martin-Luescher Dictionary
NEW EDITION

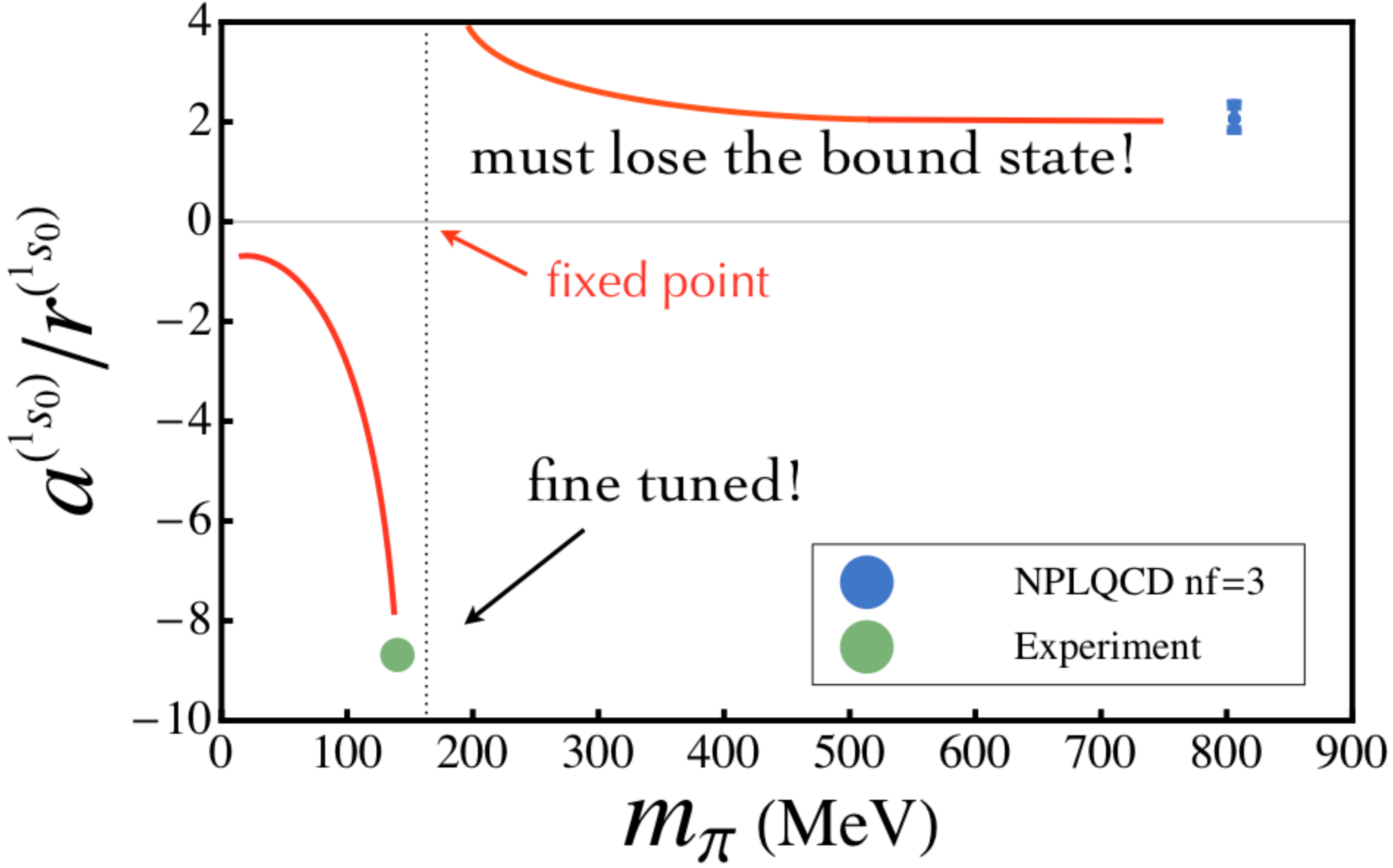

L=24 , |P|=0
L=32 , |P|=0
di−neutron
NN singlet
$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}$ fm
$r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}$ fm
two-parameter fit
k cotδ/m_π
$(k/m_\pi)^2$
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.1
0
0.1

NN singlet



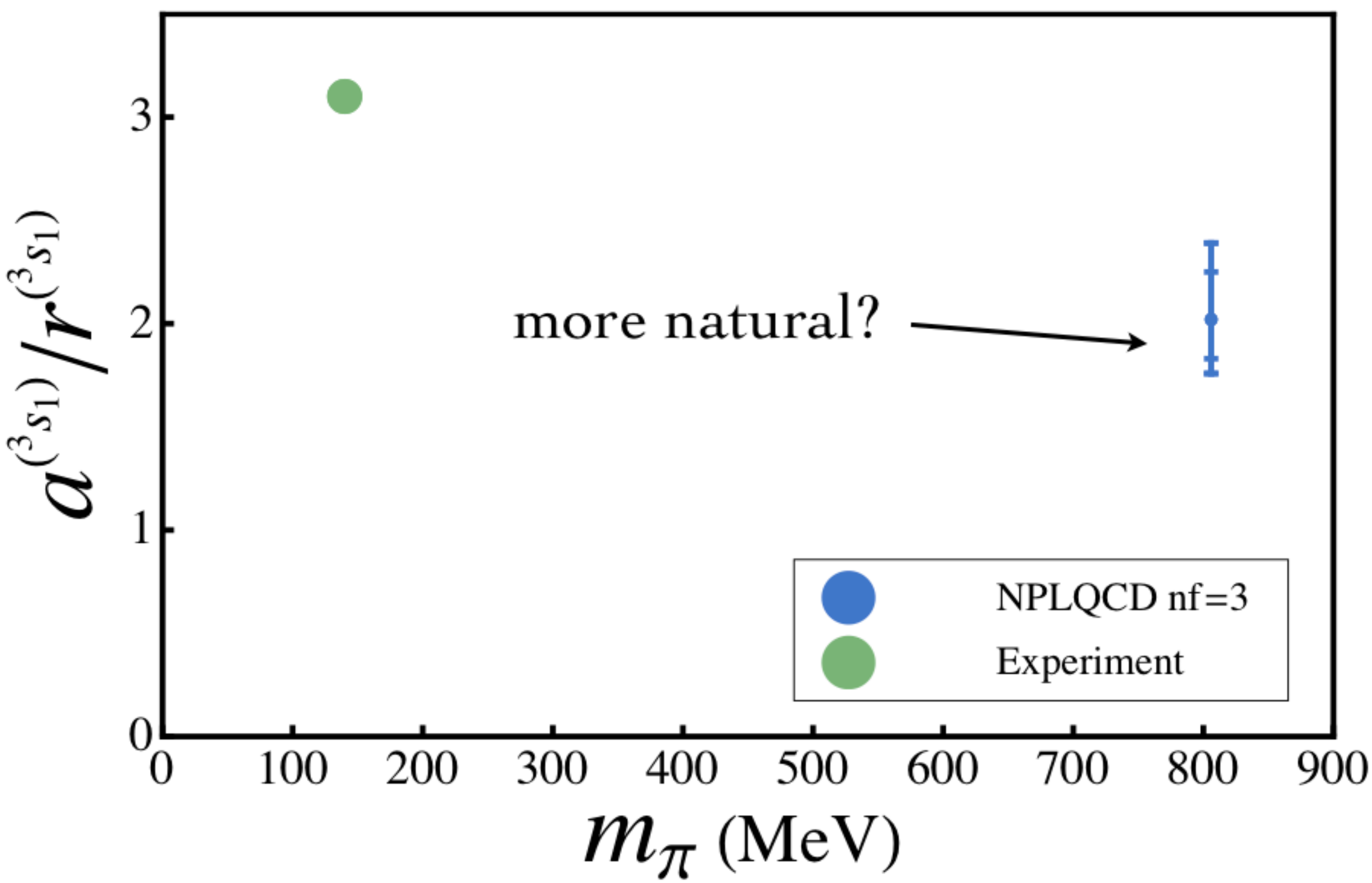
$a^{(^1s_0)}/r^{(^1s_0)}$
m_π (MeV)
fine tuned!
NPLQCD nf=3
Experiment
4
2
0
−2
−4
−6
−8
−10
0
100
200
300
400
500
600
700
800
900

NN singlet



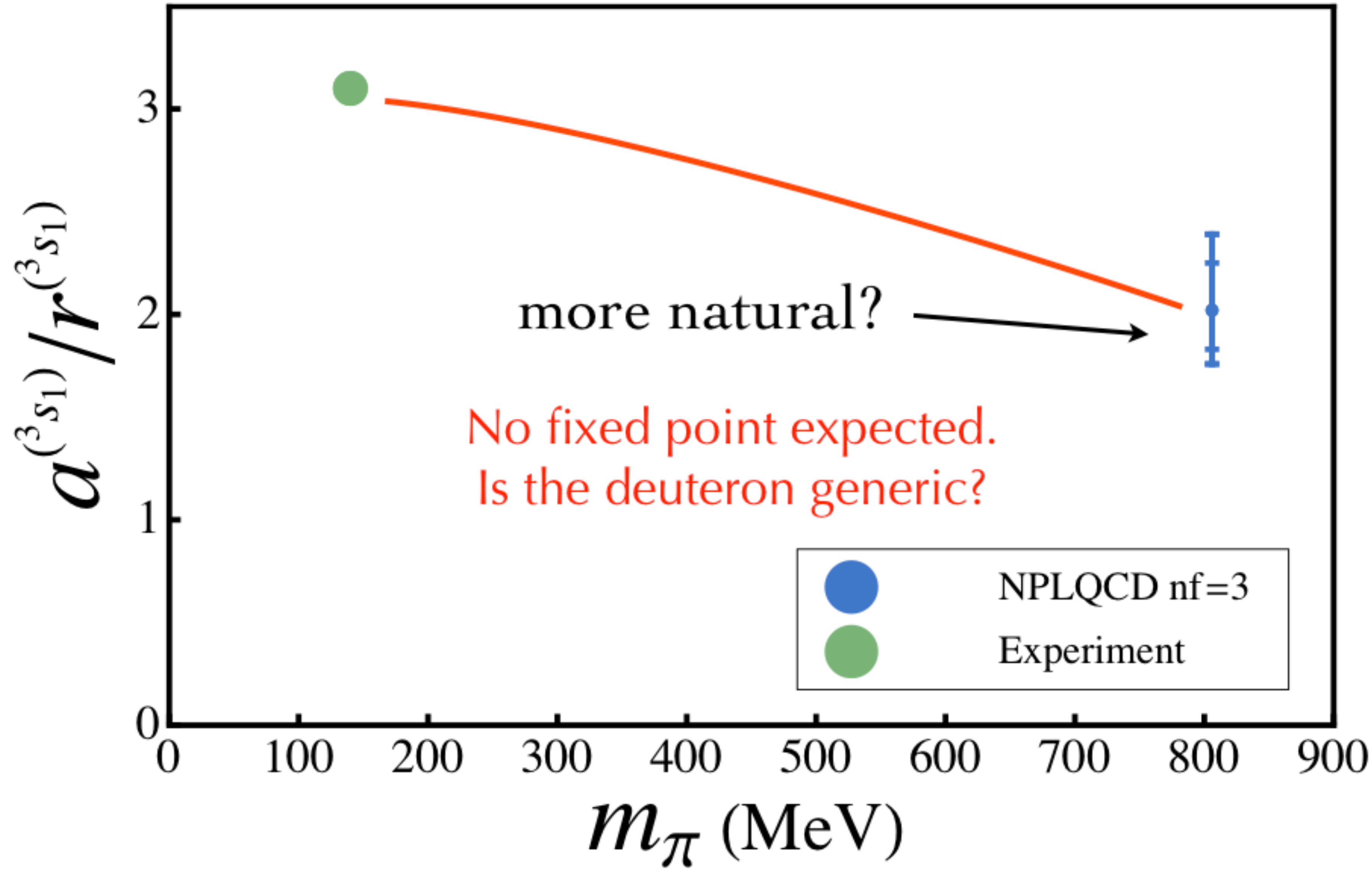
$a^{(^1s_0)}/r^{(^1s_0)}$
m_π (MeV)
must lose the bound state!
fixed point
fine tuned!
NPLQCD nf=3
Experiment
4
2
0
−2
−4
−6
−8
−10
0
100
200
300
400
500
600
700
800
900

NN triplet

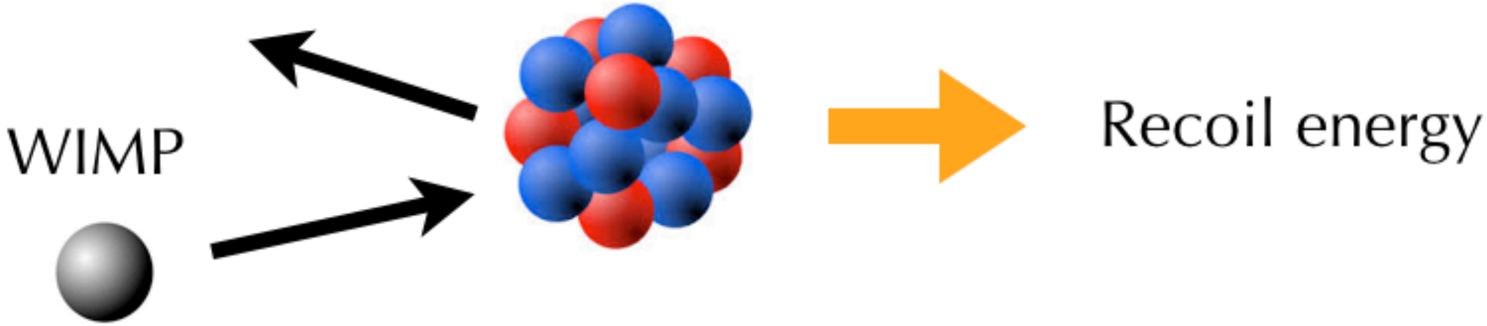


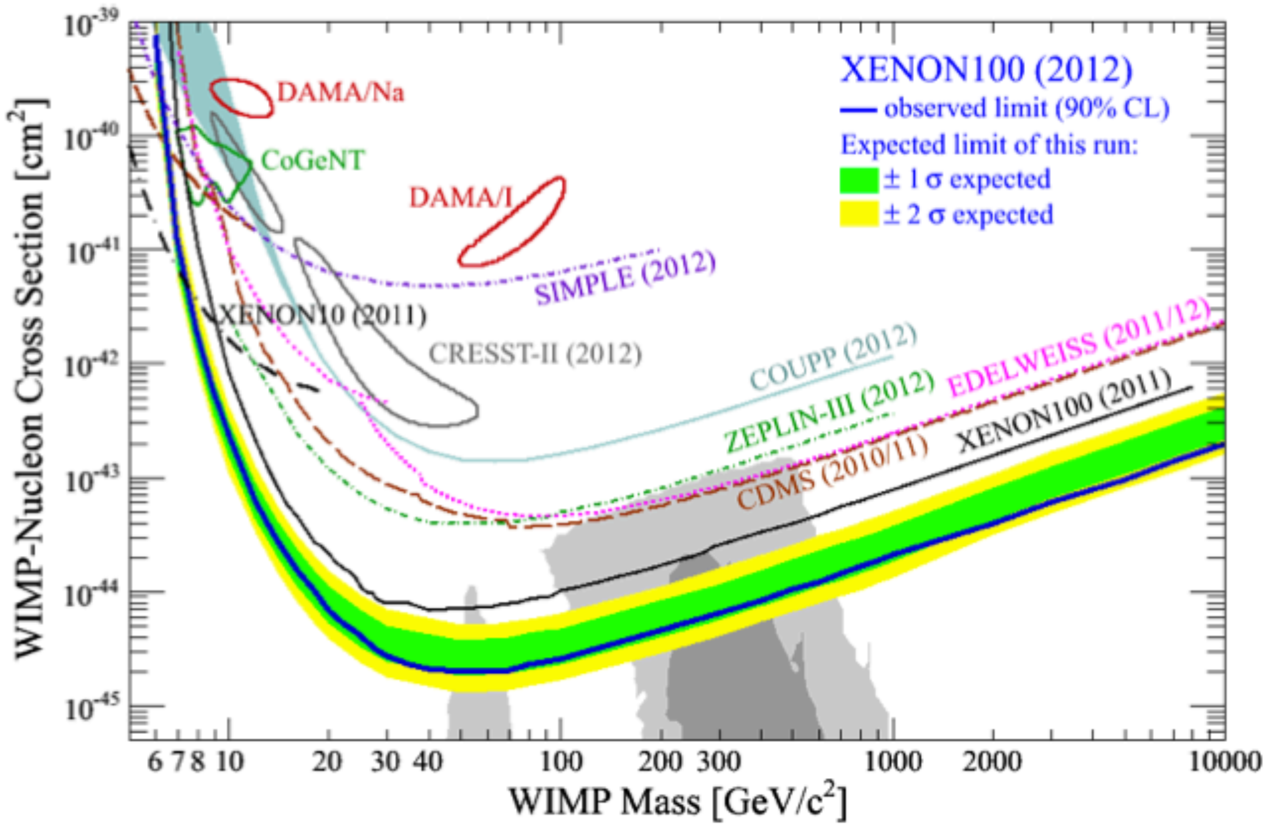
$a^{(^3s_1)}/r^{(^3s_1)}$
more natural?
NPLQCD nf=3
Experiment
0
1
2
3
0
100
200
300
400
500
600
700
800
900
m_π (MeV)

NN triplet



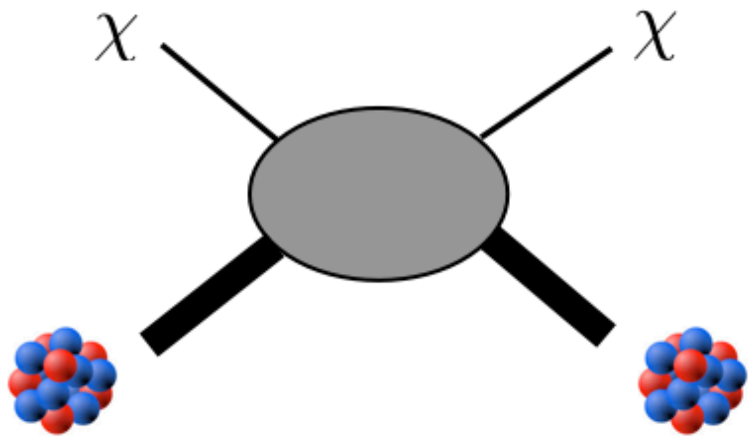
$a^{(^3s_1)}/r^{(^3s_1)}$
3
2
1
0
0
100
200
300
400
500
600
700
800
900
m_π (MeV)
more natural?
No fixed point expected.
Is the deuteron generic?
NPLQCD nf=3
Experiment



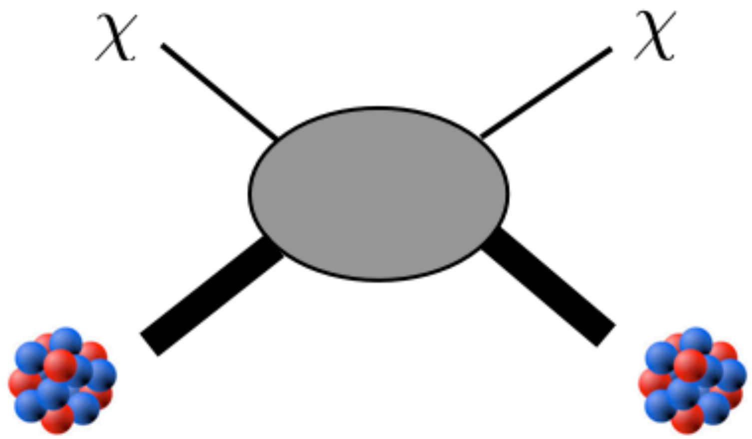


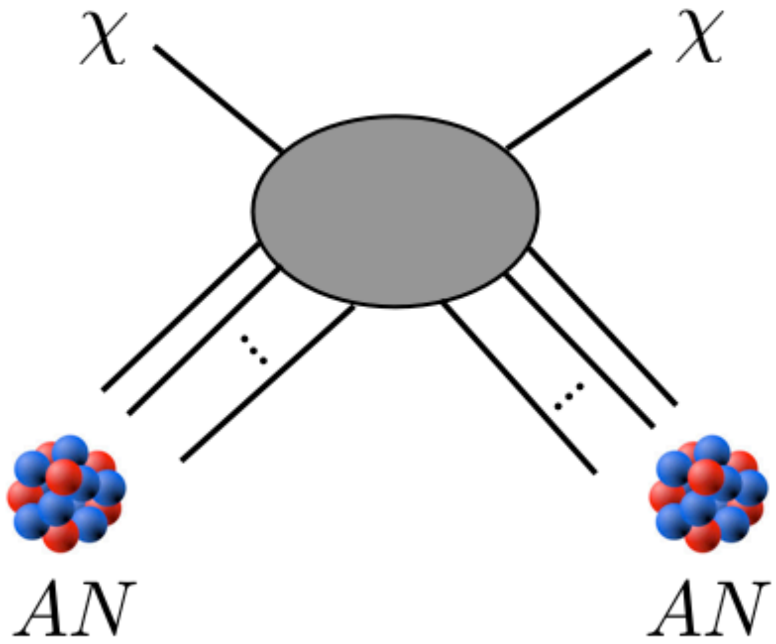
$$\sigma_{SI} > 2 \times 10^{-45}\ \text{cm}^2 \quad \text{(for 55 GeV WIMPS)}$$

Need nuclear
matrix elements



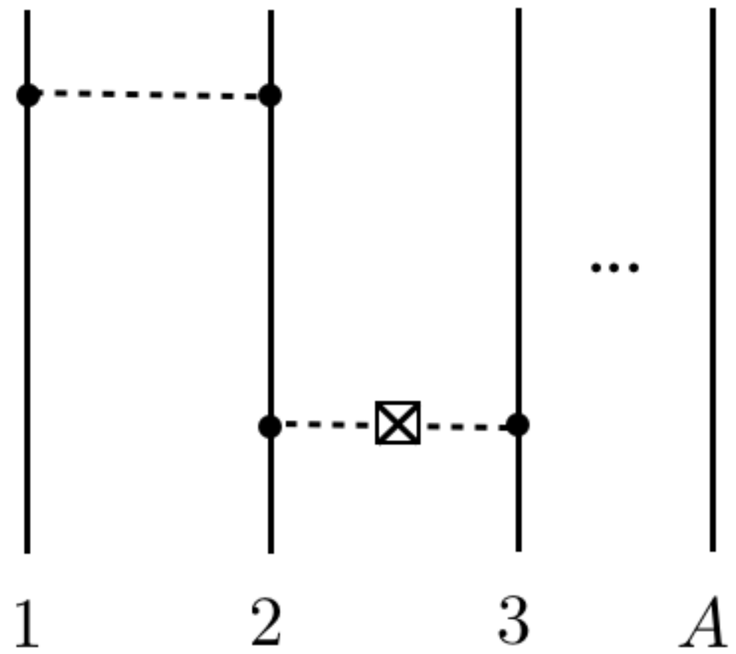
Need nuclear matrix elements



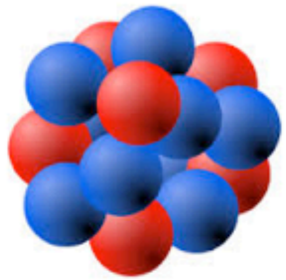


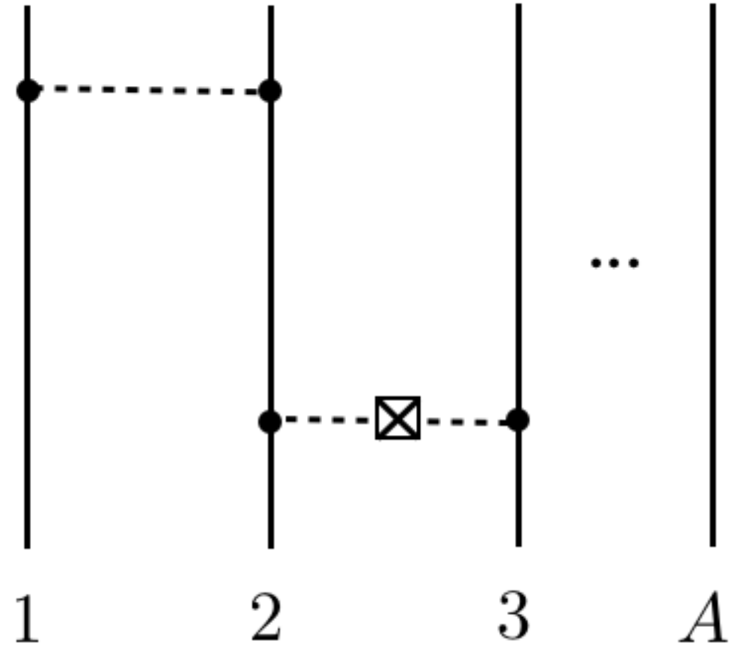
How good is the impulse approximation?

$$g_{Z,N} = g_p Z + g_n N$$

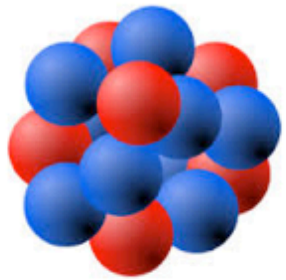


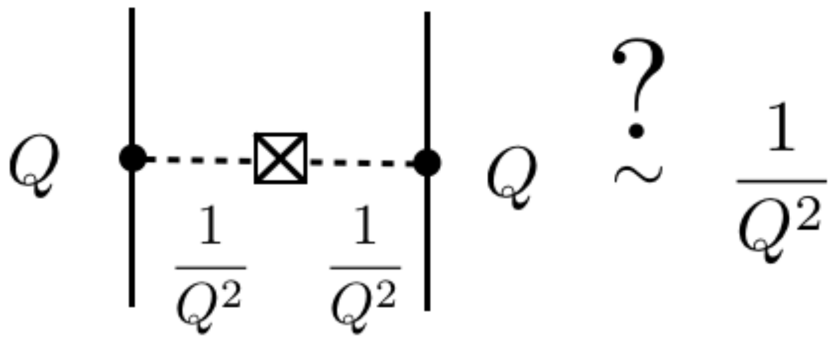
Can there be enhanced meson exchange currents??





Can there be enhanced meson exchange currents??

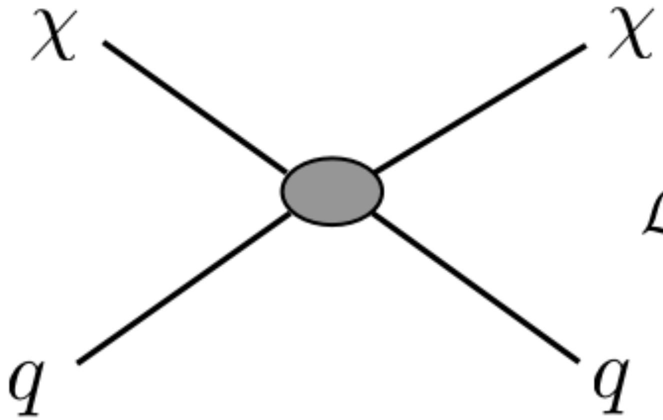




If yes, then big systematic in comparing experiments with different target nuclei.

WIMP-QCD EFT

Spin-independent WIMP-quark interactions

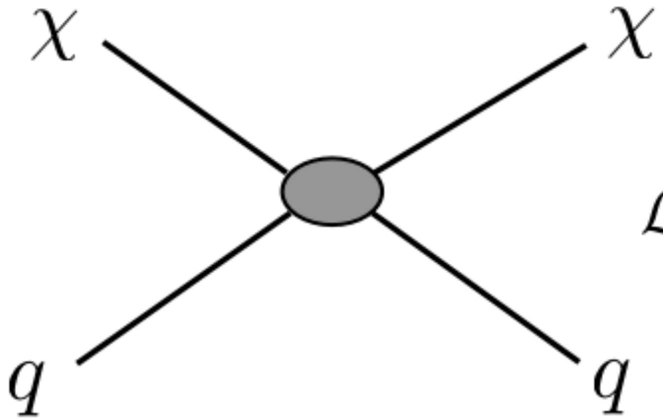


$$\mathcal{L} = G_F \, \bar{\chi}\chi \sum_q a_S^{(q)} \, \bar{q}q$$

dim-3 operator transforms like quark masses

WIMP-QCD EFT

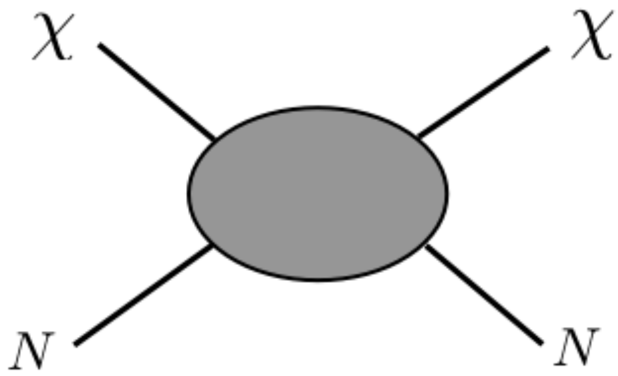
Spin-independent WIMP-quark interactions



$$\mathcal{L} = G_F \, \overline{\chi}\chi \sum_q a_S^{(q)} \, \overline{q}q$$

dim-3 operator transforms like quark masses

low-scale

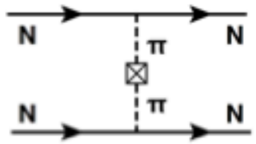


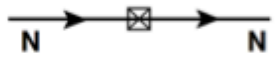
$$\sim G_F \, \bar{\chi}\chi \, \langle N|\bar{q}q|N\rangle$$

sigma term

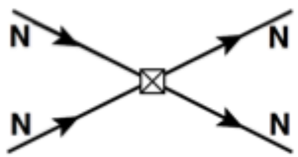
WIMP-QCD EFT

$$\mathcal{L} \;=\; \frac{G_F}{2}\,\overline{\chi}\chi\;\left[(a_S^{(u)} + a_S^{(d)})\overline{q}q \;+\; (a_S^{(u)} - a_S^{(d)})\overline{q}\tau^3 q \;+\; a_S^{(s)}\overline{s}s \;+\; \ldots\right]$$





$$\mathcal{L} \;\to\; G_F\,\overline{\chi}\chi\;\Big(\frac{1}{4}\langle 0|\overline{q}q|0\rangle\;\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N\,\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$-\;\frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N\,\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;-\; 4N^\dagger a_{S,\xi}N\right) \;+\; \ldots\Big)$$



$$-G_F\overline{\chi}\chi\left(D_{S,1}\left(N^\dagger N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; D_{S,2}N^\dagger N N^\dagger a_{S,\xi}N\right.$$
$$\left.+D_{T,1}\left(N^\dagger\sigma^a N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; D_{T,2}N^\dagger\sigma^a N N^\dagger\sigma^a a_{S,\xi}N\right)$$

Nuclear Sigma Terms

$$\sigma_{Z,N} = \overline{m}\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d\ |Z, N(\mathrm{gs})\rangle = \overline{m}\frac{d}{d\overline{m}}E_{Z,N}^{(gs)}$$

$$= \left[\ 1 + \mathcal{O}\left(m_\pi^2\right)\ \right] \frac{m_\pi}{2}\frac{d}{dm_\pi}E_{Z,N}^{(\mathrm{gs})}$$

↑

LQCD: small even for large pion masses

Nuclear Sigma Terms

$$\sigma_{Z,N} = \overline{m}\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d\ |Z, N(\mathrm{gs})\rangle = \overline{m}\frac{d}{d\overline{m}}E_{Z,N}^{(gs)}$$

$$= \left[\, 1 + \mathcal{O}\left(m_\pi^2\right)\,\right] \frac{m_\pi}{2}\frac{d}{dm_\pi}E_{Z,N}^{(\mathrm{gs})}$$

LQCD: small even for large pion masses

$$E_{Z,N}^{(\mathrm{gs})} = AM_N - B_{Z,N}$$

$$M_N = a_0 + a_1\, m_\pi$$

Nuclear Sigma Terms

$$\sigma_{Z,N} = \overline{m}\langle Z,N(\mathrm{gs})|\ \overline{u}u + \overline{d}d\ |Z,N(\mathrm{gs})\rangle = \overline{m}\frac{d}{d\overline{m}}E_{Z,N}^{(gs)}$$

$$= \left[\,1 + \mathcal{O}\left(m_\pi^2\right)\,\right]\frac{m_\pi}{2}\frac{d}{dm_\pi}E_{Z,N}^{(\mathrm{gs})}$$

LQCD: small even for large pion masses

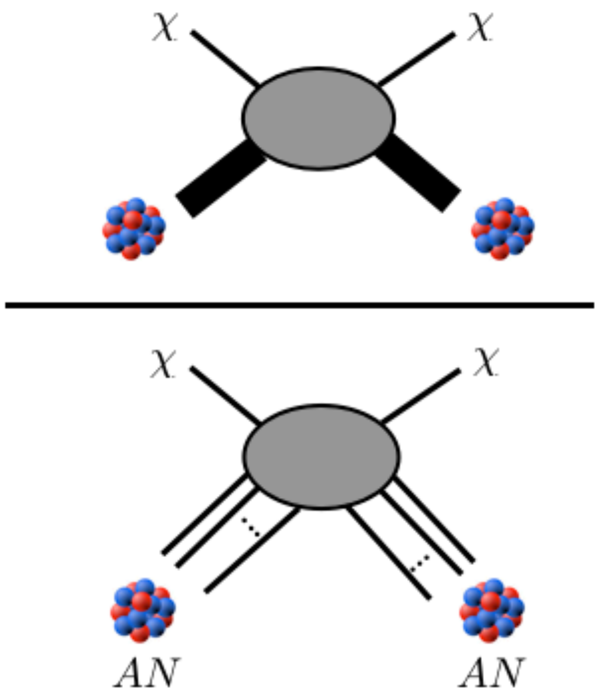
$$E_{Z,N}^{(\mathrm{gs})} = AM_N - B_{Z,N}$$

$$M_N = a_0 + a_1\, m_\pi$$

$$\sigma_{Z,N} = A\sigma_N + \sigma_{B_{Z,N}} = A\sigma_N - \frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

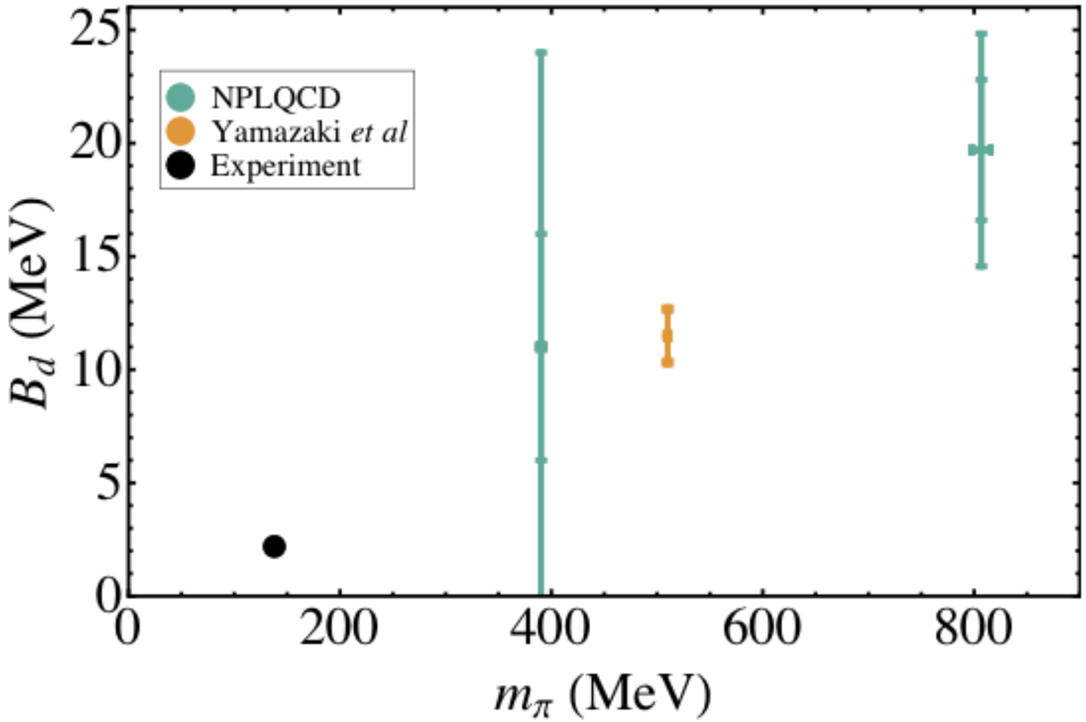
impulse approximation

$$\delta\sigma_{Z,N} = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N} = \frac{\langle Z,N(gs)|\ \overline{u}u+\overline{d}d|Z,N(gs)\rangle}{A\ \langle N|\ \overline{u}u+\overline{d}d|N\rangle} - 1$$

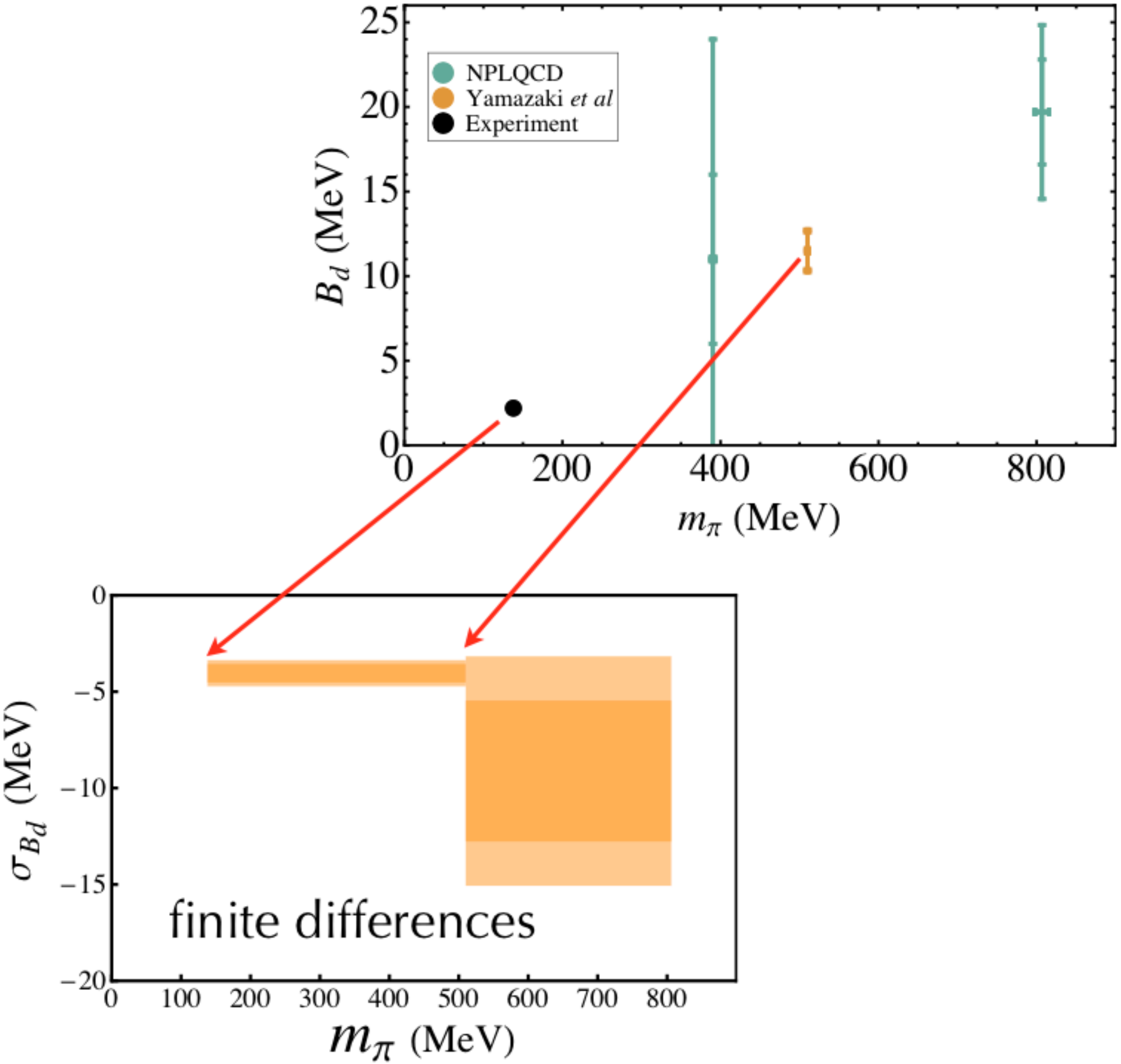


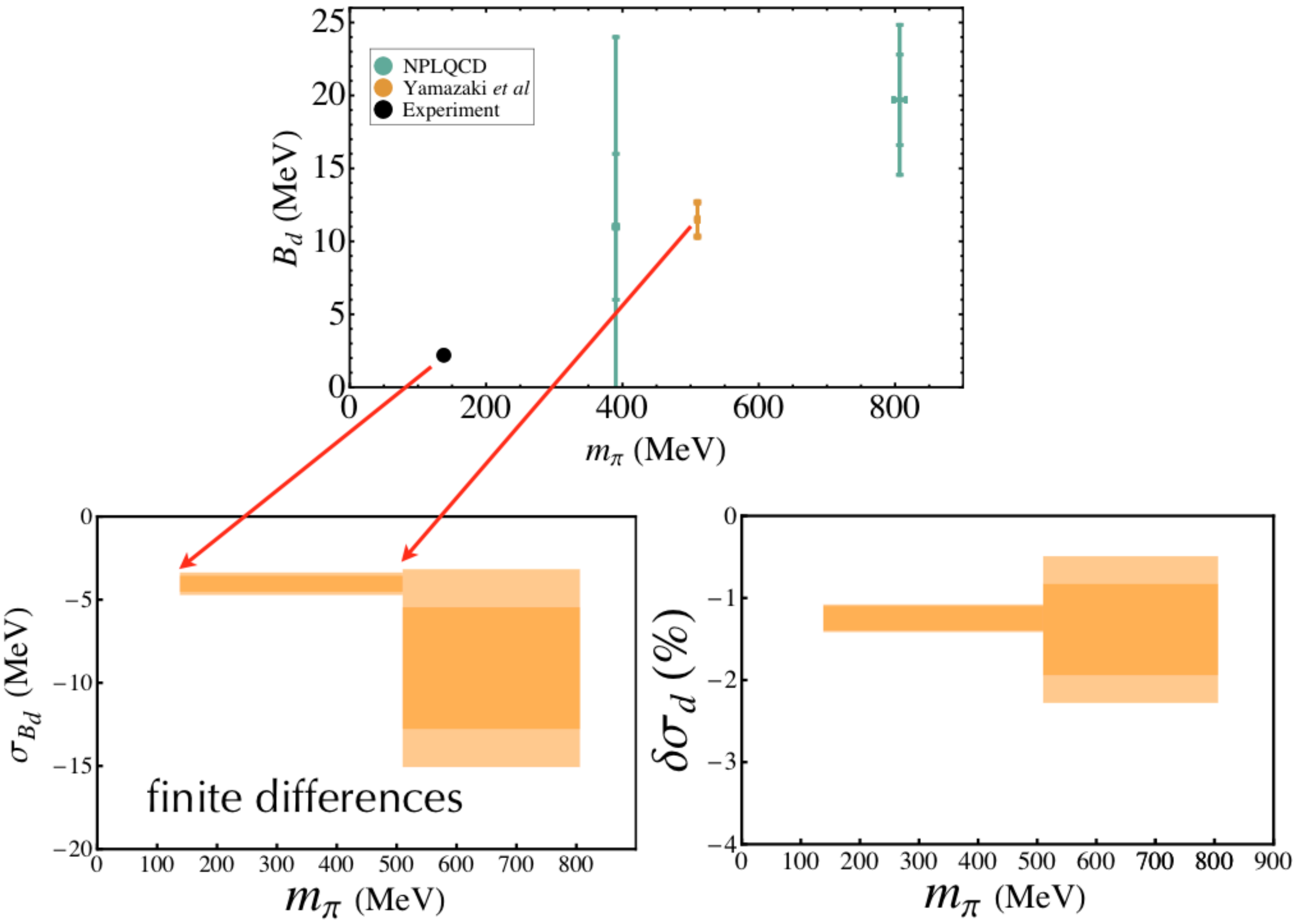
Deviation of scalar-isoscalar WIMP-nucleus scattering at zero momentum transfer!

$\delta\sigma_{Z,N}$: Measure of nuclear effects: meson exchange currents

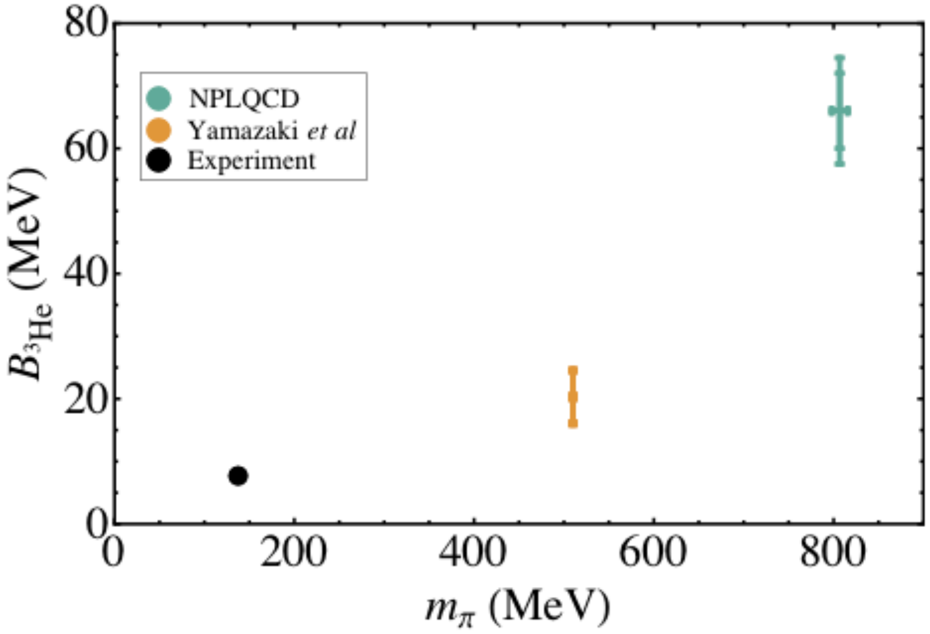


NPLQCD
Yamazaki *et al*
Experiment
B_d (MeV)
m_π (MeV)
0
5
10
15
20
25
0
200
400
600
800

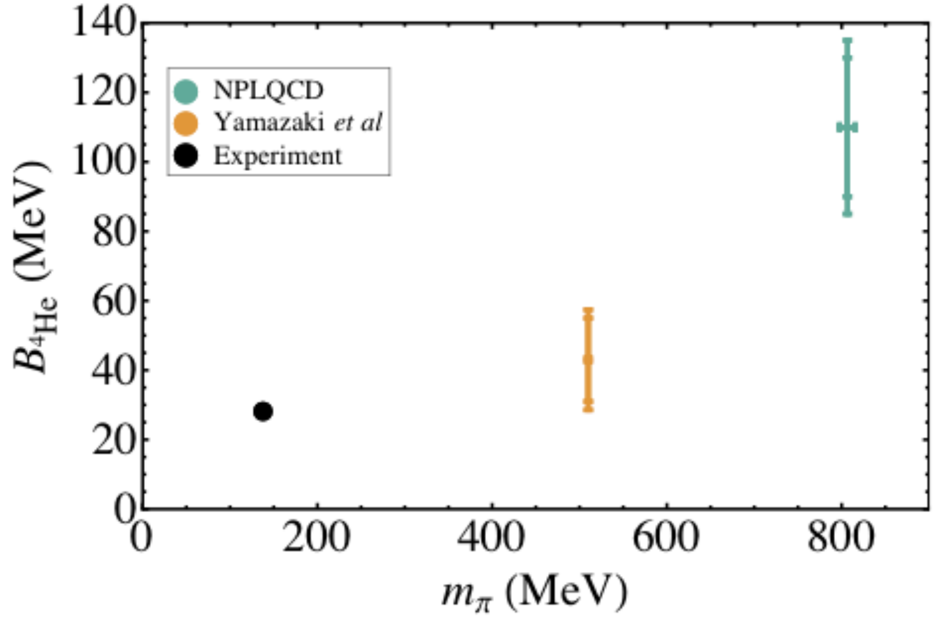

NPLQCD
Yamazaki et al
Experiment
B_d (MeV)
m_π (MeV)
σ_{B_d} (MeV)
finite differences
m_π (MeV)



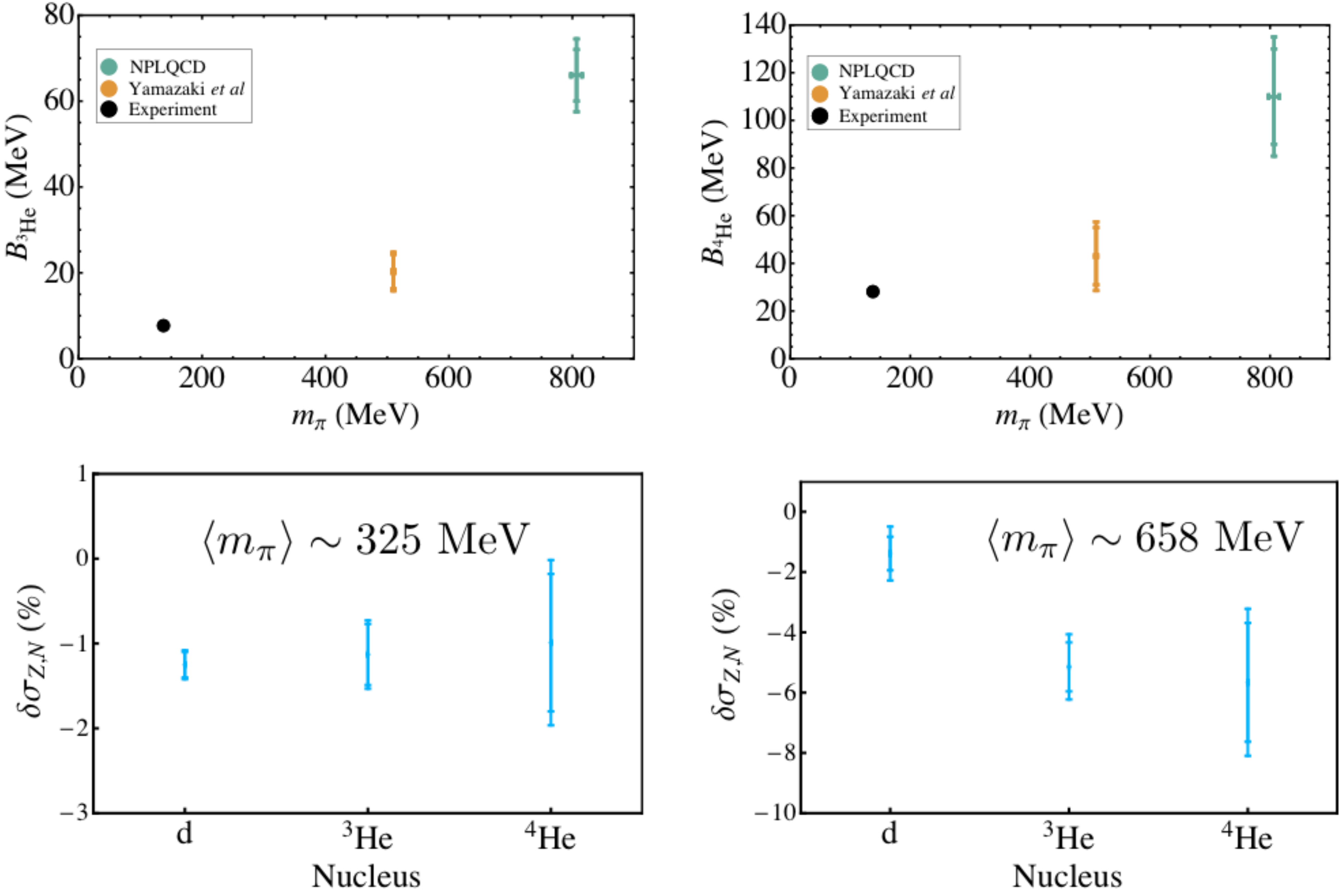
NPLQCD
Yamazaki et al
Experiment
B_d (MeV)
m_π (MeV)
σ_{B_d} (MeV)
finite differences
m_π (MeV)
$\delta\sigma_d$ (%)
m_π (MeV)



80
60
40
20
0
0
200
400
600
800
$B_{^3\mathrm{He}}$ (MeV)
m_π (MeV)
NPLQCD
Yamazaki *et al*
Experiment

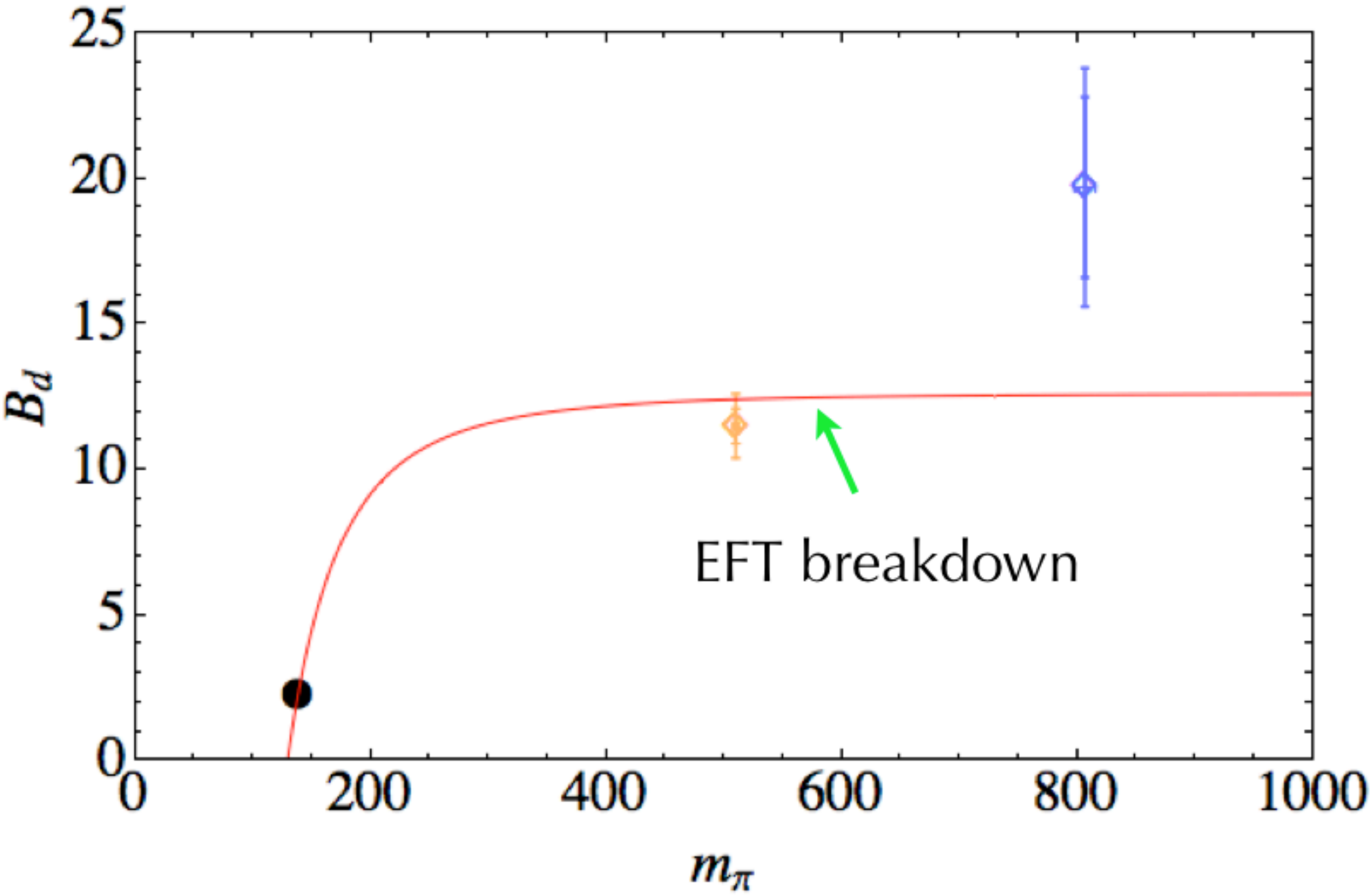


140
120
100
80
60
40
20
0
0
200
400
600
800
$B_{^4\mathrm{He}}$ (MeV)
m_π (MeV)
NPLQCD
Yamazaki *et al*
Experiment

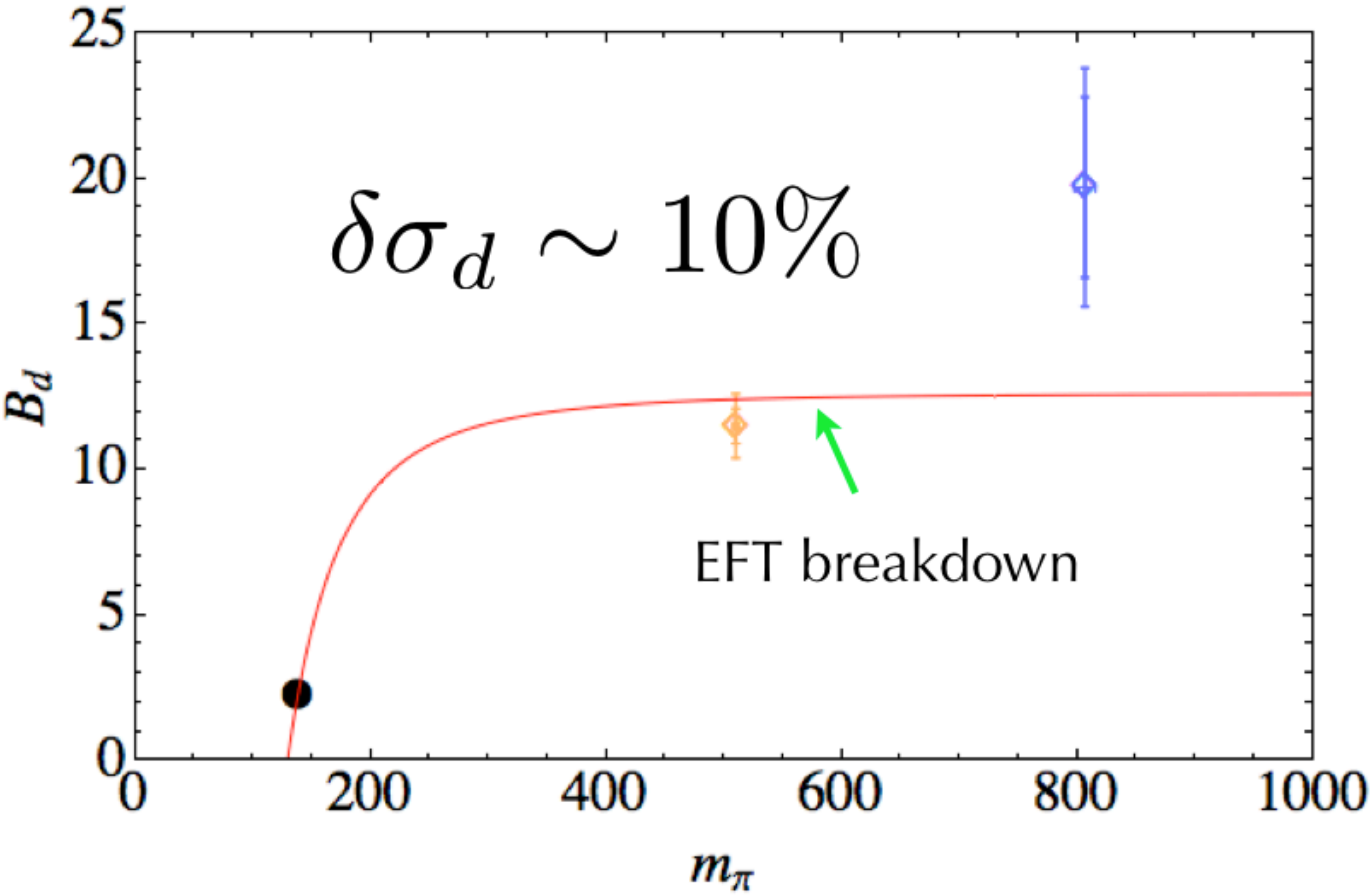


No enhanced MECs!

CAVEAT: LARGE CURVATURE



Caveat: Large curvature

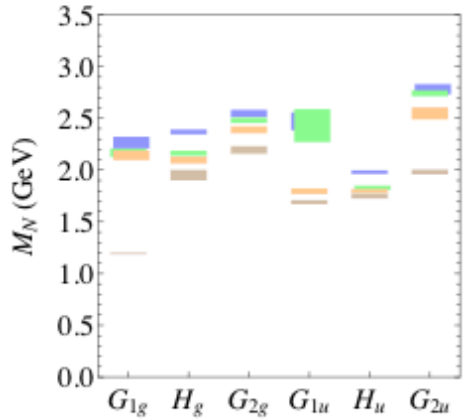


Even unlikely curvature will not help!

Lattice QCD
for
nuclear
physics

nucleon structure

e.g. excited baryons

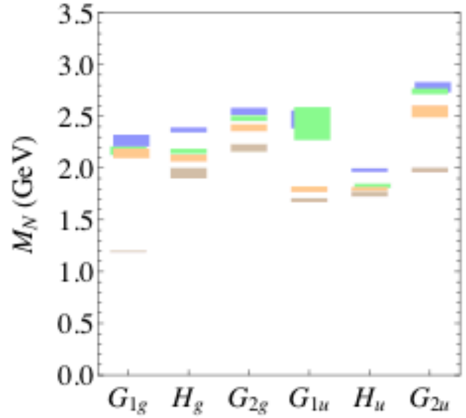


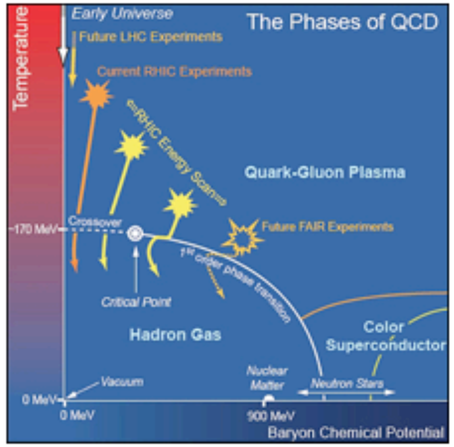
Lattice QCD for nuclear physics

nucleon structure

Lattice QCD for nuclear physics

e.g. excited baryons





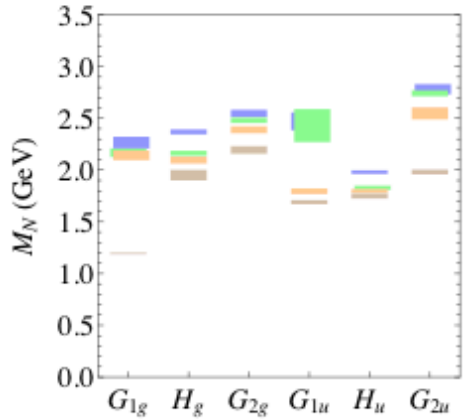
e.g. critical point

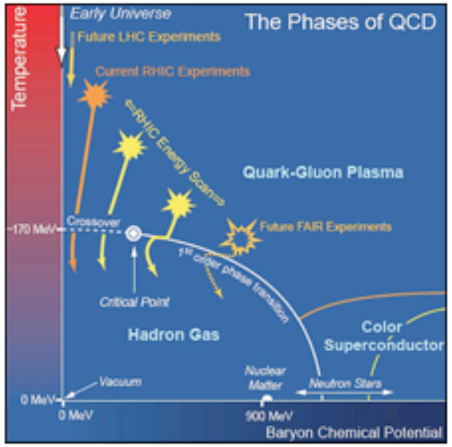
equation of state

Lattice QCD for nuclear physics

nucleon structure

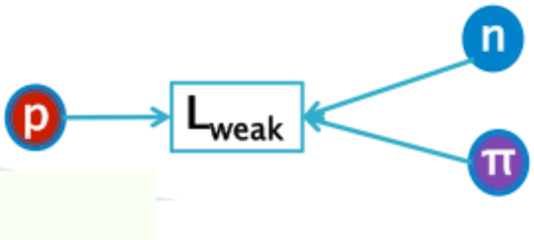
e.g. excited baryons





e.g. critical point

equation of state



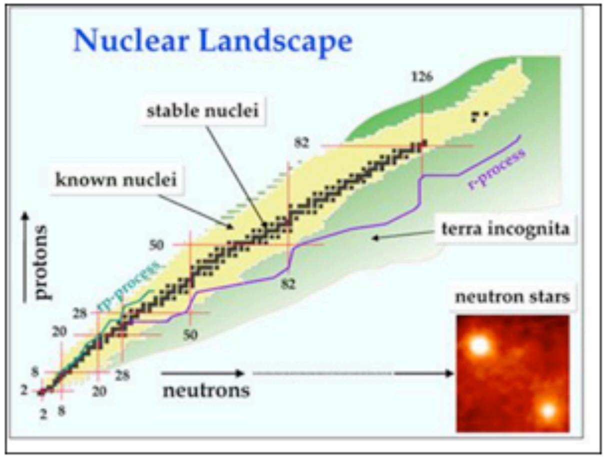
e.g. $h_{\pi NN}$

precision electroweak

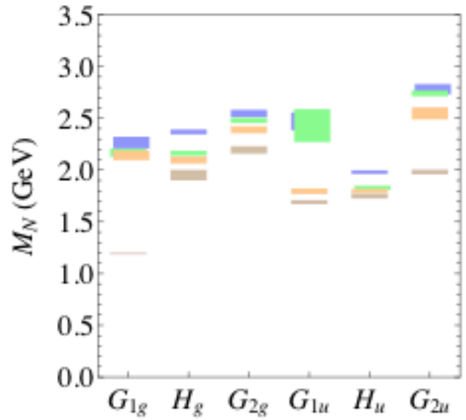
nuclear structure

nucleon structure

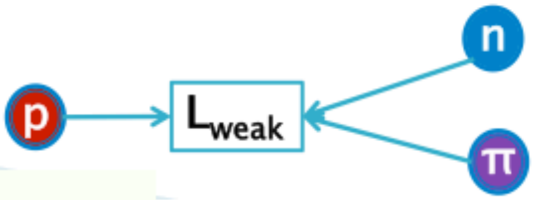
e.g. nnn

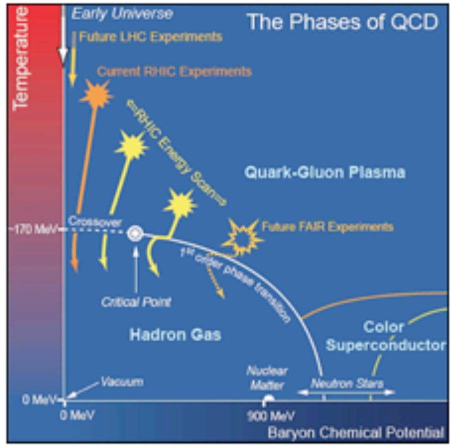


e.g. excited baryons



Lattice QCD for nuclear physics





e.g. $h_{\pi NN}$

e.g. critical point

precision electroweak

equation of state

✦ Remarkable progress is being made in understanding the visible matter in the Universe from first principles.

We have only just begun!